Lattice-QCD validation of hadron mass and trace-anomaly decomposition sum rules

Hadron mass decomposition in a common $\overline{\text{MS}}$ scheme for η_c and J/ψ

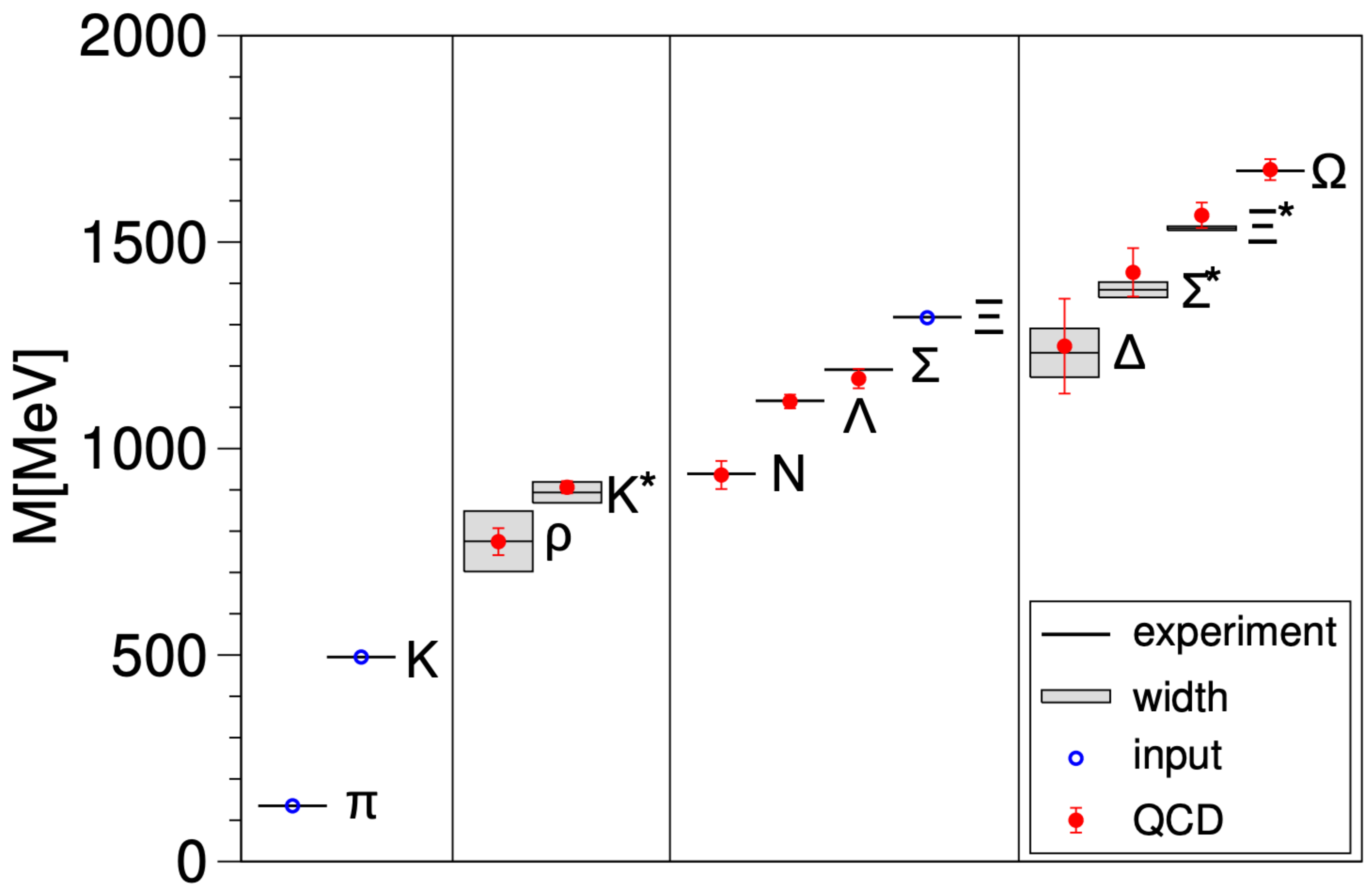
Heng-Tong Ding (丁亨通)
Central China Normal University (华中师大)

Dennis Bollweg, HTD, 高翔, 罗冉, Swagato Mukherjee, arXiv: 2601.13070 ,PRL in press

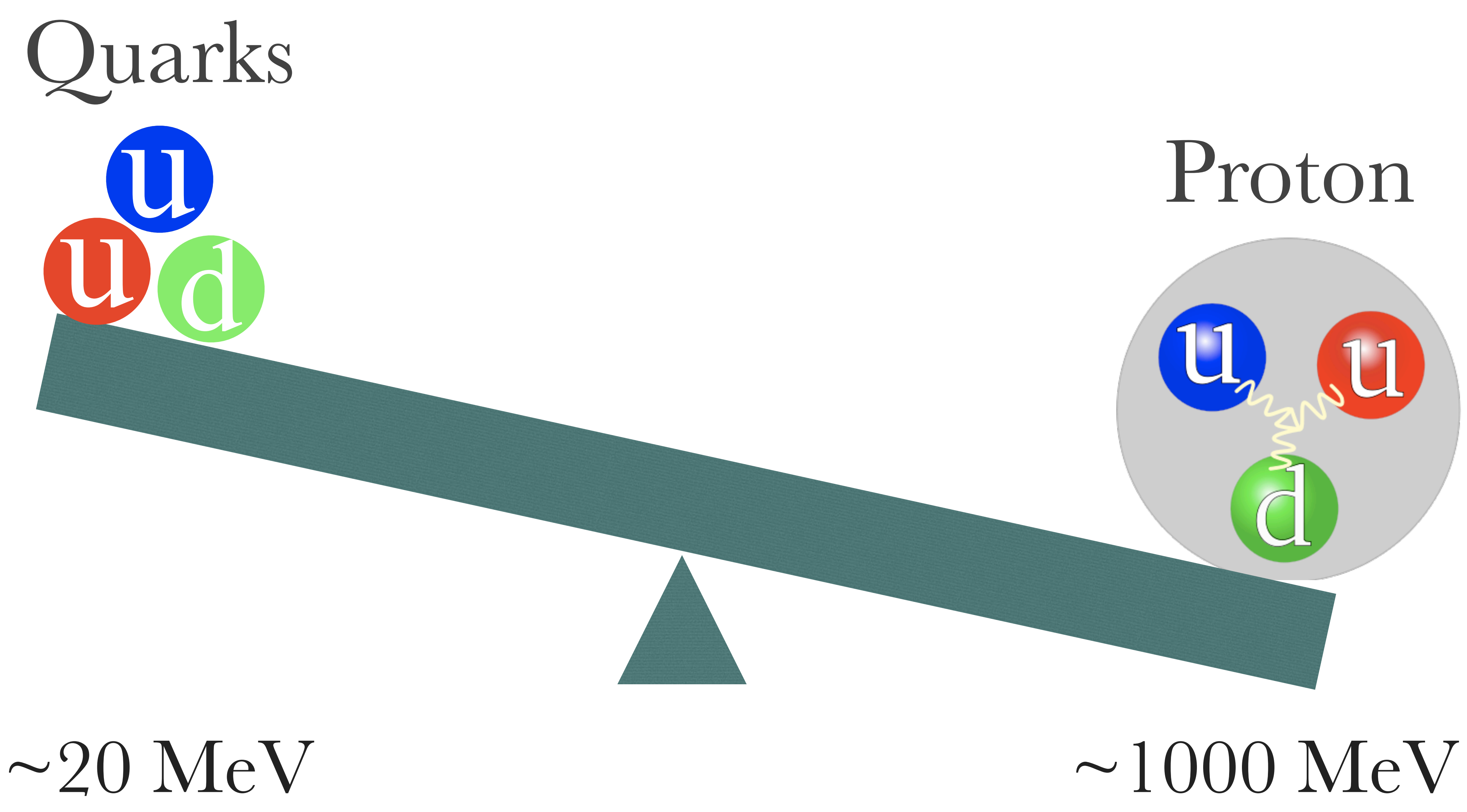
第一届强相互作用自旋物理会议

July 27-30, 2026@青岛

What generates the hadron mass?



Dürr et al., Science 322:1224-1227,2008



Energy-Momentum Tensor (EMT): from QCD operator to hadron-mass sum rules

$$\langle H(p) \,|\, T^{\mu\nu} \,|\, H(p) \rangle = 2p^\mu p^\nu$$

$$T^{\mu\nu} = \underbrace{\left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^\alpha_{\ \alpha} \right)}_{\text{Traceless}} + \underbrace{\frac{1}{4} g^{\mu\nu} T^\alpha_{\ \alpha}}_{\text{Trace}}$$

For QCD:

$$T^{\mu\nu}_{\rm QCD} = T^{\mu\nu}_q + T^{\mu\nu}_g = \underbrace{\underbrace{M_E}_{\text{quark kinetic energy}} + \underbrace{M_m}_{\text{explicit quark mass}}}_{\text{Quark part}} + \underbrace{\underbrace{M_g}_{\text{gluon energy}} + \underbrace{M_a}_{\text{anomaly}}}_{\text{Gluon part}}$$

Energy-Momentum Tensor (EMT): from QCD operator to hadron-mass sum rules

$$\langle H(p) | T^{\mu\nu} | H(p) \rangle = 2p^\mu p^\nu$$

$$T^{\mu\nu} = \underbrace{\left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^\alpha_{\ \alpha} \right)}_{\text{Traceless}} + \underbrace{\frac{1}{4} g^{\mu\nu} T^\alpha_{\ \alpha}}_{\text{Trace}}$$

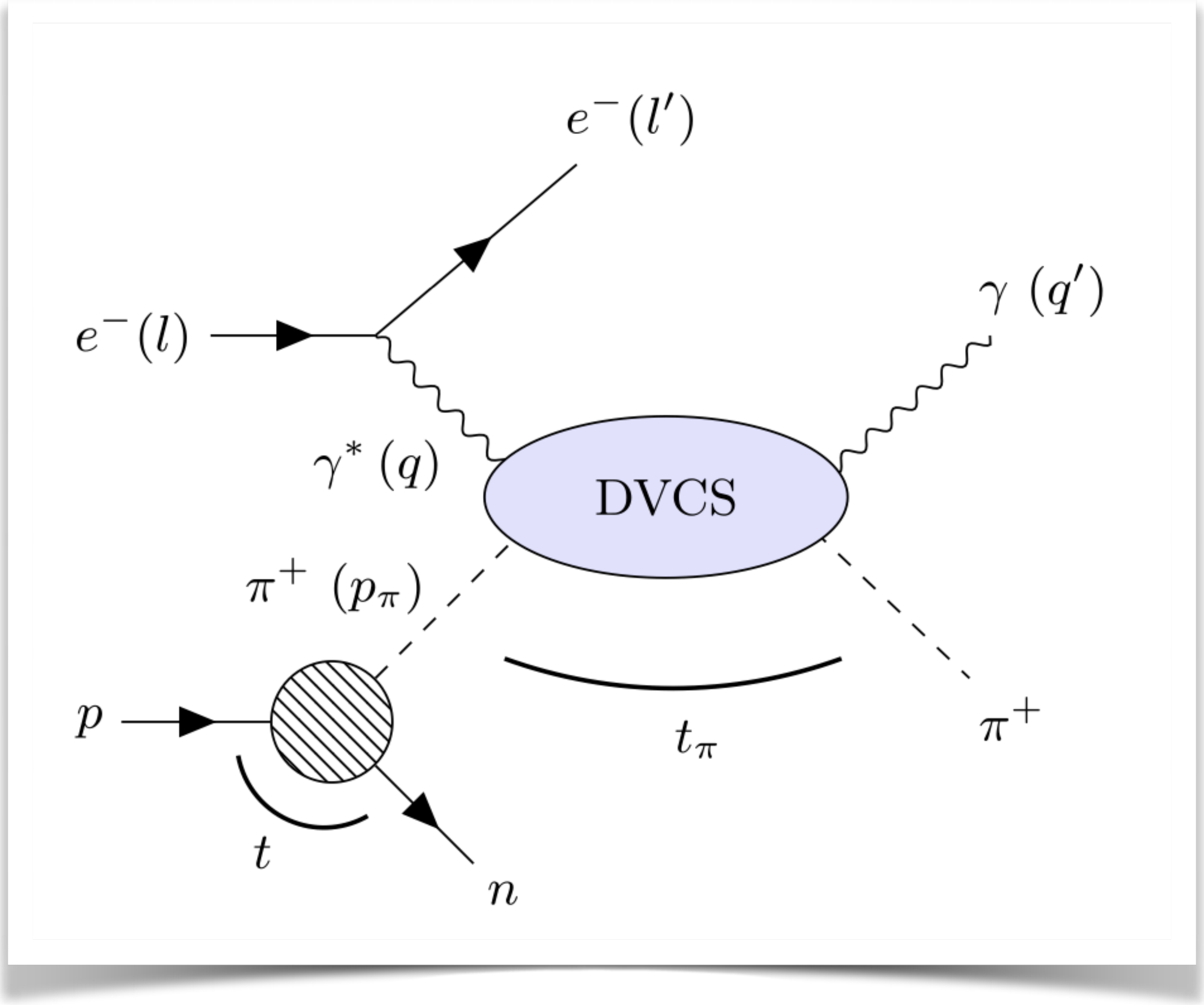
same EMT information$\longrightarrow$ different organizations $\longrightarrow$ different hadron-mass sum rules

Ji | HRT | Lorcé | MPR

Ji, PRL 74 (1995) 1071 ;Hatta, Rajan, & Tanaka, JHEP 12 (2018),008 ; Lorcé, EPJC 78 (2018)120; Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

Current status: partial EMT access from experiment and lattice QCD

Deeply virtual exclusive processes
$\Rightarrow$ partial GFF / traceless EMT access

