

Analyses of Nucleon TMDs

第一届强相互作用自旋物理会议

2026年7月30日 @ 青岛

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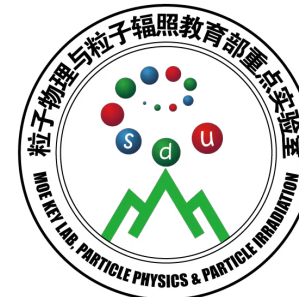
Southern Center for Nuclear-Science Theory, IMP, CAS

Transverse Nucleon Tomography Collaboration:

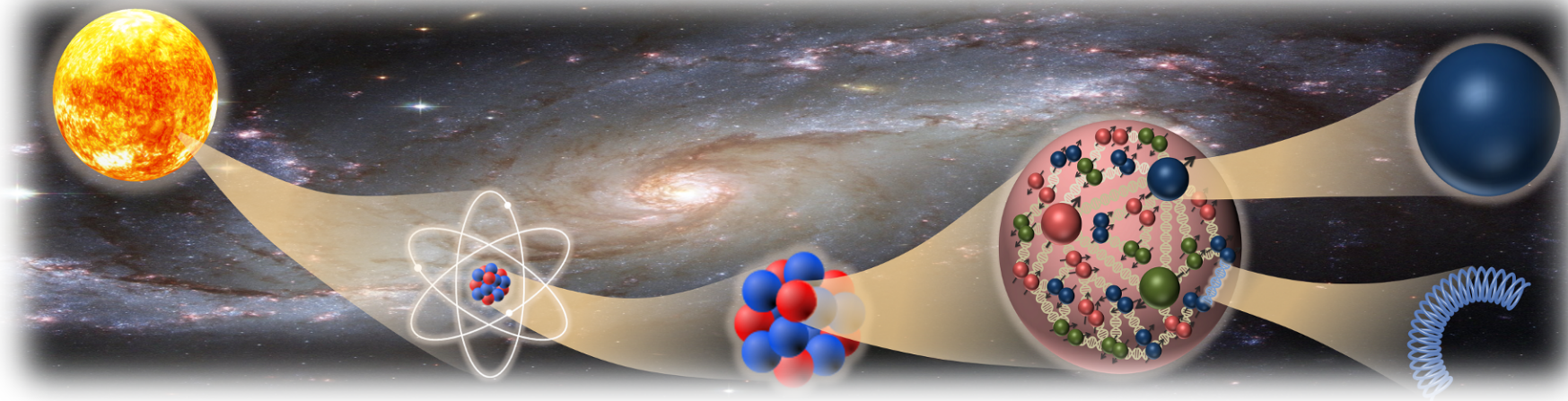
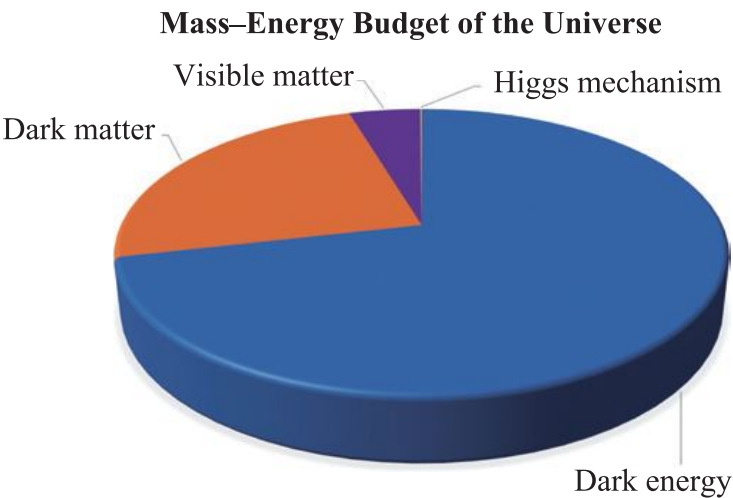
Hongxin Dong, TL, Bo-Qiang Ma, Peng Sun, Ke Yang, Chunhua Zeng, Yuxiang Zhao



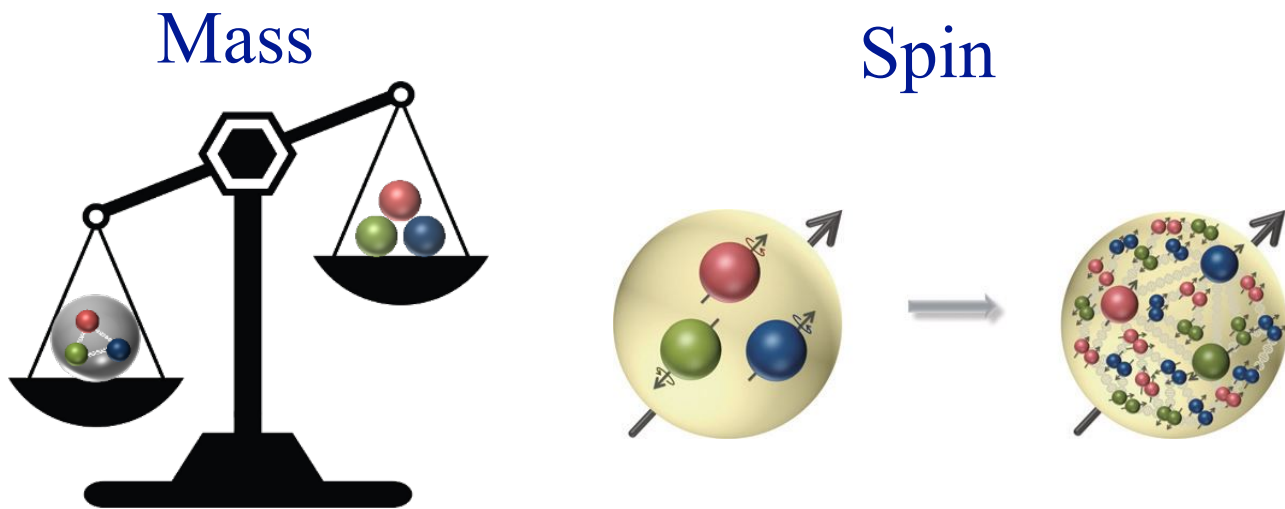
山东大学
SHANDONG UNIVERSITY



How much do we understand our world?



