



Towards precision calculations for polarized SIDIS and extraction of helicity PDFs

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based on works of the NPC collaboration, and works in collaboration with L. Dong, S. Fang, HT Li, DY Shao, HX Zhu, YJ Zhu, B. Zhou

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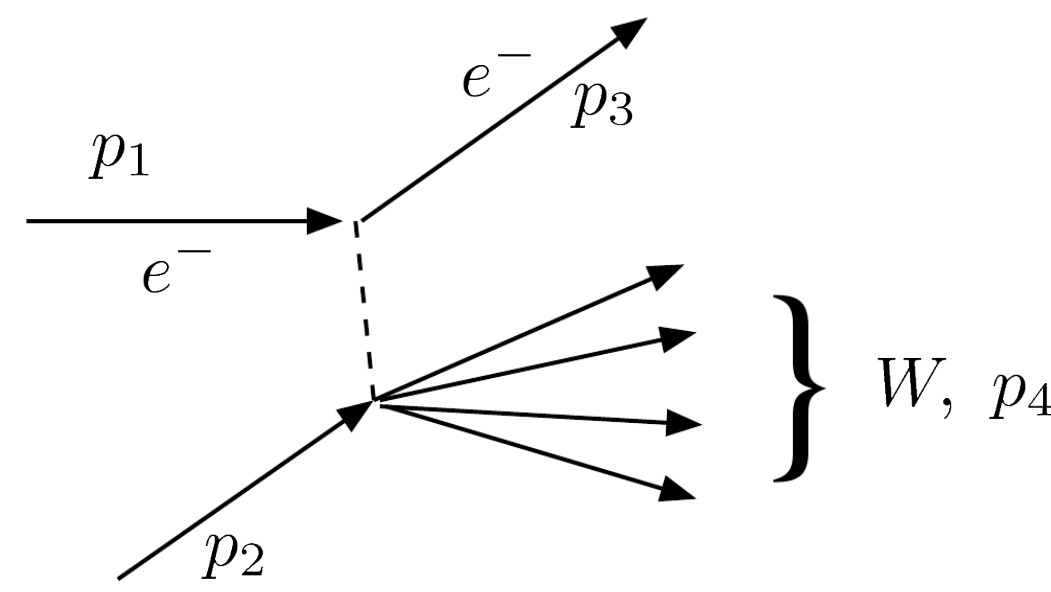
Outline

- ◆ 1. Deep inelastic scattering and nucleon structures
- ◆ 2. NPC23 analysis on FFs of light charged hadrons
- ◆ 3. Precision calculations for (polarized) SIDIS up to N3LO
- ◆ 4. NPC analysis on helicity PDFs of nucleons
- ◆ 5. Summary

Deep inelastic scattering

- ✦ Deep inelastic scattering of high-energy leptons (~ 20 GeV) on fixed-target reveals the internal structure of nucleons, consisting of quarks and gluons (QPM) described by parton distribution functions (PDFs), and lead to the establishment of QCD

DIS kinematics



lab frame - proton at rest before collision:
 $p_2 = (M, 0, 0, 0)$

$$W^2 = M^2 + 2M\nu + q^2 = M^2 + Q^2(1/x - 1)$$

$W=M \rightarrow$ elastic scattering

$1 < W < 2 \text{ GeV} \rightarrow$ inelastic scattering
 (excitation of resonances)
 $W > 2 \text{ GeV} \rightarrow$ deep inelastic scattering

lorentz invariant form

energy loss of
incoming particle

$$\nu = E_1 - E_3$$

$$\nu = \frac{p_2 q}{M}$$

Bjorken x

$$x = \frac{Q^2}{2M\nu} \quad \text{x in } [0,1]$$

$$x = \frac{Q^2}{2p_2q}$$

fractional energy loss
of incoming particle

$$y = 1 - \frac{E_3}{E_1} \quad y \text{ in } [0,1]$$

$$y = \frac{p_2 q}{p_2 p_1}$$

4-momentum transfer

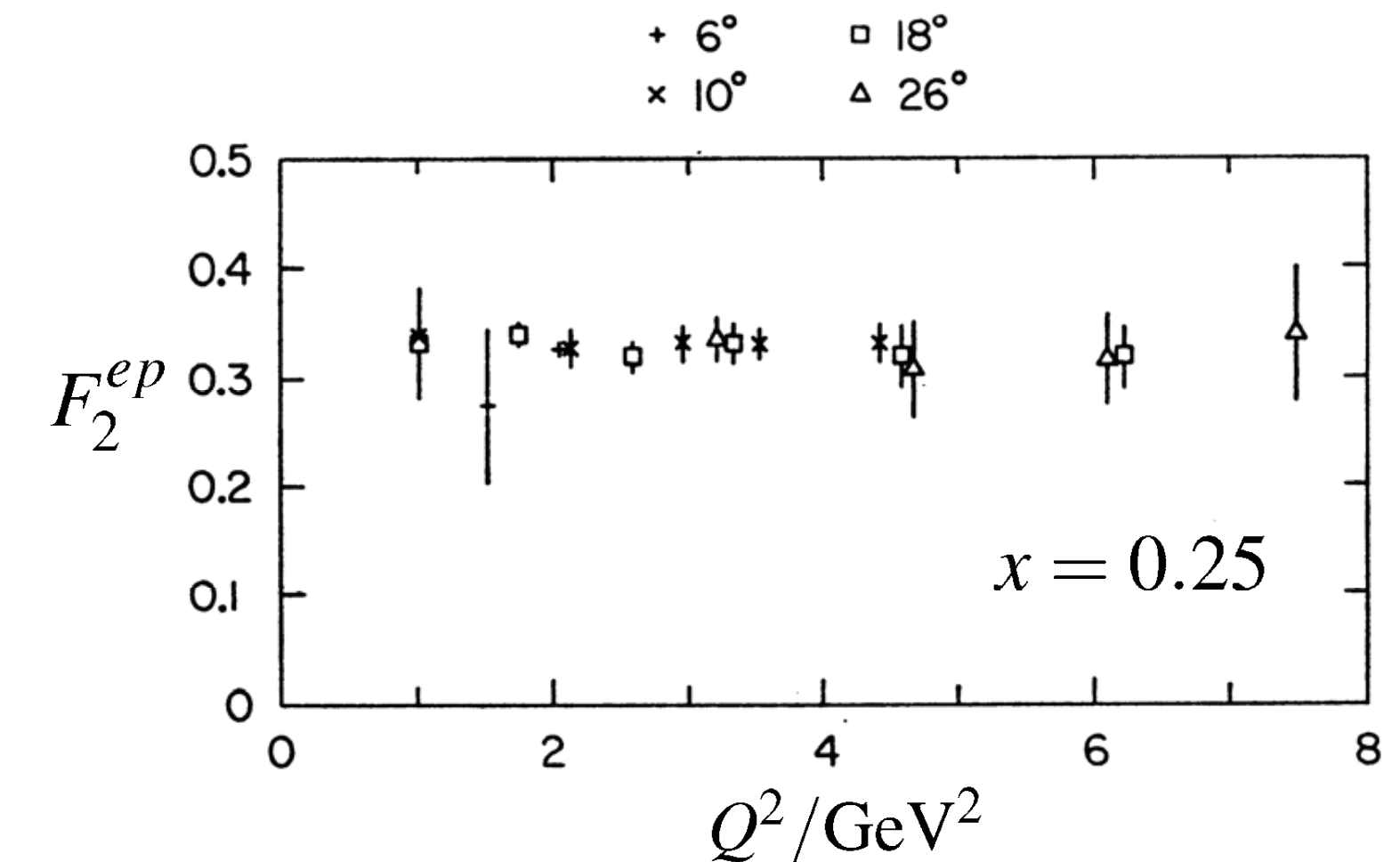
$$q^2 = (p_1 - p_3)^2 \quad Q^2 = -q^2 \quad q^2 = (p_1 - p_3)^2$$

$$q^2 = (p_1 - p_3)^2$$

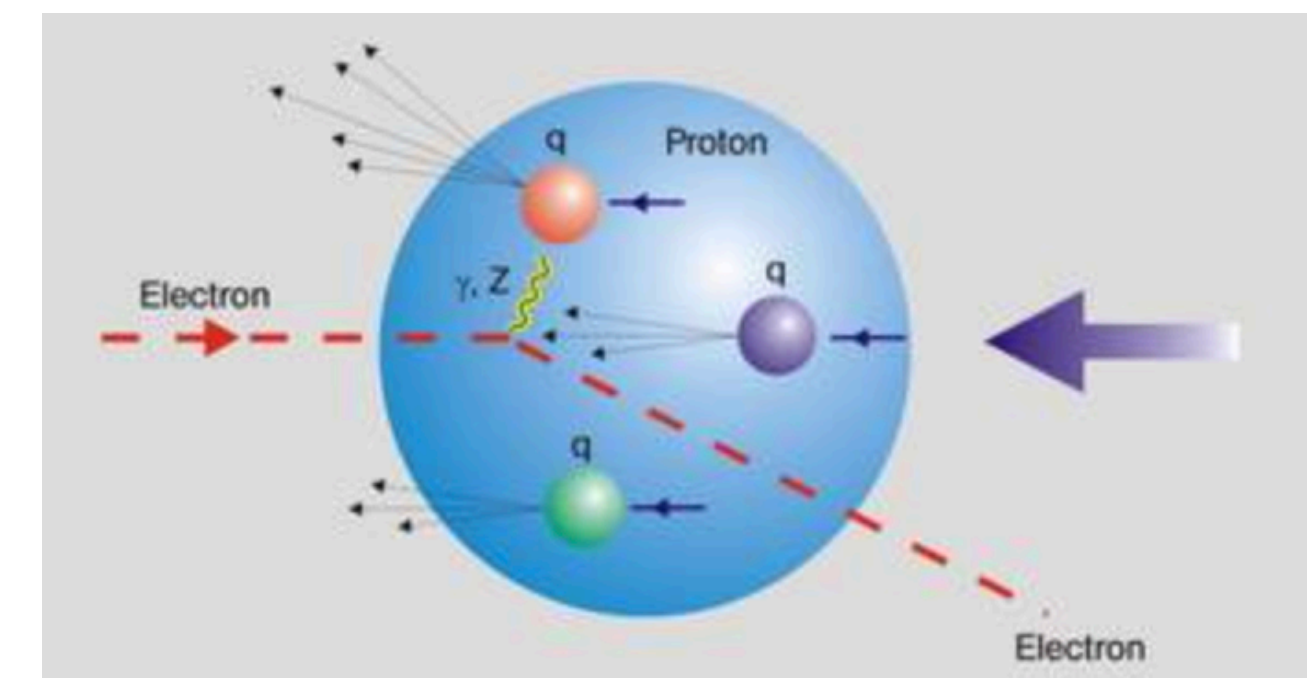
DIS cross sections and structure functions:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

Bjorken scaling of SFs and QPM



$p_g = xp$, with probability density $\sim f(x)$

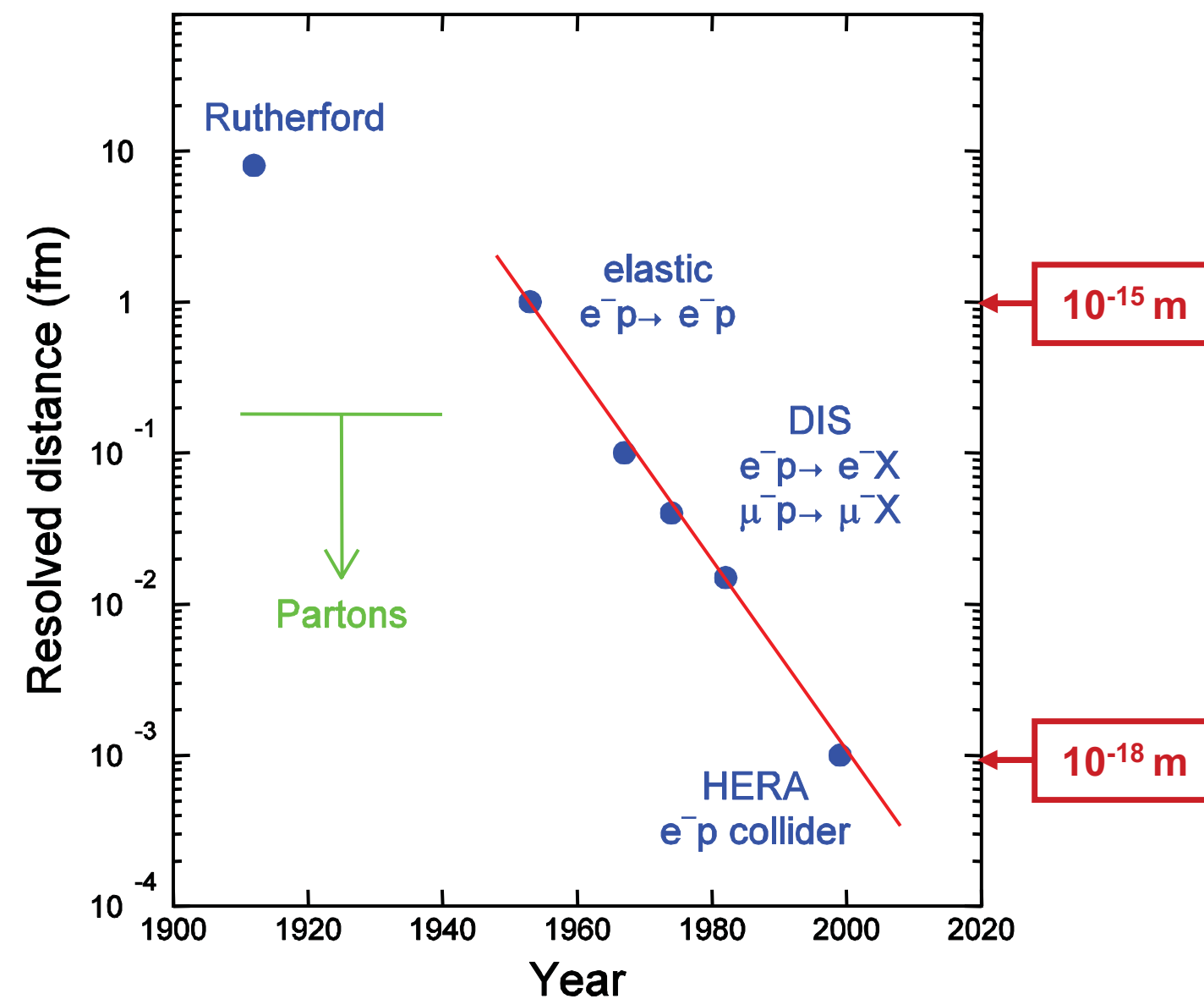


Parton distribution functions (PDFs)

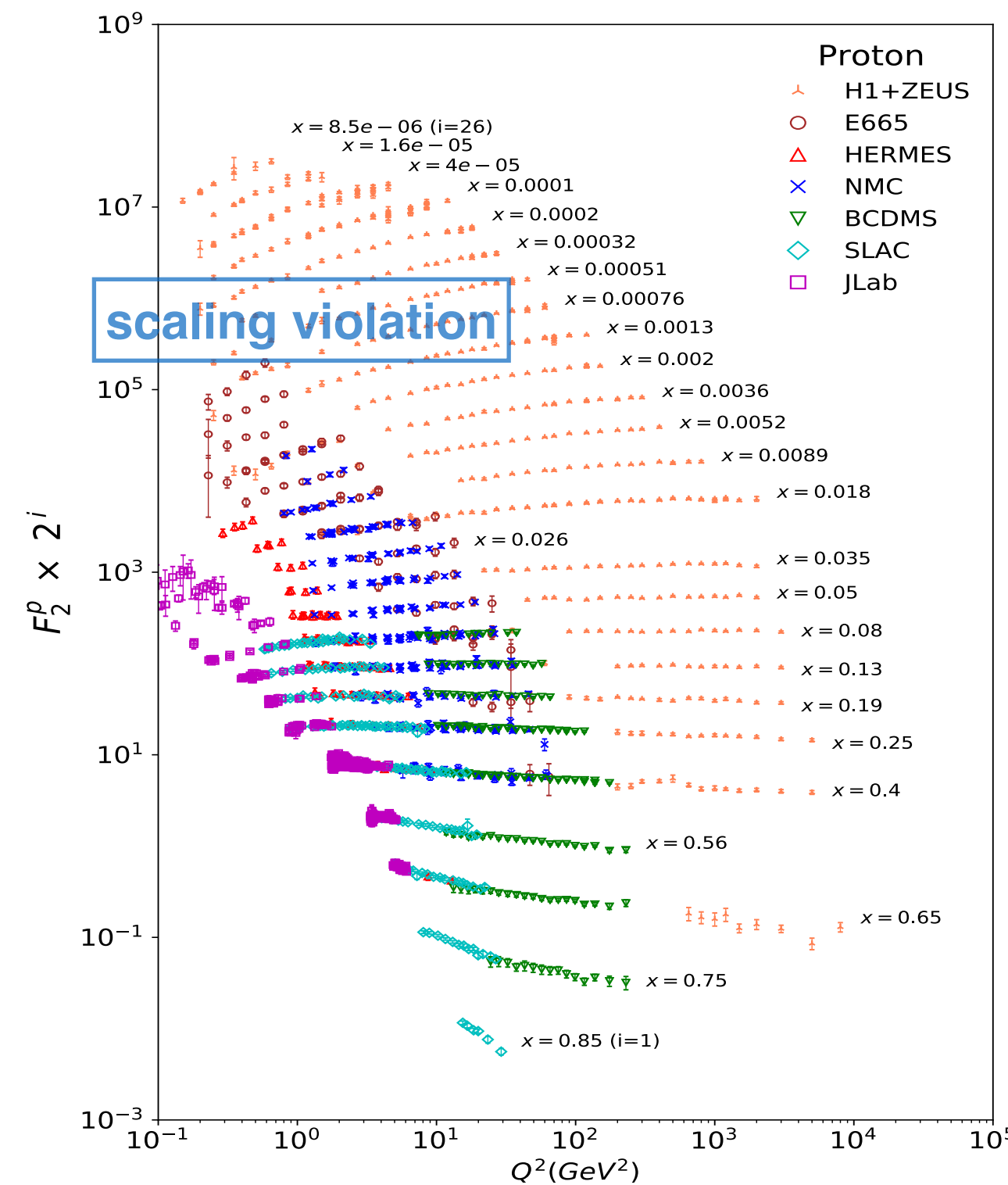
Deep inelastic scattering

- ✦ The HERA collider together with fixed-target DIS experiments provide a complete and precise scan on the nucleon structure functions or PDFs, which are successfully described by DGLAP evolution equation from perturbative QCD due to asymptotic freedoms at high-energies

Legacy of HERA ep DIS



Evolution of DIS experiments and a summary of measured F_2



factorization of structure functions in QCD

$$F_2(x, Q^2) = \sum_{i=q, \bar{q}, g} \int_0^1 d\xi C_2^i(x/\xi, Q^2/\mu_r^2, \mu_f^2/\mu_r^2, \alpha_s(\mu_r^2)) \times f_{i/h}(\xi, \mu_f) \quad \text{[Collins, Soper, Sterman]}$$

prediction of perturbative QCD on PDFs

$$\frac{d}{d \ln Q^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q \leftarrow q} & P_{q \leftarrow g} \\ P_{g \leftarrow q} & P_{g \leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$



DGLAP evolution

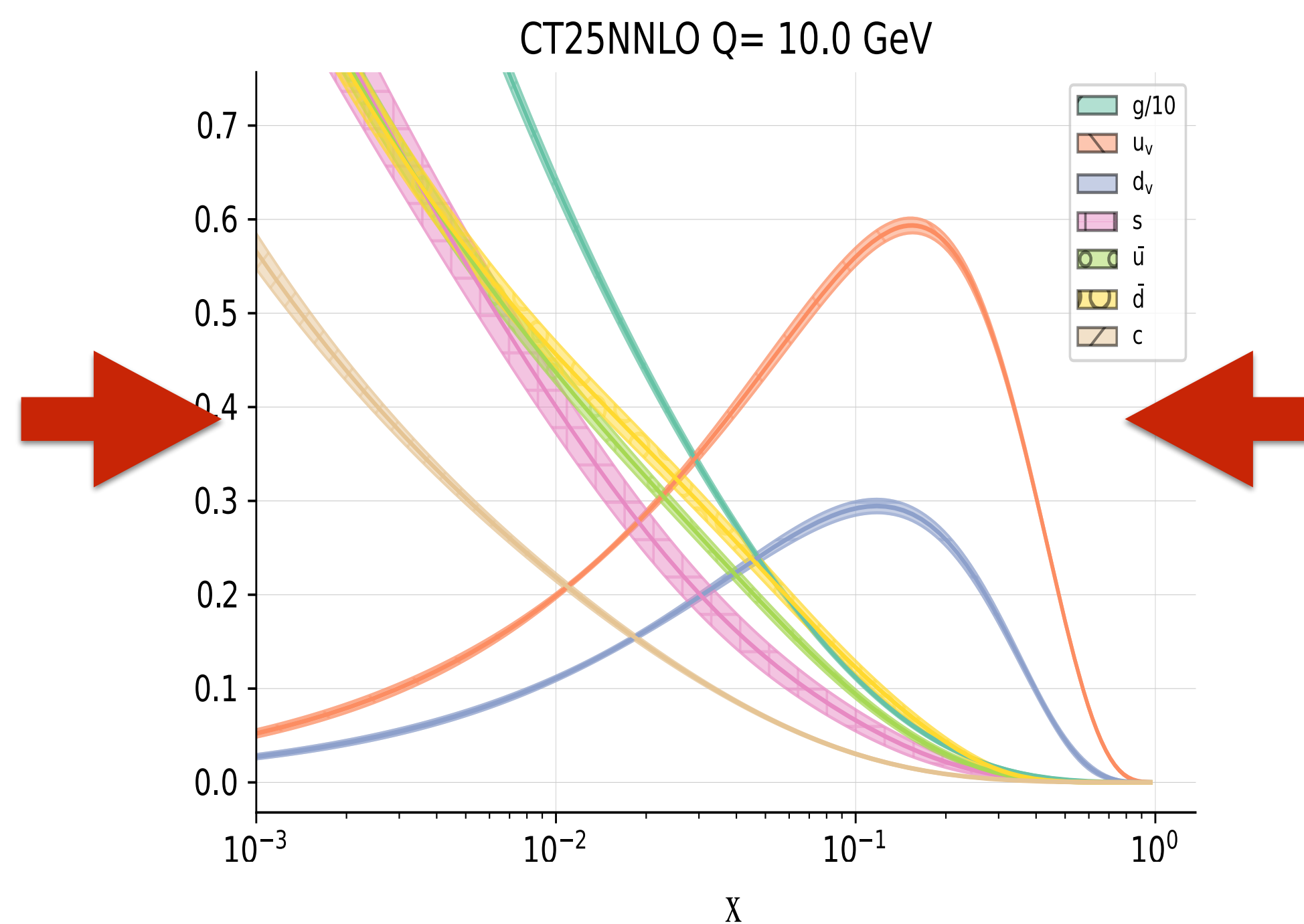
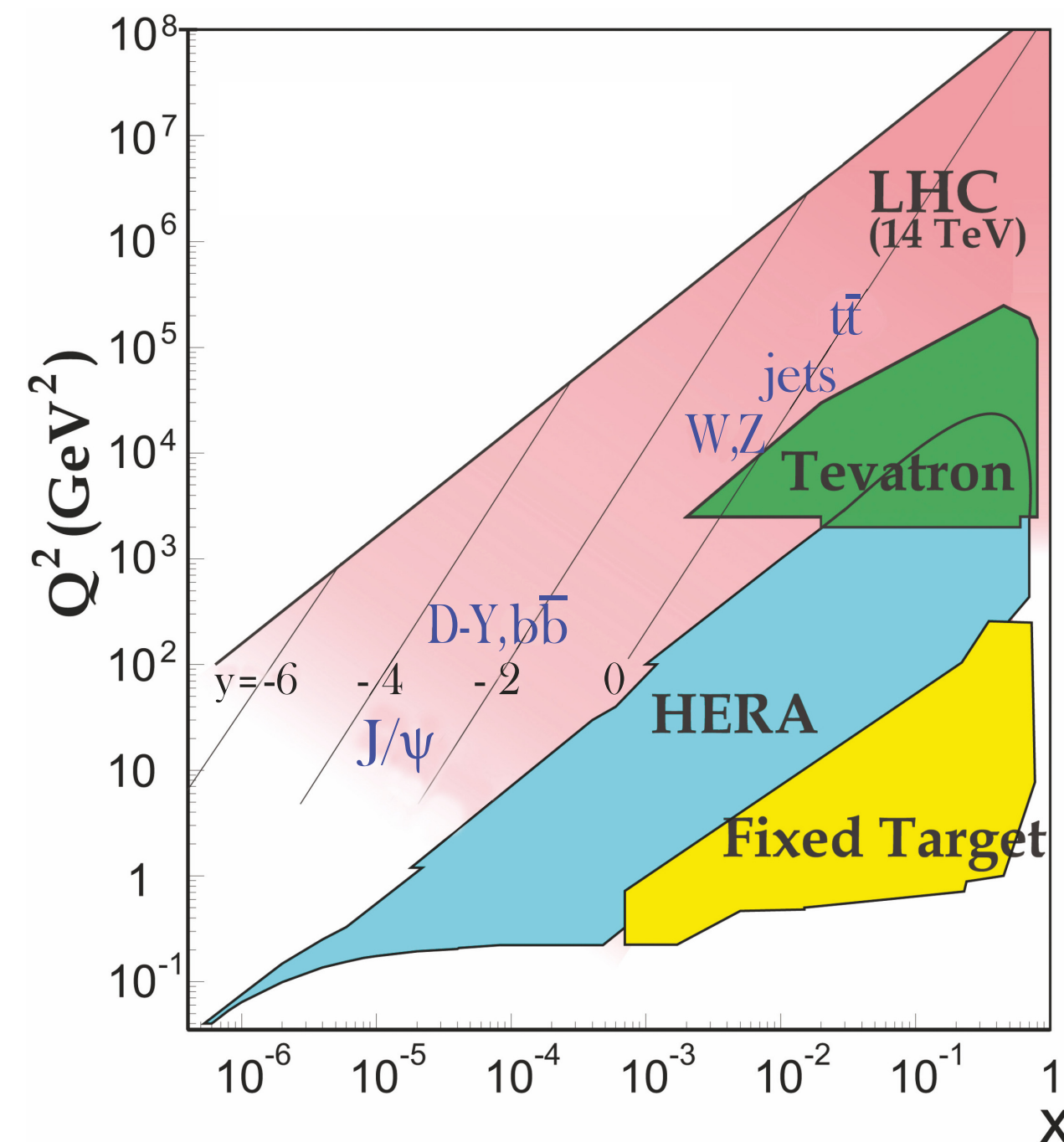


splitting kernels

Global analysis of PDFs

- ✦ Modern PDFs are usually extracted from global analysis on variety of data, e.g., DIS, Drell-Yan, jets and top quark productions at fixed-target and collider experiments, with increasing weight from LHC

