

# Probing the Color-Octet Mechanism via Dihadron Fragmentation in $\chi_b$ Decays

Base on: Zhi-Guo He, **Guanghui Li**, Yu-Jie Tian, Xin-Kai Wen, and Bin Yan, *arXiv*: 2603.18874

**Guanghui Li 李广辉**

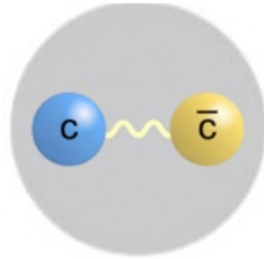
**Institute of High Energy Physics**

# Heavy Quarkonium and NRQCD

- Heavy quarkonium: the non-relativistic bound state

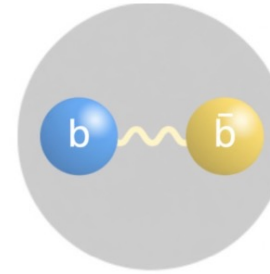
Charmonium

$$v_c^2 \simeq 0.23$$



Bottomonium

$$v_b^2 \simeq 0.08$$



- Color-Singlet Mechanism

$Q\bar{Q}$  with the same quantum numbers as the final bound state  $^{2S+1}L_J^{[c]}$

$$c\bar{c}(^3S_1^{[1]}) \rightarrow J/\Psi \quad b\bar{b}(^3S_1^{[1]}) \rightarrow \Upsilon$$

- Theoretical issues

P-wave (decays into light hadrons):infrared divergence( $\alpha_s^3$ )

- Experiment issues

Significant discrepancy between  $\sigma_{exp}$  and  $\sigma_{CSM}$  in  $pp \rightarrow J/\Psi$

F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 79, 578 (1997).

# Heavy Quarkonium and NRQCD

## ➤ Color-Octet Mechanism

$$\begin{aligned} |H(^{2S+1}L_J)\rangle &= O(1)|Q\bar{Q}(^{2S+1}L_J^{[1]})\rangle \\ &+ O(v)|Q\bar{Q}(^{2S+1}(L\pm 1)_{J'}^{[8]})g\rangle \quad \text{E1} \\ &+ O(v^2)|Q\bar{Q}(^{2S'+1}L_{J'}^{[8]})g\rangle \quad \text{M1} \\ &+ O(v^2)|Q\bar{Q}(^{2S+1}L_J^{[1,8]})gg\rangle \quad \text{E1} \cdot \text{E1} \\ &+ \dots \end{aligned}$$

G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D 51, 1125 (1995)

## ➤ NRQCD factorization

$$\Gamma(H \rightarrow LH) = \sum_n \frac{2 \operatorname{Im} f_n(\Lambda)}{M^{d_n-4}} \langle H | \mathcal{O}_n(\Lambda) | H \rangle$$

Double expansion:  $\frac{2 \operatorname{Im} f_n(\Lambda)}{M^{d_n-4}}$  short distance coefficients  $\alpha_s$

$\langle H | \mathcal{O}_n(\Lambda) | H \rangle$  long distance matrix elements  $v^2$

# LDMEs power counting and $\chi_{QJ}$ Decay

## ➤ LDMEs power counting

$$\langle H | \mathcal{O}_n(\Lambda) | H \rangle \sim v^{2i+j} \quad \begin{array}{l} i \text{ for Fock state} \\ j \text{ for operator} \end{array}$$

For P-wave quarkonium

$$^3P_J^{[1]} \quad i=0, j=2 \quad \langle \chi_{QJ} | O_1(^3P_J) | \chi_{QJ} \rangle \sim v^2$$

$$^3S_1^{[8]} \quad i=0, j=1 \quad \langle \chi_{QJ} | O_8(^3S_1) | \chi_{QJ} \rangle \sim v^2$$

## ➤ Decay rate

$$\Gamma(\chi_{QJ} \rightarrow \text{LH}) = \frac{2 \text{Im } f_1(^3P_J)}{M^4} \langle \chi_{QJ} | O_1(^3P_J) | \chi_{QJ} \rangle + \frac{2 \text{Im } f_8(^3S_1)}{M^2} \langle \chi_{QJ} | O_8(^3S_1) | \chi_{QJ} \rangle + O(v^2\Gamma), \quad J = 0, 1, 2$$

Color octet also contributes to the leading order!

