

# One-point Energy/Charge Correlator

## Probes of Nucleon Transversity

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Based on: Mei-Sen Gao(高梅森), Zhong-Bo Kang, WL, Ding Yu Shao(邵鼎煜),  
Phys. Rev. Lett. 136, 151902 (2026),

WL, Xiaohui Liu(刘晓辉), Ding Yu Shao(邵鼎煜),  
arXiv: 2607.16392.

第一届强相互作用自旋物理会议  
青岛, 山东 — Jul 29th, 2026



# RHIC's talk on DIS2026, Bologna, Italy

- Renee Fatemi, *The 25 years RHIC legacy of QCD on DIS 2026*.

## Energy-Energy Correlators in $p^\uparrow p$

EECs are infrared and collinear safe jet substructure observables that probe the evolution from the perturbative parton shower to non-perturbative hadronization.

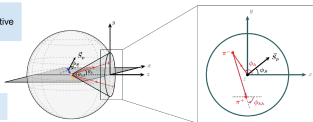
$$\Sigma_{\text{EEC}}(\theta_h, \phi_h) = \frac{d \left( \sum_{\text{jet}} \sum_i z_{h,i} \right)}{d\theta_h d\phi_h}$$

The One-Point Energy Correlator is the energy weighted angular distribution of energy flux inside of a jet

$$\frac{d\Sigma}{d\theta_h d\phi_h d\eta dP_T} = Z_{UV} + \sin(\phi_s - \phi_n) Z_{UT}$$

Clean probe of transversity and Collins FF when  $\theta_h \gg \Lambda_{\text{QCD}}/p_{T,\text{jet}}$

$$Z_{UT} = \frac{\alpha_s^2}{8} p_T^2 \theta_n \sum_{a,b,c} \int \frac{dx_1}{x_1} h_1^a(x_1, \mu) \int \frac{dx_2}{x_2} f_{2/B}(x_2, \mu) \times p_T \theta_n \mathcal{J}_{T,\perp}^a(\theta_n, Q) H_{ab \rightarrow c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u})$$



Increasing the power of the energy weight should increase the size of the asymmetry and probe higher Collins Mellin moments!

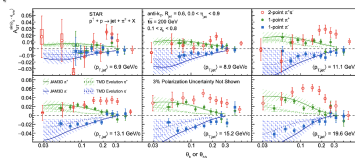
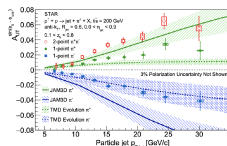
$$\Sigma_{\text{EEC}}(\theta_{hh}, \phi_{hh}) = \frac{d \left( \sum_{\text{jet}} \sum_{i,j} z_{h,i} z_{h,j} \right)}{d\theta_{hh} d\phi_{hh}}$$



OPEC @ 200 GeV

$$A_{UT}^{\sin(\phi_s - \phi_n)} = \frac{Z_{UT}}{Z_{UU}}$$

$$A_{UT} = \frac{1}{P} \frac{\Sigma_{\text{EEC}}^+(\phi_s - \phi) - \Sigma_{\text{EEC}}^-(\phi_s - \phi)}{\Sigma_{\text{EEC}}^+(\phi_s - \phi) + \Sigma_{\text{EEC}}^-(\phi_s - \phi)}$$



Theory curves *PRL* 136, 151902 (2026)

- JAM3D global analysis w/o evolution effects *Phys. Rev. D* 106, 034014 (2022).
- Parameterization of transversity and Collins from global analysis of  $e^+e^-$  and SIDIS data and implement TMD evolution as described in *Phys. Rev. D* 93, 014009 (2016).

- Experiments paper: arXiv.2604.15543 from STAR collaboration.

# Outline

- ① Motivation and process.
- ② One point energy/charge correlator:
  - Operator definition,
  - Spin decomposition,
  - Operator product expansion,
  - Factorization.
- ③ Phenomenology on RHIC.
- ④ Conclusion and outlook.

# Motivation: Transversity

- Three basic parton distribution functions (PDFs) for quark  $q$  in hadron  $h$ :
  - unpolarized PDF:  $f_{q/h}(x)$ ,
  - longitudinal polarized PDF  $\Delta f_{q/h}(x)$ ,
  - transversity PDF  $h_1^q(x)$   
(or noted as  $\delta f_{q/h}(x)$ ).

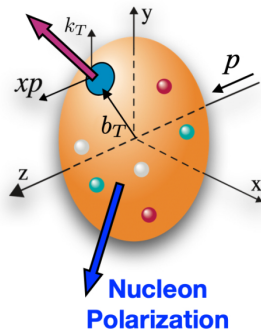
$$f_{q/h}(\xi) = \frac{1}{2} \left[ \left| \begin{array}{c} S \rightarrow \\ P \rightarrow \end{array} \begin{array}{c} \nearrow \xi P \\ \searrow \end{array} \right|^2 + \left| \begin{array}{c} S \rightarrow \\ P \rightarrow \end{array} \begin{array}{c} \nearrow \xi P \\ \searrow \end{array} \right|^2 \right]$$

$$\Delta f_{q/h}(\xi) = \frac{1}{2} \left[ \left| \begin{array}{c} S \rightarrow \\ P \rightarrow \end{array} \begin{array}{c} \nearrow \xi P \\ \searrow \end{array} \right|^2 - \left| \begin{array}{c} S \rightarrow \\ P \rightarrow \end{array} \begin{array}{c} \nearrow \xi P \\ \searrow \end{array} \right|^2 \right]$$

$$\delta f_{q/h}(\xi) = \frac{1}{2} \left[ \left| \begin{array}{c} S \uparrow \\ P \rightarrow \end{array} \begin{array}{c} \nearrow \xi P \\ \searrow \end{array} \right|^2 - \left| \begin{array}{c} S \uparrow \\ P \rightarrow \end{array} \begin{array}{c} \nearrow \xi P \\ \searrow \end{array} \right|^2 \right]$$

Figures from TMD handbook.