[see JG, Harland-Lang, Rojo 1709.04922 (Phys.Rept) for a review]



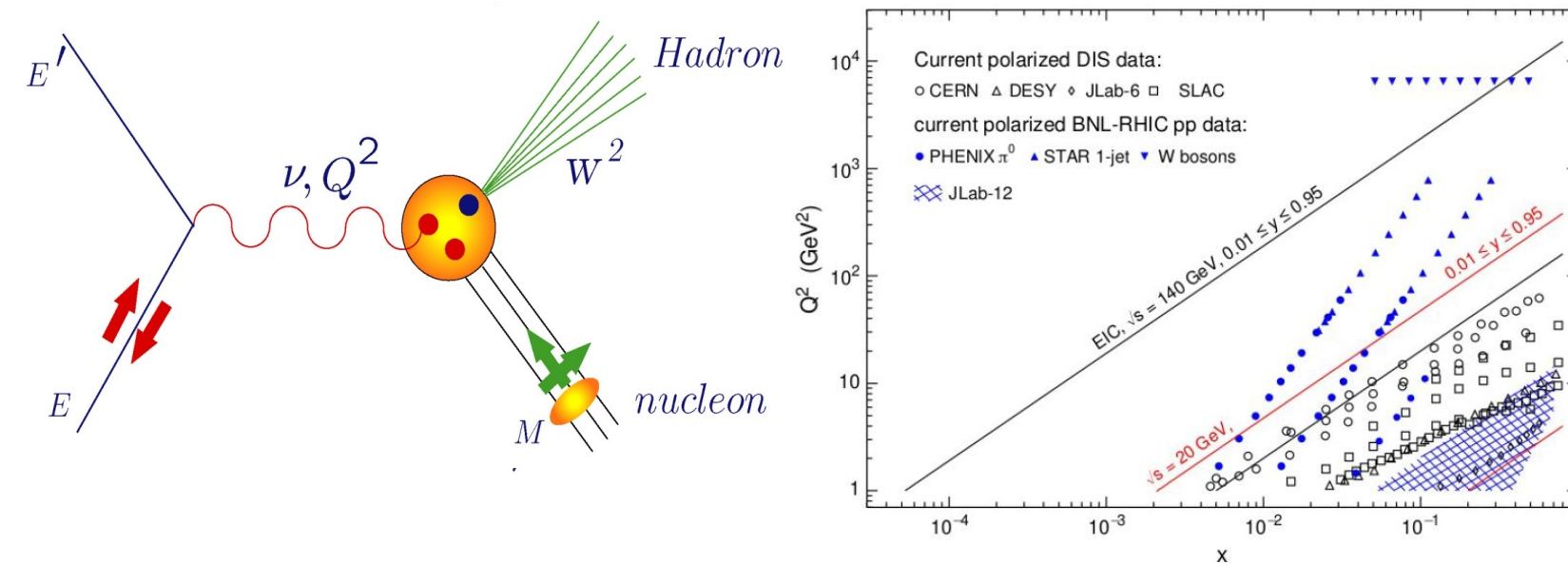
parameter variations
$\alpha_s(M_Z)$
M_c, M_b, M_t
QCD/EW corrections
nuclear corrections
EW parameters
New Physics

- ❖ diversity of the analysed data are important to ensure flavor separation and to avoid theoretical/experimental bias; possible extensions to include EW parameters and possible new physics for a self-consistent determination
- ❖ alternative approach from lattice QCD simulations, for various PDF moments or PDFs directly calculated in x-space with large momentum effective theory or pseudo-PDFs [\[2004.03543\]](#)

Polarized deep inelastic scattering

- ✦ Asymmetric cross section or structure functions measured in polarized deep inelastic scattering can access the PDFs of polarized parton inside a polarized nucleon (pPDFs), especially the longitudinal/helicity PDFs are directly related to understanding of nucleon spin

Polarized Deep Inelastic Electron Scattering



Polarized PDF

$$q = u, d, s \dots$$

helicity (longitudinal spin) distribution

$$\Delta q(x, Q^2) =$$

unpolarised distribution

$$q(x, Q^2) =$$

Quarks

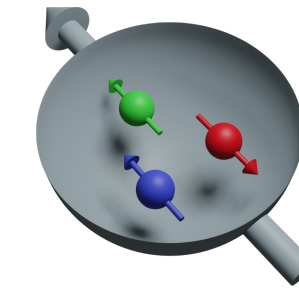
Proton	Quark
1	2/3
2	2/3
3	2/3
4	2/3
5	2/3
6	2/3
7	2/3
8	2/3
9	2/3
10	2/3
11	2/3
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93	2/3
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97	2/3
98	2/3
99	2/3
100	2/3

double-spin asymmetry

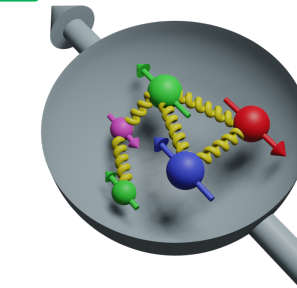
$$A_{LL}^h \equiv \frac{\sigma_{+-}^h - \sigma_{++}^h + \sigma_{-+}^h - \sigma_{--}^h}{\sigma_{+-}^h + \sigma_{++}^h + \sigma_{-+}^h + \sigma_{--}^h} \sim \frac{\Delta q(x)}{q(x)}$$

The incomplete nucleon: spin puzzle

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \underbrace{\Delta G}_{\text{green}} + \underbrace{(L_q + L_g)}_{\text{blue}} \quad \text{Ji, 96}$$

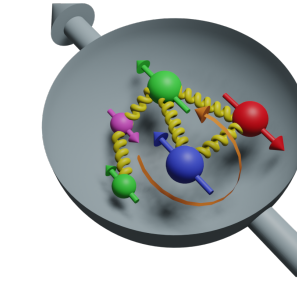


Quark helicity
Best known



Gluon helicity

Start to know

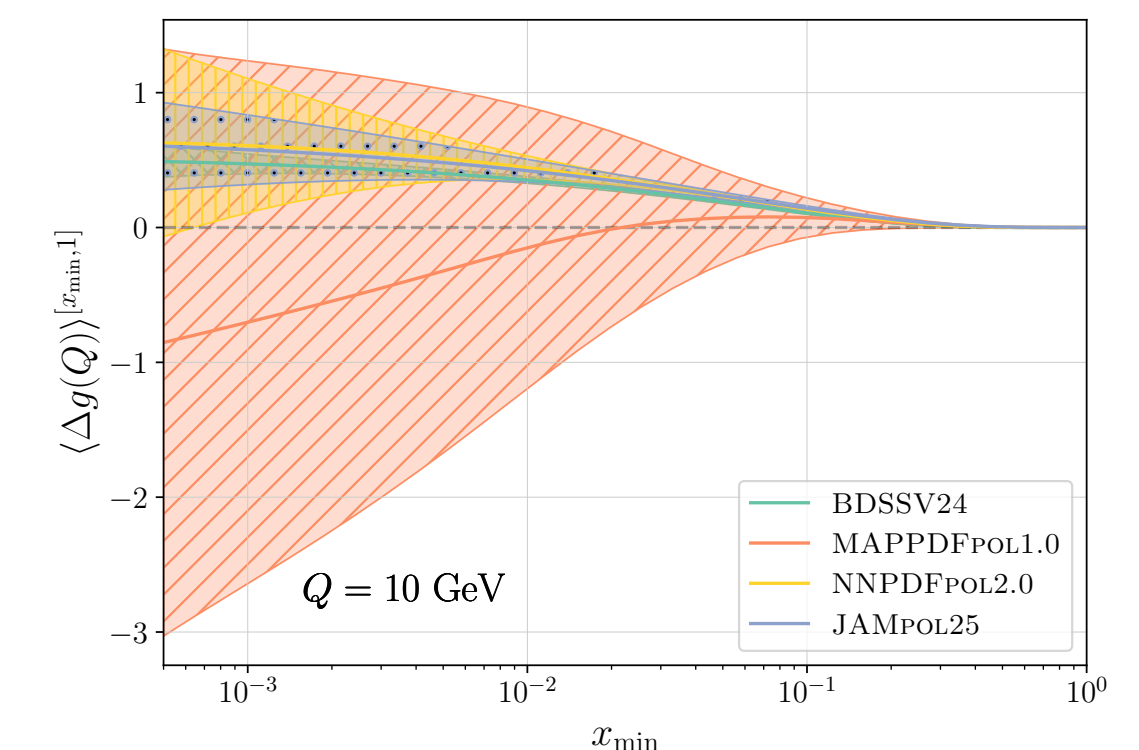
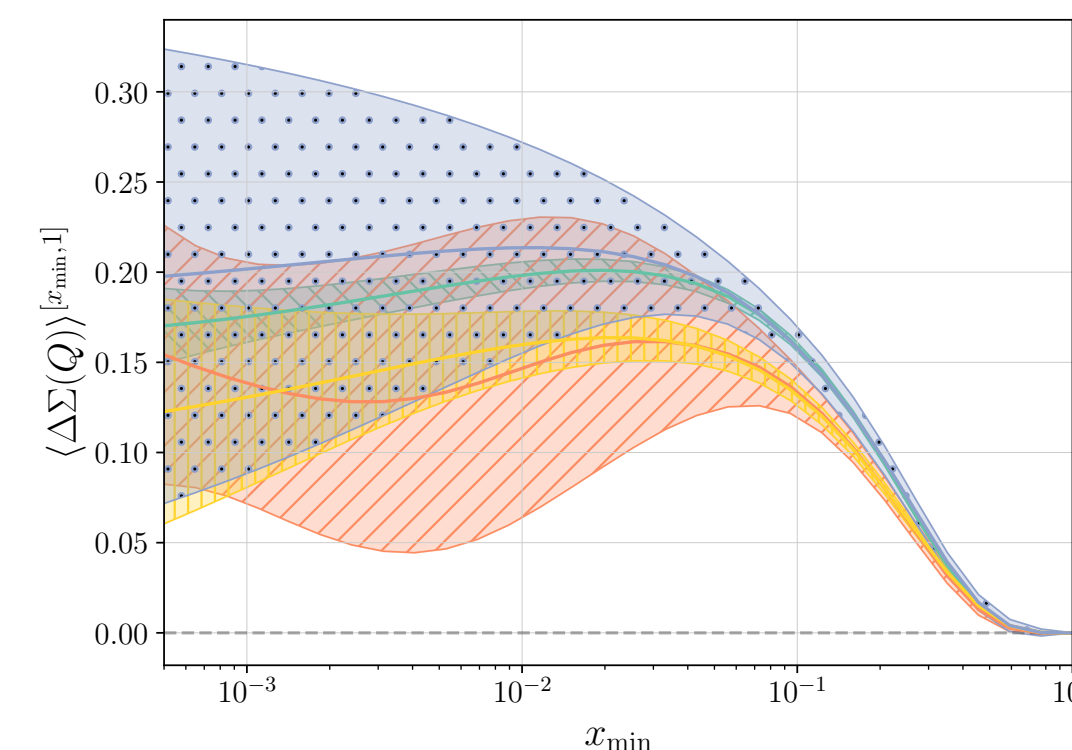


**Orbital Angular
Momentum of quarks
and gluons**
Little known

$$\frac{1}{2} \int dx (\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s})$$

$$\Delta G = \int dx \Delta g(x)$$

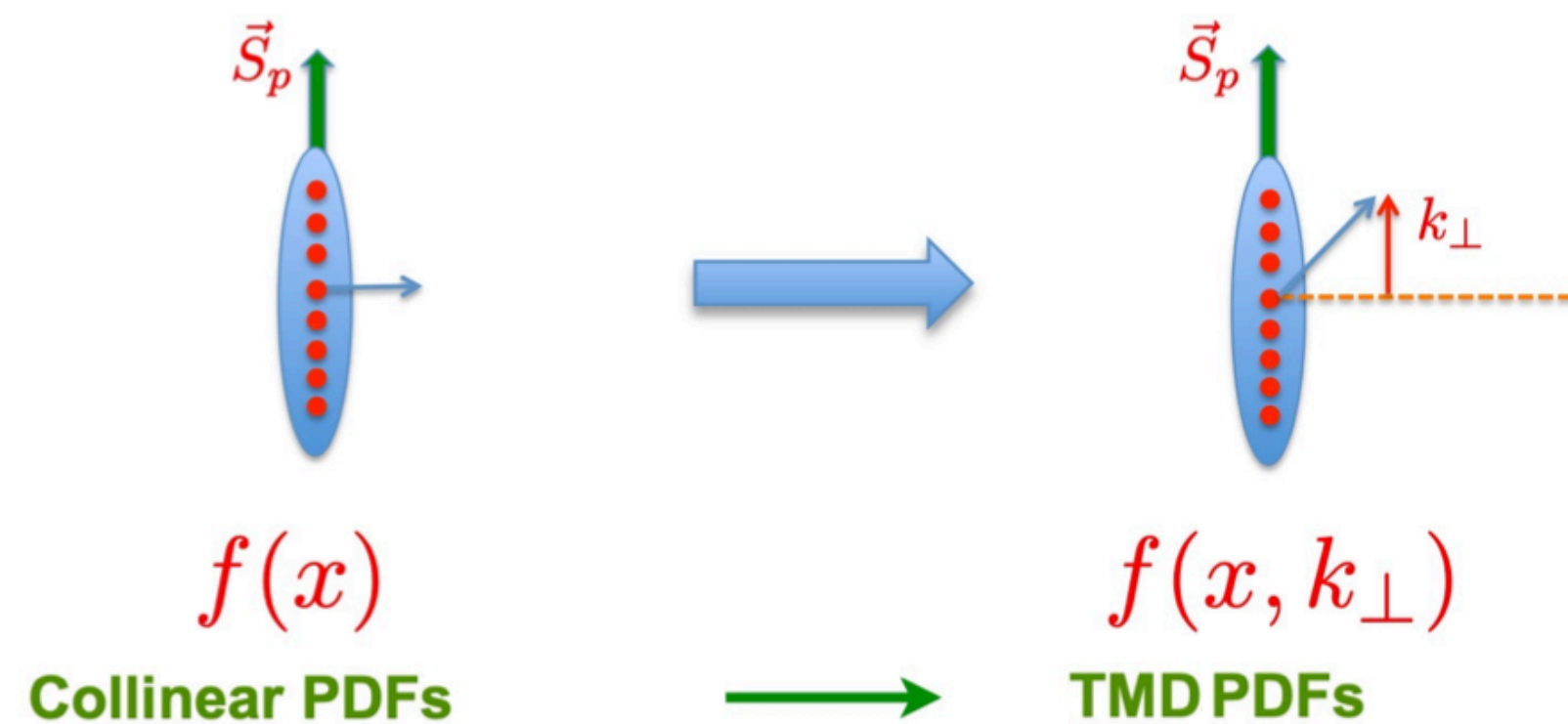
first moments of helicity PDFs [2604.04765]



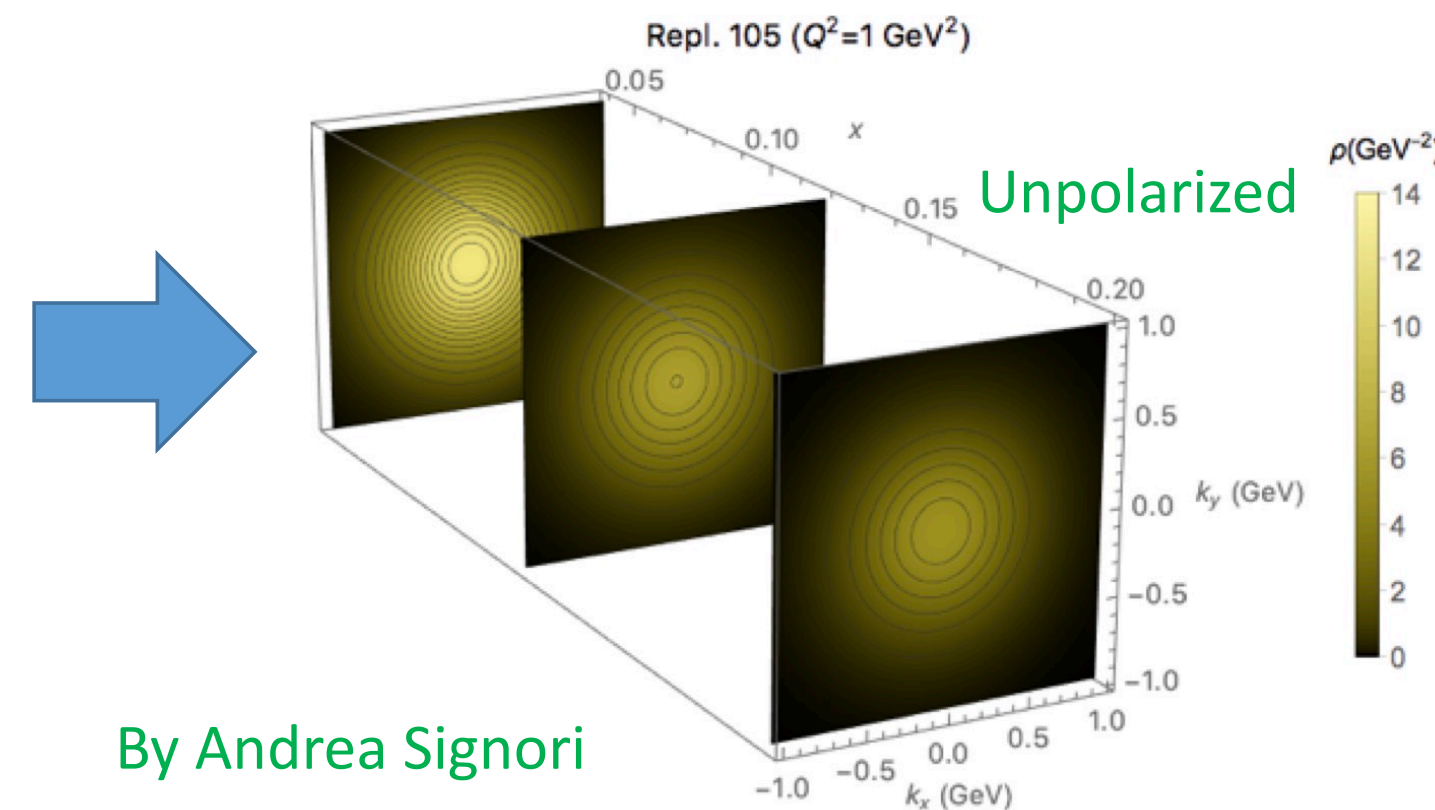
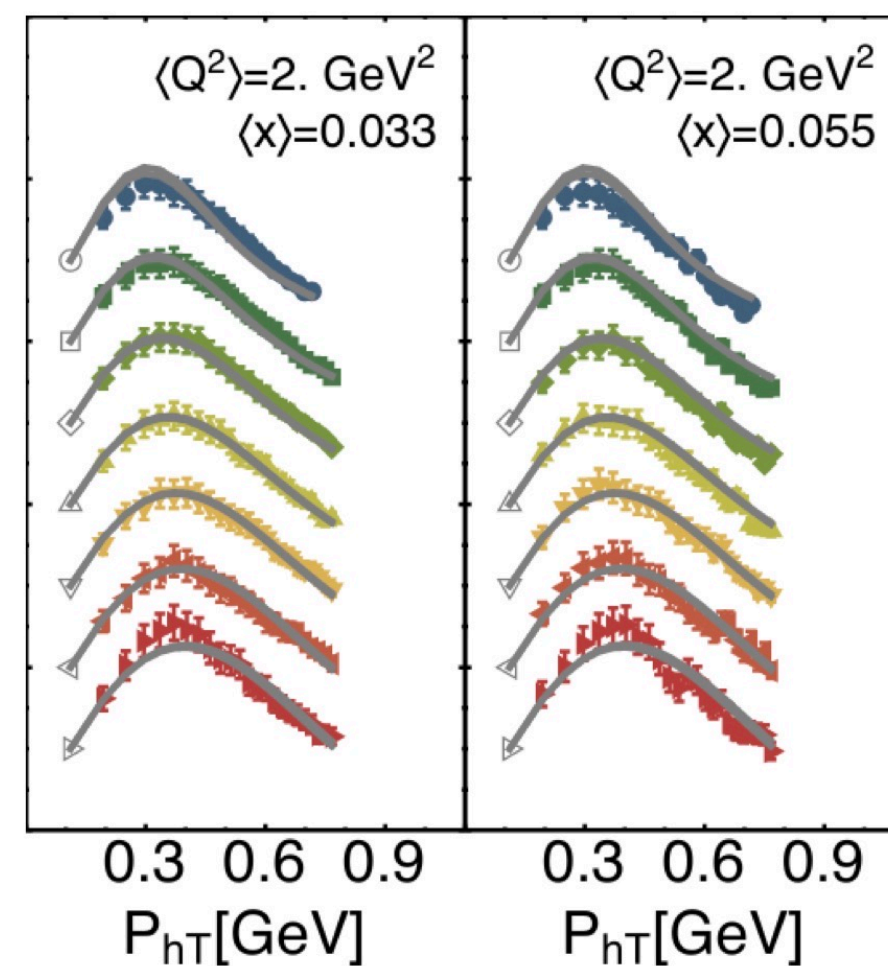
Semi-inclusive deep inelastic scattering

- ◆ Semi-inclusive DIS (SIDIS) play an essential role in study of multi-dimensional structure of nucleon and dynamics of hadronization which rely on complementary information from additional hadrons or jets apart from the scattered lepton