Three generations of matter			Force carriers	
Quarks	I	II	III	
	Up quark Mass ≈ 2.2 MeV Charge = $2/3$ Spin = $1/2$	Charm quark Mass ≈ 1.28 GeV Charge = $2/3$ Spin = $1/2$	Top quark Mass ≈ 173.1 GeV Charge = $2/3$ Spin = $1/2$	Gluon Mass = 0 Charge = 0 Spin = 1
	Down quark Mass ≈ 4.7 MeV Charge = $-1/3$ Spin = $1/2$	Strange quark Mass ≈ 96 MeV Charge = $-1/3$ Spin = $1/2$	Bottom quark Mass ≈ 4.18 GeV Charge = $-1/3$ Spin = $1/2$	Photon Mass = 0 Charge = 0 Spin = 1
Leptons				
	Electron Mass ≈ 0.511 MeV Charge = -1 Spin = $1/2$	Muon Mass ≈ 105.66 MeV Charge = -1 Spin = $1/2$	Tau Mass ≈ 1.7768 GeV Charge = -1 Spin = $1/2$	Z boson Mass ≈ 91.19 GeV Charge = 0 Spin = 1
	Electron neutrino Mass < 1 eV Charge = 0 Spin = $1/2$	Muon neutrino Mass < 0.17 MeV Charge = 0 Spin = $1/2$	Tau neutrino Mass ≈ 18.2 MeV Charge = 0 Spin = $1/2$	W boson Mass ≈ 80.39 GeV Charge = ± 1 Spin = 1
				Higgs Mass ≈ 124.97 GeV Charge = 0 Spin = 0
				Scalar bosons
				Gauge bosons Vector bosons

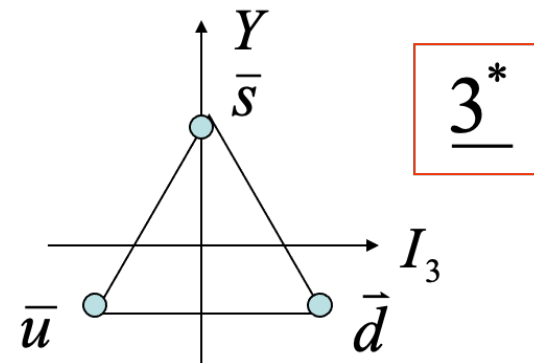
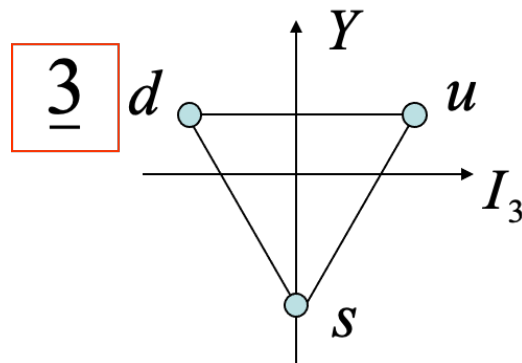


*How do quarks and gluons make up a nucleon?
How can nucleon properties be explained at
quarks and gluons degrees of freedom?*

Proton Spin Structure in Naïve Quark Model

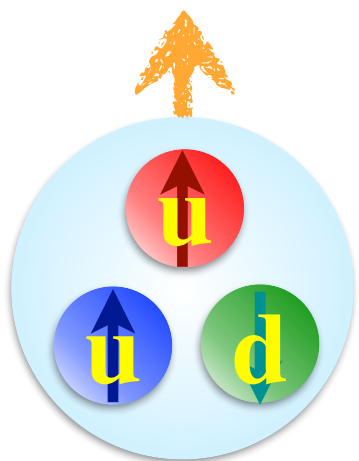
Quark model:

M. Gell-Mann, Phys. Lett. 8, 214 (1964);
G. Zweig, CERN Report No. TH-401 (1964).



ordinary baryons: $|qqq\rangle$, mesons: $|q\bar{q}\rangle$

Spin-flavor wave function of the proton:



$$|p_{\uparrow}\rangle = \frac{1}{\sqrt{18}} \left[2 |u_{\uparrow}d_{\downarrow}u_{\uparrow}\rangle + 2 |u_{\uparrow}u_{\uparrow}d_{\downarrow}\rangle + 2 |d_{\downarrow}u_{\uparrow}u_{\uparrow}\rangle - |u_{\uparrow}u_{\downarrow}d_{\uparrow}\rangle \right. \\ \left. - |u_{\uparrow}d_{\uparrow}u_{\downarrow}\rangle - |u_{\downarrow}d_{\uparrow}u_{\uparrow}\rangle - |d_{\uparrow}u_{\downarrow}u_{\uparrow}\rangle - |d_{\uparrow}u_{\uparrow}u_{\downarrow}\rangle - |u_{\downarrow}u_{\uparrow}d_{\uparrow}\rangle \right].$$

$$\Delta u = u_{\uparrow} - u_{\downarrow} = \frac{4}{3}$$

$$\Delta d = d_{\uparrow} - d_{\downarrow} = -\frac{1}{3}$$

The sum of quark spins gives the proton spin.

Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer:

