# Quantum Simulation of Generalized Parton Distributions in the Schwinger Model

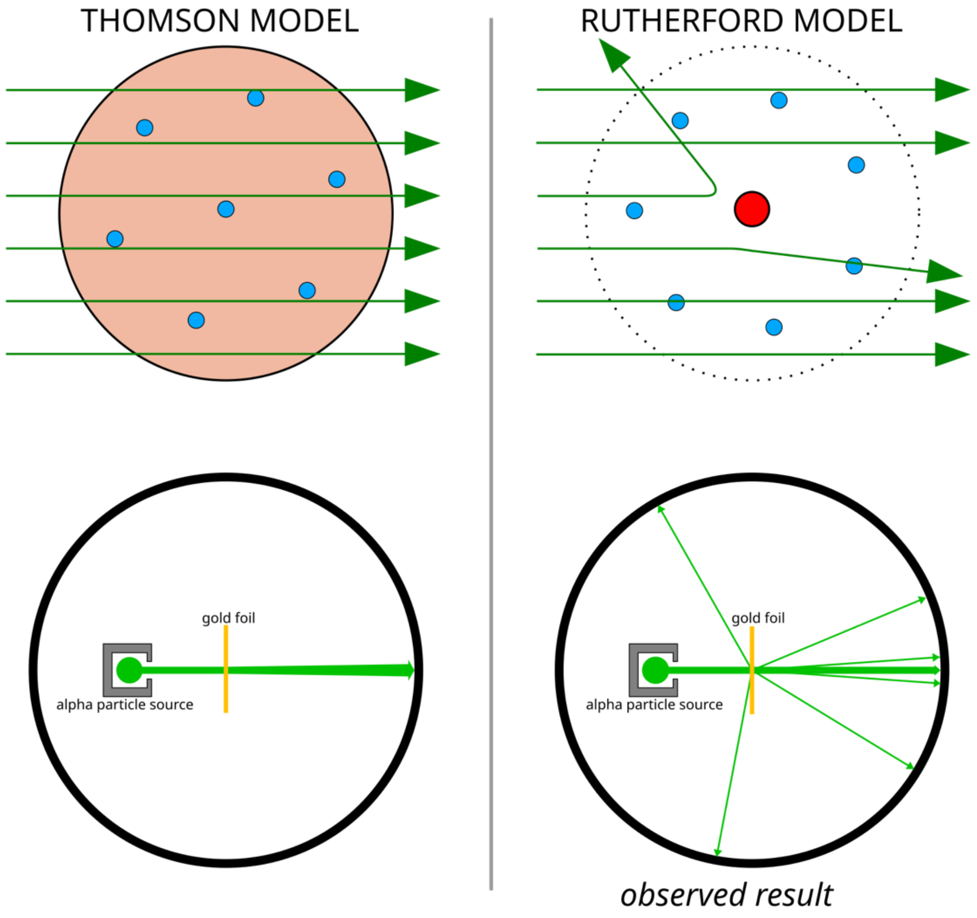
SPEAKER: TIANYIN LI （李天胤）

INSTITUTE: RIKEN, ITHEMS

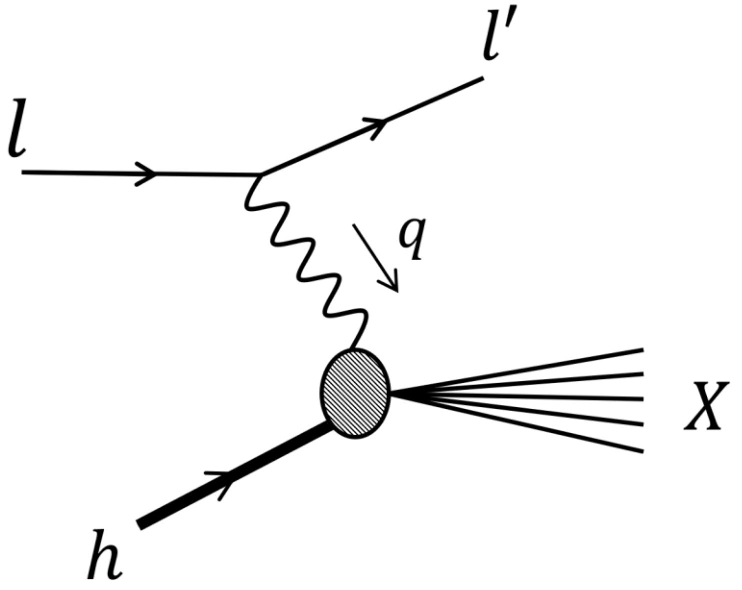
第一届强相互作用自旋物理会议

# From atom structure to hadron structure

Rutherford scattering: probe the structure of the atom




Deep inelastic scattering: probe the structure of the proton (hadron)

