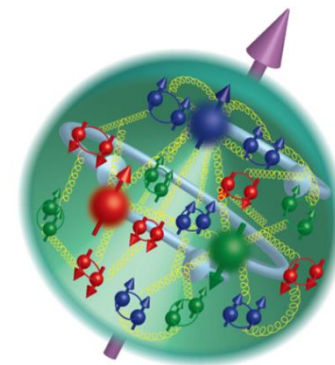




Axial-vector and spin structures of the proton

陈毅

Based on:

[PRD 110, L091503 (2024)]

[JHEP 04, 132 (2025)]

[arXiv: 2512.06801 [hep-ph]]

[arXiv: 2607.15050 [hep-ph]]

[arXiv: 2608.xxxxx (to appear)]

thanks to:

Qing Chen, Feng-Kun Guo, Yang Li, Cédric Lorcé

Oleksandr Tomalak, Qun Wang, Bing-Song Zou

第一届强相互作用自旋物理会议 · 山东青岛(07.26-07.31, 2026)

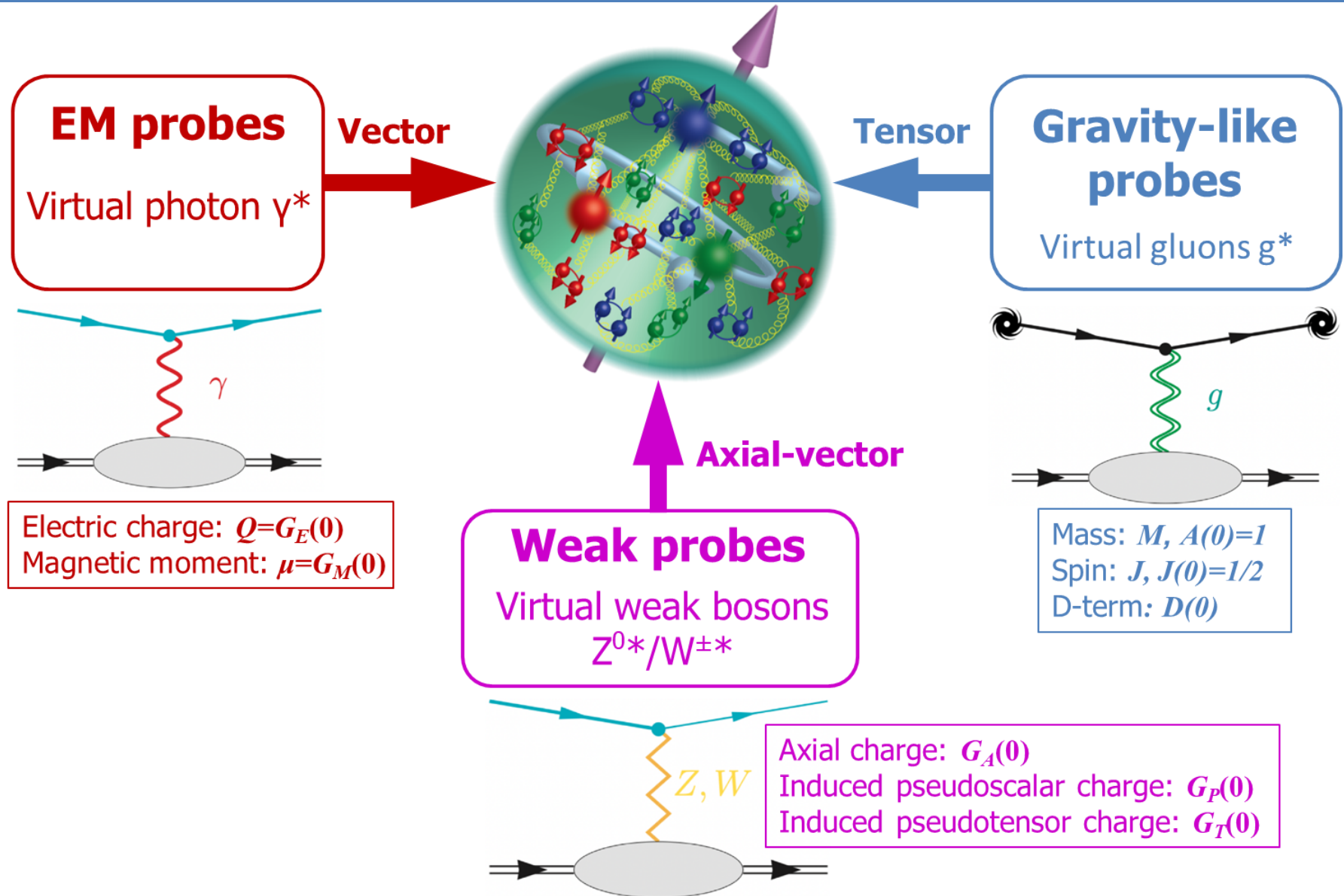
Outline

1. Introduction and motivation
2. Neutrino-proton and antineutrino-proton elastic scattering
3. Relativistic axial-vector and spin structures of a nucleon
4. Summary and outlook

致谢：

特别感谢“第一届强相互作用自旋物理会议”会务组和组委会！
感谢您给了我一个交流和展示近期研究工作的机会！

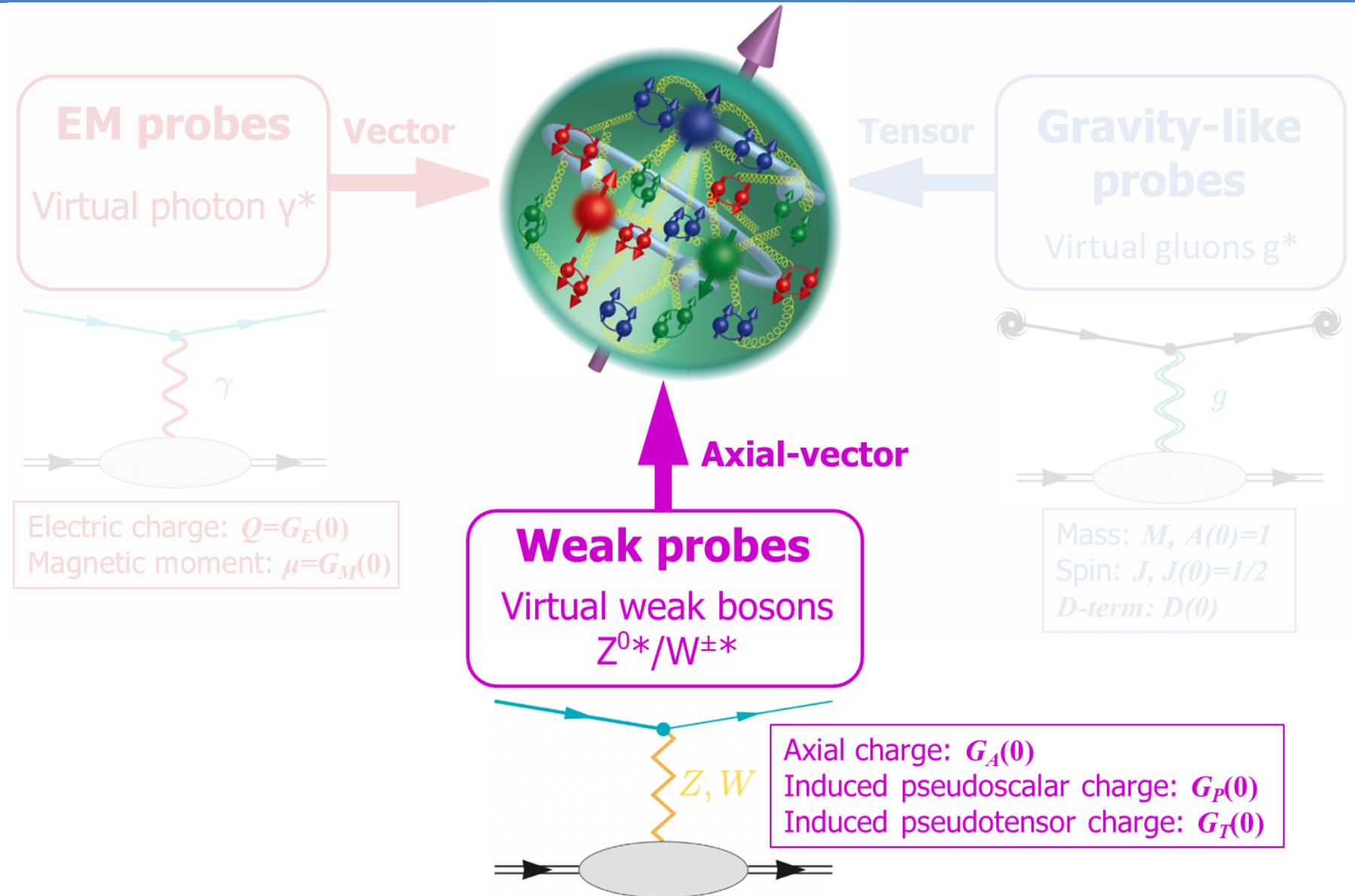
Probing the internal structures of a hadron



[M. Polyakov, & P. Schweitzer. IJMPA 33, 1830025 (2018)]

[Burkert, Elouadrhiri, Girod, Lorcé, Schweitzer, & Shanahan. RMP 95, 041002 (2023)] ...

Probing the internal structures of a hadron



[M. Polyakov, & P. Schweitzer. IJMPA 33, 1830025 (2018)]
 [V. Bernard, L. Elouadrhiri, & U. Meissner. J. Phys. G 28, R1 (2002)] ...

Structure dictates properties

■ Analogy:

"Symmetry dictates interactions"

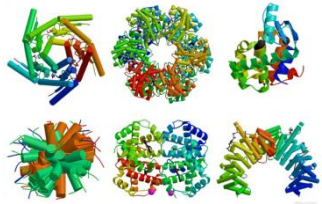
对称性决定动力学

[C.-N. Yang. Phys. Today 33N6, 42 (1980)]

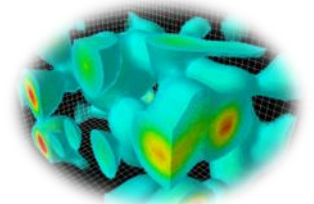
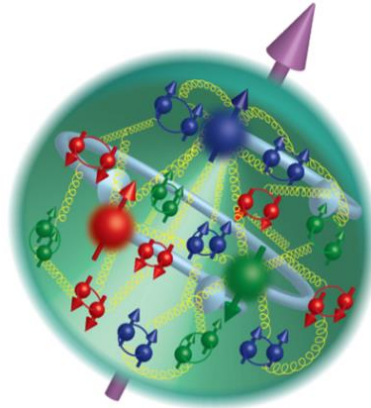


"Structure dictates properties"

结构决定性质



different proteins



QCD vacuum

■ Hadron structures are highly non-trivial and complicated!

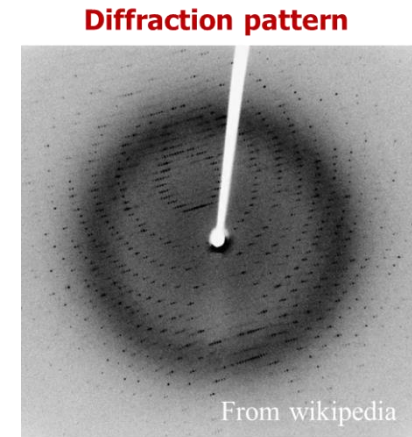
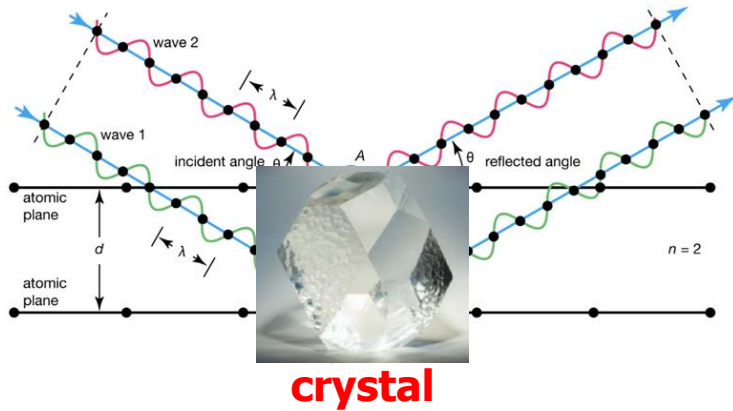
(1). Hadron structures are closely associated with the nonperturbative QCD dynamics between the internal quark and gluon degrees of freedom.

(2). The QCD vacuum itself is also highly non-trivial, due to quantum fluctuations (loop effects, pair creations and annihilations), non-trivial topologies, instanton/sphaleron transitions, and etc.

(3). On top of the complicated QCD dynamics and non-trivial QCD vacuum structure, hadron structures are also affected by electromagnetic and weak interactions.

How to probe the internal structure of a system?

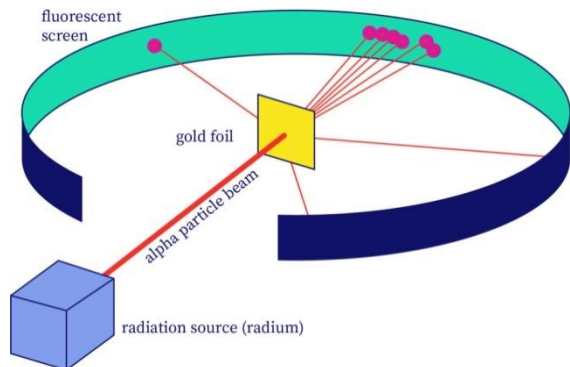
◆ Classically, e.g. using x-ray diffraction on crystals:



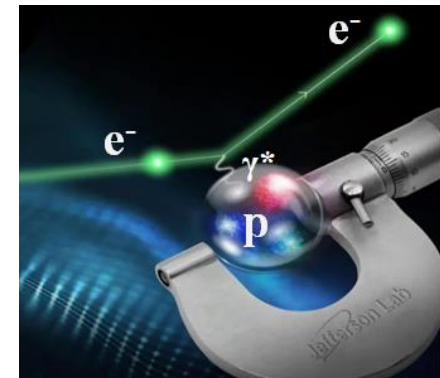
$$\propto |\mathcal{A}_{\text{scatt}}|^2$$

Scattering amplitude: $\mathcal{A}_{\text{scatt}} \propto \underbrace{F(q)}_{\text{Form factor}} = \int d^3r e^{iq \cdot r} \underbrace{\rho(r)}_{\text{Target distribution}} \quad q = k - k'$

◆ Experimentally, e.g. using modern Rutherford scattering:

