NANJING UNIVERSITY

Accessing the Infrared Structure of the Pion TMD with Continuum Schwinger Function Methods

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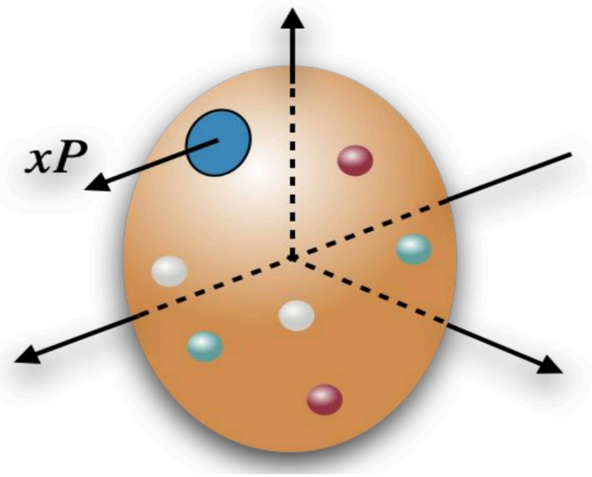
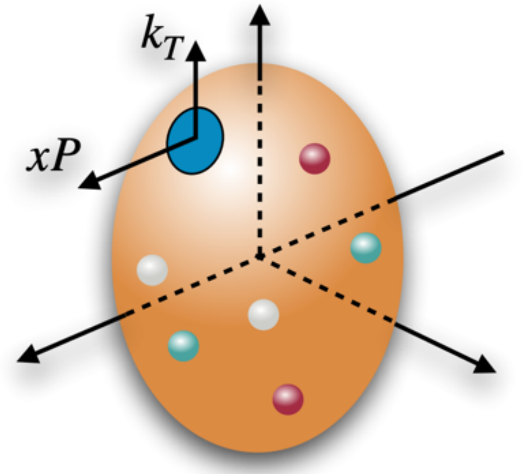
Intrinsic longitudinal/transverse momentum of quarks/gluons in hadrons

- Parton Distribution functions (PDFs) $q(x)$:
  - The probability density for finding a particle with a certain *longitudinal momentum* fraction $x = k^+/P^+$ at resolution scale.



- Transverse momentum dependent distribution functions (TMDs) $f(x, k_\perp)$:
  - It gives the joint probability density to find a parton inside a fast-moving hadron carrying longitudinal momentum fraction x and *transverse momentum* $k_\perp$ (defined perpendicular to the hadron's light-cone direction).

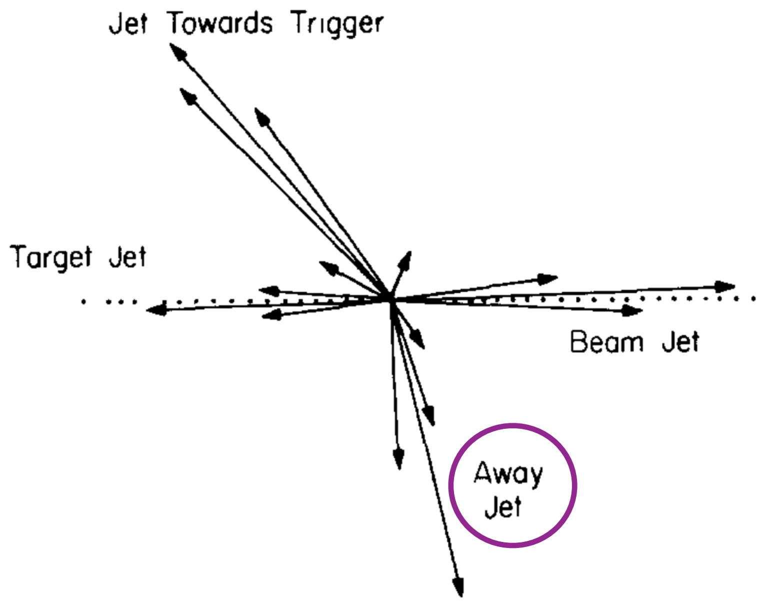


Intrinsic transverse momentum of quarks/gluons in hadrons

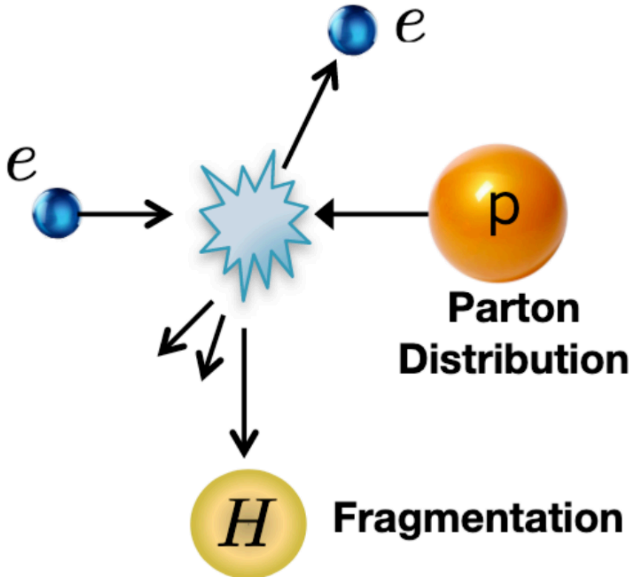
➢ 49 years ago

Correlations among particles and jets produced with large transverse momenta,
R.P. Feynman, R.D. Field, G.C. Fox, Nucl. Phys. B128, 1, 1977.

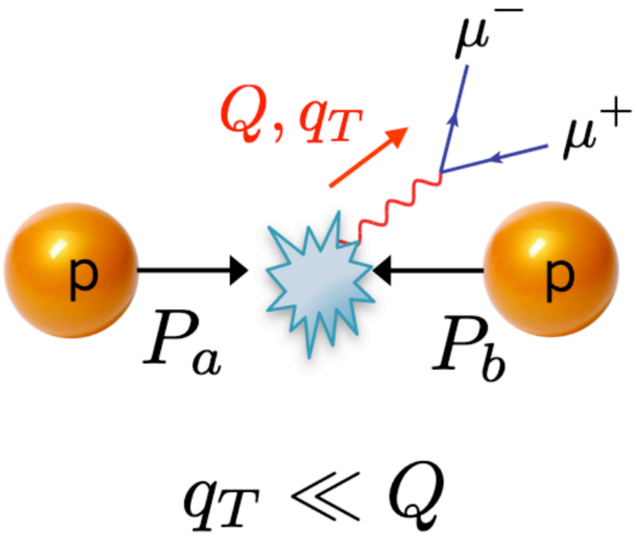
- Drell-Yan process: $A + B \to h_1 + h_2 + X$
- Abstract: "…If we allow the quarks within the initial hadrons to have ***a large mean transverse momentum***, $\langle k_\perp \rangle_{h\to q}$, then good agreement with data is found…".
- An assumption: the transverse momentum distribution is independent of x, $f(x, k_\perp) = q(x)\mathscr{F}(k_\perp)$.
- An exponential form was chosen $\mathscr{F}(k_\perp) = b^2 \exp(-b|k_\perp|)/2\pi$.
- The mean momentum $\langle k_\perp \rangle = \int dk_\perp k_\perp \mathscr{F}(x, k_\perp) = 2/b$.
- Typical choices are $\langle k_\perp \rangle = 300$ MeV and $\langle k_\perp \rangle = 500$ MeV.



