Nucleon Gluon PDFs from Lattice QCD

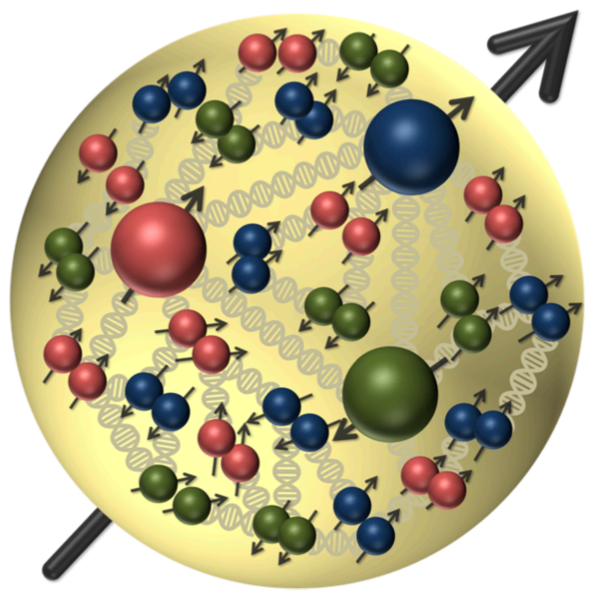
Liuming Liu(刘柳明)
Institute of Modern Physics, CAS

In collaboration with
Chen Chen, Chunhua Zeng, Shiyi Zhong, Hong-Xin Dong,
Peng Sun, Yi-Bo Yang, Fei Yao, Jian-Hui Zhang

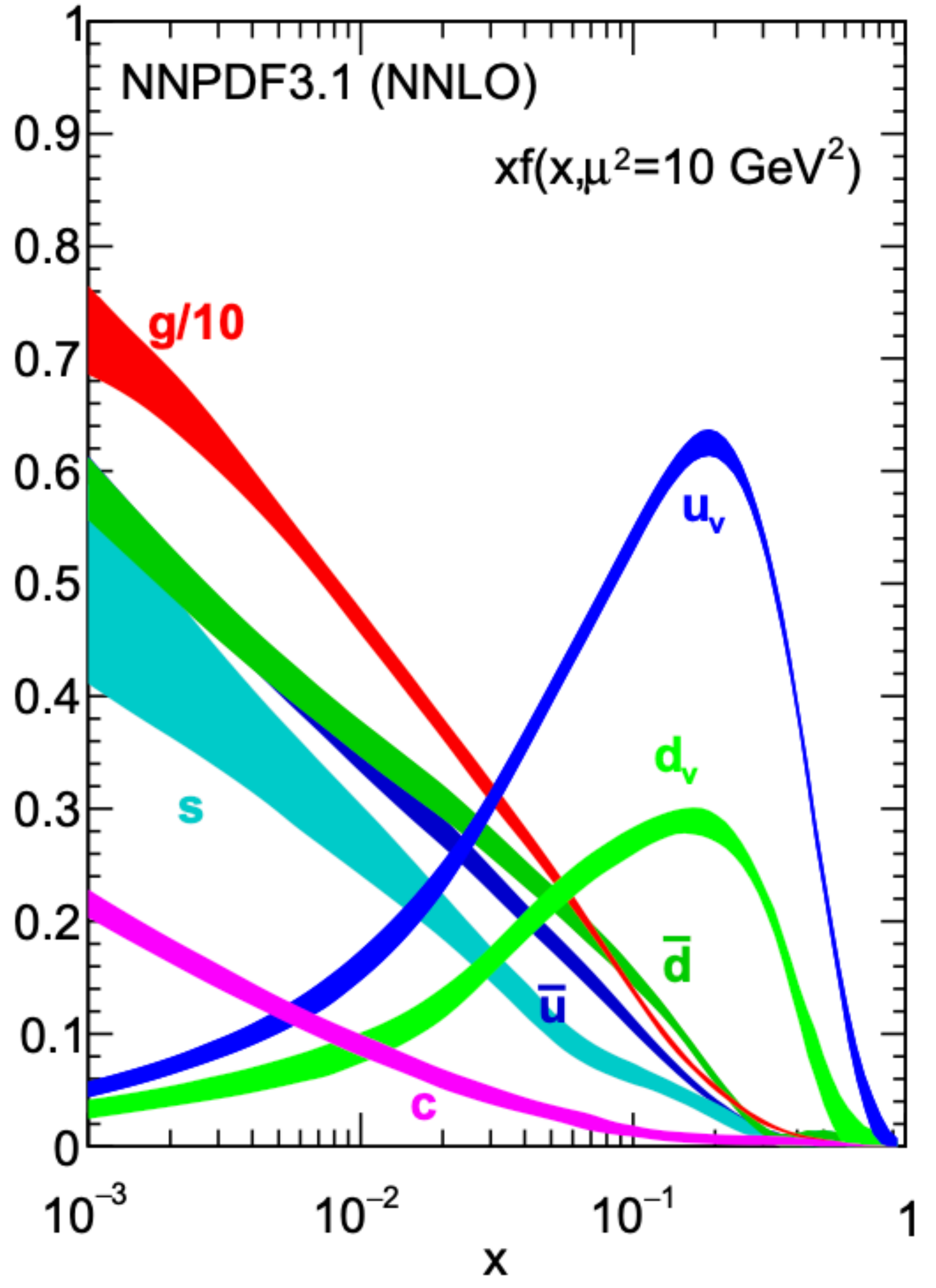
第一届强相互作用自旋物理会议

2026年7月26-31日，青岛

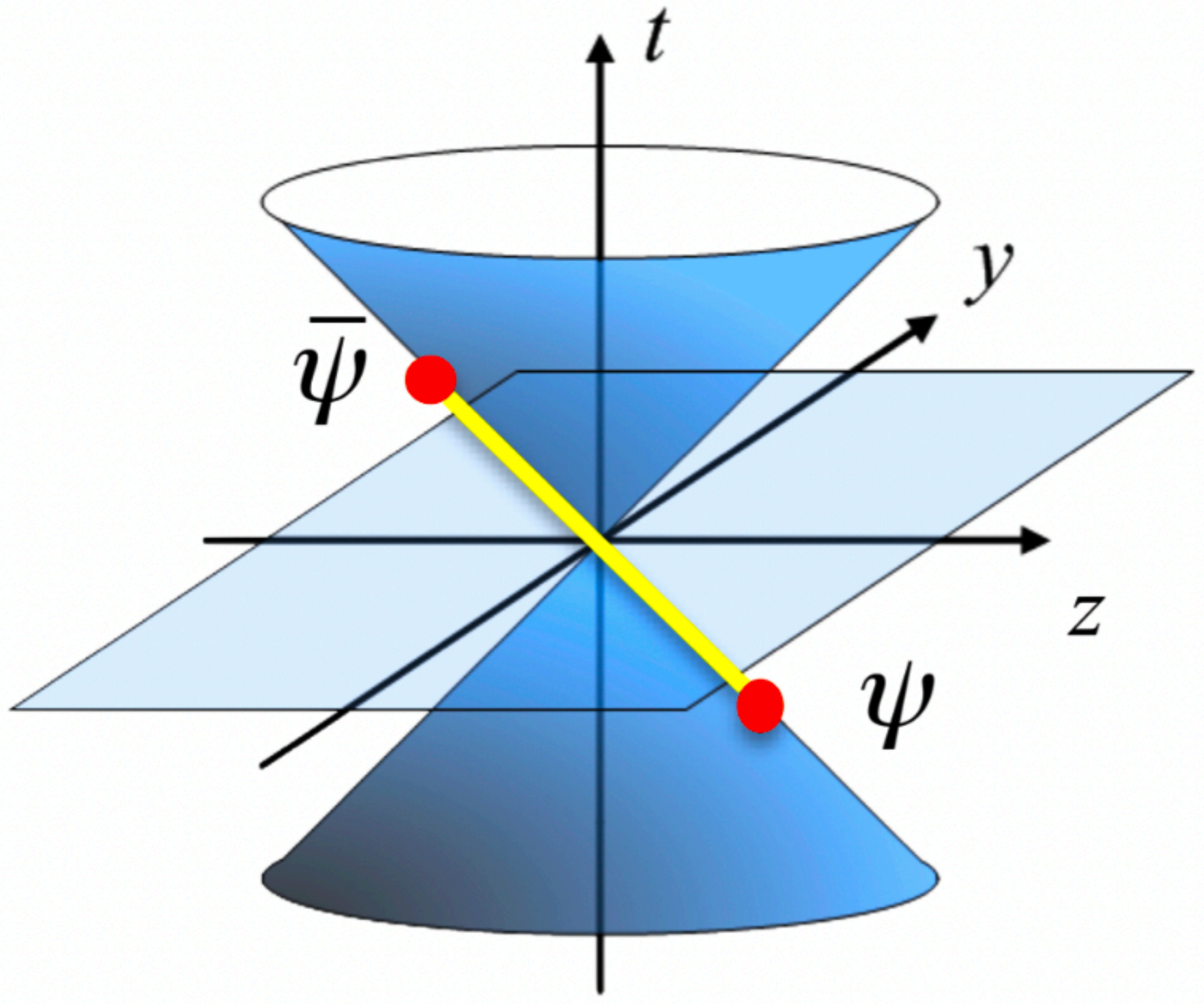
PDFs



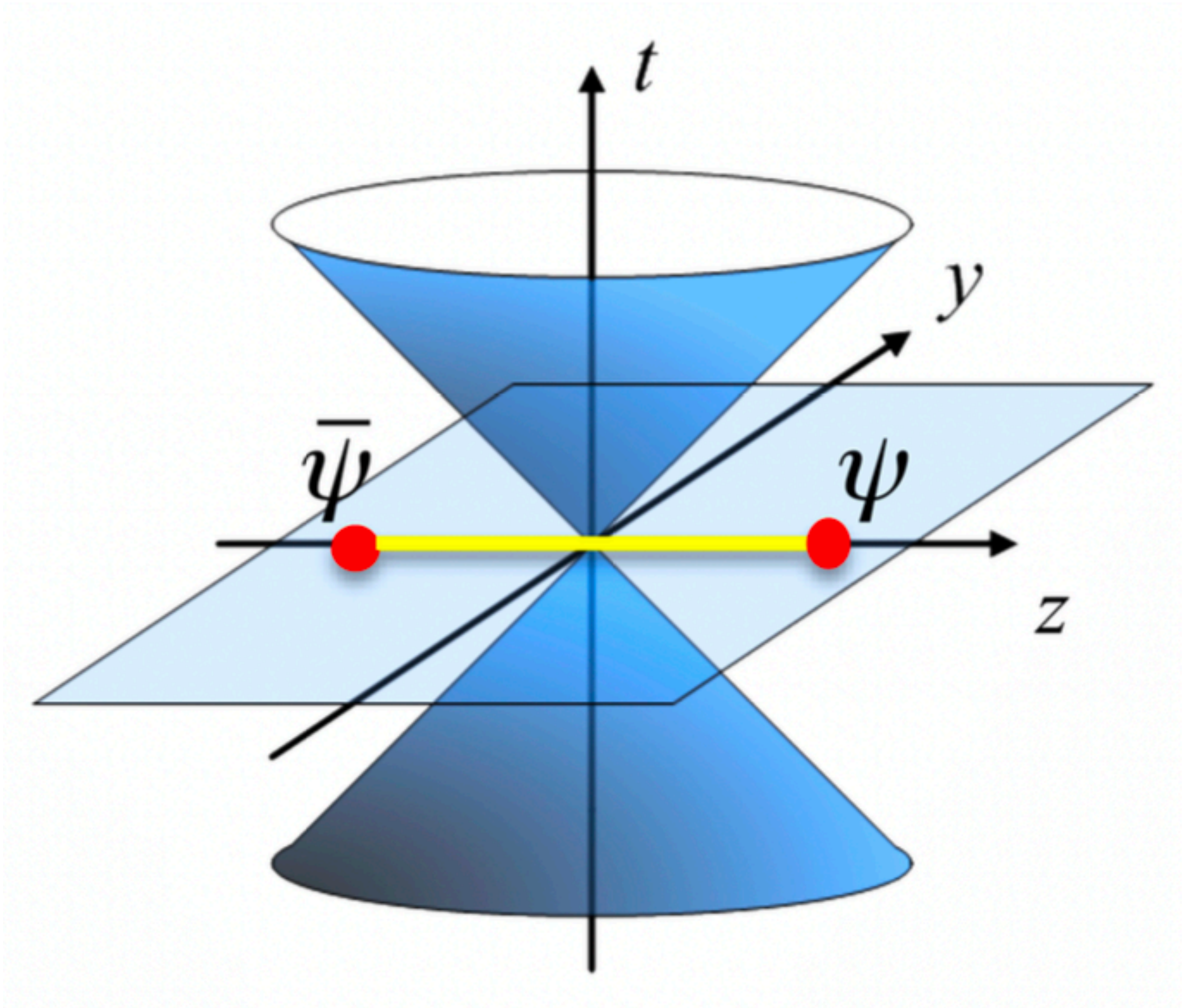
- Nucleons are complicated dynamical systems of quarks and gluons (partons).
- PDFs: describe the propability density of finding a parton with momentum fraction x in a nucleon.
- Global fit: experimental data, model dependence
- Lattice QCD: provides unique *ab initio* access to PDFs
- Gluon PDFs on lattice: suffer from noise signal.