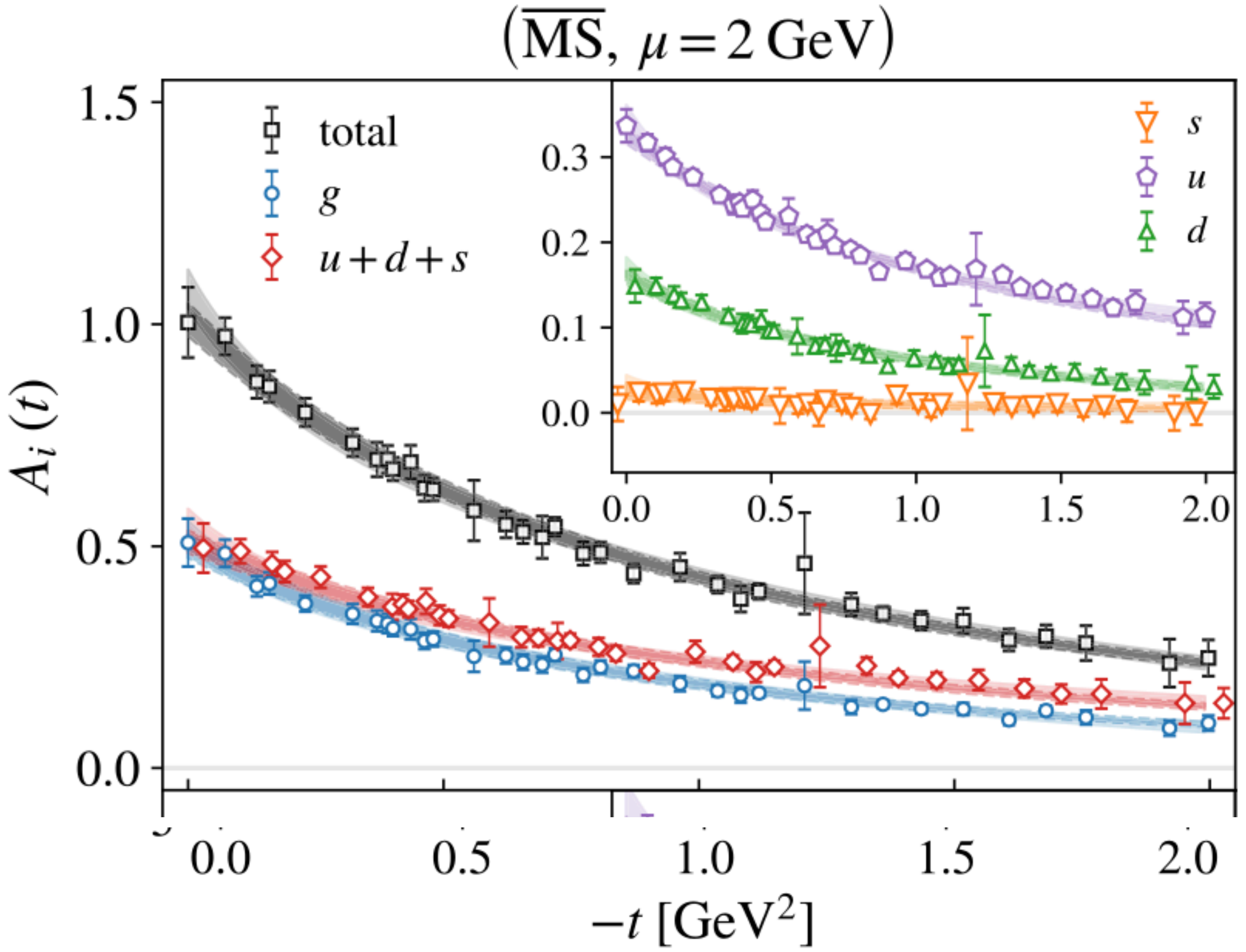


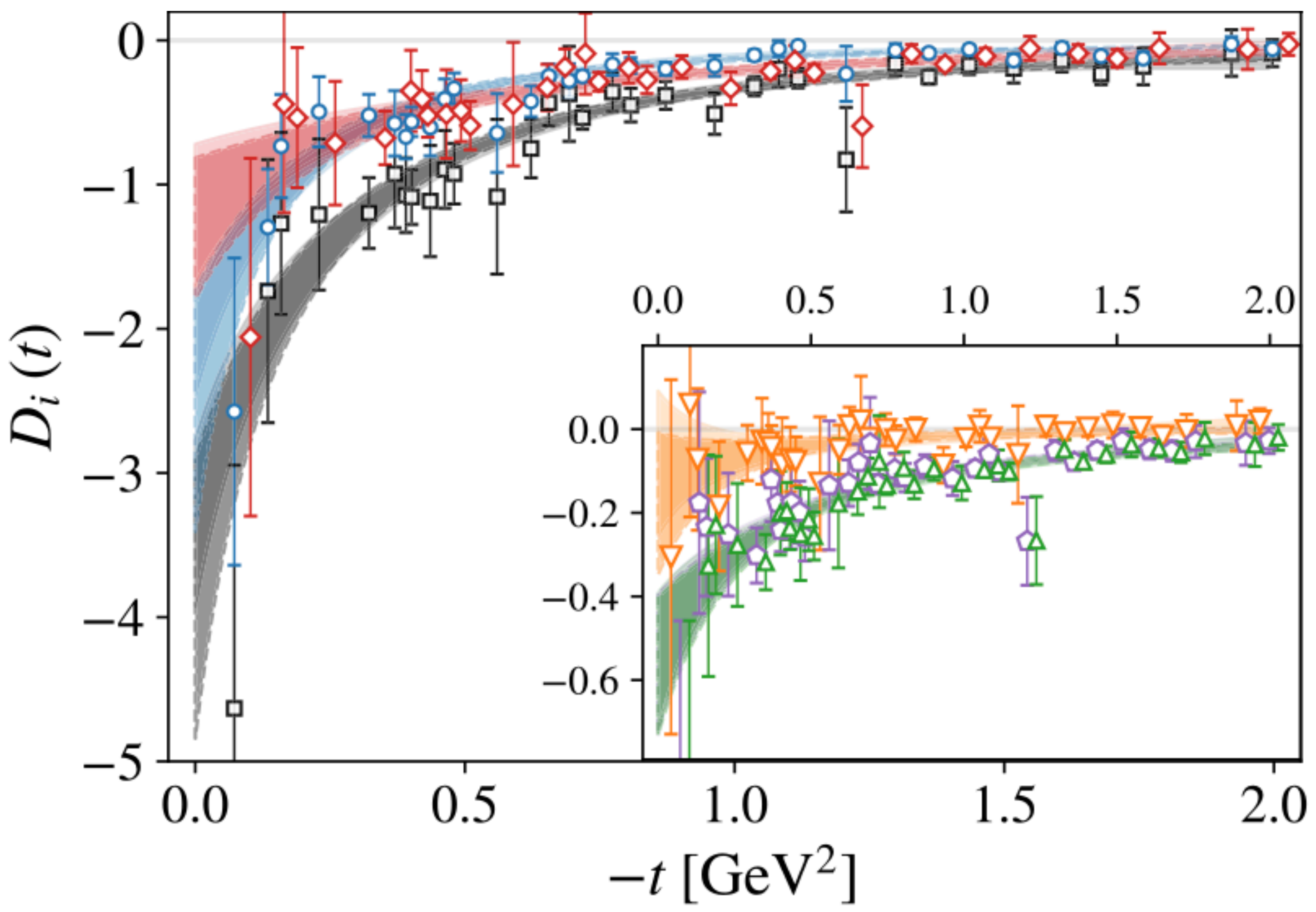
See EXP extracted pressure distribution in e.g.
Burkert, Elouadrhiri & Girod, Nature 557 (2018)396

Recent lattice QCD:
GFFs and spatial structure are accessible but the trace sector remained difficult

Traceless — Trace

$$\langle p'|T^{\mu\nu}|p\rangle = \bar{u}(p')\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)\frac{iP^{(\mu}\sigma^{\nu)\rho}\Delta_\rho}{2m} + D(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m} + \bar{C}(t)mg^{\mu\nu}\right]u(p).$$





Hackett et al., PRL 132 (2024) 251904

Current status: partial EMT access from experiment and lattice QCD

Earlier lattice QCD

Hadron mass decompositions relied on partial EMT input or indirect closure

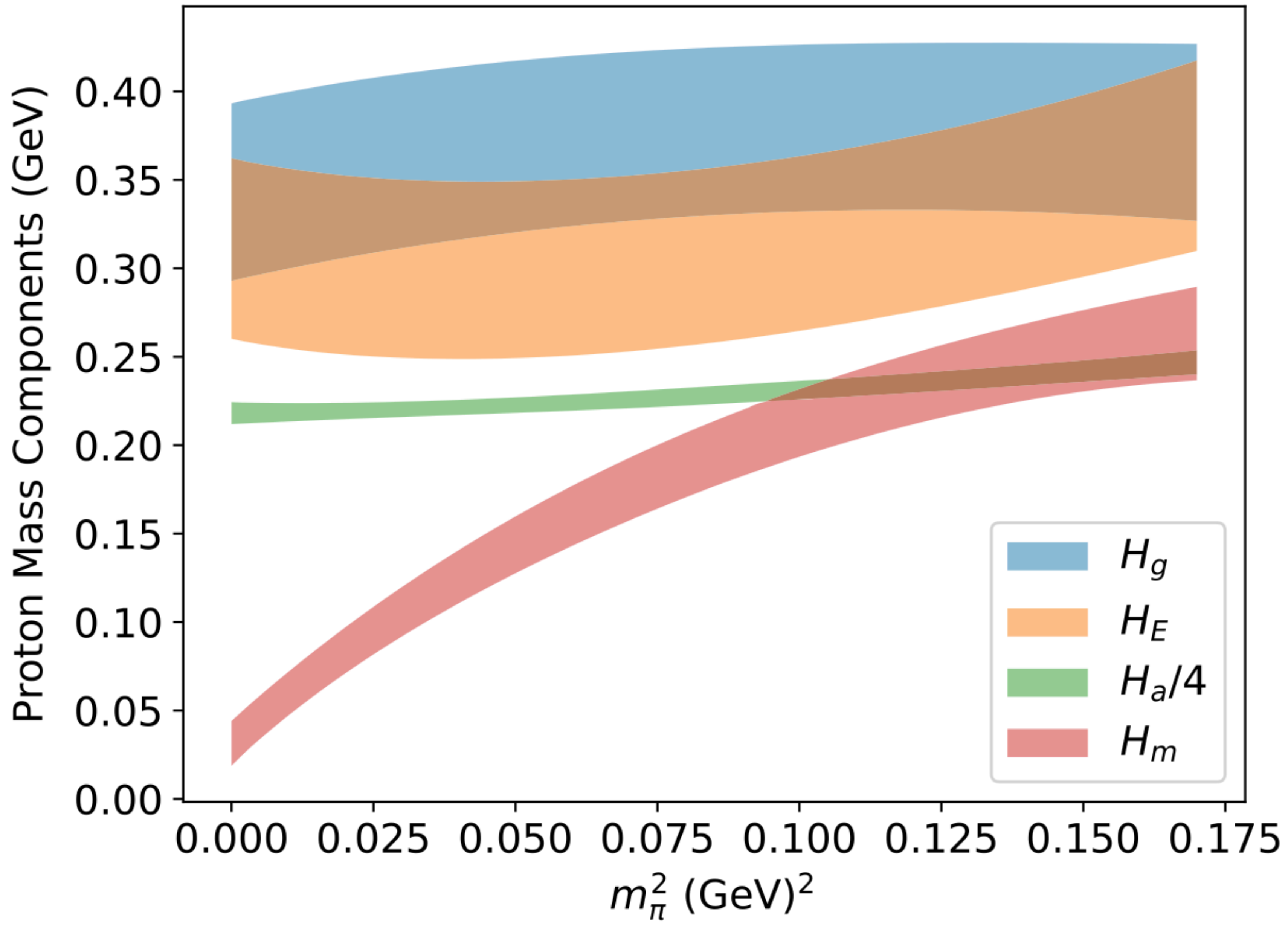
$$T^{\mu}_{\mu} = \underbrace{\sum_f m_f \bar{\psi}_f \psi_f}_{\text{explicit quark-mass term}} + \underbrace{\left[\frac{\beta(g)}{2g} F^2 + \gamma_m \sum_f m_f \bar{\psi}_f \psi_f\right]}_{\text{quantum trace anomaly}}.$$

Trace

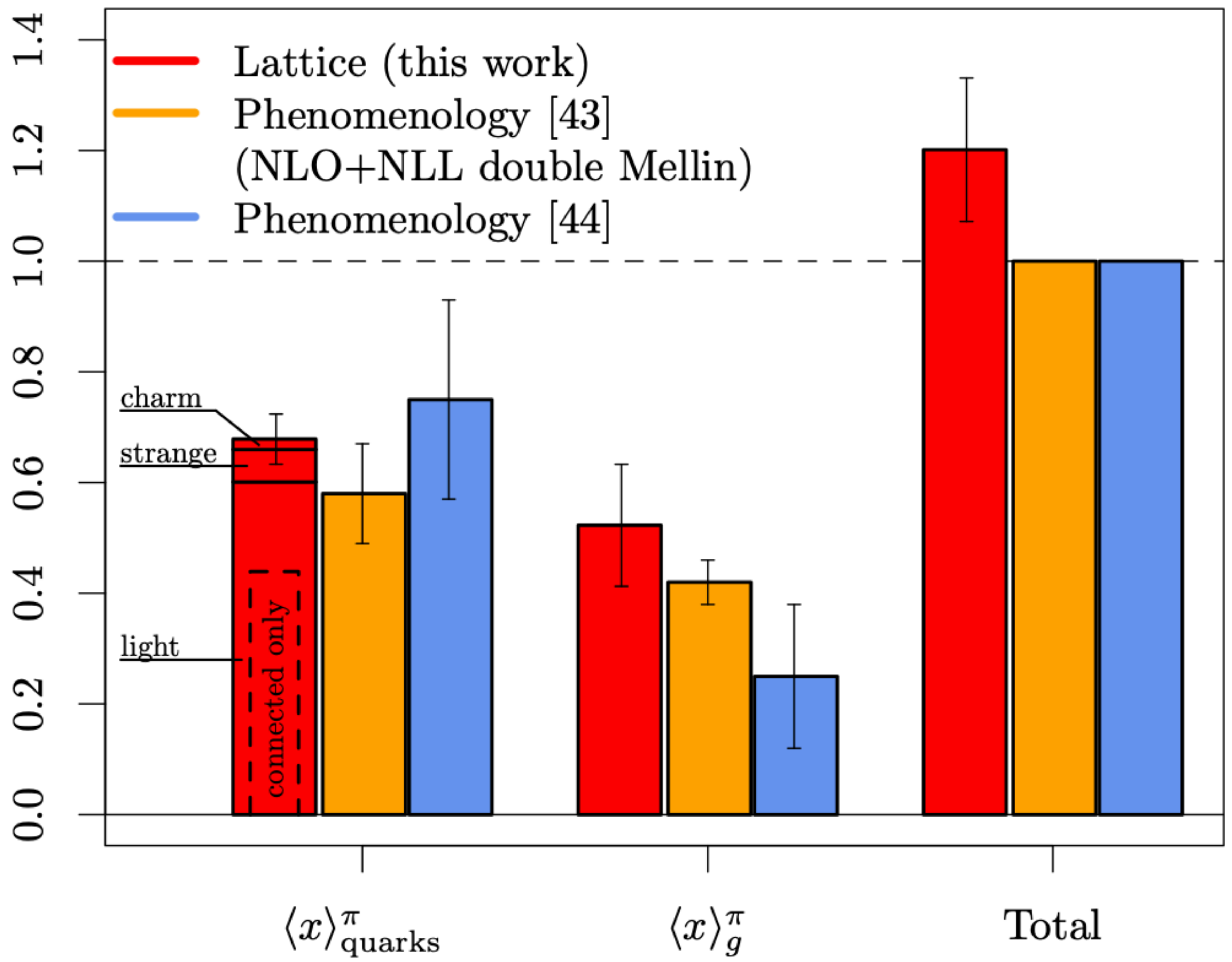
$$\langle \pi(\mathbf{p}) | \bar{T}^{X}_{\mu\nu} | \pi(\mathbf{p}) \rangle = 2\langle x \rangle^{X} \left(p_\mu p_\nu - \delta_{\mu\nu} \frac{p^2}{4} \right)$$