**Quark Polarization**



**Nucleon Polarization**

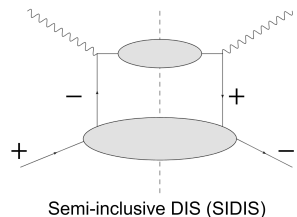
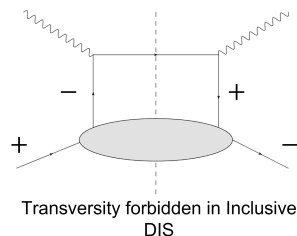


# Motivation: Transversity

- Transversity distribution  $h_1^q$  is a fundamental, yet less known, parton distribution.
- Its chiral-odd nature prevents access via inclusive DIS.

$$\delta q \equiv \int_0^1 dx [h_1^q(x) - h_1^{\bar{q}}(x)]$$

- Nucleon tensor charge  $\delta q$  from  $h_1^q$  is essential for:
  - Nucleon spin structure,  
C. A. Aidala, S. D. Bass, D. Hasch, G. K. Mallot, 2012.
  - Lattice QCD benchmarks,  
X. Gao, A. D. Hanlon, S. Mukherjee, P. Petreczky, Q. Shi, S. Syritsyn, and Y. Zhao, 2024.
  - BSM probes, i.e. neutron  $\beta$ -decay.  
J. D. Jackson, S. B. Treiman, and H. W. Wyld, 1957.



Collins function corresponds to the transverse momentum dependent (TMD) fragmentation function (FF) with transversely polarized quark that fragmented into unpolarized hadron .

Leading Quark TMDFFs

 Hadron Spin     Quark Spin

|                                 |   | Quark Polarization                                                                                                                                                                                       |                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------------------------------|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                 |   | Un-Polarized (U)                                                                                                                                                                                         | Longitudinally Polarized (L)                                                                                                                                                                 | Transversely Polarized (T)                                                                                                                                                                                                                                                                                                                                                                        |
| Unpolarized (or Spin 0) Hadrons |   | $D_1 = $ <br>Unpolarized                                                                                                |                                                                                                                                                                                              | $H_1^\perp = $  - <br>Collins                                                                                                                                                                                               |
|                                 | L |                                                                                                                                                                                                          | $G_1 = $  - <br>Helicity | $H_{1L}^\perp = $  -                                                                                                                                                                                                        |
| Polarized Hadrons               | T | $D_{1T}^\perp = $  - <br>Polarizing FF | $G_{1T}^\perp = $  -     | $H_1 = $  - <br>Transversity<br>$H_{1T}^\perp = $  -  |

Figure from TMD handbook.

# Probes of the fragmentation

- The TMD fragmentation function is defined

$$d_{h/q}(z, \vec{k}_T) = \int d\xi d^2x_T e^{i\omega\xi} \langle 0 | \chi_n(\xi, x_T) \mathcal{M}_h \bar{\chi}_n(0) | 0 \rangle,$$

where

$$\mathcal{M}_h(z, \vec{k}_T) = \not{\!\!\!\!\!\int}_X \sum_{a \in X} \delta_{ah} \delta \left( z - \frac{E_a}{\omega} \right) \delta^{(2)}(\vec{k}_T - \vec{k}_{aT}) |X, \text{out}\rangle \langle X, \text{out}|,$$

measures the energy fraction  $z$  and transverse momentum  $\vec{k}_T$  of identified hadron  $h$ .

- We can replace the measurement.

# Energy and Charge flow

- Angular correlator measurements:

- 1) Do not require tagging hadrons,
- 2) Do not measure energy fraction  $z$ ,
- 3) Measure angle instead of transverse momentum,
- 4) Proper weights.

$$\mathcal{J}_\omega(\hat{n}_T) = \int d\xi d^2x_T e^{i\omega\xi} \langle 0 | \chi_n(\xi, x_T) \hat{O}(\hat{n}) \bar{\chi}_n(0) | 0 \rangle.$$

## OPEC: energy flow

$$\hat{\mathcal{E}}(\hat{n})|X\rangle = \sum_{a \in X} \frac{E_a}{\omega} \delta^{(2)}(\hat{n} - \hat{n}_a)|X\rangle.$$

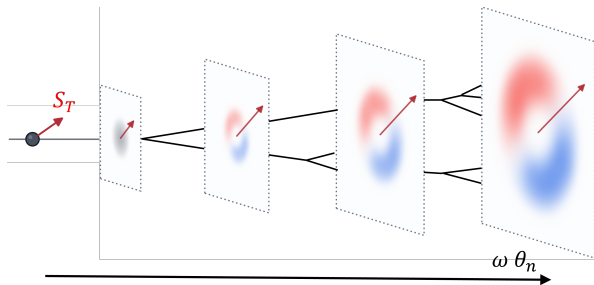
- Energy Weight  $z_a = E_a/\omega$ .
- $\omega$  is a hard scale.
- Energy weighting ensures IRC safety.

## OPCC: charge flow

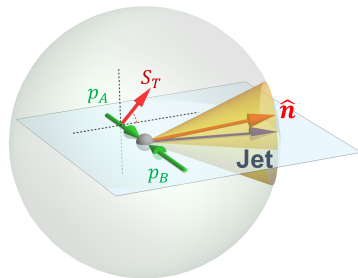
$$\hat{\mathcal{Q}}(\hat{n})|X\rangle = \sum_{a \in X} Q_a \delta^{(2)}(\hat{n} - \hat{n}_a)|X\rangle.$$

- Charge Weight  $Q_a$ .
- On “the Edge of Safety” (Monni, Vita, Xu, Zhu, 2025).
- Making use of track detector.