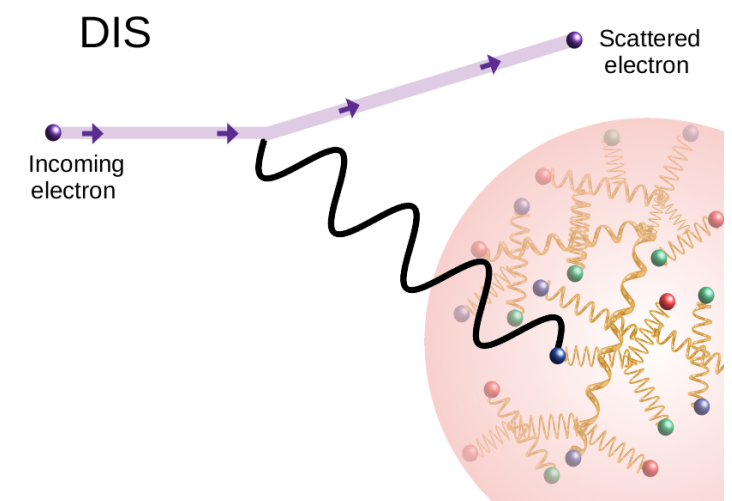
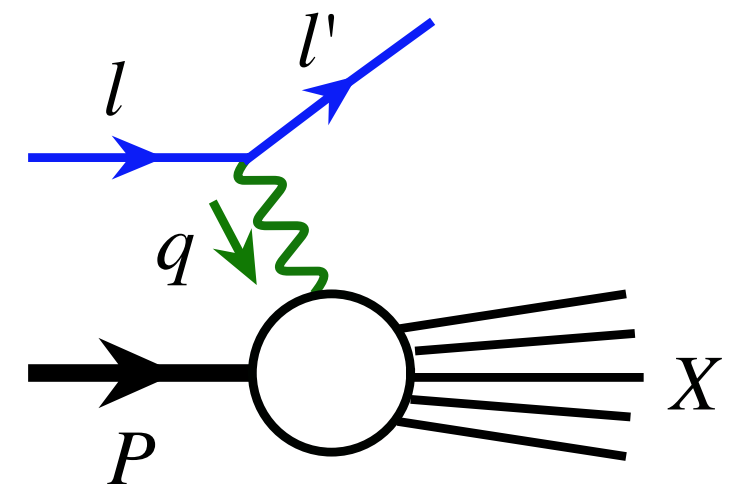
- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale $\sim 1/\text{fm}$
- collinear factorization:

$$\sigma \propto H(Q) \otimes f_{i/P}(x, \mu^2)$$

- overall corrections suppressed by $1/Q^n$
- indirectly “see” quarks, gluons and their dynamics
- predictive power relies on
 - precision of the probe
 - universality of $f_{i/P}(x, \mu^2)$

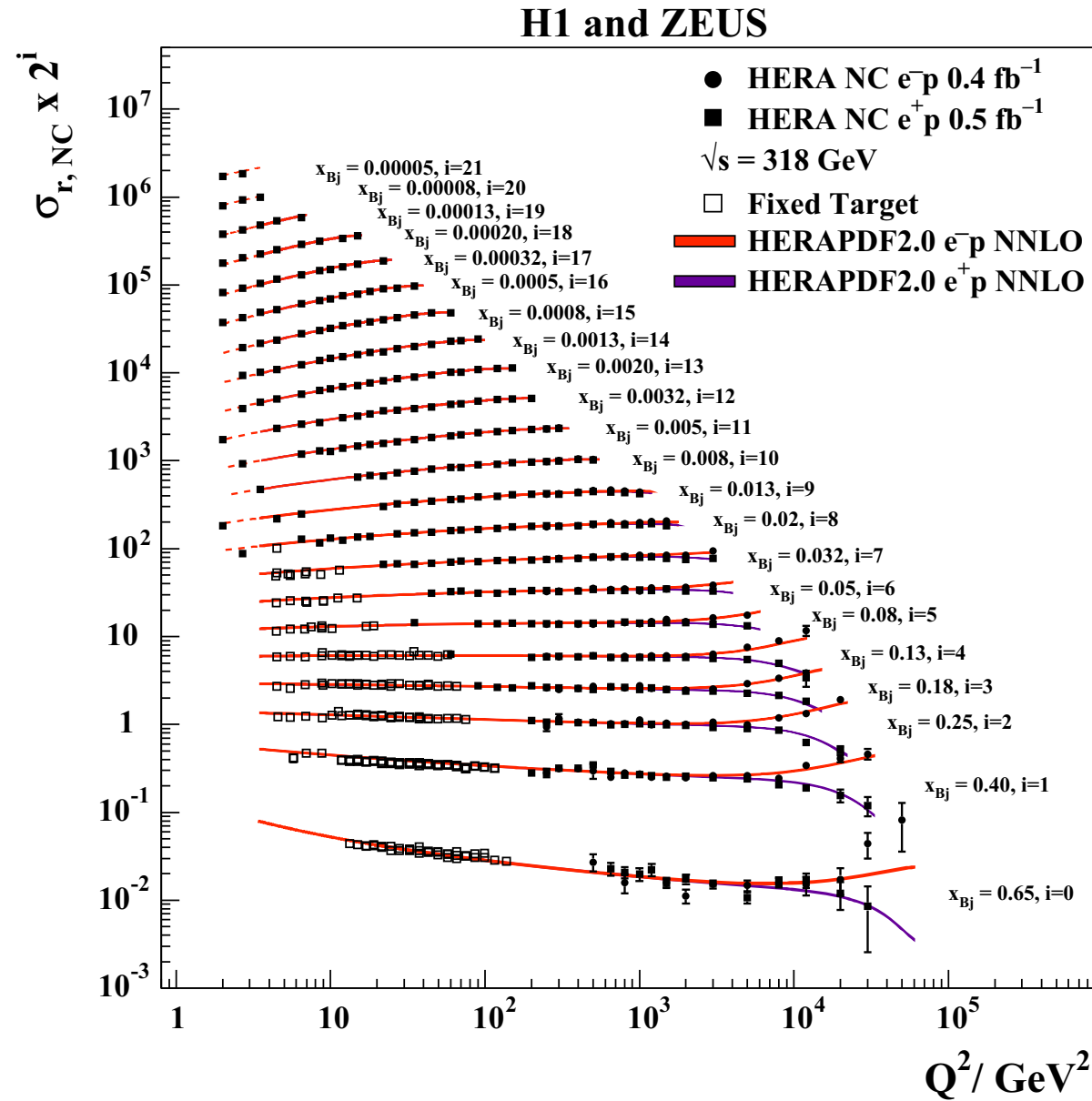
$$Q \gg \Lambda_{\text{QCD}}$$

Modern “Rutherford” experiment.