Transverse momentum dependent PDFs

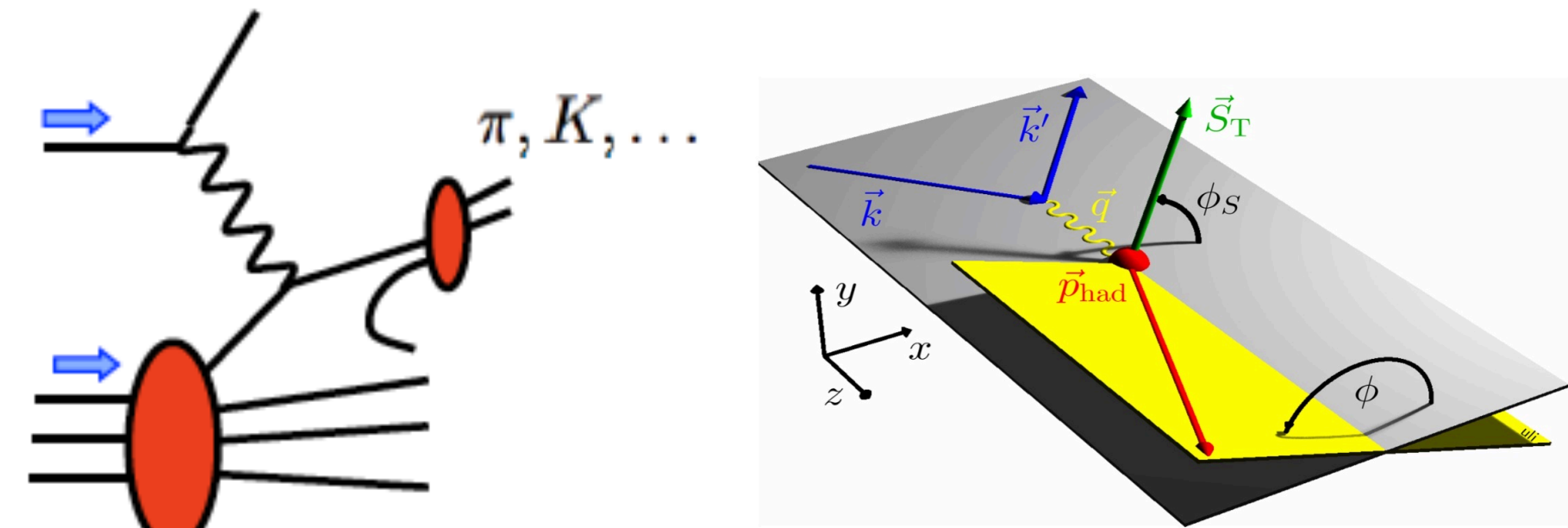


extraction of TMD PDFs

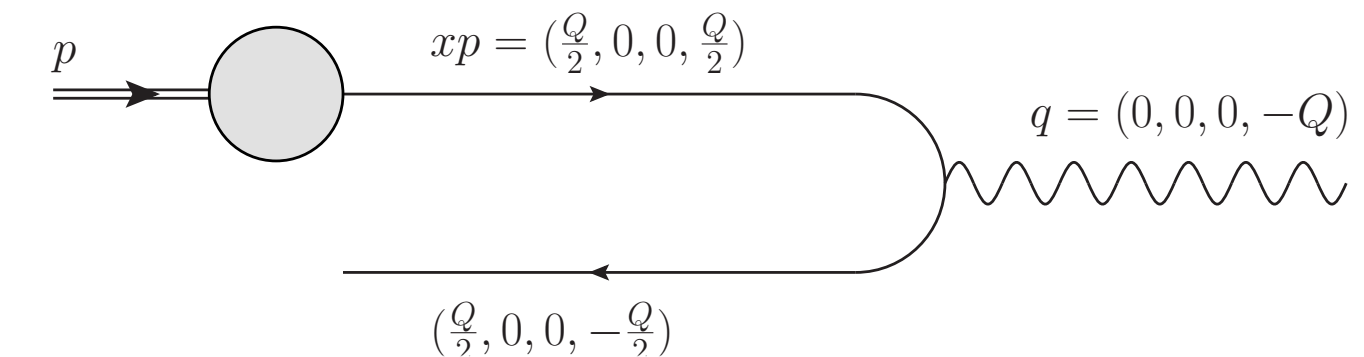


By Andrea Signori

semi-inclusive DIS



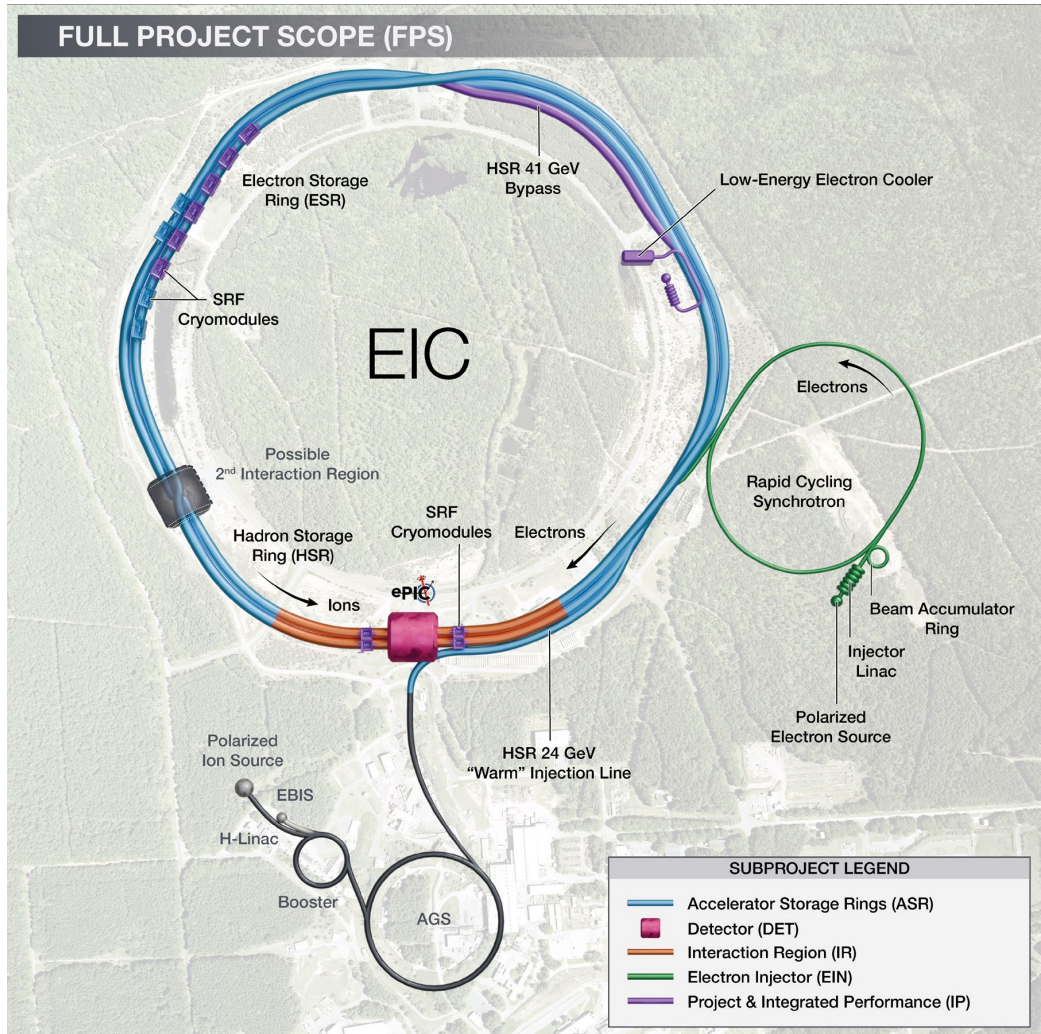
- ◆ Transverse momentum of final state hadrons provide the access to the TMD PDFs of initial partons
- ◆ tagged hadron also help with flavor separation of initial partons



Electron Ion collides

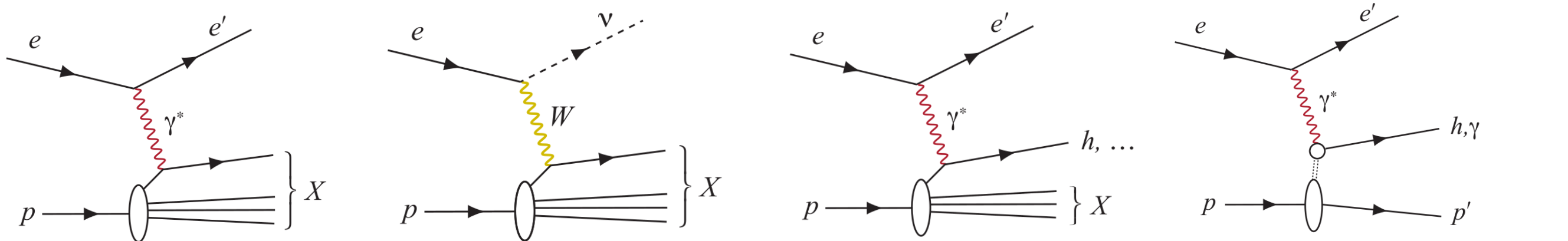
- ♦ The EIC will be the only operating particle collider in the US and will bring in precision (polarized) DIS data at center of mass energies of 20~140 GeV and luminosity of 10~100 fb⁻¹ per year; a similar collider in China (EICs) runs at center of mass energy of ~16 GeV is also planned

Preliminary Project Schedule



Experimental Processes to Access EIC Physics

DIS event kinematics - **scattered electron** or **final state particles** (CC DIS, low y)



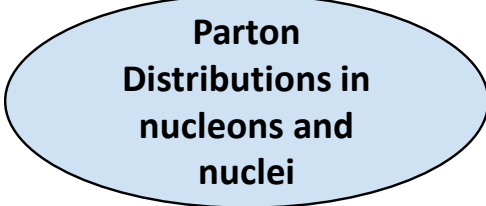
Neutral Current DIS

Charged Current DIS

Semi-Inclusive DIS

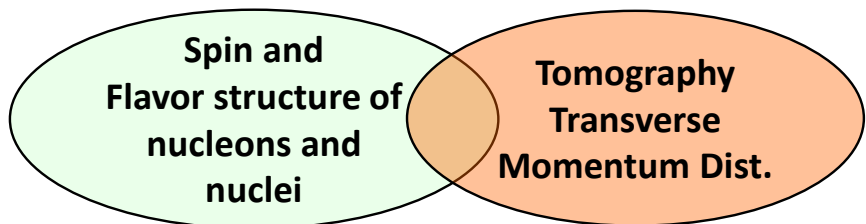
Deep Exclusive Processes

NUCLEI



SPIN

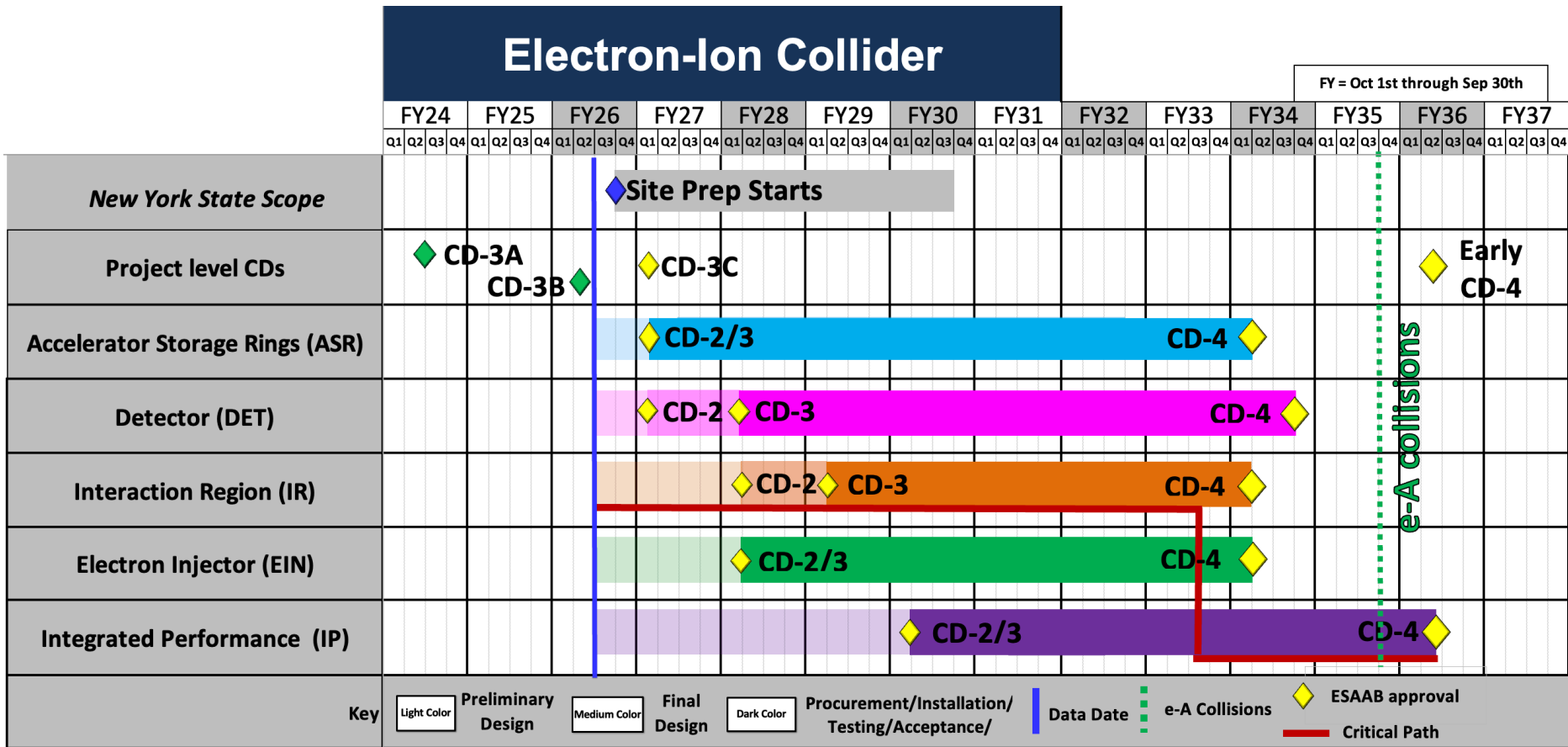
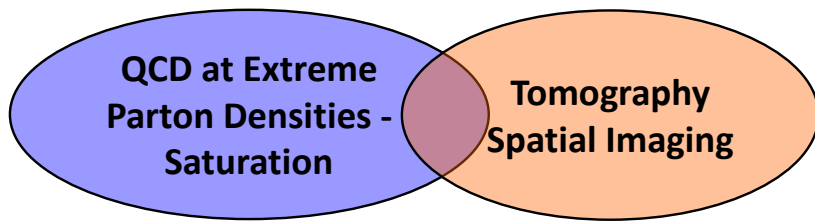
IMAGING



GLUONS

MASS

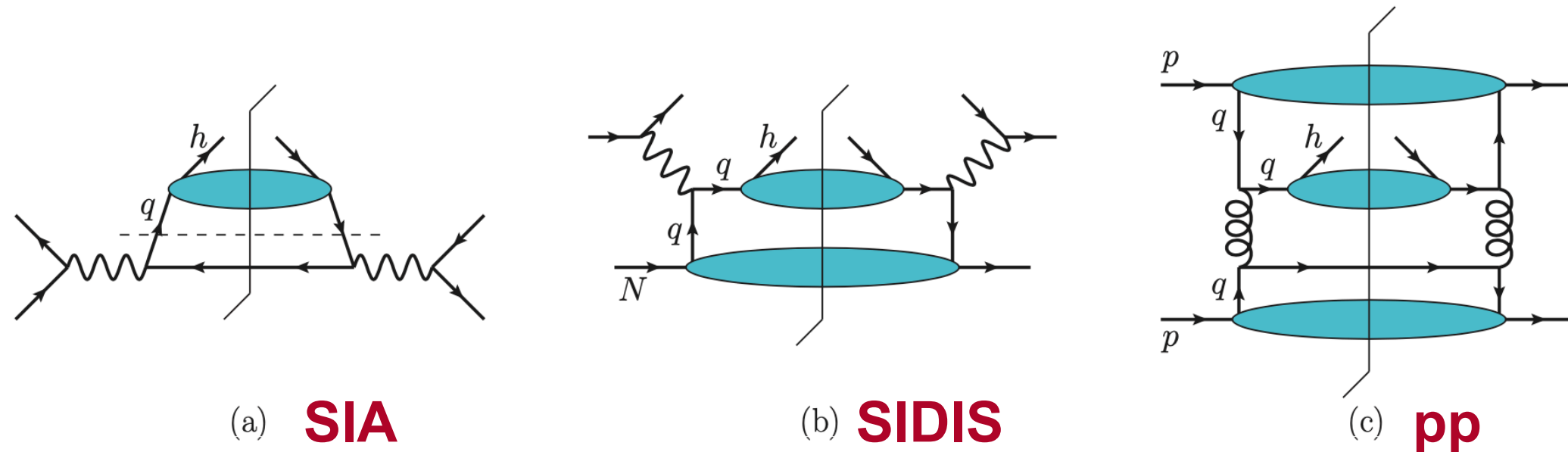
IMAGING



- ♦ High precision of the EIC calls for a timely improvements on various theoretical ingredients, especially for SIDIS

QCD collinear factorization

- ✦ QCD collinear factorization ensures universal separation of long-distance and short-distance contributions in high energy scatterings involving initial/final state hadrons, and enables predictions on cross sections



- ✦ coefficient functions, hard scattering; infrared (IR) safe, calculable in pQCD, independent of the hadron
- ✦ FFs/PDFs, reveal inner structure of hadrons or parton-hadron transition; NP origin, universal, e.g. DIS vs. pp collisions; fitted from data
- ✦ runnings of FFs/PDFs with scales μ_D/μ_f are governed by the DGLAP equation

$$\frac{d^3\sigma^{\ell p \rightarrow \ell h X}}{dx dy dz} = \frac{2\pi\alpha_{\text{em}}^2}{Q^2} \left(\frac{1 + (1-y)^2}{y} 2F_1^h(x, z, Q^2) + \frac{2(1-y)}{y} F_L^h(x, z, Q^2) \right)$$

$$2F_1^h(x, z, Q^2) = \sum_q e_q^2 \left(f_1^{q/p} D_1^{h/q} + \frac{\alpha_s(Q^2)}{2\pi} \left(\boxed{f_1^{q/p}} \otimes \boxed{C_1^{qq} \otimes D_1^{h/q}} + f_1^{q/p} \otimes C_1^{gq} \otimes D_1^{h/g} + f_1^{g/p} \otimes C_1^{qg} \otimes D_1^{h/q} \right) \right),$$

The focus of this talk!

unpolarized collinear FFs, operator definition

$$D_1^{h/q}(z) = \frac{z}{4} \sum_X \int \frac{d\xi^+}{2\pi} e^{ik^-\xi^+} \text{Tr} \left[\langle 0 | \mathcal{W}(\infty^+, \xi^+) \psi_q(\xi^+, 0^-, \vec{0}_T) | P_h, S_h; X \rangle \times \langle P_h, S_h; X | \bar{\psi}_q(0^+, 0^-, \vec{0}_T) \mathcal{W}(0^+, \infty^+) | 0 \rangle \gamma^- \right].$$

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^{e^+e^- \rightarrow hX}}{dz} = \frac{1}{\sum_q e_q^2} (2F_1^h(z, Q^2) + F_L^h(z, Q^2))$$

$$2F_1^h(z, Q^2) = \sum_q e_q^2 \left(D_1^{h/q}(z, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \left(C_1^q \otimes D_1^{h/q} + C_1^g \otimes D_1^{h/g} \right)(z, Q^2) \right)$$

$$\frac{d}{d \ln \mu^2} D_1^{h/i}(z, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_z^1 \frac{du}{u} P_{ji}(u, \alpha_s(\mu^2)) D_1^{h/j} \left(\frac{z}{u}, \mu^2 \right)$$

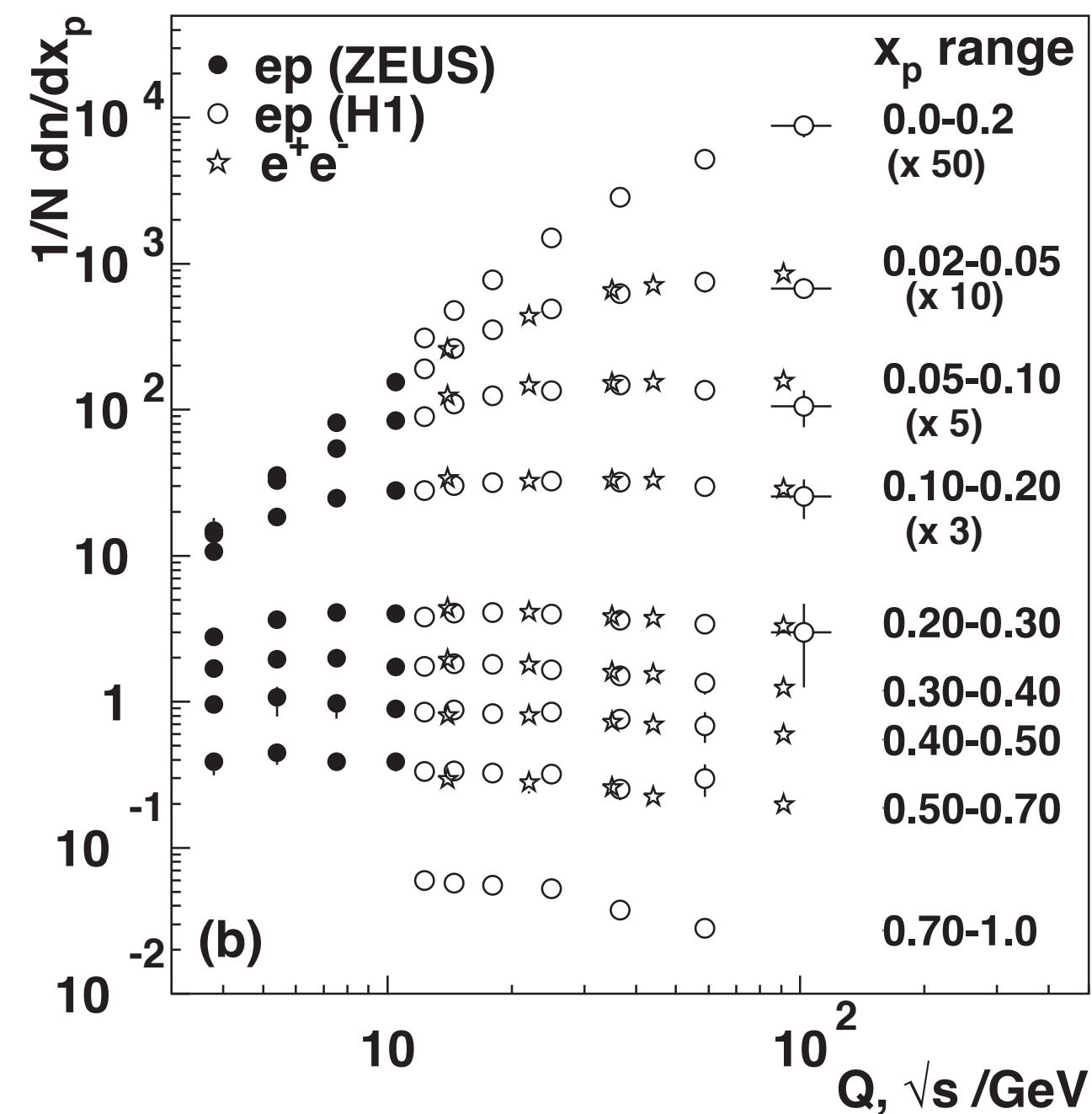
[Collins, Soper, Sterman]