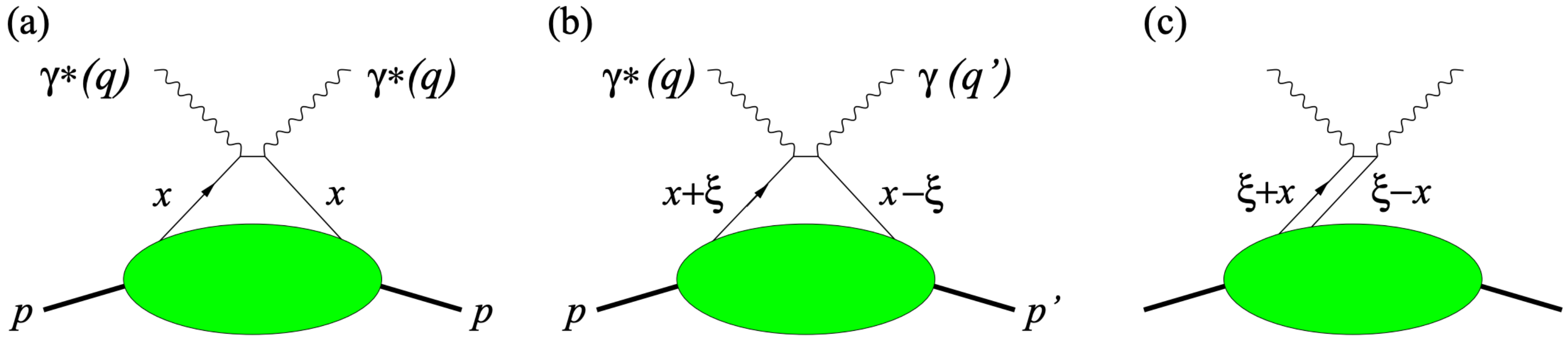



$$l(p) + h(k) \to l(p') + X$$

$$-q^2 \gg m_h^2$$

# Generalized Parton Distributions (GPDs): 3D structure of hadron

- DIS process can only probe the 1D structure of hadrons.
- Deeply virtual Compton scattering (DVCS) can probe the 3D structure of hadrons.




M. Diehl, arXiv:hep-ph/0307382

DVCS process: $\gamma^*(q) + h(p) \to \gamma(q') + h(p')$.

- QCD factorization:

$$\mathcal{H} = C \otimes H$$

# Operator definition of GPDs

We focus on the quark unpolarized GPDs:

$$H_{q/h}(x,\xi,t)=\int\frac{dz}{4\pi}e^{-ixz(n\cdot P)}e^{-iz(\Delta\cdot n)/2}\langle h(p')|\bar{\psi}(zn)W(zn,0)(n\cdot\gamma)\psi(0)|h(p)\rangle$$

- $n=(1,0,0,1)$: Light cone vector.
- $P=\frac{1}{2}(p'+p')$: Average momentum of hadron.
- $\Delta=p'-p$: Momentum transfer.
- $\xi=-\frac{n\cdot\Delta}{2n\cdot P}$: Longitudinal Momentum transfer.
- $H_{q/h}$ is related to the spin decomposition of hadron. (Ji sum rule)
- $H_{q/h}$ is time dependance non-perturbative quantities.

**We can use quantum computing method to simulate $H_{q/h}$!**

# Procedures of studying GPDs on quantum computers

The light-cone correlation function $\widetilde{H}_{q/h}$ need to be simulated on quantum computers:

$$\widetilde{H}_{q/h} = \langle h(p') | \bar{\psi}(zn) W(zn, 0)(n \cdot \gamma) \psi(0) | h(p) \rangle$$

- Discretization of the fermion field and gauge field.
- Discretization of the Wilson line $W(zn, 0)$.
- Preparation of the hadron state $|h(p)\rangle$.
- Emulation of the light cone correlation function $\widetilde{H}_{q/h}$.