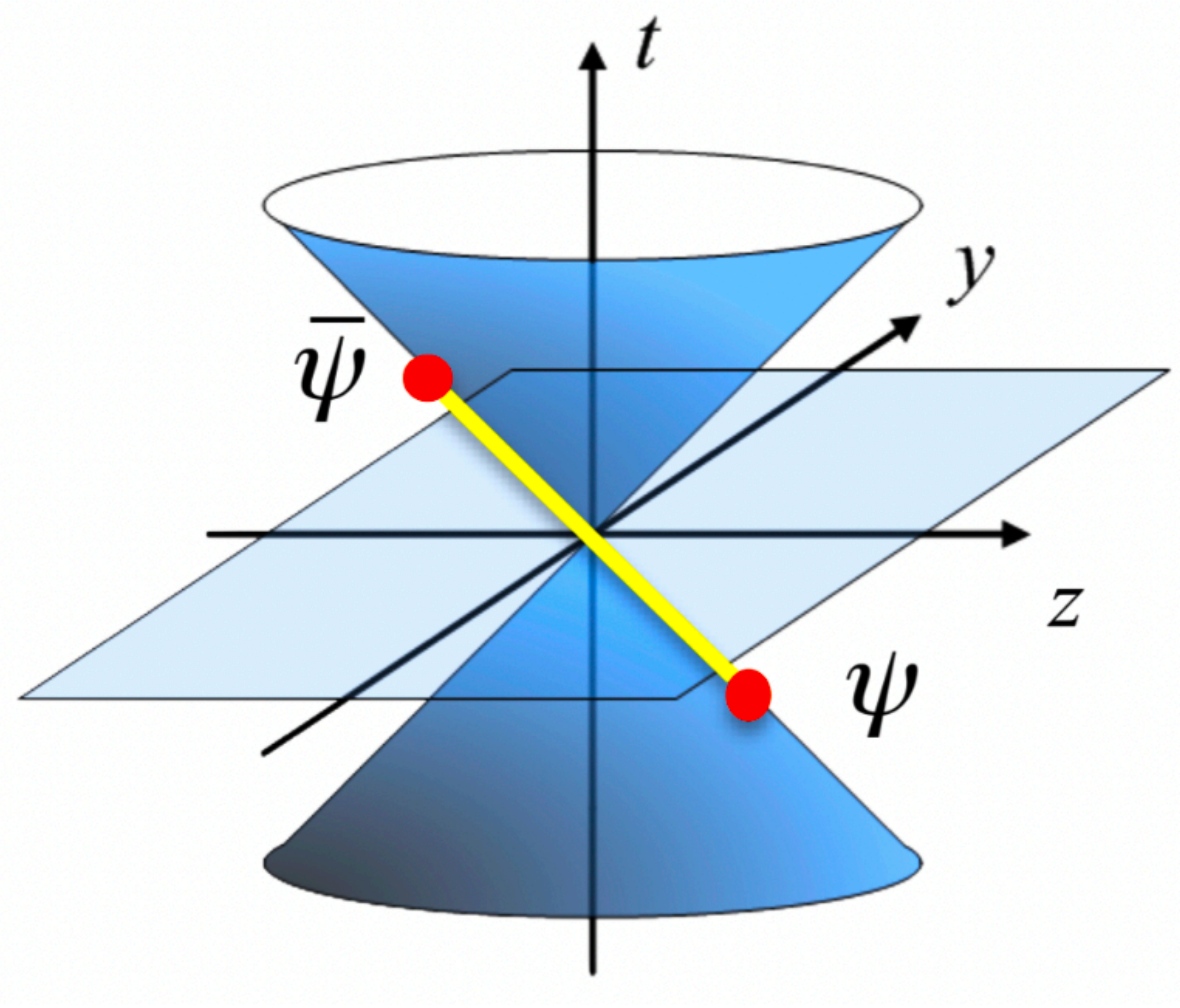


PDFs on lattice



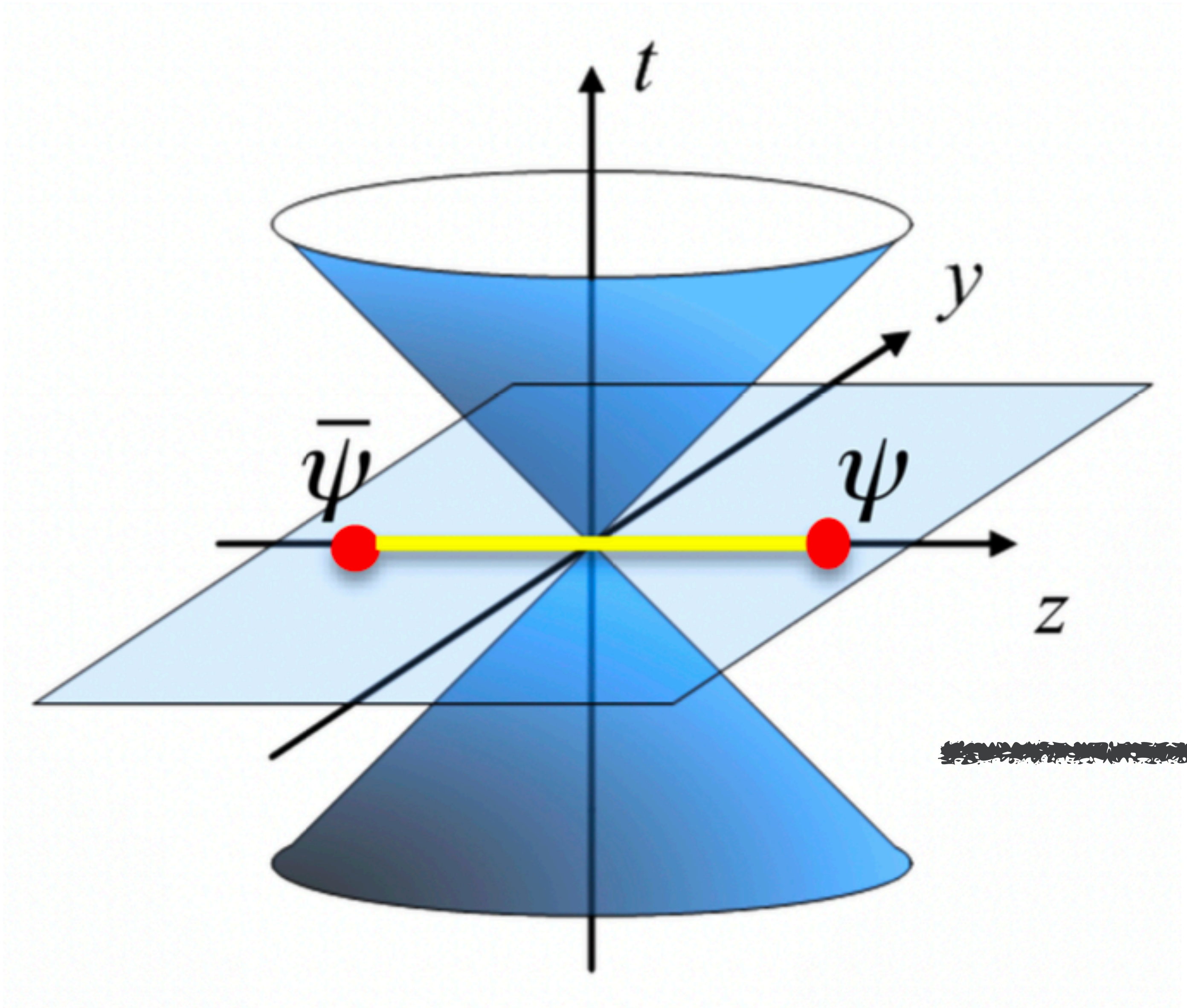
PDFs on lattice



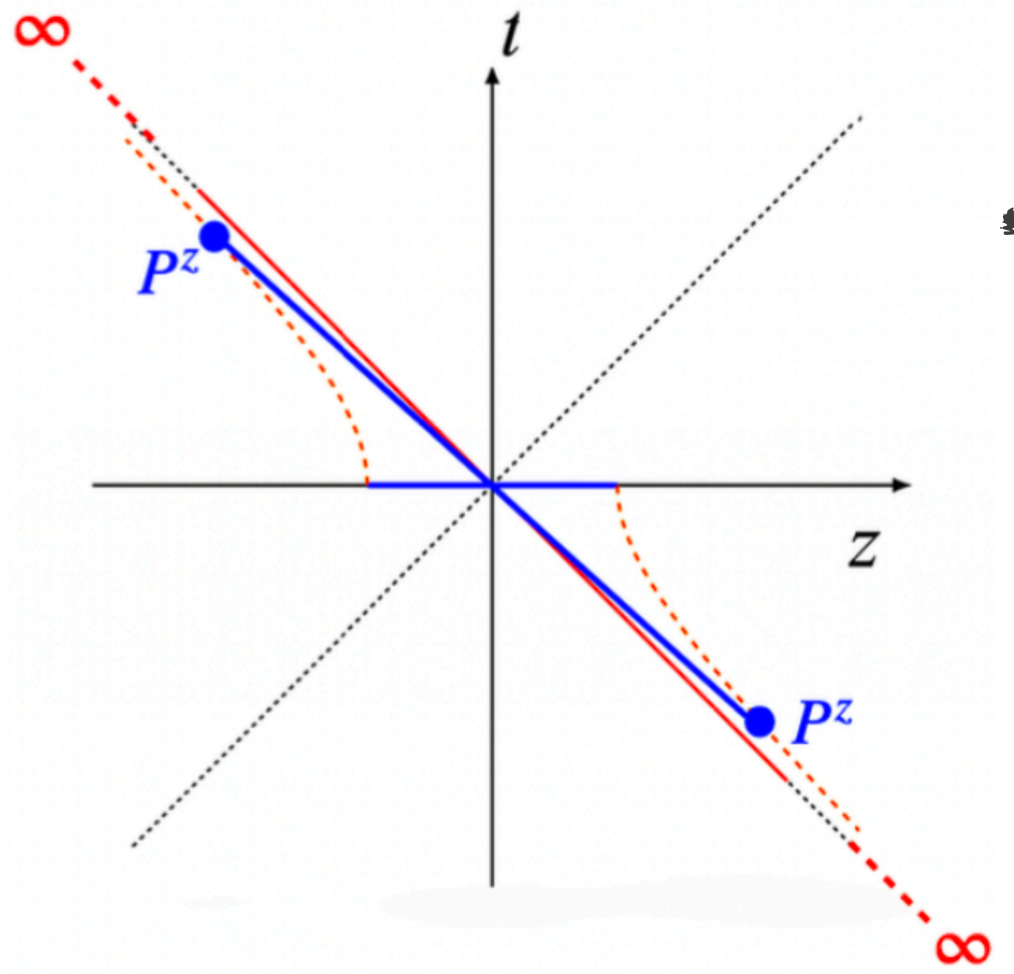


PDFs on lattice

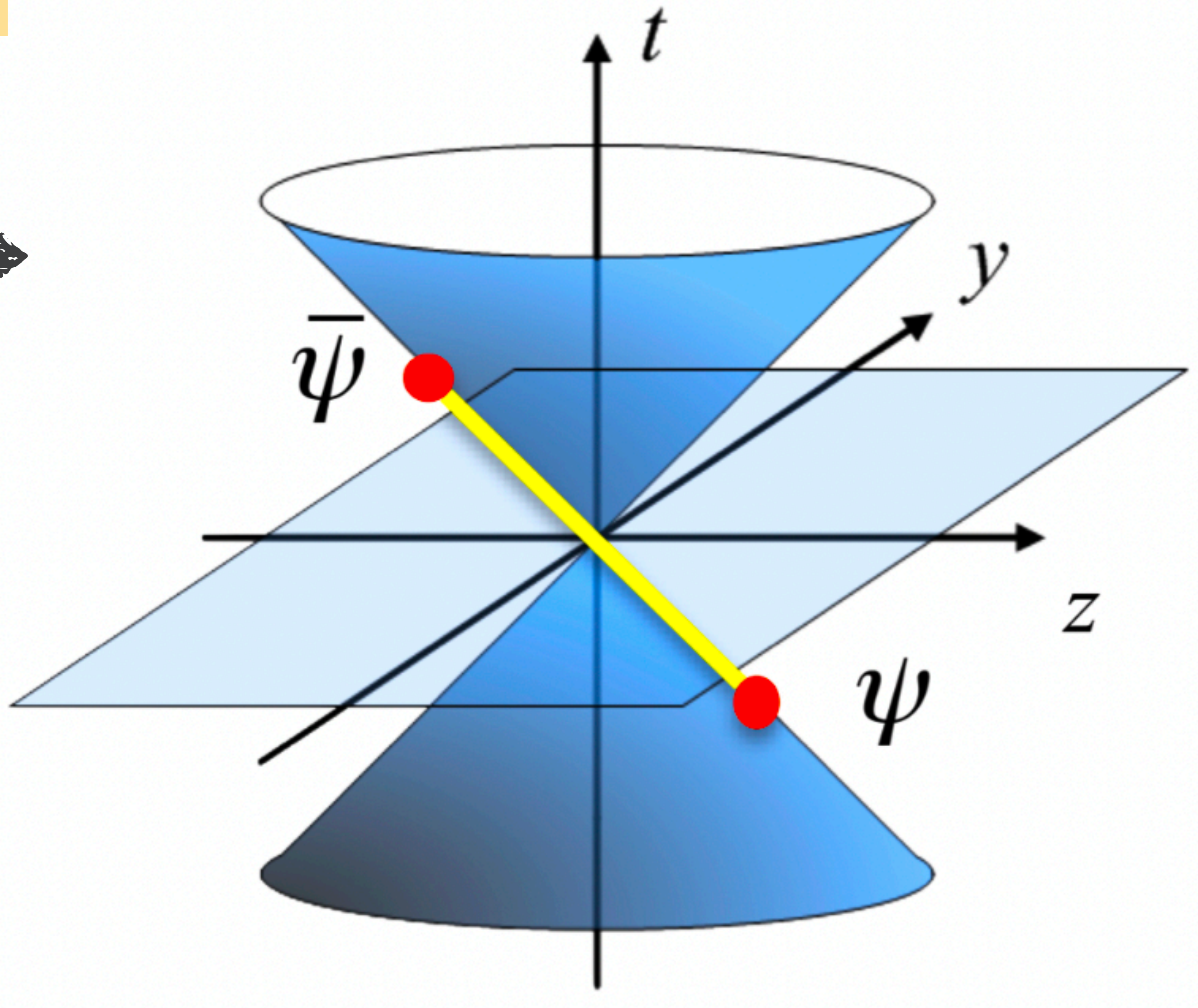
Large momentum effective theory



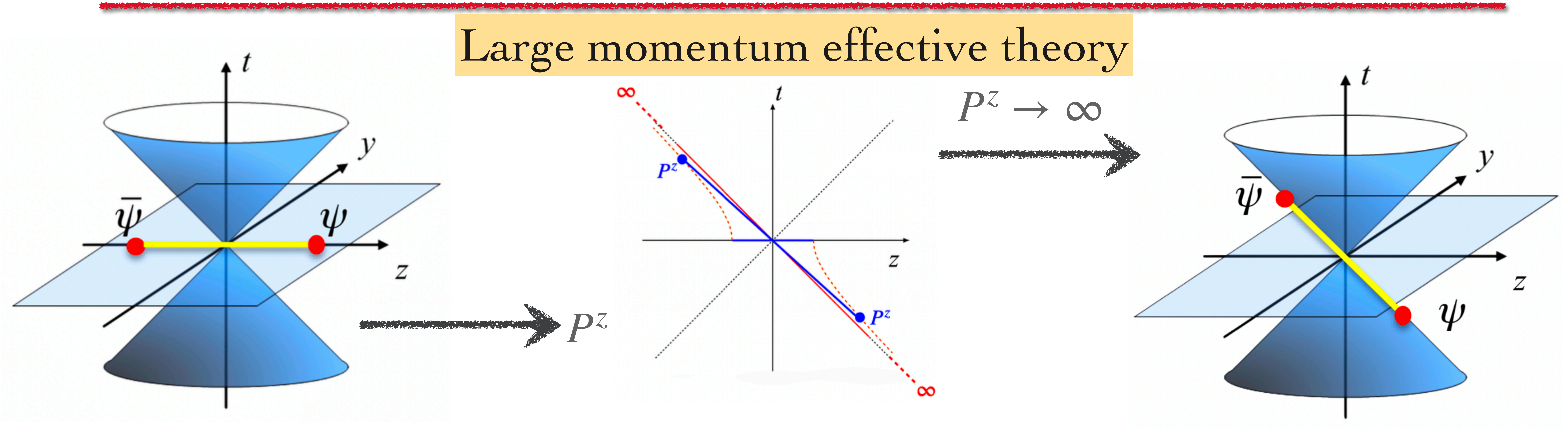
P^z



$P^z \to \infty$



PDFs on lattice

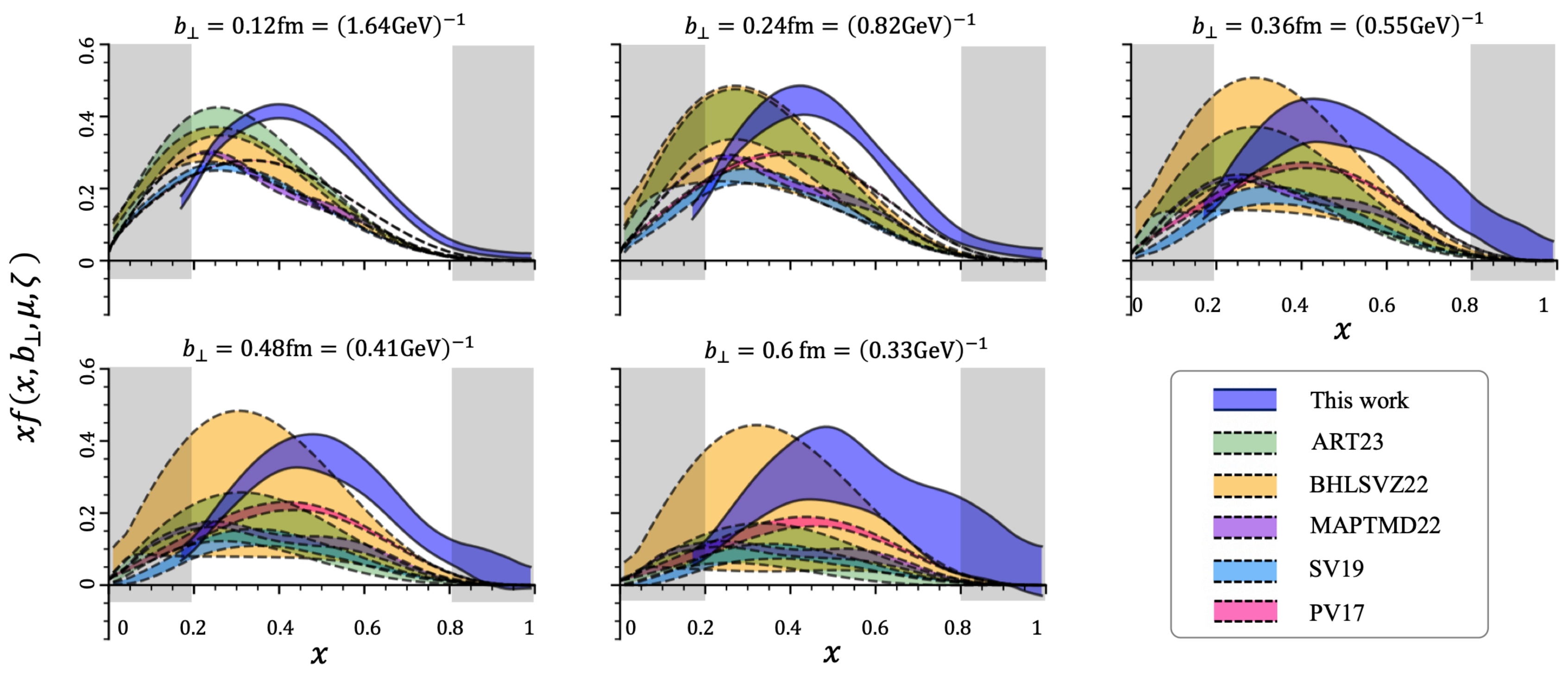


- Light-cone: take limit $P_z \to \infty$ before the renormalization.
- Lattice: renormalize before taking limit $P_z \to \infty$
- The two limits do no commute.
- The difference is UV physics, perturbative matching can connect the quasi-PDFs (lattice) and the light-cone PDFs.