✦ 2. NPC23 analysis on FFs of light charged hadrons

Global analysis of FFs

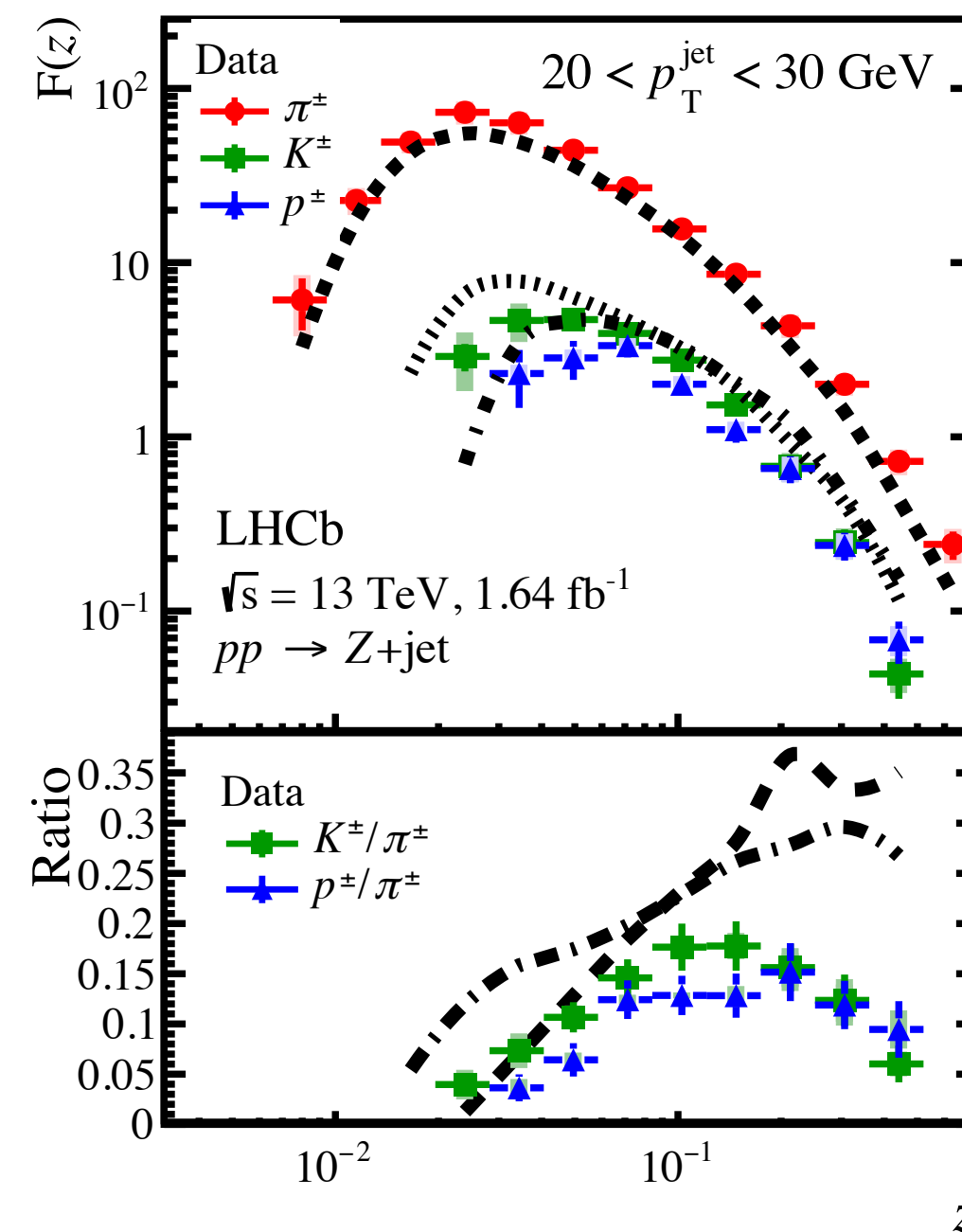
- Measurements are available from colliders SLAC, LEP, HERA, RHIC, LHC and fixed-target HERMES, COMPASS experiments for various light charged hadrons as well as neutral hadrons; many groups provide phenomenological FFs from global analysis at NLO/aNNLO in QCD

single incl. production of unidentified charged hadrons (SIA & SIDIS)



[Particle data group]

jet fragmentation to light charged hadrons (LHCb)



[2208.11691]

$$z = \frac{\mathbf{p}_{\text{had}} \cdot \mathbf{p}_{\text{jet}}}{|\mathbf{p}_{\text{jet}}|^2}, \quad F(z) = \frac{1}{N_{Z+\text{jet}}} \frac{dN_{\text{had}}(z)}{dz}$$

global analysis

[1607.02521 for a review]

- major groups/families include BKK, AKK, HKNS, DSS, NNFF, MAPFF, JAM, SAK etc.
- mostly done at NLO in QCD since exact NNLO coefficient functions only known recently for SIDIS
- different determination can be quite different due to selection of data sets as well as theory treatments, not converge as well as the case of PDF fits

pinning down uncertainties of FFs is crucial for EIC(c) physics !

The first NPC23 analysis of FFs

- ✦ Establishing a new framework on global analysis of fragmentation functions to identified charged hadrons, including charged pion, kaon and proton, using most recent data from SIA, SIDIS, and pp collisions

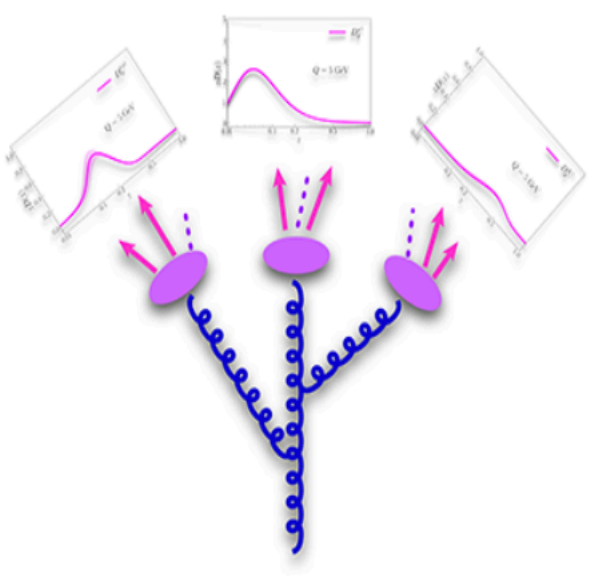
parametrization of FFs to charged pion/kaon/
proton at an initial scale ($Q=5$ GeV):

$$zD_i^h(z, Q_0) = z^{\alpha_i^h} (1-z)^{\beta_i^h} \exp \left(\sum_{n=0}^m a_{i,n}^h (\sqrt{z})^n \right)$$

parton-to- π^+	avored	α	β	a_0	a_1	a_2	d.o.f.
u	Y						5
$d \simeq u$	Y	-	-		-	-	1
$\bar{u} = \bar{d}$	N					x	4
$s = \bar{s} \simeq \bar{u}$	N	-				x	3
$c = \bar{c}$	N					x	4
$b = \bar{b}$	N					x	4
g	N		F				4

- ✦ a **joint determination** of FFs to charged pion, kaon and proton (via ratios or sum) at NLO in QCD (63 parameters) including estimation of uncertainties with Hessian sets
- ✦ apply a **strong selection criteria** on the kinematics of fragmentation processes to ensure validity of LT factorization and perturbative calculations ($E_h/p_{T,h} > 4$ GeV, and $z > 0.01$)
- ✦ including **theory uncertainties** (residual scale variations) into the covariance matrix
- ✦ use fast interpolation techniques as in FMNLO for calculations of cross sections which largely increase efficiency of the global fit

[JG, Liu, Shen, Xing, Zhao, 2401.02781 (PRL), 2407.04424 (PRD Editors' suggestion)]



EDITORS' SUGGESTION Global analysis of fragmentation functions to charged hadrons with high-precision data from the LHC
12 December, 2024

This paper presents a global analysis of fragmentation functions for light charged hadrons, which describe the production of these states from partons. The fit includes for the first time jet data from proton-proton collisions at the LHC. The authors find good agreement with data, but note significant differences with previous work and the need for careful experimental definitions for future efforts.

Jun Gao *et al.*
[Phys. Rev. D **110**, 114019 \(2024\)](#)

Impact of the LHC data

- ✦ For the first time the jet fragmentation data from LHC have been incorporated into the global analysis of FFs to light charged hadrons, including from processes of incl. jet, dijet, Z or photon tagged jet productions, due to the development of FMNLO

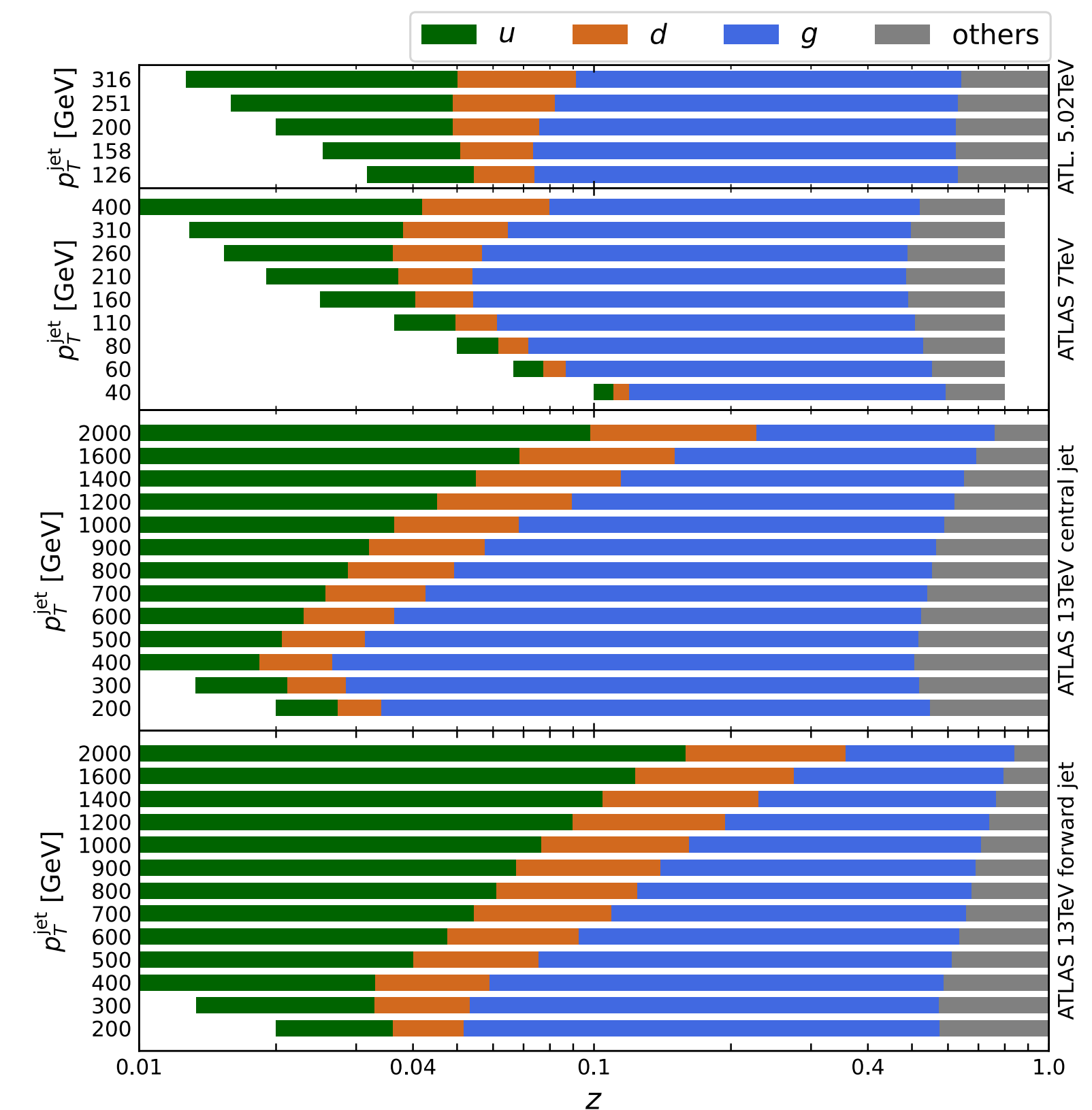
LHC measurements for hadron inside jet measurements (jet fragmentation)

exp.	$\sqrt{s}$ (TeV)	luminosity	hadrons	final states	R_j	cuts for jets/hadron	observable	N_{pt}
ATLAS[60]	5.02	25 pb ⁻¹	$h^\pm$	$\gamma + j$	0.4	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{dp_{T,h}}$	6
CMS[61]	5.02	27.4 pb ⁻¹	$h^\pm$	$\gamma + j$	0.3	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	4
ATLAS[62]	5.02	260 pb ⁻¹	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{3}{4}\pi$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{dp_{T,h}}$	9
CMS[63]	5.02	320 pb ⁻¹	$h^\pm$	$Z + h$	no jet	$\Delta\phi_{h,Z} > \frac{7}{8}\pi$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{dp_{T,h}}$	11
LHCb[64]	13	1.64 fb ⁻¹	$\pi^\pm, K^\pm, p/\bar{p}$	$Z + j$	0.5	$\Delta\phi_{j,\gamma} > \frac{7\pi}{8}, \Delta R_{h,j} < R_j$	$\frac{1}{n_Z} \frac{dN_{\text{ch}}}{d\xi}$	20
ATLAS[65]	5.02	25 pb ⁻¹	$h^\pm$	inc. jet	0.4	-	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	63
ATLAS[66]	7	36 pb ⁻¹	$h^\pm$	inc. jet	0.6	$\Delta R_{h,j} < R_j$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	103
ATLAS[67]	13	33 fb ⁻¹	$h^\pm$	dijet	0.4	$p_T^{\text{lead}}/p_T^{\text{sublead}} < 1.5$	$\frac{1}{N_{\text{jet}}} \frac{dN_{\text{ch}}}{d\xi}$	280

❖ LHC measurements on hadron inside jet provide essential inputs for u/d/g flavor separation with wide kinematic coverages, both in energy scale Q and in momentum fraction z

❖ A best-fit with good agreements to the global data sets (1370 points in total) are found, $\chi^2/N \sim 0.9$, **proven universality of FFs in different collisions at highest energies**

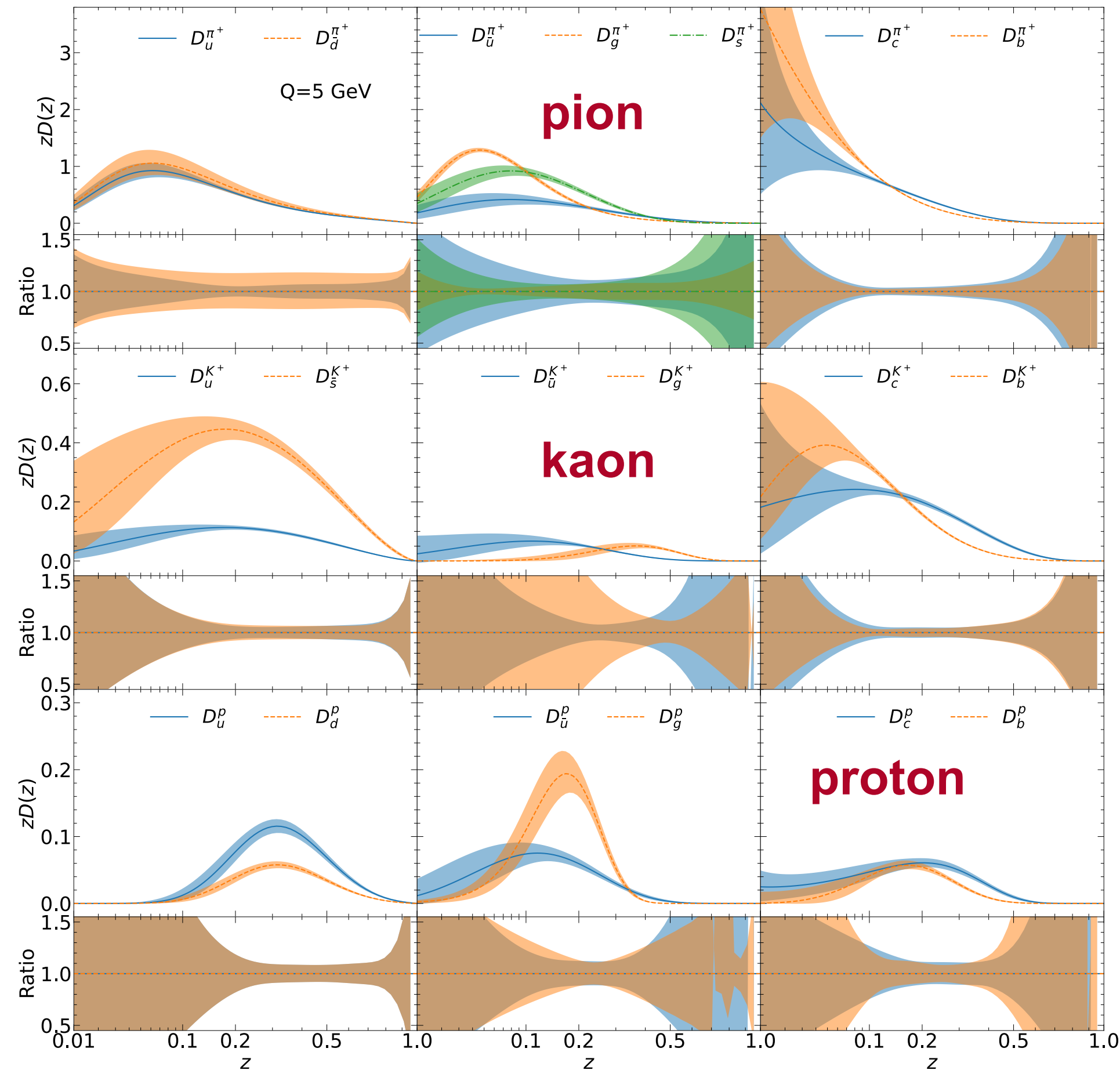
kinematic/flavor coverage (LO) for ATLAS jet fragmentation



direct probe of gluon FFs!

NPC23 FFs to charged hadrons

- ✦ We arrive at a best-fit of the charged pion, kaon and proton FFs together with 126 Hessian error FFs, two for each of the eigenvector direction; FFs are generally well constrained in the region with $z \sim 0.1-0.7$



FFs (positively charged) vs. momentum fraction

- ✦ our results show an uncertainty of 3%, 4% and 8% for FFs of gluon to pion at $z=0.05, 0.1$ and 0.3 , respectively
- ✦ similarly an uncertainty of 4%, 4% and 7% for FFs of u-quark to pion, kaon and proton at $z=0.3$, respectively
- ✦ FFs of heavy-quarks are well constrained for z between $0.1 \sim 0.5$ due to the tagged SIA events of Z-pole measurements
- ✦ high precision of gluon FFs is mostly due to the data of jet fragmentation from the LHC
- ✦ a preference for larger FFs of s quark to pion due to pulls from SIA data

NPC23 analysis at NNLO

- ✦ A first Global analysis of FFs to light charged hadrons at full NNLO with SIA and SIDIS data, especially with recent BESIII data, focusing on low momentum transfer region and test of collinear factorization

parametrization form (pi+/K+)

$$zD_i^h(z, Q_0) = z^{\alpha_i^h} (1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^m a_{i,n}^h z^{n/2}\right)$$

flavor	avored	a_0	α	β	a_1	a_2
$u = \bar{d}$	✓	✓	✓	✓	✓	✓
$d = \bar{u}$	✗	✓	✓	✓	✓	✓
$s = \bar{s}$	✗	✓	$= \alpha_d$	✓	✓	✓
$c = \bar{c}$	✗	✓	✓	✓	✓	✓
$b = \bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

flavor	avored	a_0	α	β	a_1	a_2
u	✓	✓	✓	✓	✓	✓
$\bar{s}$	✓	✓	$= \alpha_u$	$= \beta_u$	✓	✓
$s = \bar{u} = d = \bar{d}$	✗	✓	✓	✓	✓	✗
$c = \bar{c}$	✗	✓	✓	✓	✓	✓
$b = \bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

BESIII charged pion/kaon [2502.16084]

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb ⁻¹)	$N_{\text{had}}^{\text{tot}}$	N_{bkg}
2.0000	10.074	350298 ± 592	8722 ± 94
2.2000	13.699	445019 ± 668	10737 ± 104
2.3960	66.869	1869906 ± 1368	47550 ± 219
2.6444	33.722	817528 ± 905	21042 ± 146
2.9000	105.253	2197328 ± 1483	56841 ± 239
3.0500	14.893	283822 ± 533	7719 ± 88
3.5000	3.633	62670 ± 251	1691 ± 42
3.6710	4.628	75253 ± 275	6461 ± 81

[JG, Shen, Xing, Zhao, Zhou, 2502.17837 (PRL)]