# Proof of concept study: Schwinger model

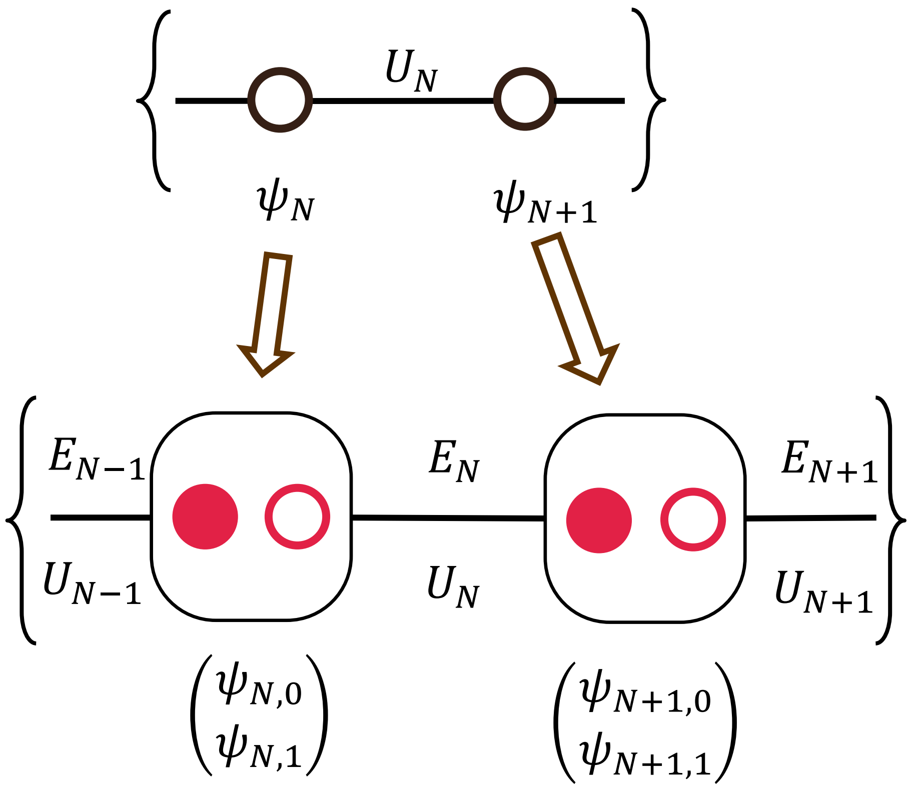
Hamiltonian of Schwinger model on lattice:

$$H = -\sum_N \frac{i}{2}\bar{\psi}_N\gamma^1[U_N(r+i\gamma^1)\psi_{N+1} - h.c.]$$

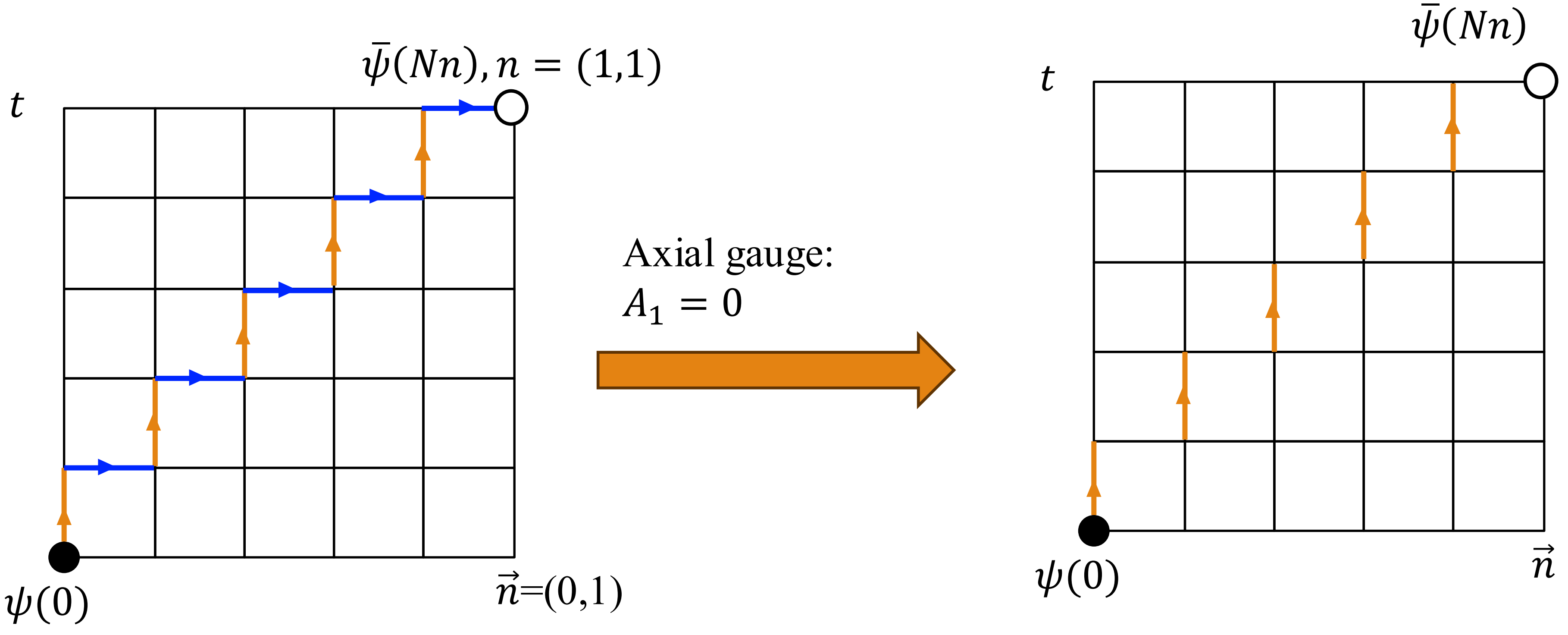
$$+(m+r)\sum_N \bar{\psi}_N\psi_N + \frac{g^2}{2}\sum_N E_N^2 = H_{kin} + H_m + H_E$$