PDFs on lattice

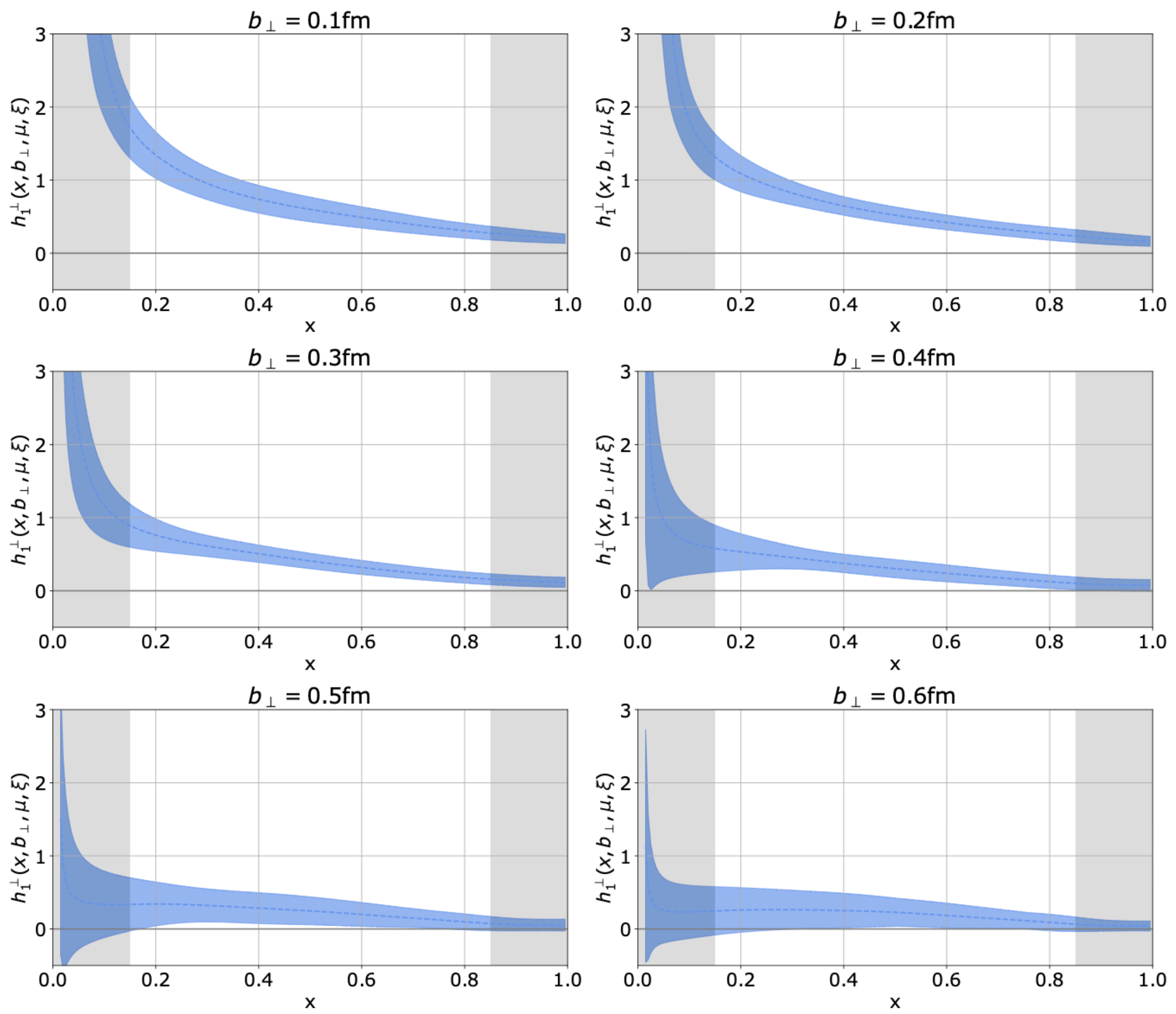
Nucleon TMD PDFs

Jin-Chen He et al., PRD109(2024),114513



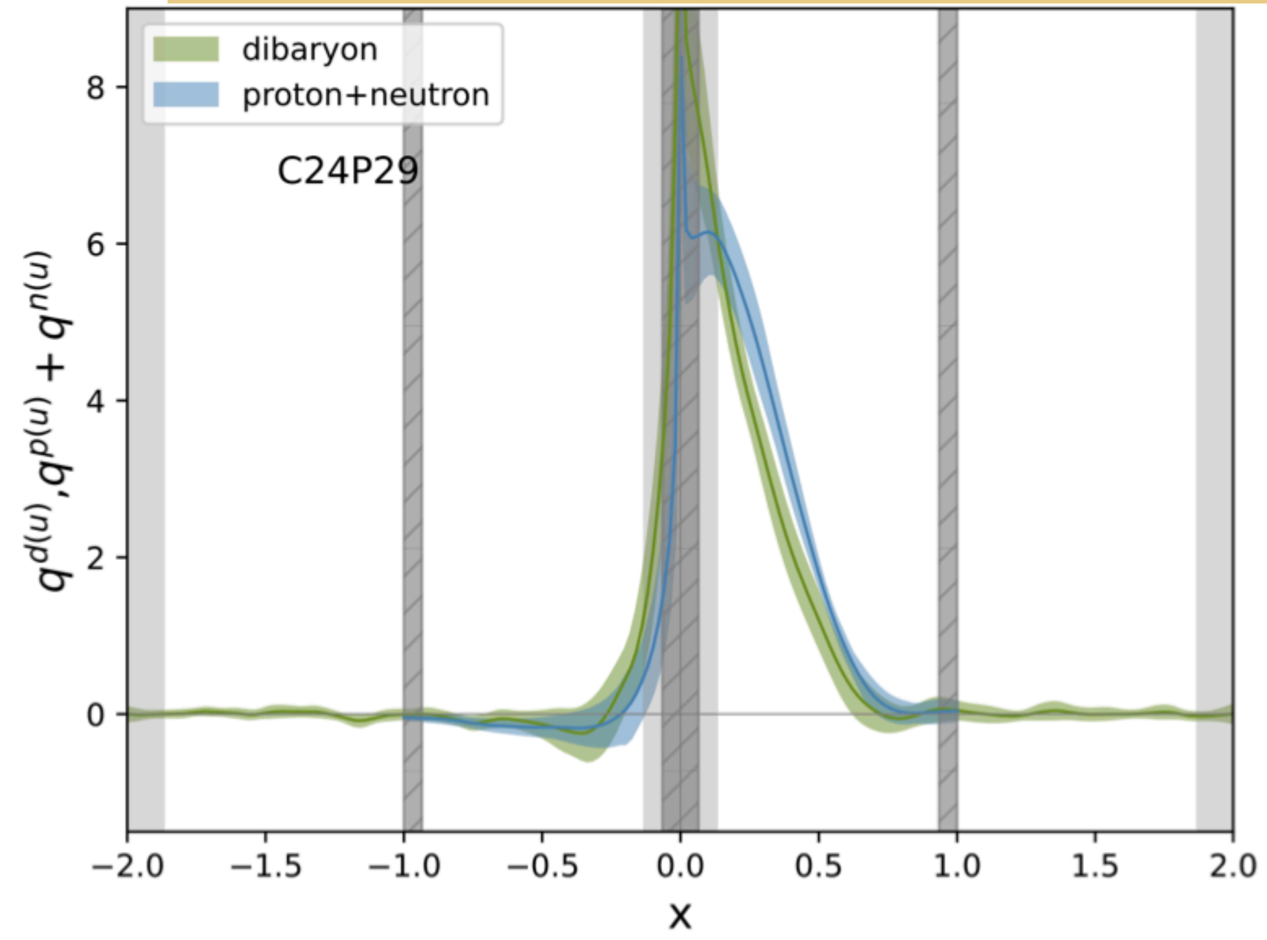
Pion Boer-Mulders function

Lisa Walter et al., PRD111(2025),094507



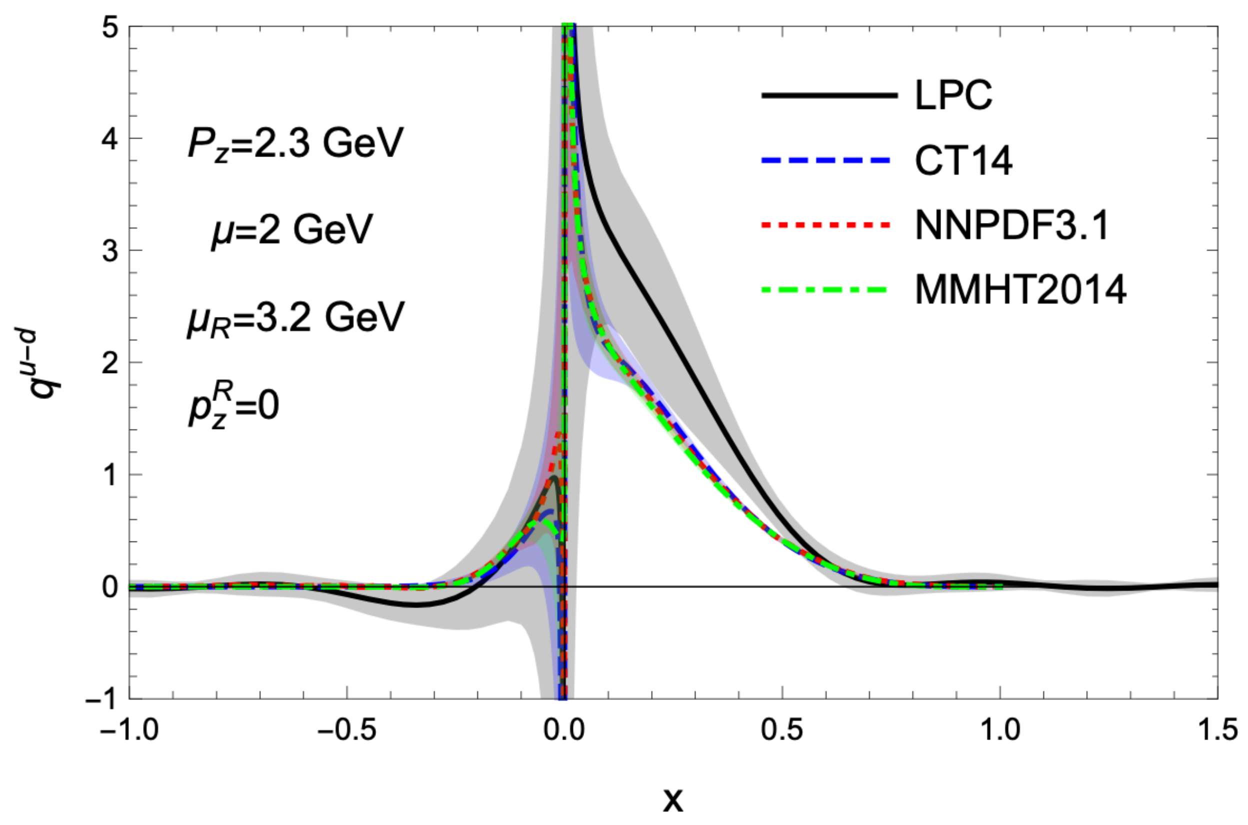
Deuteron PDF

Chen Chen et al., PRD111(2025),074506



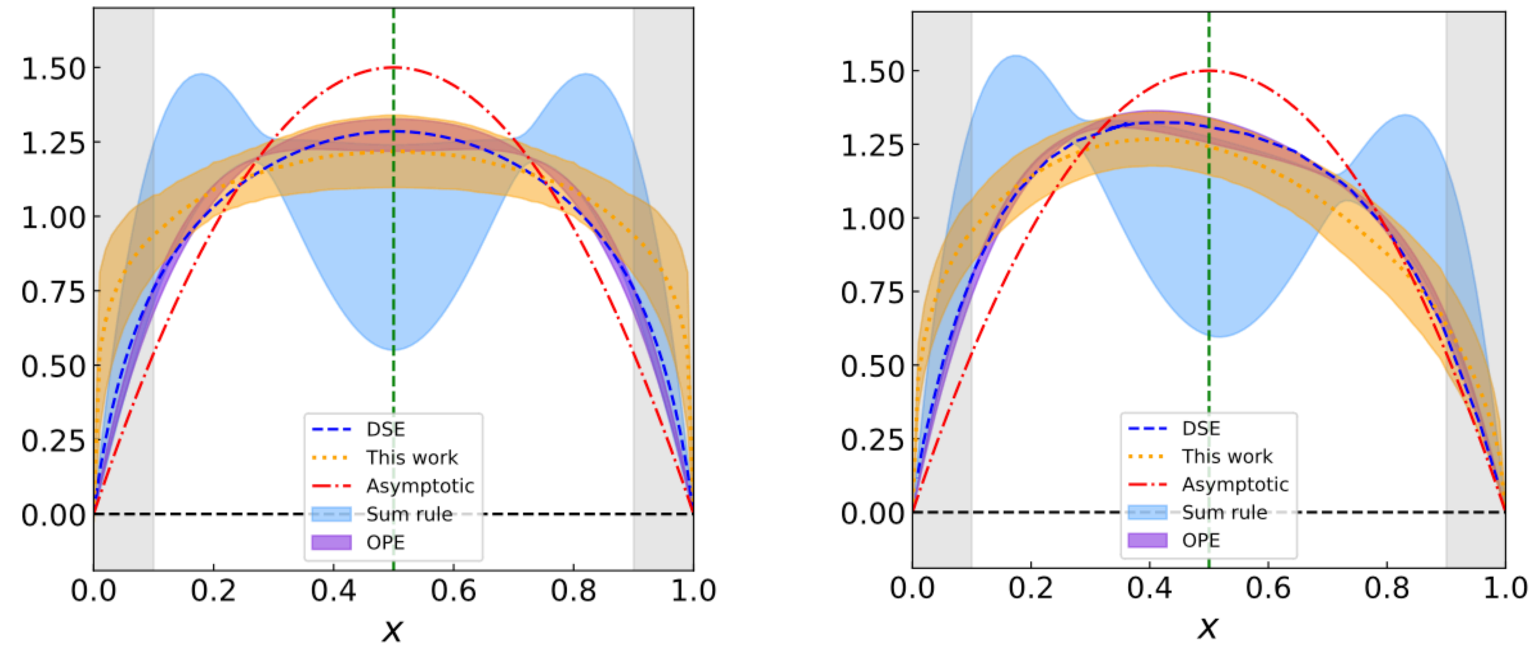
Unpolarized quark PDF

Yu-Sheng Liu et al., PRD101(2020), 034020



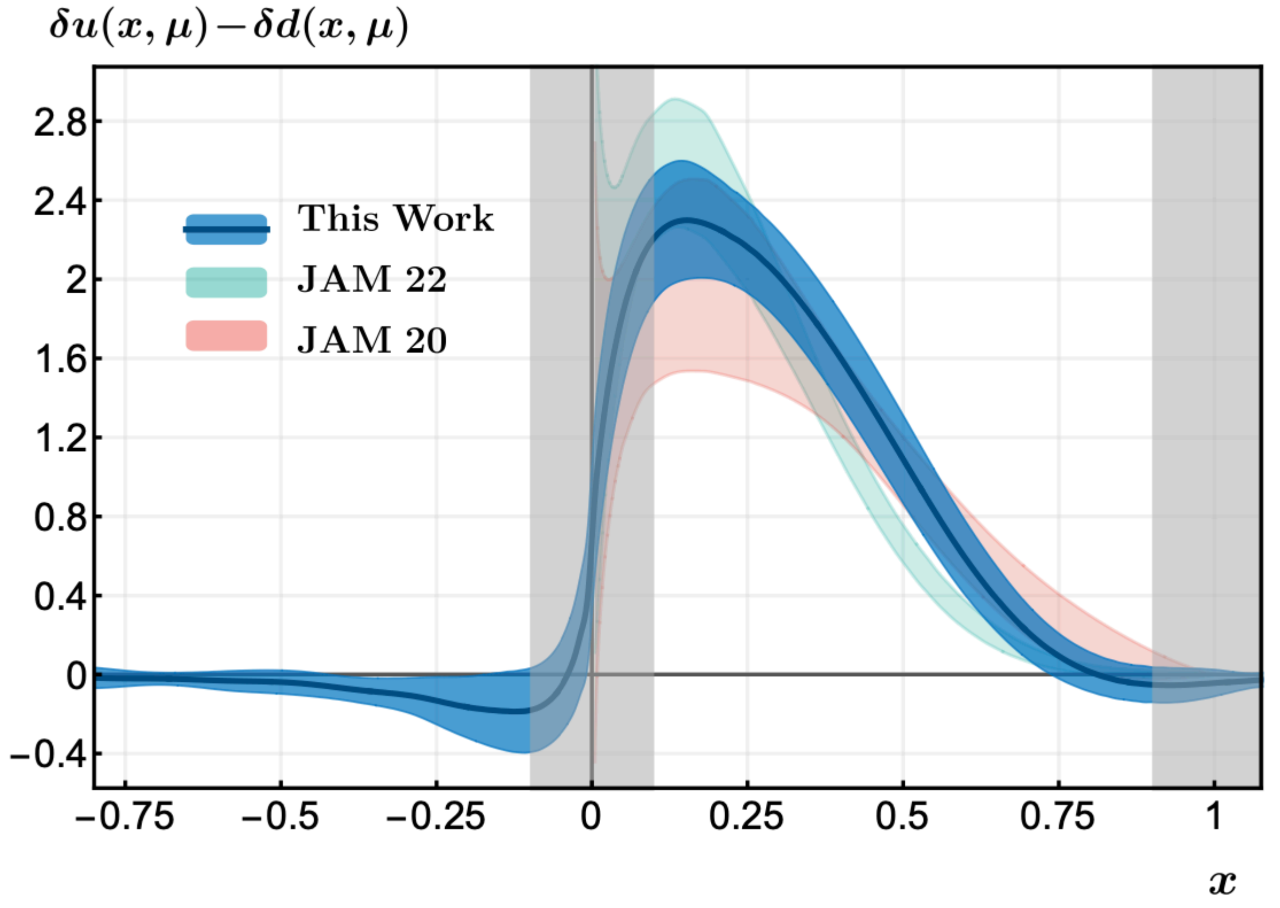
Pion and Kaon DA

Jun Hua et al., PRL129(2022),132001



Transversity distribution

Fei Yao et al., PRL131(2023),261901



CLQCD Configurations

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
0.105 fm	$24^3 \times 72$	292	1000
	$32^3 \times 64$		1000
	$48^3 \times 96$		800
	$64^3 \times 128$		70
	$32^3 \times 64$	225	450
	$48^3 \times 96$		700
	$48^3 \times 96$	135	700
	$64^3 \times 128$		200
0.090 fm	$32^3 \times 64$	287	900
0.077 fm	$24^3 \times 72$	300	250
	$32^3 \times 96$		480
	$48^3 \times 96$		200
	$32^3 \times 64$	210	460
	$48^3 \times 96$		200
	$64^3 \times 128$	135	180
0.069 fm	$36^3 \times 108$	300	700
0.052 fm	$48^3 \times 144$	317	1000
	$64^3 \times 128$		100

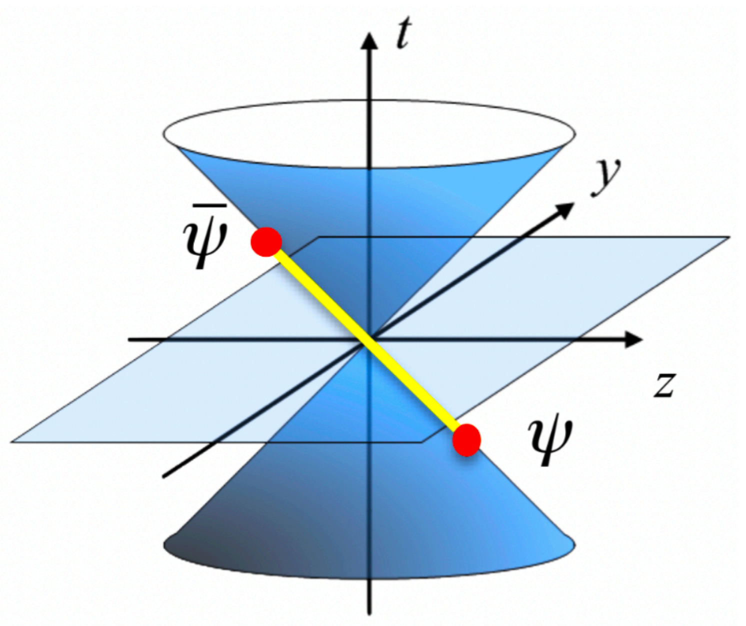
- $2+1$ dynamical quark flavor
- Symanzik-improved gauge action.
- Wilson-clover quark action.
- Quark masses m_u, m_d, m_s are computed based on these ensembles and the results agree with FLAG average value.
- Quark propagators using distillation smearing.