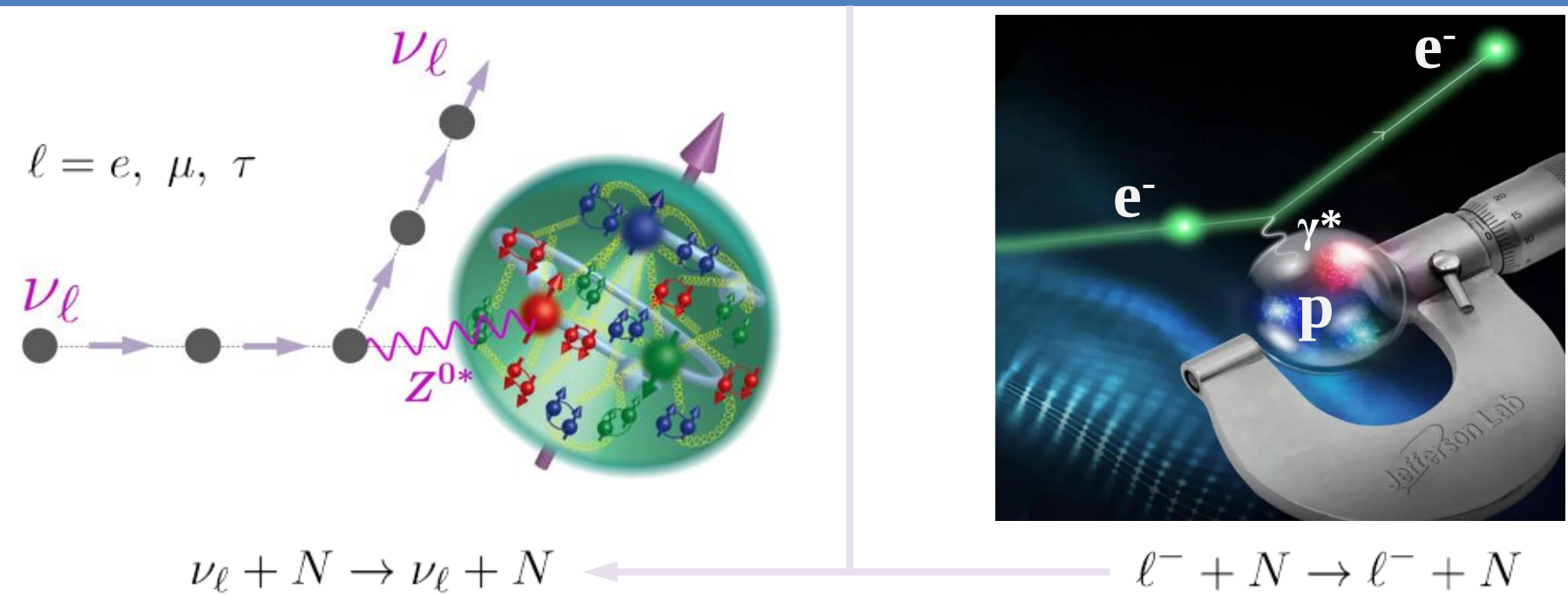


Rutherford alpha scattering experiment (1909)



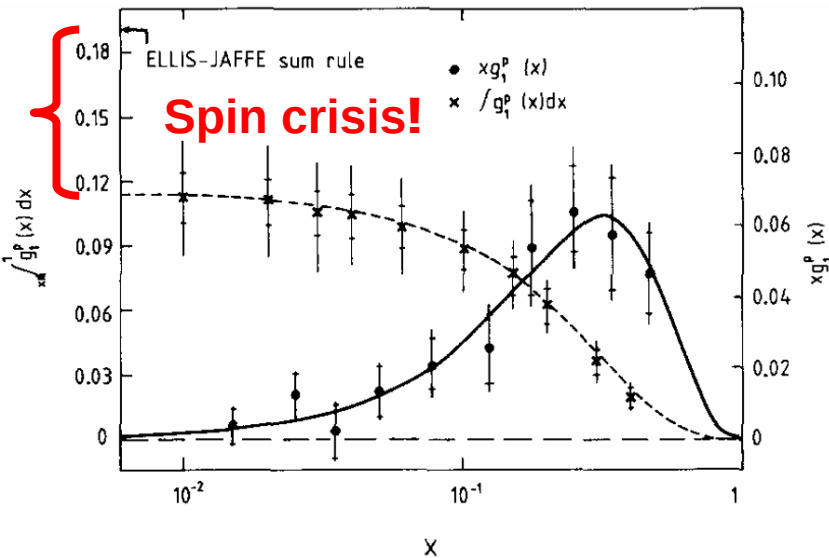
**Jefferson lab
e-p elastic scattering exps. 6**

Elastic (anti)neutrino-proton scattering and why it?



- (1). By a simple analogy with the elastic electron-nucleon scattering.
- (2). Very clean, since (anti)neutrinos participate only* in weak interactions.
- (3). Key QCD bound-states (nucleons) in the weak sector: the weak content of the most important baryonic matter in Nature.
- (4). Strangeness contributions to the nucleon spin $\rightarrow$ addressing the spin crisis!
- (5). $G_A^s(0)$ is only* accessible in weak neutral-current elastic scattering!
- (6). Constraining uncertainties in neutrino oscillation or P-violating exps.

The proton spin crisis



EMC experiment: [PLB 206, 364 (1988)]
[NPB 328, 1 (1989)]

Total quark spin contribution:

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s + \dots$$

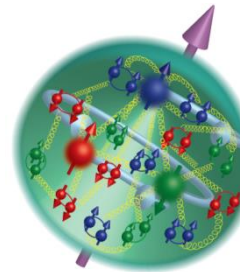
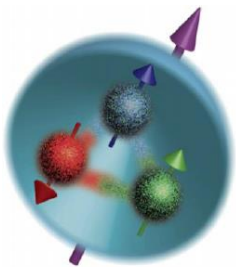
$$= G_A^u(0) + G_A^d(0) + G_A^s(0) + \dots$$

$$G_A^Z(0) = \frac{1}{2} [\Delta u - \Delta d - \Delta s + \Delta c + \dots]$$

→ **(u, d, s) quarks contribute only $(12 \pm 9 \pm 14)\%$ of the total proton spin!**

$$\frac{1}{2} \Delta\Sigma(\overline{Q}^2 = 10.7 \text{ GeV}^2) = 0.060 \pm 0.047 \pm 0.069$$

$$\int_0^1 dx g_1^p(x) = 0.123 \pm 0.013 \pm 0.019$$



$$\Delta\Sigma_{\text{EJ}} = (\Delta u + \Delta d) \approx 0.6 \pm 0.12$$

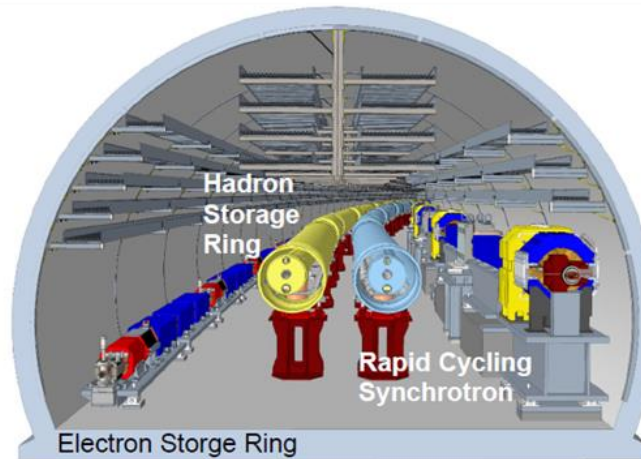
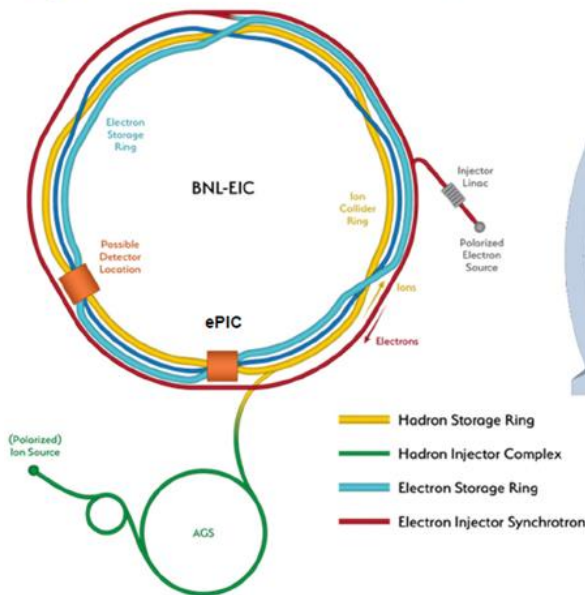
Ellis-Jaffe sum rule

$$\langle J_z \rangle^p = \frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_z^q + L_z^g$$

Jaffe-Manohar IMF sum rule

Crucial for EICs/JLab experiments and scientific goals

- ◆ One of the key scientific goals of EICs is to perform 3D tomography or imaging of the internal structures of nucleon/nuclei.



- ◆ Related to many big scientific questions:

- (1). Proton spin crisis and proton 3D spin structure;
- (2). Proton charge radius puzzle; (see, e.g. Prof. Randolph Pohl's recent works)
- (3). QCD confinement mechanism;
- (4). Mass origin of the proton;
- (5). Matter-antimatter asymmetry of the Universe (MAU/BAU); ...

Some related facilities and experiments



(MiniBooNE, MINERvA, MicroBooNE, NOvA, ICARUS, ANNIE)



(Super-Kamiokande, T2K)



@ LHC near ATLAS



...



EicC



...

Elastic (anti)neutrino-proton scattering

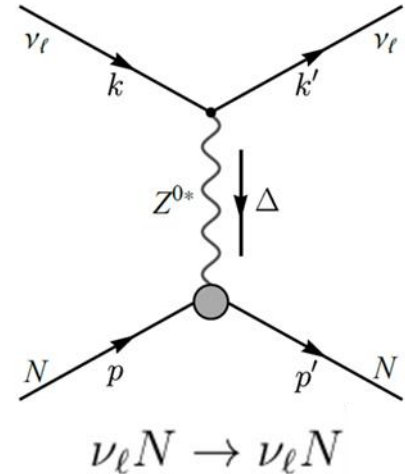
- In general, a spin-1/2 hadron has **6 weak neutral form factors (FFs)**

$$\langle p', s' | \hat{j}_Z^\mu(0) | p, s \rangle = \bar{u}(p', s') \Gamma_Z^\mu(P, \Delta) u(p, s)$$

$$\Gamma_Z^\mu(P, \Delta) = \Gamma^\mu(P, \Delta) - \Gamma_5^\mu(P, \Delta),$$

$$\Gamma^\mu(P, \Delta) \equiv \gamma^\mu G_M^Z + \frac{P^\mu (G_E^Z - G_M^Z)}{M(1 + \tau)} + \frac{i\Delta^\mu}{2M} G_S^Z,$$

$$\Gamma_5^\mu(P, \Delta) \equiv \left[\gamma^\mu G_A^Z + \frac{\Delta^\mu}{2M} G_P^Z - \frac{\sigma^{\mu\nu} \Delta_\nu}{2M} G_T^Z \right] \gamma^5.$$



$$\hat{j}_Z^\mu(x) \equiv \sum_q \hat{q}(x) \gamma^\mu (g_V^q - g_A^q \gamma^5) \hat{q}(x)$$

$$g_A^{u_i} = \frac{1}{2}, \quad g_A^{d_i} = -\frac{1}{2}$$

$$g_V^{u_i} = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W, \quad g_V^{d_i} = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W$$

$$q = u, d, s, c, b, t$$

$$u_i = u, c, t, \quad d_i = d, s, b$$

- **3 vector + 3 axial-vector** weak neutral FFs

$$G_{E,M,S}^Z \longrightarrow \text{electric, magnetic, induced scalar}$$

$$G_{A,P,T}^Z \longrightarrow \text{axial, induced pseudoscalar, induced pseudotensor}$$

Differential cross sections at tree level

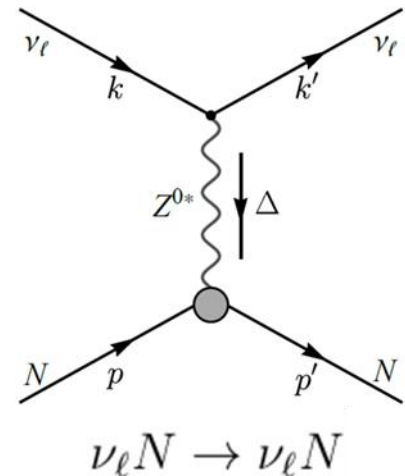
■ Elastic (anti)neutrino-proton scattering at tree level:

$$\frac{d\sigma^\pm}{dQ^2} = \frac{G_F^2 M^2}{8\pi E_\nu^2} \left(\frac{M_Z^2}{M_Z^2 + Q^2} \right)^2 \left[A(Q^2) \pm \frac{s-u}{M^2} B(Q^2) + \frac{(s-u)^2}{M^4(1+\tau)} C(Q^2) \right]$$

$$A(Q^2) \equiv 4\tau \left[(\mathcal{F}_A^Z)^2 - (\mathcal{F}_T^Z)^2 + \tau (G_M^Z)^2 - (G_E^Z)^2 \right],$$

$$B(Q^2) \equiv 4\tau G_A^Z G_M^Z,$$

$$C(Q^2) \equiv \frac{1}{4} \left[(\mathcal{F}_A^Z)^2 + (\mathcal{F}_T^Z)^2 + \tau (G_M^Z)^2 + (G_E^Z)^2 \right].$$



■ Definition of some variables

$$\Delta = p' - p = k - k' \quad \sigma^+ = \sigma(\nu p \rightarrow \nu p), \quad \sigma^- = \sigma(\bar{\nu} p \rightarrow \bar{\nu} p)$$

$$P = (p' + p)/2 \quad \mathcal{F}_A^Z \equiv \sqrt{1+\tau} G_A^Z \quad s \equiv (p+k)^2$$

$$Q^2 = -\Delta^2 \geq 0 \quad \mathcal{F}_T^Z \equiv \sqrt{\tau(1+\tau)} G_T^Z \quad u \equiv (k' - p)^2$$