# One Point Energy Correlator (OPEC)



- Angular pattern of charge flow from a transversely polarized quark fragmentation.



- Energy/charge flow in the jet substructure.

# Spin Decomposition of OPEC and OPCC

- We do not require hadron tagging and hadron polarization detection in the energy/charge flow measure.
- Therefore, at leading twist TMD, the only two channels from the spin decomposition are

$$\begin{aligned}\omega^{-2} \mathcal{J}_{\gamma^+, \omega}(\hat{n}_T) &= J_D(\omega\theta_n), \\ \omega^{-2} \mathcal{J}_{i\sigma^\alpha + \gamma_5, \omega}(\hat{n}_T) &= \epsilon_T^{\alpha\beta} \omega \hat{n}_{T\beta} J_{1,\perp}(\omega\theta_n).\end{aligned}$$

- $J_D$  is unpolarized jet function.
- $J_{1,\perp}$  is transversely polarized jet function.
- $\hat{n} = (\hat{n}_T, \cos\theta_n)$ , where  $\hat{n}_T = \sin\theta_n(\cos\phi_n, \sin\phi_n)$  is measured from the fragmenting quark.

# Operator Product Expansions

- The jet functions can be matched onto their collinear counterparts via OPEs.

$$\tilde{J}_D^q(b_T, \mu, \zeta) = \sum_j \tilde{C}_{j/q}^D(b_T, \mu, \zeta) D_j(\mu),$$

$$\tilde{J}_{1,\perp}^q(b_T, \mu, \zeta) = \sum_j \tilde{C}_{j/q}^{\text{Collins}}(b_T, \mu, \zeta) \mathcal{D}_j^\perp(\mu).$$

- While it is possible to derive the matching from scratch (see appendix in [arXiv: 2607.16392](#)), a easier way to understand this is to connect the OPEC/OPCC jet functions to TMD FFs,

$$\text{OPEC : } J_\Gamma^q(b_T) = \sum_h \int dz z \tilde{d}_{\Gamma,h/q}(z, -\vec{b}_T)$$

$$\text{OPCC : } J_\Gamma^q(b_T) = \sum_h \int dz Q_h \tilde{d}_{\Gamma,h/q}(z, -\vec{b}_T)$$

# Operator Product Expansions

## OPEC

$$D_j(\mu) = \sum_h \int dz_h z_h D_{h/j}(z, \mu) = 1,$$

$$\mathcal{D}_j^\perp(\mu) = \sum_h \int dz_h z_h M_h H_{1h/j}^{\perp(1)}(z, \mu).$$

## OPCC

$$D_j(\mu) = \sum_h \int dz_h Q_h D_{h/j}(z, \mu) = Q_{\text{flow},j},$$

$$\mathcal{D}_j^\perp(\mu) = \sum_h \int dz_h Q_h M_h H_{1h/j}^{\perp(1)}(z, \mu).$$

- $D_{h/j}(z, \mu)$  is the unpolarized collinear FF.  $H_{1h/j}^{\perp(1)}(z, \mu)$  is the first moment of the Collins function.
- The OPEC matching coefficient is related to the TMD case by

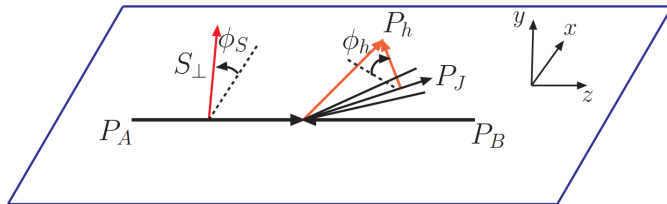
$$\tilde{C}_{j/q}(b_T, \mu, \zeta) = \int_0^1 dz z C_{j/q}(z, b_T, \mu, \zeta)$$

- To ensure infrad safety, OPCC needs to combine quark and antiquark matchings

$$\tilde{C}_{j/q}(b_T, \mu, \zeta) = \int_0^1 dz [C_{j/q}(z, b_T, \mu, \zeta) - C_{\bar{j}/q}(z, b_T, \mu, \zeta)].$$



# Jet Production in Transversely Polarized $p^\uparrow p$ Collision



- Proposed by F. Yuan, *Phys.Rev.Lett.* 100, 032003 (2008), the differential cross section comes with an azimuthal modulation,

$$\frac{d\sigma}{d\mathcal{PS}} = F_{UU} + \sin(\phi_s - \phi_h) F_{UT}.$$

- The reaction plane is spanned by the two incoming protons and the jet axis.
  - $\phi_s$ : the azimuthal angle of transverse polarization vector  $S_\perp$ , with respect the to reaction plane.
  - $\phi_h$ : the azimuthal angle of hadron  $h$  in jet, with respect the to reaction plane.

# Factorization

- We consider the OPEC in inclusive jet production in  $p^\uparrow + p \rightarrow J + X$

$$\frac{d\Sigma}{d\theta_n d\phi_n d\eta dp_T} = \sum_{h \in J} \int_0^1 dz_h \int d^2\Omega_h \delta(\phi_n - \phi_h) \delta(\theta_n - \theta_h) \mathcal{O}_h \frac{d\sigma}{dz_h d^2\Omega_h d\eta dp_T}$$

- The weight factor  $\mathcal{O}_h = z_h$  for OPEC, and  $\mathcal{O}_h = Q_h$  for OPCC.
- Integral on  $z_h$  converts the final-state  $z_h$  distribution into a number!
- The jet is characterized by the rapidity  $\eta$  and transverse momentum  $p_T$ .
- The azimuthal dependent OPEC inclusive jet production can be described by

$$\frac{d\Sigma}{d\theta_n d\phi_n d\eta dp_T} = Z_{UU} + \sin(\phi_s - \phi_n) Z_{UT}.$$