Z.-C.Hu el al., (CLQCD), PRD109(2024)5,054507

Gluon quasi-PDF matrix element

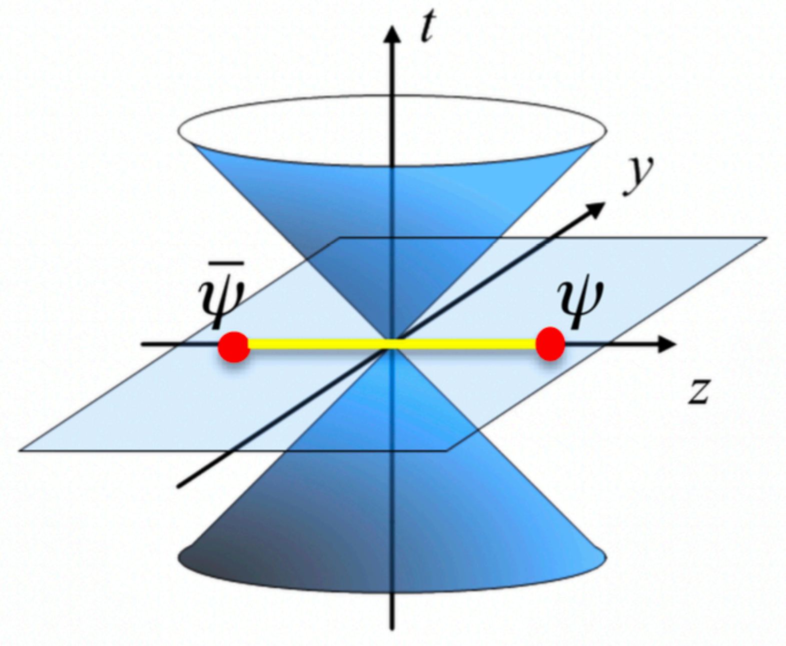
- The light-cone gluon PDF is defined as:

$$g(x,\mu) = \frac{1}{2xP^+}\int_{-\infty}^{+\infty}\frac{d\xi^-}{2\pi}e^{-ixP^+\xi^-}\langle P|F_a^{+\mu}(\xi^-)\mathcal{W}_{ab}(\xi^-,0)F^+_{b,\mu}(0)|P\rangle$$



- According to LaMET, the gluon pdf can be extracted from a spatially correlated matrix element:

$$C_{3pt}(z,P_z,t,t_i) = \langle 0|N(t,P_z)\mathcal{O}_G(z,t_i)\bar{N}(t_0,P_z)|0\rangle$$

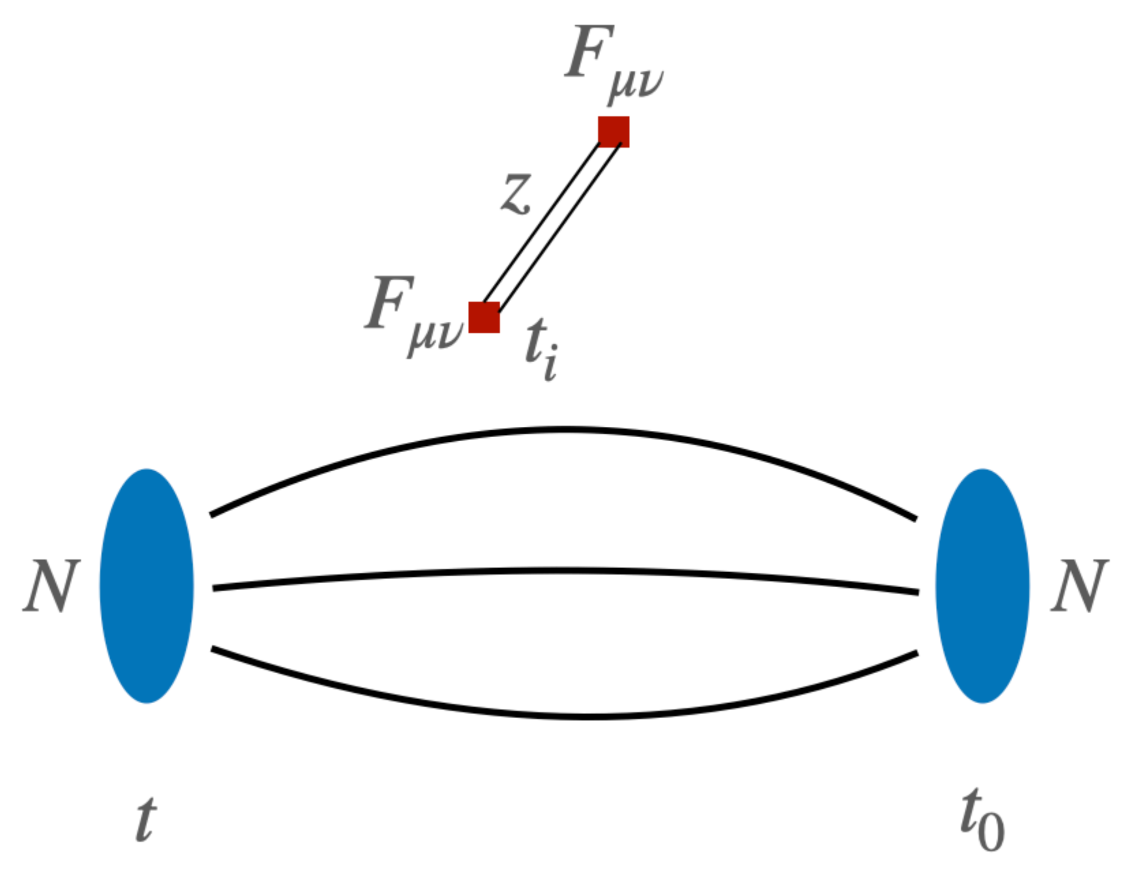


Gluon quasi-PDF matrix element

Unpolorized gluon operator:

$$\mathcal{O}_{\mu\nu}(z,t_i) = \sum_x Tr[F_{\mu\nu}(x,t_i)L(x,x+z)F_{\nu\mu}(x+z,t_i)L(x+z,x)]$$

$$\mathcal{O}_G = \mathcal{O}_{tx} + \mathcal{O}_{ty} - 2\mathcal{O}_{xy}$$



Polorized gluon operator:

$$\mathcal{O}_{\mu\lambda,\nu\rho}(z,t_i) = \sum_x Tr[F_{\mu\lambda}(x,t_i)L(x,x+z)\tilde{F}_{\nu\rho}(x+z,t_i)L(x+z,x)]$$

$$\mathcal{O}_G = \mathcal{O}_{tx,tx} + \mathcal{O}_{ty,ty} - 2\mathcal{O}_{xy,xy}$$

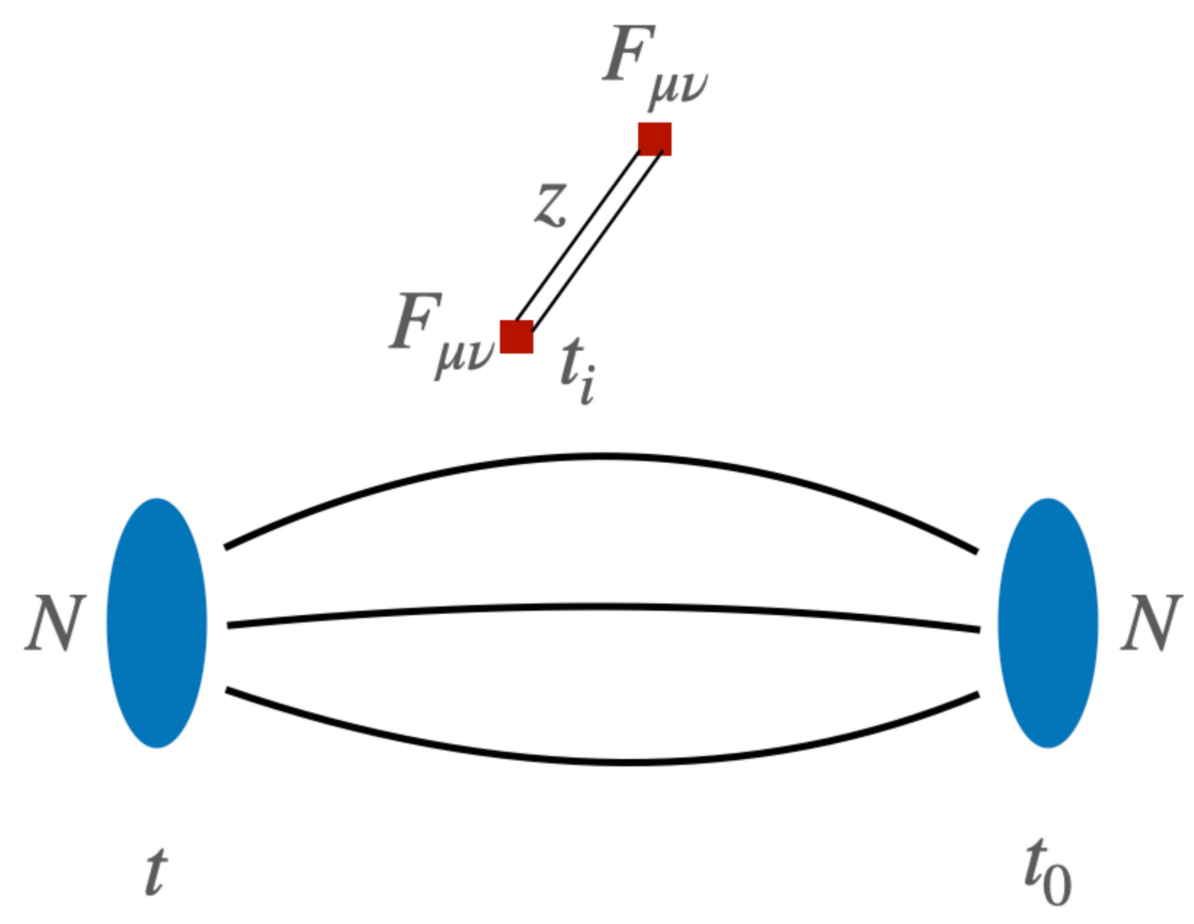
Nucleon operator: $N_{\text{polarized}} = (1 + i\gamma_3\gamma_5)\, P_+(u^T C\gamma_5\gamma_t d)u$

Noise reduction techniques

❖ Distillation quark smearing method

- Smearing: an operator that effectively projects onto the hadronic states of interest.
- Distillation smearing operator: $\square = \sum_{k=1}^{N} v_k \otimes v_k^*$

  v_k is the k'th eigenvector of the laplacian operator defined on the gauge configuration.



- Improve precision, suppress excited-states contaminations, all-to-all propagators, efficient computation with many interpolating operators…

- Momentum smear on distillation: improve the signal for large-momentum nucleon.

Unpoloraized gluon pdf results

Chen Chen, Chunhua Zeng *et al.*, arXiv:2510.26425

Lattice setup

Ensemble	a (fm)	m_π (MeV)	P_z (GeV)	$N_{\mathrm{conf.}}$
C24P29	0.105	292	0, 1.95	878
C32P29	0.105	292	1.84, 2.21	980
C32P23	0.105	228	1.84, 2.21	597
C48P14	0.105	136	1.72, 1.96, 2.21, 2.46	347
E32P29	0.0897	287	0, 1.72, 2.16, 2.59	1700
F32P30	0.0775	300	0, 2.0, 2.5, 2.99	777
G36P29	0.0688	297	0, 2.0, 2.5, 3.0	349
H48P32	0.0519	317	0, 1.99, 2.48, 2.98	650

- **Continuum limit**

  Five different lattice spacings

- **Physical pion mass**

  Pion mass: 136 - 317MeV

- **Infinite-momentum limit**

  Various momenta up to 3GeV

Bare matrix elements

Keeping the ground state and the first excited-state, the three- and two-point functions can be decomposed as:

$$C^{3pt}(z; P_z; t, t_{sep}) = |A_0|^2 \langle 0 | \mathcal{O}_G(z; t) | 0 \rangle e^{-E_0 t_{sep}} + |A_1|^2 \langle 1 | \mathcal{O}_G(z; t) | 1 \rangle e^{-E_1 t_{sep}}$$

$$+ A_1 A_0^* \langle 1 | \mathcal{O}_G(z; t) | 0 \rangle e^{-E_1(t_{sep}-t)} E^{-E_0 t} + A_0 A_1^* \langle 0 | \mathcal{O}_G(z; t) | 1 \rangle e^{-E_0(t_{sep}-t)} E^{-E_1 t} + \cdots$$

$$C^{2pt}(P_z; t_{sep}) = |A_0|^2 e^{-E_0 t_{sep}} + |A_1|^2 e^{-E_1 t_{sep}} + \cdots$$

The ground state matrix elements can be extracted from the ratio of three- to two-point function :

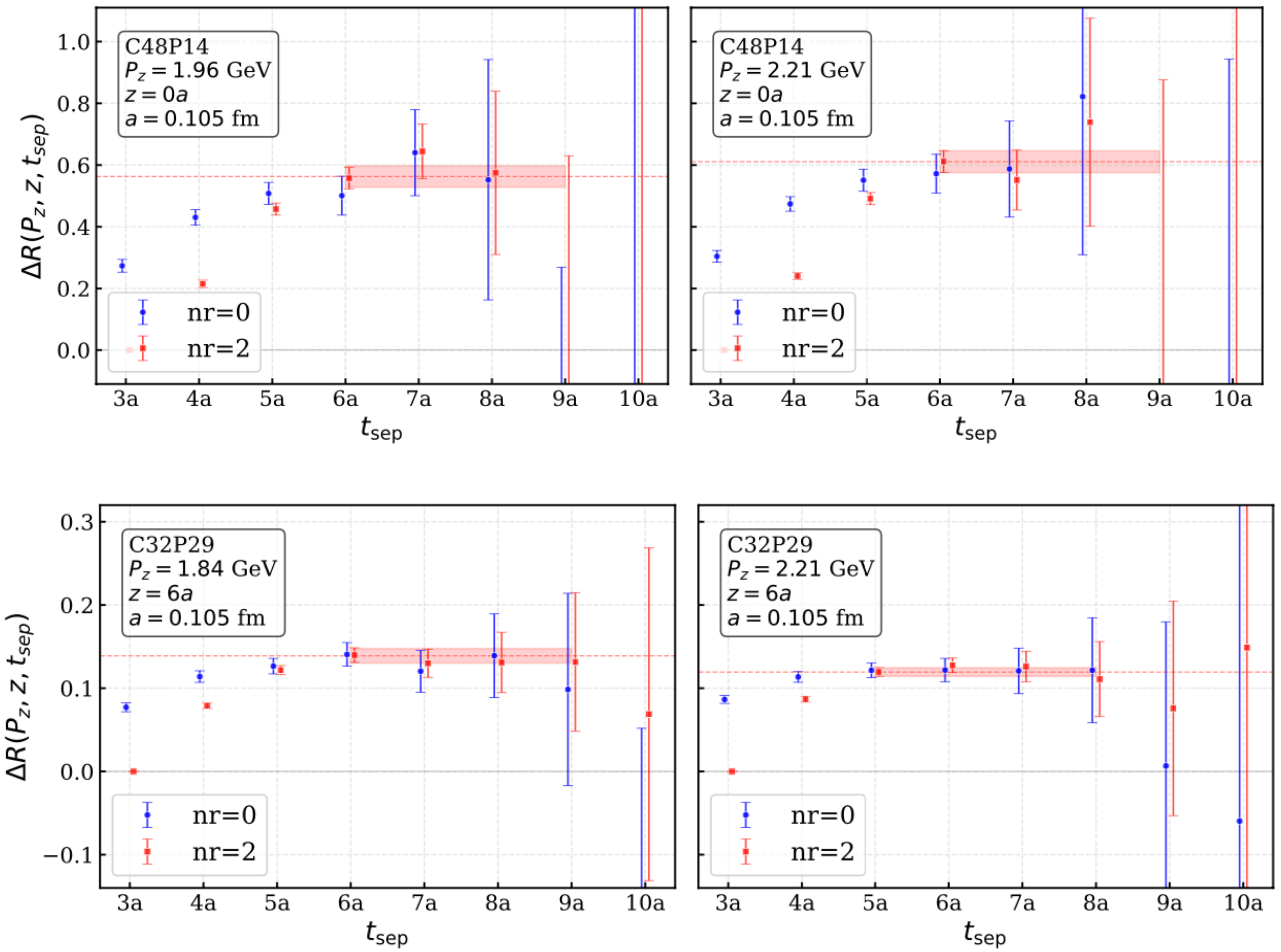
$$R(t, t_{sep}) = \frac{C^{3pt}}{C^{2pt}} = c_0 + c_1 e^{-\Delta E (t_{sep}-t)} + c_1 e^{-\Delta E\, t} + c_2 e^{-\Delta E\, t_{sep}} + \cdots$$