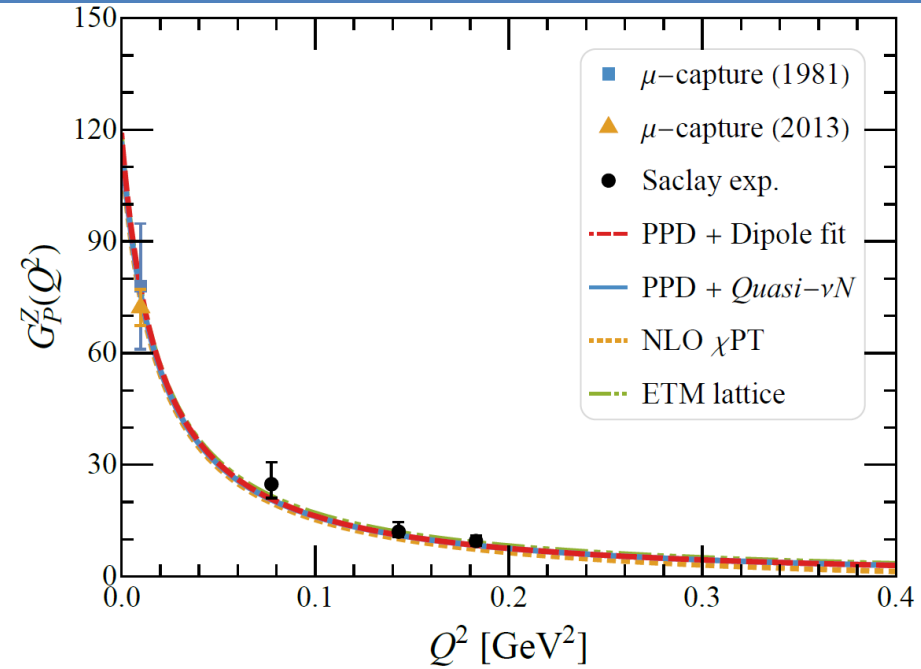
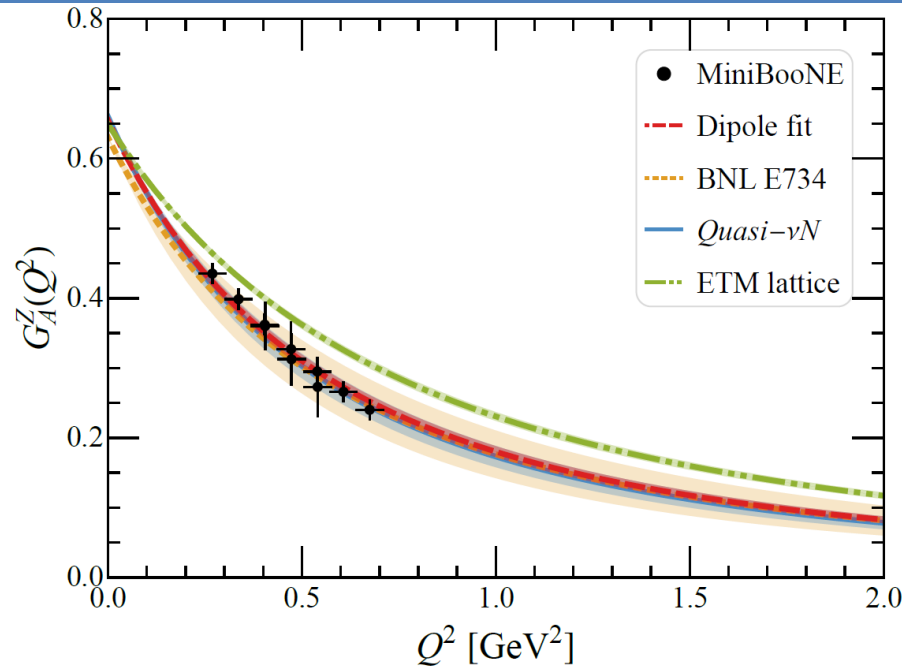
$$\tau \equiv Q^2/(4M^2)$$

■ Effectively, 4 FFs contribute explicitly to the differential cross sections.

$$G_E^Z, \quad G_M^Z, \quad G_A^Z, \quad G_T^Z$$

■ In particular, nonzero $G_A^Z(Q^2)$ implies the P violation, and nonzero $G_T^Z(Q^2)$ implies both P and CP violations $\Rightarrow$ matter-antimatter asymmetry/MAU. 12

Weak-neutral axial-vector FFs of the nucleon: $G_{A,P,T}^Z(Q^2)$



- | | | |
|---------------------------------|---|--|
| BNL E734 | { | [1]. Horstkotte, et al. Phys. Rev. D 25, 2743 (1982);
[2]. Ahrens, et al. Phys. Rev. D 35, 785 (1987); |
| MiniBooNE | { | [3]. Aguilar-Arevalo, et al. Phys. Rev. D 82, 092005 (2010);
[4]. Aguilar-Arevalo, et al. Phys. Rev. D 91, 012004 (2015); |
| ETM lattice | { | [5]. Alexandrou, et al. Phys. Rev. D 103, 034509 (2021);
[6]. Alexandrou, et al. Phys. Rev. D 104, 074503 (2021);... |
| μ-capture | { | [7]. Bardin, et al. Phys. Lett. B 104, 320 (1981);
[8]. Andreev, et al. Phys. Rev. Lett. 110, 012504 (2013); |
| Saclay exp. | { | [9]. Choi, et al. Phys. Rev. Lett. 71, 3927 (1993); |

[Bernard, Kaiser & Meissner. PRD 50, 6899 (1994)]
 [R. Sufian, K. Liu & D. Richards. JHEP 01, 136 (2020)]
 [YC. JHEP 04, 132 (2025); 2512.06801 [hep-ph]]

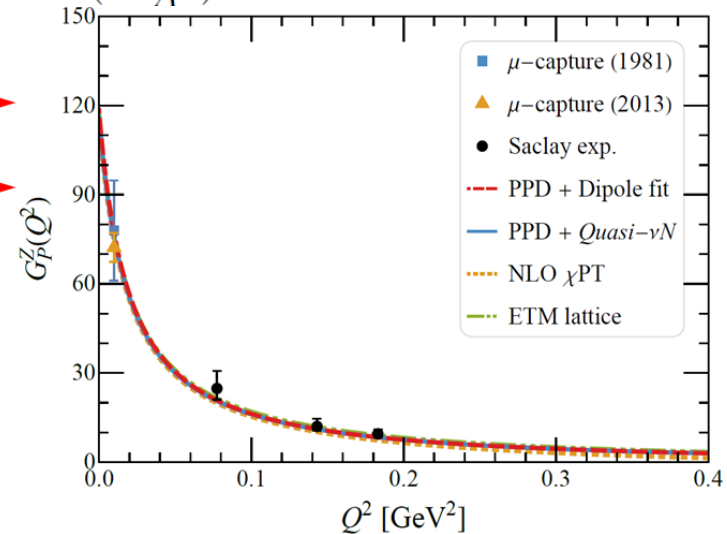
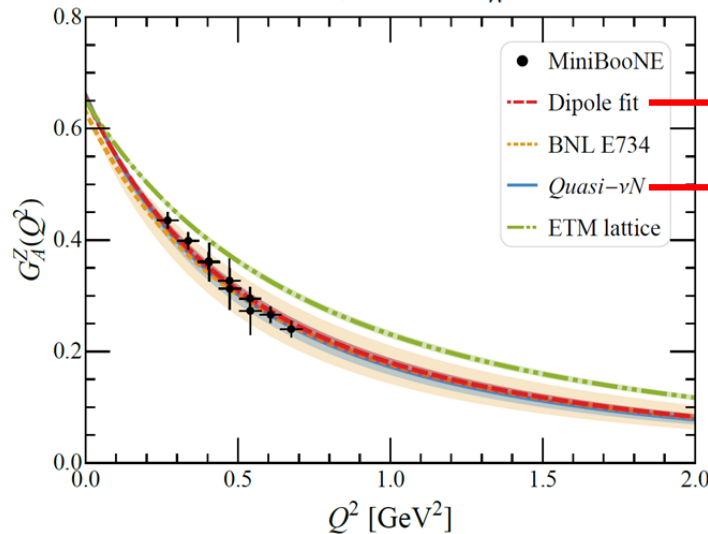
Pion-pole dominance (PPD) hypothesis and scaling ansatz

● **PPD:** $G_P^Z(Q^2) = \frac{4M^2}{M_\pi^2 + Q^2} G_A^Z(Q^2)$ ← **PCAC and Goldberger-Treiman relations**
pion pole

● **NLO ChPT (χ PT):**

$$g_{\pi^\pm pn} = \frac{\sqrt{4\pi}(M_p + M_n)}{M_\pi} f_{\pi^\pm pn} \approx (13.22613 \pm 0.04369)$$

$$G_P^W(Q^2) = g_{\pi^\pm pn} \frac{2(M_p + M_n)F_\pi}{Q^2 + M_\pi^2} - 2G_A^W(0) \frac{(M_p + M_n)^2}{(M_A^W)^2} + \mathcal{O}(Q^2; M_\pi^2)$$



● **Induced pseudotensor FF (scaling ansatz):** $G_T^Z(Q^2) = \kappa_T \cdot G_A^Z(Q^2), \quad \kappa_T \approx 0.1$

[Y. Jang, R. Gupta, B. Yoon, & T. Bhattacharya. PRL 124, 072002 (2020)]

[C. Alexandrou [ETM], et al. PRD 103, 034509 (2021)]

[Bernard, Kaiser & Meissner. PRD 50, 6899 (1994); Reinert, et al. PRL 126, 092501 (2021)]

[M. Day & K. McFarland. PRD 86, 053003 (2012)]

[C. Chen, C. Fischer, C. Roberts, & J. Segovia. PRD 105, 094022 (2022); EPJA 58, 206 (2022)]

Nucleon 3D mean-square axial radius $\langle r_A^2 \rangle \neq R_A^2$

- Standard definition of **3D mean-square axial (charge) radius** $\langle r_A^2 \rangle$:

$$\langle r_A^2 \rangle \equiv \frac{\int d^3r r^2 J_{5,B}^0(\mathbf{r})}{\int d^3r J_{5,B}^0(\mathbf{r})}$$

$$\langle r_E^2 \rangle \equiv -\frac{6}{G_E(0)} \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q=0}$$

- **Naïve traditional definition** of axial (charge) radius R_A^2 , **by a simple analogy** with the traditional definition of the mean-square proton charge radius:

$$R_A^2 \equiv -\frac{6}{G_A(0)} \frac{dG_A(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

Justification of the naïve 3D mean-square nucleon axial (charge) radius R_A^2 has never been rigorously discussed since 1986!

(for almost 40 years!!!)

[Meissner, & Kaiser. PLB 180, 129 (1986)]
[Meissner, Kaiser, & Weise. NPA 466, 129 (1986)]
[A1 Collaboration. PLB 468, 20 (1999)]
[Hill, et al. Rept. Prog. Phys. 81, 096301 (2018)]
[MINERvA Collaboration. Nature 614, 48 (2023)]

...

3D mean-square axial radius $\langle r_A^2 \rangle$ in the Breit frame

(1). Assuming **G-parity** invariance,

$${}_N \langle p', s' | \hat{j}_5^\mu(0) | p, s \rangle_N = \bar{u}(p', s') \left[\gamma^\mu \underbrace{G_A}_{\text{Axial}} + \frac{\Delta^\mu}{2M} \underbrace{G_P}_{\text{Induced pseudoscalar}} \right] \gamma^5 u(p, s)$$

In the 3D BF: ${}_N \langle p', s' | \hat{j}_5^0(0) | p, s \rangle_N = 0 \quad \longrightarrow \quad J_{5,B}^0(r) = 0$

$\longrightarrow$ Totally vanishing 3D axial charge distribution, thus no 3D axial radius!

(2). Without assuming **G-parity** invariance,

$${}_N \langle p', s' | \hat{j}_5^\mu(0) | p, s \rangle_N = \bar{u}(p', s') \left[\gamma^\mu \underbrace{G_A^Z}_{\text{Axial}} + \frac{\Delta^\mu}{2M} \underbrace{G_P^Z}_{\text{Induced pseudoscalar}} - \frac{\sigma^{\mu\nu} \Delta_\nu}{2M} \underbrace{G_T^Z}_{\text{Induced pseudo-tensor}} \right] \gamma^5 u(p, s)$$

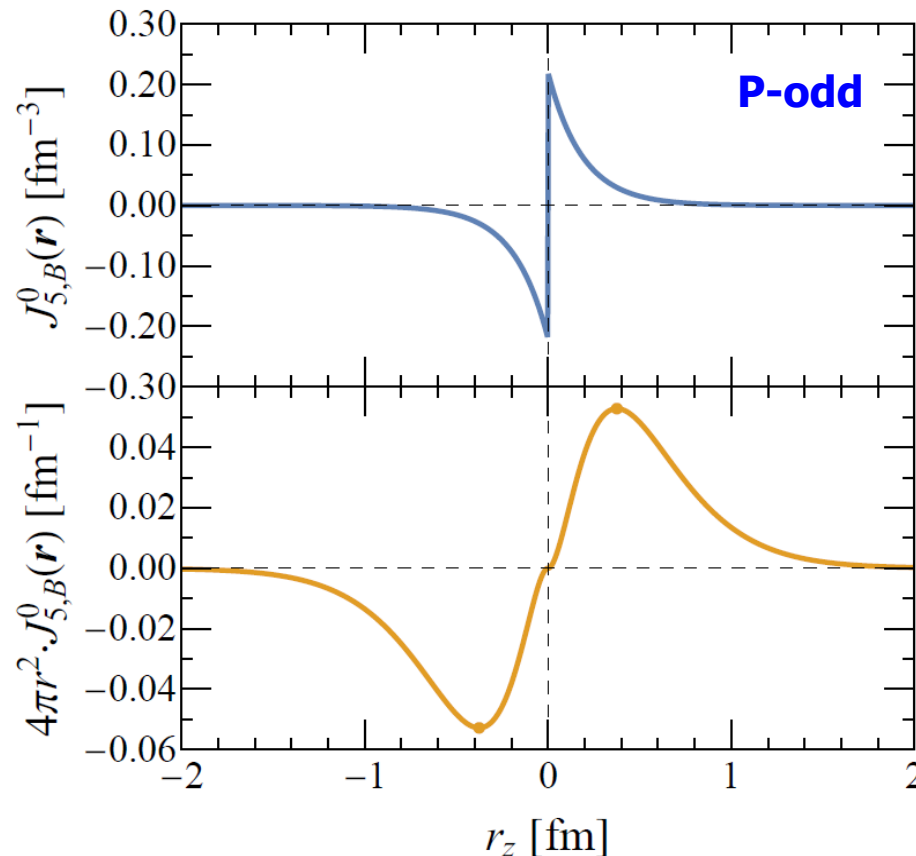
In the 3D BF: ${}_N \langle p', s' | \hat{j}_5^0(0) | p, s \rangle_N = \sqrt{1 + \tau} \underbrace{(\boldsymbol{\sigma} \cdot i\boldsymbol{\Delta})}_{\text{P-odd}} G_T^Z(\Delta^2)$

$\longrightarrow$ 1). 3D axial charge distribution is related to $G_T^Z(Q^2)$ rather than $G_A^Z(Q^2)$;
2). It is **parity-odd**, thus there is no 3D mean-square axial radius!

Nucleon 3D axial charge distribution & axial radius $\langle r_A^2 \rangle$

In either cases, i.e. $G_T^Z(Q^2) = 0$ or $G_T^Z(Q^2) \neq 0$, **3D mean-square axial radius for a spin-1/2 hadron does not exist.** This is dictated by the parity symmetry.