- We use **Wilson fermion** to discretize the fermion field because it preserves both **charge conjugation (C) and parity (P) symmetry**.
- We fix the axial gauge $A_1^a = 0$ to eliminate the gauge field:
  - $U_N = \exp(igA_1^a(N)t_a) = I$
  - Gauss's law $E_N - E_{N-1} = \bar{\psi}_N\gamma^0\psi_N$
  - Map to qubits by Jordan-Wigner transformation.

- Example: two space points



# Wilson line of Schwinger model on lattice



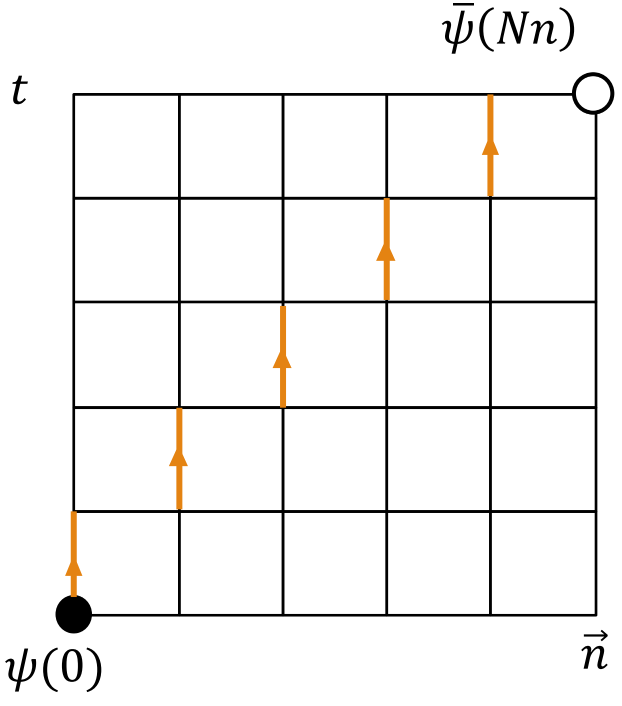

$= \exp[-igaA_0(M, M\vec{n})]$

$= \exp[igaA_1(M+1, M\vec{n})]$

$= \exp[-igaA_0(N, N)]$

$$W(Nn, 0) = \prod_{N'=0}^{N-1} \left[e^{-iH} \exp(-igA_0(N'))\right]$$

# Wilson line of Schwinger model on lattice




$= \exp[-igaA_0(N,N)]$

$$\boldsymbol{W(Nn,0) = \prod_{N'=0}^{z-1}\left[e^{-iH}\exp(-igA_0(N'))\right]}$$

- How to determined $A_0(N)$?