# Factorization with OPEC in Inclusive Jet Production

- The energy-weighted unpolarized and transversely polarized structure functions  $Z_{UU}$  and  $Z_{UT}$  admit the following factorized forms:

$$\begin{aligned}
 Z_{UU} &= \frac{\alpha_s^2}{s} p_T^2 \theta_n \sum_{a,b,c} \int \frac{dx_1}{x_1} f_{a/A}(x_1, \mu) \int \frac{dx_2}{x_2} f_{b/B}(x_2, \mu) \\
 &\quad \times J_D^c(p_T \theta_n) H_{ab \rightarrow c}^U(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}), \\
 Z_{UT} &= \frac{\alpha_s^2}{s} p_T^2 \theta_n \sum_{a,b,c} \int \frac{dx_1}{x_1} h_1^a(x_1, \mu) \int \frac{dx_2}{x_2} f_{b/B}(x_2, \mu) \\
 &\quad \times p_T \theta_n J_{1,\perp}^c(p_T \theta_n) H_{ab \rightarrow c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}).
 \end{aligned}$$

- Hard factors  $H$  for the partonic subprocess  $ab \rightarrow c$  for  $UU$  and  $UT$ .
- The transversity PDF  $h_1^a$  and unpolarized PDF  $f$ .

# TMD Evolution

- In the perturbative limit, the jet functions can be evaluated with OPE and TMD evolution

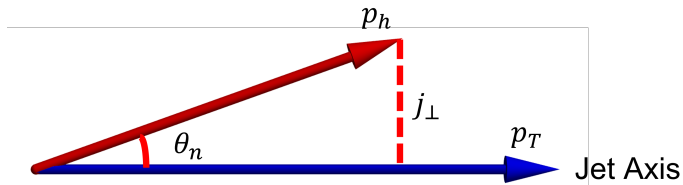
$$J_D^q(p_T\theta_n) = \int_0^\infty \frac{db_T b_T}{2\pi} J_0(p_T\theta_n b_T) \hat{C}_{j/q}^D(b_T) D_j(\mu_b) e^{-\frac{1}{2}S_{\text{pert}}(Q,b_T)},$$

$$p_T\theta_n J_{1,\perp}^q(p_T\theta_n) = \int_0^\infty \frac{db_T b_T^2}{2\pi} J_1(p_T\theta_n b_T) \delta\hat{C}_{i\leftarrow q}^{\text{collins}}(b_T) \mathcal{D}_j^\perp(\mu_b) e^{-\frac{1}{2}S_{\text{pert}}(Q,b_T)},$$

- The jet algorithm and radius dependencies can be further incorporated in the semi-inclusive fragmenting jet function formalism.

(Z.B. Kang, X. Liu, F. Ringer, and H. Xing, 2017; Z.B. Kang, K. Lee, and F. Zhao, 2020)),

## Kinematics of OPEC Hadron in Jet



- The geometry yields

$$p_h^z = |\mathbf{p}_J| z_h = |\mathbf{p}_h| \cos \theta_n,$$

- In the collinear limit, we further see:

$$\theta_n \approx \sin \theta_n = \frac{j_\perp}{p_h^z} = \frac{j_\perp}{p_T z_h},$$

- Note in the centra rapidity, jet  $|\mathbf{p}_J| = p_T$ .

$j_\perp$  or  $\theta_n$ ?

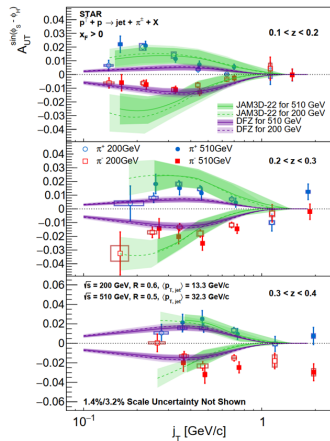
- Compared with standard non-weighted TMD analysis, we have a different overall factor

$$z_h dj_\perp^2 = p_T^2 z_h^3 d\theta_n^2,$$

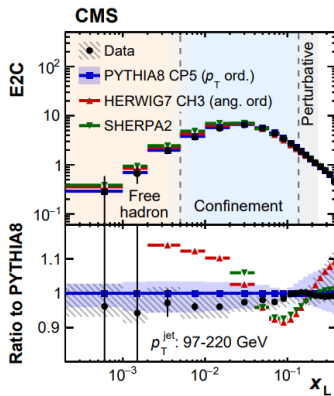
due to  $j_\perp \simeq p_T z_h \theta_n$  in the collinear limit.

Why switch to  $\theta_n$ ?

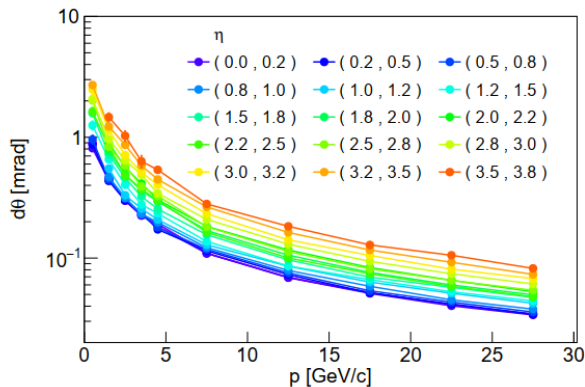
- In one STAR configuration,
  - $\sqrt{s} = 510$  GeV,  $p_T = 32.3$  GeV,
  - at lower cut  $z = 0.1$ ,
  - $j_\perp$  can be measured in about  $(0.1, 1)$  GeV.
- $j_\perp$  only corresponds to  $\theta_n$  about  $(0.03, 0.3)$  rad.