Bare matrix elements

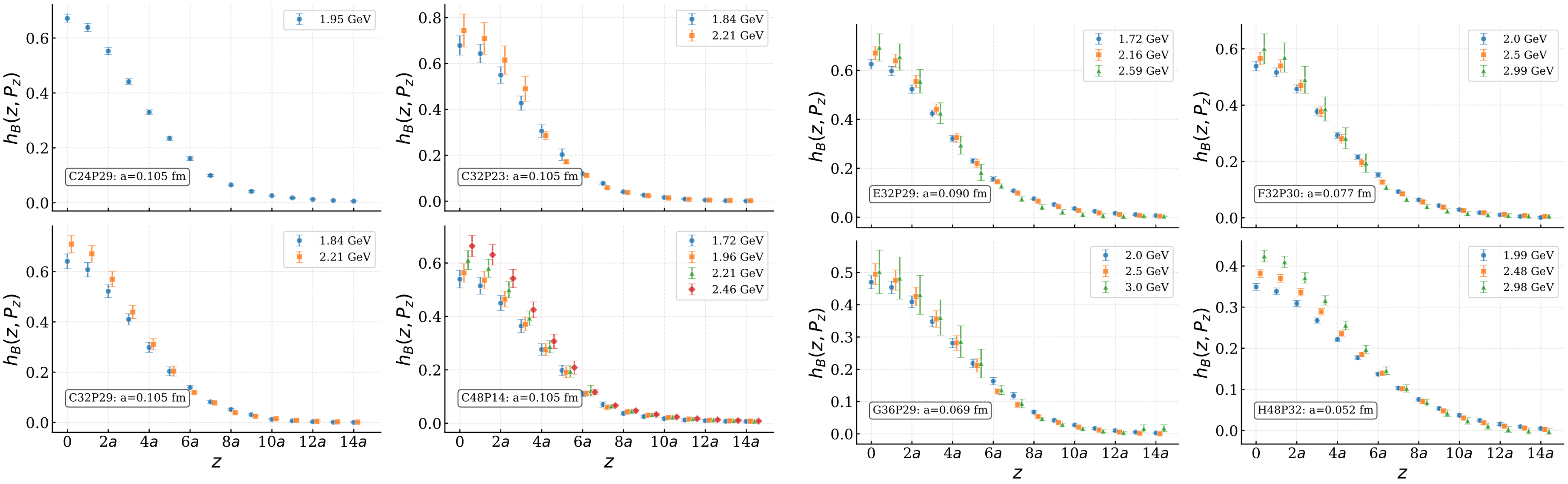
Feynman-Hellmann theory:

$$S(t_{sep}) = \sum_{t_i} R(t_i, t_{sep}) = c + t_{sep} c_0 + \cdots$$

$$\Delta R(t_{sep}) = S(t_{sep}) - S(t_{sep} - 1) \xrightarrow{t_{sep}\to\infty} c_0$$



Bare matrix elements



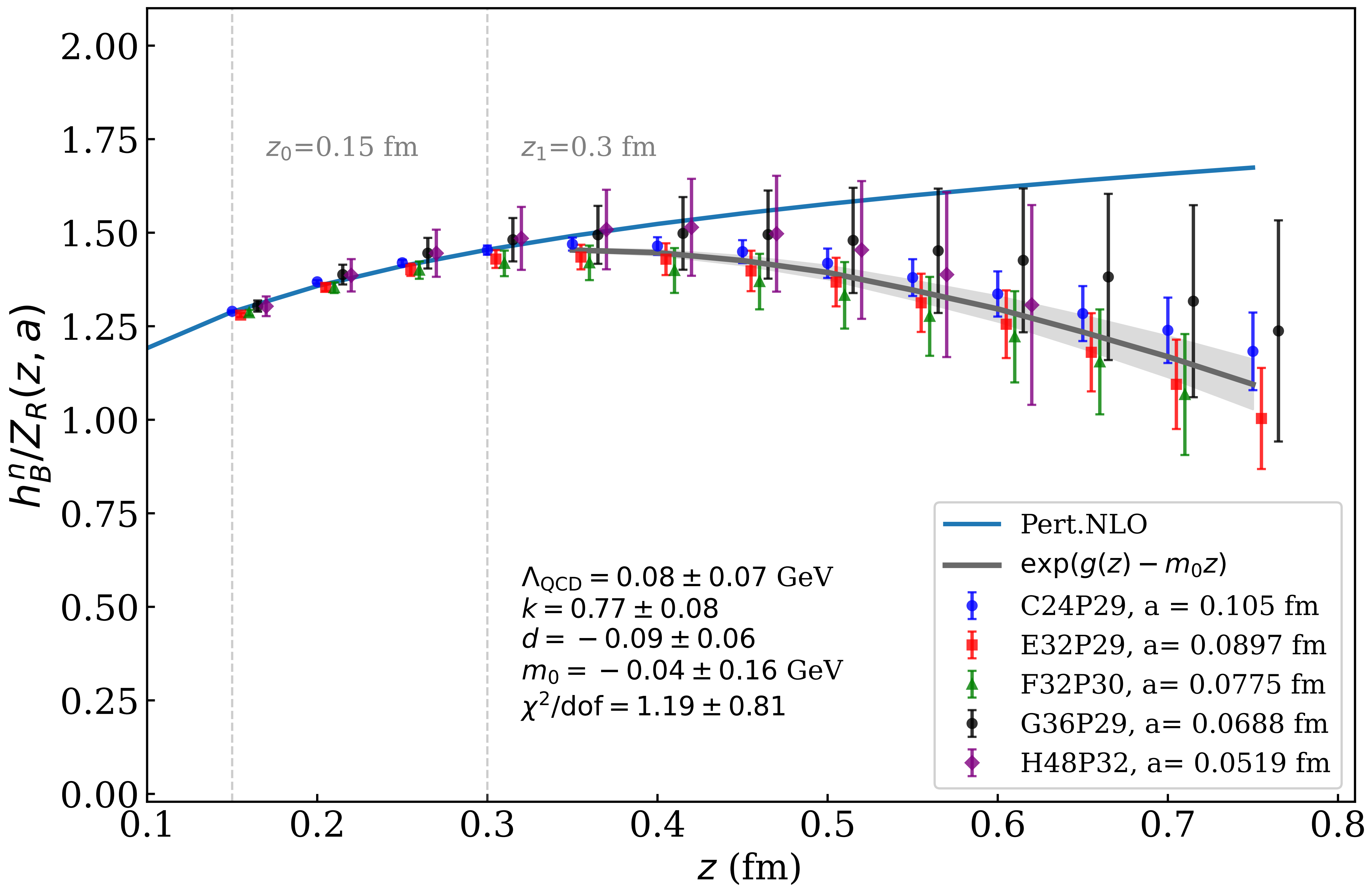
Renormalization

Hybrid renormalization：

$$\tilde{h}_R(z,P_z) = \frac{\tilde{h}_B^n(z,P_z,1/a)}{\tilde{h}_B^n(z,P_z=0,1/a)}\theta(z_s-|z|) \text{ (Ratio scheme)}$$

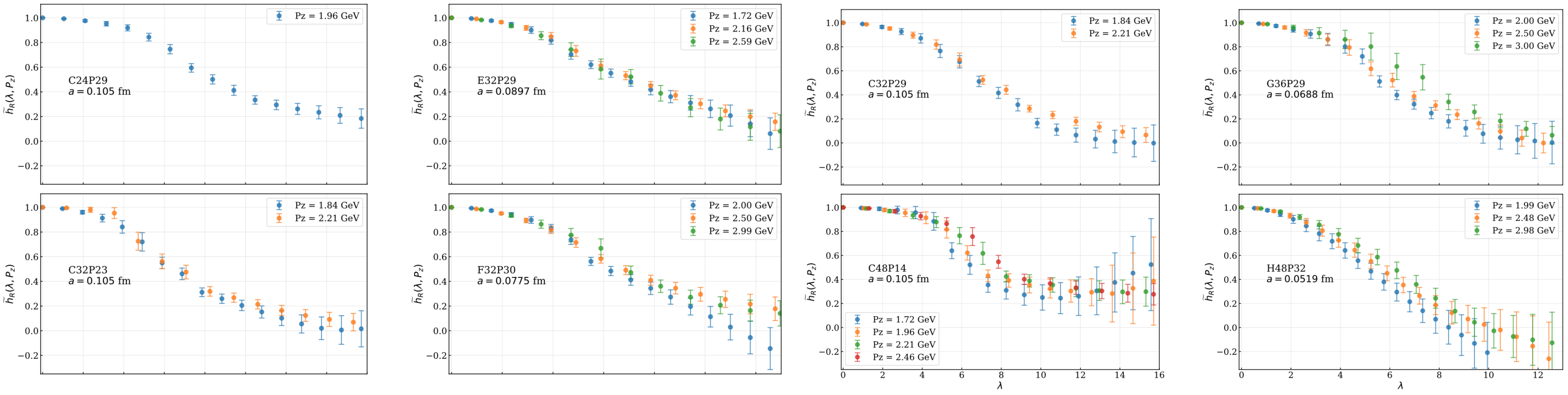
$$+\eta_s\frac{\tilde{h}_B^n(z,P_z,1/a)}{Z_R(z,1/a)}\theta(|z|-z_s) \text{ (Self-renormalization)}$$

$$Z_R(z,1/a,\mu) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{QCD}]}+m_0 z + \frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{QCD})}{\ln(\mu/\Lambda_{QCD})}\right]+\frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{QCD}]}\right]^2\right\}$$



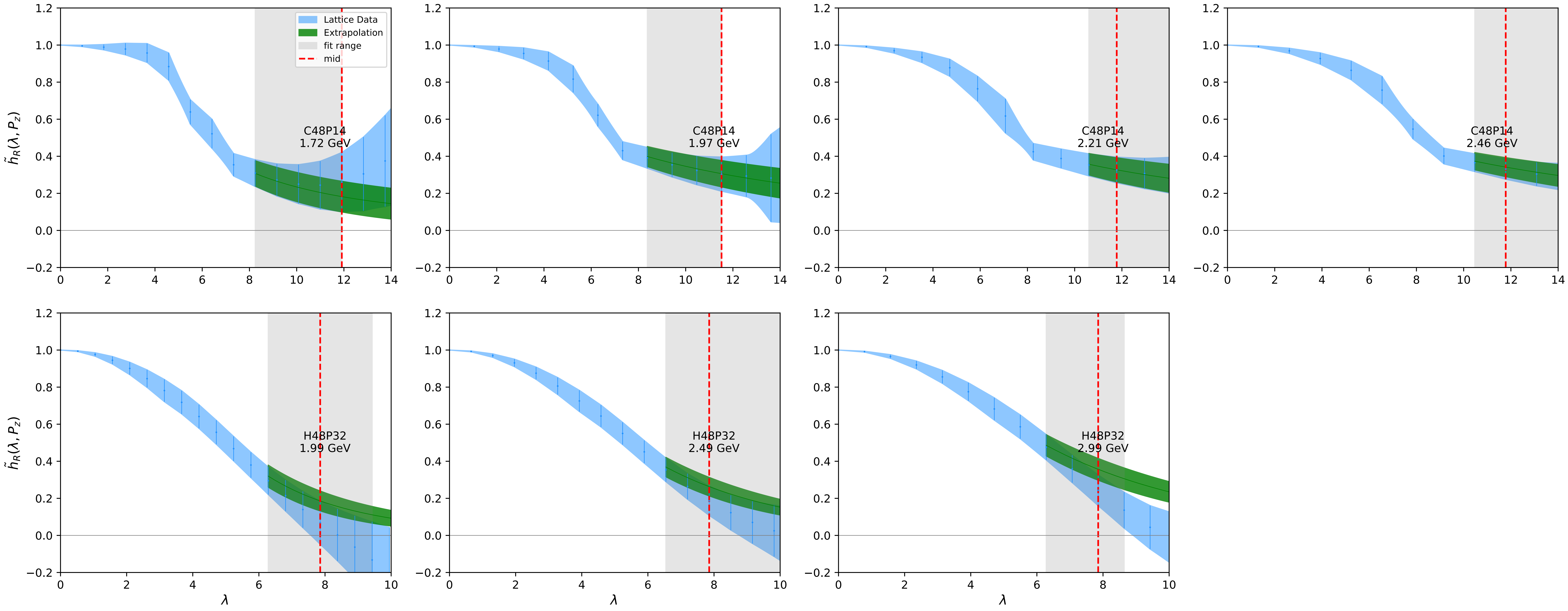
Renormalization

Renormalized matrix elements



λ extrapolation

Large $\lambda\,(zP_z)$ extrapolation: $\tilde{h}_R(\lambda) = l_1 \lambda^{-a_1} e^{-\lambda/\lambda_0}$



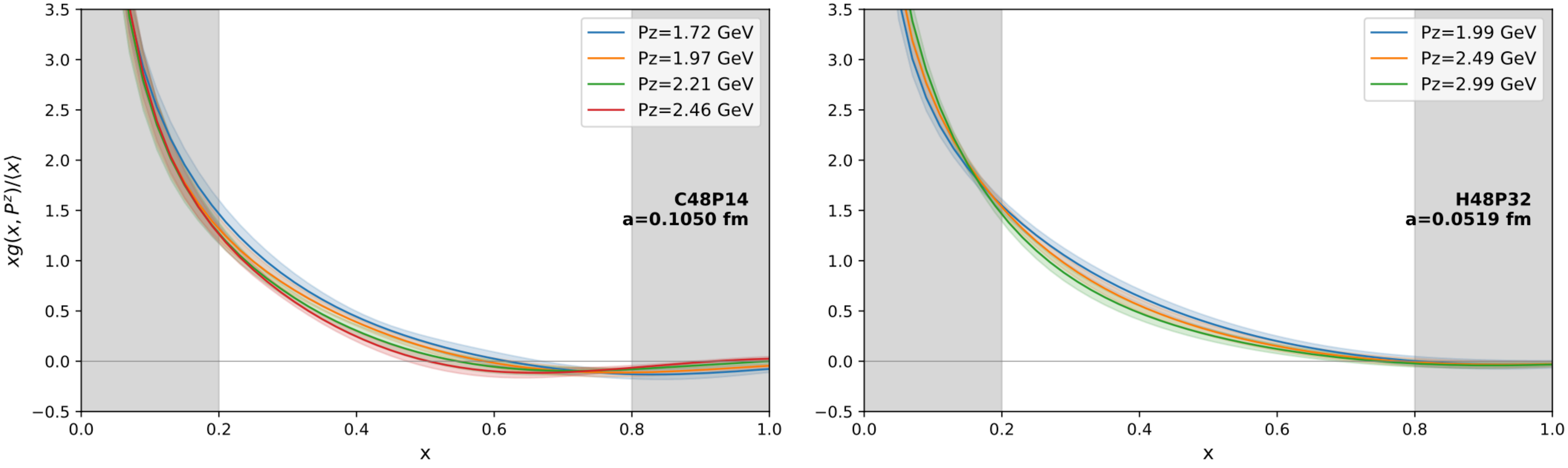
Matching to light-cone PDF

$$x\tilde{g}(x,P^z) = \int_{-1}^{1} \frac{dy}{|y|} C_{\text{hybrid}}\left(\frac{x}{y}, \lambda_s, \frac{\mu}{yP^z}\right) yg(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(xP^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x)P^z)^2}\right)$$

Quasi PDF (Lattice)

Matching kernel (Perturbative)