Gauss law: $E_N - E_{N-1} = Q_N$

Gauge field$A_0$: $A_0(N+1) - A_0(N) = E_N$

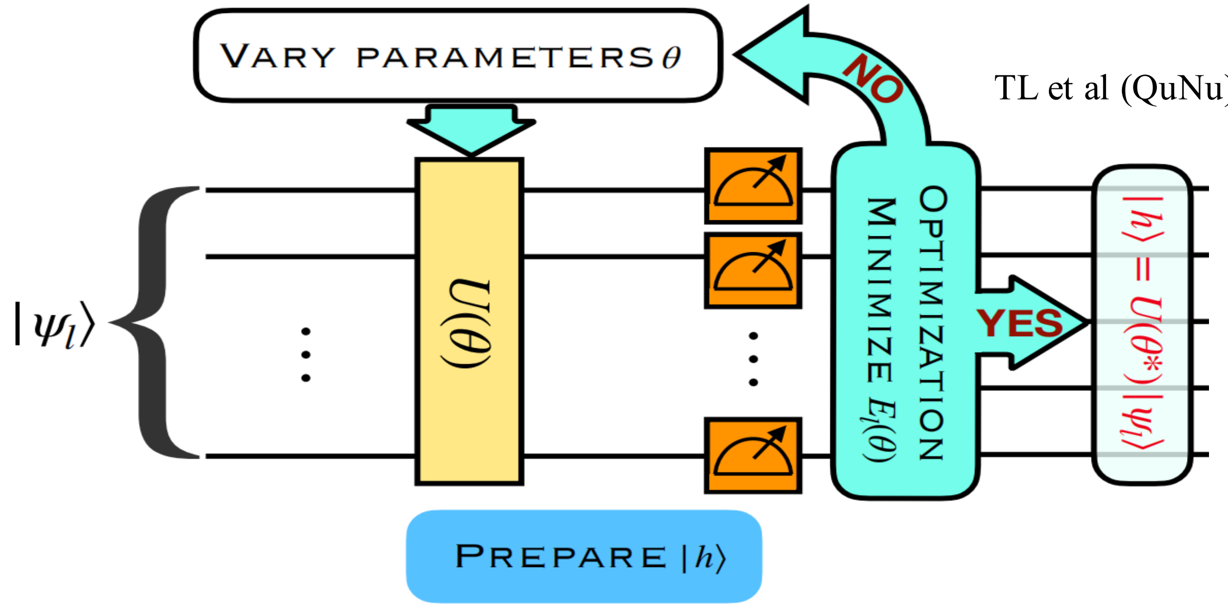
Differential equation for $A_0$:

$$\boldsymbol{\partial_1^2 A_0(N) = Q_N \Rightarrow A_0(N) = \sum_{N'}[V(N-N') - V(0)]Q_{N'}}$$

Period boundary condition:

$$V(N-N') = \frac{1}{\mathcal{N}}\sum_{k=1}^{\mathcal{N}}\frac{e^{ip_k(N-N')}}{4\sin^2\frac{p_k}{2}}$$