Traceless

EMT conservation and the traceless twist-2 operators



Yang, et al.,[χQCD], PRL 121 (2018) 212001



Alexandrou, et al.,[ETMC], PRL 127 (2021) 25, 252001

Current status: partial EMT access from experiment and lattice QCD

Earlier lattice QCD

Hadron mass decompositions relied on partial EMT input or indirect closure

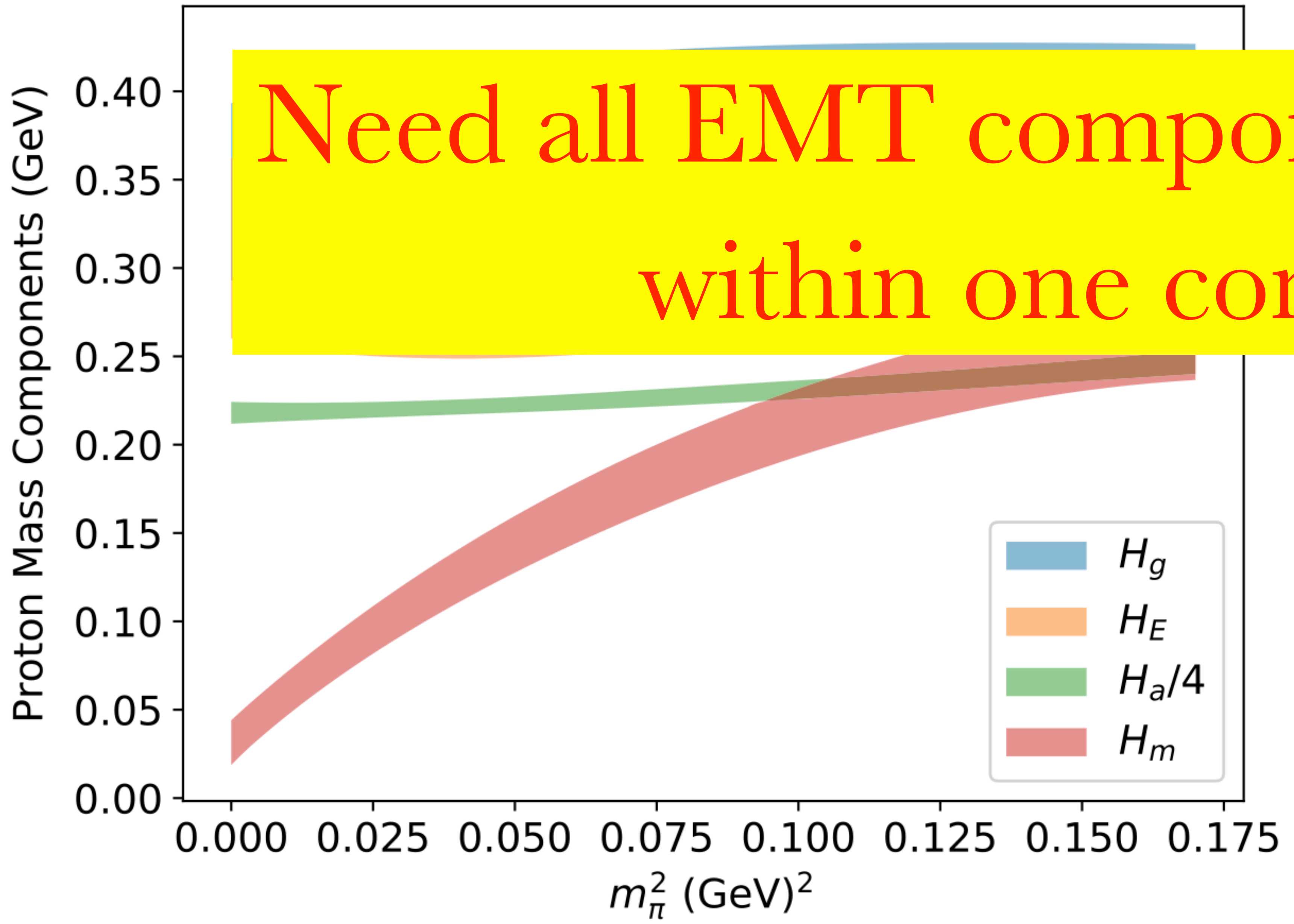
$$T^{\mu}_{\mu} = \underbrace{\sum_f m_f \bar{\psi}_f \psi_f}_{\text{explicit quark-mass term}} + \underbrace{\left[\frac{\beta(g)}{2g}F^2 + \gamma_m \sum_f m_f \bar{\psi}_f \psi_f\right]}_{\text{quantum trace anomaly}}.$$

Trace

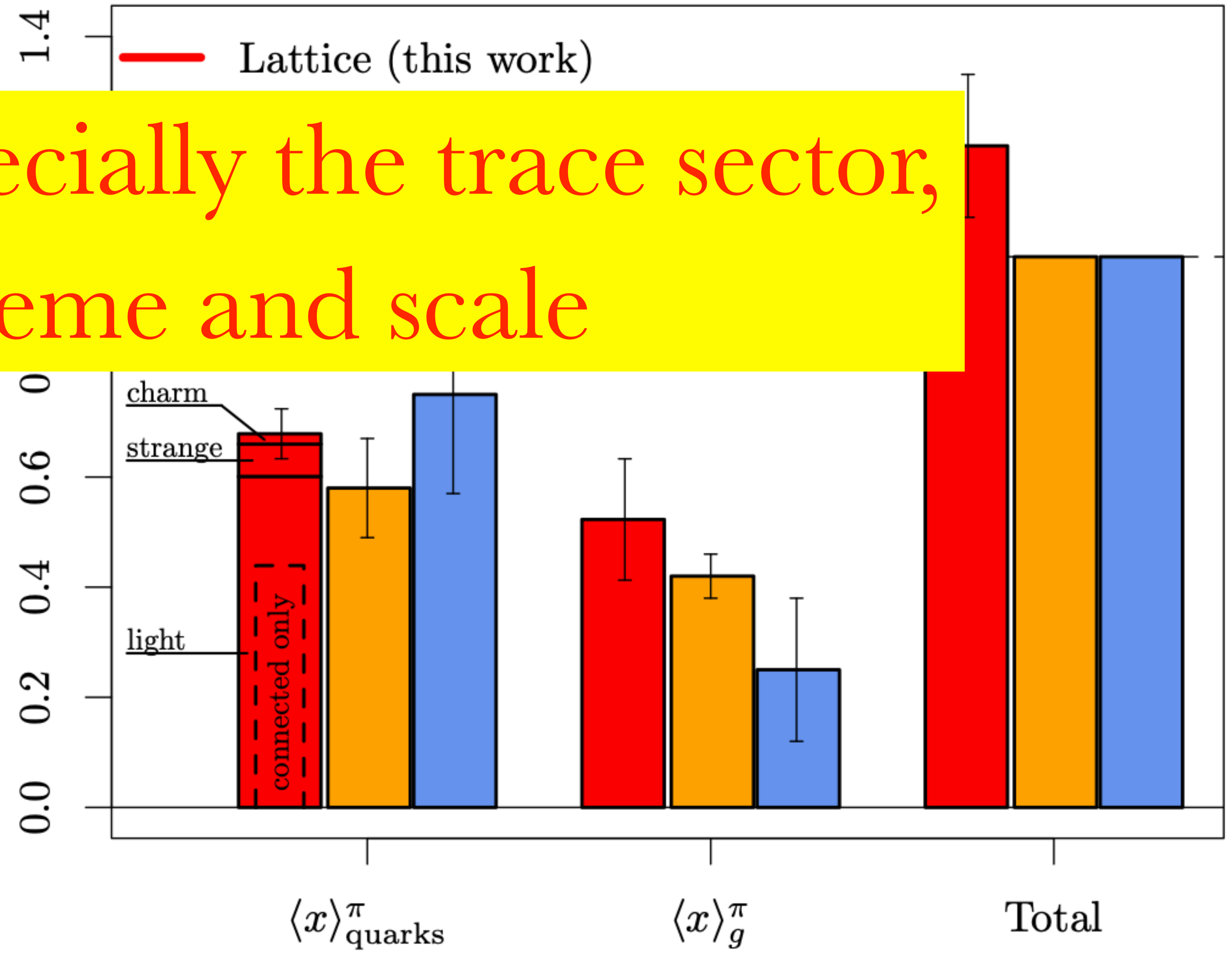
$$\langle \pi(\mathbf{p}) | \bar{T}^{X}_{\mu\nu} | \pi(\mathbf{p}) \rangle = 2\langle x \rangle^X \left(p_\mu p_\nu - \delta_{\mu\nu}\frac{p^2}{4} \right)$$

Traceless

EMT conservation and the traceless twist-2 operators



Yang, et al.,[χQCD], PRL 121 (2018) 212001



Alexandrou, et al.,[ETMC], PRL 127 (2021) 25, 252001

Need all EMT components, especially the trace sector, within one common scheme and scale

What was difficult before

$$T_{\rho\sigma}(x) = \underbrace{\mathcal{O}_{1,\rho\sigma}(x) - \frac{1}{4}\mathcal{O}_{2,\rho\sigma}(x)}_{\text{Gluon part: } T_{g,\rho\sigma}} + \underbrace{\frac{1}{4}\mathcal{O}_{3,\rho\sigma}(x) - \frac{1}{2}\mathcal{O}_{4,\rho\sigma}(x) - \mathcal{O}_{5,\rho\sigma}(x)}_{\text{Quark part: } T_{q,\rho\sigma}}$$

$$\mathcal{O}_{1,\rho\sigma} = \frac{2}{g_0^2}\mathrm{Tr}^c\left[F_{\rho\omega}F_{\sigma\omega}\right], \quad \mathcal{O}_{2,\rho\sigma} = \frac{2}{g_0^2}\delta_{\rho\sigma}\mathrm{Tr}^c\left[F_{\omega\xi}F_{\omega\xi}\right]$$

$$\mathcal{O}_{3,\rho\sigma} = \sum_f \mathcal{O}_{3,\rho\sigma,f} = \sum_f \bar{\psi}_f\left(\gamma_\rho \overleftrightarrow{D}_\sigma + \gamma_\sigma \overleftrightarrow{D}_\rho\right)\psi_f$$

$$\mathcal{O}_{4,\rho\sigma} = \delta_{\rho\sigma}\sum_f \bar{\psi}_f \overleftrightarrow{\not{D}}\psi_f\,, \quad \mathcal{O}_{5,\rho\sigma} = \delta_{\rho\sigma}\sum_f m_f\bar{\psi}_f\psi_f$$

Tensor operators: $\mathcal{O}_1, \mathcal{O}_3$ Pure-trace operators: $\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5$

- EMT trace lies in the scalar channel under lattice hypercubic symmetry $\Rightarrow$ mixing with lower-dimensional operators
- power-divergent subtractions are required
- conventional RI/MOM-type renormalization: gauge dependent/IR sensitive/ noisy

Belinfante, Phys. Rev. 128 (1962) 2832; Belitsky & Radyushkin, Phys. Rept. 418 (2005) 1; Freese, Phys. Rev. D 106 (2022) 125012,…

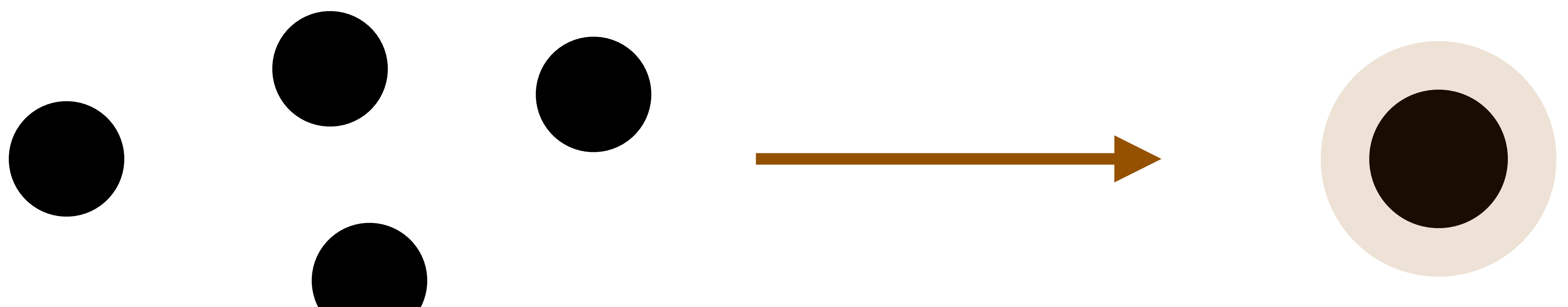
Our strategy: from gradient flow to a common $\overline{\text{MS}}$ EMT scheme

Gradient Flow (GF): Lüscher et al., 10', 11', 14'

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(0,x) = A_\mu(x)$$

Flow time t_f smooth UV fluctuations

over a radius of $\sqrt{8t_f}$ $\quad B_\mu(t,p) \simeq e^{-tp^2} A_\mu(p)$



rough field

flowed/smeared field

Small-flow-time EMT reconstruction

$$T^{\overline{\text{MS}}}_{\rho\sigma}(x,\mu) = \lim_{t_f \to 0} \Sigma_i c_i(t_f,\mu)\bar{\mathcal{O}}_{i,\rho\sigma}(t_f,x)$$

flowed operators are matched to the physical EMT;
the coefficients c_i are known perturbatively

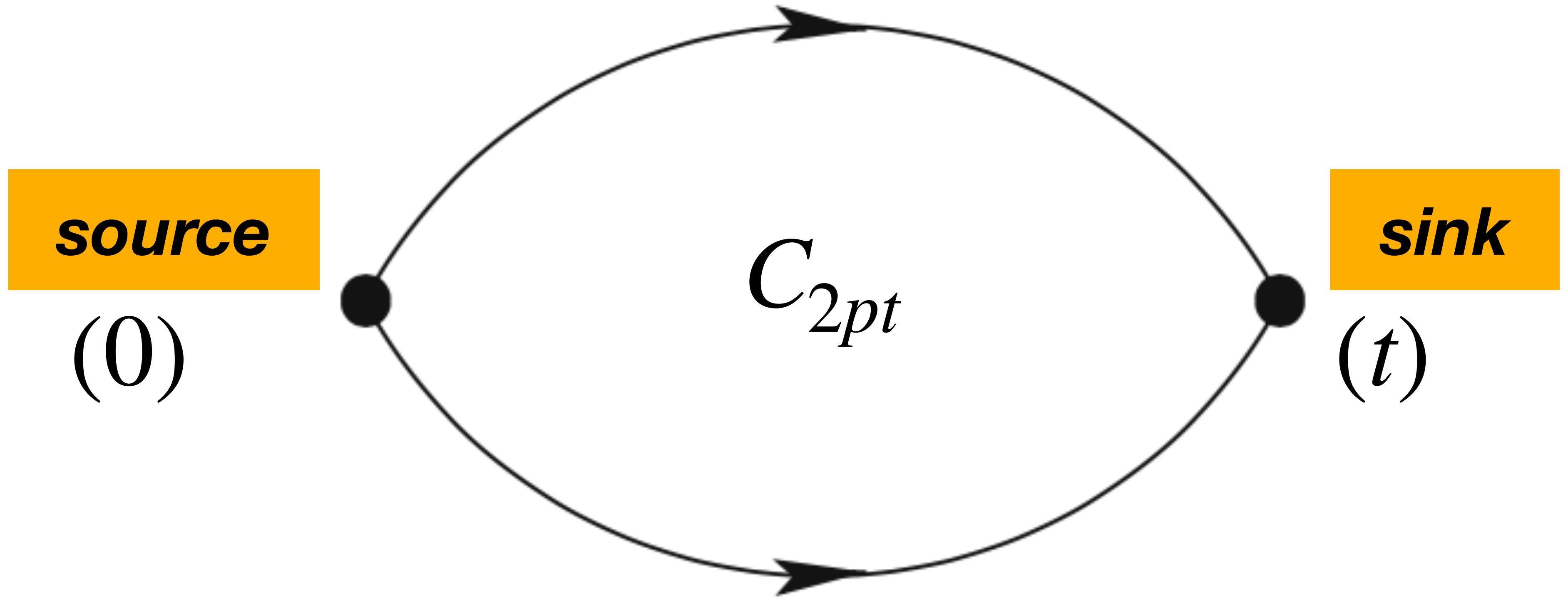
- simplifies renormalization and avoids power-divergent subtractions
- puts traceless and trace sectors into one common framework