# Testing Color-Octet Mechanism

## ➤ Lattice VS Experiment

$$\rho_8(m_Q) = H_8^Q(m_Q)m_Q^2/H_1^Q$$

$$H_1^Q = \langle \chi_{QJ} | \mathcal{O}(^3P_J^{[1]}) | \chi_{QJ} \rangle$$

$$H_8^Q(\mu_\Lambda) = \langle \chi_{QJ} | \mathcal{O}(^3S_1^{[8]}, \mu_\Lambda) | \chi_{QJ} \rangle$$

Charmonium: in good agreement

$$\rho_8 = 0.128(2)(9)^{+61}_{-47} \text{ (Lattice)}$$

$$\rho_8 = 0.095(43) \text{ (Exp)}$$

Bottomonium: significantly different

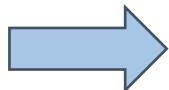
$$\rho_8 = 0.044 \pm 0.015 \text{ (Lattice)}$$

$$\rho_8 = 0.16^{+0.071}_{-0.047} \text{ (Exp)}$$

**Mismatch!**

G. T. Bodwin, E. Braaten, D. Kang, and J. Lee, Phys. Rev. D 76, 054001 (2007), 0704.2599.

R. A. Briere et al. (CLEO), Phys. Rev. D 78, 092007 (2008), 0807.3757.

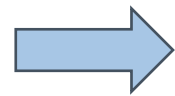


**An independent cross-check of  $\rho_8$  is important!**

## How to separate CO from CS?

$$b\bar{b}(^3S_1^{[8]}) \rightarrow q\bar{q}$$

$$b\bar{b}(^3P_J^{[1]}) \rightarrow gg$$



The spin of quark and gluon is different!

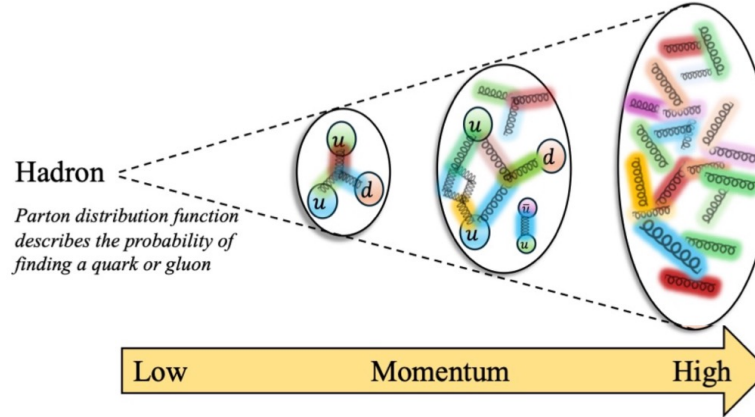
# Fragmentation Functions and Spin Effect in QCD