STAR collaboration arXiv.2507.16355

$j_\perp$  or  $\theta_n$ ?

CMS (2024)



J. Arrington et al., EIC (2021)

- However, modern colliders could achieve angular resolution at 1 mrad!

# Phenomenology Study on Energy-Weighted $\pi^\pm$ Production in Jet

- The OPEC Collins azimuthal asymmetry is defined as

$$A_{UT}^{\sin(\phi_s - \phi_n)} = \frac{Z_{UT}}{Z_{UU}}.$$

- We consider pion production in jets for two kinematic settings extensively studied by the STAR Collaboration:
  - (a)  $\sqrt{s} = 510$  GeV,  $p_T = 32.3$  GeV;
  - (b)  $\sqrt{s} = 200$  GeV,  $p_T = 13.3$  GeV.



# Evaluation Frameworks

We present evaluation in two approaches:

- A full TMD evolution framework
  - Parametrization of the transversity PDF, Collins functions, and non-perturbative Sudakov factor from [Kang, Prokudin, Sun, Yuan \(2016\)](#) ,
  - The extraction is done for SIDIS and  $e^+e^-$  annihilation,
  - The application on  $p^\uparrow p$  collision can be a complementary test of the TMD universality.
- The JAM3D-22 global QCD analysis: [JAM \(2022\)](#)
  - The TMD functions are modeled by Gaussian ansätze,
  - The corresponding collinear components evolve with only DGLAP.

# $p_T$ Distribution of the OPEC Collins Asymmetry

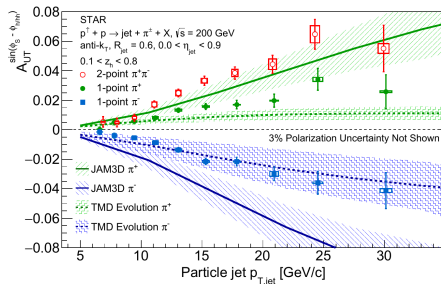
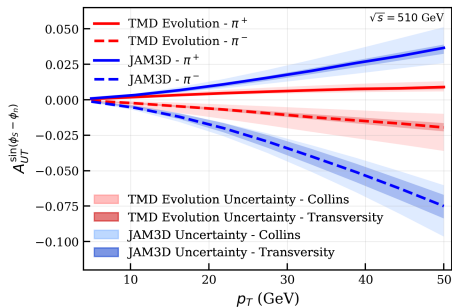
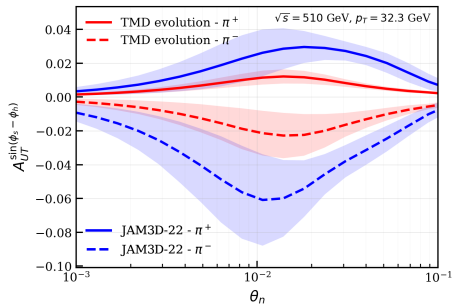


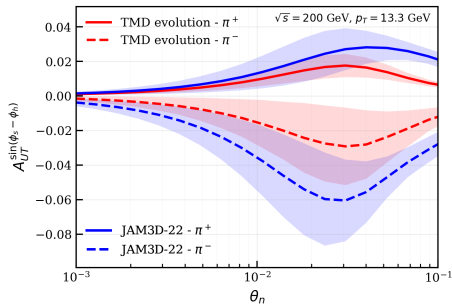
Figure from STAR Collaboration, 2604.15543.

- JAM3D analysis gives overall larger Collins asymmetry than TMD evolution results.
- The error propagated from the parametrization of the Collins function is larger than from the transversity PDF.
- $\theta_n$  is integrated on (0, 0.1).

# $\theta_n$ Distribution of the OPEC Collins Asymmetry



(a)  $\sqrt{s} = 510$  GeV,  $p_T = 32.3$  GeV



(b)  $\sqrt{s} = 200$  GeV,  $p_T = 13.3$  GeV

- Error is estimated from transversity PDF and Collins function combined.
- Two scenarios differ on the positions of their peak in  $\theta_n$ .
- Current parametrizations do not yet allow us to resolve the effects of TMD evolution.

# $\theta_n$ Distribution of the OPEC Collins Asymmetry

- STAR recent paper on OPEC Asymmetry with  $\sqrt{s} = 200$  GeV.

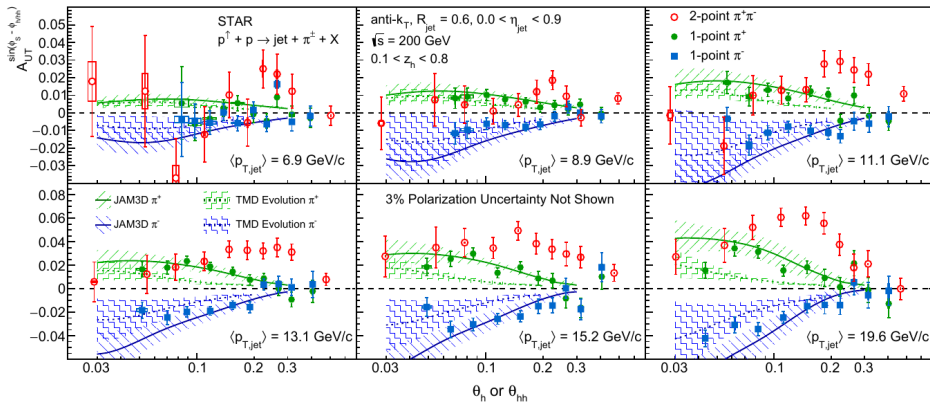
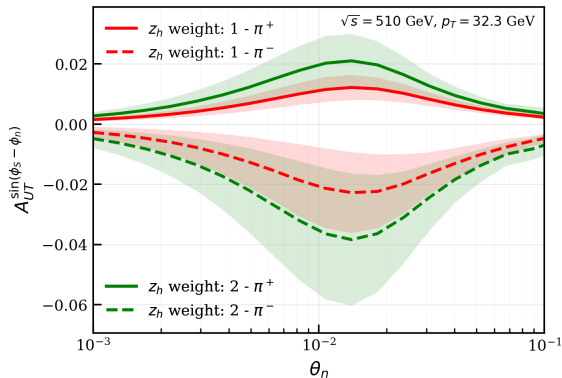


Figure from STAR Collaboration, 2604.15543.