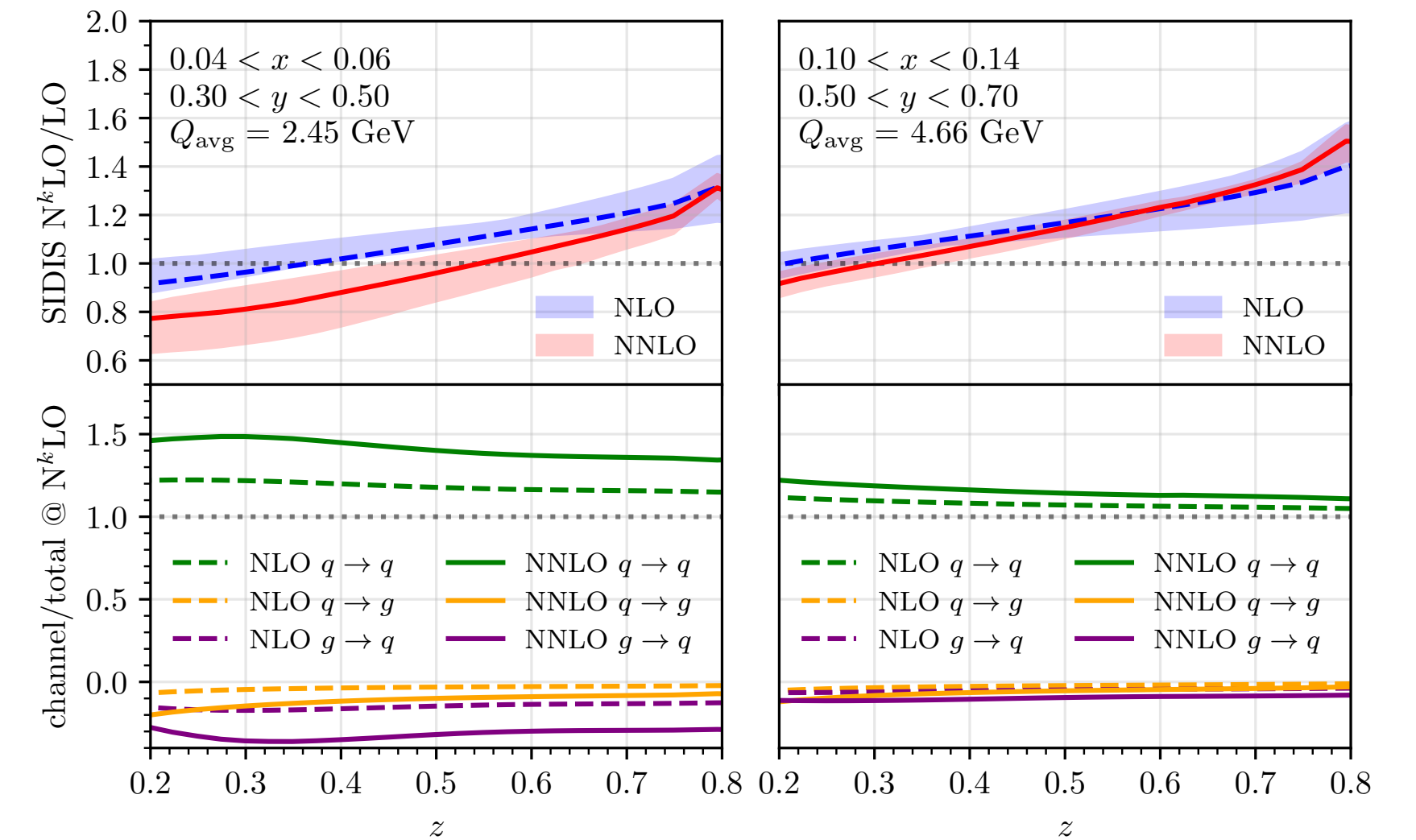
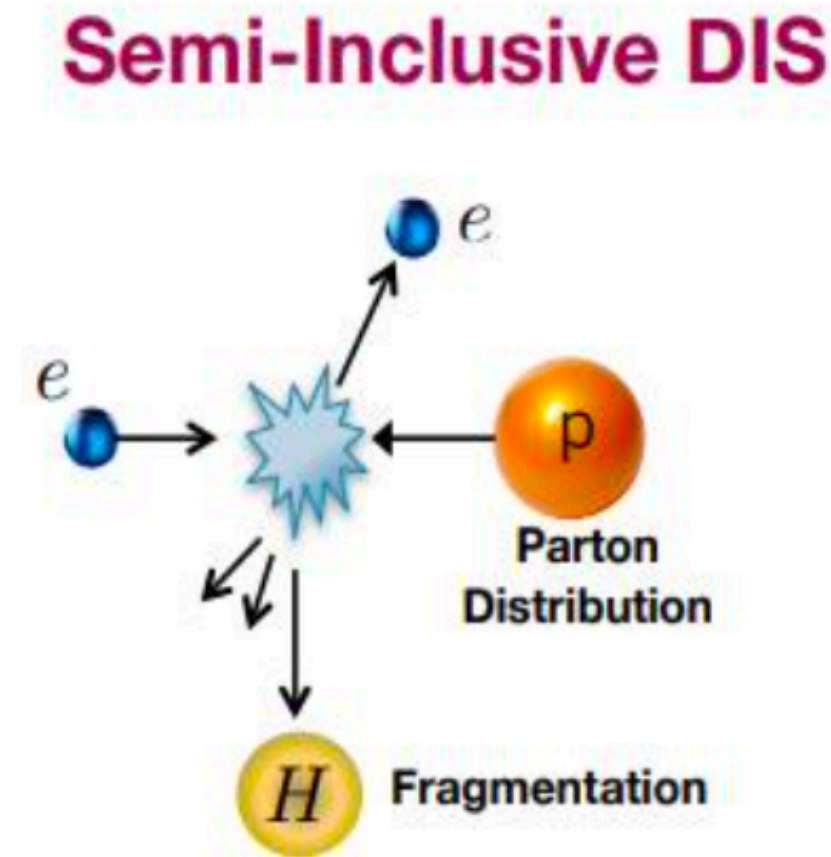
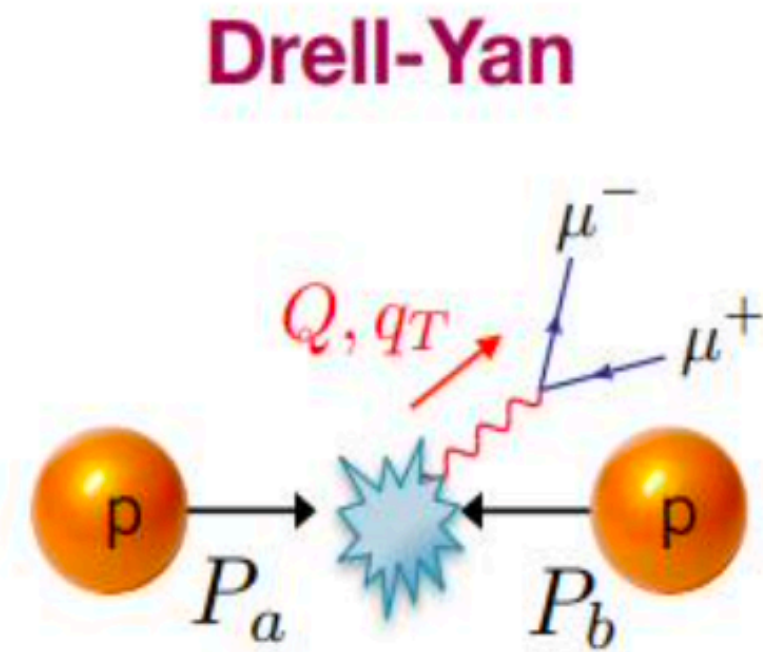
- ✦ a joint determination of FFs to charged pion, kaon at NNLO in QCD (54 parameters) with Hessian uncertainties
- ✦ parametrization at $Q_0=1.4$ GeV for light flavors and at mass threshold for heavy quarks; 3-loop DGLAP evolution from [Mitov, Moch, Vogt, Almasy; Chen, Yang, Zhu, Zhu] implemented in HOPPET
- ✦ SIDIS coefficient functions from [Bonino, Gehrmann, Stagnitto; Goyal, Moch, Pathak, Rana, Ravindran]; SIA with hadron mass corrections; include residual scale variations into the covariance matrix
- ✦ select SIA and SIDIS data with $z>0.01$, $Q>2$ GeV, and $E_h > E_{h,\text{min}}$ (0.8 GeV default) with later varied

first FFs of light hadrons at full NNLO ensuring a successful test of factorization at low Q^2 !

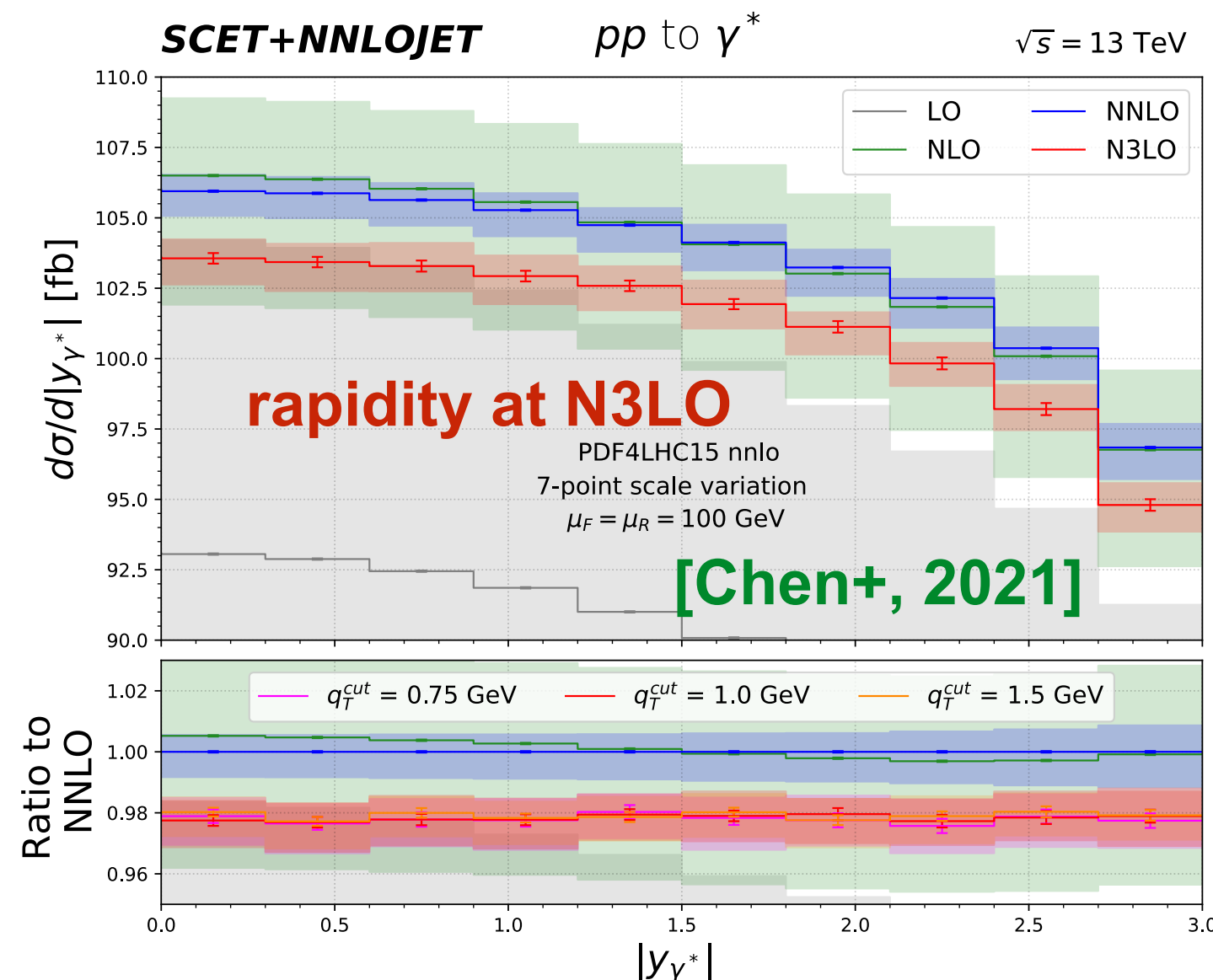
◆ 3. Precision calculations for (p)SIDIS up to N3LO

Frontier of the perturbative QCD calculations

- ✦ The LHC has pushed the pQCD precision on calculations of hard cross sections/coefficient functions to NNLO as a standard and to N3LO for a few processes, e.g. Drell-Yan production; while it is less matured for processes in ep collisions



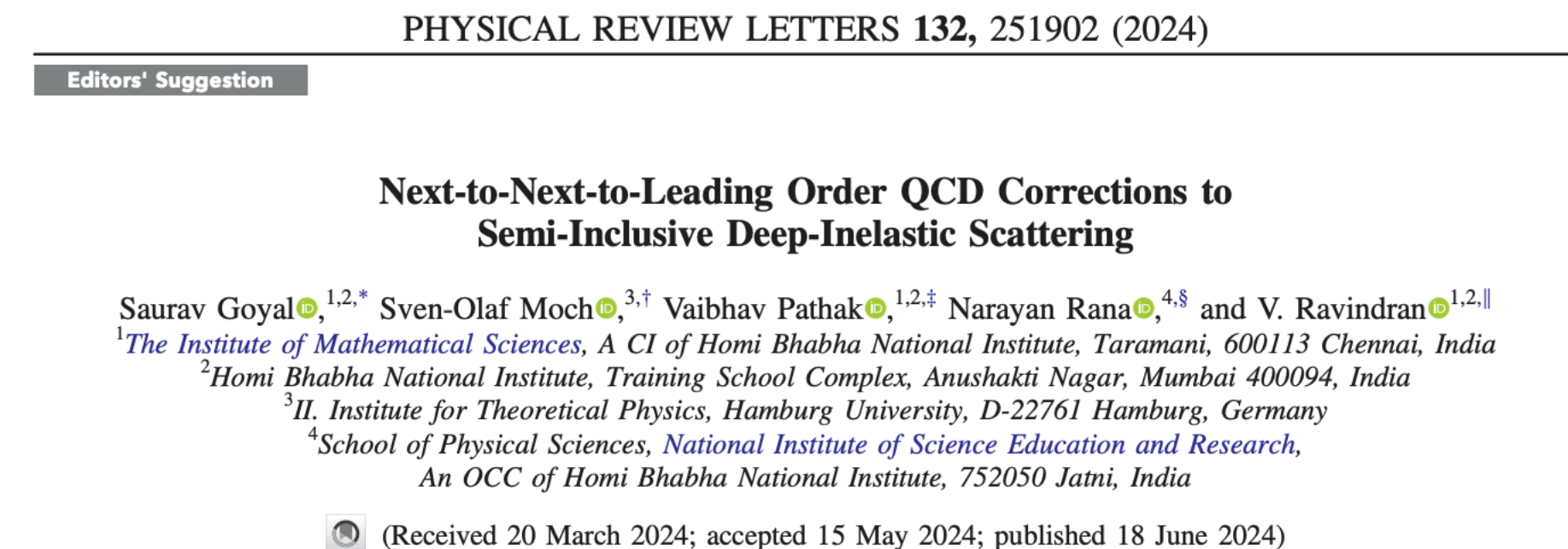
[Bonino+, Goyal+, 2024]



- ✦ N3LO corrections in Drell-Yan show unexpected behavior of being outside the NNLO uncertainty band
- ✦ NNLO corrections to SIDIS are recently calculated in an analytic form; extension to N3LO is challenging due to complications from fragmentations

The story of higher-order calculations for SIDIS

- ✦ In early **2023**, I collaborated with HuaXing on a NLO calculation for hard functions of Energy-energy correlations. Soon we realized that the same method can be applied to hadron production as well.
- ✦ Around late **2023**, we started working on the NNLO QCD calculations for SIDIS with subtraction method. Somehow we got stuck with unreasonable numerical results...
- ✦ In the middle of **2024**, two other groups finished the analytical calculation of SIDIS at NNLO both published as Editors' Suggestion in PRL



- ✦ In late **2025**, after a long cooling down, we finally find a bug in our numerical code and immediately we are able to reproduce the known NNLO corrections
- ✦ Running to **2026**, through a series of works, we now made a step forward of leveraging the perturbative precision to N3LO in QCD for (polarized) SIDIS

FMNLO (fragmentation at NLO in QCD)

- FMNLO is a program for automated and fast calculations of fragmentation cross sections of arbitrary hard processes. It is based on a hybrid scheme of phase-space slicing method and local subtraction method, accurate to NLO in QCD

- automation of fragmentation calculations for arbitrary hard processes up to NLO, within SM and BSMs via MG5_aMC@NLO
- fast convolution of partonic cross sections with FFs without repeating the time consuming MC integrations using interpolation grids
- generalizations: transverse observables, NNLO corrections

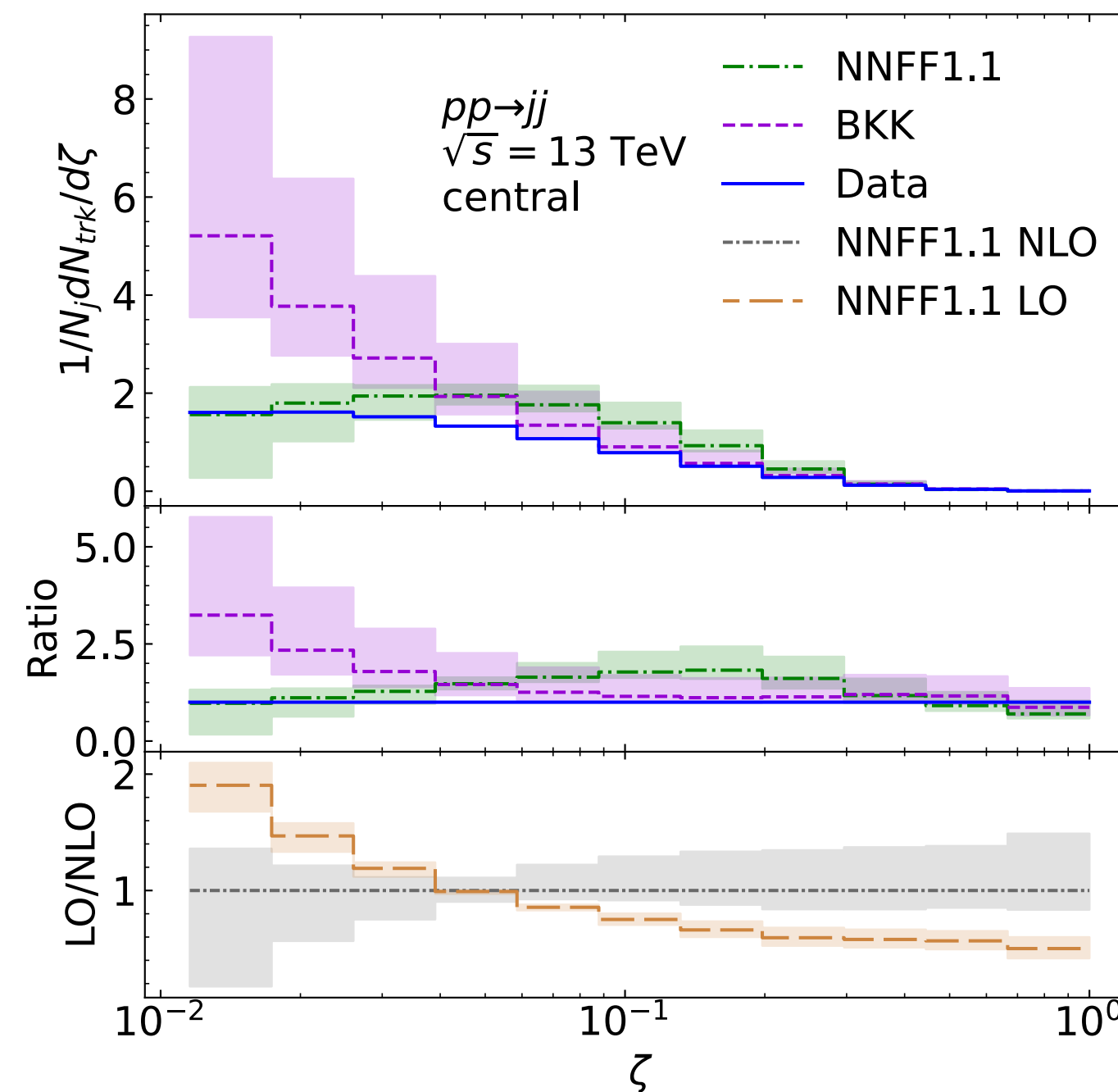
🔥 News

2024.07: 🎉🎉 [FMNLOv2.1](#) NNLO calculations are available for limited cases, SIA, decay of the Higgs boson to gluons, and SIDIS.

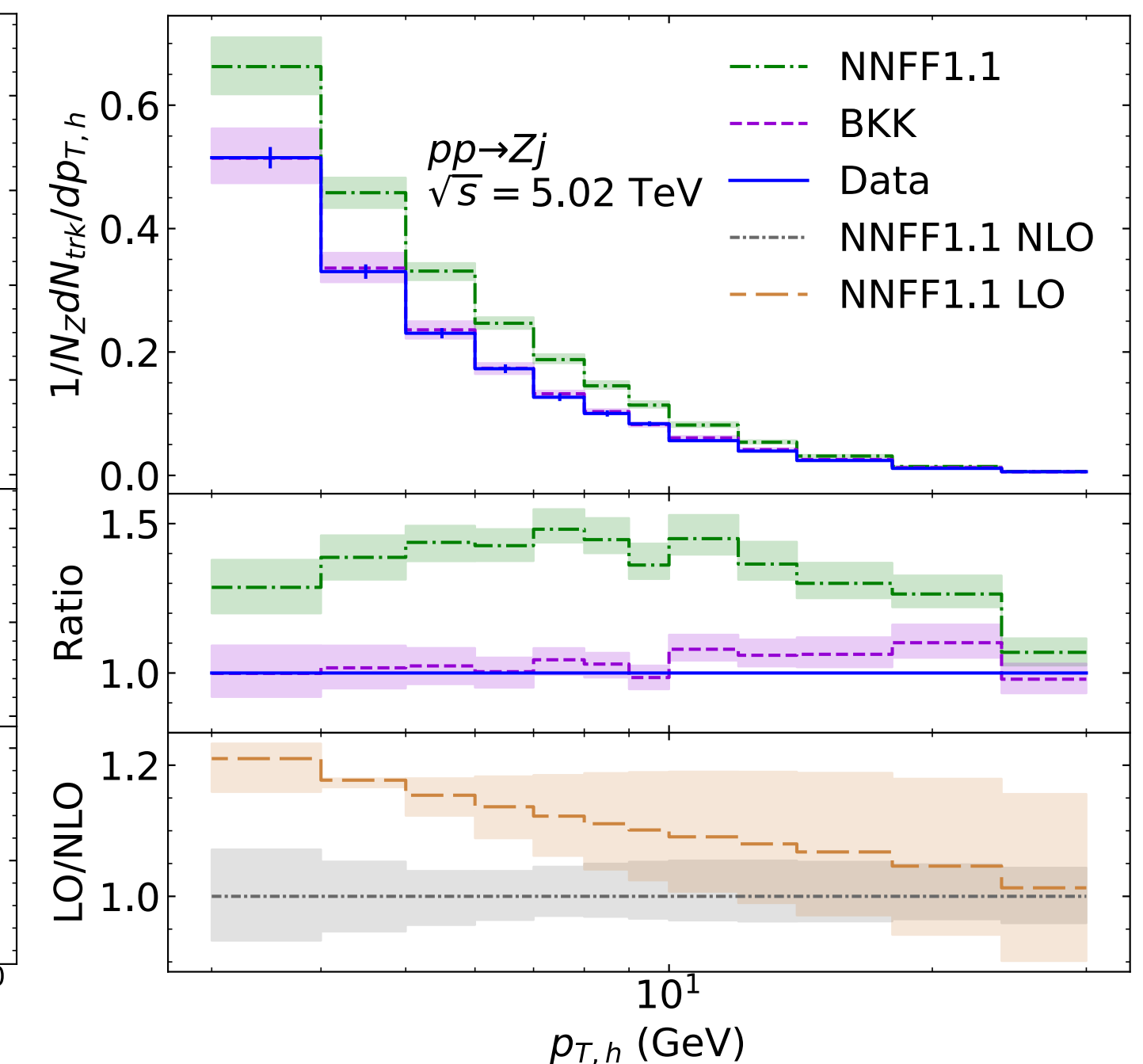
2024.07: 🎉🎉 [FMNLOv2.0](#) include a **SIDIS** module for calculations of SIDIS at NLO.

2023.05: 🎉🎉 [FMNLOv1.0](#) first release of **FMNLO** interfaced with **MG5_aMC@NLO**.