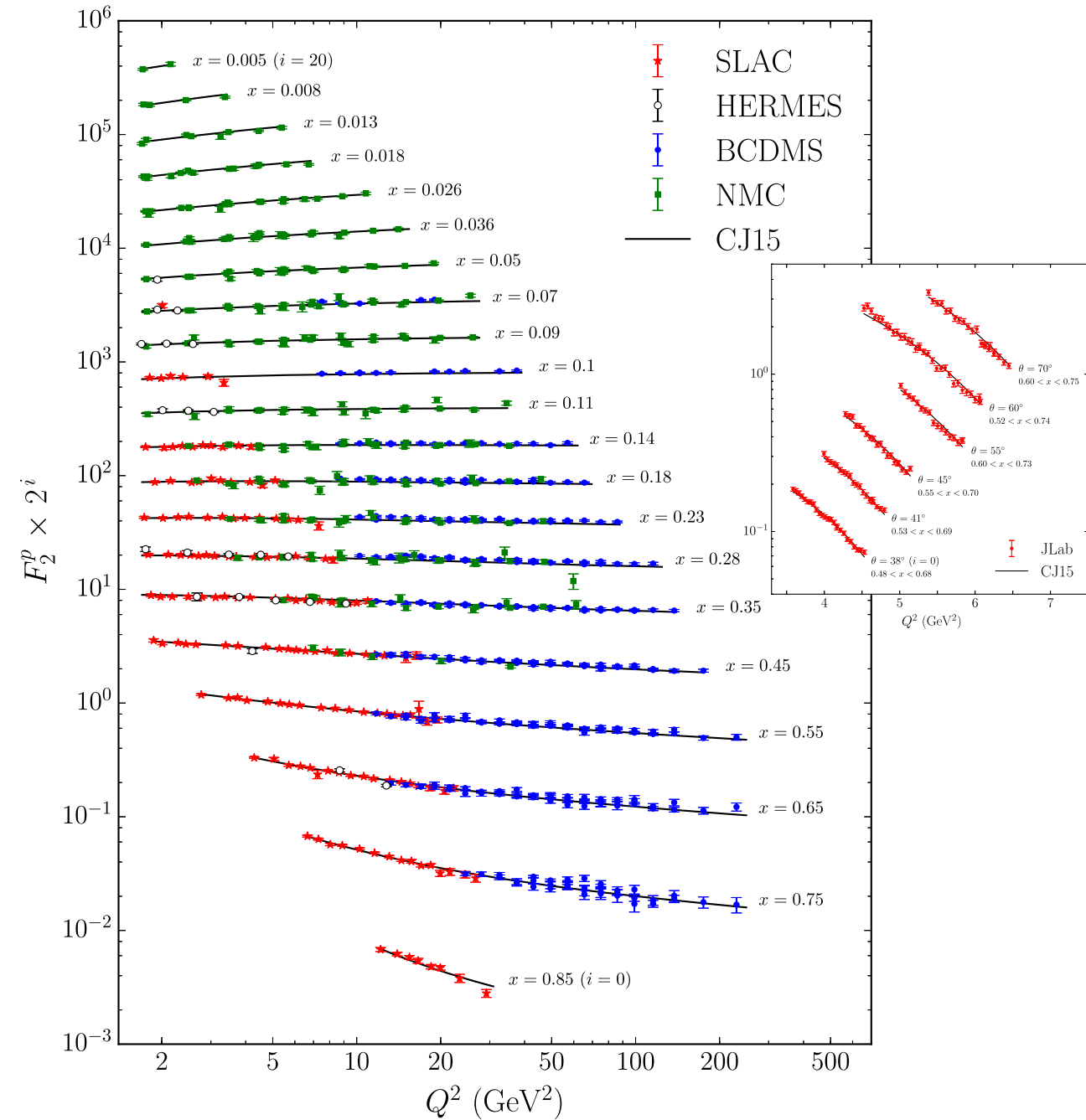


[Figure from DESY-21-099]