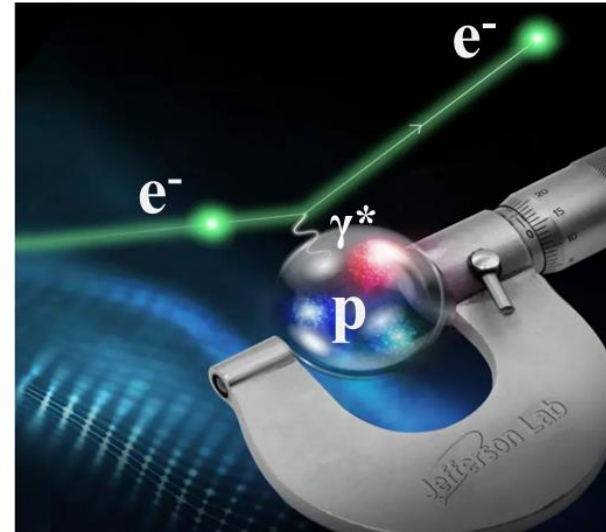
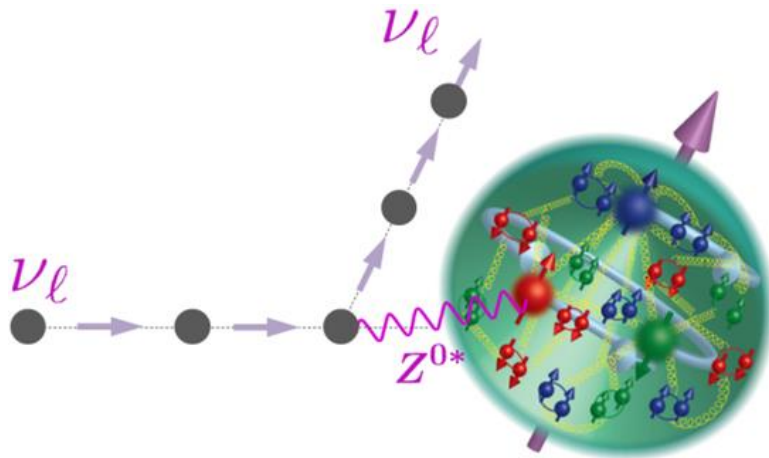
$$J_{5,B}^0(\mathbf{r}) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \frac{(\boldsymbol{\sigma} \cdot i\boldsymbol{\Delta})}{2M} G_T^Z(\Delta^2) \quad \int d^3r J_{5,B}^0(\mathbf{r}) = 0$$



[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]

[YC. JHEP 04, 132 (2025); 2512.06801 [hep-ph]]

Different nature of axial-vector and vector four-currents



$$\textcircled{P_\mu \Gamma_5^\mu = 0} \quad \hat{j}_5^\mu = \hat{\psi} \gamma^\mu \gamma^5 \hat{\psi}$$

spacelike four-current J_5^μ

intrinsic part

$$\mathbf{J}_5 = 2\mathbf{S} \quad \text{twice of spin distribution}$$

$$P = (p' + p)/2$$

$$\Delta = p' - p$$

$$\textcircled{\Delta_\mu \Gamma^\mu = 0} \quad \hat{j}^\mu = \hat{\psi} \gamma^\mu \hat{\psi}$$

timelike four-current J^μ

intrinsic part

$$J^0 = \rho_{\text{ch}} \quad \text{electric charge distribution}$$

- [C. Lorcé. PRL 125, 232002 (2020)]
 [YC, & Cédric Lorcé. PRD 106, 116024 (2022)]
 [YC, & Cédric Lorcé. PRD 107, 096003 (2023)] ...
 [YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]
 [YC. JHEP 04, 132 (2025); 2512.06801 [hep-ph]]

Relation between spin tensor and axial-vector four-current

- Using the **QCD equations of motion**, one can explicitly show that

$$\underline{\hat{S}^{\mu\alpha\beta}(x)} = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda} \underline{\hat{\psi}(x)\gamma_\lambda\gamma^5\hat{\psi}(x)} = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda} \underline{\hat{j}_{5\lambda}^\mu}$$

with the **axial-vector four-current operator** given by $\underline{\hat{j}_5^\mu(x)} \equiv \hat{\psi}(x)\gamma^\mu\gamma^5\hat{\psi}(x)$

[E. Leader & C. Lorcé. Physics Reports 541, 163 (2014)]

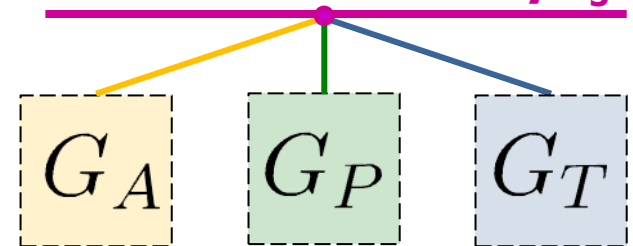
- Spin density operator and spin vector density (spin current)

$$\hat{S}^i(x) = \frac{1}{2}\epsilon^{ijk}\hat{S}^{0jk} = \frac{1}{2}\hat{j}_5^i(x)$$



$$S = \frac{1}{2}J_5$$

➡ **Spin vector density S** is so closely related to the **axial current density J₅**.



[Lorcé, & Mantovani, & Pasquini. PLB 776, 38 (2018)]

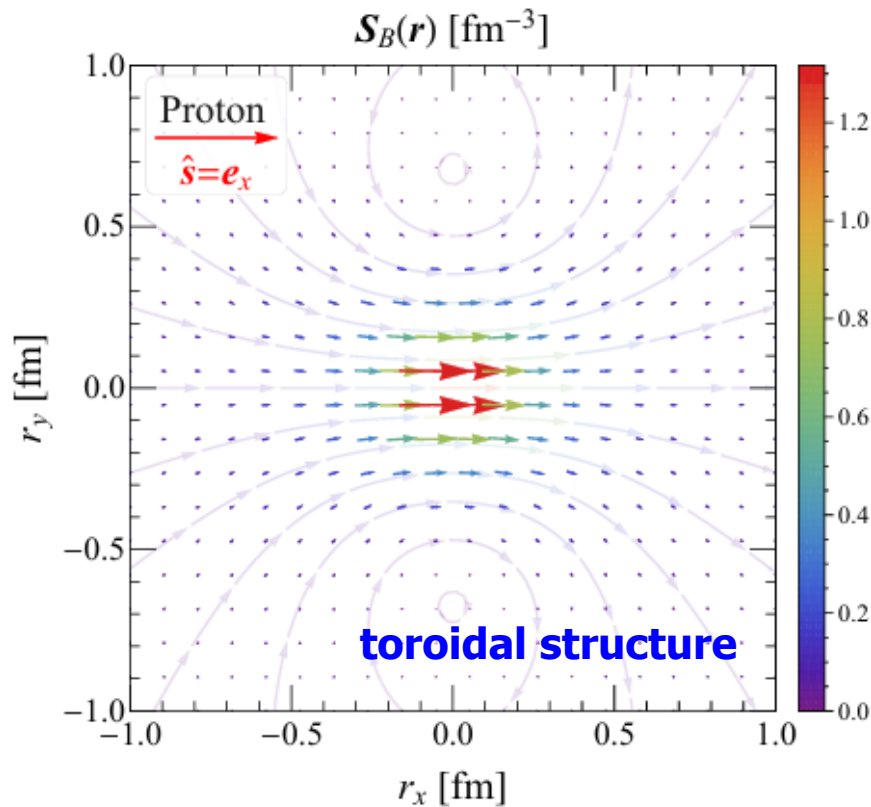
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]

[YC. JHEP 04, 132 (2025); 2512.06801 [hep-ph]]

Relativistic 3D intrinsic spin structure of the nucleon

◆ **3D BF spin vector density** $S_B(\mathbf{r}) = \frac{1}{2} \mathbf{J}_{5,B}(\mathbf{r})$

$$\mathbf{J}_{5,B}(\mathbf{r}) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \left\{ \left[\boldsymbol{\sigma} - \frac{\Delta(\Delta \cdot \boldsymbol{\sigma})}{4P_B^0(P_B^0 + M)} \right] G_A^Z(\Delta^2) - \frac{\Delta(\Delta \cdot \boldsymbol{\sigma})}{4MP_B^0} G_P^Z(\Delta^2) \right\}$$



■ **Input from MiniBooNE @ Fermilab (anti)neutrino data:**

$G_A^Z(Q^2)$: MiniBooNE data;
 $G_P^Z(Q^2)$: MiniBooNE data of $G_A^Z(Q^2)$ +
 PPD (pion pole dominance);

■ **Multipole decomposition:**

$$S_B(\mathbf{r}) = S_B^{(M)}(\mathbf{r}) + S_B^{(Q)}(\mathbf{r})$$

Up-down: mirror-symmetric

Left-right: mirror-antisymmetric

[MiniBooNE. PRD 82, 092005 (2010); PRD 91, 012004 (2015)]

[YC, Y. Li, C. Lorcé, Q. Wang. PRD 110, L091503 (2024); YC. JHEP 04, 132 (2025); 2512.06801]

[Sufian, et al. JHEP 01, 136 (2020)]

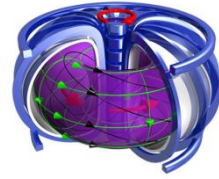
[P. Neumann-Cosel, et al. PRL 133, 232502 (2024)]

Toroidal structure of ^{208}Pb & proton

PHYSICAL REVIEW LETTERS **133**, 232502 (2024)

Editors' Suggestion

Featured in Physics



Candidate Toroidal Electric Dipole Mode in the Spherical Nucleus ^{58}Ni

P. von Neumann-Cosel^{1,*}, V. O. Nesterenko^{2,3,†}, I. Brandherm¹, P. I. Vishnevskiy^{1,2,4}, P.-G. Reinhard⁵, J. Kvasil⁶,
H. Matsubara^{7,8}, A. Repko⁹, A. Richter¹, M. Scheck^{10,11} and A. Tamii⁷

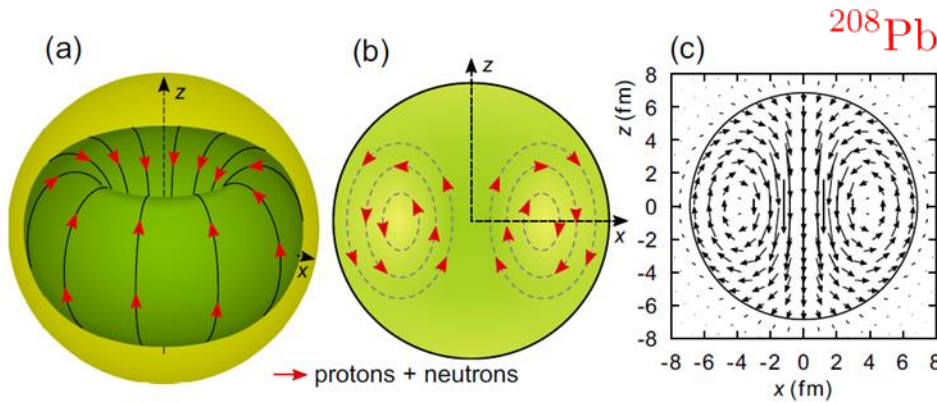
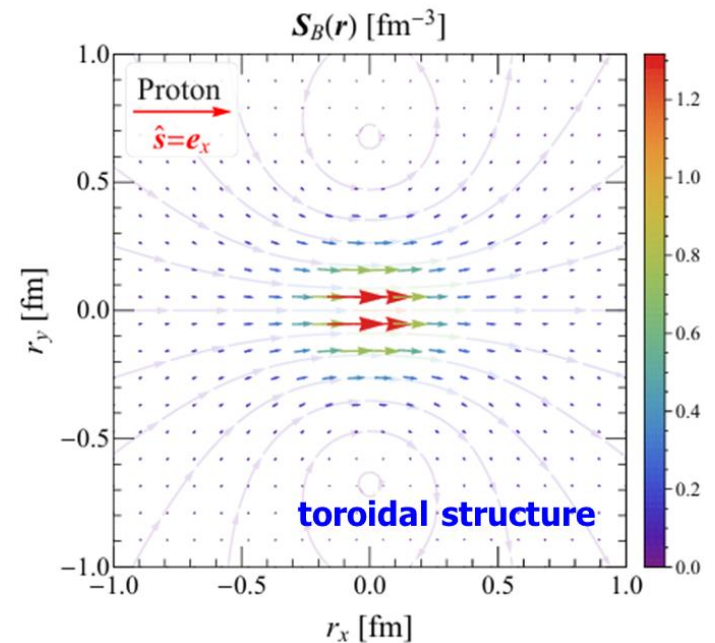


FIG. 1. Nuclear toroidal excitations. (a) Schematic view and (b) its cut in the x - z plane. (c) Same as (b) for the toroidal mode predicted in the nucleus ^{208}Pb [14]. The arrows mark the current along stream lines and their length is a measure of the current density.



➡ **Nucleon intrinsic spin structure is naturally of toroidal structure!**

[P. Neumann-Cosel, et al. PRL 133, 232502 (2024)]

[YC, Y. Li, C. Lorcé, Q. Wang. PRD 110, L091503 (2024); YC. JHEP 04, 132 (2025); 2512.06801] **21**

Nucleon 3D mean-square spin radius $\langle r_{\text{spin}}^2 \rangle$

- Physically meaningful **3D mean-square spin radius**:

$$\langle r_{\text{spin}}^2 \rangle \equiv \frac{\int d^3r r^2 \hat{s} \cdot \mathbf{S}_B(\mathbf{r})}{\int d^3r \hat{s} \cdot \mathbf{S}_B(\mathbf{r})} = R_A^2 + \frac{1}{4M^2} \left[1 + \frac{2G_P^Z(0)}{G_A^Z(0)} \right] \quad \text{Model-independent!}$$

Based on **recent lattice QCD results** and **PPD hypothesis**, one can show that

$$\frac{G_P(0)}{G_A(0)} \gg 1$$

the last term $\frac{1}{2M^2} \frac{G_P(0)}{G_A(0)}$ **actually plays an dominant role!**

- Recently, R_A^2 was measured with high-precision by the **MINERvA Collaboration** at Fermilab. **[MINERvA Collaboration. Nature 614, 48 (2023)]**
- However, R_A^2 **alone is not enough** to fix the meaningful 3D mean-square spin radius $\langle r_{\text{spin}}^2 \rangle$. **One still needs to determine the ratio $G_P^Z(0)/G_A^Z(0)$.**
- This **provides an additional key motivation for ongoing lattice QCD and model calculations or future experimental measurements** of the nucleon induced pseudoscalar form factor $G_P^Z(Q^2)$.