## ➤ PDF and FFs

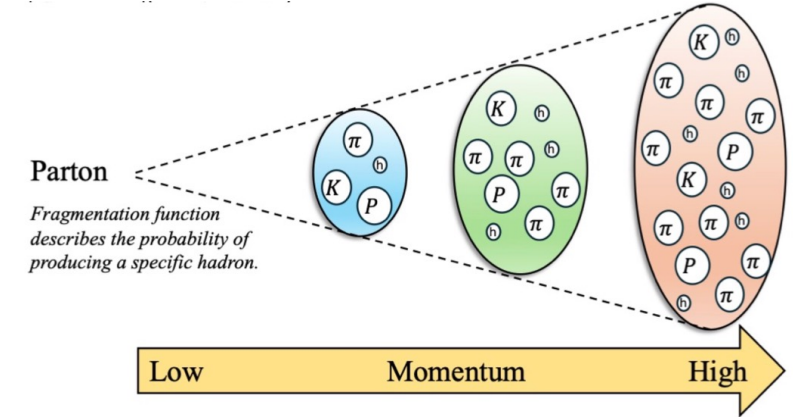
J. Datta et al, PRL 134 (2025) 111902

$$\sigma = f \otimes \hat{\sigma} \otimes D$$

$f$ : parton distribution functions

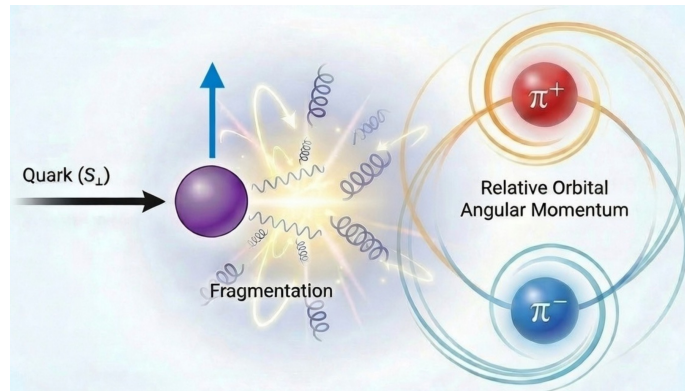


$D$ : fragmentation functions



The process from partons to hadrons can be described with un-perturbative **fragmentation functions**. 🙌

## ➤ Parton fragmentation carries transverse spin correlations

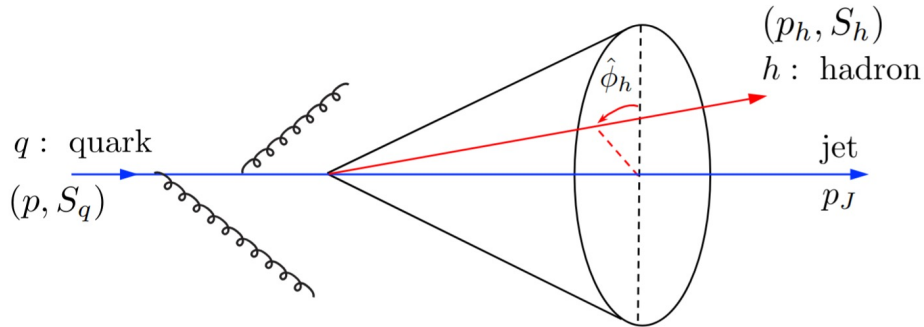


$$b\bar{b}({}^3S_1^{[8]}) \rightarrow q\bar{q} \rightarrow h + X$$

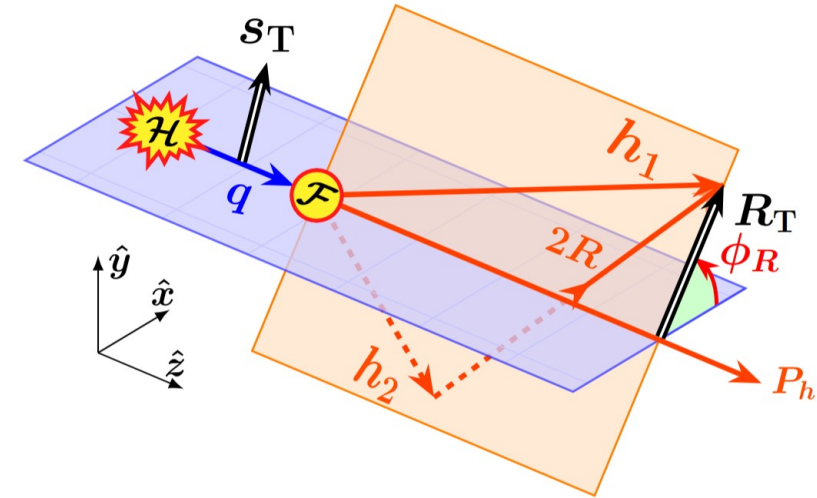
$$b\bar{b}({}^3P_J^{[8]}) \rightarrow gg \rightarrow h + X$$

# Fragmentation Functions and Spin Effect in QCD

TMD fragmentation:



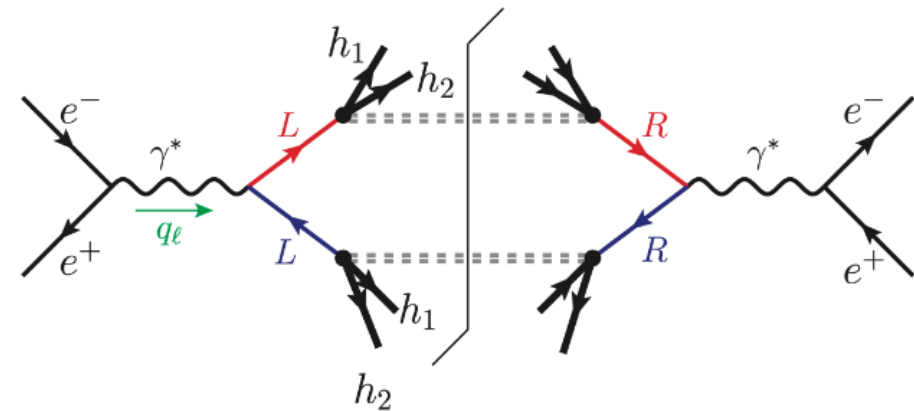
Di-hadron fragmentation:



Leading Quark TMDFFs

○ → Hadron Spin    ● → Quark Spin

|                                 |   | Quark Polarization                                                                              |                                                                                   |                                                                                                                                                                         |
|---------------------------------|---|-------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                 |   | Un-Polarized (U)                                                                                | Longitudinally Polarized (L)                                                      | Transversely Polarized (T)                                                                                                                                              |
| Unpolarized (or Spin 0) Hadrons |   | $D_1 = \text{○} \rightarrow \text{○}$<br>Unpolarized                                            |                                                                                   | $H_1^\perp = \text{○} \rightarrow \text{○} - \text{○} \rightarrow \text{○}$<br>Collins                                                                                  |
|                                 |   |                                                                                                 |                                                                                   |                                                                                                                                                                         |
| Polarized Hadrons               | L |                                                                                                 | $G_1 = \text{○} \rightarrow \text{○} - \text{○} \rightarrow \text{○}$<br>Helicity | $H_{1L}^\perp = \text{○} \rightarrow \text{○} - \text{○} \rightarrow \text{○}$                                                                                          |
|                                 | T | $D_{1T}^\perp = \text{○} \rightarrow \text{○} - \text{○} \rightarrow \text{○}$<br>Polarizing FF | $G_{1T}^\perp = \text{○} \rightarrow \text{○} - \text{○} \rightarrow \text{○}$    | $H_1 = \text{○} \rightarrow \text{○} - \text{○} \rightarrow \text{○}$<br>Transversity<br>$H_{1T}^\perp = \text{○} \rightarrow \text{○} - \text{○} \rightarrow \text{○}$ |



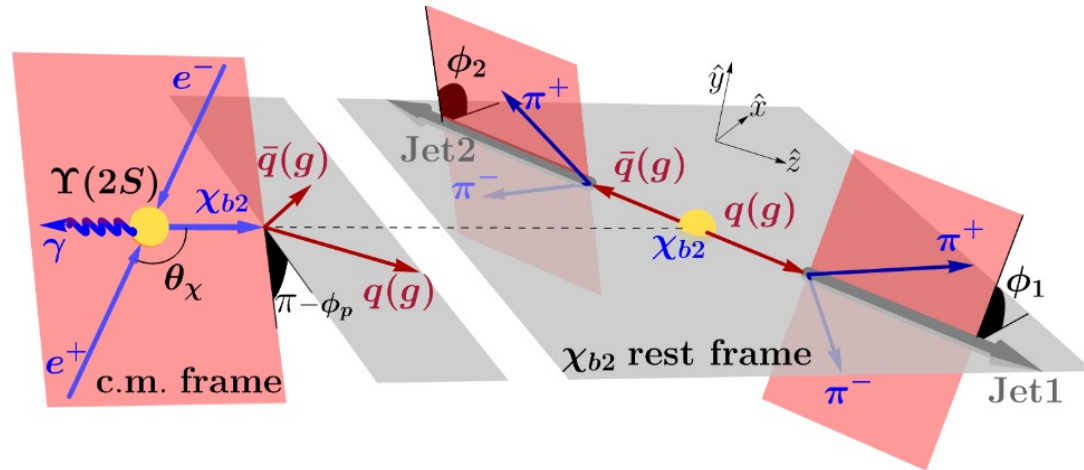
Kang, Prokudin, Sun, Yuan, *Phys.Rev.D* **93** (2016) 014009;  
Zeng, Dong, Liu, Sun, Zhao, *Phys.Rev.D* **109** (2024) 056002;  
TMD Handbook, *arXiv*: 2304.03302; .....