Lepton-Hadron Deep Inelastic Scattering

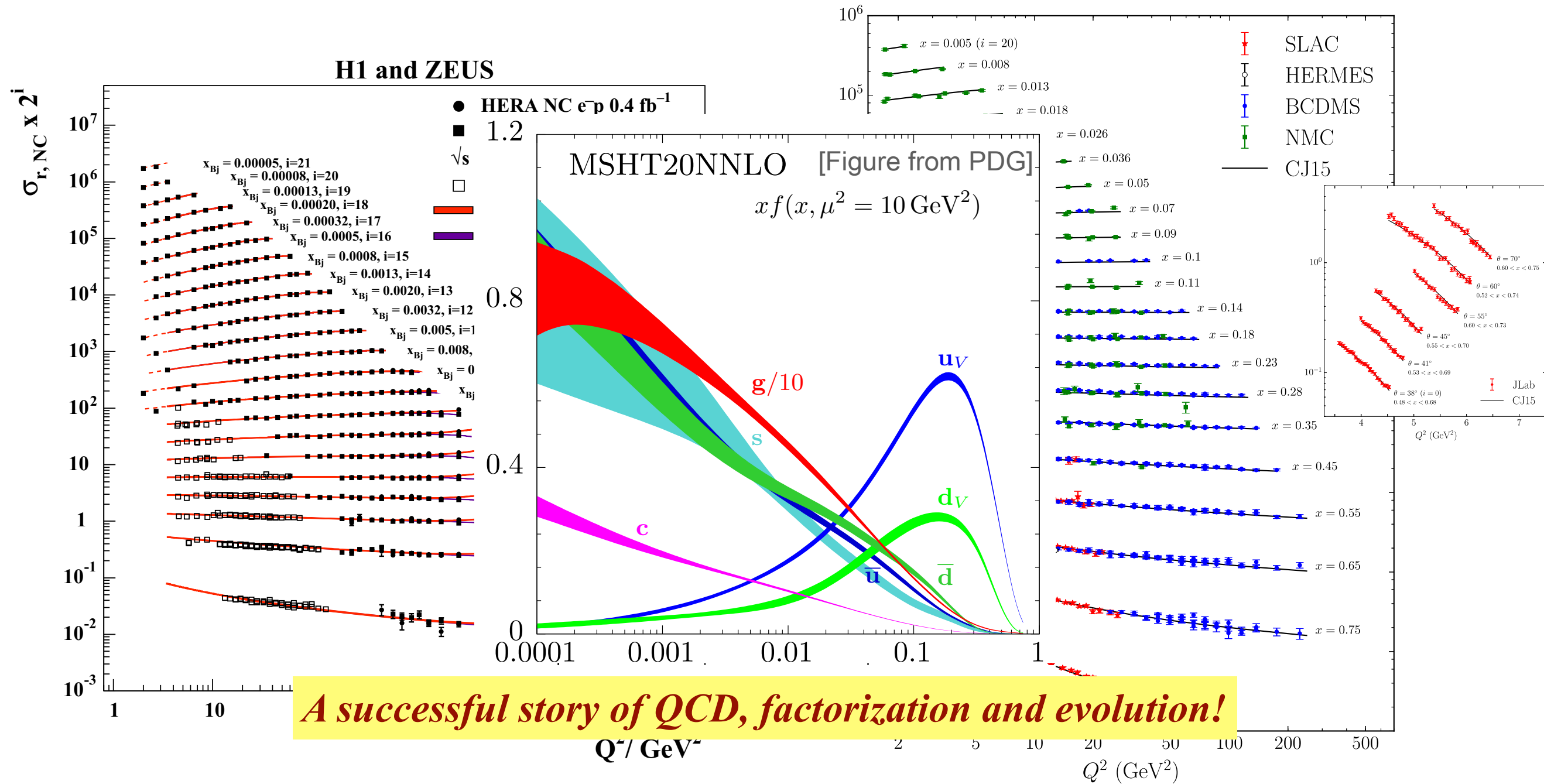


H. Abramowicz *et al.*, EPJC 78, 580 (2015).



A. Accardi *et al.*, PRD 93, 114017 (2016).

Lepton-Hadron Deep Inelastic Scattering



H. Abramowicz *et al.*, EPJC 78, 580 (2015).

A. Accardi *et al.*, PRD 93, 114017 (2016).

Nucleon Spin Structure

Proton spin puzzle

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \sim 0.3$$

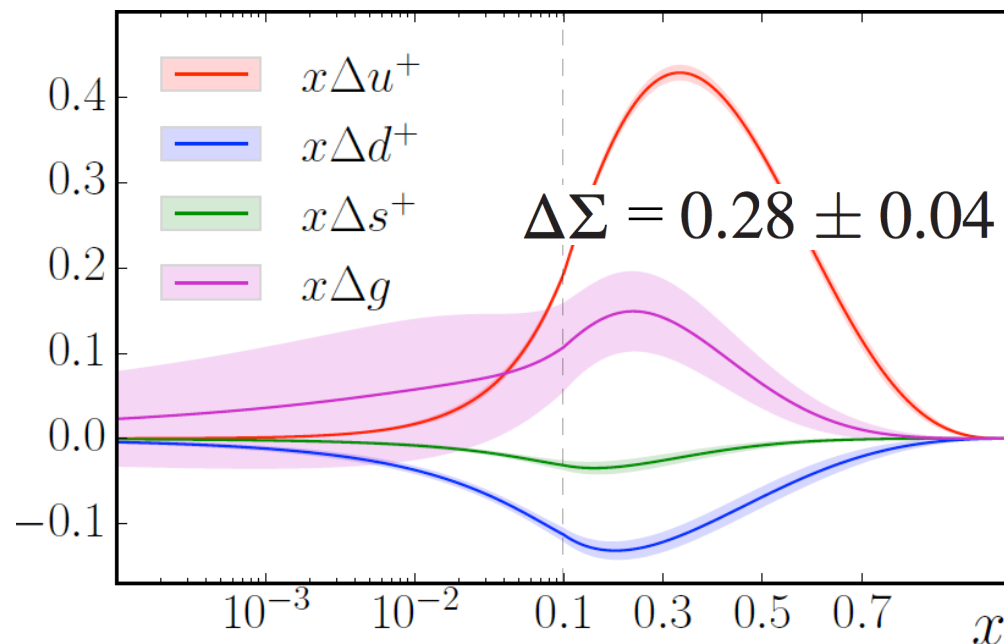
Spin decomposition

$$J = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$= \frac{1}{2}\Delta\Sigma + L_{q,\text{kin}} + J_g$$

R.L. Jaffe, A. Manohar, NPB 337, 509 (1990).

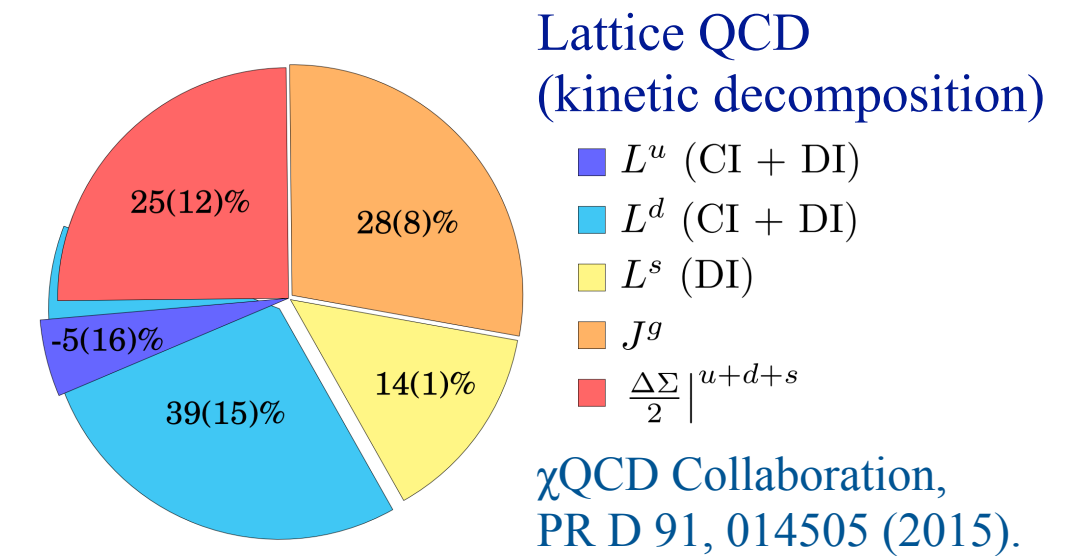
X. Ji, PRL 78, 610 (1997).



JAM Collaboration, PRD 93, 074005 (2016); PRL 119, 132001 (2017).

Quark spin only contributes a small fraction to the nucleon spin.