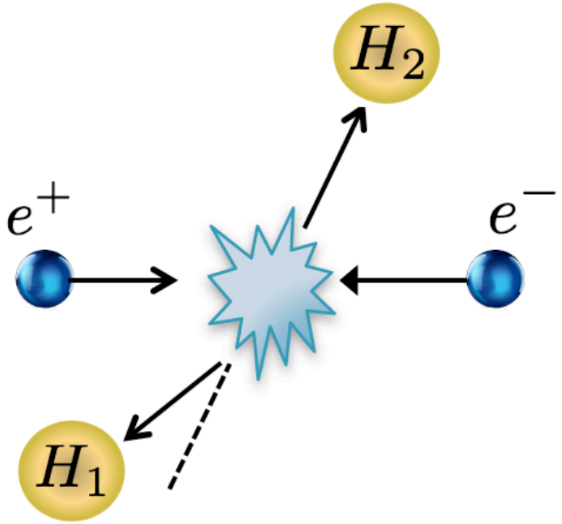
Intrinsic transverse momentum of quarks/gluons in hadrons



- ➢ Semi-Inclusive Deep Inelastic Scattering (SIDIS)



- ➢ Drell-Yan Process

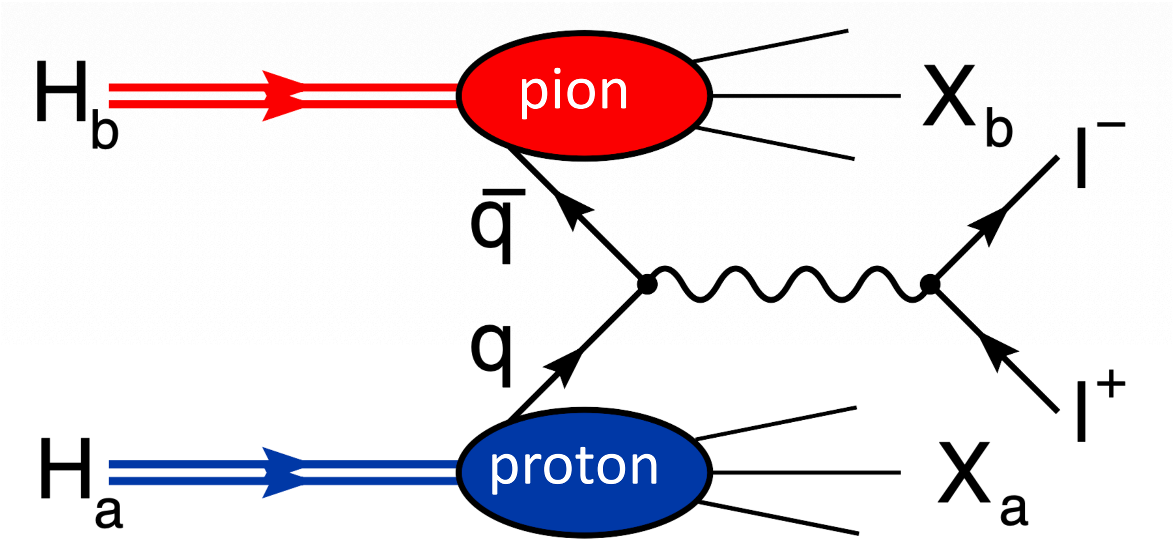


- ➢ Dihadron production in e^+e^-

– The *transverse momentum of the observed hadron or lepton pair is measured* in the $q_T \ll Q$ region, where it cannot arise from hard scattering alone and is instead dominated by intrinsic parton transverse momentum and soft radiation.

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

➢ DY cross section with pions and **polarized proton** is described in terms of *six structure functions* Bastami, Gamberg, Parsamyan, Pasquini, Prokudinb, and Schweitzer, JHEP 02, (2021), 166.



$$f_{\bar{u}|\pi} \otimes f_{u|p}$$

pion proton

$$F^1_{UU} \sim f_{1,\pi} \otimes f_{1,p}$$

$$F^{\cos 2\phi}_{UU} \sim h^\perp_{1,\pi} \otimes h^\perp_{1,p}$$

$$F^{\sin 2\phi}_{UL} \sim h^\perp_{1,\pi} \otimes h^\perp_{1L,p}$$

$$F^{\sin \phi_S}_{UT} \sim f_{1,\pi} \otimes f^\perp_{1T,p}$$

$$F^{\sin(2\phi-\phi_S)}_{UT} \sim h^\perp_{1,\pi} \otimes h_{1,p}$$

$$F^{\sin(2\phi+\phi_S)}_{UT} \sim h^\perp_{1,\pi} \otimes h^\perp_{1T,p}$$

Drell-Yan process with pions and polarized nucleons $\pi^- + p^\uparrow \to \mu^+\mu^- + X$

- DY cross section with pions and **polarized proton** is described in terms of *six structure functions*

pion		proton	
unpolarized TMD	$\otimes$	unpolarized TMD	$F_{UU}^{1} \sim f_{1,\pi} \otimes f_{1,p}$
Boer–Mulders function	$\otimes$	Boer–Mulders function	$F_{UU}^{\cos 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}^{\perp}$
Boer–Mulders function	$\otimes$	worm-gear L	$F_{UL}^{\sin 2\phi} \sim h_{1,\pi}^{\perp} \otimes h_{1L,p}^{\perp}$
unpolarized TMD	$\otimes$	Sivers function	$F_{UT}^{\sin \phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^{\perp}$
Boer–Mulders function	$\otimes$	transversity	$F_{UT}^{\sin(2\phi-\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}$
Boer–Mulders function	$\otimes$	pretzelosity	$F_{UT}^{\sin(2\phi+\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1T,p}^{\perp}$

transversely polarized proton target

COMPASS Drell-Yan experiment for single-spin asymmetries in 2015

$$A^{\sin\phi_S} \sim f_{1,\pi} \otimes f_{1T,p}^{\perp}$$

pion unpolarized TMD $\otimes$ proton Sivers function

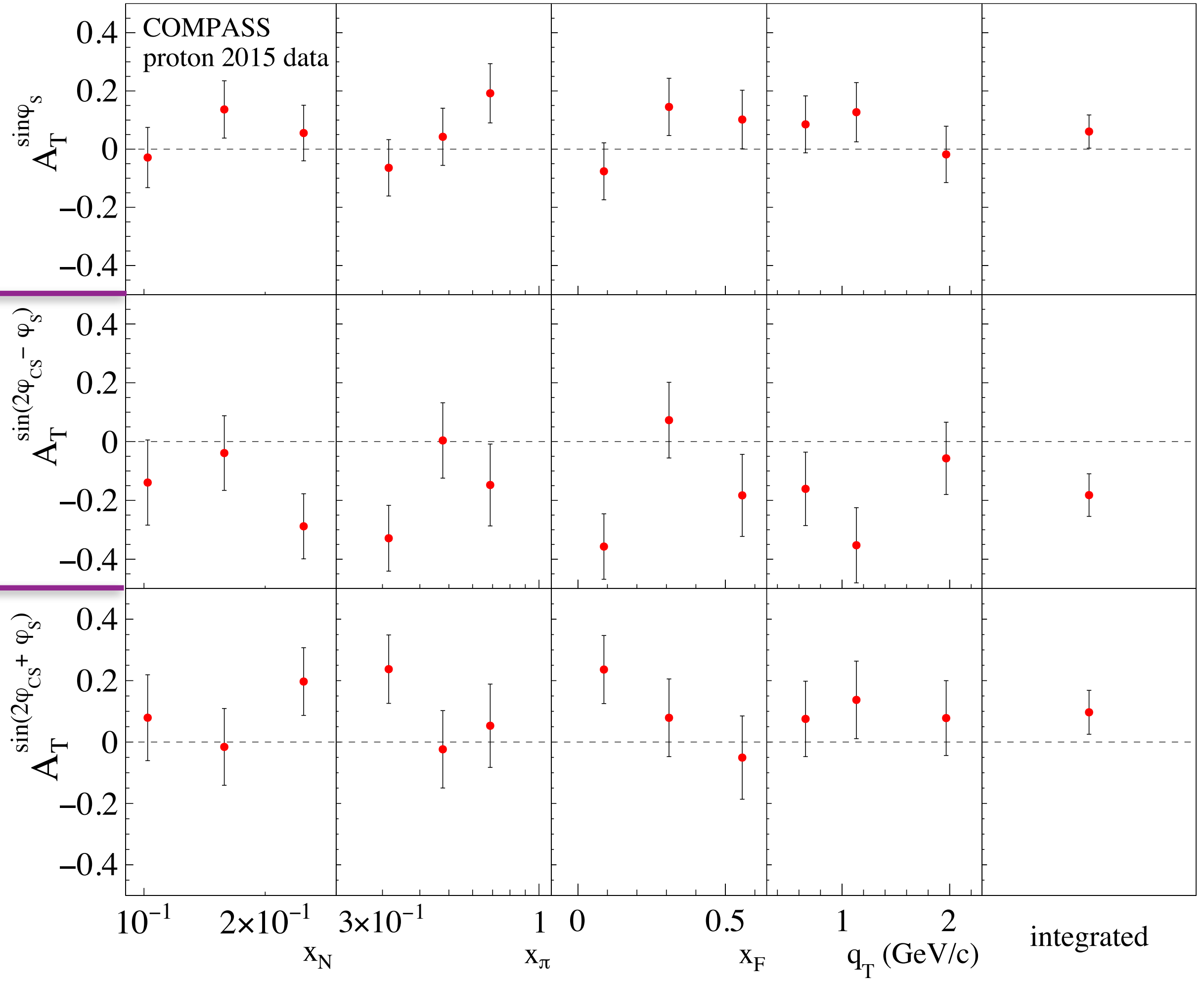
$$A^{\sin(2\phi_{CS}-\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1,p}$$