Light-cone PDF



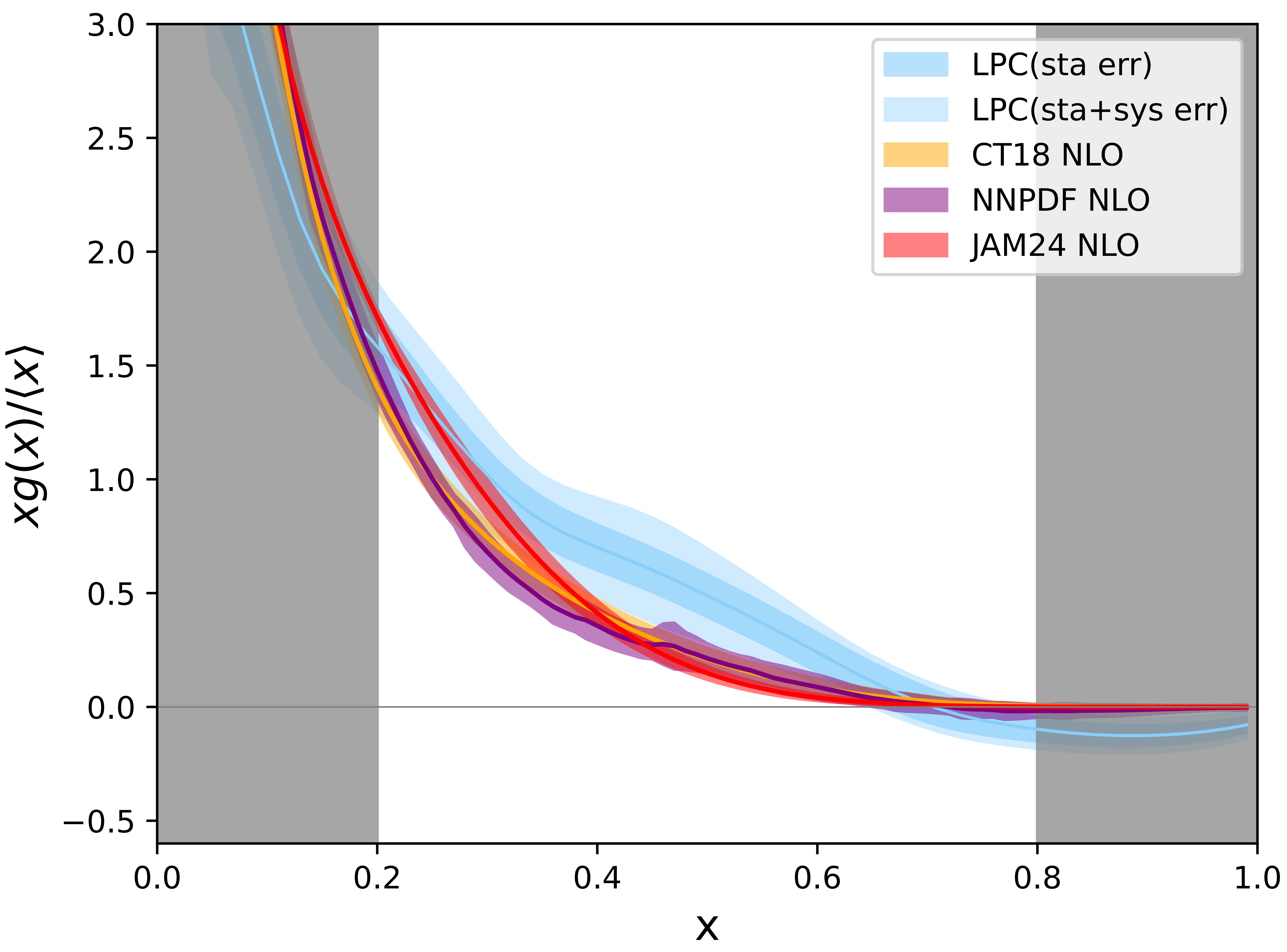
Extrapolations

✦ Infinite momentum ✦ Continuum limit ✦ Pion mass

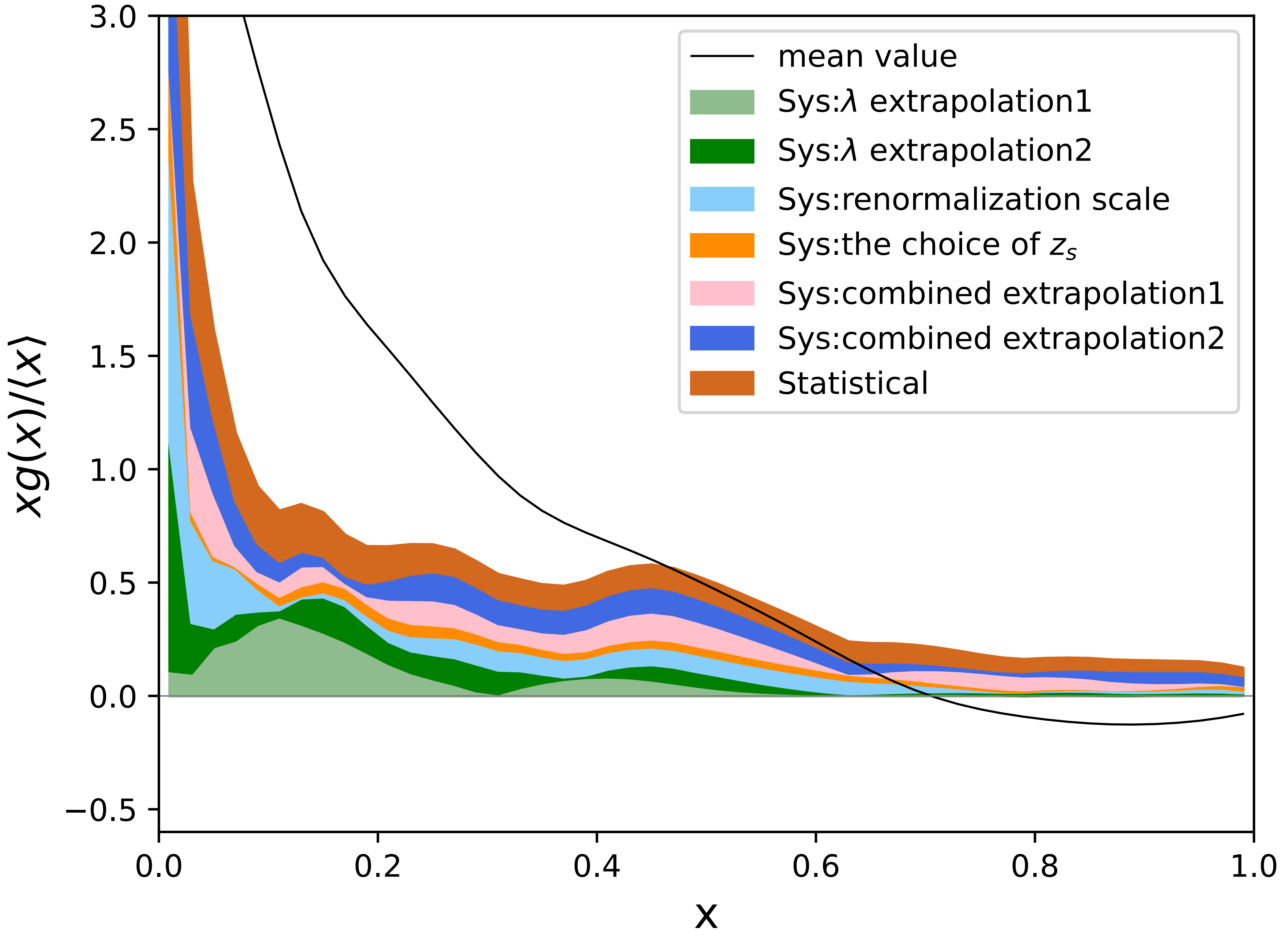
$$xg(x,P_z,a) = xg_0(x) + a^2 f(x) + \frac{d(x)}{P_z^2} + k(x)(m_\pi^2 - m_{\pi,phy}^2)$$

$$xg(x,P_z,a) = xg_0(x) + a f(x) + \frac{d(x)}{P_z^2} + k(x)(m_\pi^2 - m_{\pi,phy}^2)$$

$$xg(x,P_z,a) = xg_0(x) + a^2 f(x) + a^2 P_z^2 h(x) + \frac{d(x)}{P_z^2} + k(x)(m_\pi^2 - m_{\pi,phy}^2)$$



Systematic uncertainties



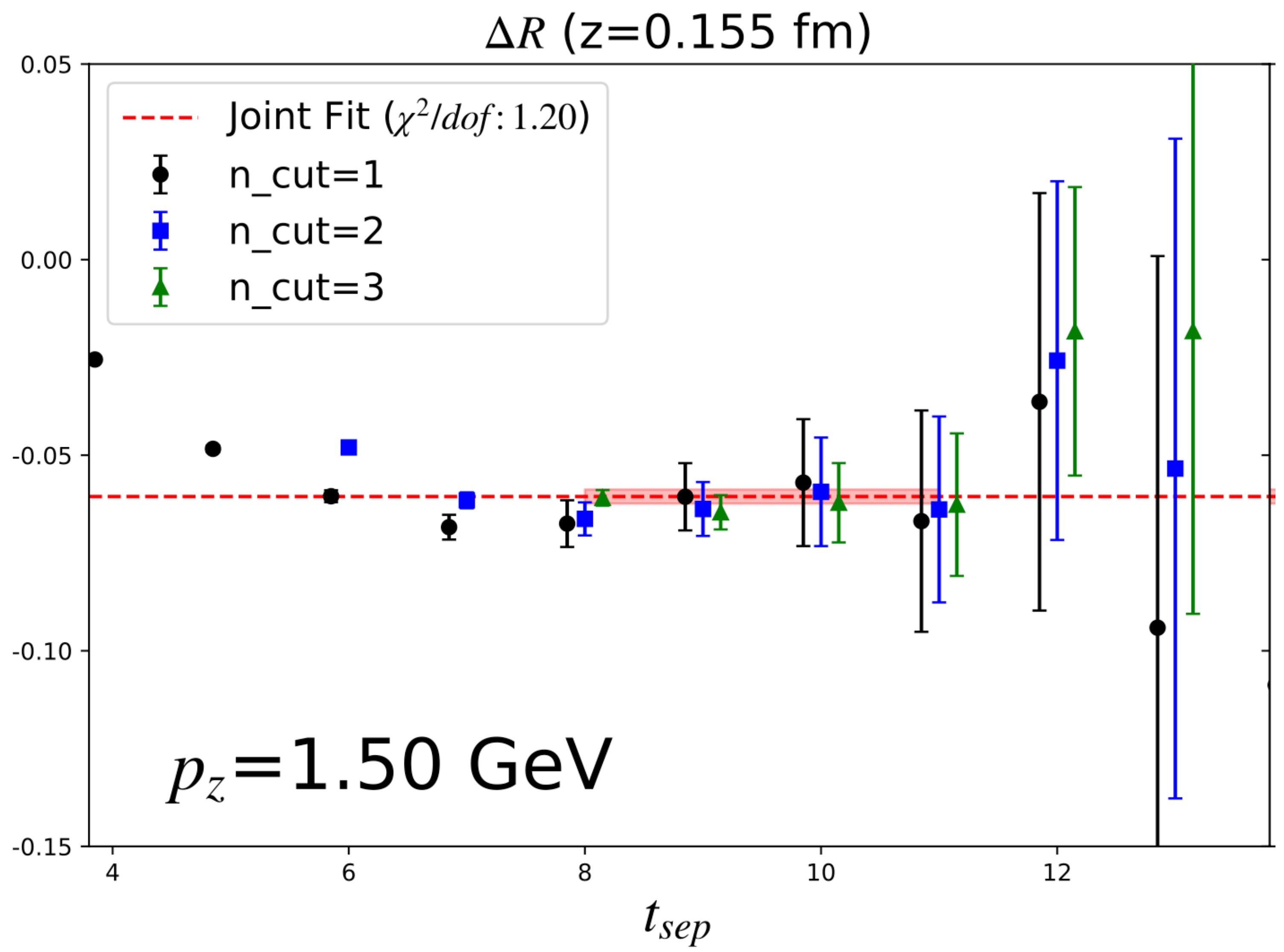
Polarized gluon PDF results

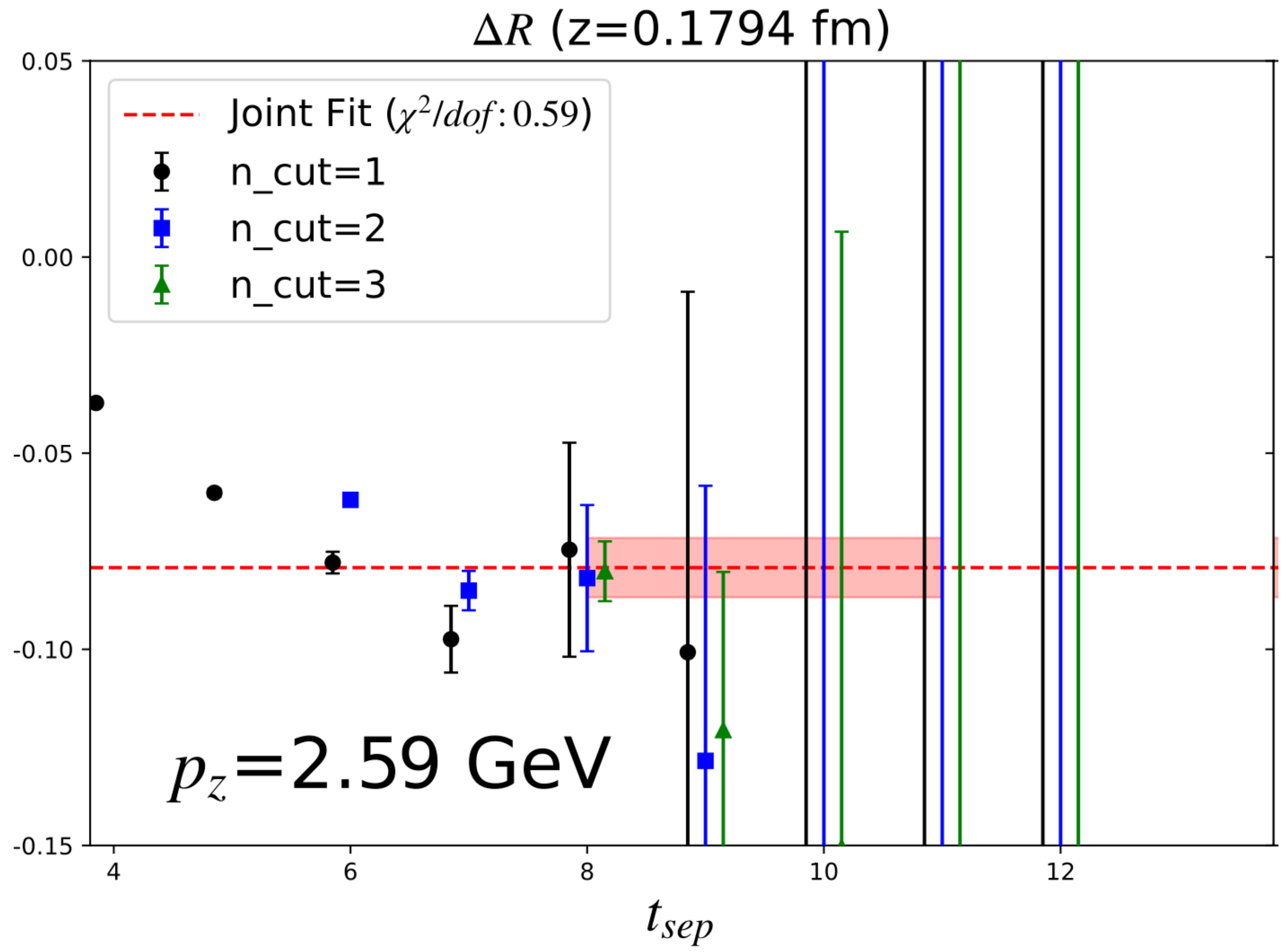
Shiyi Zhong *et al.*, arXiv:2608.xxxxx

Bare matrix elements

Lattice setup: three different lattice spacings, pion mass ~300MeV

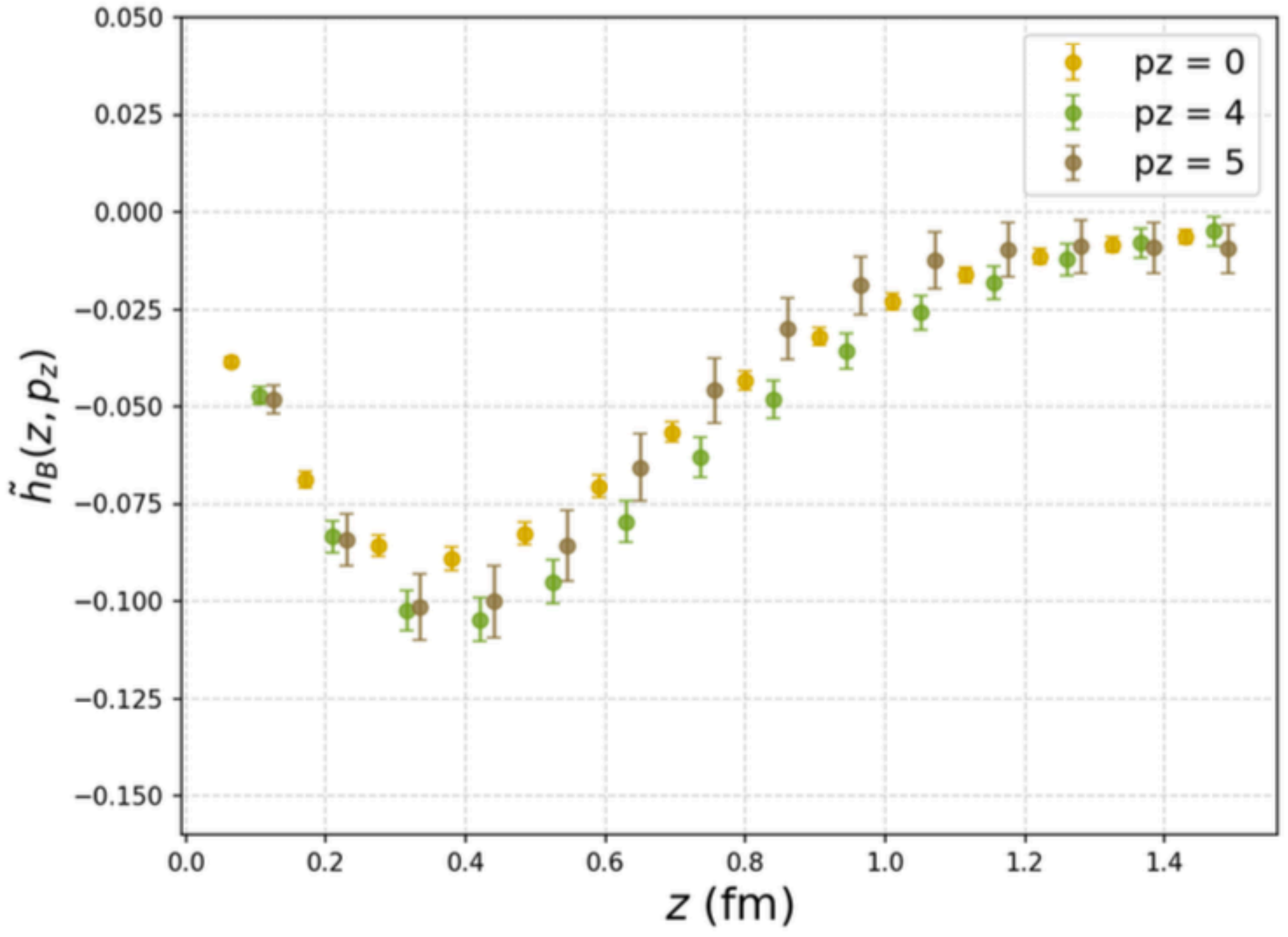
Ensemble	a(fm)	$L^3 \times T$	m_π(MeV)	$m_\pi L$	$N_{\text{conf.}}$
C32P29	0.105	$32^3 \times 64$	292.4(1.1)	5.01	854
E32P29	0.0897	$32^3 \times 64$	287.3(2.5)	4.179	1700
F32P30	0.0775	$32^3 \times 96$	300.4(1.2)	3.780	777