Operationally in this work

Flowed EMT at fixed t_f ⇨ Take $a \to 0$ at each fixed t_f ⇨ 2-loop matching GF → MSbar ⇨ Zero-Flow-time extrapolation $t_f \to 0$ ⇨ **All EMT ingredients in one common scheme**

common scheme-and-scale determination of traceless and trace contributions

Hadron ground states from 2-pt correlators



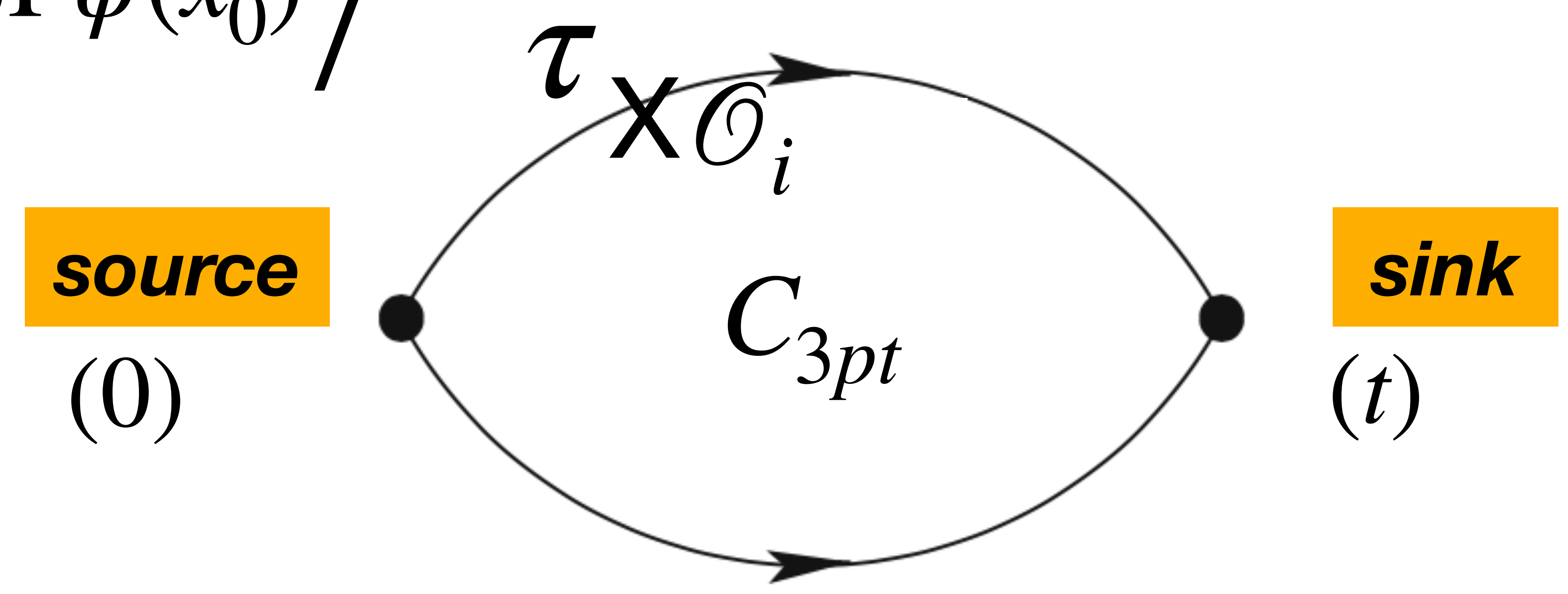
$$C_\Gamma^{2\text{pt}}(t) \equiv \sum_x \left\langle \bar{\psi}(x)\Gamma\psi(x)\ \bar{\psi}(x_0)\Gamma\psi(x_0) \right\rangle$$

- Determine energy of states from the energy decomposition:

$$C^{2\text{pt}}(t) = \sum_{n=0}^{n_{\max}} |Z_n|^2 \left[e^{-E_n t} + e^{-E_n(N_t a - t)}\right]$$

EMT matrix elements from the ratio of 3pt to 2pt correlators

- 3-pt correlation function $C^{\rho\sigma}_{q,\Gamma}(t,\tau) \equiv \sum_{x} \left\langle \bar{\psi}(x)\Gamma\psi(x)\, \tilde{\mathcal{O}}_{3,\rho\sigma}(\tau)\, \bar{\psi}(x_0)\Gamma\psi(x_0) \right\rangle$



Spectral decomposition

$$C^{\rho\sigma}_{X,\Gamma}(t,\tau) = \sum_{m,n} Z^*_m Z_n \frac{\langle m|\widetilde{\mathcal{O}}_{i,\rho\sigma}|n\rangle}{2M} e^{-\tau E_n} e^{-(t-\tau)E_m}$$

- 2-pt correlation function $C^{2\text{pt}}(t) = \sum_{n=0}^{n_{\max}} |Z_n|^2 \left[e^{-E_n t} + e^{-E_n(N_t a - t)}\right]$

Ratio of 3pt- to 2pt- correlator

$$R^{\rho\sigma}_{X,\Gamma}(t,\tau) = \frac{1}{2M} \frac{\langle 0|\widetilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle - \frac{\mathrm{Re}(Z_1^* Z_0 \langle 0|\widetilde{\mathcal{O}}_{i,\rho\sigma}|1\rangle)}{|Z_0|^2}\left[e^{-\Delta E\tau} + e^{-\Delta E(t-\tau)}\right]}{1 + (|Z_1|^2/|Z_0|^2)e^{-\Delta E t}}$$

Wanted matrix element

asymptotic limit

t ≫ τ ≫ 0

3-point correlation function: Demonstration for the gluonic part

- 3-pt corr: $$C^{\rho\sigma}_{g,\Gamma}(t,\tau)=\sum_{\boldsymbol{x},\boldsymbol{y}}\langle\mathcal{O}_\Gamma(\boldsymbol{x},t)\widetilde{\mathcal{O}}_{1,\rho\sigma}(\boldsymbol{y},\tau)\mathcal{O}^\dagger_\Gamma(\boldsymbol{0},0)\rangle-\langle C^{\text{2pt}}_\Gamma(t)\rangle\sum_{\boldsymbol{y}}\langle\widetilde{\mathcal{O}}_{1,\rho\sigma}(\boldsymbol{y},\tau)\rangle,$$
- Gluon operator $$\widetilde{\mathcal{O}}_{1,\rho\sigma}(t_\text{f},x)\equiv 2\text{Tr}^\text{c}\left[F_{\rho\omega}(t_\text{f},x)F_{\sigma\omega}(t_\text{f},x)\right]$$

  Clover-type discretization: $$F_{\rho\sigma}(x)=-i\left[F'_{\rho\sigma}(x)-\frac{\text{Tr}^\text{c}F'_{\rho\sigma}(x)}{N_\text{c}}I\right]$$

  $$F'_{\rho\sigma}(x)=\frac{1}{8}\left[P_{\rho\sigma}(x)-P^\dagger_{\rho\sigma}(x)\right],\ P_{\rho\sigma}(x)=U_{\rho\sigma}(x)+U_{\sigma,-\rho}(x)+U_{-\rho,-\sigma}(x)+U_{-\sigma,\rho}(x)$$

  $$U_{\rho\sigma}(x)=U_\rho(x)U_\sigma(x+\hat\rho)U^\dagger_\rho(x+\hat\sigma)U^\dagger_\sigma(x).$$

- Gauge links $U_\rho(x)$ are replaced by the flowed counterparts

$$\partial_{t_\text{f}}B_\mu(t_\text{f},x)=D_\nu F_{\nu\mu}(t_\text{f},x),\qquad B_\mu(0,x)=A_\mu(x)$$

3-point correlation function: Demonstration for the quark part

- 3-pt corr: $$C^{\rho\sigma}_{q,\Gamma}(t,\tau) \equiv \sum_{x} \left\langle \bar{\psi}(x)\Gamma\psi(x)\, \tilde{\mathcal{O}}_{3,\rho\sigma}(\tau)\, \bar{\psi}(x_0)\Gamma\psi(x_0) \right\rangle \,, \qquad \widetilde{\mathcal{O}}_{3,\rho\sigma}(\tau) = \sum_{\boldsymbol{y}} \widetilde{\mathcal{O}}_{3,\rho\sigma}(y)$$

- Quark operator $$\widetilde{\mathcal{O}}_{3,\rho\sigma}(t_{\mathrm{f}},x) \equiv \sum_{f} \mathring{\bar{\chi}}_f(t_{\mathrm{f}},x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \mathring{\chi}_f(t_{\mathrm{f}},x)$$

$$\boxed{\begin{aligned} &\partial_{t_{\mathrm{f}}}\chi(t_{\mathrm{f}},x) = \Delta\chi(t_{\mathrm{f}},x) \\ &\chi(0,x) = \psi(x) \end{aligned}}$$

Ringed fermion fields: cancel the wave function renormalization by construction

$$\mathring{\bar{\chi}}_f(t_{\mathrm{f}},x) \equiv \sqrt{\frac{-6}{(4\pi t_{\mathrm{f}})^2 \langle \bar{\chi}_f(t_{\mathrm{f}},x) \overleftrightarrow{\not{D}} \chi_f(t_{\mathrm{f}},x)\rangle}}\, \bar{\chi}_f(t_{\mathrm{f}},x) \qquad \mathring{\chi}_f(t_{\mathrm{f}},x) \equiv \sqrt{\frac{-6}{(4\pi t_{\mathrm{f}})^2 \langle \bar{\chi}_f(t_{\mathrm{f}},x) \overleftrightarrow{\not{D}} \chi_f(t_{\mathrm{f}},x)\rangle}}\, \chi_f(t_{\mathrm{f}},x)$$

- Contractions among flowed and un-flowed fermion fields: No additional Dirac inversions are needed

$$\left\langle \bar{\chi}(t_f,y)\,\psi(x) \right\rangle = \sum_{z} K(t_f,y;0,z)\, S_\psi(z\,|\,x)\,, \qquad \left\langle \psi(x)\,\bar{\chi}(t_f,y) \right\rangle = \sum_{z} S_\psi(x\,|\,z)\, K^\dagger(t_f,y;0,z), \qquad \lim_{t_f\to 0} K(t_f,y;0,z) = \delta_{yz}\,.$$

Is J/ψ simply two charm quarks?

$$M_{J/\psi} = 3.097\ \text{GeV}$$

Current quark mass: $m_c^{\overline{MS}}(m_c) \approx 1.27\ \text{GeV}$

Pole/Constituent-like mass:

$$m_c^{\text{pole}} = m_c^{\overline{\text{MS}}}(m_c)\left(1 + \frac{4}{3}\frac{\alpha_s}{\pi} + \cdots\right) \approx 1.5\ \text{GeV}$$

Much less challenging computationally compared with light mesons or proton

Lattice setup and extracted matrix elements

Ensembles

- 2+1 flavor HISQ, HotQCD ensembles
- $a \approx 0.06, 0.08, 0.12$ fm
- $64 \times 48^3, 64^4, 64^4$
- physical strange, near-physical light quarks
- physical charmonium: η_c & J/ψ

Measured quantities

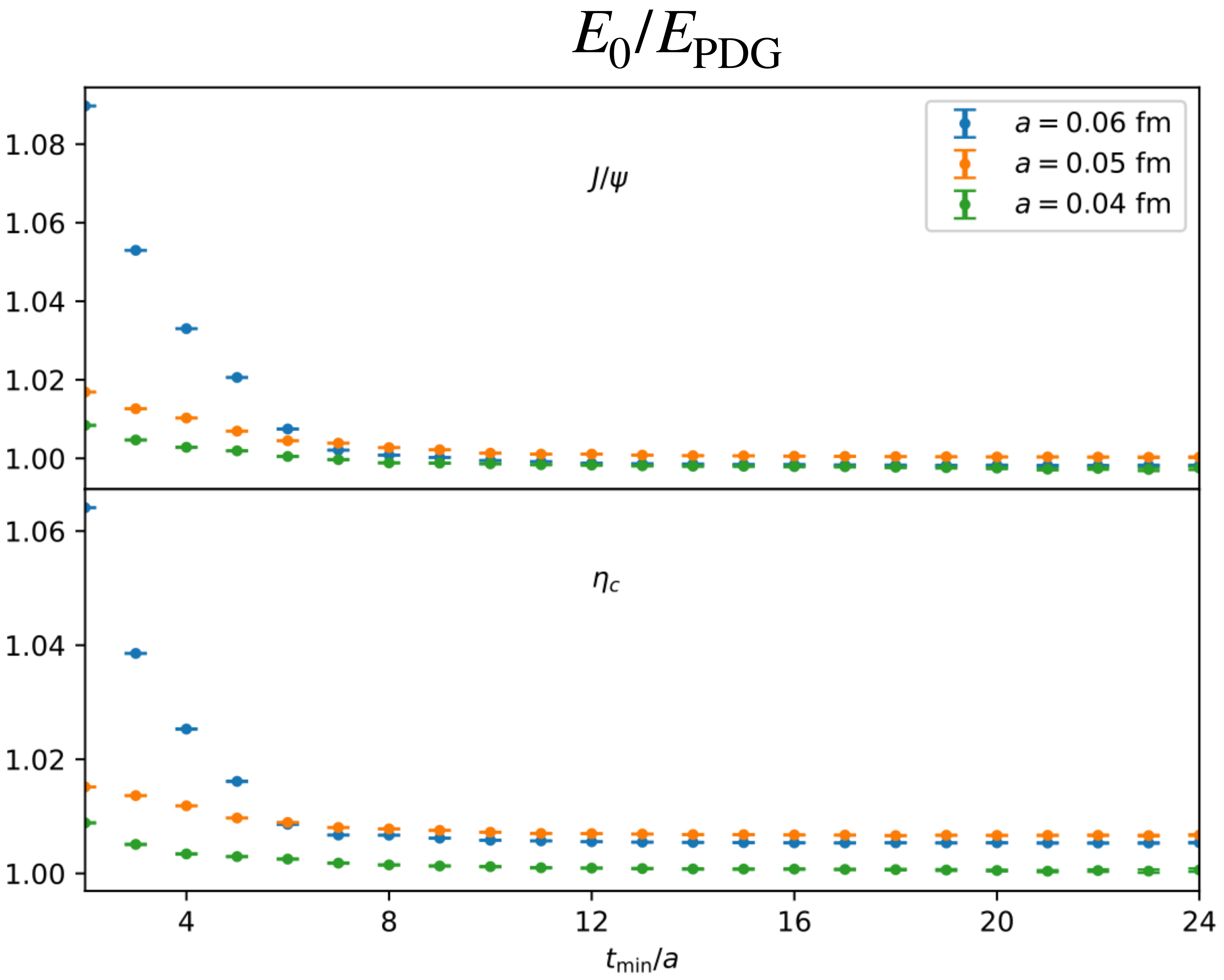
- flowed EMT operators $\mathcal{O}_1, \cdots \mathcal{O}_5$
- flowed quark and gluon matrix elements
- both traceless & trace sectors