QCD inclusive dijets at LHC



Z-boson tagged jet



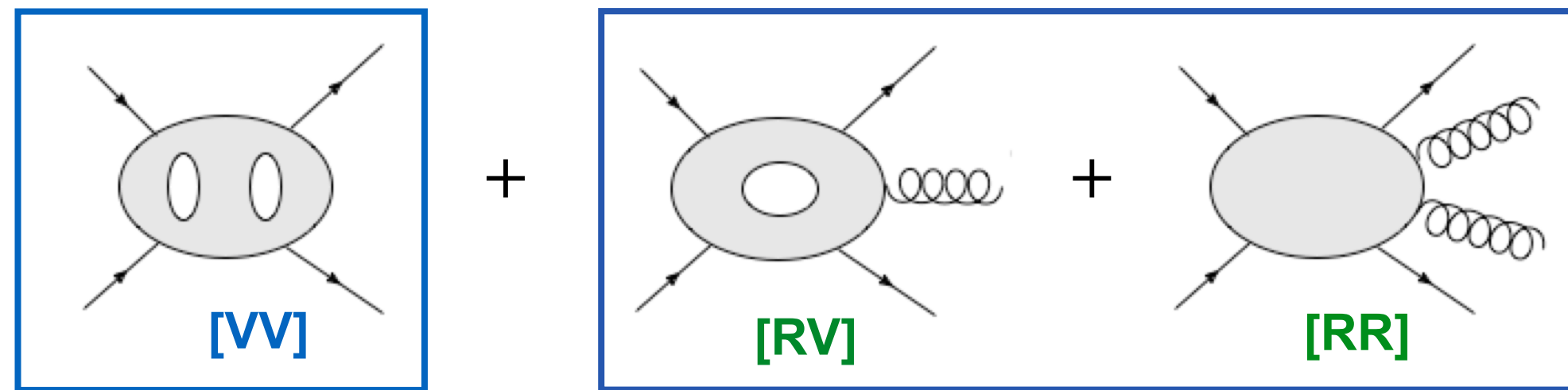
<https://fmnlo.sjtu.edu.cn/~fmnlo/>

[JG, Liu, Shen, Zhou, 2305.14620 (JHEP)]

Transverse momentum subtraction for SIDIS

- Generalizing the qT-subtraction formalism (Catani–Grazzini) from Drell-Yan to SIDIS using **hadron transverse momentum qT (Breit frame)** as the slicing variable, validated against existing analytic NNLO results

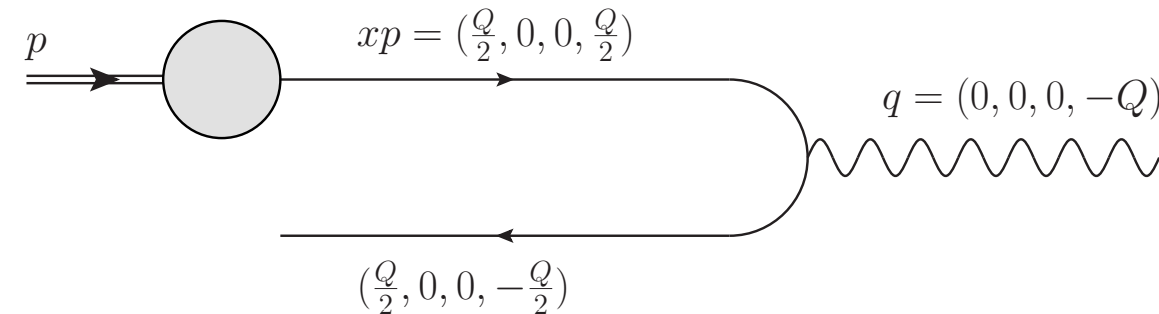
components of NNLO corrections



qT subtraction/slicing method

NNLO → NLO

$$\frac{d\sigma}{d\mathcal{O}} = \int_{q_{T,\text{cut}}}^{q_{T,\text{max}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}} + \int_0^{q_{T,\text{cut}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}}$$

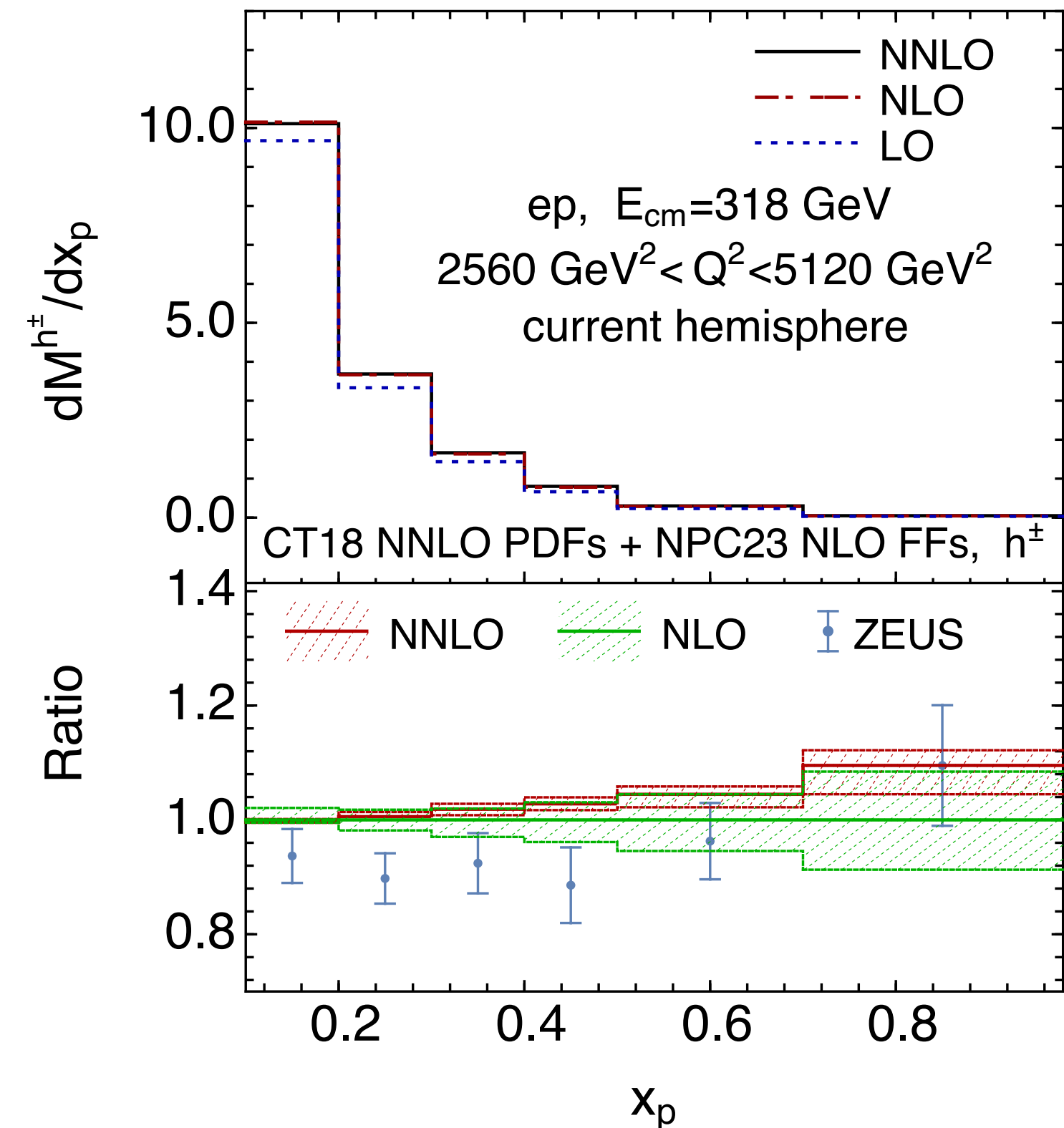


TMD factorization of the unresolved part

$$\frac{1}{\sigma_0} \frac{d\sigma_{\ell+N \rightarrow \ell'+h+X}}{d^2\vec{q}_\perp dx dy dz} \simeq \sum_q H_q(Q^2, \mu) \times \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{q}_\perp} f_1^q(x, b_\perp, \xi_0^n, \mu_0^n) \times D_1^q\left(z, \frac{b_\perp}{z}, \xi_0^{\bar{n}}, \mu_0^{\bar{n}}\right) \\ \times \prod_i e^{-2K_{\text{cusp}}^i(\mu_0^i, \mu) + A_H^i(\mu_0^i, \mu)} \times \left(\frac{\xi^i}{\mu_0^i}\right)^{A_{\text{cusp}}^i(\mu_0^i, \mu)} \times \left(\frac{\sqrt{\xi^i}}{\sqrt{\xi_0^i}}\right)^{K^i(b_\perp, \mu_0^i)},$$

[JG, Li, Zhu, Zhu, 2602.06364]

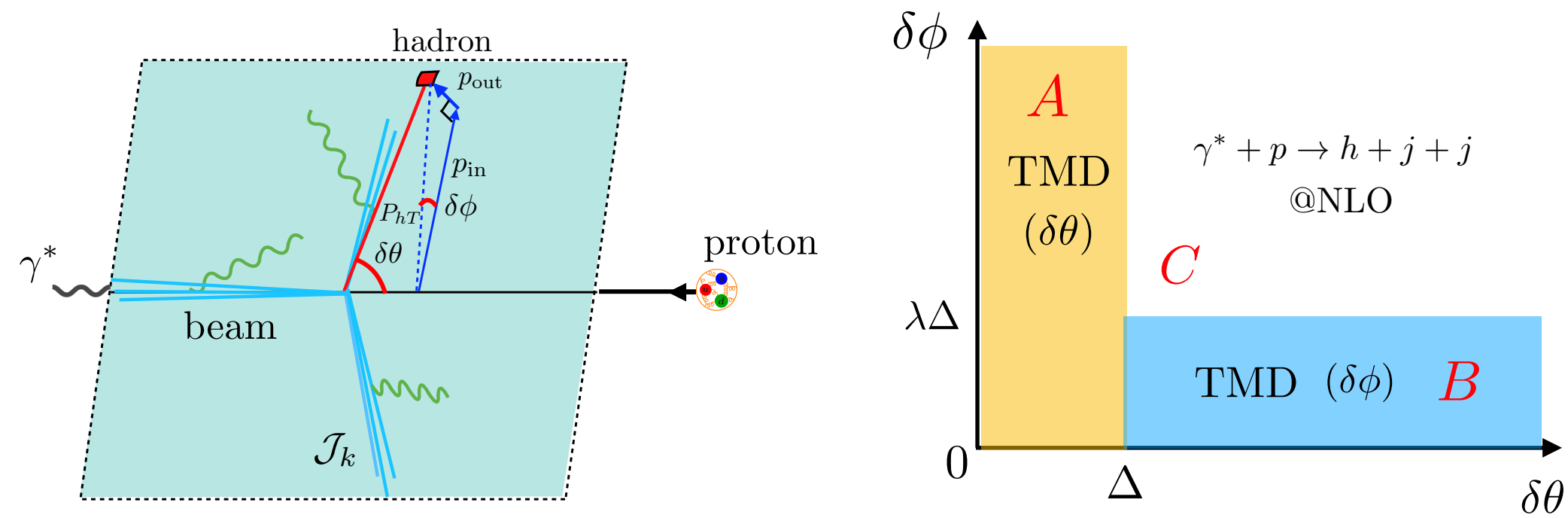
charged hadron multiplicity at HERA



- excellent agreements with NNLO analytical results for all partonic channels; our **calculations are fully differential and can be extended to N3LO**

Two-dimensional transverse momentum subtraction

- Introduction of the two-dimensional transverse momentum subtraction formalism for identified hadron production, using both **conventional and out-of-plane q_T** as slicing variables, resulting a first calculation of unpolarized SIDIS at N3LO in QCD
hadron kinematics and slicing variables

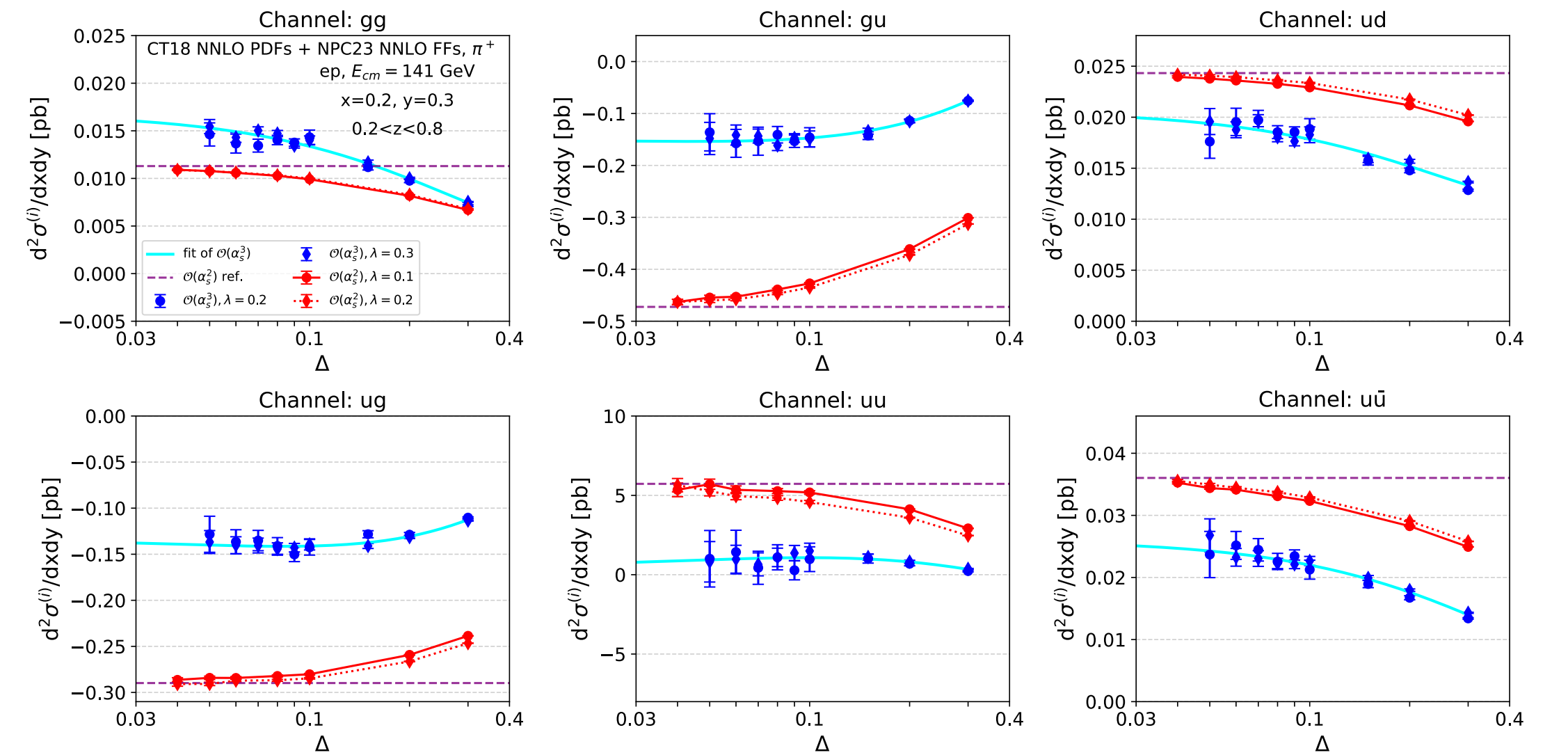


double slicing and two cutoffs **N3LO $\rightarrow$ NLO**

$$\frac{d\sigma}{d\mathcal{O}} = \int_0^\Delta d\delta\theta \frac{d\sigma^A}{d\delta\theta d\mathcal{O}} + \int_\Delta^{\delta\theta^{\max}} d\delta\theta \times \left(\int_0^{\lambda\Delta} d\delta\phi \frac{d\sigma^B}{d\delta\theta d\delta\phi d\mathcal{O}} + \int_{\lambda\Delta}^{\delta\phi^{\max}} d\delta\phi \frac{d\sigma^C}{d\delta\theta d\delta\phi d\mathcal{O}} \right)$$

[Dong, Fang, JG, Li, Shao, Zhu, Zhu, 2602.22972(PRL), 2603.29673]

pion cross sections vs. resolution parameter

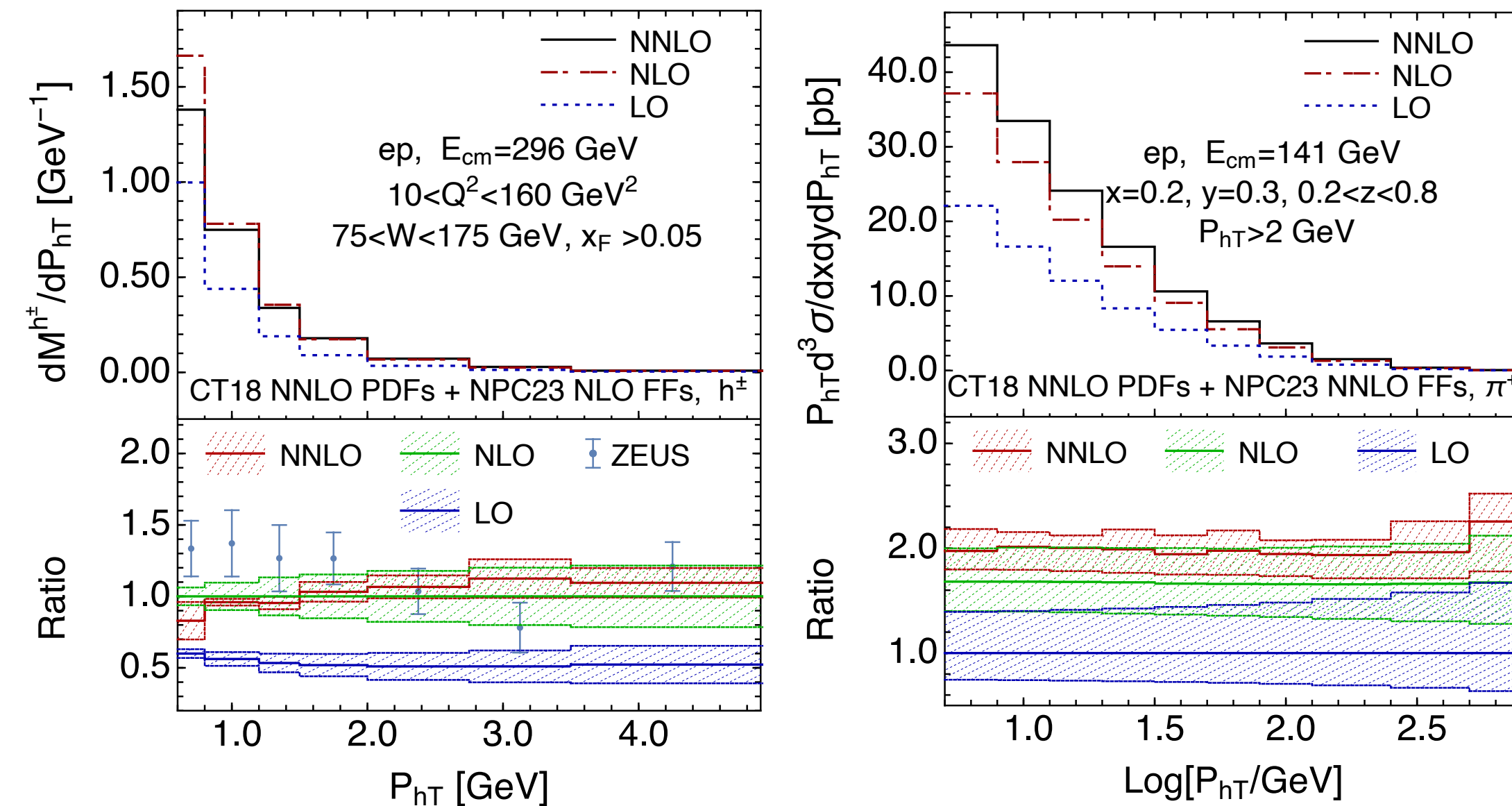


- good convergence of the QCD corrections wrt. both cutoffs for all partonic channels, numerically challenging for small cutoffs
- N3LO corrections (blue) are much smaller than NNLO ones (red) for three major channels uu, gu and ug

Implication for Electron-ion colliders

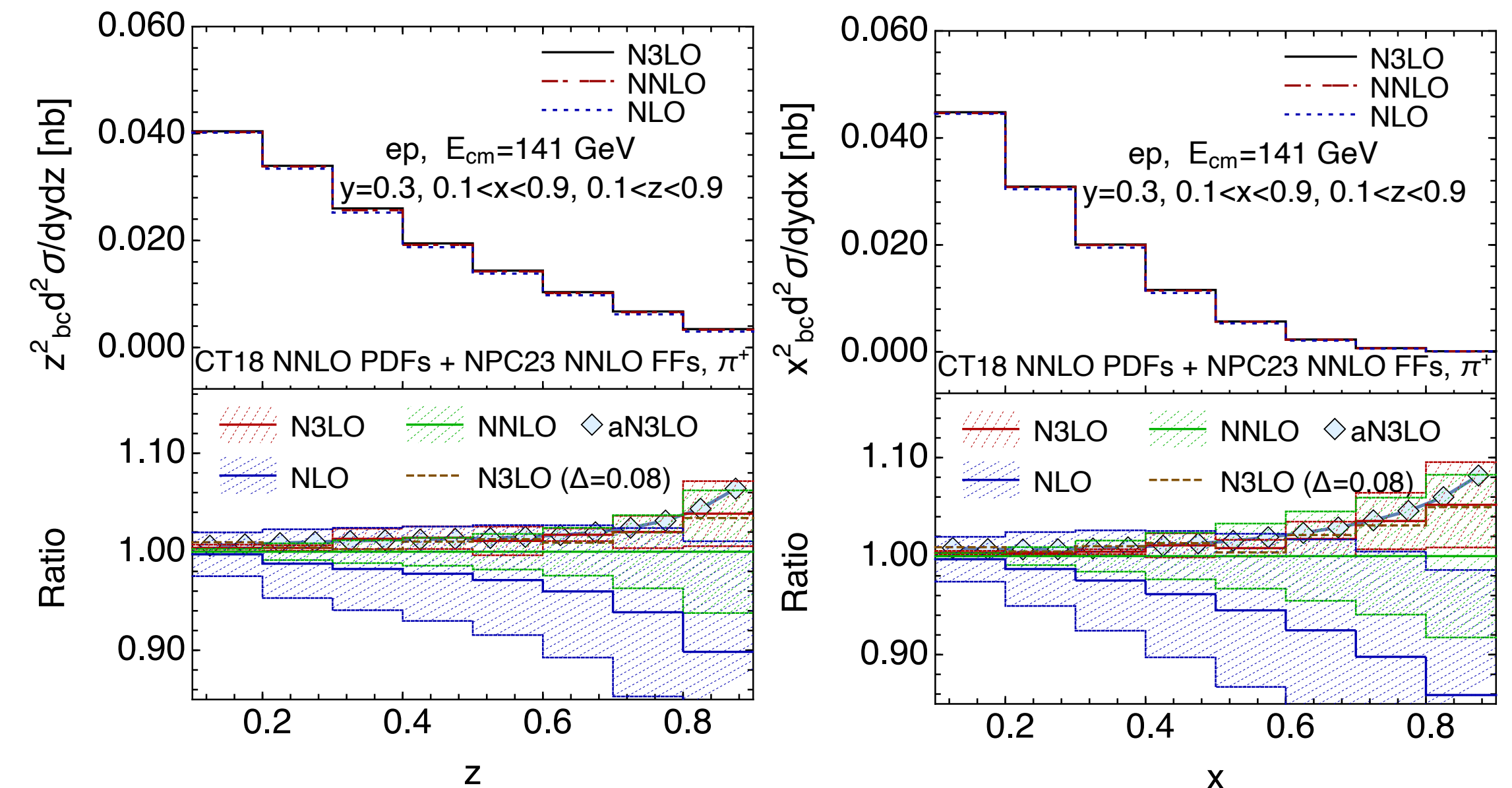
- ◆ New perturbative QCD corrections for **unpolarized** SIDIS exhibit good perturbative convergence and reduced scale variations for differential cross sections at future electron-ion collider, providing state-of-the-art predictions for study of nucleon structure and hadronizations at EIC

hadron transverse momentum spectrum
at HERA and EIC upto NNLO



[Dong, Fang, JG, Li, Shao, Zhu, Zhu, 2602.22972(PRL), 2603.29673]

distribution of Bjorken-x and z for
SIDIS at EIC upto N3LO

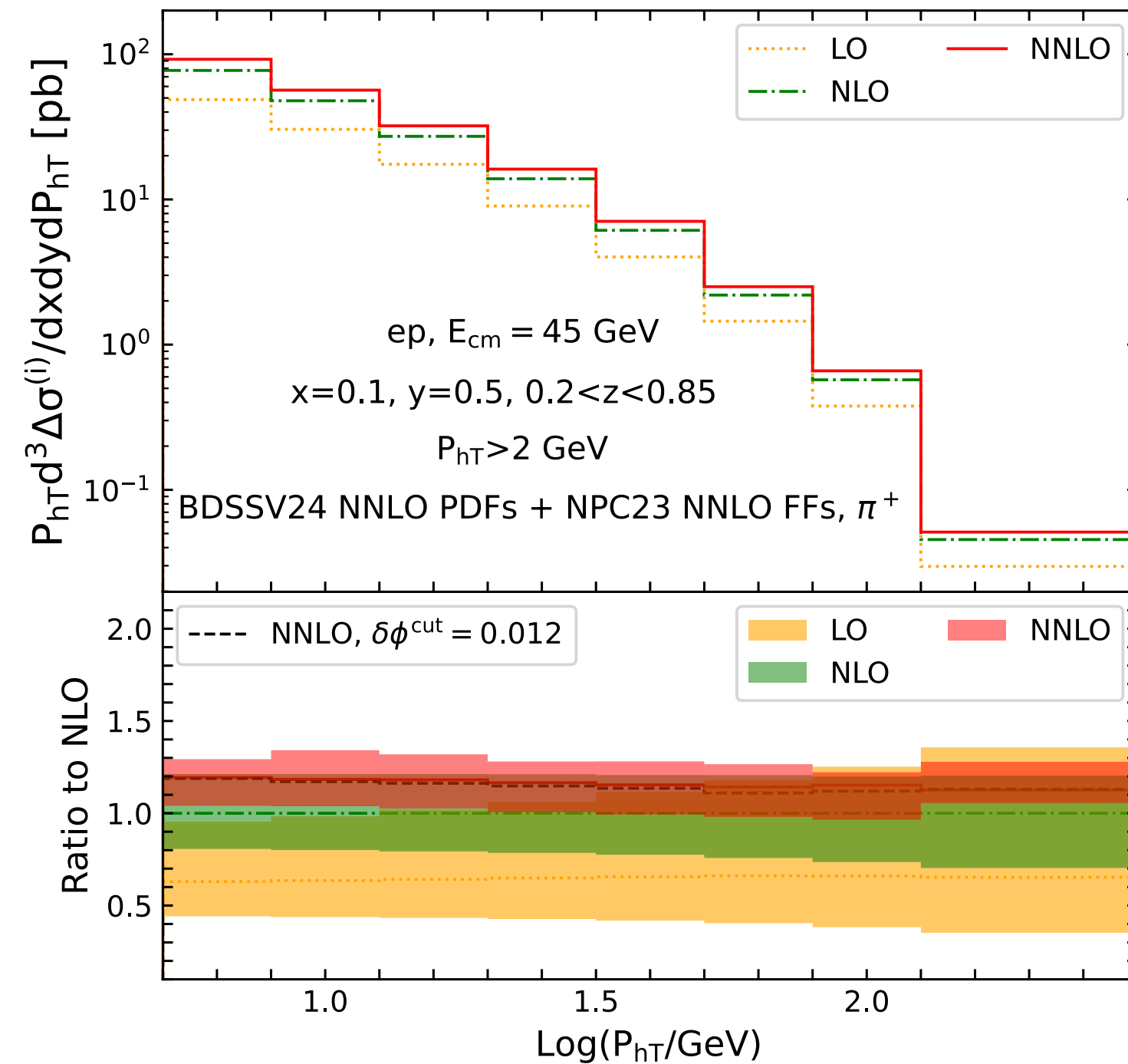


approximated N3LO from [Goyal, Moch, Pathak, Rana, Ravindran 2025]