pion Boer–Mulders function $\otimes$ proton transversity

$$A^{\sin(2\phi_{CS}+\phi_S)} \sim h_{1,\pi}^{\perp} \otimes h_{1T,p}^{\perp}$$

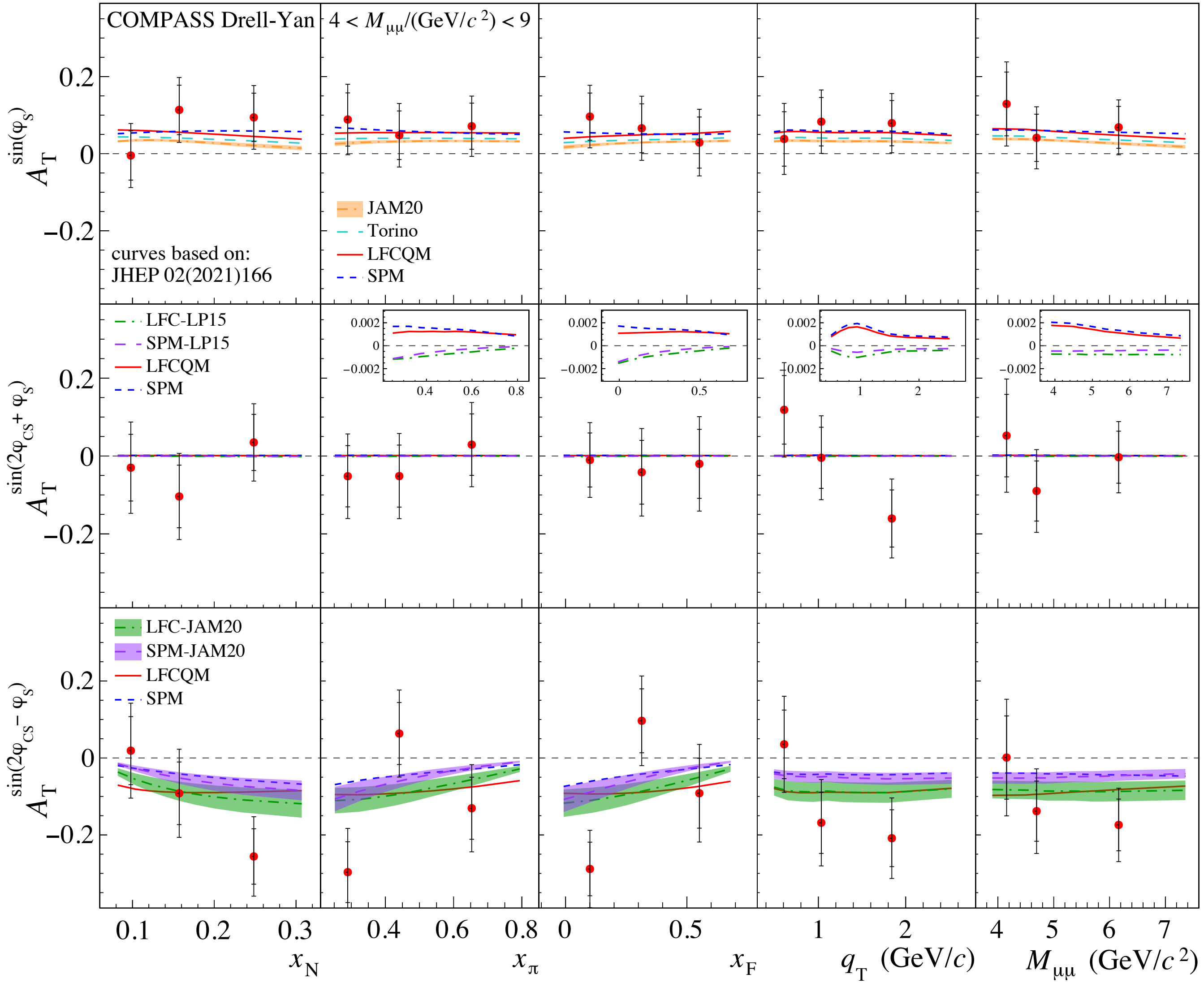
pion Boer–Mulders function $\otimes$ proton pretzelosity

$$A_{XY}^{\text{weight}} = \frac{F_{XY}^{\text{weight}}}{F_{UU}^{1}}$$



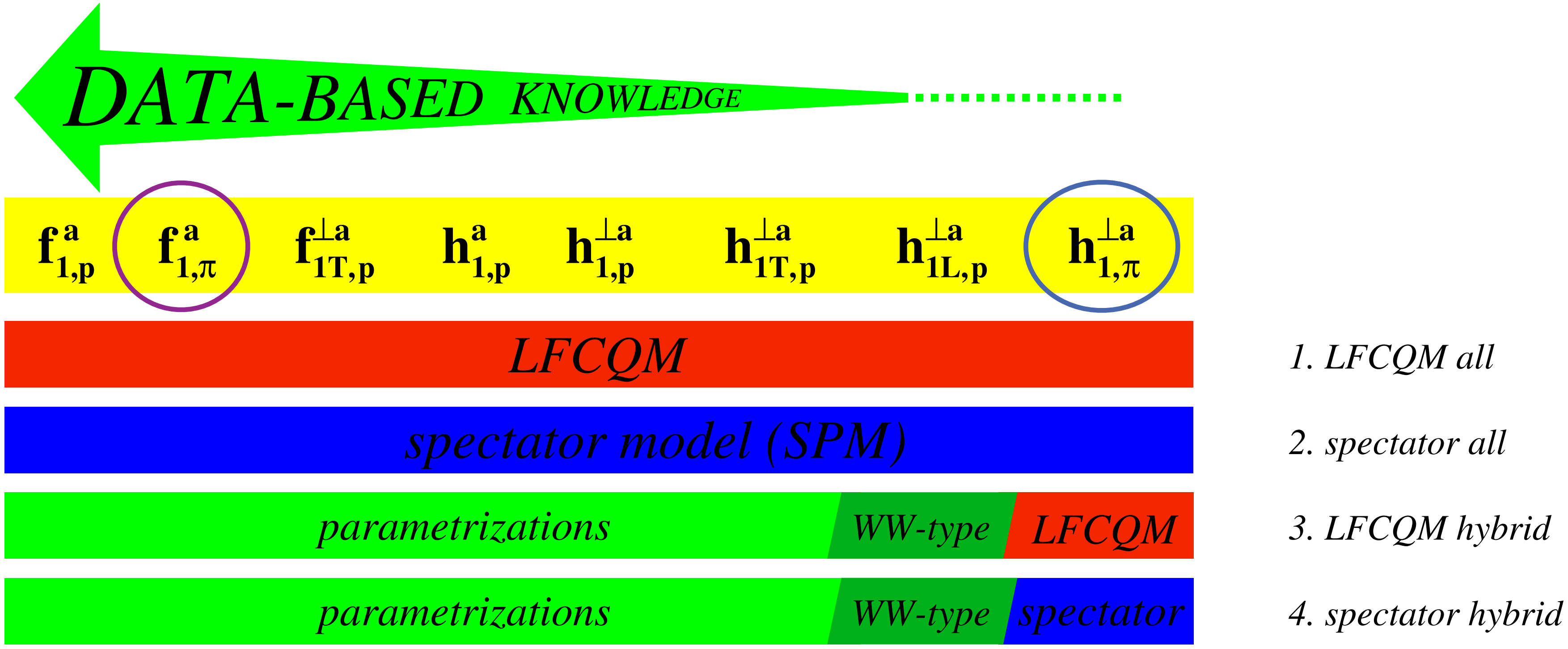
First measurement of transverse-spin-dependent azimuthal asymmetries in the Drell-Yan process, COMPASS Collaboration, Phys. Rev. Lett. 119, 112002 (2017)

COMPASS Drell-Yan experiment for single-spin asymmetries in 2018



Final COMPASS Results on the Transverse-Spin-Dependent Azimuthal Asymmetries in the Pion-Induced Drell-Yan Process, COMPASS Collaboration, Phys. Rev. Lett. 133 (2024) 071902