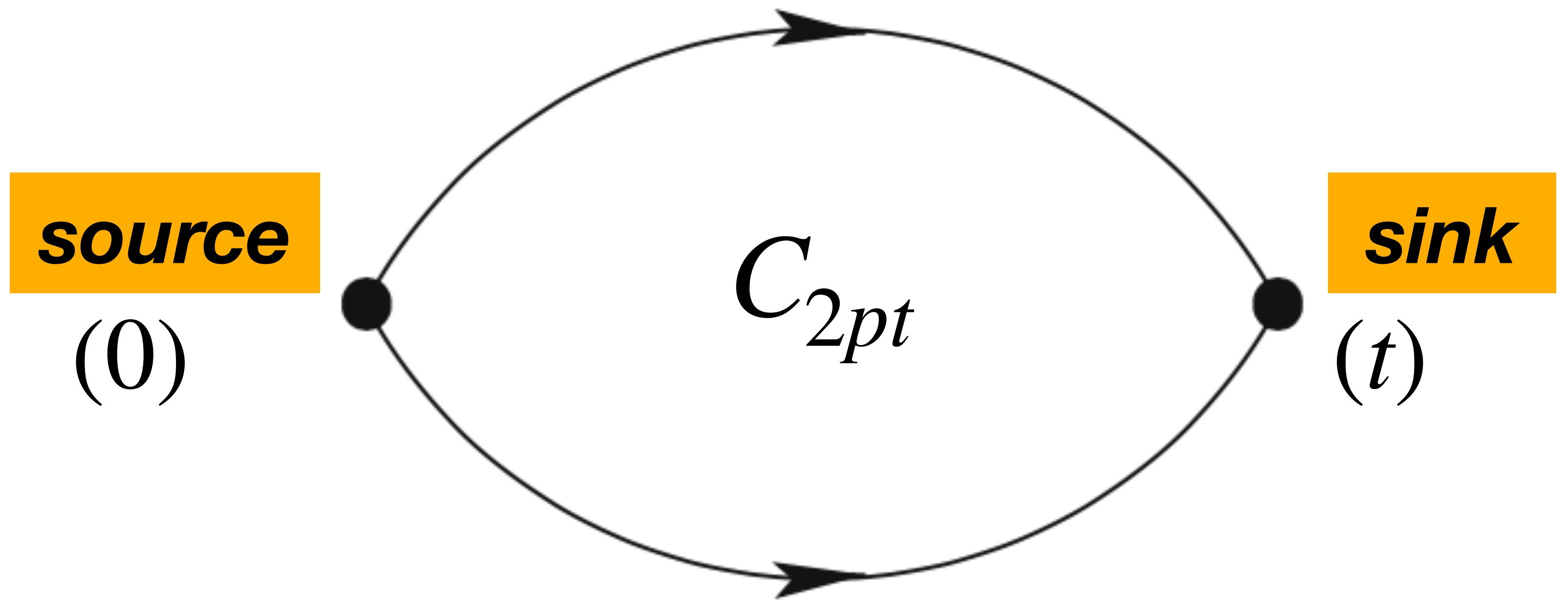
Extraction

- 2-point & 3-point correlators $\to < H | \mathcal{O}_i | H >$
- $a \to 0$ at fixed t_f
- GF $\to \overline{\text{MS}}$ matching
- $t_f \to 0$

Dennis Bollweg, HTD, Xiang Gao, Ran Luo, Swagato Mukherjee, arXiv: 2601.13070, PRL in press

Construction of quarkonia states via 2-pt correlation function





$$C_\Gamma^{\rm 2pt}(t) \equiv \sum_{\boldsymbol{x}} \langle \bar{c}(x)\Gamma c(x)\bar{c}(x_0)\Gamma c(x_0)\rangle$$

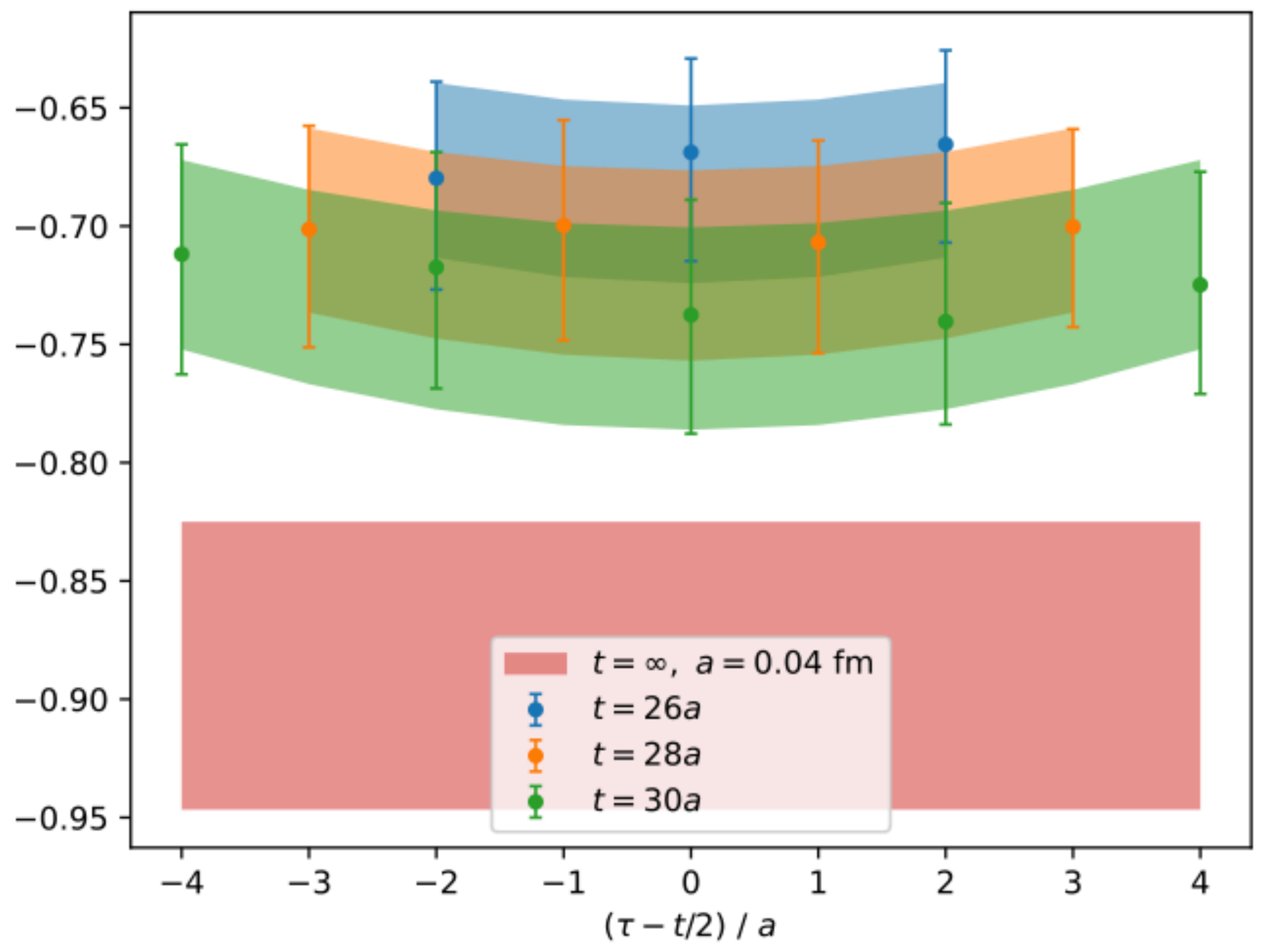
$$C^{\rm 2pt}(t) = \sum_{n=0}^{n_{\rm max}} |Z_n|^2 \left[e^{-E_n t} + e^{-E_n(N_t a - t)}\right]$$

- Ratios are remarkably close to unity

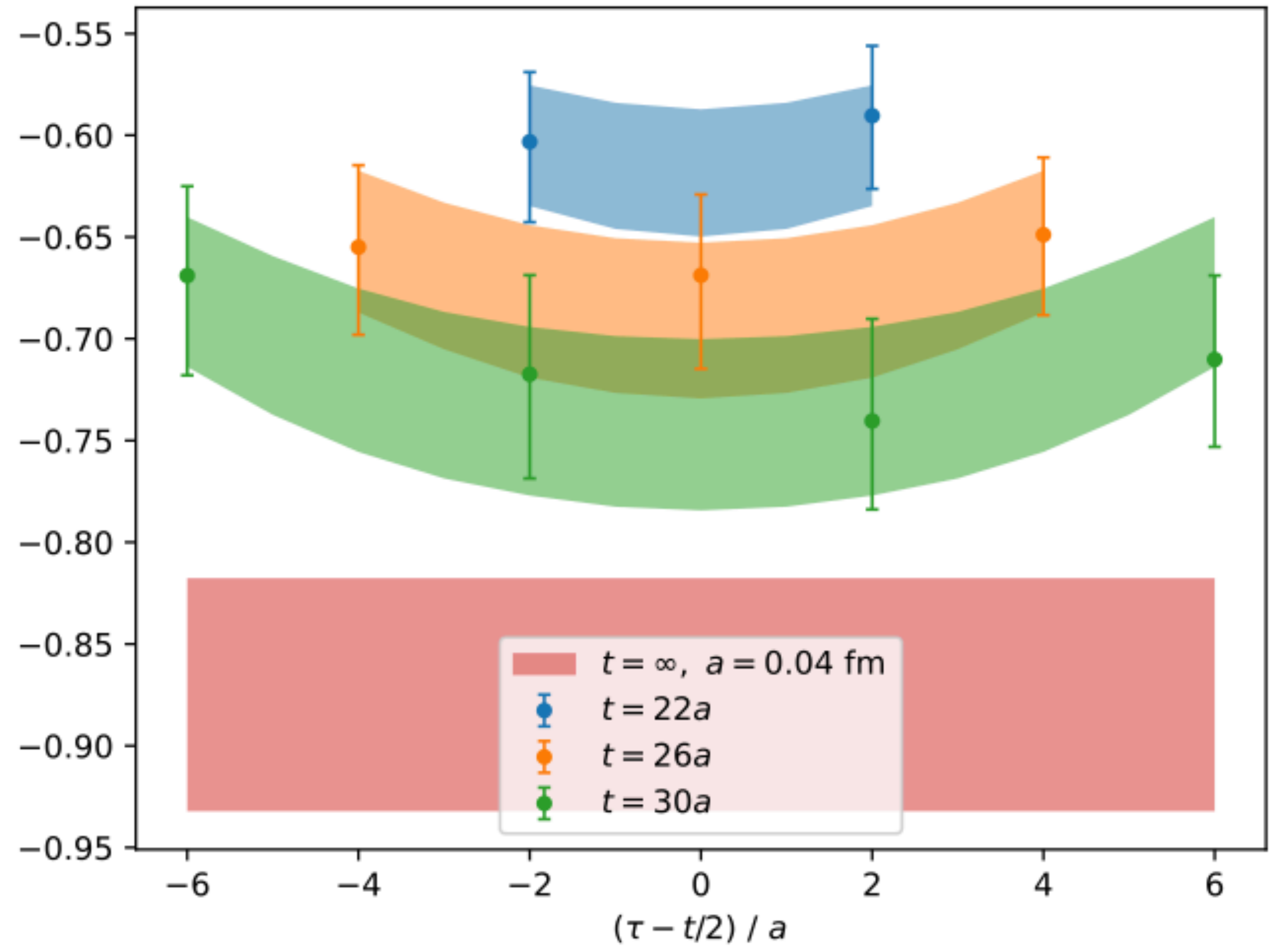
- Bare charm-quark mass is tuned to reproduce the physical charmonium states

Demonstration: Extraction of gluon matrix elements $<\eta_c|\tilde{\mathcal{O}}_{1,kk}|\eta_c>$ at a= 0.04 fm & $t_f = 5\epsilon_f$

2-state fits: $$R^{\rho\sigma}_{X,\Gamma}(t,\tau) = \frac{1}{2M}\frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle - \frac{\mathrm{Re}(Z_1^* Z_0\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|1\rangle)}{|Z_0|^2}\left[e^{-\Delta E\tau}+e^{-\Delta E(t-\tau)}\right]}{1+(|Z_1|^2/|Z_0|^2)e^{-\Delta E t}}$$



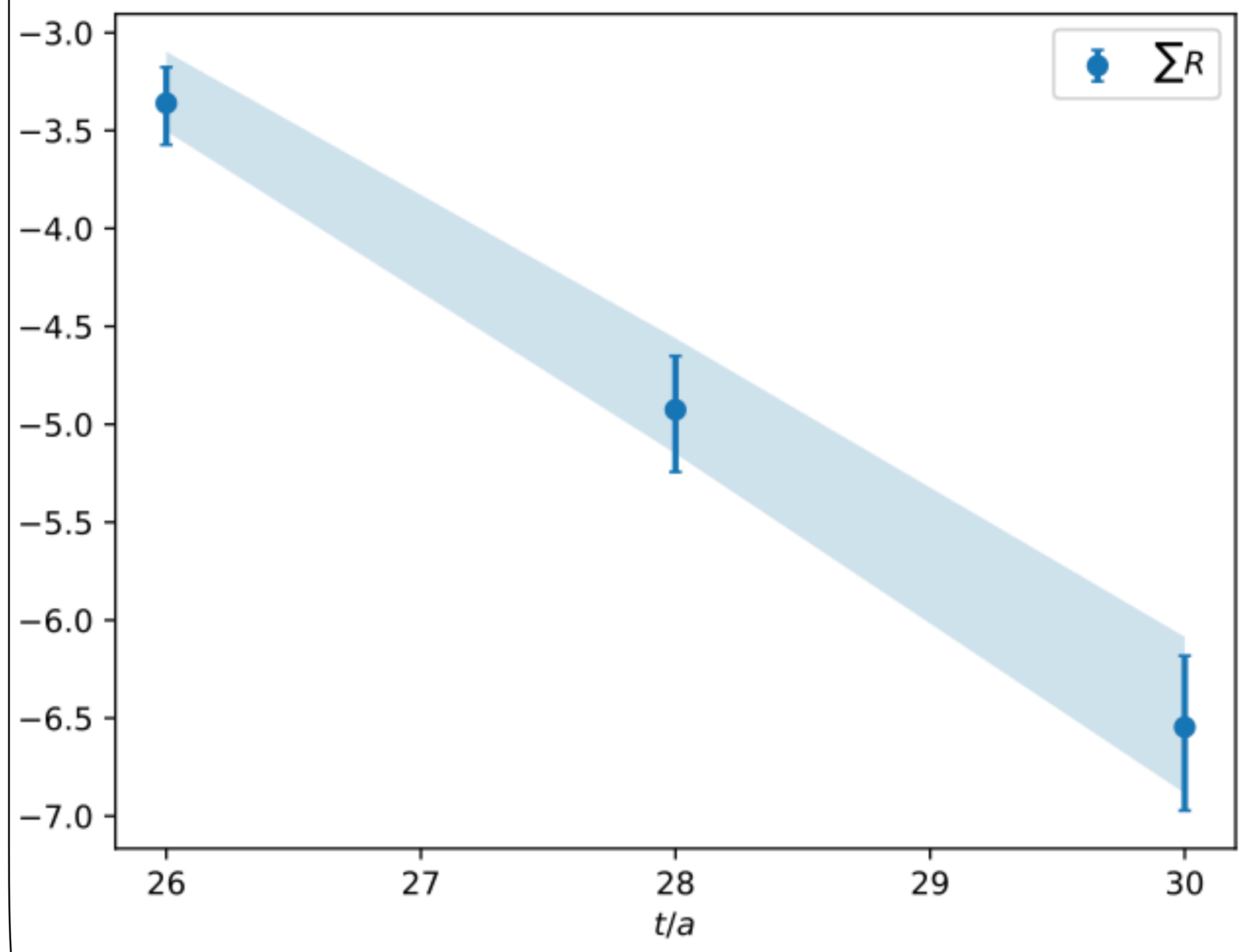
2-stride fit ($N_{\mathrm{stride}} = 2$)



4-stride fit ($N_{\mathrm{stride}} = 4$)

$\tau = N_{\mathrm{skip}}, N_{\mathrm{skip}} + N_{\mathrm{stride}}, \cdots, \tau \in [N_{\mathrm{skip}}, t - N_{\mathrm{skip}}]$