# Preparation of a hadron state $|h(p)\rangle$ by VQE

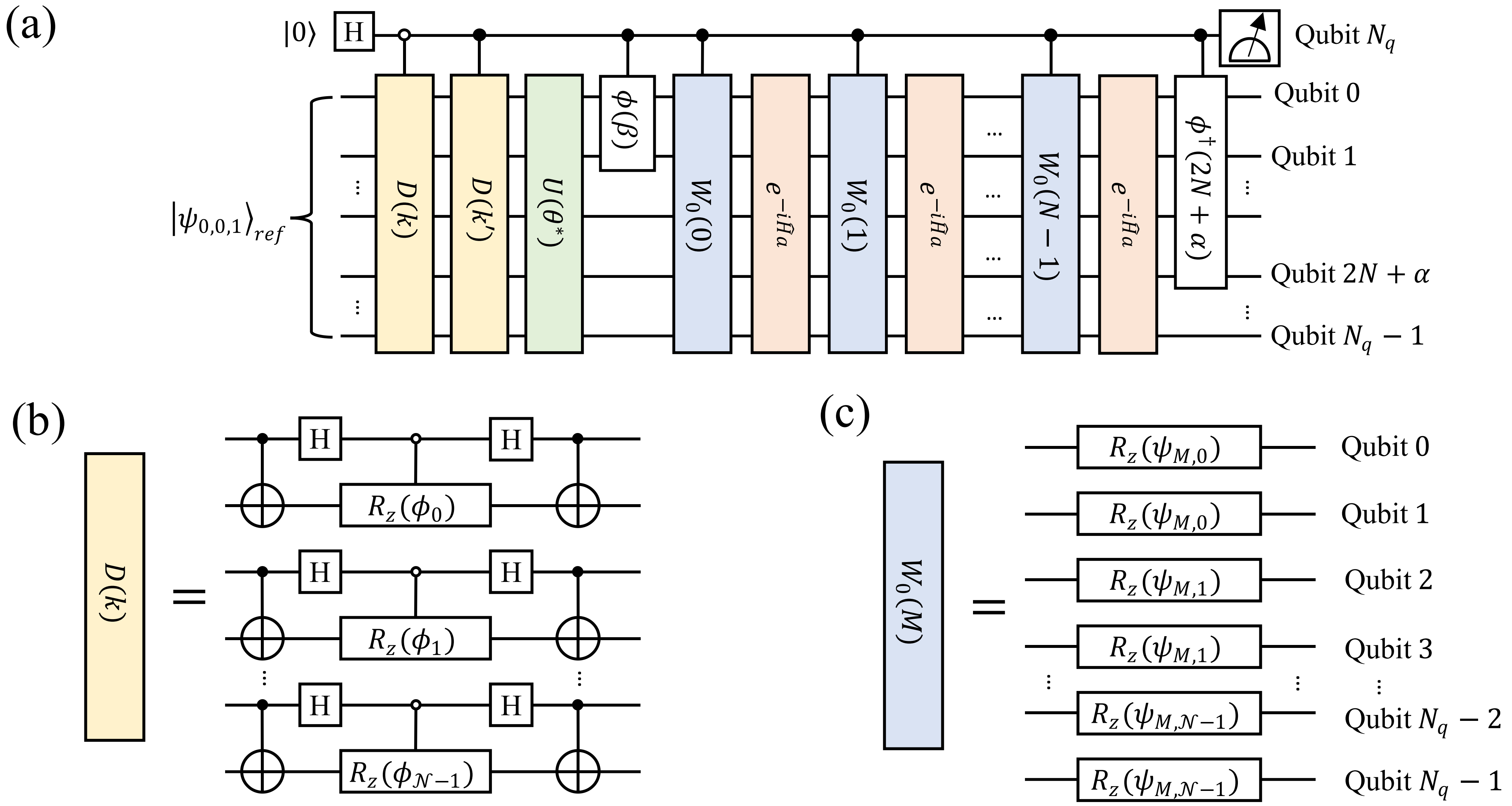



TL et al (QuNu), PRD (letter, 2022)

- $|\psi_{Q,p,i}\rangle_{ref}$ : initial reference states, have the same quantum number with hadron state $|h\rangle$. (Dicke state)
- $U(\theta)$: VQE ansatz, preserves the quantum number.
- $E_{Q,p}(\theta)$: loss function, need to be minimized.
- Hadron state: $|h(p)\rangle = U(\theta^*)|\psi_{Q,p,i}\rangle_{ref}$.