Pion and Proton TMDs: From Best to Least Constrained

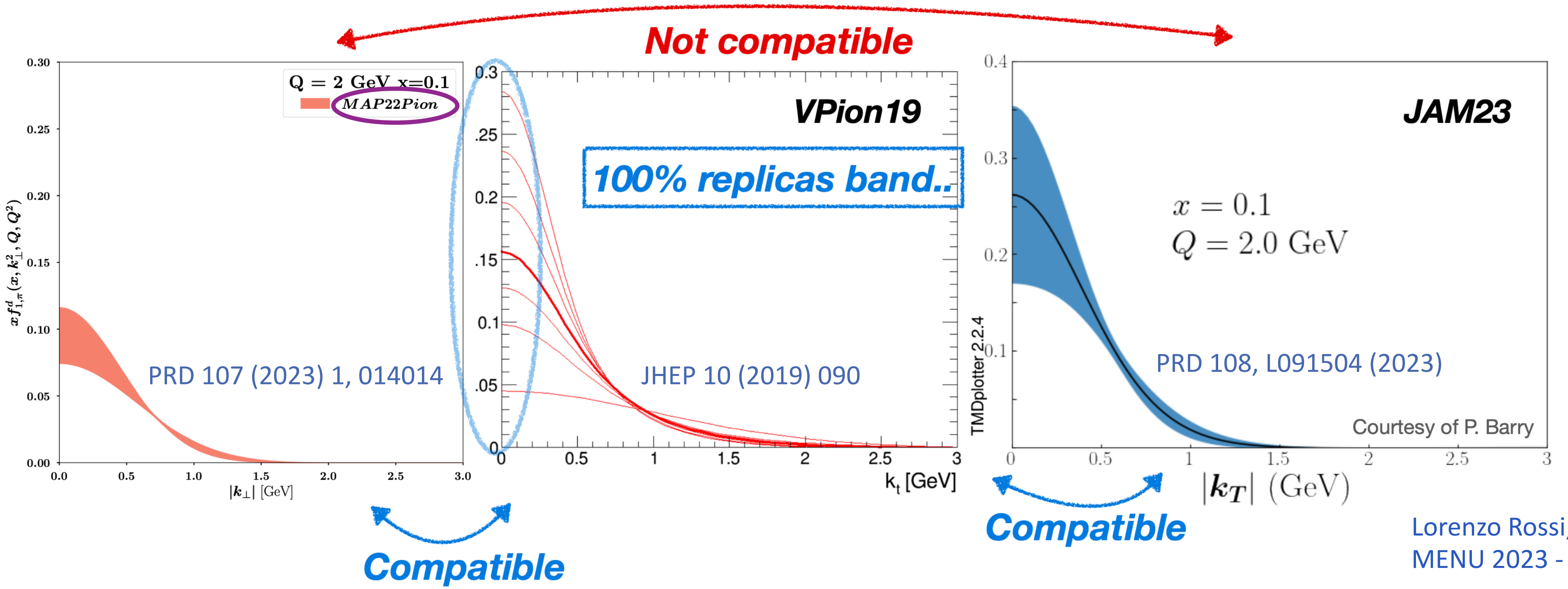


Bastami, Gamberg, Parsamyan, Pasquini, Prokudinb, and Schweitzer, JHEP 02, (2021), 166.

- The pion unpolarized TMDs $f_{1,\pi}(x, k_\perp)$ have been studied and much progress was achieved in incorporating effects of QCD evolution.
- The pion Boer-Mulders function $h^\perp_{1,\pi}(x, k_\perp)$ is the *least known of the TMDs* needed to describe the pion-proton DY process at leading twist. No extractions are currently available for this TMD.

Unpolarized Pion TMD: Phenomenological Extractions

Comparison of pion TMD PDFs

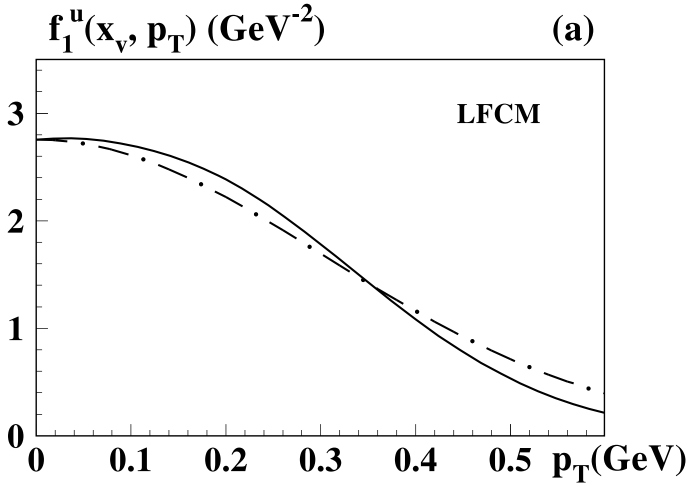


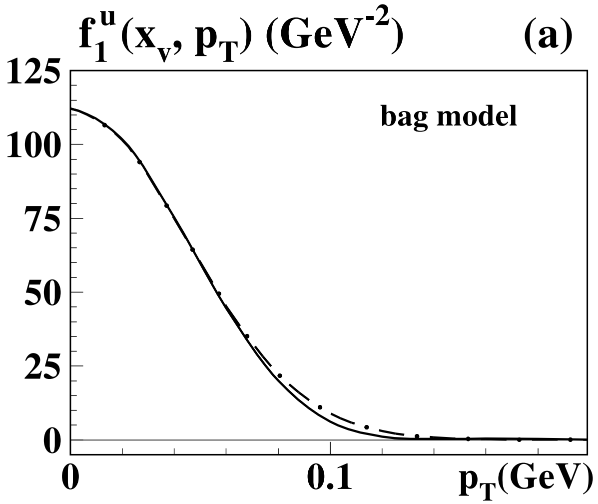
Lorenzo Rossi, MAP Collaboration, MENU 2023 - Mainz

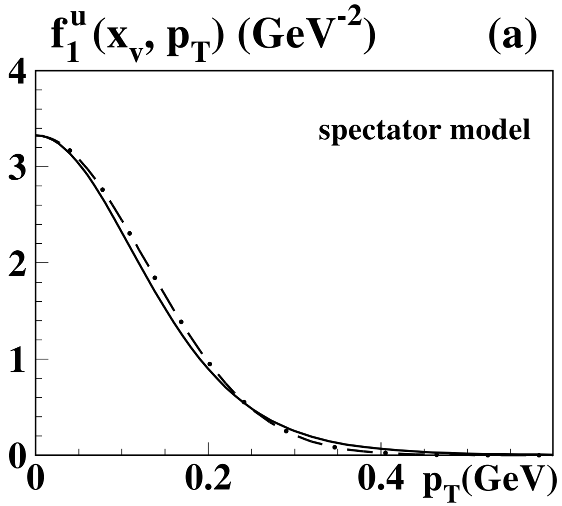
➢ **Unpolarised** Drell-Yan lepton-pair production, **E615 and E537 experiments data**.

Extraction of pion transverse momentum distributions from Drell-Yan data, MAP Collaboration, Phys. Rev. D 107, 014014 (2023)

Unpolarized Pion TMD: in models







(a) Pion	LFCM		
TMD	$\langle p_T \rangle$	$\langle p_T^2 \rangle^{1/2}$	R_G
f_1^q	0.28	0.32	0.99
(b) Pion	LFCM	Bag	Spectator
TMD	$\langle p_{T,v}^2 \rangle^{1/2}$	$\langle p_{T,v}^2 \rangle^{1/2}$	$\langle p_{T,v}^2 \rangle^{1/2}$
f_1^q	0.420	0.063	0.180

$$R_G = \frac{2}{\sqrt{\pi}} \frac{\langle p_T \rangle}{\langle p_T^2 \rangle^{1/2}}$$

Gaussian approximations

- light-front constituent model (LFCM),
- bag model,
- spectator model.

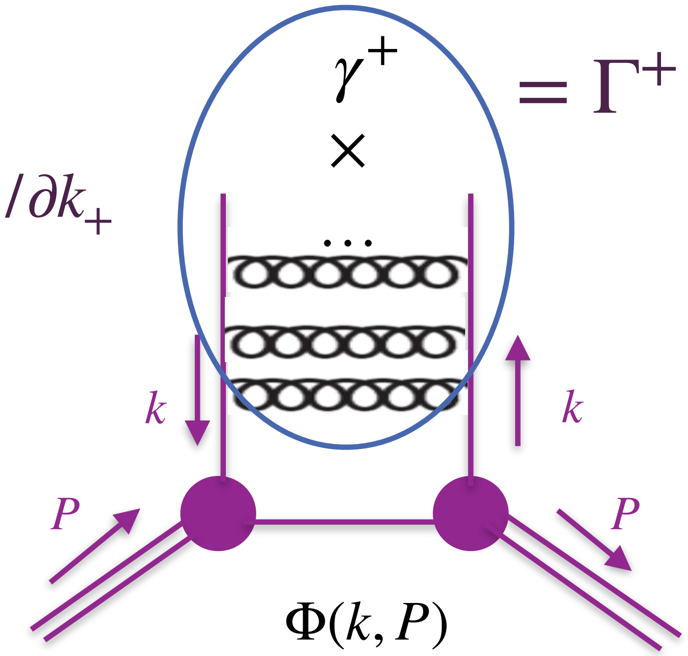
Lorcé, Pasquini, Schweitzer, Transverse pion structure beyond leading twist in constituent models, Eur. Phys. J. C 76 (2016) 7, 415.