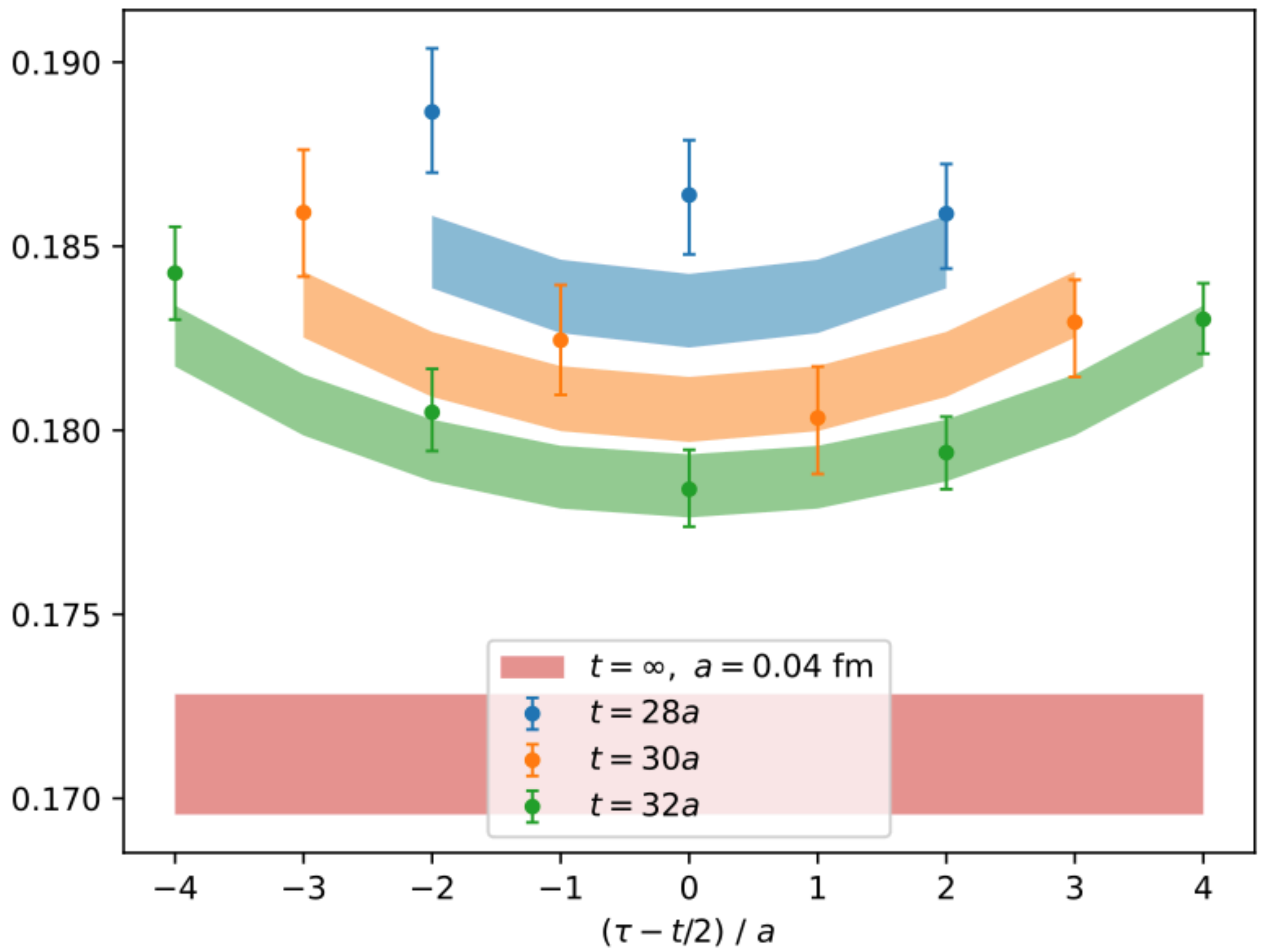
$$\sum_{\tau=aN_{\mathrm{skip}}}^{t-aN_{\mathrm{skip}}} R^{\rho\sigma}_{X,\Gamma}(t,\tau) \xrightarrow[t\to\infty]{} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle}{2M}\, t + B\,.$$



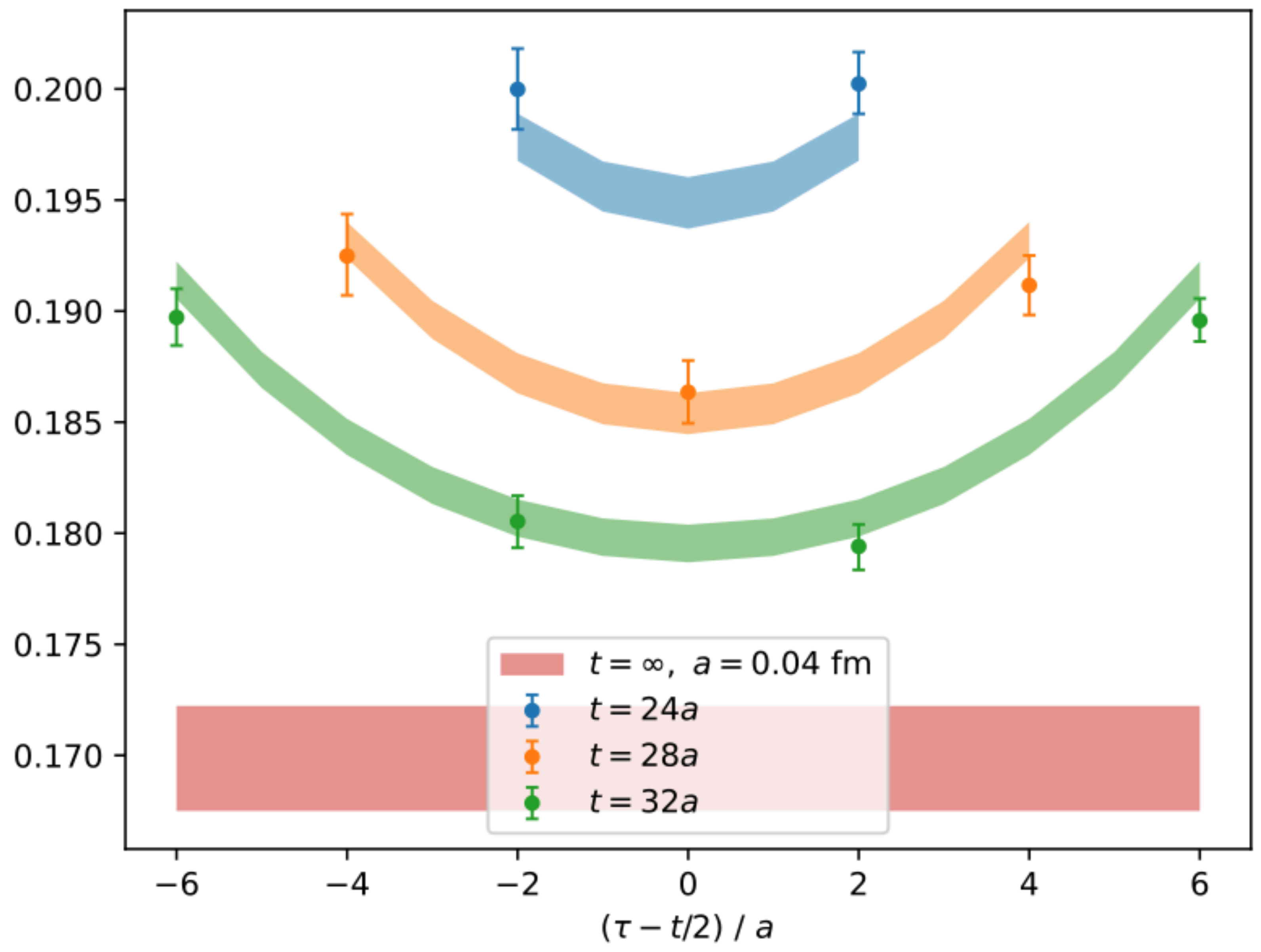
Summation fit

Demonstration: Extraction of quark matrix elements $< \eta_c | \tilde{\mathcal{O}}_{3,kk} | \eta_c >$ at a= 0.04 fm & $t_f = 5\epsilon_f$

2-state fits: $$R^{\rho\sigma}_{X,\Gamma}(t,\tau) = \frac{1}{2M} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle - \frac{\mathrm{Re}(Z_1^* Z_0 \langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|1\rangle)}{|Z_0|^2}\left[e^{-\Delta E\tau} + e^{-\Delta E(t-\tau)}\right]}{1 + (|Z_1|^2/|Z_0|^2)e^{-\Delta E t}}$$



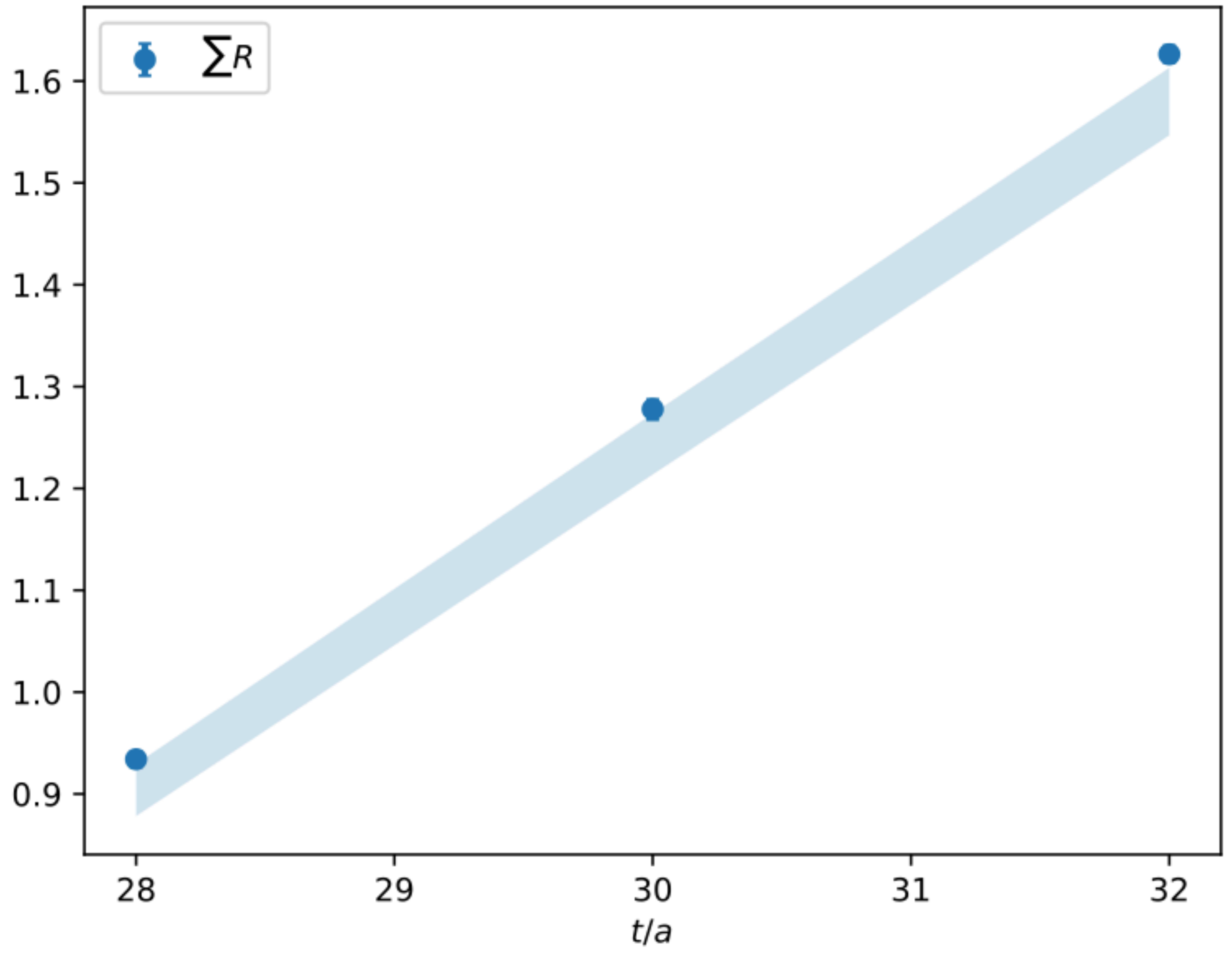
2-stride fit ($N_{\mathrm{stride}} = 2$)



4-stride fit ($N_{\mathrm{stride}} = 4$)

$\tau = N_{\mathrm{skip}}, N_{\mathrm{skip}} + N_{\mathrm{stride}}, \cdots, \tau \in [N_{\mathrm{skip}}, t - N_{\mathrm{skip}}]$

$$\sum_{\tau=aN_{\mathrm{skip}}}^{t-aN_{\mathrm{skip}}} R^{\rho\sigma}_{X,\Gamma}(t,\tau) \xrightarrow[t\to\infty]{} \frac{\langle 0|\tilde{\mathcal{O}}_{i,\rho\sigma}|0\rangle}{2M} t + B\,.$$