Covariant Lorentz transformations (based on Poincaré symmetry)

◆ Axial-vector four-current case:

$$\langle p', s' | \hat{j}_5^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s's'_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu \langle p'_B, s'_B | \hat{j}_5^\nu(0) | p_B, s_B \rangle}_{\text{Lorentz mixing}}$$

◆ Exactly the same angular conditions as before* for the Wigner rotation angle θ :

$$\cos \theta = \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \quad \sin \theta = -\frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}}$$

◆ Elastic frame (EF) axial-vector four-current distributions:

$$\begin{aligned} J_{5,\text{EF}}^0(b_\perp; P_z) &= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \left[\frac{(i\Delta_\perp \cdot \sigma_\perp)_{s's}}{2M} G_T^Z(\Delta_\perp^2) + \left(\frac{P_z}{P^0} \right) (\sigma_z)_{s's} G_A^Z(\Delta_\perp^2) \right] \\ J_{5,\text{EF}}^z(b_\perp; P_z) &= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \left[\left(\frac{P_z}{P^0} \right) \frac{(i\Delta_\perp \cdot \sigma_\perp)_{s's}}{2M} G_T^Z(\Delta_\perp^2) + (\sigma_z)_{s's} G_A^Z(\Delta_\perp^2) \right] \\ J_{5,\text{EF}}^\perp(b_\perp; P_z) &= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \left\{ -\frac{\Delta_\perp (\Delta_\perp \cdot \sigma_\perp)_{s's}}{4P^0} \left[\frac{G_A^Z(\Delta_\perp^2)}{P^0 + M} + \frac{G_P^Z(\Delta_\perp^2)}{M} \right] \right. \\ &\quad \left. + \frac{\sqrt{P^2}}{P^0} \left[\cos \theta (\sigma_\perp)_{s's} - \frac{(\mathbf{e}_z \times i\Delta_\perp)_\perp}{2M\sqrt{\tau}} \sin \theta \delta_{s's} \right] G_A^Z(\Delta_\perp^2) \right\} \end{aligned}$$

} free from Wigner spin rotation
} suffer from Wigner spin rotation

[Durand, De Celles, Marr. PR 126, 1882 (1962)]

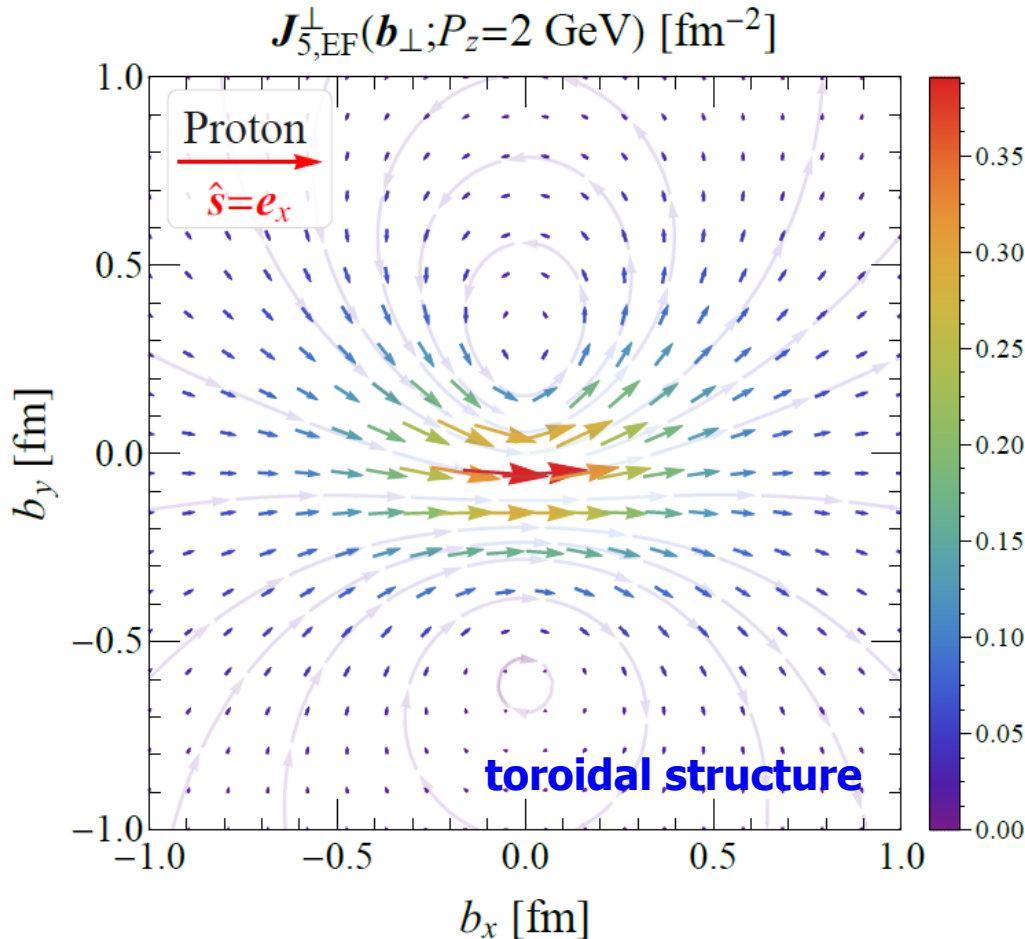
[Cédric Lorcé. PRL 125, 232002 (2020)]

[YC, & Cédric Lorcé. PRD 106, 116024 (2022); PRD 107, 096003 (2023)]*

[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]

[YC. JHEP 04, 132 (2025)]

Relativistic EF distributions of a moving proton



$$S_{EF}^{\perp} = \frac{1}{2} J_{5,EF}^{\perp}$$

$$J_{5,EF}^{\perp} = J_{5,EF}^{\perp(M)} + \mathbf{J}_{5,EF}^{\perp(D)} + J_{5,EF}^{\perp(Q)}$$

← **Dipole contribution** breaks explicitly the up-down symmetry!

Up-down: mirror-asymmetric

Left-right: mirror-antisymmetric

■ Both $S_{\perp,EF}$ and $J_{5,EF}^{\perp}$ are **free from Lorentz mixing effect** but **suffer from the spin Wigner rotation!**

[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]

[YC. JHEP 04, 132 (2025)]

[P. Neumann-Cosel, et al. PRL 133, 232502 (2024)]

Proton's 2D transverse mean-square axial and spin radii

- **Axial radius** (for a longitudinally polarized, moving spin-1/2 hadron).

$$\langle b_A^2 \rangle_{\text{EF}}(P_z) = \frac{1}{2E_P^2} + \frac{2}{3}R_A^2$$

- **Longitudinal spin radius** (for a longitudinally polarized spin-1/2 hadron).

$$\langle b_{\text{spin},L}^2 \rangle_{\text{EF}}(P_z) \equiv \frac{\int d^2b_{\perp} b^2 S_{\text{EF}}^z(\mathbf{b}_{\perp}; P_z)}{\int d^2b_{\perp} S_{\text{EF}}^z(\mathbf{b}_{\perp}; P_z)} = \frac{2}{3}R_A^2$$

- **Transverse spin radius** (for a transversely polarized spin-1/2 hadron).

$$\langle b_{\text{spin},T}^2 \rangle_{\text{EF}}(P_z) = \frac{2}{3}R_A^2 + \frac{1}{2M^2} \frac{G_P(0)}{G_A(0)} - \frac{1}{2M(E_P + M)} + \frac{1}{2E_P^2}$$

Conclusion: the second-class current contribution [**associated with $G_T^Z(Q^2)$**], although explicitly included, does not contribute in fact to the mean-square axial and spin radii.

Relativistic light-front (LF) distributions

◆ **In the IMF limit ($P_z \rightarrow \infty$):** $J_{5,\text{LF}}^+(\mathbf{b}_\perp; P^+) = J_{5,\text{EF}}^0(\mathbf{b}_\perp; \infty) = J_{5,\text{EF}}^z(\mathbf{b}_\perp; \infty)$

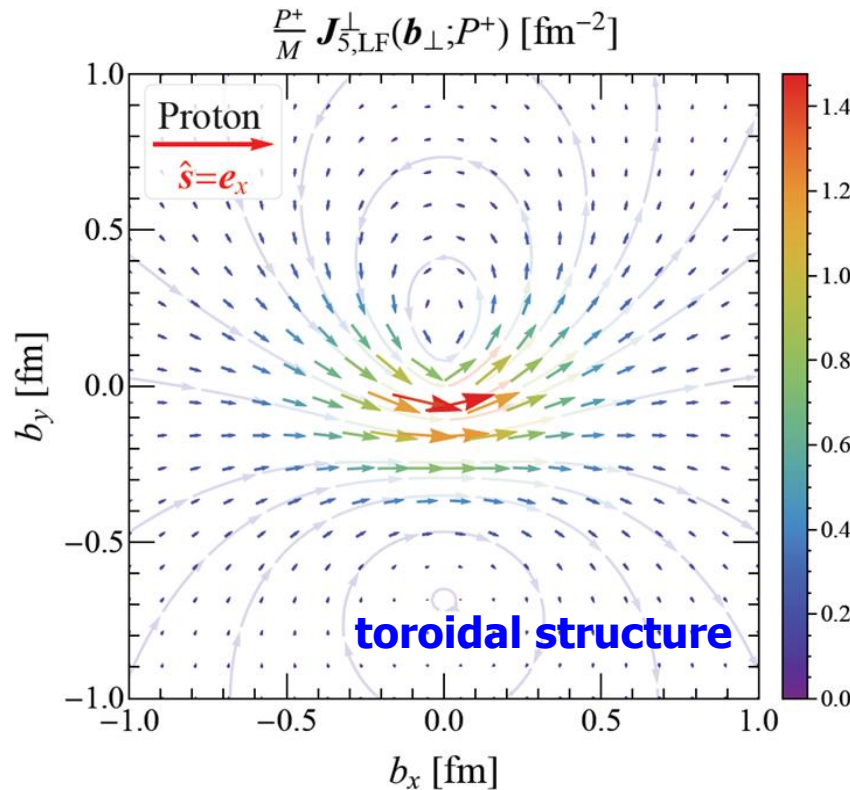
$$\langle b_A^2 \rangle_{\text{LF}}(P^+) = \langle b_{\text{spin},L}^2 \rangle_{\text{LF}}(P^+) = \frac{2}{3} R_A^2$$

◆ **Mean-square LF radii:**

$$\langle b_{\text{spin},T}^2 \rangle_{\text{LF}}(P^+) = \frac{2}{3} R_A^2 + \frac{1}{2M^2} \frac{G_P^Z(0)}{G_A^Z(0)}$$

$\sim$ independent of G_T^Z
(second-class current)

◆ **Scaled transverse LF axial current distribution:**



$$\mathbf{S}_{\text{LF}}^\perp = \frac{1}{2} \mathbf{J}_{5,\text{LF}}^\perp$$

Transversely polarized proton

Up-down: mirror-asymmetric

Left-right: mirror-antisymmetric

LF amplitudes via EF amplitudes at proper IMF limit

Conjecture. Any light-front (LF) amplitudes for well-defined LF distributions in principle can be explicitly reproduced from the corresponding elastic frame (EF) amplitudes in the proper infinite-momentum frame (IMF) limit.

◆ EF amplitudes

$$\begin{aligned}\mathcal{A}_{\text{EF}}^0 &= 2P^0 \left[\frac{(i\Delta_{\perp} \cdot \sigma_{\perp})}{2M} G_T^Z(\Delta_{\perp}^2) + \frac{P_z}{P^0} \sigma_z G_A^Z(\Delta_{\perp}^2) \right], \\ \mathcal{A}_{\text{EF}}^z &= 2P^0 \left[\frac{P_z}{P^0} \frac{(i\Delta_{\perp} \cdot \sigma_{\perp})}{2M} G_T^Z(\Delta_{\perp}^2) + \sigma_z G_A^Z(\Delta_{\perp}^2) \right], \\ \mathcal{A}_{\text{EF}}^{\perp} &= 2\sqrt{P^2} \left[\frac{P^0 + M(1+\tau)}{(P^0 + M)\sqrt{1+\tau}} \sigma_{\perp} + \frac{(e_z \times i\Delta_{\perp})_{\perp}}{2M} \frac{P_z}{(P^0 + M)\sqrt{1+\tau}} \right] G_A^Z(\Delta_{\perp}^2) \\ &\quad - \frac{\Delta_{\perp}(\Delta_{\perp} \cdot \sigma_{\perp})}{2} \left[\frac{G_A^Z(\Delta_{\perp}^2)}{P^0 + M} + \frac{G_P^Z(\Delta_{\perp}^2)}{M} \right],\end{aligned}$$

◆ LF amplitudes

$$\begin{aligned}\mathcal{A}_{\text{LF}}^+ &= 2P^+ \left[\frac{(i\Delta_{\perp} \cdot \sigma_{\perp})_{\lambda'\lambda}}{2M} G_T^Z(\Delta_{\perp}^2) + (\sigma_z)_{\lambda'\lambda} G_A^Z(\Delta_{\perp}^2) \right], \\ \mathcal{A}_{\text{LF}}^- &= 2P^- \left[\frac{(i\Delta_{\perp} \cdot \sigma_{\perp})_{\lambda'\lambda}}{2M} G_T^Z(\Delta_{\perp}^2) - (\sigma_z)_{\lambda'\lambda} G_A^Z(\Delta_{\perp}^2) \right], \\ \mathcal{A}_{\text{EF}}^{\perp} &= 2M \left\{ \left[(\sigma_{\perp})_{\lambda'\lambda} + \frac{(e_z \times i\Delta_{\perp})_{\perp}}{2M} \delta_{\lambda'\lambda} \right] G_A^Z(\Delta_{\perp}^2) - \frac{\Delta_{\perp}(\Delta_{\perp} \cdot \sigma_{\perp})_{\lambda'\lambda}}{4M^2} G_P^Z(\Delta_{\perp}^2) \right\}\end{aligned}$$

This conjecture has been verified independently in:

(1). Electromagnetic four-current case;

(2). Polarization-magnetization and energy-momentum tensor cases;

(3). Axial-vector four-current cases (with/without G_T^Z);

...

supporting **LaMET**?

(X. Ji, F. Yuan, Y. Zhao, Y. Liu, ...)

[Cédric Lorcé. PRL 125, 232002 (2020)]

[YC, & Cédric Lorcé. PRD 106, 116024 (2022); PRD 107, 096003 (2023)]

[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]

[YC. JHEP 04, 132 (2025)]

Summary and outlook

- **Hadron structures are highly non-trivial and complicated!** They are closely related to many ongoing and new scientific programs and facilities. E.g. EIC, EicC, JLab 22 GeV upgrade, HAIFF, etc.
- **Neutrino/antineutrino elastic or quasi-elastic scatterings offer us a clean way to probe the internal structures of hadrons/nuclei in the weak sector. In particular, relativistic axial-vector and spin structures of nucleon are discussed in detail in terms of axial-vector FFs $G_{A,P,T}^Z(Q^2)$.** New facilities, e.g. Fermilab, DUNE, JUNO, FASER, COHERENT, etc., can be well expected.
- **Due to parity symmetry, physically meaningful 3D axial (charge) radius $\langle r_A^2 \rangle$ does not exist for any spin-1/2 hadrons. However, 3D mean-square nucleon spin radius $\langle r_{\text{spin}}^2 \rangle$ is a well-defined, physically meaningful quantity that better characterizes the size of spatial extension of weak content of nucleon.**
- **Any relativistic spatial distortions of a moving spinning hadron can be understood as a combination of Lorentz transformation and Wigner spin rotation.** This is precisely protected by the Poincaré symmetry.

Thank you!