Twist-2 PDF of pion

$$\Phi_{ij}(x) = \int dk^- d^2\mathbf{k}_\perp \Phi_{ij}(k,P)\Big|_{k^+=xP^+} = q(x)\gamma^+$$

$$q(x) \sim \int dk^- dk_\perp^2 \mathrm{Tr}\left[\gamma^+\Phi(k,P)\right]\Big|_{k^+=xP^+} \sim \int d^4k \mathrm{Tr}\left[\gamma^+\Phi(k,P)\right]\delta(k^+ - xP^+)$$

- Because of gluon dressing, the dressed quark-photon vertex is used, $\gamma^+ \to \Gamma^+$, which is constrained by Ward's identity $\Gamma^+(k) = \partial S^{-1}(k)/\partial k_+$
- The rest is $S(k)\Gamma_\pi(k,-P)S(k+P)\Gamma_\pi(k,P)S(k)$



$$q_A(x) \sim \int d^4k \mathrm{Tr}\big[\Gamma^+(k)S(k)\Gamma_\pi(k,-P)S(k+P)$$
$$\times \Gamma_\pi(k,P)S(k)\big]\delta(k^+ - xP^+)$$

Twist-2 PDF of pion

- DF: (term A) $$q_A(x) = \int d^4k \mathrm{Tr}\left[n^\mu \frac{\partial S(k)}{\partial k_\mu}\Gamma_\pi(k,-P)S(k+P)\Gamma_\pi(k,P)\right]\delta(n\cdot k - xn\cdot P)\,.$$

- A second contribution, referred to as "term BC", is introduced to restore symmetry: the pion PDF at hadron scale is expected to be a symmetric function if we consider isospin symmetry: $q_{\zeta_H}(x) = q_{\zeta_H}(1-x)\,.$

$$q_{BC}(x) = \int d^4k \mathrm{Tr}\left[n^\mu S(k)\frac{\partial \Gamma_\pi(k,-P)}{\partial k_\mu}S(k+P)\Gamma_\pi(k,P)\right]\delta(n\cdot k - xn\cdot P)\,.$$

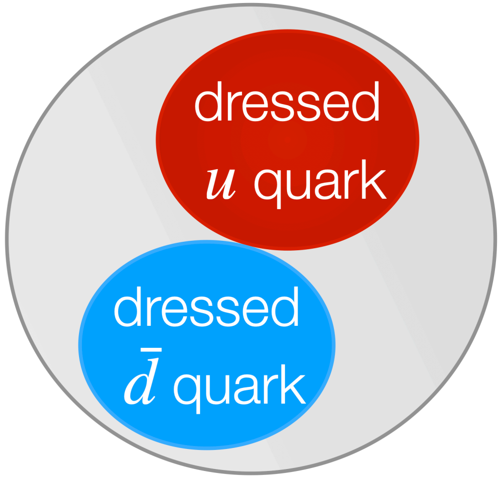
- DF:

$$q(x) = q_A(x) + q_{BC}(x) = \int d^4k \mathrm{Tr}\left[n^\mu \frac{\partial [S(k)\Gamma_\pi(k,-P)]}{\partial k_\mu}S(k+P)\Gamma_\pi(k,P)\right]\delta(n\cdot k - xn\cdot P)\,.$$

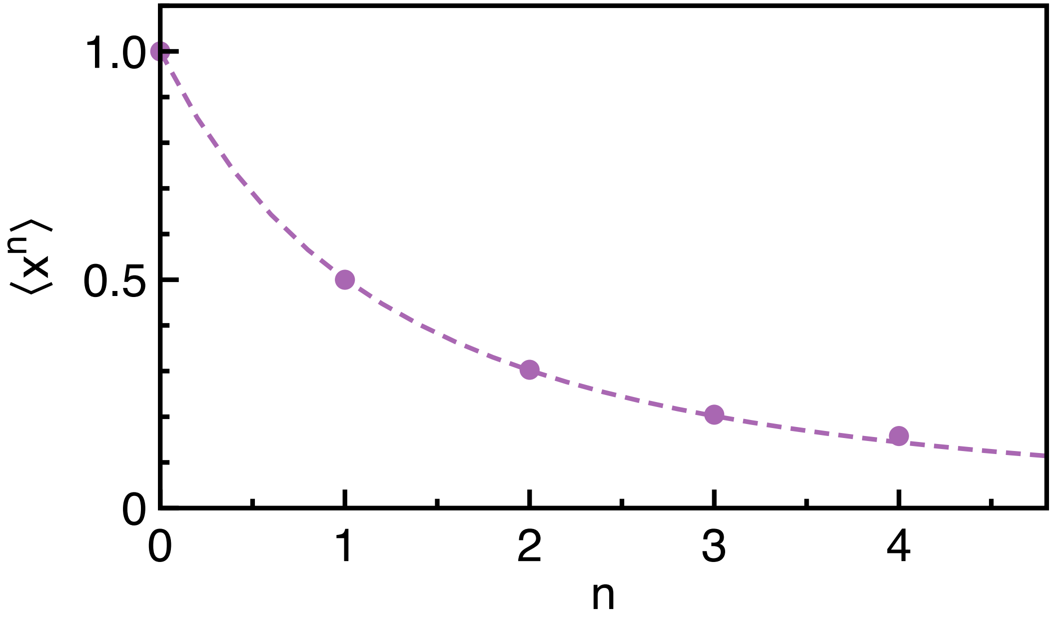
Chang, Mezrag, Moutarde, Roberts, Rodríguez-Quintero, Tandy, Phys. Lett. B737 (2014) 23-29

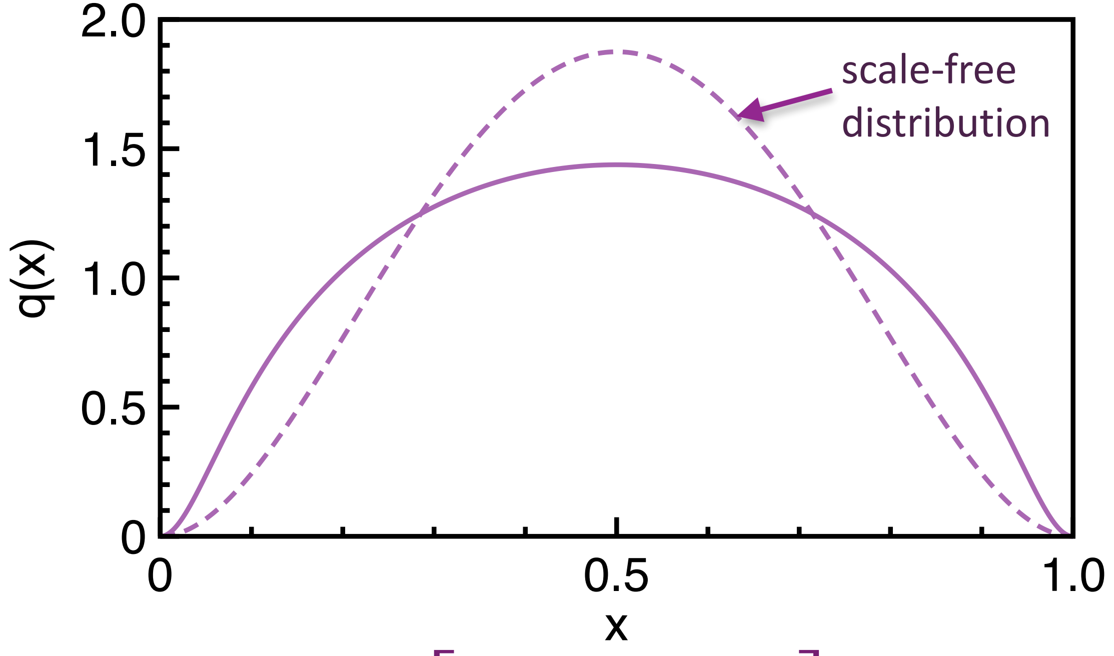
Twist-2 PDF of pion

- PDF $$q(x) = \int d^4k \mathrm{Tr}\left[\gamma^+ \Phi(k,P)\right] \delta(k^+ - xP^+)$$
- Mellin moments $$\langle x^n \rangle = \int dx x^n q(x) \sim \int d^4k \mathrm{Tr}\left[\gamma^+ \Phi(k,P)\right] (k^+)^n/(P^+)^n$$
- The calculation only requires the quark propagator and meson Bethe-Salpeter amplitude, which are calculated using the conventional method.
- Gauge sector: $g(k^2) = g_{IR}(k^2) + g_{UV}(k^2)$, with $g_{IR}(k^2) = \dfrac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2}$ and $g_{UV}(k^2)$ is from one-loop perturbative calculation.
- At a hadron scale $\zeta_H = 0.331$ GeV, dressed valence-quarks carry all the pion's light-front momentum and the glue and sea distributions vanish.
- In isospin limit $m_u = m_{\bar{d}}$, $q(1-x, \zeta_H) = q(x, \zeta_H)$.