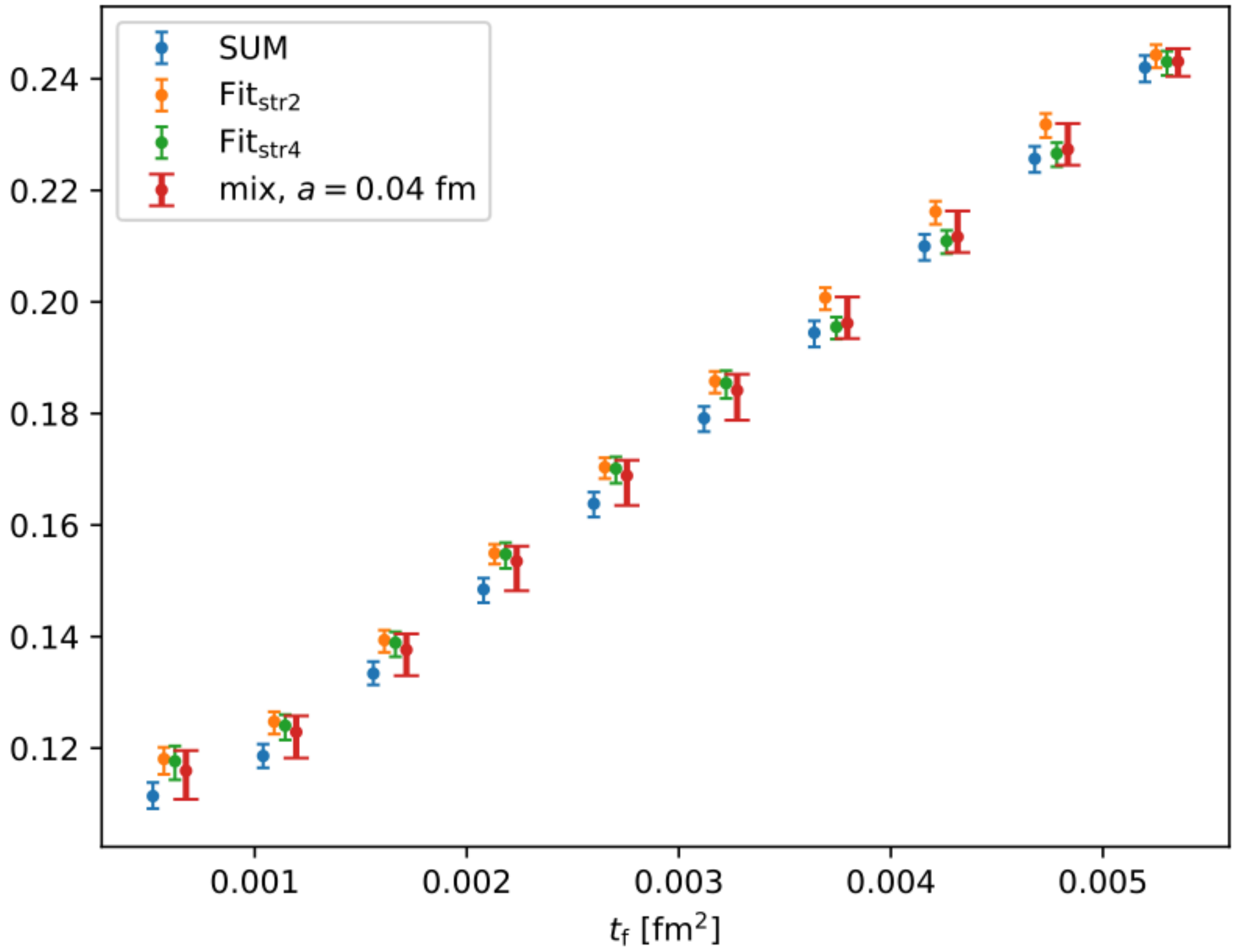


Summation fit

Stable extraction of flowed EMT matrix elements

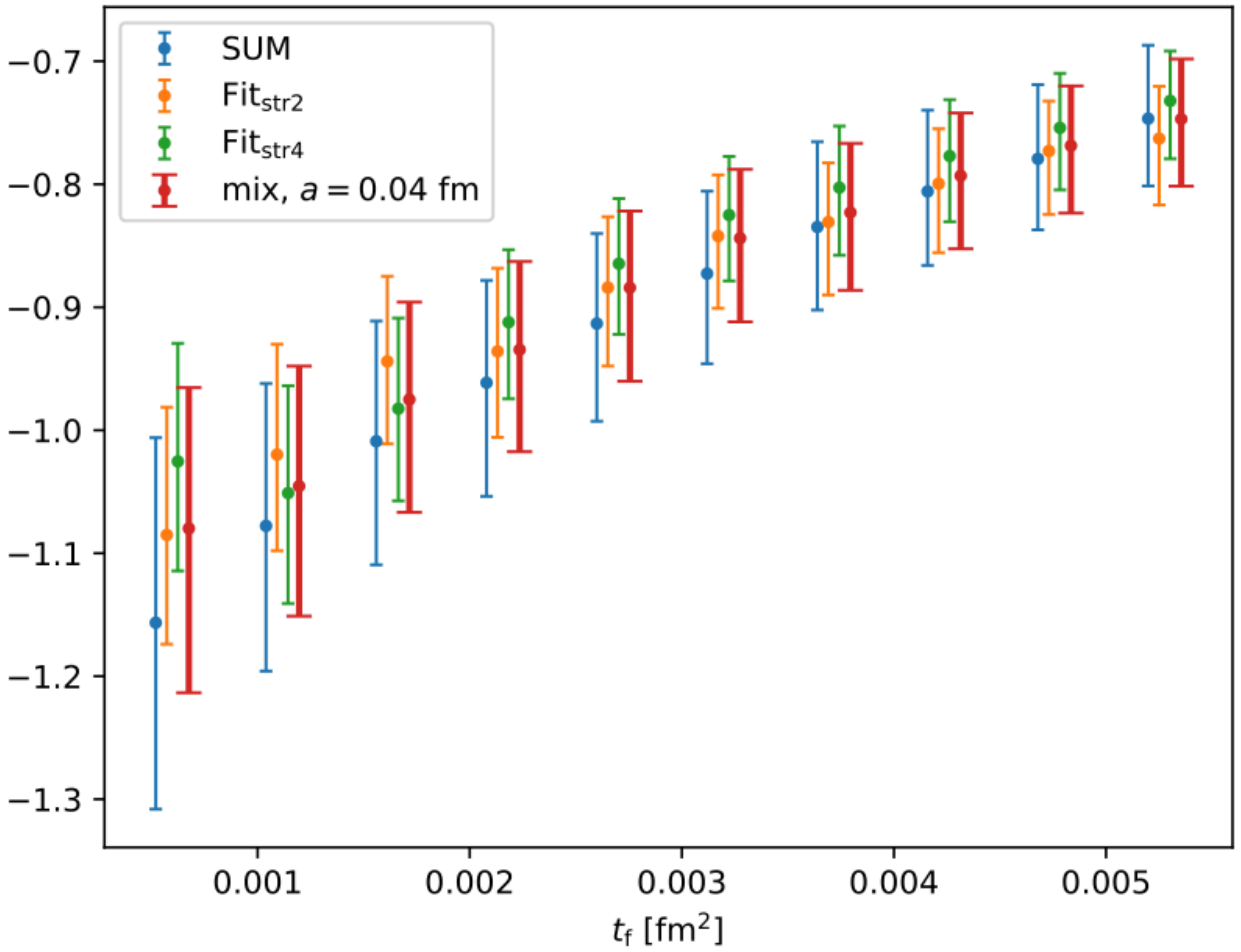
quark matrix element

$\frac{1}{2M}\sum_{k=1}^{3}\langle\eta_c|\widetilde{\mathcal{O}}_{3,kk}/3|\eta_c\rangle$



gluon matrix element

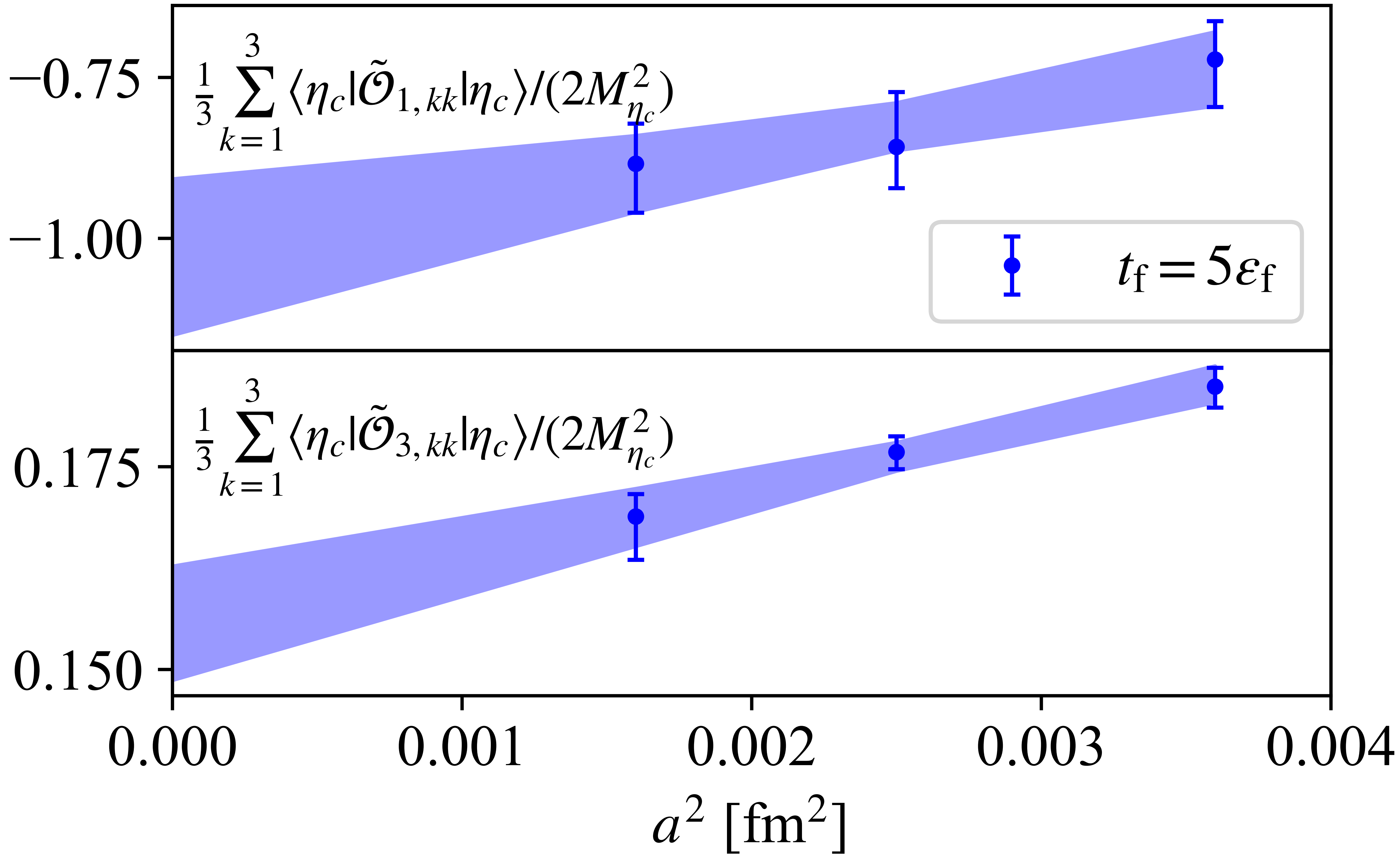
$\frac{1}{2M}\sum_{k=1}^{3}\langle\eta_c|\widetilde{\mathcal{O}}_{1,kk}/3|\eta_c\rangle$



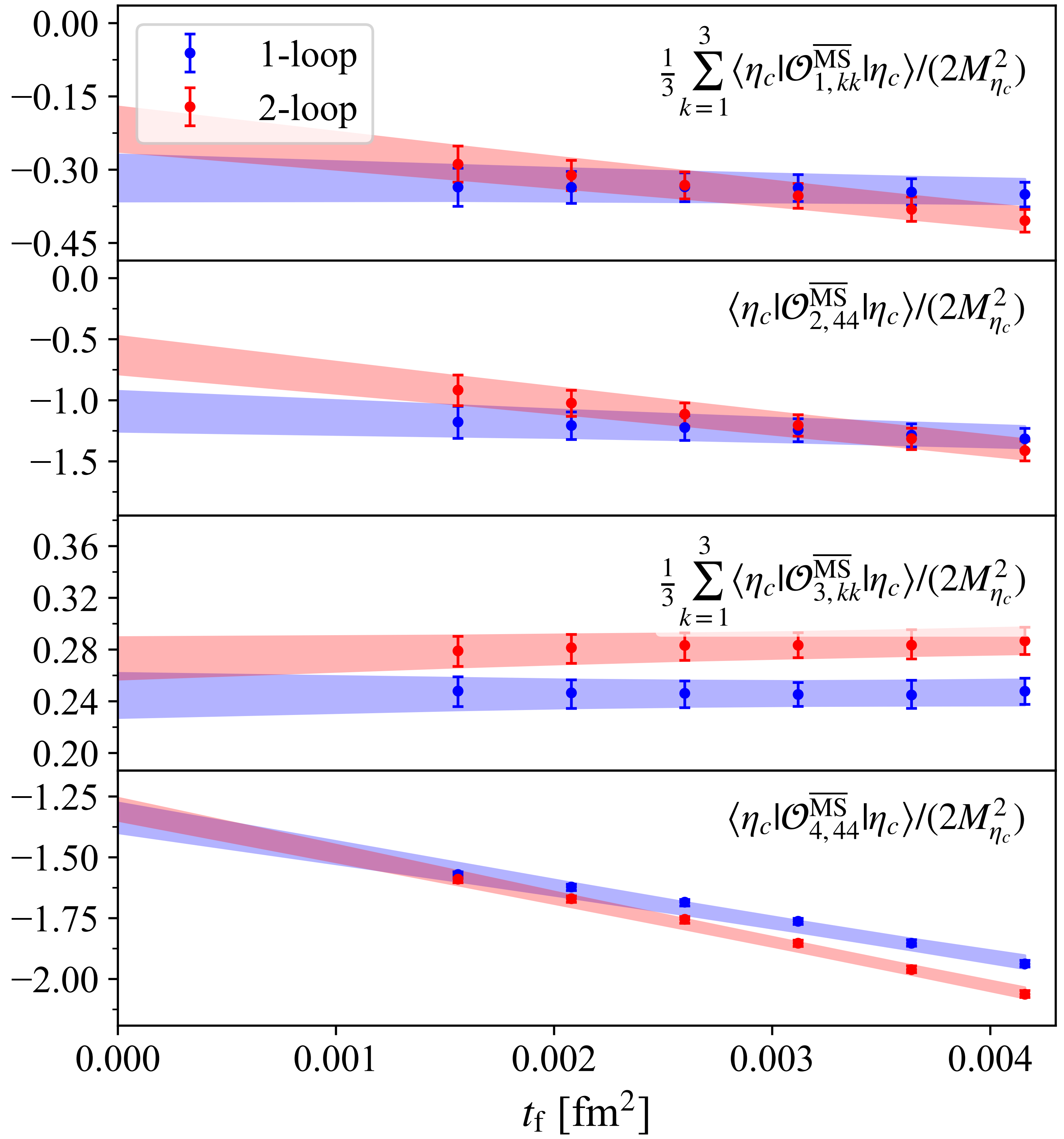
Flowed quark and gluon EMT matrix elements are extracted stably from standard 3pt/2pt ratios

From flowed matrix elements to the physical EMT

Continuum extrapolation at fixed flow time t_f



GF → $\overline{\text{MS}}$ matching and $t_f \to 0$ extrapolation



$\sim \sigma_f$

Pert. matching from: Suzuki, 14'15'; Harlander et al., 18'

Smooth continuum, matching and $t_f \to 0$ behavior enables a common determination of all EMT components

Forward-limit sum rules for GFF

$A(0)$: momentum fraction type term;$\bar{C}(0)$: trace-related GFF ingredient; $\bar{f}(0)$: extra spin-1 structure for J/ψ

Spin-0 hadron

$$\langle \eta_c | T^{\overline{\text{MS}}}_{X,00} | \eta_c \rangle = 2M^2_{\eta_c}[A_X(0) + \bar{C}_X(0)]$$

$$\sum_{k=1}^{3} \langle \eta_c | T^{\overline{\text{MS}}}_{X,kk} | \eta_c \rangle = -6M^2_{\eta_c}\bar{C}_X(0)$$

Poincaré symmetry $\sum_X A_X(0) = 1$

Conservation of the total EMT $\sum_X \bar{C}_X(0) = 0$

X=c,g

Spin-1 hadron

$$\langle J/\psi | T^{\overline{\text{MS}}}_{X,11} | J/\psi \rangle = 2M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) + \frac{3}{4}\bar{f}_X(0)\right]$$

$$\langle J/\psi | T^{\overline{\text{MS}}}_{X,00} | J/\psi \rangle = 2M^2_{J/\psi}[A_X(0) - \frac{1}{2}\bar{C}_X(0) + \frac{1}{4}\bar{f}_X(0)]$$

$$\sum_{k=2,3} \langle J/\psi | T^{\overline{\text{MS}}}_{X,kk} | J/\psi \rangle = 4M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) - \frac{1}{4}\bar{f}_X(0)\right]$$

Rotational symmetry & polarization structure

$$\sum_X \bar{f}_X(0) = 0$$

Ji, PRL 78 (1997) 610, Hackett et al., PRD108 (2023) 114504, Pefkou et al., 105 (2022) 054509, Polyakov & Sun, PRD 100 (2019) 036003

Forward-limit GFF ingredients at rest for η_c and J/ψ

$A(0)$: momentum fraction type term; $\bar{C}(0)$: trace-related GFF ingredient; $\bar{f}(0)$: extra spin-1 structure for J/ψ

	η_c		J/ψ		
	$A(0)$	$\bar{C}(0)$	$A(0)$	$\bar{C}(0)$	$\bar{f}(0)$
gluon(g)	0.084(45)	0.058(18)	0.077(35)	-0.146(29)	-0.012(32)
charm(c)	0.918(17)	-0.068(4)	0.926(34)	0.176(87)	0.023(5)
total	1.000(46)	−0.010(18)	0.997(43)	0.030(30)	0.010(32)

$$\langle \eta_c | T^{\overline{\mathrm{MS}}}_{X,00} | \eta_c \rangle = 2M^2_{\eta_c}[A_X(0) + \bar{C}_X(0)]$$

$$\sum_{k=1}^{3} \langle \eta_c | T^{\overline{\mathrm{MS}}}_{X,kk} | \eta_c \rangle = -6M^2_{\eta_c}\bar{C}_X(0)$$

$$\langle J/\psi | T^{\overline{\mathrm{MS}}}_{X,11} | J/\psi \rangle = 2M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) + \frac{3}{4}\bar{f}_X(0)\right]$$

$$\langle J/\psi | T^{\overline{\mathrm{MS}}}_{X,00} | J/\psi \rangle = 2M^2_{J/\psi}[A_X(0) - \frac{1}{2}\bar{C}_X(0) + \frac{1}{4}\bar{f}_X(0)]$$

$$\sum_{k=2,3} \langle J/\psi | T^{\overline{\mathrm{MS}}}_{X,kk} | J/\psi \rangle = 4M^2_{J/\psi}\left[\frac{1}{2}\bar{C}_X(0) - \frac{1}{4}\bar{f}_X(0)\right]$$