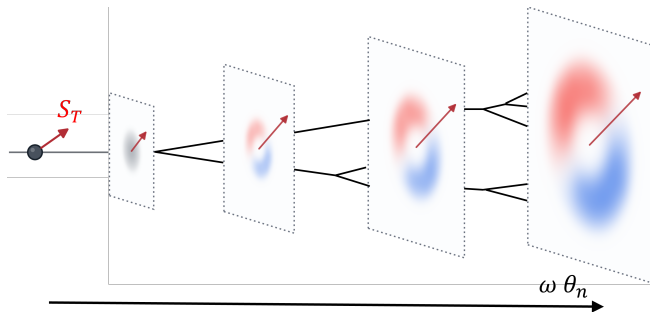
# $\theta_n$ Distribution of the OPEC Collins Asymmetry for Different Weights



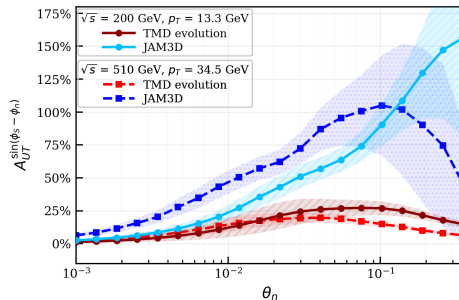
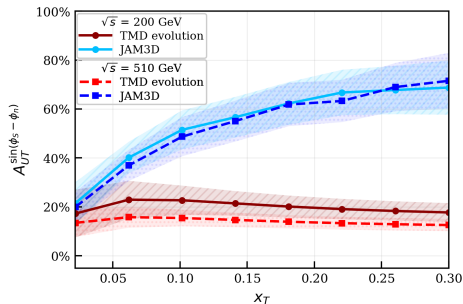
- Consistently enhancement of the Collins asymmetry for a higher power of the weight  $z_h$  (i.e. higher Mellin moment).

# Transverse charge distribution from the OPCC

- **Transverse charge distribution:** A characteristic pattern of the charge distribution on the transverse plane from the fragmenting quark.
  - The unpolarized channel gives a rotationally invariant transverse charge distribution.
  - The transversely polarized quark fragmentation yields a charge dipole distribution.
  - Oppositely charged quark gives a charge dipole align also in the opposite way, thus the total charge dipole is enhanced (about doubled).

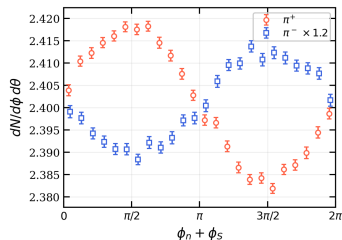


# Charge Weighting Strongly Enhances the RHIC Asymmetry

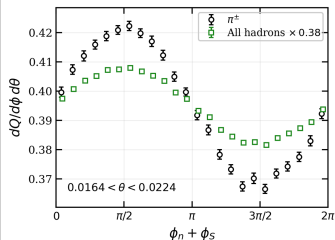


- At fixed  $x_T = 2p_T/\sqrt{s}$ , the two collision energies give similar predictions; the TMD-evolution asymmetry reaches about 20%–25%, while JAM3D becomes much larger at wide angles.
- Opposite charges cancel in the signed monopole  $Z_{UU}^Q$  but add coherently in  $Z_{UT}^Q$ ; therefore values above unity do not imply a positivity violation.

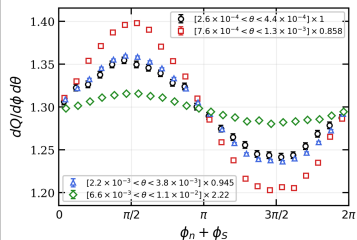
# SIDIS Simulations Show the Same Coherent Enhancement



(a) COMPASS, non-weighted



(b) COMPASS, charged-weighted



(c) EIC, charged-weighted

- In COMPASS kinematics, the charge-weighted pion modulation is about an order of magnitude larger than the separate  $\pi^+$  and  $\pi^-$  modulations.
- The charge-weighted distribution retains comparable statistical precision; at EIC kinematics the asymmetry reaches roughly 10% after dividing out the effective depolarization factor.



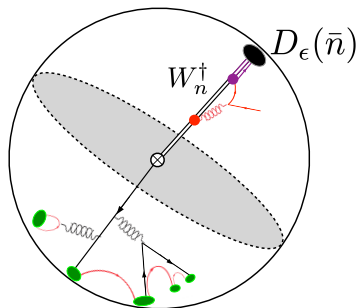
# Conclusion

- We propose the one-point energy/charge correlator (OPEC/OPCC) as probes of nucleon transversity.
- For inclusive jet production in transversely polarized  $p^\uparrow p$  collisions, both energy flow and charge flow measurements give an enhanced  $\sin(\phi_s - \phi_n)$  asymmetry.
- Compared with standard TMD studies, OPEC/OPCC has two major benefits on determination of the transversity distribution:
  - The model dependency on final state fragmentation is reduced by taking the Mellin moment of the fragmentation functions.
  - OPEC/OPCC estimates the Collins asymmetry by energy flow polar angle  $\theta_n$ , accessing a much broader kinematic range of the jet substructure. In particular, OPCC is decided by the charge trackers, unaffected by neutral hadrons.
  - OPEC/OPCC enhance the Collins asymmetry.