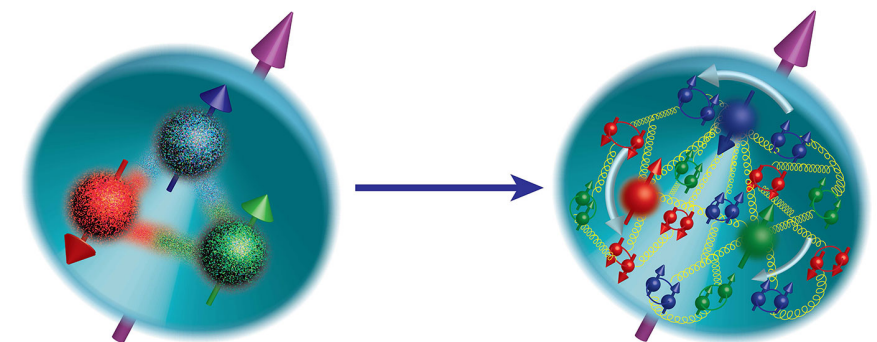
J. Ashman *et al.*, PLB 206, 364 (1988); NP B328, 1 (1989).



Gluon spin from LQCD: $S_g = 0.251(47)(16)$

50% of total proton spin

Y.-B. Yang *et al.* (χ QCD Collaboration), PRL 118, 102001 (2017).



Wigner Rotation Effect

Melosh-Wigner rotation

quark spin in a rest proton $\neq$ quark spin in a moving proton

If applying a kinetic boost, one may relate the spin states in *proton rest frame* to the spin states in *infinite momentum frame*

$$\begin{aligned}\chi_T^\uparrow &= w \left[(k^+ + m) \chi_F^\uparrow - (k^1 + ik^2) \chi_F^\downarrow \right] \\ \chi_T^\downarrow &= w \left[(k^+ + m) \chi_F^\downarrow + (k^1 - ik^2) \chi_F^\uparrow \right]\end{aligned}\quad \begin{aligned}k^+ &= k^0 + k^3 \\ w &= \left[2k^+ (k^0 + m) \right]^{-1/2}\end{aligned}$$

E.P. Wigner, Ann. Math 40 (1939) 149; H.J. Melosh, Phys. Rev. D 9 (1974) 1095.

The effect on quark polarization

$$\Delta q = \int d^3\mathbf{k} \mathcal{M} \left[q^\uparrow(k) - q^\downarrow(k) \right] \quad \mathcal{M} = \frac{(k^+ + m)^2 - k_T^2}{2k^+(k^0 + m)}$$

B.-Q. Ma, J. Phys. G 17 (1991) L53-L58; B.-Q. Ma, Q.-R. Zhang, Z. Phys. C 58 (1993) 479.

It predicts decreasing polarization with k_T , which should be tested by data.

This interpretation is based on a kinetic boost, but a complete boost including QCD dynamics is challenging.

Semi-inclusive Deep Inelastic Scattering

SIDIS process $l + N \rightarrow l' + h + X$

Differential cross section
in terms of 18 structure functions

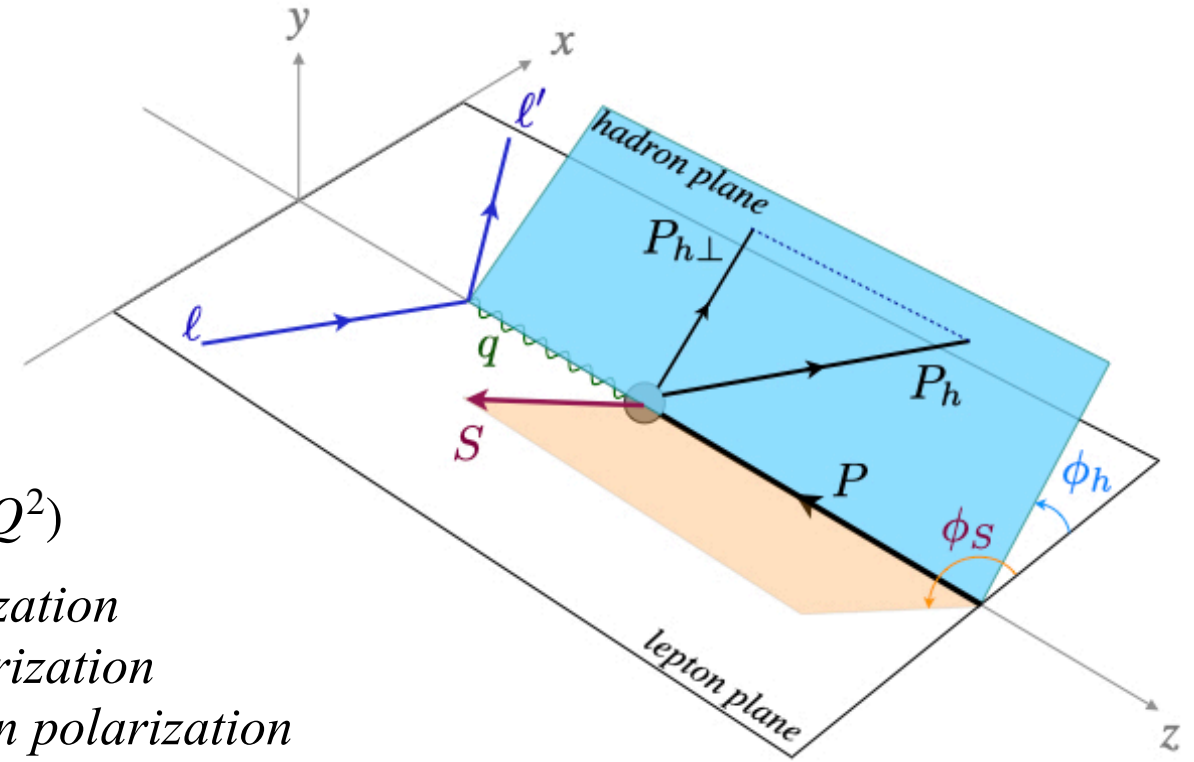
$$\begin{aligned}
 & \frac{d\sigma}{dx_B dy dz dP_{hT}^2 d\phi_h d\phi_S} \\
 &= \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x_B} \right) \\
 & \times \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} F_{UU}^{\cos \phi_h} \cos \phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h + \lambda_e \sqrt{2\epsilon(1-\epsilon)} F_{LU}^{\sin \phi_h} \sin \phi_h \right. \\
 & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} F_{UL}^{\sin \phi_h} \sin \phi_h + \epsilon F_{UL}^{\sin 2\phi_h} \sin 2\phi_h \right] + \lambda_e S_L \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} F_{LL}^{\cos \phi_h} \cos \phi_h \right] \\
 & + S_T \left[\left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \sin(\phi_h - \phi_S) + \epsilon F_{UT}^{\sin(\phi_h + \phi_S)} \sin(\phi_h + \phi_S) \right. \\
 & + \epsilon F_{UT}^{\sin(3\phi_h - \phi_S)} \sin(3\phi_h - \phi_S) + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin \phi_S} \sin \phi_S + \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin(2\phi_h - \phi_S)} \sin(2\phi_h - \phi_S) \Big] \\
 & + \lambda_e S_T \left[\sqrt{1-\epsilon^2} F_{LT}^{\cos(\phi_h - \phi_S)} \cos(\phi_h - \phi_S) \right. \\
 & \left. + \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos \phi_S} \cos \phi_S + \sqrt{2\epsilon(1-\epsilon)} F_{LT}^{\cos(2\phi_h - \phi_S)} \cos(2\phi_h - \phi_S) \right] \Big\}
 \end{aligned}$$