- $\bar{C}(0)$ determined directly for η_c and J/ψ
- $A_g(0) + A_c(0) = 1$ for both states
- $\bar{C}_g(0) + \bar{C}_c(0) = 0$ within uncertainties
- $\bar{f}_g(0) + \bar{f}_c(0) = 0$ within uncertainties for J/ψ

Consistent with the forward-limit GFF sum rules

D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , to appear in PRL

Four sum rules of hadron mass decomposition

$$M_H = \sum_{X=q,g} M_X^{\mathrm{HRT}}, \text{ with } M_X^{\mathrm{HRT}} = \frac{\langle H| \left(T_X^{\overline{\mathrm{MS}}}\right)^{\rho}_{\rho} |H\rangle}{2M_H}.$$

Hatta, Rajan, & Tanaka, JHEP 12 (2018) 008

$$M_H = \sum_{X=q,g} M_X^{\mathrm{Lor}}, \quad \text{with} \quad M_X^{\mathrm{Lor}} = \frac{\langle H|T_{X,00}^{\overline{\mathrm{MS}}}|H\rangle}{2M_H}.$$

Lorcé, EPJC 78 (2018)120

$$M_H = \sum_{X=q,g,m} M_X^{\mathrm{MPR}}, \quad \text{with} \quad M_m^{\mathrm{MPR}} = \sum_f \sigma_f,$$

$$M_q^{\mathrm{MPR}} = M_q^{\mathrm{Lor}} - \sum_f \sigma_f, \quad M_g^{\mathrm{MPR}} = M_g^{\mathrm{Lor}}.$$

Metz, Pasquini, & Rodini, PRD 102 (2020) 114042

$$M_H = \sum_{X=q,g,m,a} M_X^{\mathrm{Ji}}, \quad \text{with} \quad M_m^{\mathrm{Ji}} = M_m^{\mathrm{MPR}},$$

$$M_q^{\mathrm{Ji}} = M_q^{\mathrm{Lor}} - \frac{1}{4}M_q^{\mathrm{HRT}} - \frac{3}{4}M_m^{\mathrm{MPR}},$$

$$M_g^{\mathrm{Ji}} = M_g^{\mathrm{Lor}} - \frac{1}{4}M_g^{\mathrm{HRT}},$$

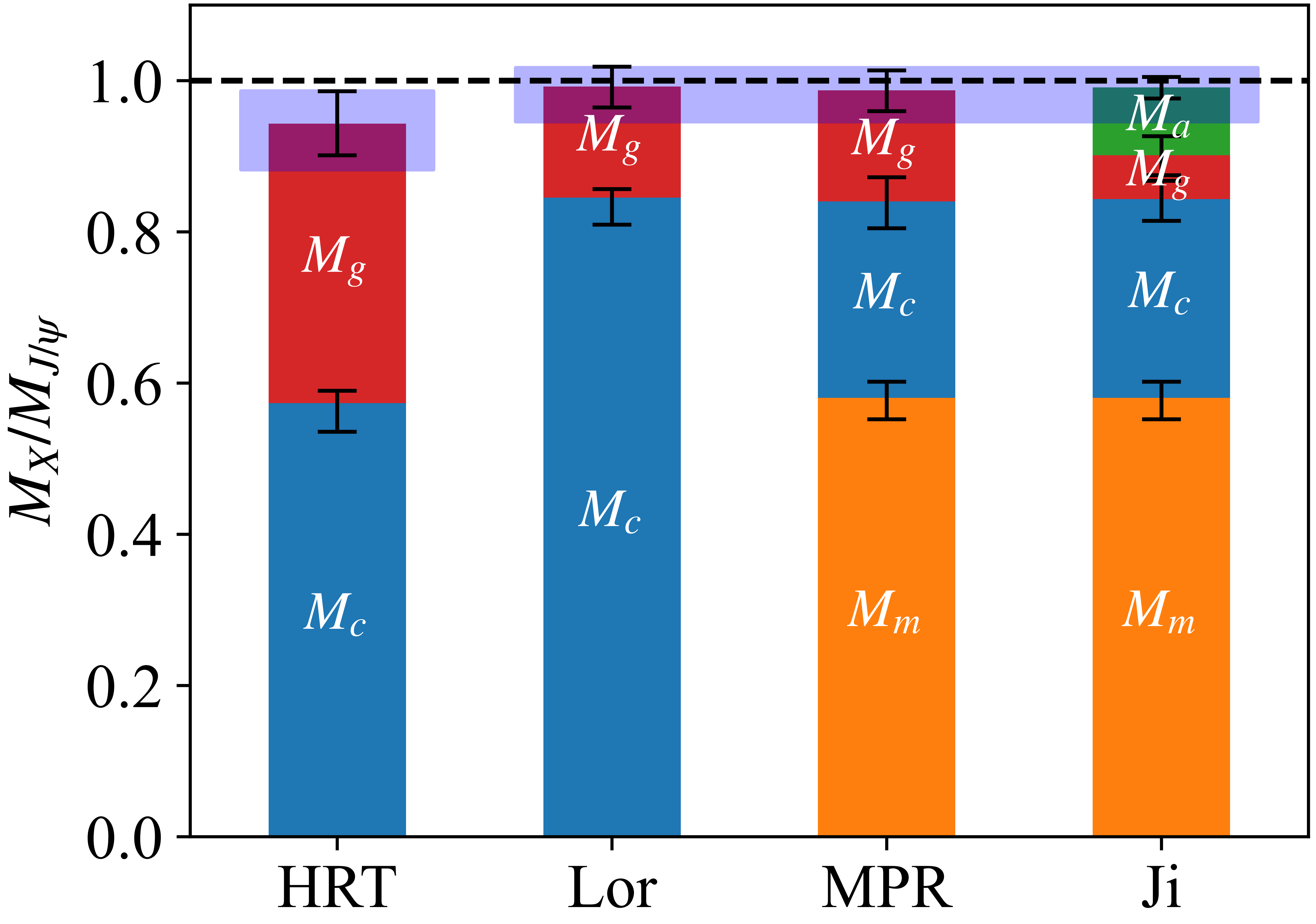
$$M_a^{\mathrm{Ji}} = \frac{1}{4}\left[M_q^{\mathrm{HRT}} + M_g^{\mathrm{HRT}} - M_m^{\mathrm{MPR}}\right].$$

Ji, PRL 74 (1995) 1071

Lattice QCD closes the J/ψ & η_c mass budget across independent sum rules

Same EMT input; HRT / Lorcé: 2-part organization; MPR /Ji: explicit trace-sector split

J/ψ mass budget



Gluon energy is sizable

About 15% in Lorce/MPR,
About 30% in HRT

The Ji anomaly term is visible

277 MeV in J/ψ , close to 9%

Forward-limit GFF tests close

$A_c + A_g = 1, \bar{C}_c + \bar{C}_g = 0$

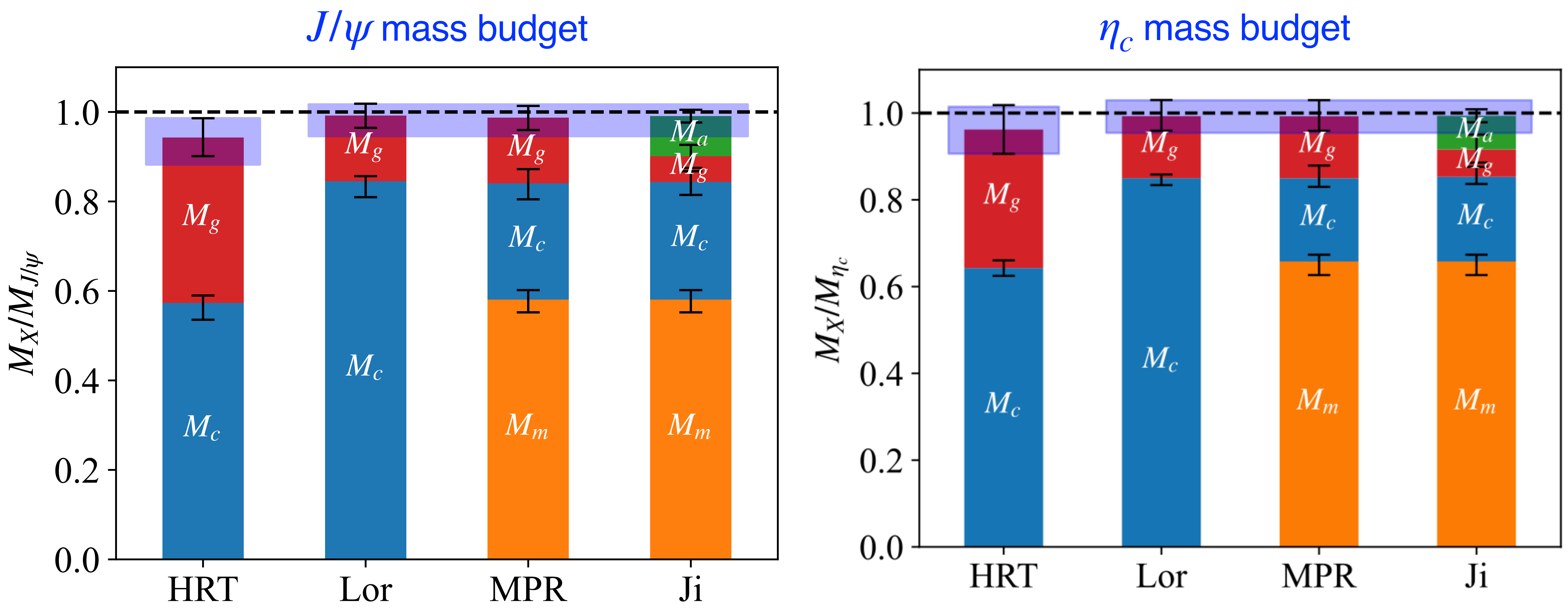
First direct results for $\bar{C}$

M_c: charm energy ; M_g: gluon; M_m: explicit charm-mass term; M_a: trace-anomaly

D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , to appear in PRL

Lattice QCD closes the J/ψ & η_c mass budget across independent sum rules

Same EMT input; HRT / Lorcé: 2-part organization; MPR /Ji: explicit trace-sector split



M_c: charm energy ; M_g: gluon; M_m: explicit charm-mass term; M_a: trace-anomaly

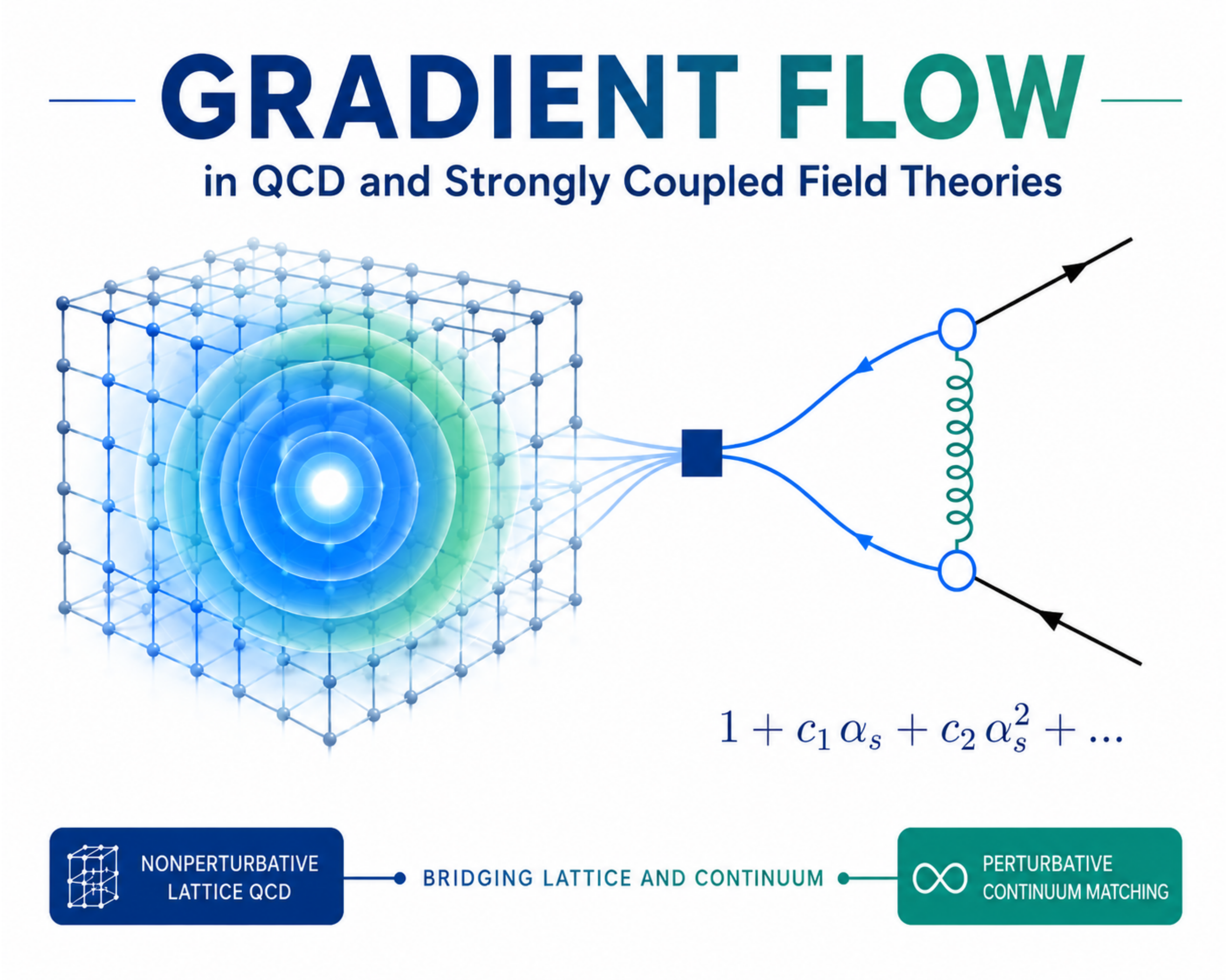
D. Bollweg, 丁亨通, 高翔, 罗冉, S. Mukherjee, arXiv: 2601.13070 , to appear in PRL

Summary

- Constructed the full QCD EMT, including the trace sector, in a common $\overline{\text{MS}}$ framework
- Provides a general first-principles framework for studying hadron mass, spin, and GFFs

- Determined forward-limit GFF ingredients for η_c and J/ψ, including $\bar{C}(0)$, for the first time
- HRT, Lorcé, MPR and Ji mass sum rules are mutually consistent within uncertainties
- Gluon, explicit charm-mass, and anomaly contributions are all nontrivial in charmonium mass decompositions

Same EMT input, different organizations - one consistent first-principles picture



梯度流应用 国际研讨会

2027年3月2-5日,华中核理论中心, 武汉

Central China
Center for Nuclear Theory
华中核理论中心

https://indico.ihep.ac.cn/event/30090/

Local organizers:

王翔鹏 (Central China Normal University)
舒海涛(Central China Normal University)
丁亨通 (Central China Normal University)