Backup: Covariant Lorentz transformation -- two key effects

- **Four-vector case (e.g., electromagnetic four-current):**

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s'_B s_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu \langle p'_B, s'_B | \hat{j}^\nu(0) | p_B, s_B \rangle}_{\text{Lorentz mixing}}$$

- **Axial four-vector case (e.g., axial-vector four-current):**

$$\langle p', s' | \hat{j}_5^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s'_B s_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu \langle p'_B, s'_B | \hat{j}_5^\nu(0) | p_B, s_B \rangle}_{\text{Lorentz mixing}}$$

- **Tensor case (e.g., polarization-magnetization tensor):**

$$\langle p', s' | \hat{P}^{\mu\nu}(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s'_B s_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta \langle p'_B, s'_B | \hat{P}^{\alpha\beta}(0) | p_B, s_B \rangle}_{\text{Lorentz mixing}}$$

(1). Lorentz mixing effect

$$\begin{pmatrix} \tilde{J}_{\text{EF}}^0 \\ \tilde{J}_{\text{EF}}^z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \tilde{J}_B^0 \\ \tilde{J}_B^z \end{pmatrix} \sim \text{momentum-space amplitudes}$$

boost $\Lambda^\mu{}_\nu$

(2). Wigner rotation effect

Fundamental reason: boost generators of Lorentz group do not commute!

$$[\hat{K}^i, \hat{K}^j] = -i\epsilon^{ijk} \hat{J}^k \longrightarrow \text{Wigner rotation } D_{s_B s}^{(j)}(p_B, \Lambda)$$

[Durand, De Celles, & Marr. PR 126, 1882 (1962)]

[Cédric Lorcé. PRL 125, 232002 (2020)]

[YC, & Cédric Lorcé. PRD 106, 116024 (2022)]

[YC, & Cédric Lorcé. PRD 107, 096003 (2023)]

[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024); JHEP 04, 132 (2025)]

Relativistic LF interpretation

PRL 99, 112001 (2007)

PHYSICAL REVIEW LETTERS

week ending
14 SEPTEMBER 2007

Charge Densities of the Neutron and Proton

Gerald A. Miller

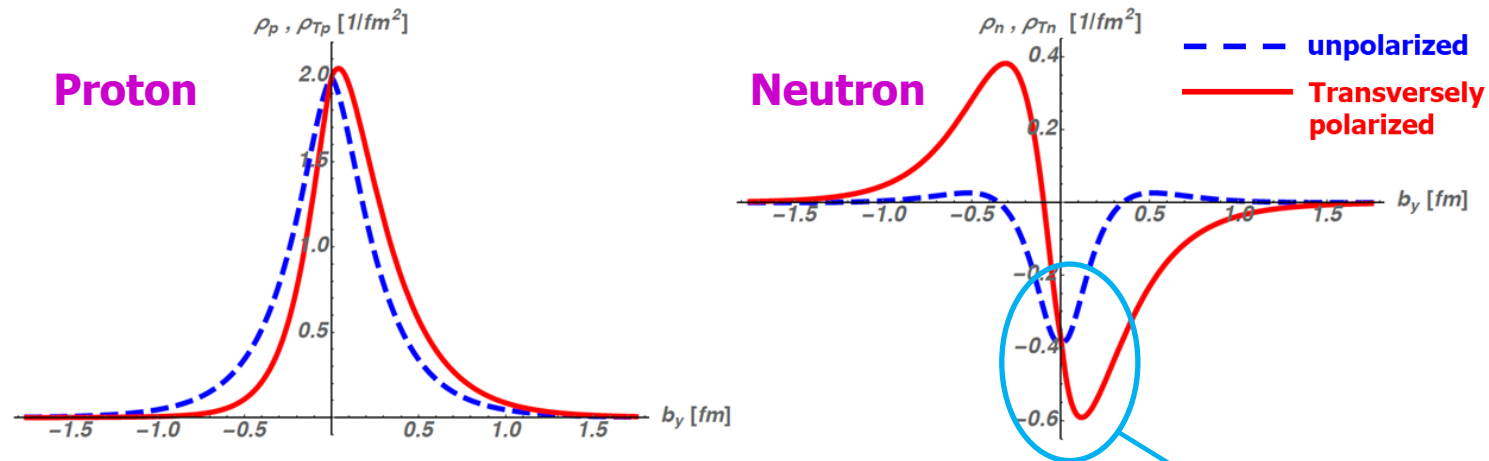
Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA

(Received 18 May 2007; published 13 September 2007)

A model-independent analysis of the infinite-momentum-frame charge density of partons in the transverse plane is presented for the nucleon. We find that the neutron-parton charge density is negative at the center, so that the square of the transverse charge radius is positive, in contrast with many expectations. Additionally, the proton's central d quark charge density is larger than that of the u quark by about 30%. The proton (neutron) charge density has a long range positively (negatively) charged component.

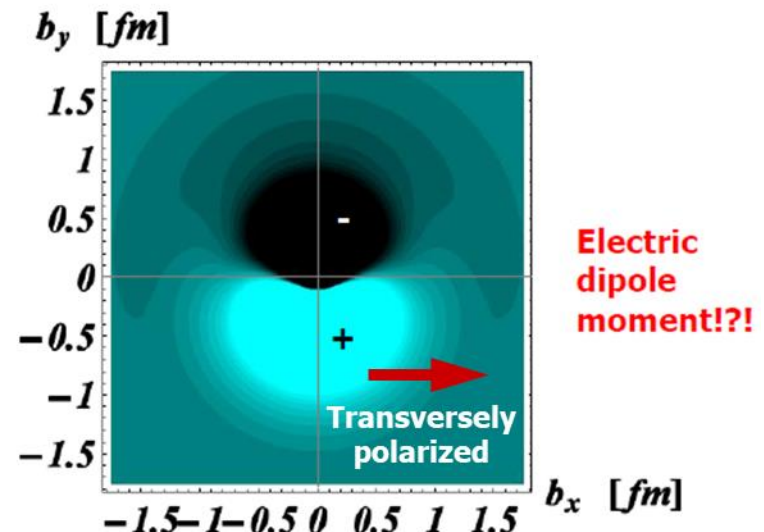
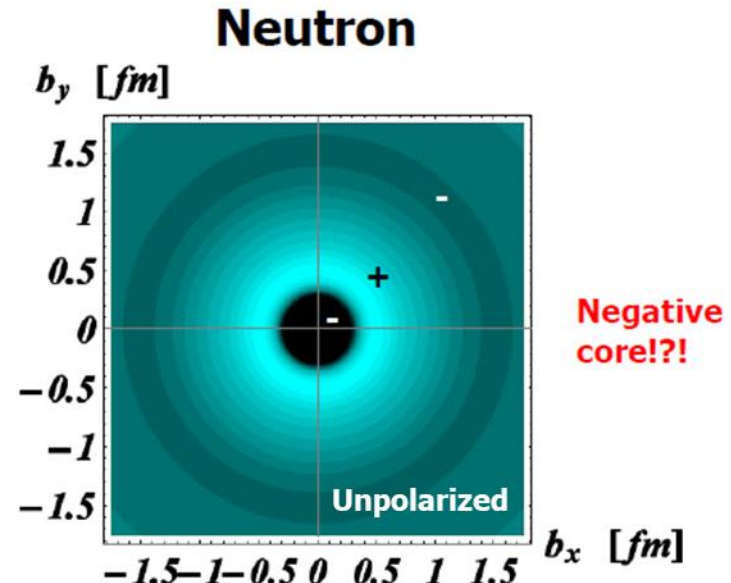
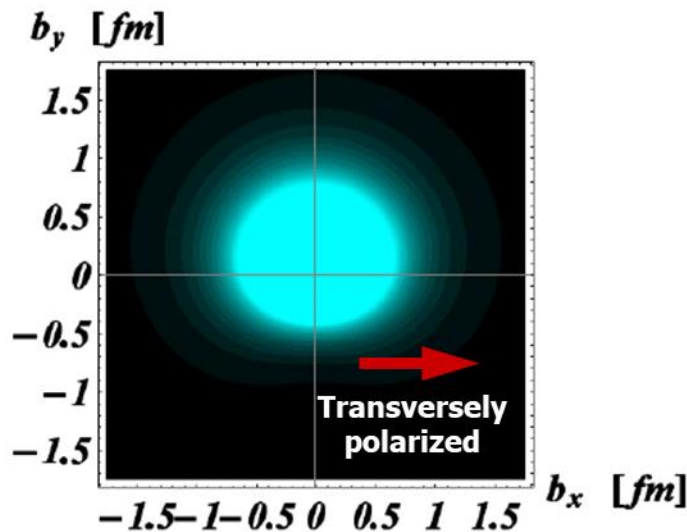
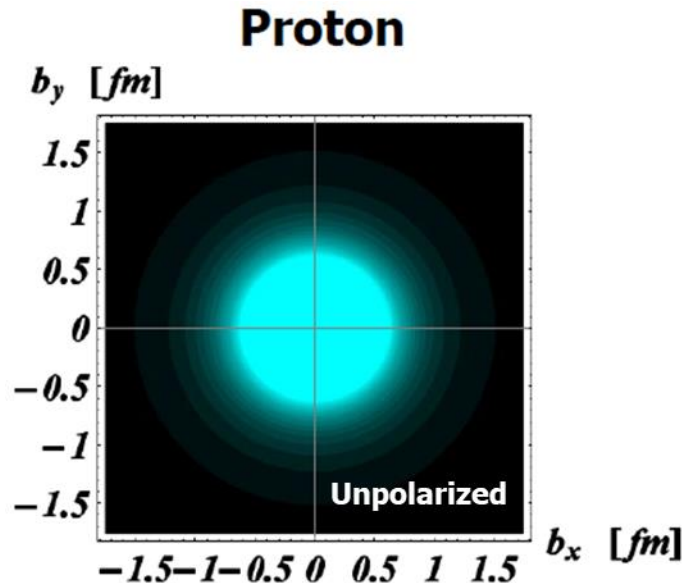
◆ Generic LF distributions (strict probabilistic!!!):

$$O_{\text{LF}}(\mathbf{b}_{\perp}; P^+) \equiv \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i \Delta_{\perp} \cdot \mathbf{b}_{\perp}} \frac{\text{LF} \langle p', \lambda' | \hat{O}(0) | p, \lambda \rangle_{\text{LF}}}{2P^+} \Big|_{\Delta^+ = |\mathbf{P}_{\perp}| = 0}$$

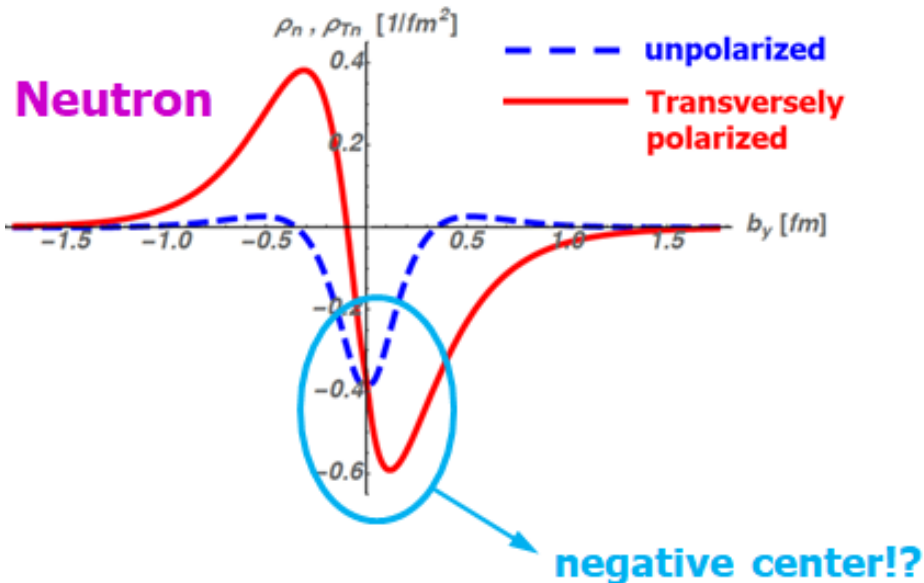
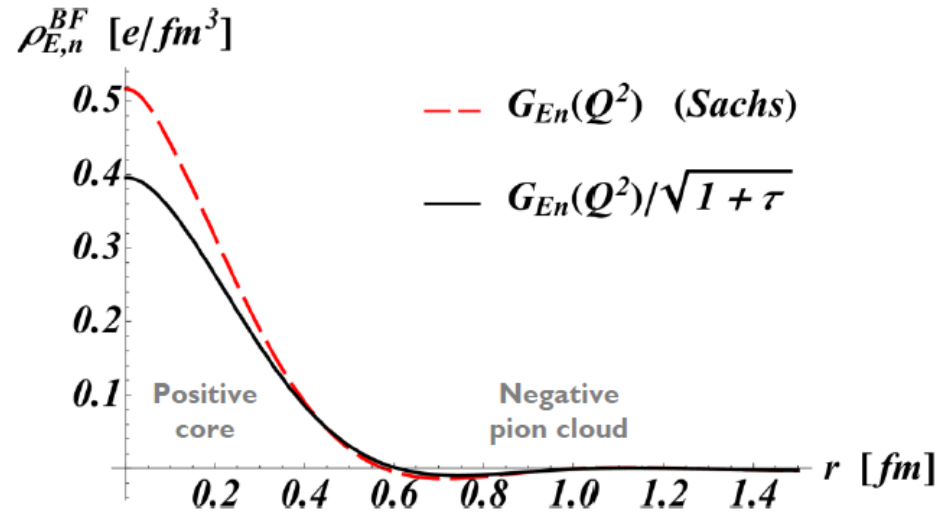


[G. A. Miller. PRL 99, 112001 (2007)]
[Carlson, & Vanderhaeghen. PRL 100, 032004 (2008)]

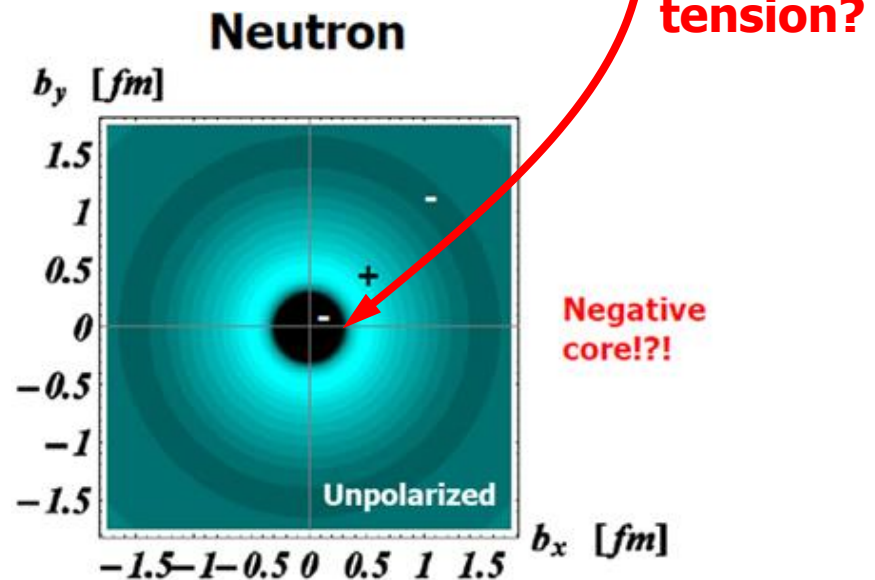
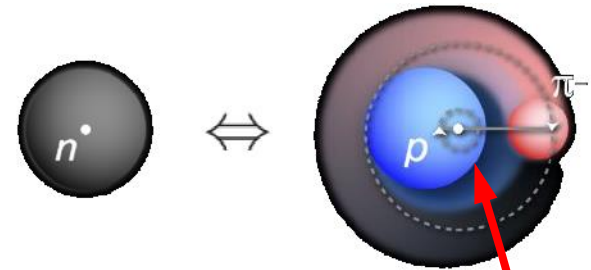
Some unusual observations on the LF



Neutron: 3D positive center → 2D negative center?!