$F_{AB,C}(x_B, z, P_{hT}^2, Q^2)$

A: lepton polarization

B: nucleon polarization

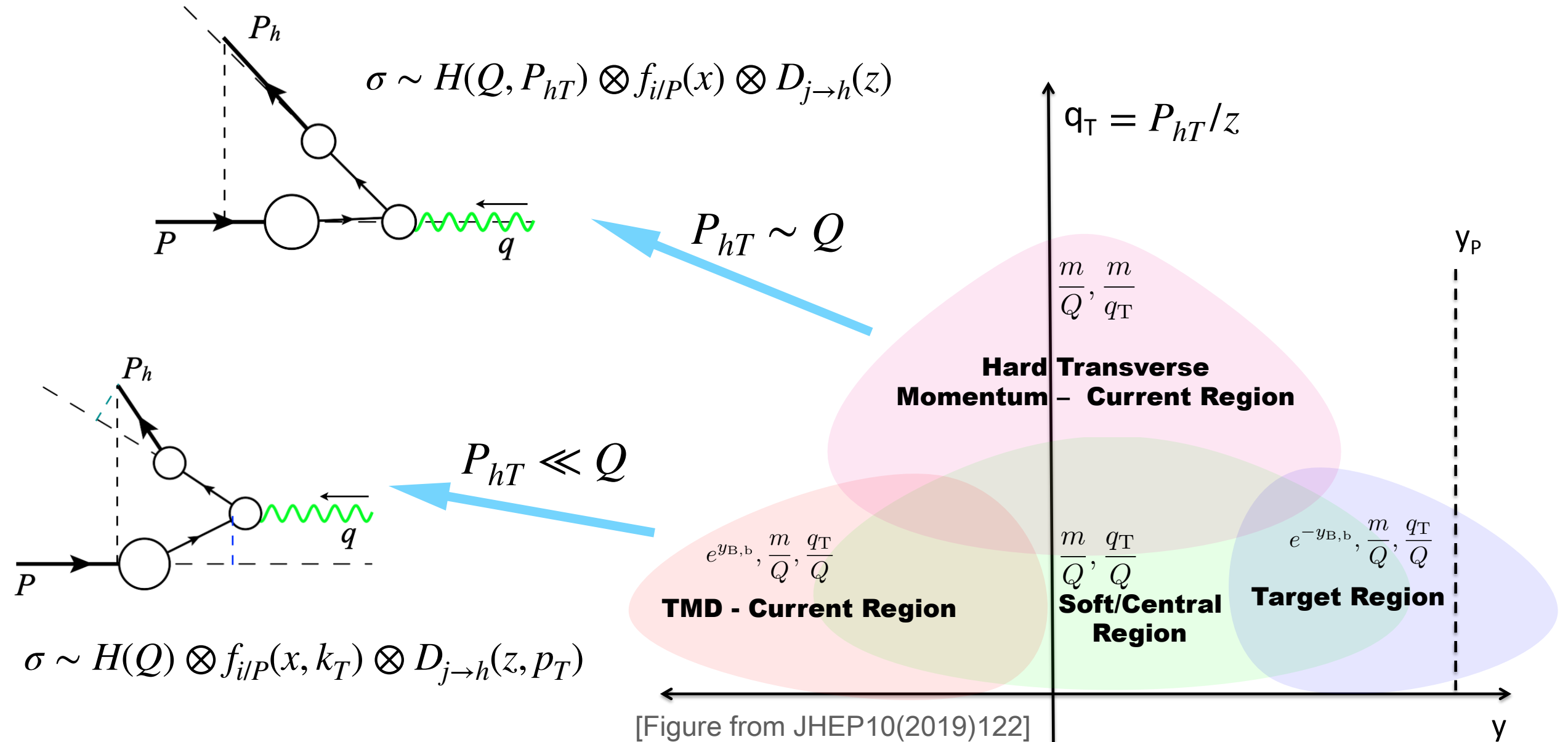
C: virtual photon polarization



$$\begin{aligned}
 x_B &= \frac{Q^2}{2P \cdot q} \\
 y &= \frac{P \cdot q}{P \cdot l} \\
 z &= \frac{P \cdot P_h}{P \cdot q} \\
 \gamma &= \frac{2x_B M}{Q}
 \end{aligned}$$