# Artru-Collins Asymmetry

## ➤ Kinematics

$$e^+e^- \rightarrow \Upsilon(2S) \rightarrow \gamma\chi_{b2}$$

$$\chi_{b2} \rightarrow q\bar{q}(gg) \rightarrow \pi^+\pi^-\pi^+\pi^- + X$$



## ➤ Collinear Factorization

J. C. Collins, S. F. Heppelmann, G. A. Ladinsky, NPB 420 (1994) 565

$$\frac{d\sigma}{\sigma_0 dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2 d\cos\theta_\chi d\cos\theta_p d\phi_p}$$

$$= H_8^b \sum_q C_q \left[ \boxed{D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2)} + \frac{1}{2} \mathcal{B} \boxed{H_1^{\triangleleft, q}(z_1, M_1) H_1^{\triangleleft, \bar{q}}(z_2, M_2)} \cos(\phi_1 + \phi_2) \right] + H_1^b C_g \boxed{D_1^g(z_1, M_1) D_1^g(z_2, M_2)}$$

Unpolarized quark diFF
Transverse polarized quark diFF
Unpolarized gluon diFF

Only quark fragmentation generate the asymmetry



Separation CO from CS!

# Artru-Collins Asymmetry

## ➤ Observable

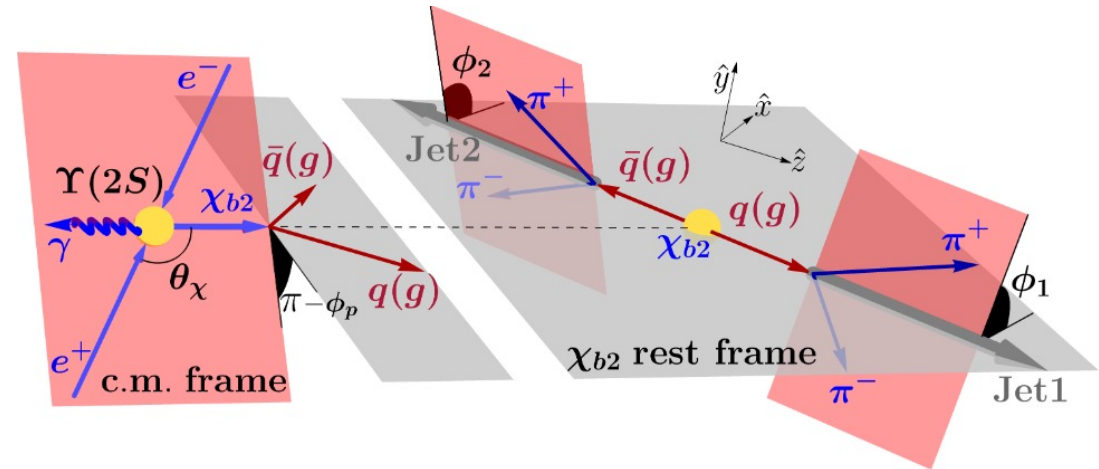
$$A_{12} \equiv 2\langle \cos(\phi_1 + \phi_2) \rangle = \frac{1}{2} \frac{\rho_8(m_b) \mathcal{B} \sum_q \mathcal{C}_q H_1^{\triangleleft, q}(z_1, M_1) H_1^{\triangleleft, \bar{q}}(z_2, M_2)}{\rho_8(m_b) \sum_q \mathcal{C}_q D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2) + m_b^2 \mathcal{C}_g D_1^g(z_1, M_1) D_1^g(z_2, M_2)}$$