# Discussions and outlooks

- Two points near-side energy correlator on Collins dihadron production.  
(see recent development: Z.B. Kang, A. Metz, D. Pitonyak, C. Zhang, arXiv:2604.28131).
- Incorporating the twist-3  $qqq$  correlator contributions.  
(see recent development: D. Scantamburlo, M. Schlegel, arXiv: 2603.10628; ).
- Clarifying the perturbative calculation on Collins function.  
(Upcoming work with Han-Qing Yu and Ding Yu Shao. ).
- Charge-Charge Correlator in  $e^+e^-$  with Collins effect.
- Calculation of OPCC in SIDIS, and new global fit of transversity PDF and Collins function with TMD evolution.  
(Upcoming work with Zhong-Bo Kang and Zhen Xu. ).

# Flow Charge and the Wilson-Line Defect



In general, quantum numbers (e.g. electric charges, baryon numbers) can escape from a collinear mode through the defect on the celestial sphere punctured by the Wilson line,

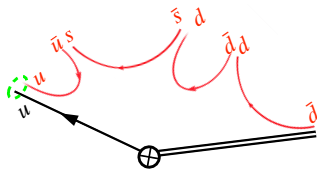
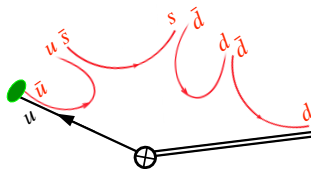
$$Q_{\text{flow}} = Q_f - Q_D.$$

In perturbation theory,

$$Q_D^{\text{pert}} = 0, \quad Q_{\text{flow}}^{\text{pert}} = Q_f.$$

- Hadronization may generate a nonperturbative boundary charge  $Q_D \neq 0$ .

# Hadronization, Charge, and Baryon Number



A simple hadronization model with  $p_u : p_d : p_s = 1 : 1 : \lambda$ .

Electric charge

$$Q_D|_f \approx \frac{1}{2+\lambda} \left( \frac{2}{3} - \frac{1}{3} \right) + \frac{\lambda}{2+\lambda} \frac{1}{3},$$

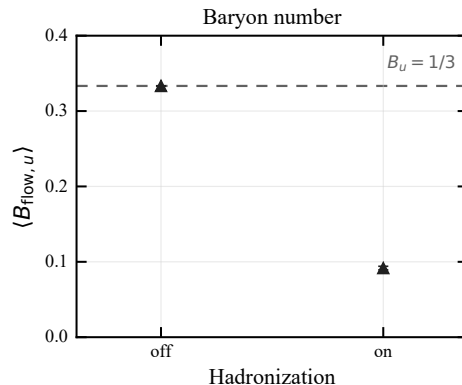
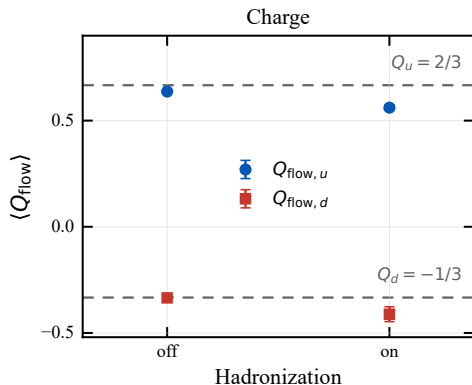
$$Q_{\text{flow}}|_f \approx Q_f - \frac{1-\lambda}{2+\lambda} \frac{1}{3}.$$

Baryon number

$$B_D|_f \approx \frac{1}{3},$$

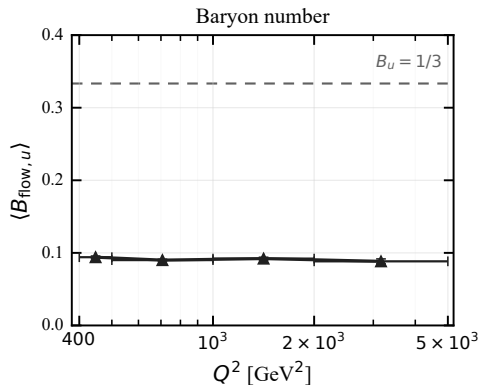
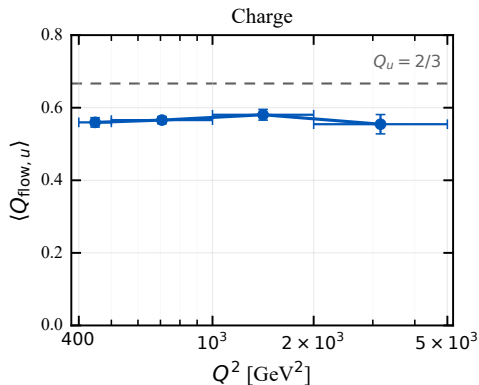
$$B_{\text{flow}}|_f \approx 0.$$

# Hadronization On/Off: Charge and Baryon Number



Hadronization modifies the charge and strongly suppresses the baryon number.

# Charge and Baryon Number vs. Hard Scale



Charge and baryon numbers are about scale independent.