Twist-2 PDF of pion





- ➢ Lowest five Mellin moments of the pion valence-quark DF.
- ➢ Scale free result $q_{\rm sf}(x) = 30x^2(1-x)^2$

✓ Large x behaviour: $q^{\pi}(x;\zeta_H) \sim (1-x)^2$

$$\mathbf{q}(x) = \frac{1}{2}\ln\left[1 + \frac{x^2(1-x)^2}{\rho_{DF}^2}\right], \quad \rho_{DF} = 0.061$$

✓ Broad function, induced by dynamical chiral symmetry breaking, and emergence of hadron mass (EHM).

Ding, Raya, Binosi, Chang, Roberts, Schmidt, Phys. Rev. D 101 (2020) 5, 054014

Twist-2 TMDs of pion

unpolarized TMD   Boer–Mulders function

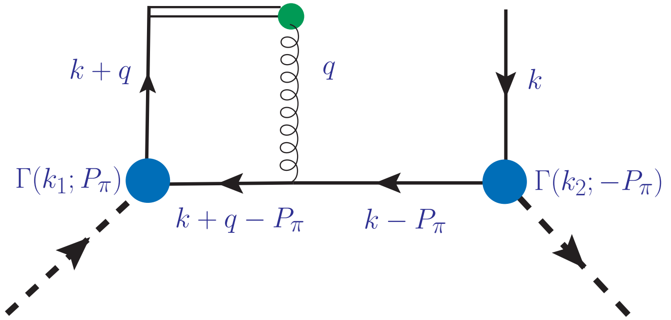
$$\Phi_{ij}(x,\boldsymbol{k}_\perp) = \int dk^-\Phi_{ij}(k,P,S)\Big|_{k^+=xP^+} = f_1(x,\boldsymbol{k}_\perp)\gamma^+ + h_1^\perp(x,\boldsymbol{k}_\perp)\sigma^{+i}$$

$$f_1(x,\boldsymbol{k}_\perp) = \int dk^-\mathrm{Tr}\left[\gamma^+\Phi(k)\right]\Big|_{k^+=xP^+} = \int d^4q\,\mathrm{Tr}\left[\gamma^+\Phi(q)\right]\delta(q^+-xP^+)\delta(k_\perp-q_\perp)$$

$$h_1^\perp(x,\boldsymbol{k}_\perp) = \int dk^-\mathrm{Tr}\left[\sigma^{+i}\Phi(k)\right]\Big|_{k^+=xP^+} = \int d^4q\,\mathrm{Tr}\left[\sigma^{+i}\Phi(k)\right]\delta(q^+-xP^+)\delta(k_\perp-q_\perp) = 0$$

- Wilson line contribution in

$$\Phi_{ij}(k,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi e^{ik\cdot\xi}\langle P,S|\bar{\psi}_j(0)\mathcal{U}(0,\xi)\psi_i(\xi)|P,S\rangle$$



- Interaction between the eikonalised quark and the spectator ⇒ nonzero $h_1^\perp(x,\boldsymbol{k}_\perp)$

Cheng, Cui, Ding, Roberts, Schmidt, Eur. Phys. J. C 85 (2025) 1, 115

Cheng, Ding, Binosi, Roberts, arXiv: 2603.25941 [hep-ph]

Twist-2 unpolarised TMD of pion

➢ TMD $\quad f_1(x, \boldsymbol{k}_\perp) = \int d^4q \mathrm{Tr}\left[\gamma^+\Phi(q)\right] \delta(q^+ - xP^+)\delta(k_\perp - q_\perp)$

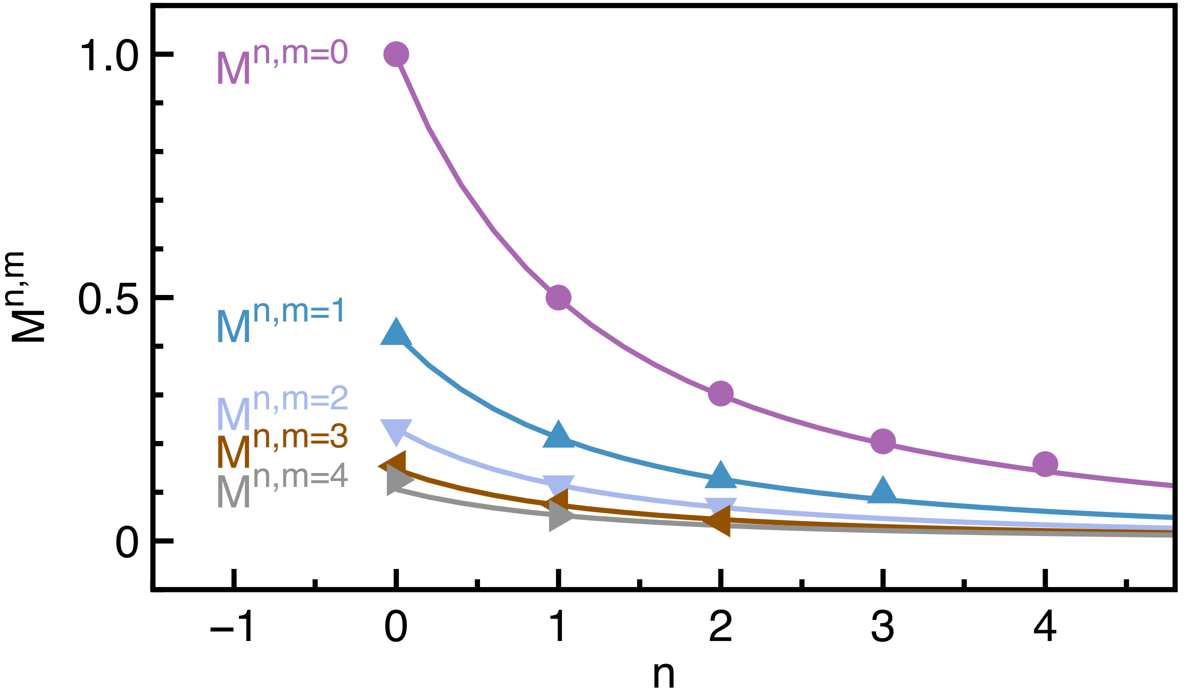
➢ Mellin-transverse moments

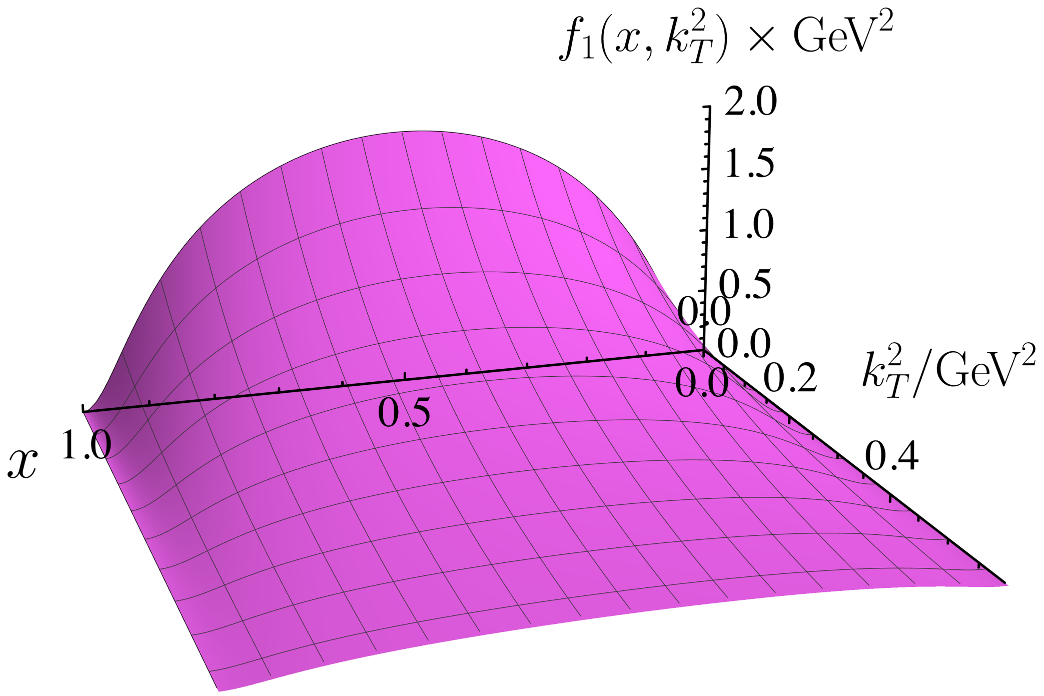
$$\mathscr{M}^{n,m} = \int_0^1 dx\, x^n \int d^2k_T\, k_T^m\, f_1(x, k_T^2)$$

$$\sim \int d^4k \mathrm{Tr}\left[\gamma^+\Phi(k,P)\right](k^+)^n/(P^+)^n(q_{Tn})^m$$

where $q_{Tn} = |\boldsymbol{q}_T|$ is the magnitude of the transverse momentum, defined using the transverse basis vectors n_{T_1} and n_{T_2} as $q_{Tn} = \sqrt{(n_{T_1}\cdot q)^2 + (n_{T_2}\cdot q)^2}$, $n_{T_1} = (0,1,0,0), n_{T_2} = (0,0,1,0)$.