Asymmetry provides a clean probe of  $\rho_8$

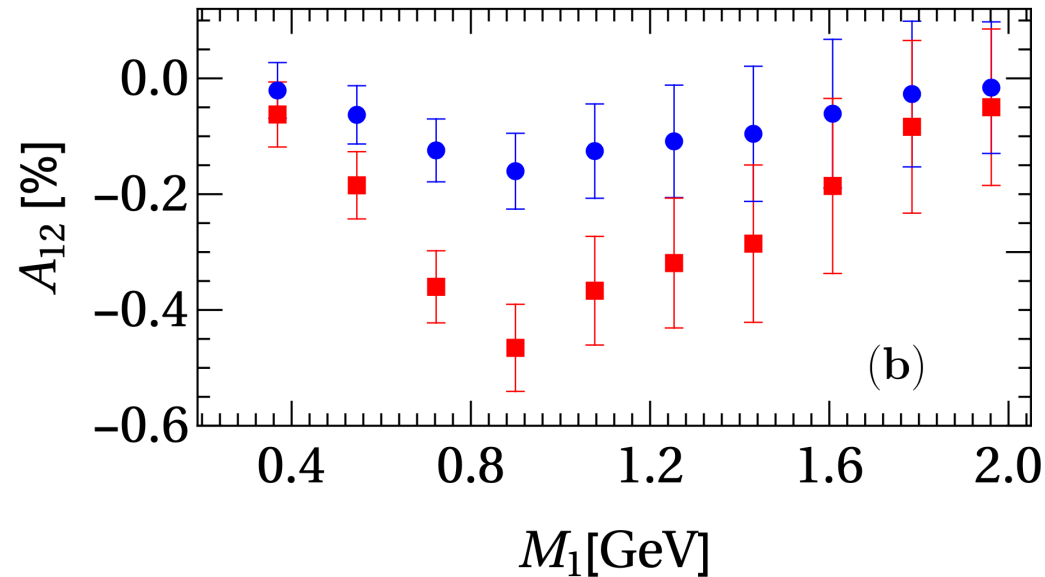
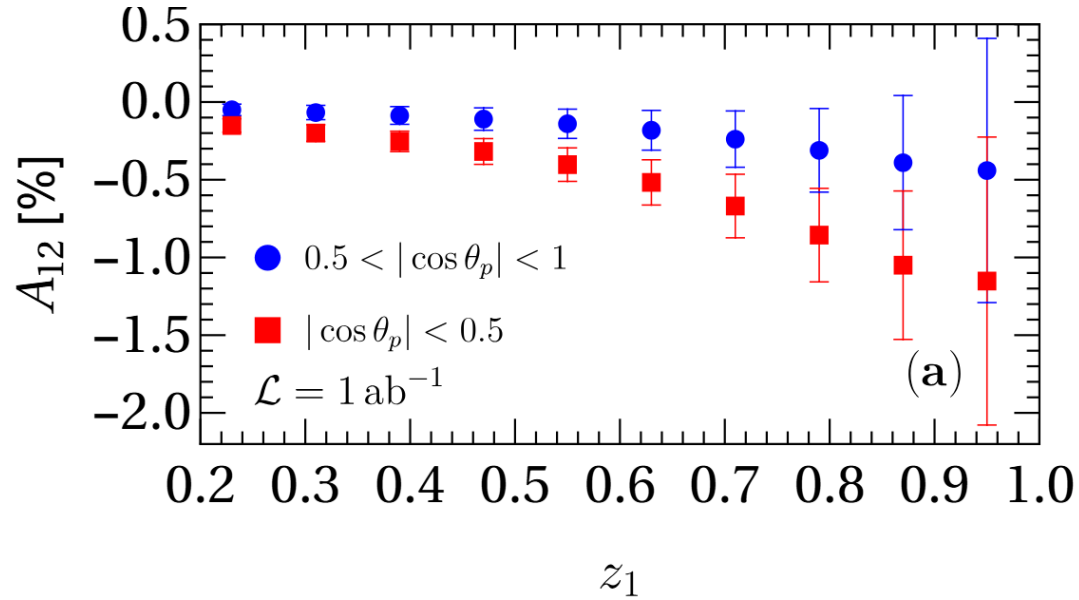
## ➤ Belle: boost effects

$$\mathcal{B} = \frac{42 \sin^2 \theta_p}{21 \cos^2 \theta_p + 73}$$

- The coefficients does not depend on  $\theta_\chi$ ,  $\phi_p$
- Enhanced in the central region