Pion cloud picture (rest frame)



Relativistic EF interpretation

◆ Poincaré symmetry:

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s' s'_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu \langle p'_B, s'_B | \hat{j}^\nu(0) | p_B, s_B \rangle}_{\text{Lorentz mixing}}$$

(1). Lorentz mixing

$$\begin{pmatrix} \tilde{J}_{\text{EF}}^0 \\ \tilde{J}_{\text{EF}}^z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \tilde{J}_B^0 \\ \tilde{J}_B^z \end{pmatrix} \quad \text{boost } \Lambda^\mu{}_\nu \quad \sim \text{momentum-space amplitudes}$$

(2). Wigner spin rotation

Key insight: boost generators do not commute!

$$[\hat{K}^i, \hat{K}^j] = -i\epsilon^{ijk} \hat{J}^k$$

$$D^{(1/2)}(p_B, \Lambda) = \begin{pmatrix} \cos \frac{\theta}{2} & -e^{-i\phi\Delta} \sin \frac{\theta}{2} \\ e^{i\phi\Delta} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \quad \theta: \text{Wigner rotation angle}$$

$$\cos \theta = \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \quad \sin \theta = -\frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}}$$

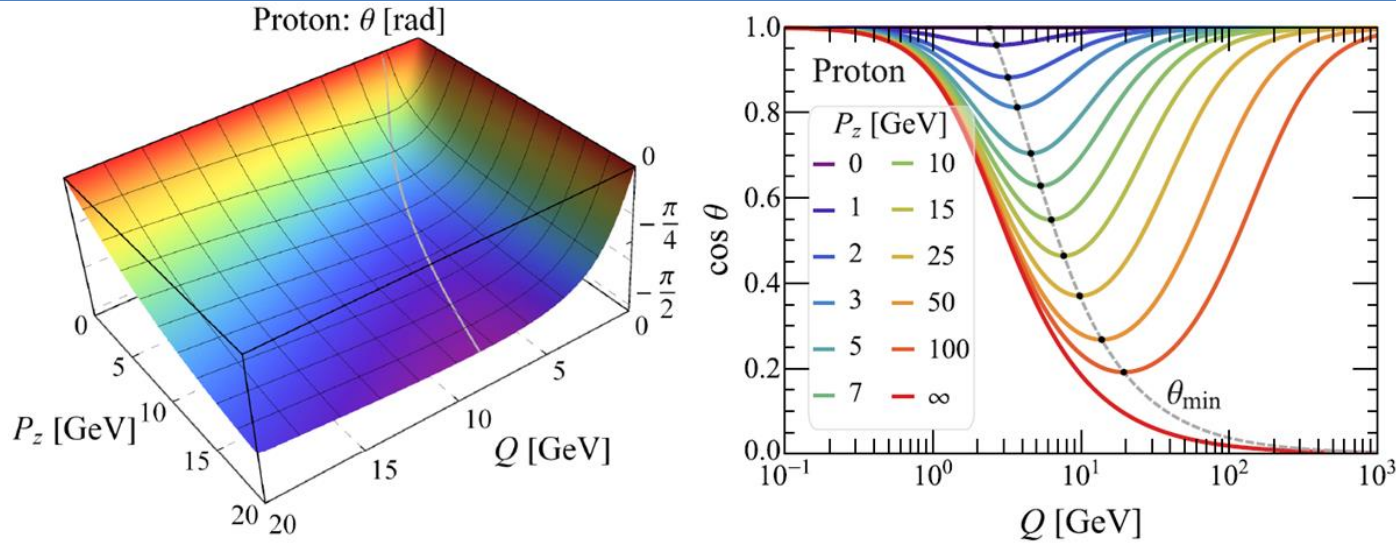
[Durand, De Celles, & Marr. PR 126, 1882 (1962)]

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[YC, & Cédric Lorcé. PRD 106, 116024 (2022)]

[YC, & Cédric Lorcé. PRD 107, 096003 (2023)]

Relativistic EF interpretation



$$-\frac{\pi}{2} \leq \theta \leq 0$$

◆ Generic angular conditions for Wigner spin rotation:

$$\cos \theta = \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \quad \sin \theta = -\frac{\sqrt{\tau}P_z}{(P^0 + M)\sqrt{1 + \tau}}$$

(1). At fixed P_z :

$$\cos \theta_{\min} = \frac{1 + 2\tau_{\min}}{(1 + \tau_{\min})^{3/2}} \longrightarrow \tau_{\min} = \frac{1}{2} + \sqrt{\frac{P_z^2}{M^2} + \frac{5}{4}}$$

(2). At $P_z \rightarrow \infty$:

$$\lim_{P_z \rightarrow \infty} \cos \theta = \frac{1}{\sqrt{1 + \tau}}, \quad \lim_{P_z \rightarrow \infty} \sin \theta = -\frac{\sqrt{\tau}}{\sqrt{1 + \tau}}$$

Relativistic EF interpretation

◆ EF distributions (2D):

Wigner spin rotation

$$\begin{aligned}
 J_{\text{EF}}^0(\mathbf{b}_\perp; P_z) &= e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\
 &\quad + e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}} \\
 J_{z,\text{EF}}(\mathbf{b}_\perp; P_z) &= e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\
 &\quad + e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}
 \end{aligned}$$

$$\underline{J_{\perp,\text{EF}}(\mathbf{b}_\perp; P_z)} = \delta_{s's} \frac{e\sigma_z}{2M} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} (\mathbf{e}_z \times i\boldsymbol{\Delta})_\perp \frac{P_B^0}{P^0} \frac{G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

free of Wigner spin rotation

suppressed by Lorentz contraction

In conclusion: Lorentz transformation and Wigner spin rotation indeed jointly play a central role for the structures of moving spinning hadrons.

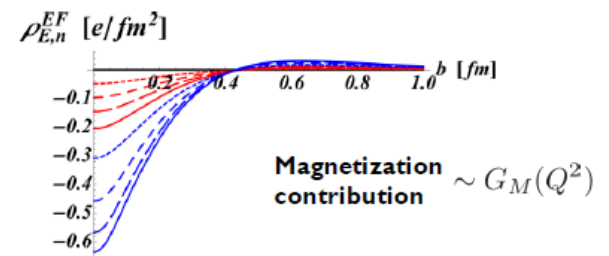
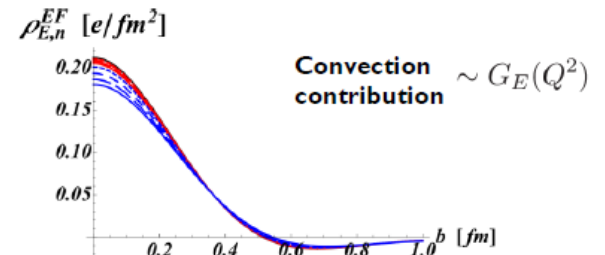
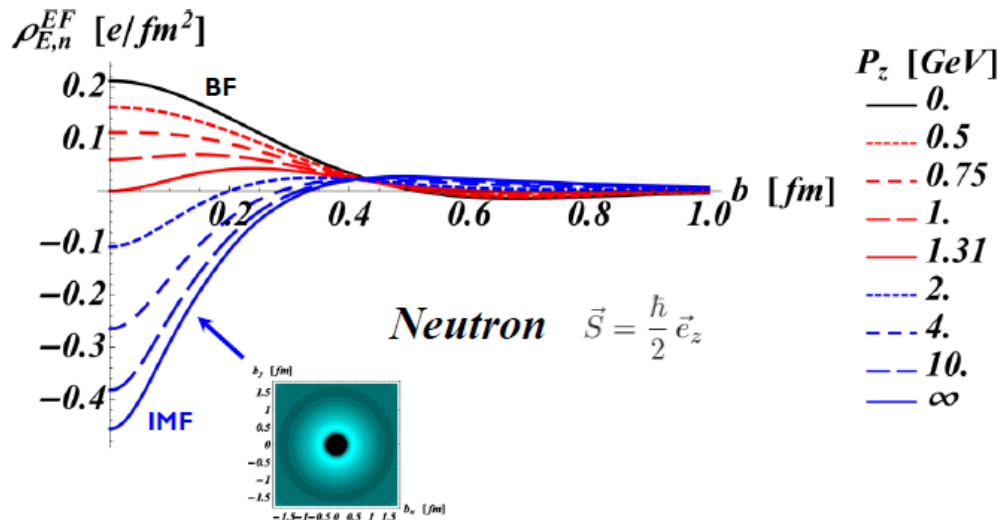
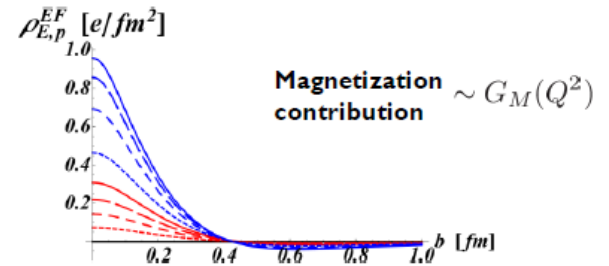
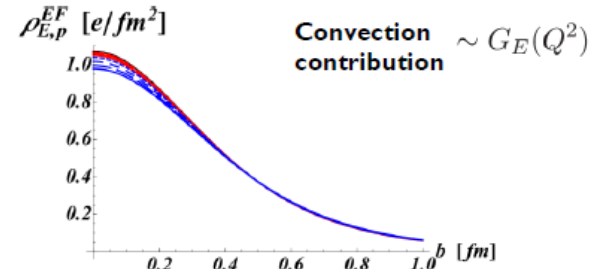
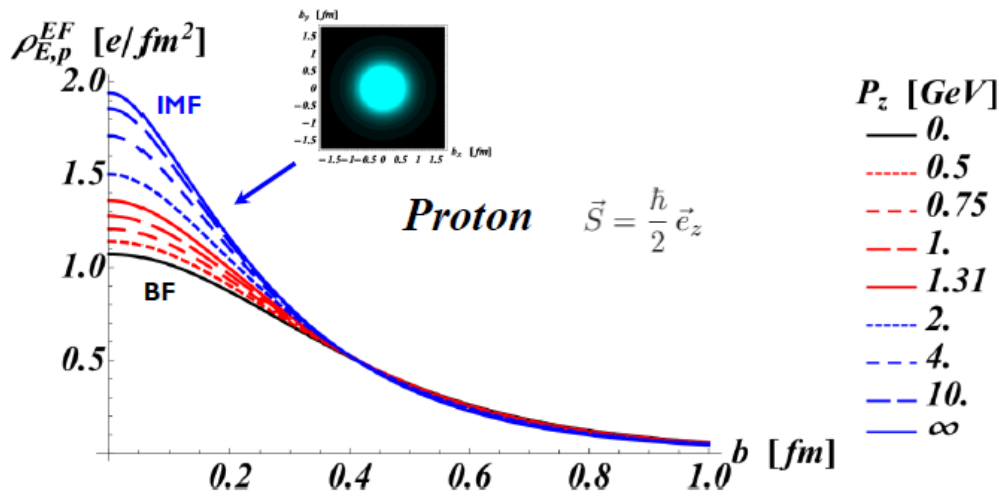
[Cédric Lorcé, & Pierre Wang. PRD 105, 096032 (2022)]

[YC, & Cédric Lorcé. PRD106, 116024 (2022)]

[YC, & Cédric Lorcé. PRD107, 096003 (2023)]

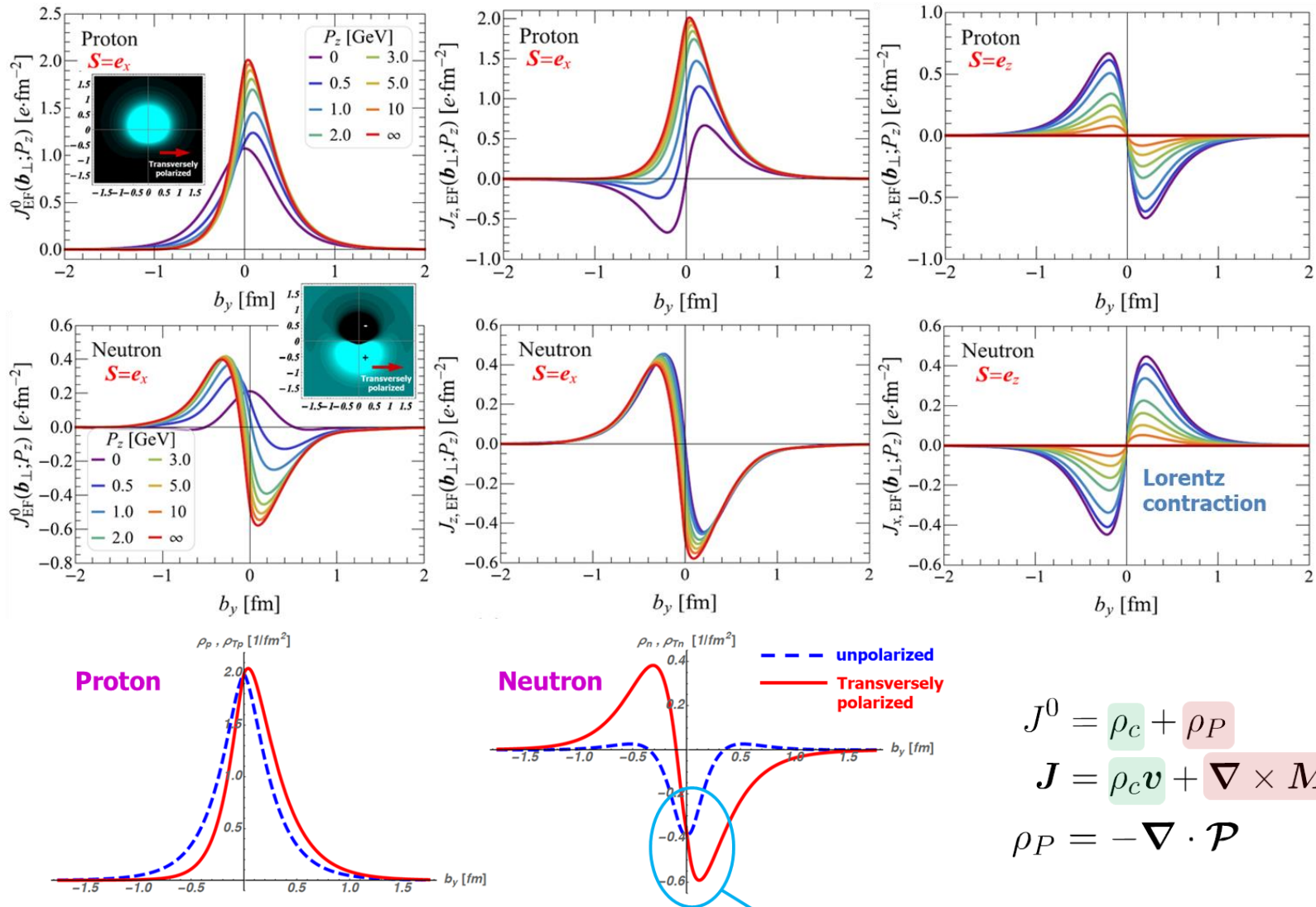
Relativistic EF interpretation

◆ EF distributions (2D): unpolarized case



Relativistic EF interpretation

◆ EF distributions (2D): polarized case



$$J^0 = \rho_c + \rho_P$$

$$\mathbf{J} = \rho_c \mathbf{v} + \nabla \times \mathbf{M} + \cancel{\partial_0 \mathcal{P}}$$

$$\rho_P = -\nabla \cdot \mathcal{P}$$

[Carlson, & Vanderhaeghen. PRL 100, 032004 (2008)]
 [YC, & Cédric Lorcé. PRD 106, 116024 (2022); PRD 107, 096003 (2023)]