# Quantum circuit for simulating gauge invariance light cone correlation functions

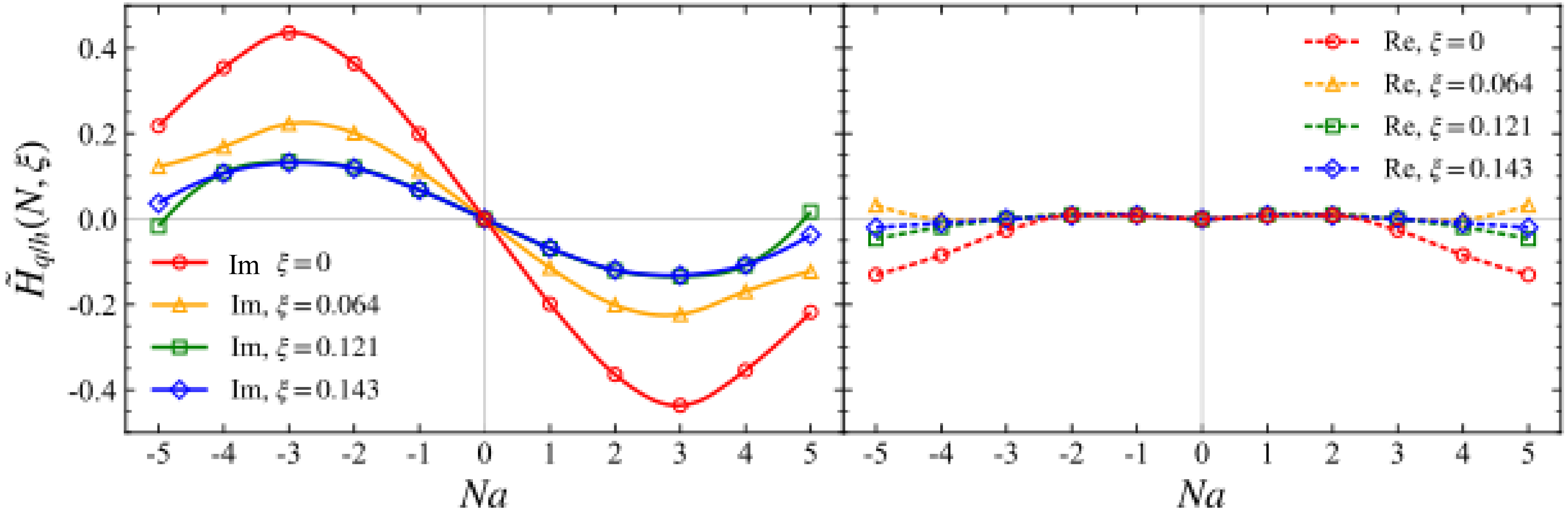
TL and H. Xing, arXiv: 2606.22602



# Results of light cone correlation functions

Gauge invariance light-cone two-point function

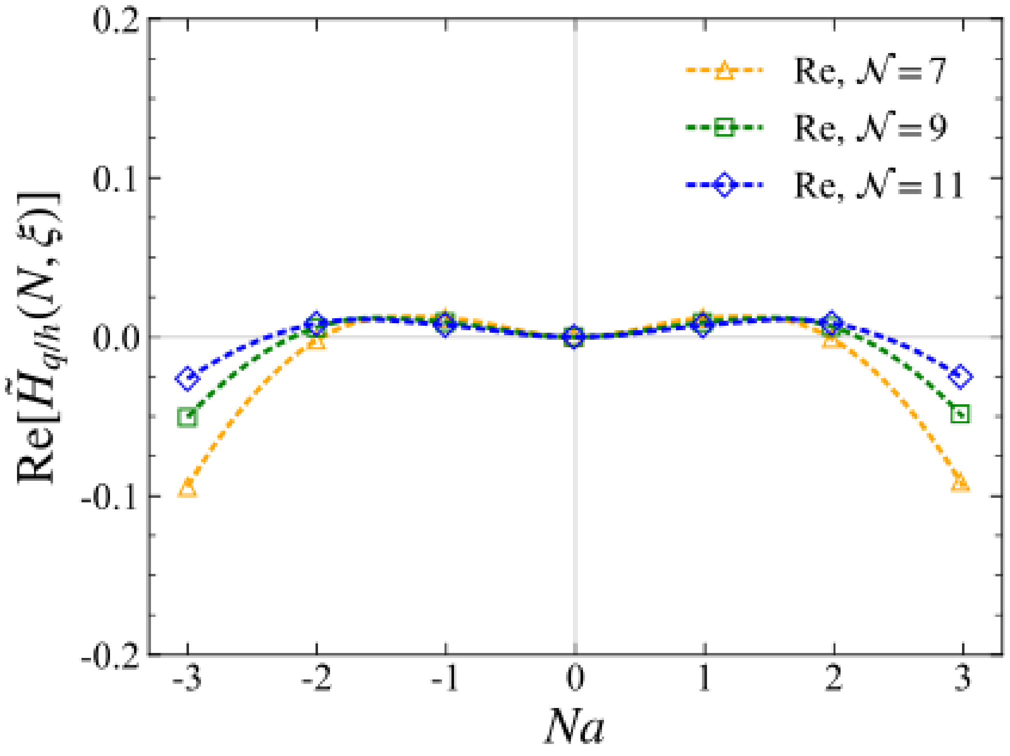
TL and H. Xing, arXiv: 2606.22602

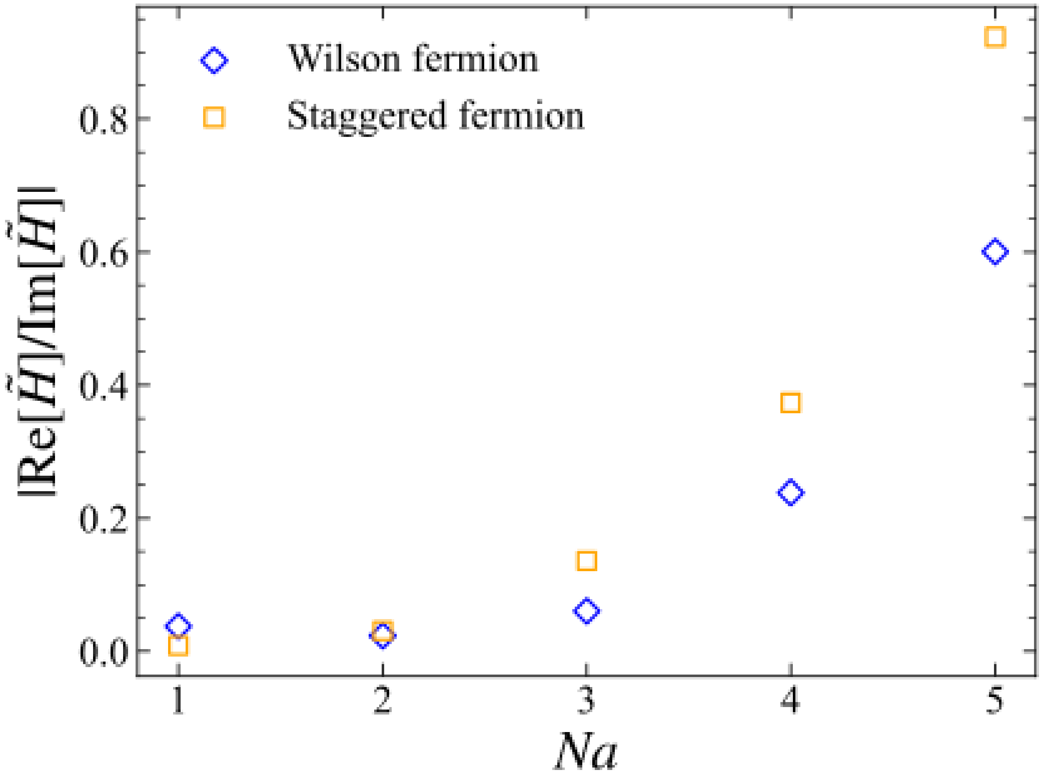



- $\xi = 0$ corresponds to PDFs $\tilde{f}_{q/h}(x)$.
- $\xi \neq 0$ corresponds to the GPDs.
- We can see the bound state behavior of meson states of Schwinger model.
- Still non-zero real part after using Wilson fermion.