# Results and Sensitivity



$$\rho_8 = 0.044$$

- Asymmetry magnitude:  $\sim 1\%$
- Enhanced in the central region

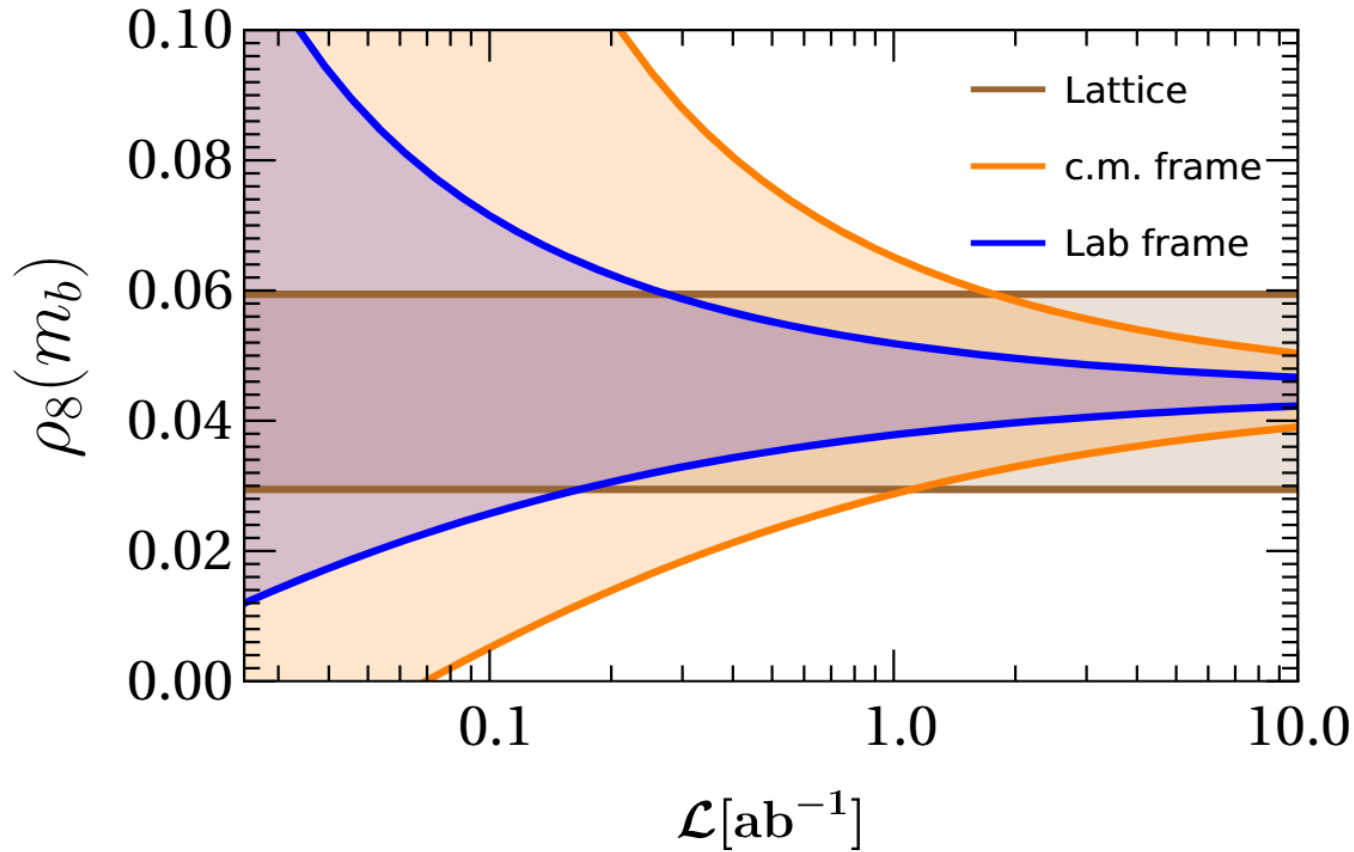
$$\mathcal{B} = \frac{42 \sin^2 \theta_p}{21 \cos^2 \theta_p + 73}$$

- Sign determined by DiFF properties

$$H_1^{\triangleleft, q} = -H_1^{\triangleleft, \bar{q}}$$

- Observable is measurable at Belle

# Results and Sensitivity



Lab frame ( $\chi_{b2}$ ): binned in  $\theta_p$

c.m. frame ( $\chi_{b2}$ ): binned in  $\theta_\chi$

- Cancellation in phase space at c.m. frame
- Projected sensitivity surpasses current lattice uncertainty with  $O(0.1) \text{ ab}^{-1}$
- The potential to **resolve the long-standing discrepancy in  $\rho_8$**

# SUMMARY

- We proposed a novel observable based on dihadron fragmentation to probe the color-octet mechanism
- Spin-dependent asymmetry provides a clean quark-gluon discriminator
- The observable is directly sensitive to the color-octet contribution
- It offers a new avenue to resolve  $\rho_8$  discrepancy

👍 A new way to test NRQCD via QCD spin effects!

Thank you