Relativistic EF interpretation

◆ In the IMF limit ($P_z \rightarrow \infty$): why it becomes $F_1(Q^2)$ and $F_2(Q^2)$ for LF J^+ ?

$$J_{\text{LF}}^+(\mathbf{b}_\perp; P^+) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{\lambda'\lambda} F_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda'\lambda} \times i\boldsymbol{\Delta})_z}{2M} F_2(\Delta_\perp^2) \right]$$

$$\lim_{P_z \rightarrow \infty} \frac{\cos \theta G_E(Q^2) - \sin \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{1+\tau}} = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau} = F_1(Q^2)$$

$$\lim_{P_z \rightarrow \infty} \frac{\sin \theta G_E(Q^2) + \cos \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{\tau} \sqrt{1+\tau}} = \frac{G_M(Q^2) - G_E(Q^2)}{1+\tau} = F_2(Q^2)$$

unpolarized

Transversely polarized

◆ Besides, we found another analytic relations for LF J^- :

$$J_{\text{LF}}^-(\mathbf{b}_\perp; P^+) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P^-}{P^+} \left[\delta_{\lambda'\lambda} G_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda'\lambda} \times i\boldsymbol{\Delta})_z}{2M} G_2(\Delta_\perp^2) \right]$$

$$\lim_{P_z \rightarrow \infty} \frac{\cos \theta G_E(Q^2) + \sin \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{1+\tau}} = \frac{(1-\tau)F_1(Q^2) - 2\tau F_2(Q^2)}{1+\tau} \equiv G_1(Q^2)$$

$$\lim_{P_z \rightarrow \infty} \frac{\sin \theta G_E(Q^2) - \cos \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{\tau} \sqrt{1+\tau}} = -\frac{2F_1(Q^2) + (1-\tau)F_2(Q^2)}{1+\tau} \equiv G_2(Q^2)$$

unpolarized

Transversely polarized

new

→ $J_{\text{LF}}^+(\mathbf{b}_\perp; P^+) = J_{\text{EF}}^0(\mathbf{b}_\perp; \infty) = J_{\text{EF}}^z(\mathbf{b}_\perp; \infty) \sim$ generic relation

LF interpretation vs. EF interpretation

◆ LF interpretation:

$$J_{\text{LF}}^+(\mathbf{b}_\perp; P^+) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{\lambda'\lambda} F_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda'\lambda} \times i\boldsymbol{\Delta})_z}{2M} F_2(\Delta_\perp^2) \right],$$

$$J_{\text{LF}}^-(\mathbf{b}_\perp; P^+) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left(\frac{P^-}{P^+} \right) \left[\delta_{\lambda'\lambda} G_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda'\lambda} \times i\boldsymbol{\Delta})_z}{2M} G_2(\Delta_\perp^2) \right],$$

$$J_{\perp, \text{LF}}(\mathbf{b}_\perp; P^+) = e(\sigma_z)_{\lambda'\lambda} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{(\mathbf{e}_z \times i\boldsymbol{\Delta})_\perp}{2P^+} G_M(\Delta_\perp^2).$$

In total,
we need **5** EMFFs.

F_1, F_2
 G_1, G_2
 G_M

◆ EF interpretation:

$$J_{\text{EF}}^0(\mathbf{b}_\perp; P_z) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\ + e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

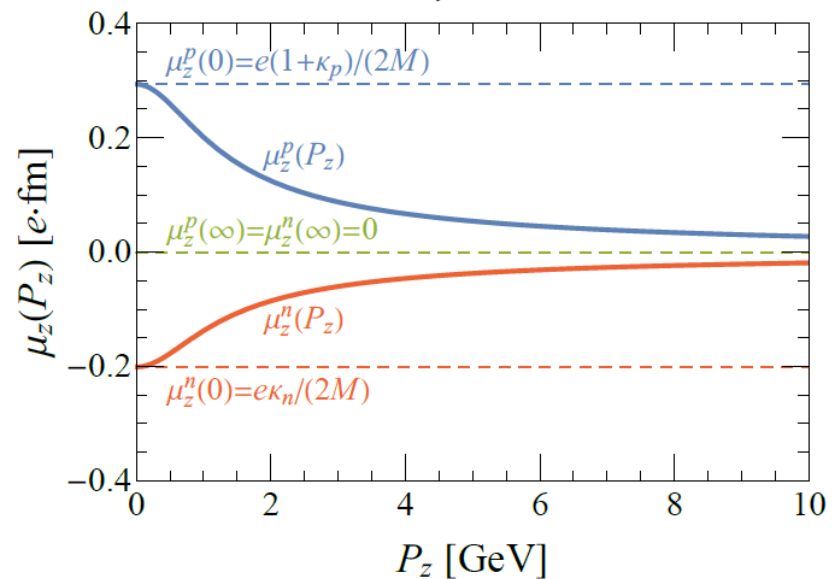
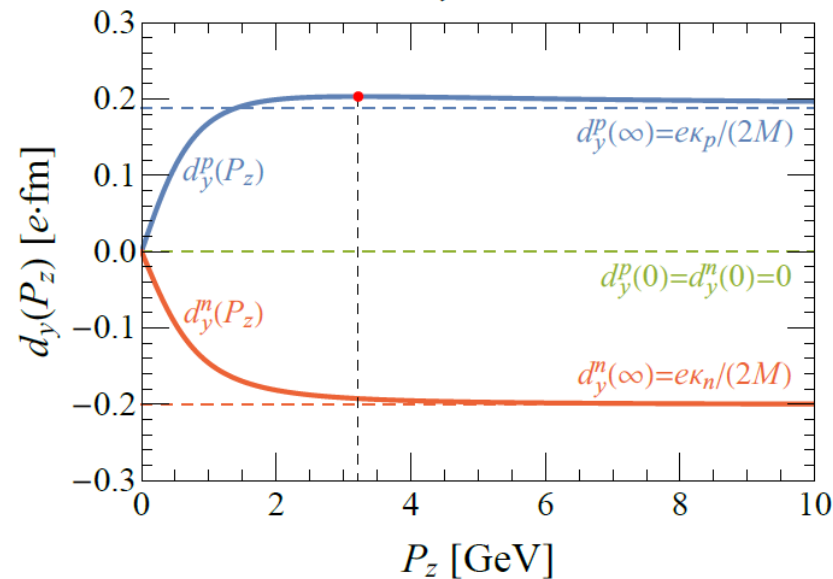
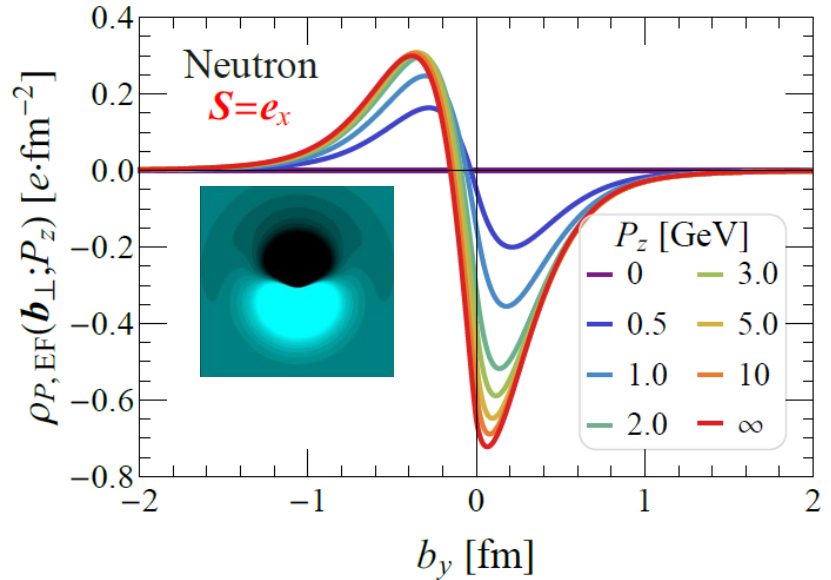
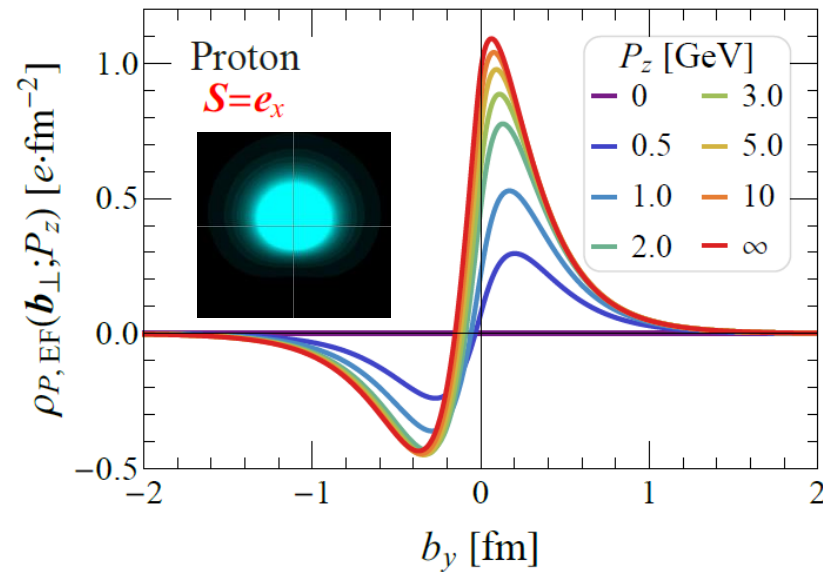
$$J_{z, \text{EF}}(\mathbf{b}_\perp; P_z) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\ + e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

$$J_{\perp, \text{EF}}(\mathbf{b}_\perp; P_z) = \delta_{s's} \frac{e\sigma_z}{2M} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} (\mathbf{e}_z \times i\boldsymbol{\Delta})_\perp \frac{P_B^0}{P^0} \frac{G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

In total,
we need **2** EMFFs
and **1** Wigner
rotation angle.

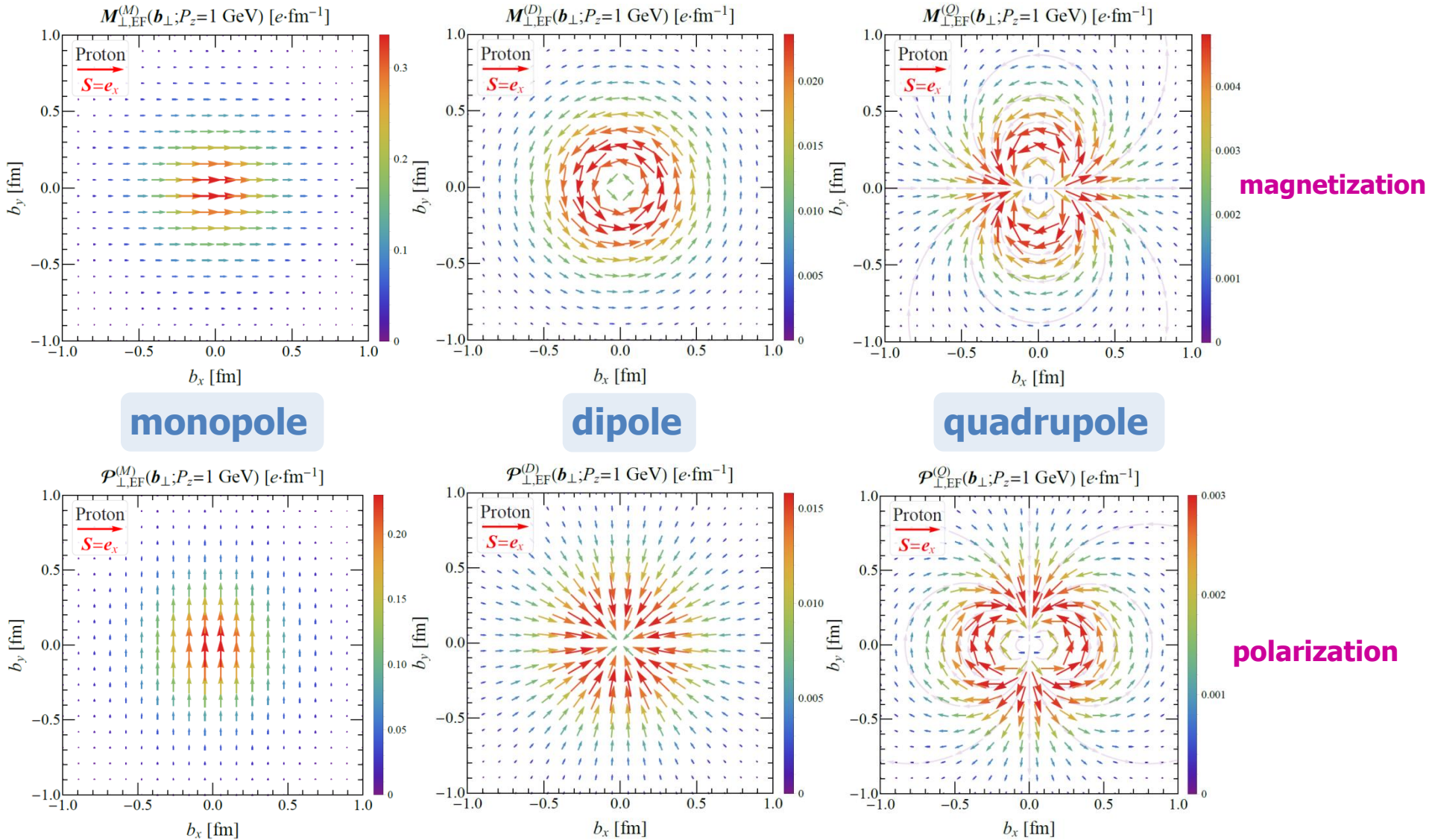
G_E, G_M, θ

Relativistic EF interpretation



Induced EDM: $d = v \times \mu \sim$ manifestation of relativistic effect!

Relativistic EF interpretation



$d = v \times \mu$ ← $\tilde{\mathcal{P}}^{(l)} = v \times \tilde{M}^{(l)} \sim \text{explicit manifestation of Ehrenfest's theorem!}$
classical level valid at quantum level (in momentum space) to all orders!