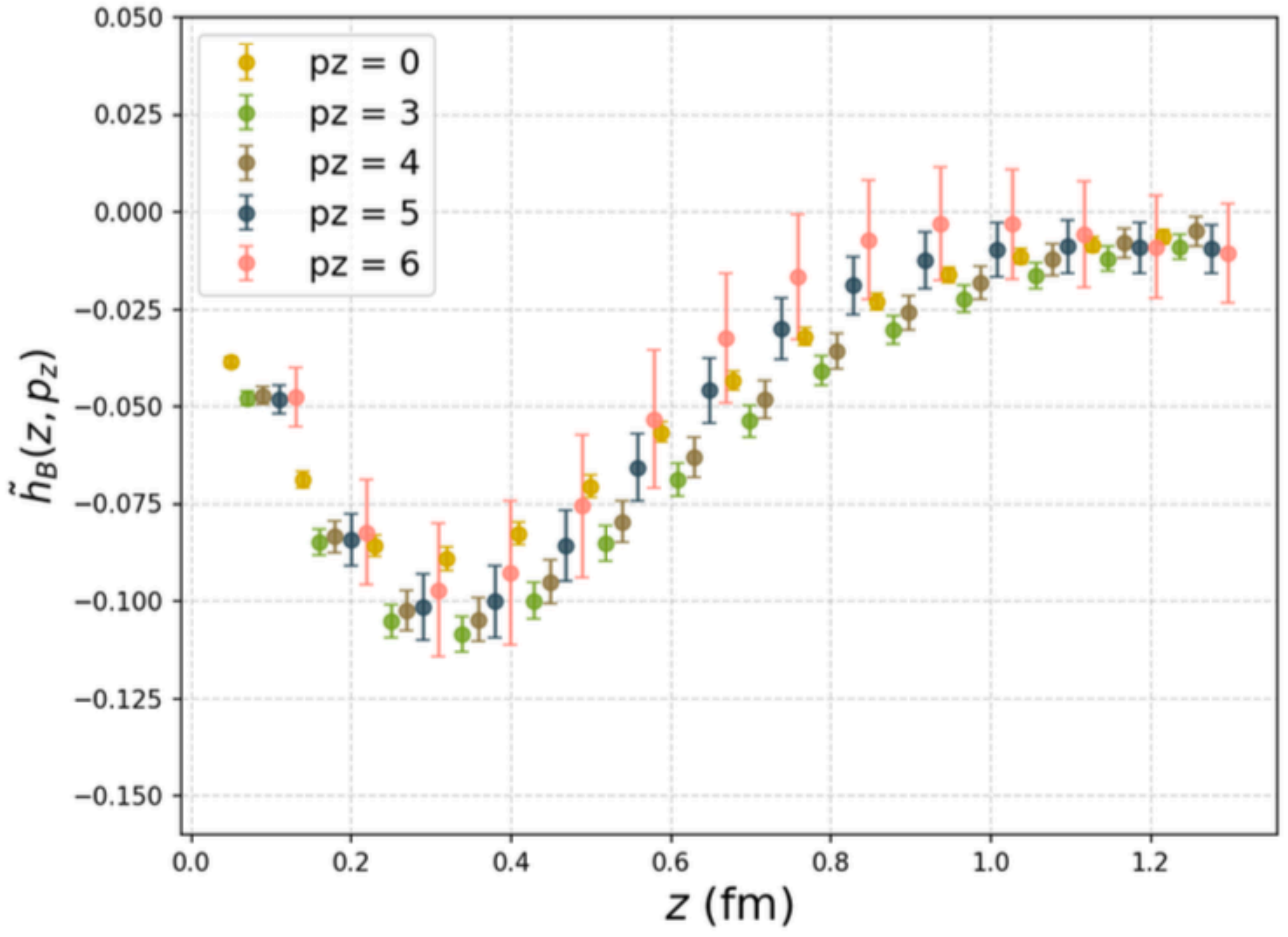


Bare matrix elements

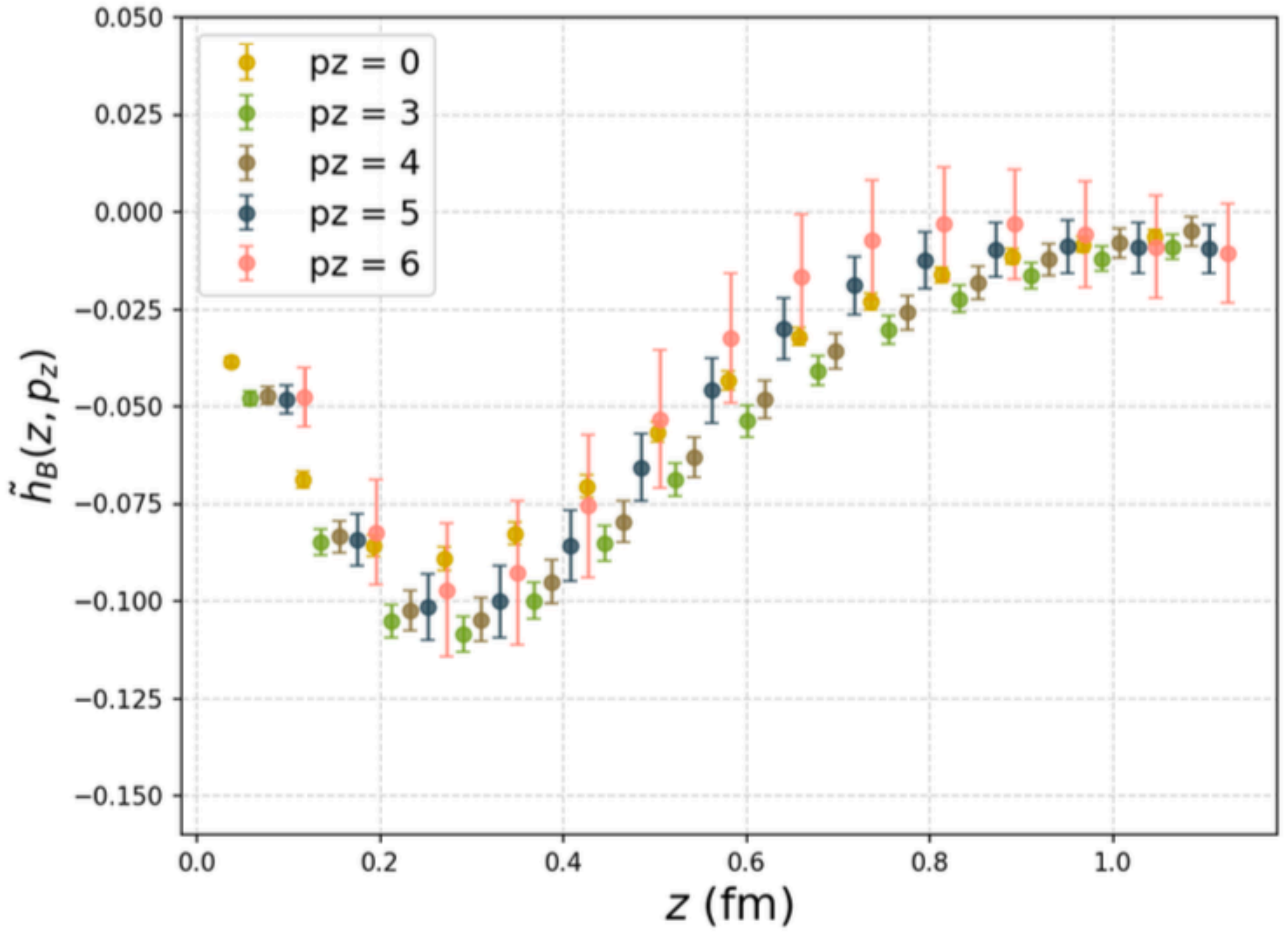
C32P29, $a = 0.105$fm



E32P29, $a = 0.0897$fm



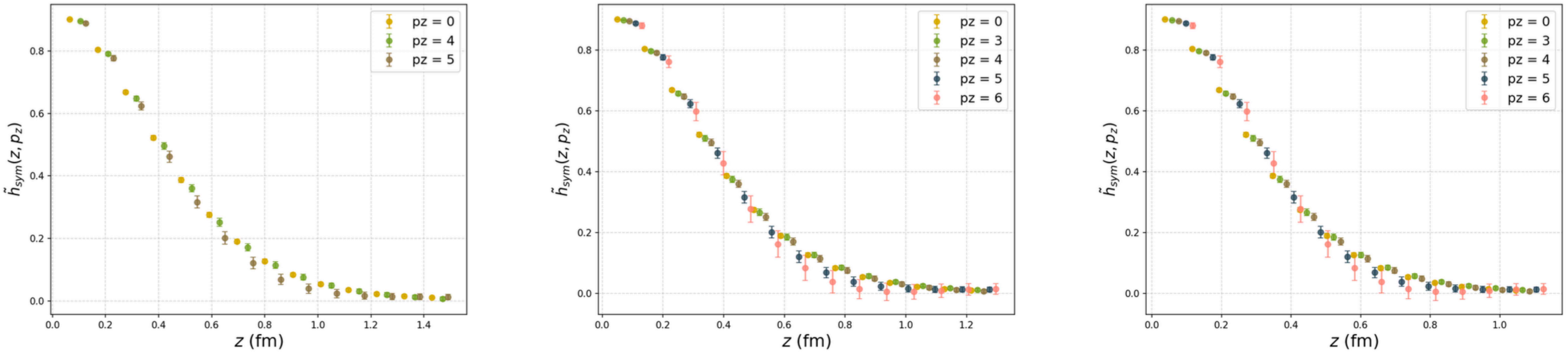
F32P30, $a = 0.0775$fm



Renormalization

- Self-renormalization procedure requires a symmetric matrix element at small z .
- Gluon helicity quasi-LF correlation is imaginary and thus antisymmetric in z .
- Construct a symmetric matrix element:

$$\tilde{h}_{sym}(z, P_z) = \frac{\tilde{h}_B(z, P_z)/z}{[\tilde{h}_B(z, P_z)/z]_{z\to 0}}$$



Renormalization

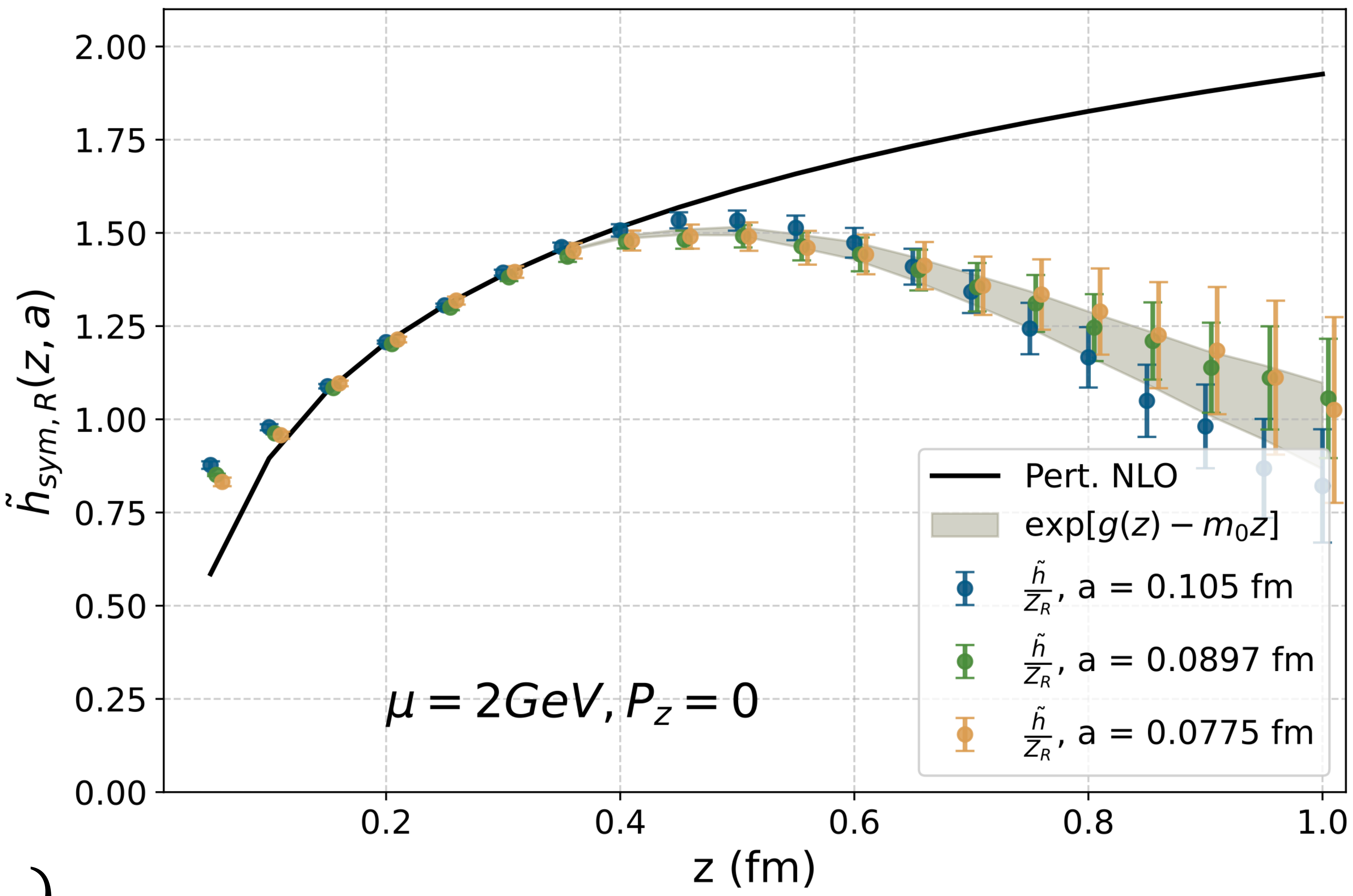
Hybrid renormalization：

$$\tilde{h}_R(z,P_z) = z\Big[\frac{\tilde{h}_{sym}(z,P_z,1/a)}{\tilde{h}_{sym}(z,P_z=0,1/a)}\theta(z_s-|z|)$$

$$+\frac{\tilde{h}_{sym}(z,P_z,1/a)}{Z_R(z,1/a)\tilde{h}^{pert}_{sym,\bar{MS}}}\theta(|z|-z_s)\Big]$$

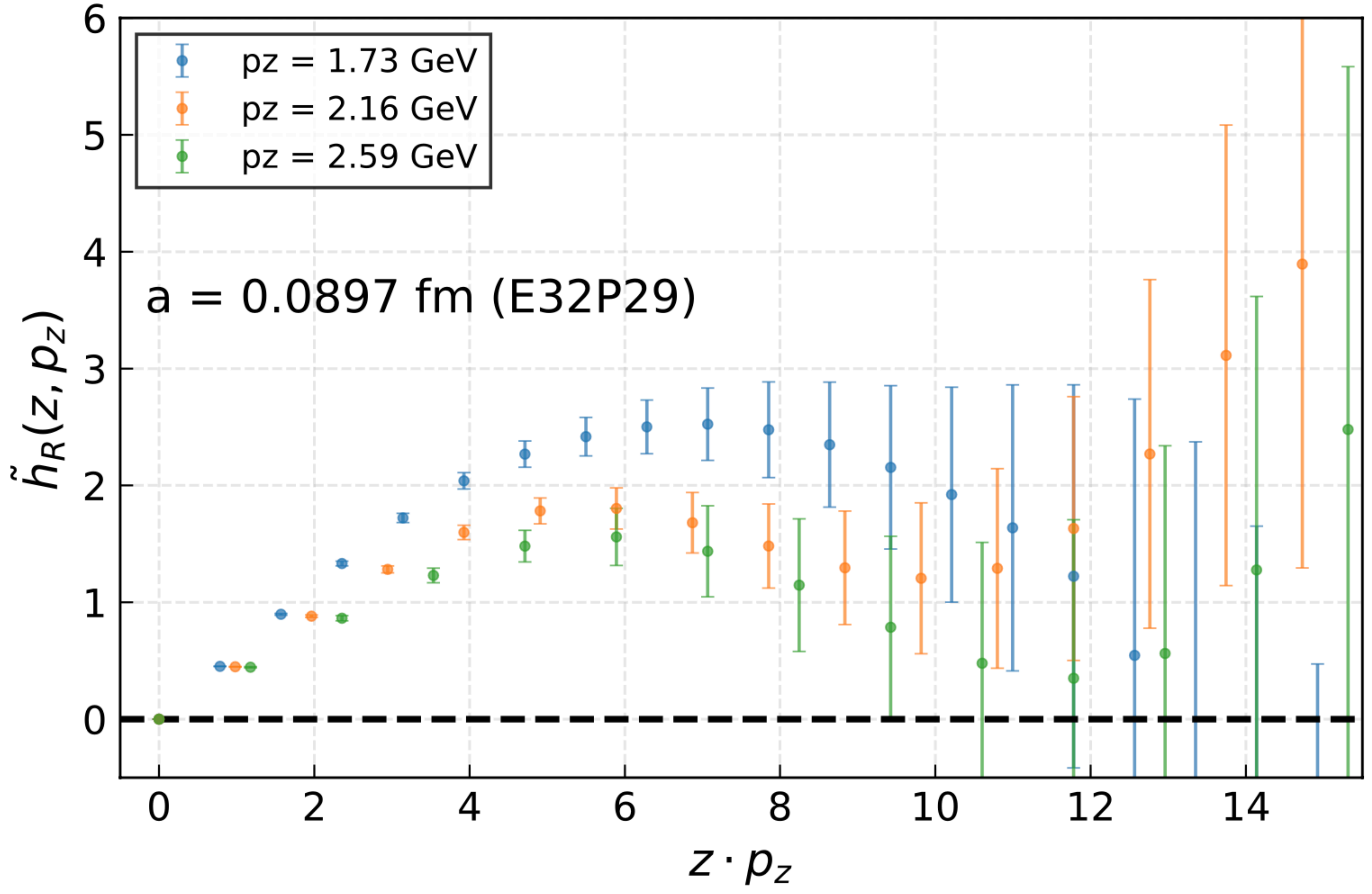
$$Z_R(z,1/a,\mu) = \exp\Big\{\frac{kz}{a\ln[a\Lambda_{\rm QCD}]}+m_0 z$$

$$+\frac{14C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right]+\frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\rm QCD}]}\right]^2\Big\}$$