# OPEC FJF Factorization

- Analogously to the TMD FJFs (Z.B. Kang, X. Liu, F. Ringer, and H. Xing, 2017; Z.B. Kang, K. Lee, and F. Zhao, 2020), the OPEC FJFs admit the factorized representations in the limit  $R \gg \theta_n$ ,

$$\begin{aligned}\mathcal{J}^c(z, \hat{\mathbf{n}}, \omega_J R, \mu) &= \mathcal{H}_{c \rightarrow i}(z, \omega_J R, \mu) \int d^2 \mathbf{l}_\perp d^2 \boldsymbol{\lambda}_\perp \delta^2(\mathbf{l}_\perp + p_T \theta_n \hat{\mathbf{n}}_T - \boldsymbol{\lambda}_\perp) \\ &\quad \times J_i^U(\mathbf{l}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R), \\ p_T \theta_n \mathcal{J}_{1,\perp}^q(z, \hat{\mathbf{n}}, \omega_J R, \mu) &= \mathcal{H}_{c \rightarrow i}^T(z, \omega_J R, \mu) \int d^2 \mathbf{l}_\perp d^2 \boldsymbol{\lambda}_\perp \delta^2(\mathbf{l}_\perp + p_T \theta_n \hat{\mathbf{n}}_T - \boldsymbol{\lambda}_\perp) \\ &\quad \times p_T \theta_n J_i^T(\mathbf{l}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R).\end{aligned}$$

- $\mathcal{H}$  is a hard matching coefficient, encodes the contributions from radiations at the jet scale  $\omega_J R$ .  $J$  is the OPEC jet function.

# OPEC jet function OPE

The  $z^{N-1}$  OPEC jet function can be matched on to the collinear counterparts

$$\begin{aligned}\tilde{J}_q^U(\mathbf{b}, \mu, \nu) &= C_{qq}(\mathbf{b}, \mu, \nu) \tilde{D}_{q/q}^N(\mu) + C_{gq}(\mathbf{b}, \mu, \nu) \tilde{D}_{g/q}^N(\mu), \\ \tilde{J}_g^U(\mathbf{b}, \mu, \nu) &= C_{gg}(\mathbf{b}, \mu, \nu) \tilde{D}_{g/g}^N(\mu) + C_{qg}(\mathbf{b}, \mu, \nu) \tilde{D}_{q/g}^N(\mu), \\ \tilde{J}_q^T(\mathbf{b}, \mu, \nu) &= \delta C_{qq}(\mathbf{b}, \mu, \nu) \tilde{H}_{q/q}^{(3)N}(\mu),\end{aligned}$$

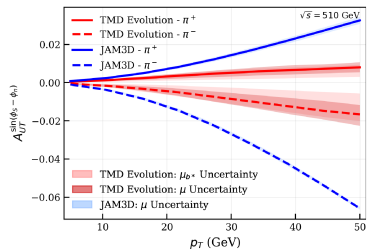
with the NLO renormalized matching coefficients

$$\begin{aligned}C_{qq}(\mathbf{b}, \mu, \nu) &= 1 + \frac{\alpha_s}{2\pi} \left[ C_F \left( \ln \frac{\mu^2}{\mu_b^2} \left( \frac{3}{2} + 2 \ln \frac{\nu}{\omega_J} \right) + \frac{1}{N(N+1)} \right) - \gamma_{qq}(N) \ln \frac{\mu^2}{\mu_b^2} - 2\gamma_{qq}^{(1)}(N) \right], \\ C_{gq}(\mathbf{b}, \mu, \nu) &= \frac{\alpha_s}{2\pi} \left[ -\gamma_{gq}(N) \ln \frac{\mu^2}{\mu_b^2} - 2\gamma_{gq}^{(1)}(N) + C_F \frac{1}{N+1} \right], \\ C_{gg}(\mathbf{b}, \mu, \nu) &= 1 + \frac{\alpha_s}{2\pi} \left[ C_A \ln \frac{\mu^2}{\mu_b^2} \left( \frac{\beta_0}{2C_A} + 2 \ln \frac{\nu}{\omega_J} \right) - \gamma_{gg}(N) \ln \frac{\mu^2}{\mu_b^2} - 2\gamma_{gg}^{(1)}(N) \right], \\ C_{qg}(\mathbf{b}, \mu, \nu) &= \frac{\alpha_s}{2\pi} \left[ -\gamma_{qg}(N) \ln \frac{\mu^2}{\mu_b^2} - 2\gamma_{qg}^{(1)}(N) + T_F \frac{2}{(N+1)(N+2)} \right], \\ \delta C_{qq}(\mathbf{b}, \mu, \nu) &= 1 + \frac{\alpha_s}{2\pi} \left[ C_F \ln \frac{\mu^2}{\mu_b^2} \left( \frac{3}{2} + 2 \ln \frac{\nu}{\omega_J} \right) - \Delta_T \gamma_{qq}(N) \ln \frac{\mu^2}{\mu_b^2} - 2\Delta_T \gamma_{qq}^{(1)}(N) \right].\end{aligned}$$

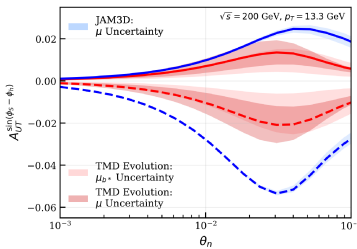


# Backup: Scale uncertainties

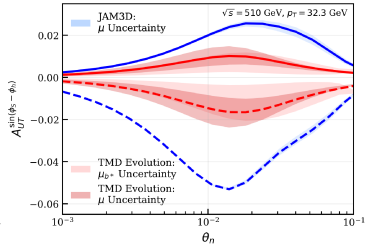
- For the TMD evolution framework, we vary both the OPE scale  $\mu_i$  and the hard scale  $\mu = Q$  independently by a factor of two around their canonical values ( $\mu_i = \mu_{b^*}$  and  $\mu = Q = p_T$ ) to estimate the perturbative uncertainty.
- For the JAM3D-based approach, where only collinear DGLAP evolution is included, we vary the hard scale  $\mu = Q = p_T$  in the same manner.



(a)



(b)



(c)