Twist-2 unpolarised TMD of pion





- ➢ Lowest seventeen moments Mellin-transverse moments of the pion unpolarised valence-quark TMD. Ding, Phys. Lett. B 879 (2026) 140677

$$f_1(x, k_T^2) = \frac{1}{2}\ln\left[1 + \frac{x^2(1-x)^2}{\rho_{TMD}^2}\right]\frac{e^{-k_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle},$$

with $\langle k_T^2\rangle = 0.231$ GeV^2 and $\rho_{\mathrm{TMD}} = \rho_{\mathrm{DF}}$.

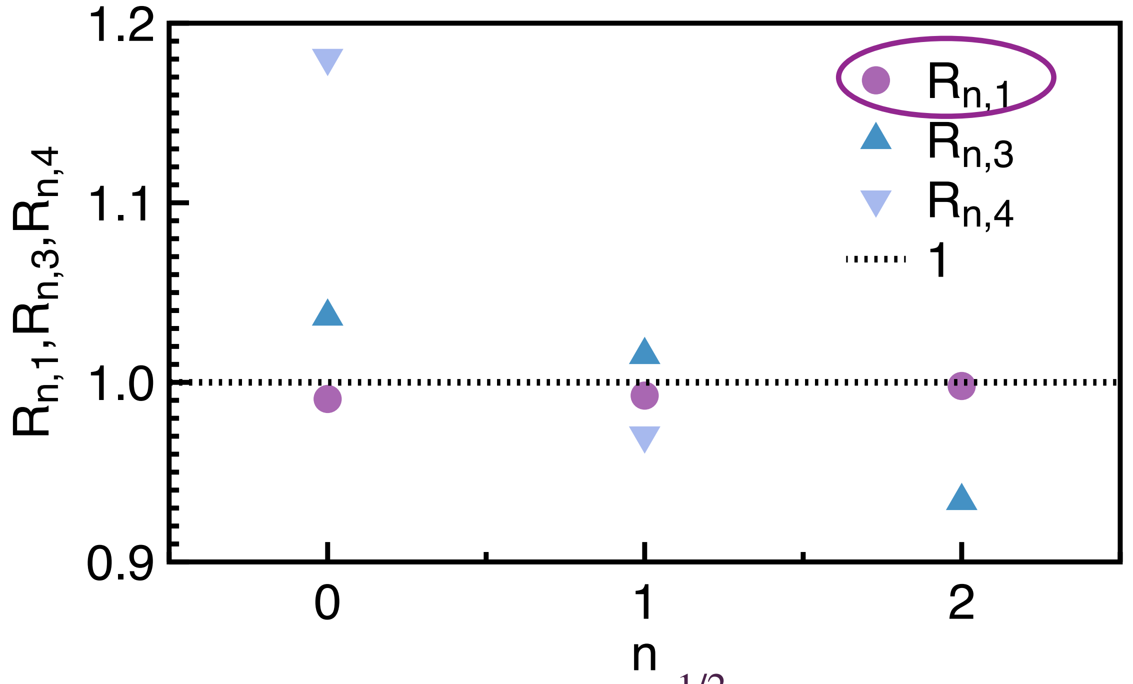
$$\langle \boldsymbol{k}_T\rangle = 0.422\ \mathrm{GeV}$$

Gaussianity of the transverse-momentum distribution

➢ To quantify deviations from a Gaussian transverse profile, we construct dimensionless ratios

$$R_{n,m} \equiv \frac{1}{\Gamma\left(1+\frac{m}{2}\right)} \frac{\mathscr{M}^{n,m}/\mathscr{M}^{n,0}}{\left(\mathscr{M}^{n,2}/\mathscr{M}^{n,0}\right)^{m/2}},$$

➢ which satisfy $R_{n,m}=1$ for all m if the transverse-momentum dependence is purely Gaussian.



➢ $\mathscr{M}^{0,1}=\langle k_T\rangle = 0.422$ GeV, $\mathscr{M}^{0,2}=\langle k_T^2\rangle = 0.231$ GeV^2. Therefore $\langle k_T\rangle \simeq \left(\frac{\pi}{4}\langle k_T^2\rangle\right)^{1/2}$ confirming ***near-Gaussian behavior of the transverse-momentum distribution at the hadron scale.***

➢ The intrinsic transverse width $\langle k_\perp^2\rangle = 0.267$ GeV^2 $(\omega = 0.4$ GeV$)$, and $\langle k_\perp^2\rangle = 0.193$ GeV^2 $(\omega = 0.6$ GeV$)$, In all cases, $\langle k_T\rangle \simeq \left(\frac{\pi}{4}\langle k_T^2\rangle\right)^{1/2}$ is satisfied to high precision.

Large k_T behavior of $f_1^{\pi}(x, \boldsymbol{k}_\perp)$

- It is possible to determine the power behavior at large $k_\perp$ for the different distributions, for example Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008) 023

$$f_1(x, k_\perp^2) \sim \frac{1}{k_\perp^2} \alpha_s \sum_{i=q,\bar{q},g} q^i(x) \otimes K_1^i$$

$$h_1^\perp(x, k_\perp^2) \sim \frac{M^2}{k_\perp^4} \alpha_s \sum_{i=q,\bar{q},g} [\text{collinear twist-three distributions}] \otimes K_3^i$$

- If this is true, then

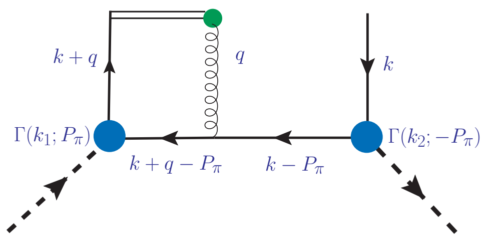
$$\langle \boldsymbol{k}_\perp^m \rangle = \int dk_\perp^2 (|\boldsymbol{k}_\perp|)^m f_1(x, k_\perp) \quad (\text{power divergence for } m \geq 1)$$

- The calculated TMD primarily describes the soft ***nonperturbative component of the transverse-momentum distribution*** and does not reproduce the perturbative large- k_T tail expected in full QCD.

Small k_T behavior of $f_1^{\pi}(x, \boldsymbol{k}_\perp)$

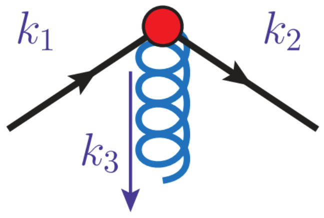
- Within the present ***impulse approximation*** framework, the TMD is determined solely by the dressed-quark propagator and pion Bethe–Salpeter amplitude, whose ultraviolet behavior leads to a substantially faster falloff.
- ***Lower transverse moments*** remain numerically stable and provide reliable information on the ***infrared structure***, whereas ***higher moments*** become increasingly sensitive to ***ultraviolet details*** and should therefore be interpreted with appropriate caution.
- A more complete description of the ultraviolet region will require

✓ explicit incorporation of gauge-link contributions,



Cheng, Cui, Ding, Roberts, Schmidt, Eur. Phys. J. C 85 (2025) 1, 115

✓ exchange-current effects beyond impulse approximation,



Costa et al, Phys. Rev. C 104 (2021) 4, 045201.