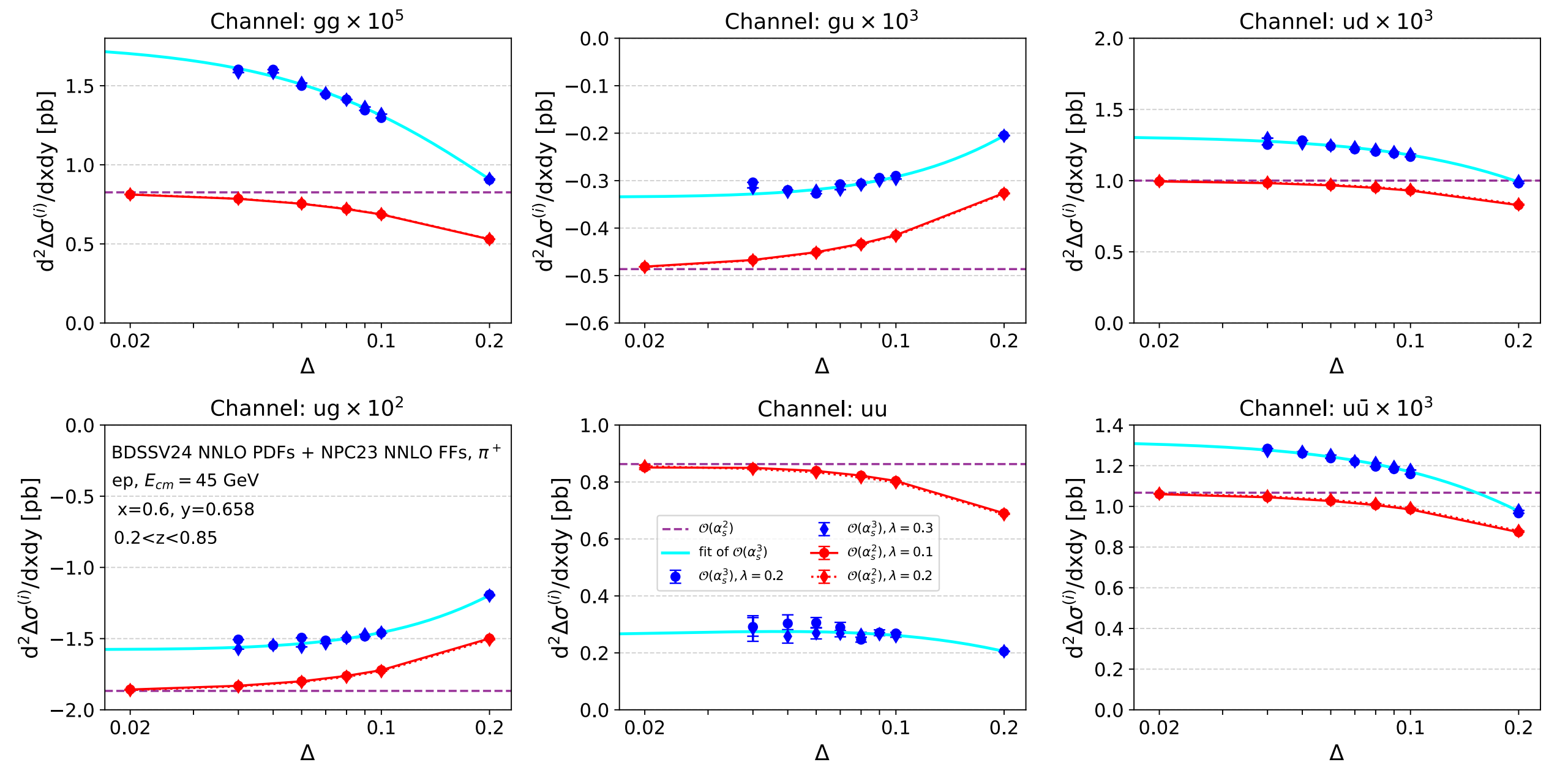
Extension to polarized SIDIS

- ✦ The same method has been successfully generalized to longitudinal **polarized** SIDS, with modifications on few ingredients, including the TMD beam functions, hard functions, as well as the subtraction scheme for multi-jets at NLO; similar convergence and performance for the asymmetric cross sections are found

asymmetric pion transverse momentum spectrum at EIC upto NNLO



asymmetric pion cross sections vs. resolution parameter at EIC upto $\mathcal{O}(\alpha_s^3)$



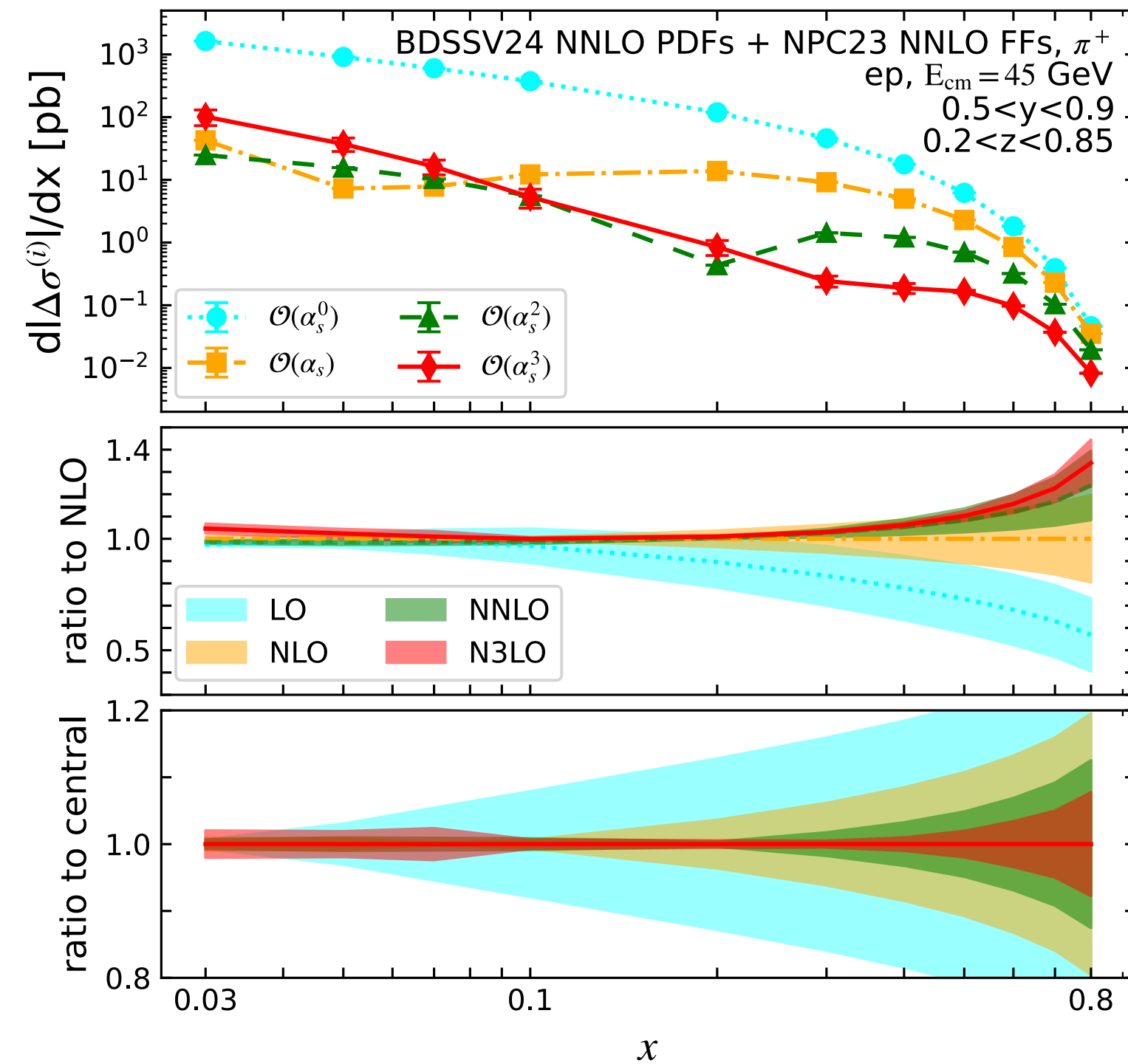
- ✦ good convergence of the QCD corrections wrt. both cutoffs for all partonic channels

[Dong, Fang, JG, Li, Shao, Zhou, Zhu, to appear soon]

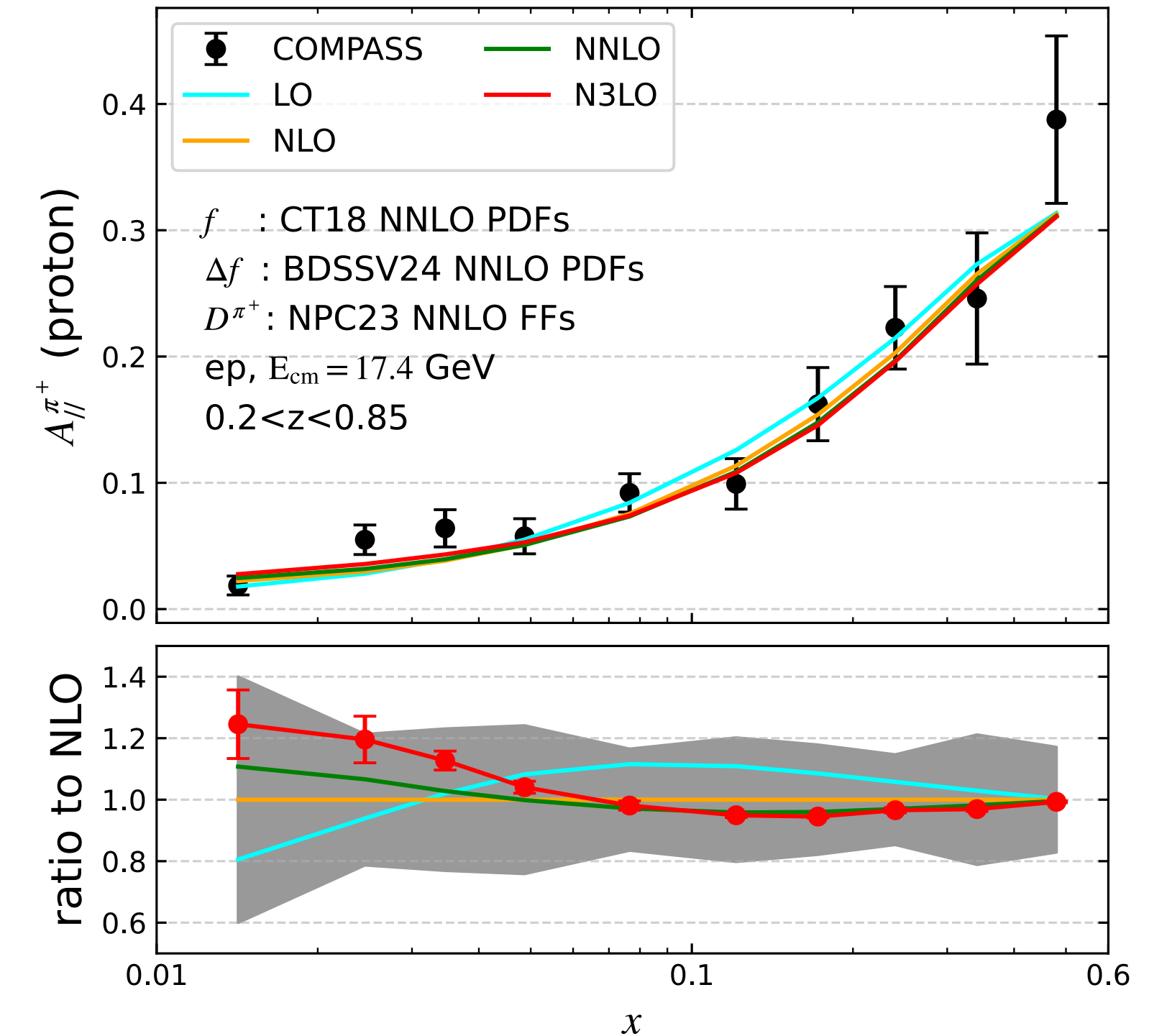
Extension to polarized SIDIS

- ✦ New perturbative QCD corrections for longitudinal **polarized** SIDIS exhibit good perturbative convergence and reduced scale variations for differential cross sections at future electron-ion collider, providing state-of-the-art predictions for study of helicity PDF at EIC

asymmetric pion cross section as a function of Bjorken-x at EIC upto N3LO



double spin asymmetry of pion as a function of Bjorken-x at COMPASS upto N3LO



[Dong, Fang, JG, Li, Shao, Zhou, Zhu, to appear soon]

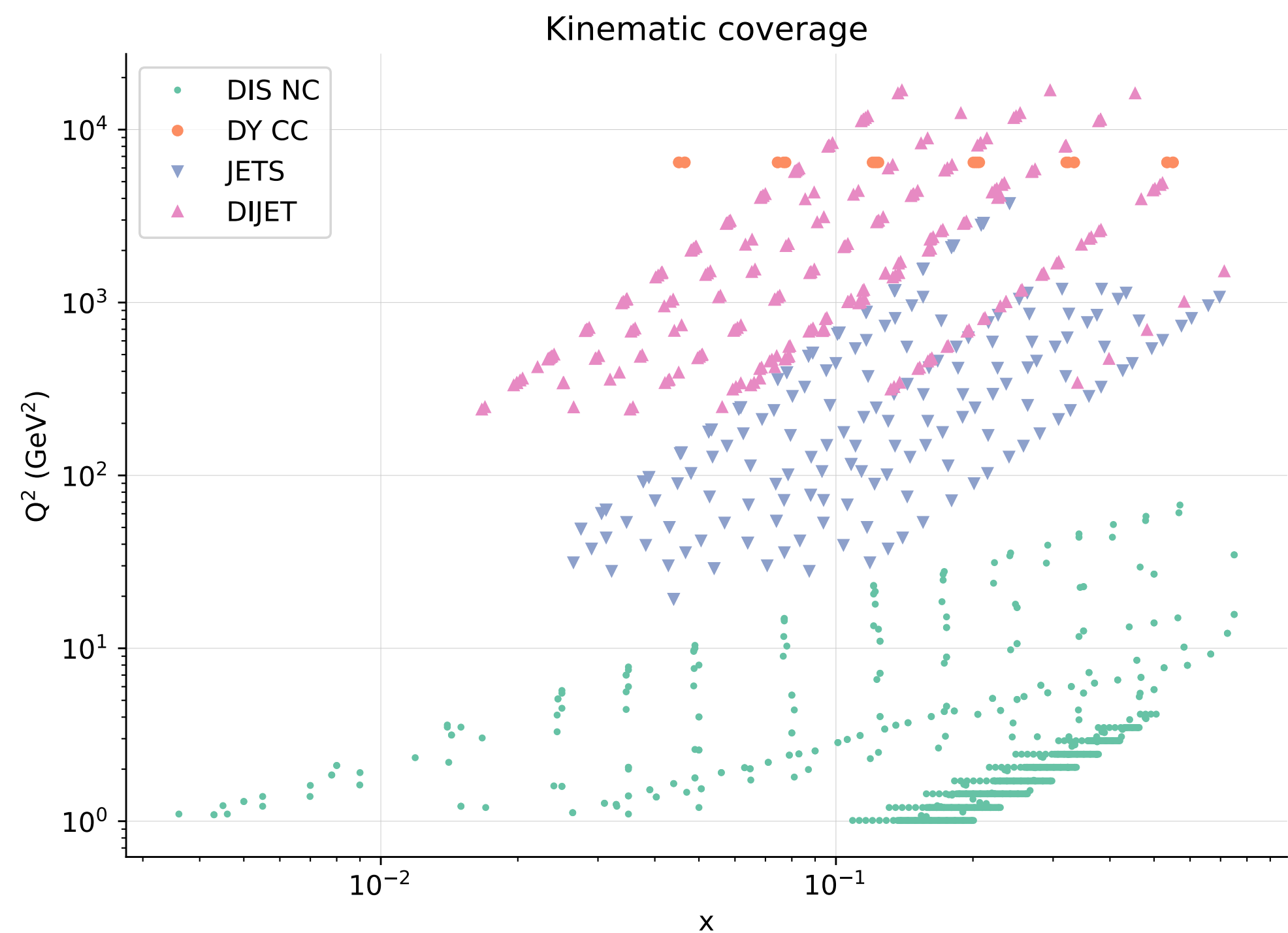
- ✦ the double spin asymmetry are rather stable at large-x due to cancellations of QCD corrections in (un)polarized SIDIS

◆ 4. NPC analysis on helicity PDFs of nucleons

Global analysis of helicity PDFs

- Measurements are available from polarized fixed-target experiments on DIS/SIDIS, polarized pp collider (RHIC) on W/Z boson, incl. hadron, incl. jet, dijets, and direct photon production; many groups provide phenomenological pPDFs from global analysis at NLO/aNNLO in QCD

typical kinematic coverage in x-Q² plane



most recent pPDFs from major analysis groups

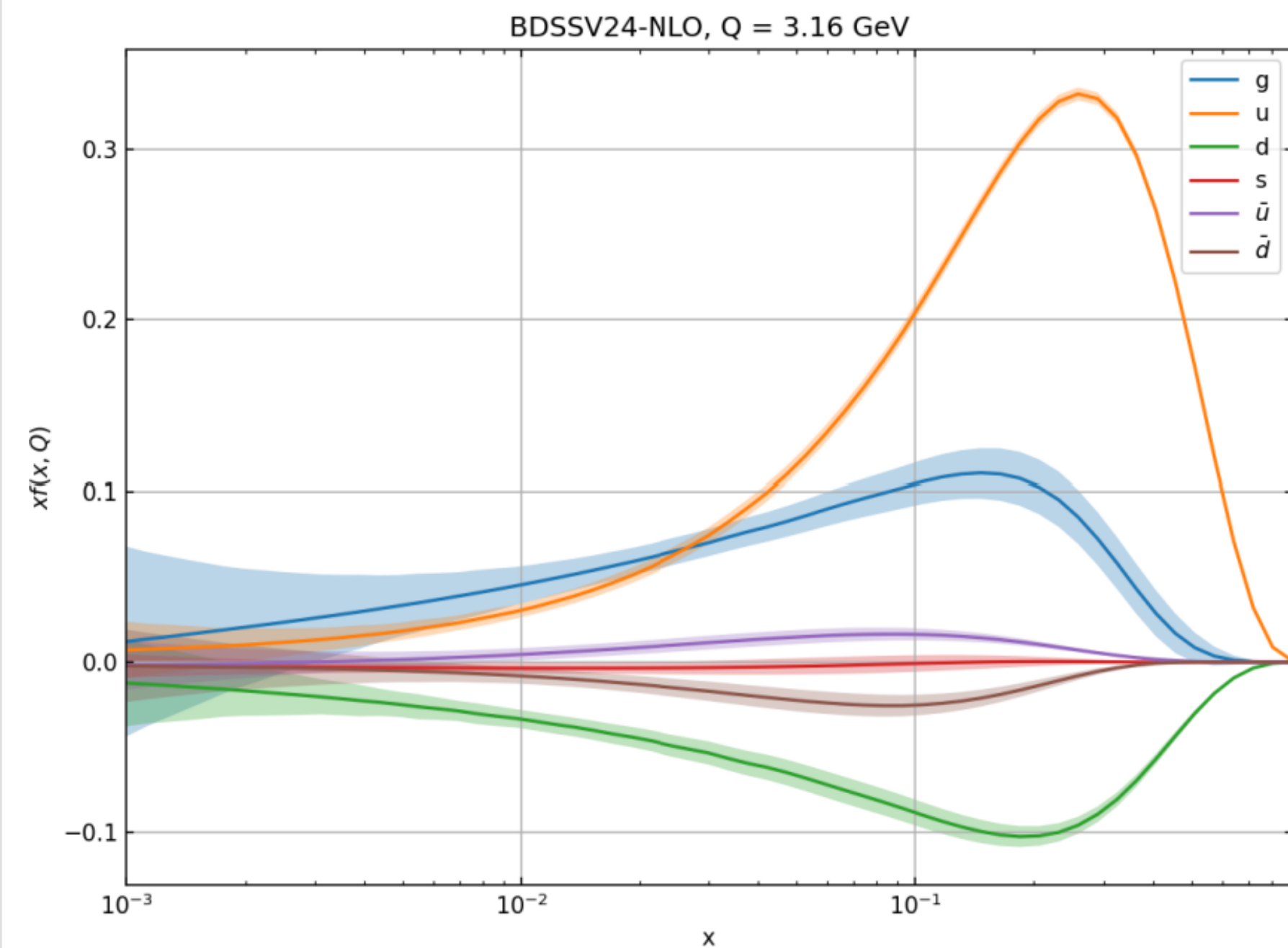
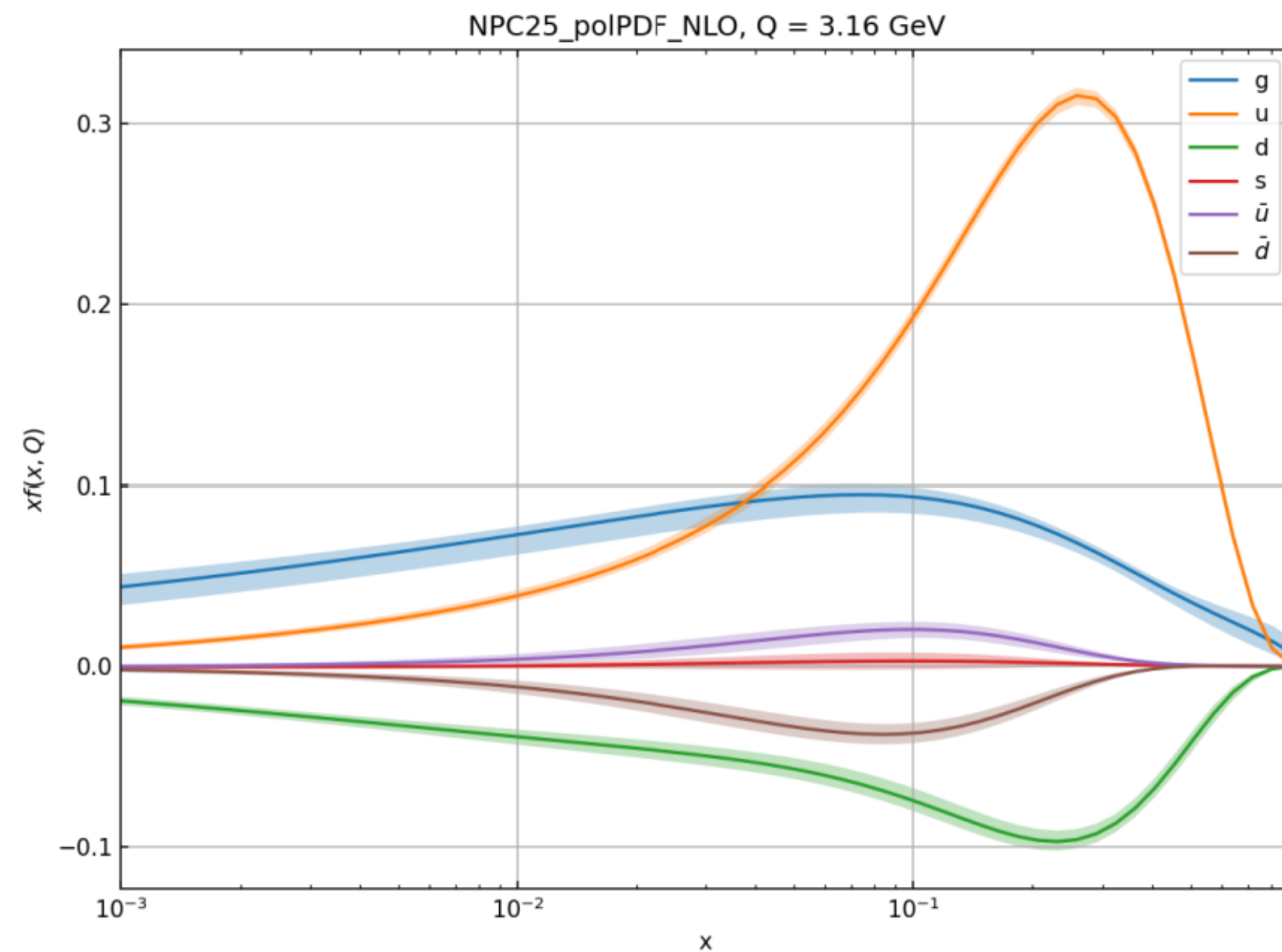
pPDFs	MAPPDFpol1.0	BDSSV24	NNPDFpol2.0	JAM25
	2404.04712	2407.11635	2503.11814	2506.1361
order	NNLO	aNNLO	aNNLO	NLO
DIS	✓	✓	✓	✓
SIDIS:h	✓	✓		✓
SIDIS:open-charm				
pp:jet prod.		✓	jet	✓
pp:DY(W)		✓	✓	✓
pp:hadron prod		✓		
lattice				✓