# Discussions of Non-zero real parts of $\tilde{H}_{q/h}$

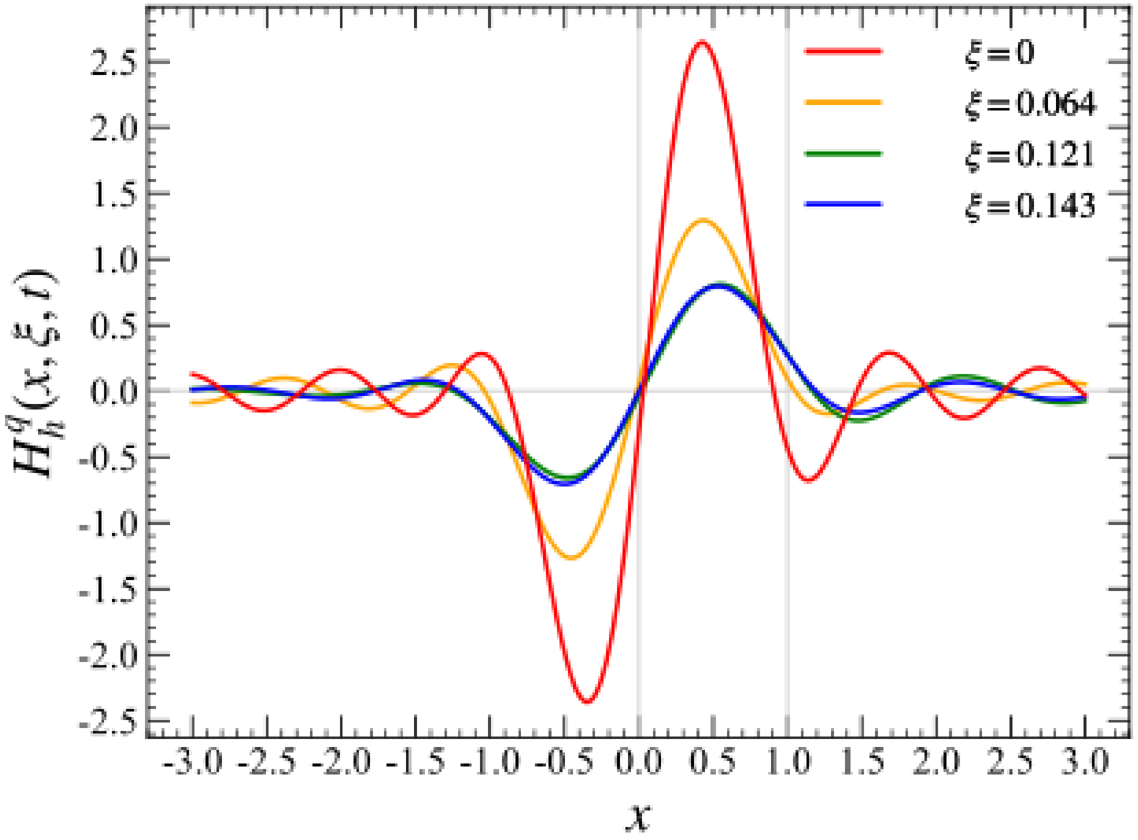
TL and H. Xing, arXiv: 2606.22602




- ➢ Wilson fermion: non-zero real part of light cone correlation function comes from lattice effect.
- ➢ Staggered fermion: non-zero real part comes from both lattice effect and violation of C symmetry.

# Results of PDFs and GPDs




TL and H. Xing, arXiv: 2606.22602

- Momentum transfer $\xi$ and $t$ are not independent in (1+1)-D.
- Both PDFs and GPDs are odd functions due to C symmetry.
- Meson electric form factor:

$$F_M(t) = \int dx\, H_h^q(x,\xi,t) = 0.$$

# Summary

- We present a quantum algorithm for simulating GPDs.
- To preserve the C symmetry, we use the Wilson fermion instead of the staggered fermion.
- We give the results for GPDs of the Schwinger model up to 11 lattice site and 22 qubits.