✓ transverse components of the dressed quark–photon vertex.

$$\Gamma_\mu(k,p) = \Gamma_\mu^L(k,p) + \Gamma_\mu^T(k,p)\,,$$

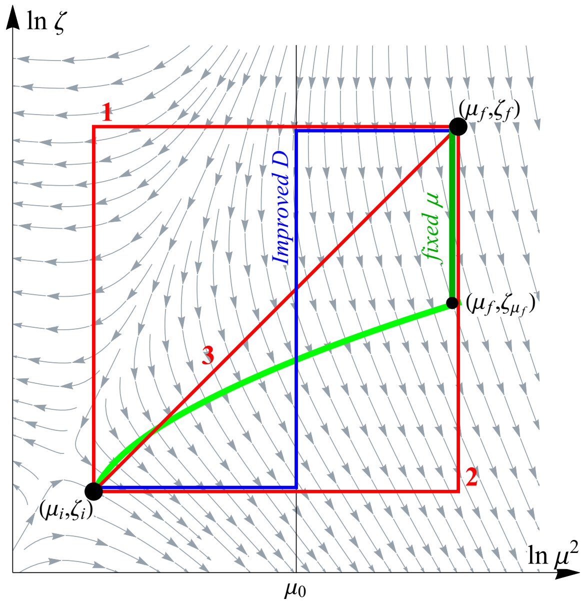
$$\Gamma_\mu^L(k,p) = \sum_{j=1}^{4} \lambda_j(k,p)\, L_\mu^j(k,p)\,,$$

$$\Gamma_\mu^T(k,p) = \sum_{j=1}^{8} \tau_j(k,p)\, T_\mu^j(k,p)\,,$$

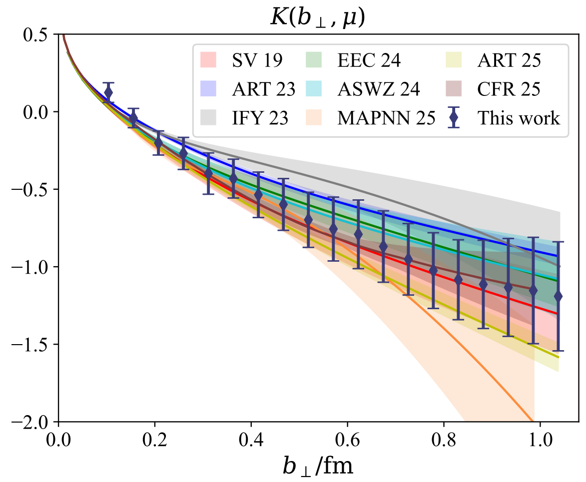
Qin, Chang, Liu, Roberts, Schmidt, Phys. Lett. B 722 (2013) 384-388.

QCD evolution

- TMDs generally depend on a renormalization scale μ and rapidity scale ζ, the latter governing *Collins–Soper evolution*.
- The present calculation is performed at the hadron scale μ_H, where the pion is described solely in terms of dressed valence quark and antiquark. The x-dependent functions correspond to collinear distributions and the exponential factors are *"primordial shapes" of TMDs at the initial scale*. This particular dependence is *often used in phenomenology*, corresponds to the Gaussian Ansatz and is supported in models.
- Consequently, the resulting TMD may be defined at a fixed hadron-scale rapidity parameter ζ_H, naturally of order μ_H^2.



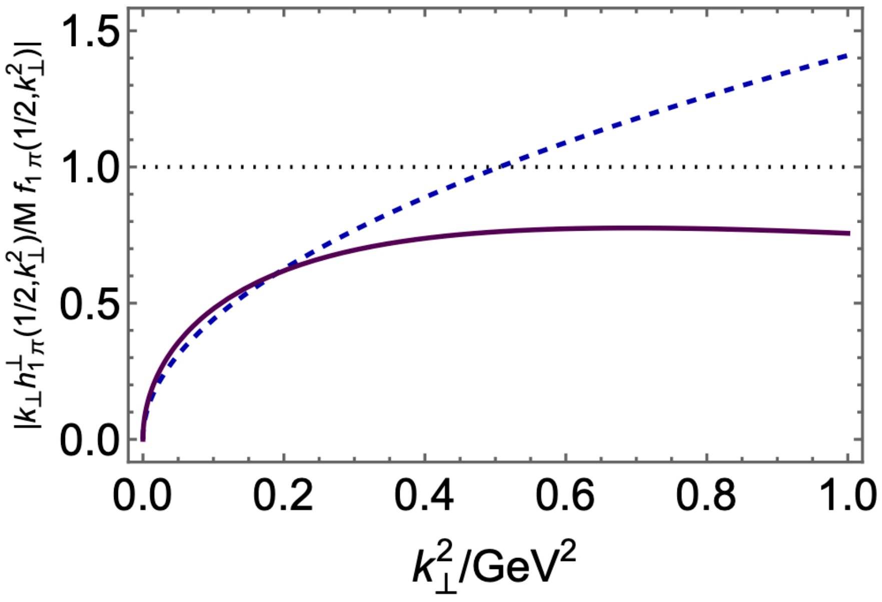
Scimemi and Vladimirov, J. High Energ. Phys. 2018, 3 (2018)

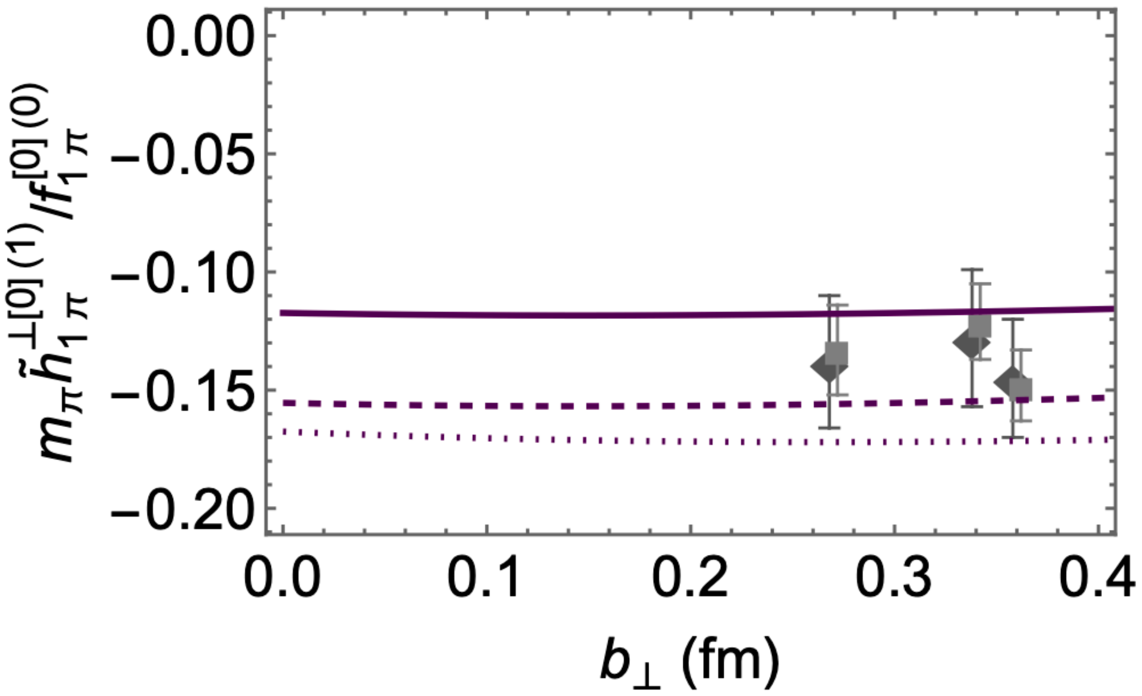


Jin-Xin Tan et al.,.Phys.Rev.D 113 (2026) 5, 054505

TMDs of pion: Boer-Mulders function

Cheng et al, *Eur. Phys. J. C* 85 (2025) 1, 115





- Positivity constraint: the bound is violated if the curve crosses the horizontal dotted line. $|k_\perp h_{1\pi}^{\perp\text{MI}}(x=1/2,k_\perp^2)/Mf_{1\pi}(x=1/2,k_\perp^2)|$ -- dashed blue cf. $|k_\perp h_{1\pi}^{\perp}(x=1/2,k_\perp^2)/Mf_{1\pi}(x=1/2,k_\perp^2)|$ solid purple.

- Generalised Boer Mulders shift. Dotted purple curve -- hadron scale physical pion mass result; dashed purple curve -- hadron scale heavy pion ($m_\pi = 0.518$ GeV); and solid purple curve -- heavy pion result evolved $\zeta_{\mathscr{H}} \to \zeta_2$. LQCD results, obtained with $m_\pi = 0.518$ GeV at a resolving scale $\zeta = \zeta_2 = 2$ GeV.

Summary

- Determination of the *infrared structure* of the *pion twist-2 unpolarized valence-quark TMD*.
- A *transverse-projection method* was proposed that enables the computation of transverse-momentum moments of the TMD.
- The method is general and readily extendable to other hadrons and transverse observables, establishing a pathway for studying three-dimensional hadron structure.

Outlook

- Boer-Mulders function with realistic interaction, Kaon, proton.
- Collins-Soper kernel, QCD evolution.
- Comparison with COMPASS 2018 Pion-Induced Polarized Drell–Yan Data.

Thank you!