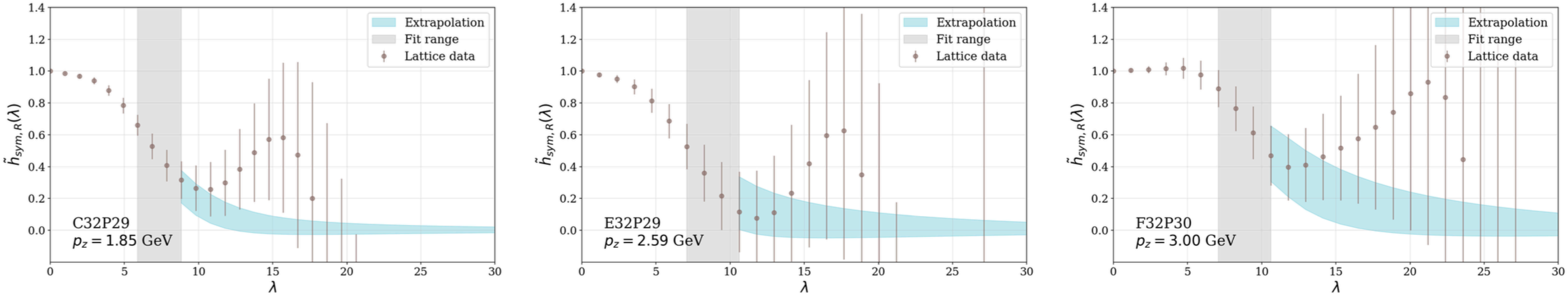
Renormalization

Renormalized matrix elements



λ extrapolation

Large $\lambda\,(zP_z)$ extrapolation: $\tilde{h}_R(\lambda) = l_1 \lambda^{-a_1} e^{-\lambda/\lambda_0}$



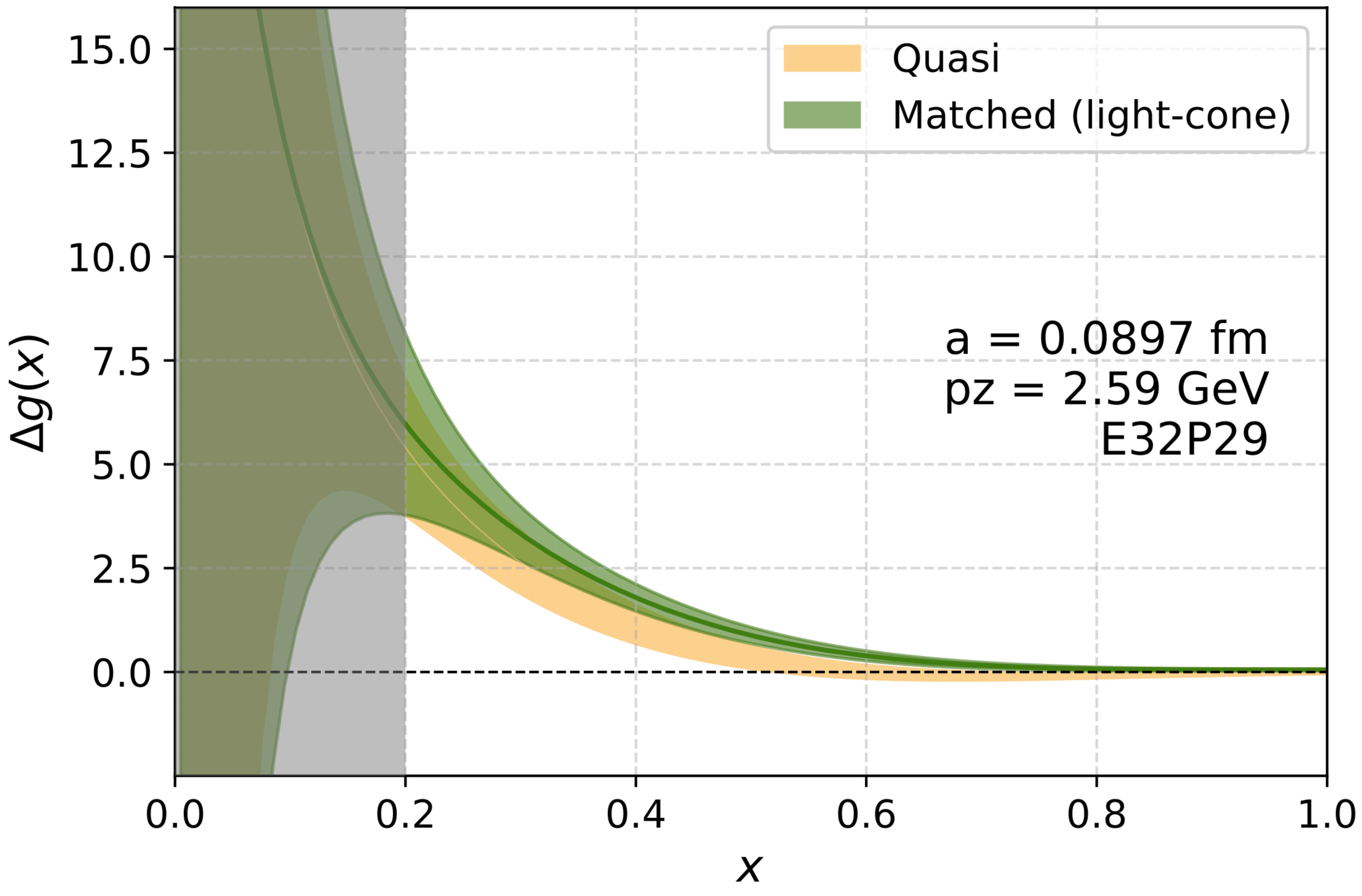
Matching to light-cone PDF

$$y\Delta g(y,\mu) = \int_{-1}^{1} \frac{dx}{|x|} C_{\text{hybrid}}\left(\frac{y}{x}, \lambda_s, \frac{\mu}{xP^z}\right) x\Delta\tilde{g}(x, P_z) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{(yP^z)^2}, \frac{\Lambda^2_{\text{QCD}}}{((1-y)P^z)^2}\right)$$

Light-cone PDF

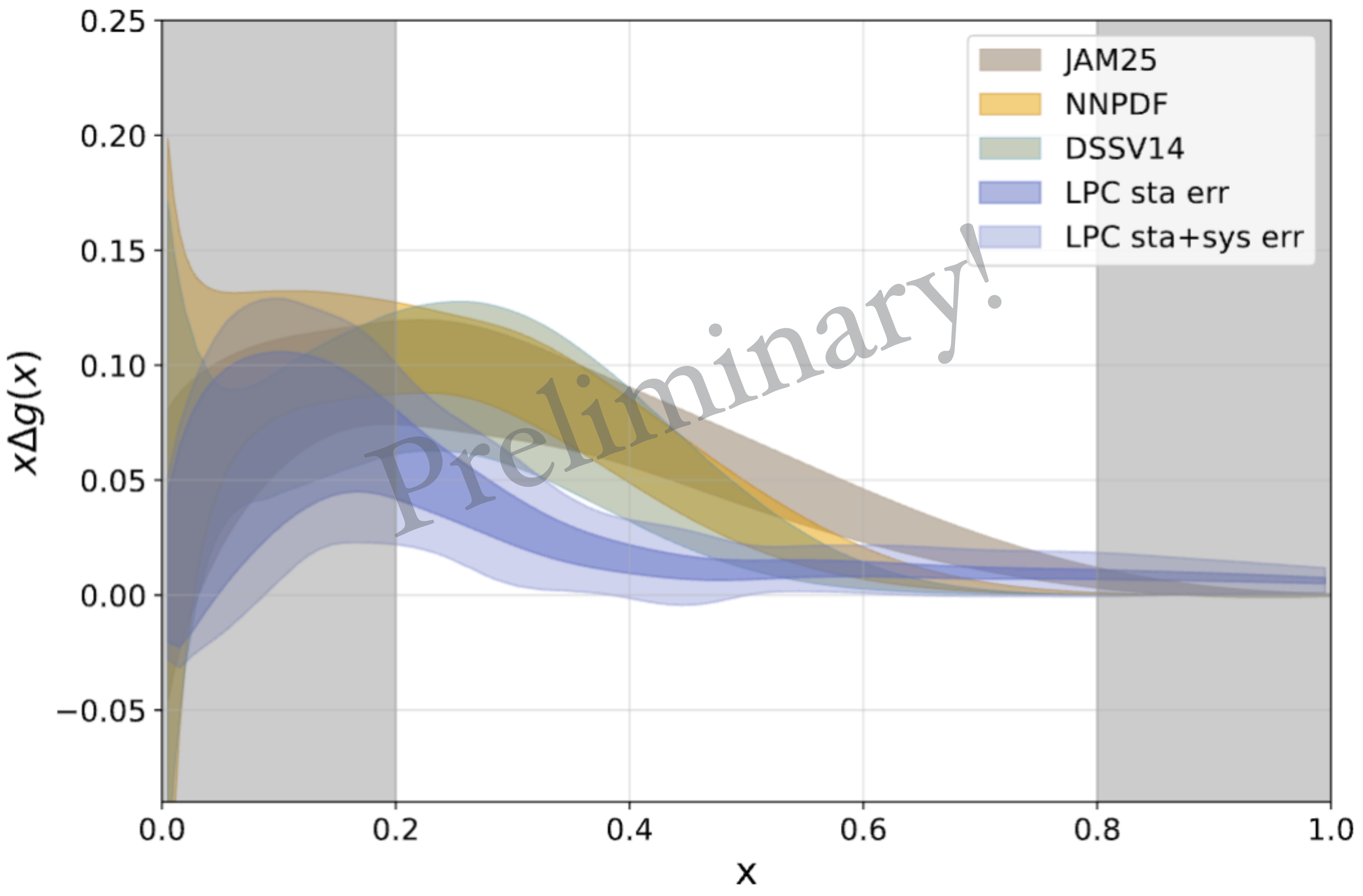
Matching kernel
(Perturbative)

Quasi PDF
(Lattice)



Extrapolations

$$x\Delta g(x, P_z, a) = x\Delta g_0(x) + a^2 f(x) + a^2 P_z^2 h(x) + \frac{d(x)}{P_z^2}$$



Summary and outlooks

- First systematic lattice study of nucleon unpolorized gluon pdf at physical pion mass, continuum limit and infinite momentum limit.
- Polorized gluon pdf is computed in LaMET for the first time.
- Mixing with quarks is ignored
- Furthur improve the precision
- Gluon GPDs, TMDs

Thanks!

Backups

Self renormalization factor(unpolarized)

$$Z_R(z,1/a,\mu) = \exp\left\{ \frac{kz}{a\ln[a\Lambda_{\mathrm{QCD}}]} + m_0 z + \frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\mathrm{QCD}})}\right] + \frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\mathrm{QCD}}]}\right]^2 \right\}$$

Parameters in Z_R is determined by fitting the zero-momentum bare matrix elements:

$$\begin{aligned} \ln\tilde{h}_B^n(z, P_z=0, 1/a) &= \frac{kz}{a\ln[a\Lambda_{\mathrm{QCD}}]} \\ &+ \frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\mathrm{QCD}})}\right] + \frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\mathrm{QCD}}]}\right]^2 \\ &+ \begin{cases} \ln\left[Z_{\overline{\mathrm{MS}}}(z,\mu)\right] + m_0 z & \text{if } z_0 \le z \le z_1 \\ g(z) & \text{if } z_1 < z \end{cases}, \end{aligned} \tag{22}$$

$$Z_{\overline{\mathrm{MS}}}(z,\mu) = 1 + \frac{\alpha_s C_A}{4\pi}\left(\frac{5}{3}\ln\frac{z^2\mu^2}{4e^{-2\gamma_E}} + 3\right)$$

Backups

Self renormalization factor(polarized)

$$Z_R(z,1/a,\mu) = \exp\left\{ \frac{kz}{a\ln[a\Lambda_{\mathrm{QCD}}]} + m_0 z \quad + \frac{14C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\mathrm{QCD}})}\right] + \frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\mathrm{QCD}}]}\right]^2 \right\}$$

Parameters in Z_R is determined by fitting the zero-momentum bare matrix elements:

$$\begin{aligned}
&\ln \tilde{h}_{\mathrm{sym}}(z, P_z = 0, 1/a) = \ln\left[Z_R(z, 1/a, \mu; k, \Lambda_{\mathrm{QCD}}, m_0, d)\right] \\
&+ \begin{cases} \ln\left[\tilde{h}^{\mathrm{pert}}_{sym,\overline{\mathrm{MS}}}(z,\mu)\right] & z_0 \le z \le z_1 \\ g(z) - m_0 z & z_1 < z \end{cases}, \qquad (21)
\end{aligned}$$

$$\tilde{h}^{\mathrm{pert}}_{sym,\overline{\mathrm{MS}}}(z,\mu) = 1 + \frac{\alpha_s(\mu)C_A}{4\pi}\left(\frac{19}{6}\ln\frac{z^2\mu^2}{4e^{-2\gamma_E}} - \frac{5}{6}\right).$$

Backups

Matching kernel(NLO, unpolarized):

$$C_{\text{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_s C_A}{2\pi}\begin{cases}\left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi>1\\ \left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\left(-\ln\frac{\mu^2}{4y^2P_z^2}+\ln(\xi(1-\xi))\right)-\frac{15-56\xi+102\xi^2-96\xi^3+48\xi^4}{6(1-\xi)}\right]_+ & 0<\xi<1\\ \left[\frac{-2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}-\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi<0,\end{cases}$$

$$C_{\text{hybrid}}(\xi,\lambda_s,\frac{\mu}{yP_z})=C_{\text{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)+\frac{\alpha_s C_A}{2\pi}\frac{5}{6}\left(-\frac{1}{|1-\xi|}+\frac{2\text{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)}\right)_+$$

Backups

Matching kernel(NLO, unpolarized):

$$C_{\mathrm{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_s C_A}{2\pi}\begin{cases}\left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi>1\\ \left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\left(-\ln\frac{\mu^2}{4y^2P_z^2}+\ln(\xi(1-\xi))\right)-\frac{15-56\xi+102\xi^2-96\xi^3+48\xi^4}{6(1-\xi)}\right]_+ & 0<\xi<1\\ \left[\frac{-2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}-\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi<0,\end{cases}$$

$$C_{\mathrm{hybrid}}(\xi,\lambda_s,\frac{\mu}{yP_z})=C_{\mathrm{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)+\frac{\alpha_s C_A}{2\pi}\frac{5}{6}\left(-\frac{1}{|1-\xi|}+\frac{2\mathrm{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)}\right)_+$$

Backups

Matching kernel(NLO, polarized):

$$C_{\text{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_s(\mu)C_A}{2\pi}\begin{cases}\left[\left(\frac{2\xi^2}{1-\xi}+4\xi(1-\xi)\right)\ln\frac{\xi}{\xi-1}-\frac{7-12\xi+12\xi^2}{3(1-\xi)}\right]_+ & \xi>1\\ \left[\left(\frac{2\xi^2}{1-\xi}+4\xi(1-\xi)\right)\left(-\ln\frac{\mu^2}{4p_z^2}+\ln(\xi(1-\xi))\right)+\frac{7-36\xi+69\xi^2-42\xi^3}{3(1-\xi)}\right]_+ & 0<\xi<1\\ \left[-\left(\frac{2\xi^2}{1-\xi}+4\xi(1-\xi)\right)\ln\left(\frac{-\xi}{1-\xi}\right)+\frac{7-12\xi+12\xi^2}{3(1-\xi)}\right]_+ & \xi<0,\end{cases} \tag{10}$$

$$C_{\text{hybrid}}(\xi,\lambda_s,\frac{\mu}{yP_z})=C_{\text{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)+\frac{\alpha_s(\mu)C_A}{2\pi}\frac{13}{6}\left(-\frac{1}{|1-\xi|}+\frac{2\text{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)}\right)_+, \tag{11}$$