- complementary constraints from different data sets are crucial for pPDF fit due to limited data sets (~1000 data points), e.g., jets on gluon and W/Z on anti-quark flavor separation

Preliminary results from NPC

- ✦ The NPC analysis on pPDF aims at a global determination using all available polarized data with exact NNLO QCD predictions, as well as implementing recent lattice QCD calculations from LPC collaboration

preliminary NPC pPDF at NLO comparing to BDSSV24

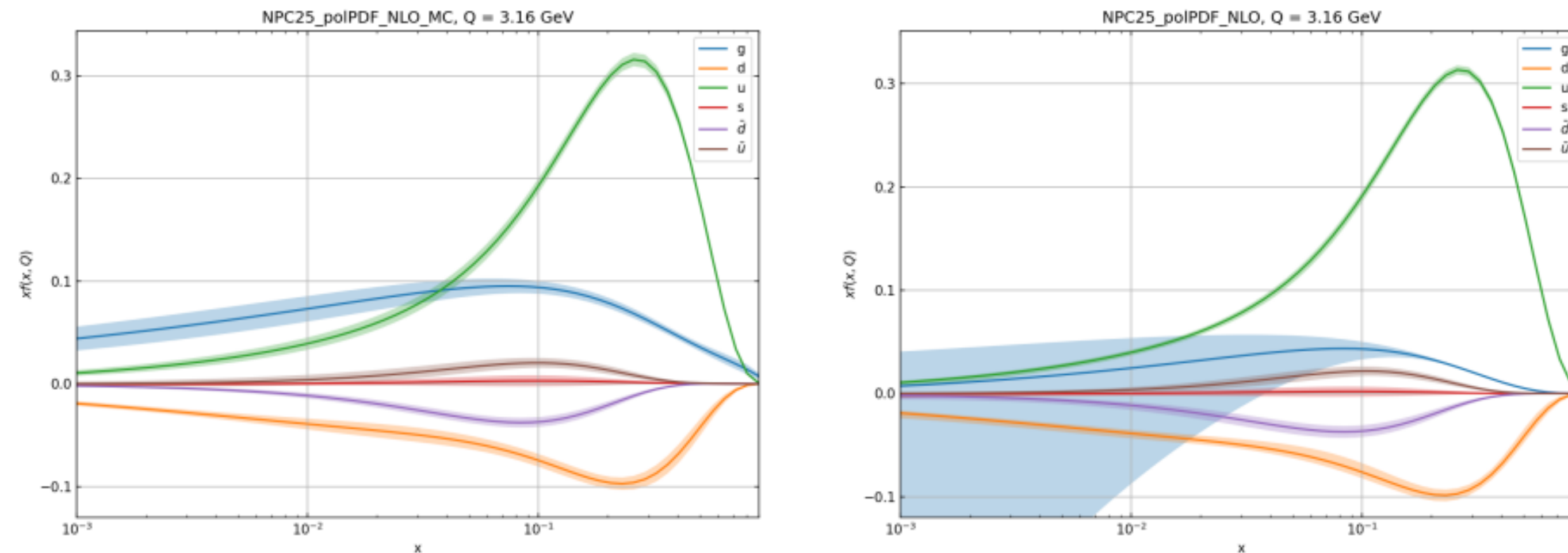


- ❖ we have tested our framework with a NLO fit using similar data and theory as BDSSV, but with different parametrization form; the two agrees well except for uncertainties at small- x to be improved latter

Preliminary results from NPC

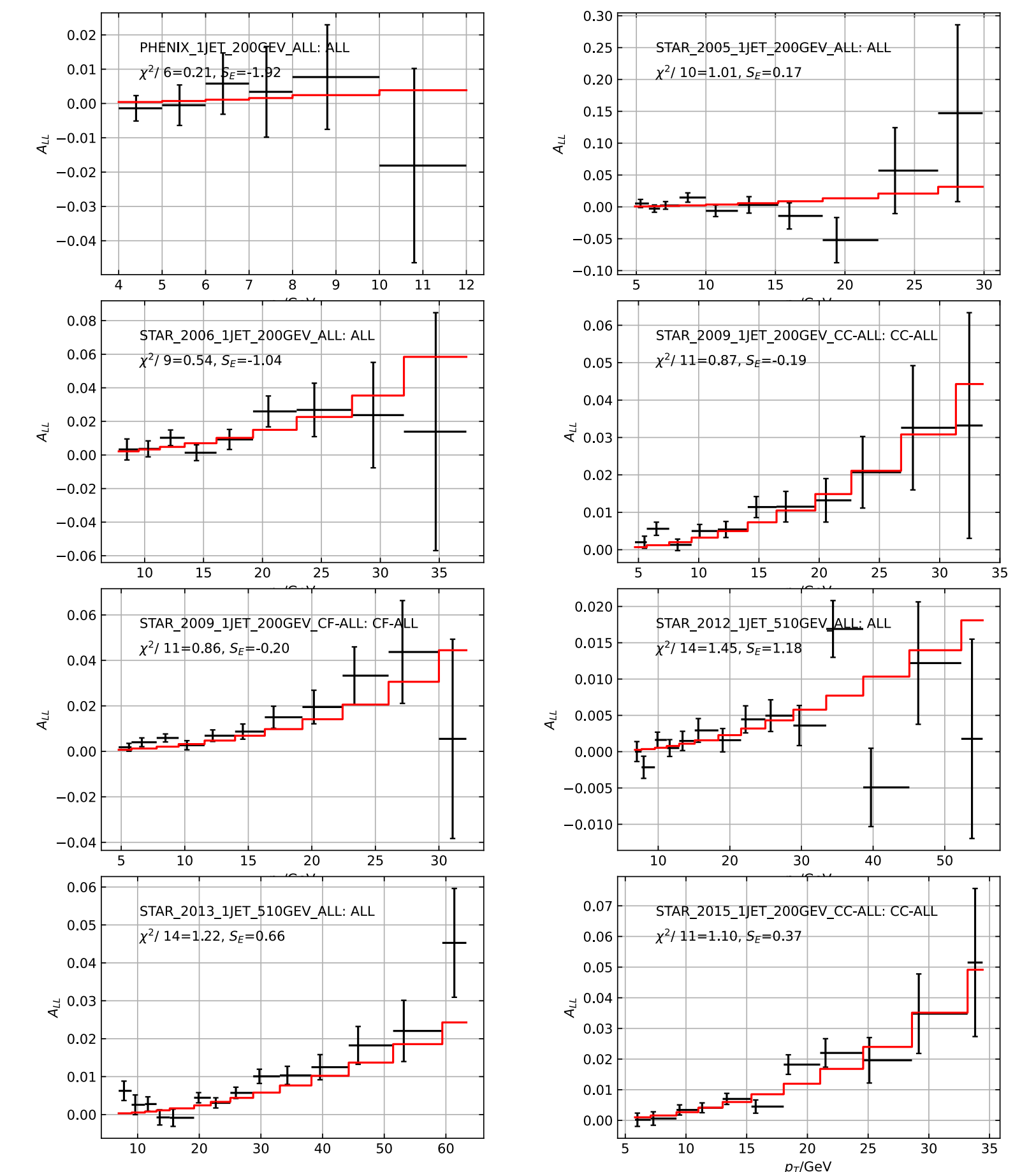
- ✦ The most challenging task for achieving NNLO precision would be calculating the NNLO QCD corrections for incl. jet and dijets production in polarized pp collisions which are critical for constraining the gluon pPDF

comparison of pPDFs with (left) and w/o (right) including jet data



- ✦ The gluon pPDF are almost unconstrained if w/o jet data (except for lattice data on gluon normalization); analysis from other groups currently use at most approximated NNLO jet cross sections from threshold expansions

Fit to the incl. jet data from PHENIX and STAR, A_{LL} vs. p_T



Summary

- ✦ Deep inelastic scatterings are essential for refined study of nucleon structure, including PDFs, helicity PDFs, and TMD PDFs, with implications not only for understanding of QCD but also for precision test of the SM
- ✦ We developed a systematic routine of subtraction/slicing methods for calculations of single inclusive hadron production, and leverage perturbative precision of (p)SIDIS to N3LO in QCD
- ✦ The NPC collaboration also provides most comprehensive/advanced global analyses on FFs of light hadrons, including on hadron species, data selections, and theoretical precisions (to NNLO)
- ✦ With the advancements on ingredients of both non-perturbative and perturbative QCD we are on the way of building a precision framework for extraction of the helicity PDFs of the nucleons

Thank you for your attention!

NPC23 analysis at NNLO

- ✦ A first Global analysis of FFs to light charged hadrons at full NNLO with SIA and SIDIS data, especially with recent BESIII data, focusing on low momentum transfer region and test of collinear factorization

quality of NNLO fit with varying hadron energy cut

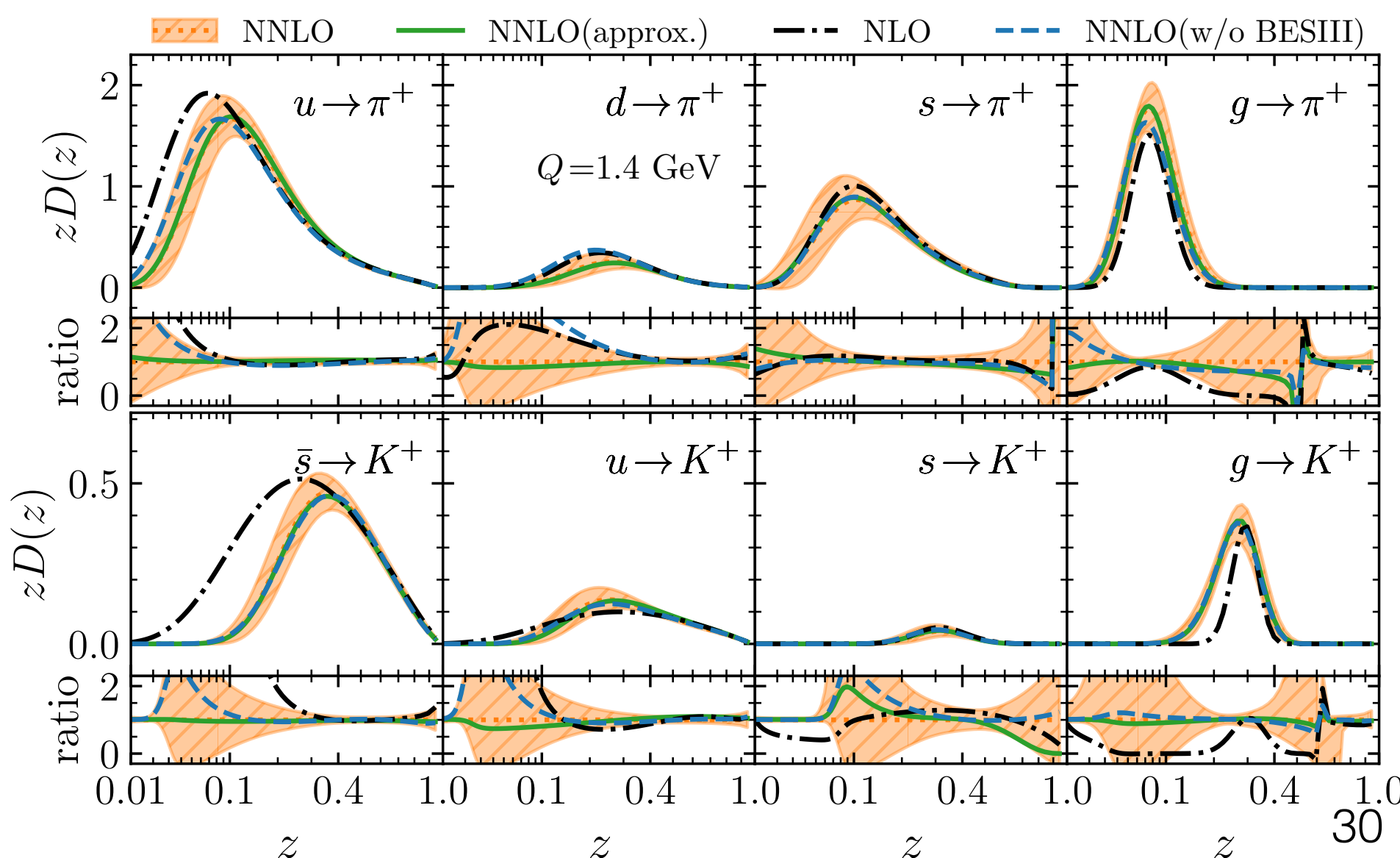
[JG, Shen, Xing, Zhao, Zhou, 2502.17837 (PRL)]

	$E_{h,min}[\text{GeV}]$	BESIII		COMPASS		B-factories		HE-SIA		global		
		N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2/N_{pt}	N_{pt}	χ^2	χ^2/N_{pt}
NNLO	0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
	0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
	0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
	0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
	0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
	1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77
NLO	0.8	152	1.03	142	1.26	218	0.54	407	0.85	919	801.6	0.87
NNLO(approx.)	0.8	152	0.96	142	1.40	218	0.53	407	0.81	919	791.5	0.86
NNLO(w/o BES)	0.8	-	-	142	1.23	218	0.52	407	0.81	767	620.2	0.81

[BES III & NPC collaboration, 2502.16084 (PRL)]

satisfactory agreements between NNLO theory and data are found for both the SIA and SIDIS data with $Q \sim 2\text{-}3$ GeV

FFs to charged pion/kaon at $Q=1.4$ GeV



- ✦ proven factorization/universality of FFs in SIA and SIDIS at lowest Q values

- ✦ significantly worsen of all χ^2 for hadron energy below 0.8 GeV; slightly larger global χ^2 for NLO or aNNLO fits compared with NNLO fit

Non-singlet FFs from charge asymmetries

- ✦ In the pion NS FF NNLO fit we find a preference on the scaling index β close to 1 rather than 2, at $Q=1.3$ GeV, despite of various systematic effects; several models are tested including Field-Feynman parametrization and CSM FFs [Xing, Bian, Cui, Roberts, 2025] [JG, Liu, Zhou, 2507.14637, CPC cover story & editors' suggestion]

various fits to the pion NS FFs

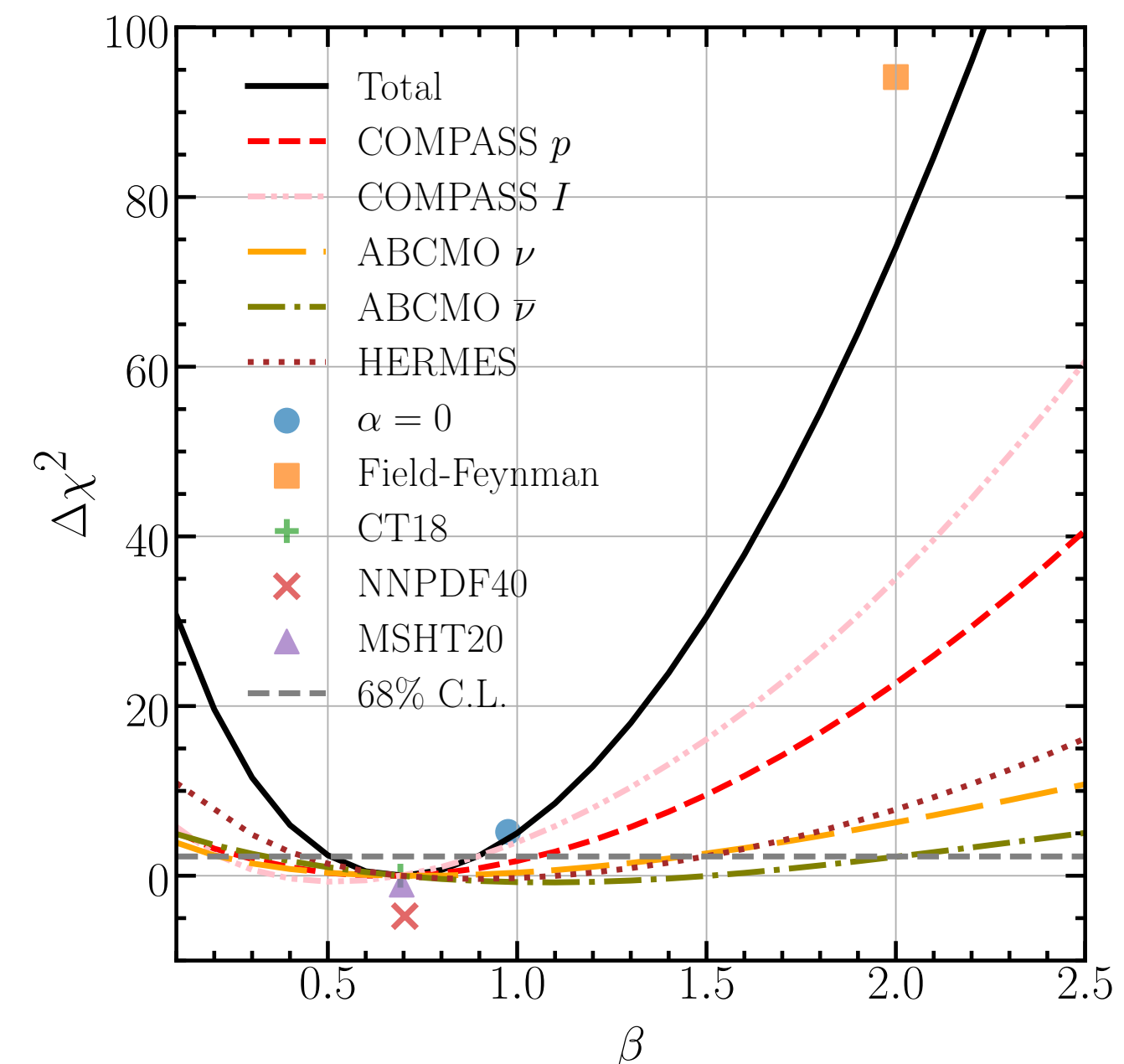
$zD_{u-}^{\pi+}$	α	β	$a_0(a_1)$	$\chi^2 (N_{pt})$
nominal	$-0.335^{+0.22}_{-0.21}$	$0.692^{+0.20}_{-0.19}$	$-1.891^{+0.30}_{-0.28}$	252.4 (249)
mod.1	0	0.976	-1.438	257.5 (249)
mod.2	-0.035	1.0	-1.460	257.4 (249)
mod.3	0.893	2.0	-0.091	326.4 (249)
mod.4	1.0	2.0	0.054	346.5 (249)
mod.5	0.681	2.298	-1.352(2.444)	563.2 (249)
sys.1	-0.323	0.703	-1.846	247.6 (249)
sys.2	-0.476	0.665	-1.984	152.7 (144)

nominal parametrization of NS FFs

$$zD_{i-}^h(z, Q_0) \equiv z(D_i^h - D_{\bar{i}}^h)(z, Q_0) = z^\alpha (1-z)^\beta \exp(a_0)$$

- ✦ 3-parameter nominal fit ($z>0.3$) shows a very good χ^2 and a scaling index of about 0.7 ± 0.2 ; changing PDFs, Q_0 , z cut, using only proton data, or switch to 4-parameter fit or NLO fit, all return a scaling index within uncertainties

a scan on the scaling index



- ✦ a surprisingly consistent preference of β close to 1 than 2 for all SIDIS data studied; see [Aichier, Schafer, Volgesang; JAM; xFitter; FantoPDF] for similar observation in pion PDFs

Hadron production cross sections at NLO

- Generic algorithms on NLO calculations of jet production cross sections have been developed for long times, based on local subtraction or phase-space slicing method; especially automation of NLO jet cross sections exists, e.g., in MG5 and Sherpa

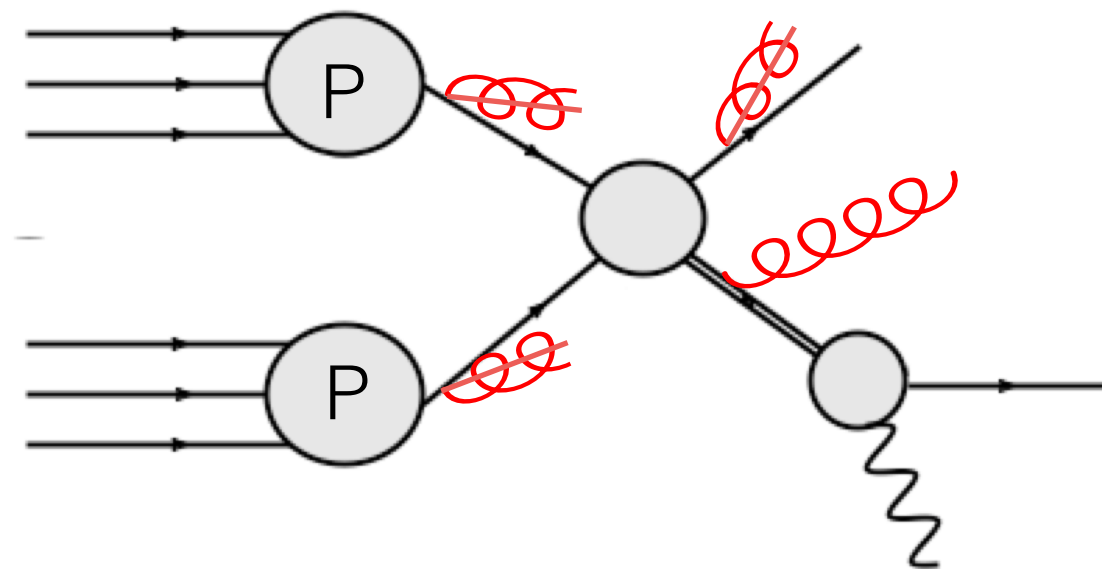
local subtraction

$$\frac{d\sigma}{dF} = \int dPS_m \left[|M|_{B,m}^2 + |M|_{V,m}^2 + |\tilde{\mathcal{I}}_m|^2 \right] \delta(\hat{F}(p_m; f_m) - F) \\ + \int dPS_{m+1} \left[|M|_{R,m+1}^2 \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) - |\mathcal{I}_{m+1}|^2 \delta(\hat{F}(\tilde{p}_m; \tilde{f}_m) - F) \right]$$

phase-space slicing

$$\frac{d\sigma}{dF} = \int dPS_m \left[|M|_{B,m}^2 + |M|_{V,m}^2 \right] \delta(\hat{F}(p_m; f_m) - F) \\ + \int dPS_{m+1} (\Theta(C - \lambda) + \Theta(\lambda - C)) \left[|M|_{R,m+1}^2 \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) \right]$$

QCD radiations



- ❖ FKS subtraction (jet), [\[Frixione, Kunszt, Signer\]](#), as implemented in Madgraph5
- ❖ Dipole subtraction (jet & hadron), [\[Catani, Seymour\]](#), as implemented in MCFM
- ❖ Two-cutoff slicing (jet & hadron), [\[Harris, Owens\]](#)
- ❖ Antenna subtraction (jet & hadron), [\[2406.09925\]](#); alternative subtraction [\[2403.14574\]](#); semi-analytical calculations, [\[1903.01529, BigkT\]](#)
- ❖ STRIPPER (jet & hadron at NNLO), [\[2503.11489\]](#)

public tools and automation on the hadron production cross sections are very much limited!