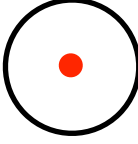
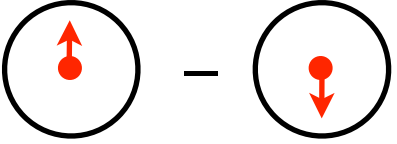
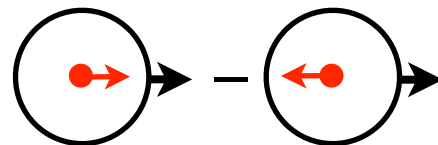
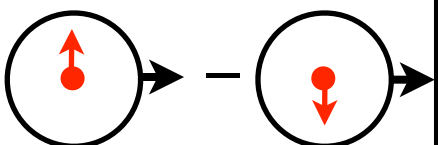
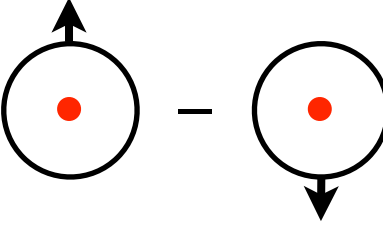
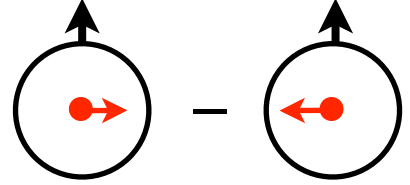
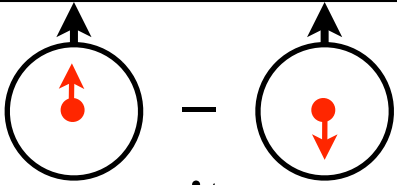
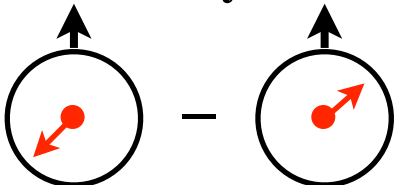
SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron



P_{hT} is defined in the photon-hadron frame

Leading Twist TMDs

		Quark Polarization		
		U	L	T
Nucleon Polarization	U	f_1  unpolarized		$h_1^\perp$  Boer-Mulders
	L		g_{1L}  helicity	$h_{1L}^\perp$  longi-transversity (worm-gear)
	T	$f_{1T}^\perp$  Sivers	g_{1T}  trans-helicity (worm-gear)	h_1  transversity $h_{1T}^\perp$  pretzelosity

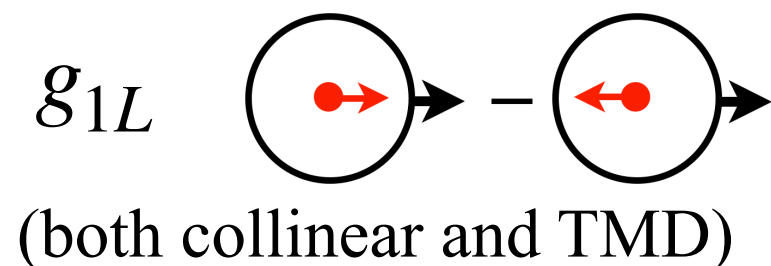
Longitudinal Double Spin Asymmetry

Longitudinal DSA in SIDIS

$$A_{LL} \equiv \frac{\sigma_{++} - \sigma_{+-} + \sigma_{--} - \sigma_{-+}}{\sigma_{++} + \sigma_{+-} + \sigma_{--} + \sigma_{-+}} = \frac{\sqrt{1 - \varepsilon^2} F_{LL}(x, z, P_{hT}^2, Q^2)}{F_{UU}(x, z, P_{hT}^2, Q^2)}$$

In TMD region: $F_{LL}(x, z, P_{hT}^2, Q^2) \sim g_{1L}(x, k_T^2) \otimes D_1(z, p_T^2)$

$$F_{UU}(x, z, P_{hT}^2, Q^2) \sim f_1(x, k_T^2) \otimes D_1(z, p_T^2)$$



Several global analyses of collinear helicity
but TMD helicity was missing for a long time!

P_{hT} dependent DSA measurements

HERMES: proton (H_2) and deuteron (D_2) targets

HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

JLab CLAS: proton (NH_3) target

CLAS Collaboration, Phys. Lett. B 782 (2018) 662.

TMD Evolution

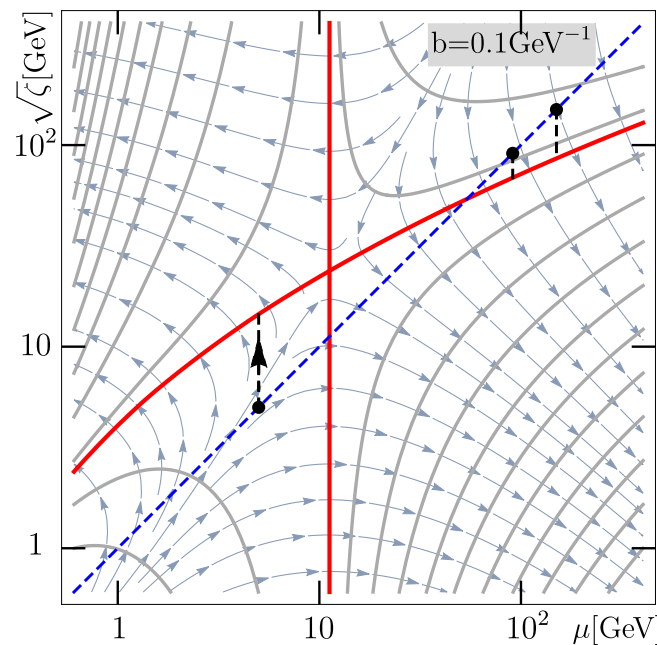
Evolution equations

$$\begin{aligned} \mu^2 \frac{dF(x, b; \mu^2, \zeta)}{d\mu^2} &= \frac{\gamma_F(\mu, \zeta)}{2} F(x, b; \mu^2, \zeta) & -\zeta \frac{d\gamma_F(\mu, \zeta)}{d\zeta} &= \mu \frac{d\mathcal{D}(\mu, b)}{d\mu} = \Gamma_{\text{cusp}}(\mu) \\ \zeta \frac{dF(x, b; \mu^2, \zeta)}{d\zeta} &= -\mathcal{D}(\mu, b) F(x, b; \mu^2, \zeta) & \gamma_F(\mu, \zeta) &= \Gamma_{\text{cusp}}(\mu) \ln \frac{\mu^2}{\zeta} - \gamma_V(\mu) \end{aligned}$$

➔

$$F(x, b; \mu_f, \zeta_f) = \exp \left[\int_P \left(\gamma_F(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}(\mu, b) \frac{d\zeta}{\zeta} \right) \right] F(x, b; \mu_i, \zeta_i)$$

ζ -prescription



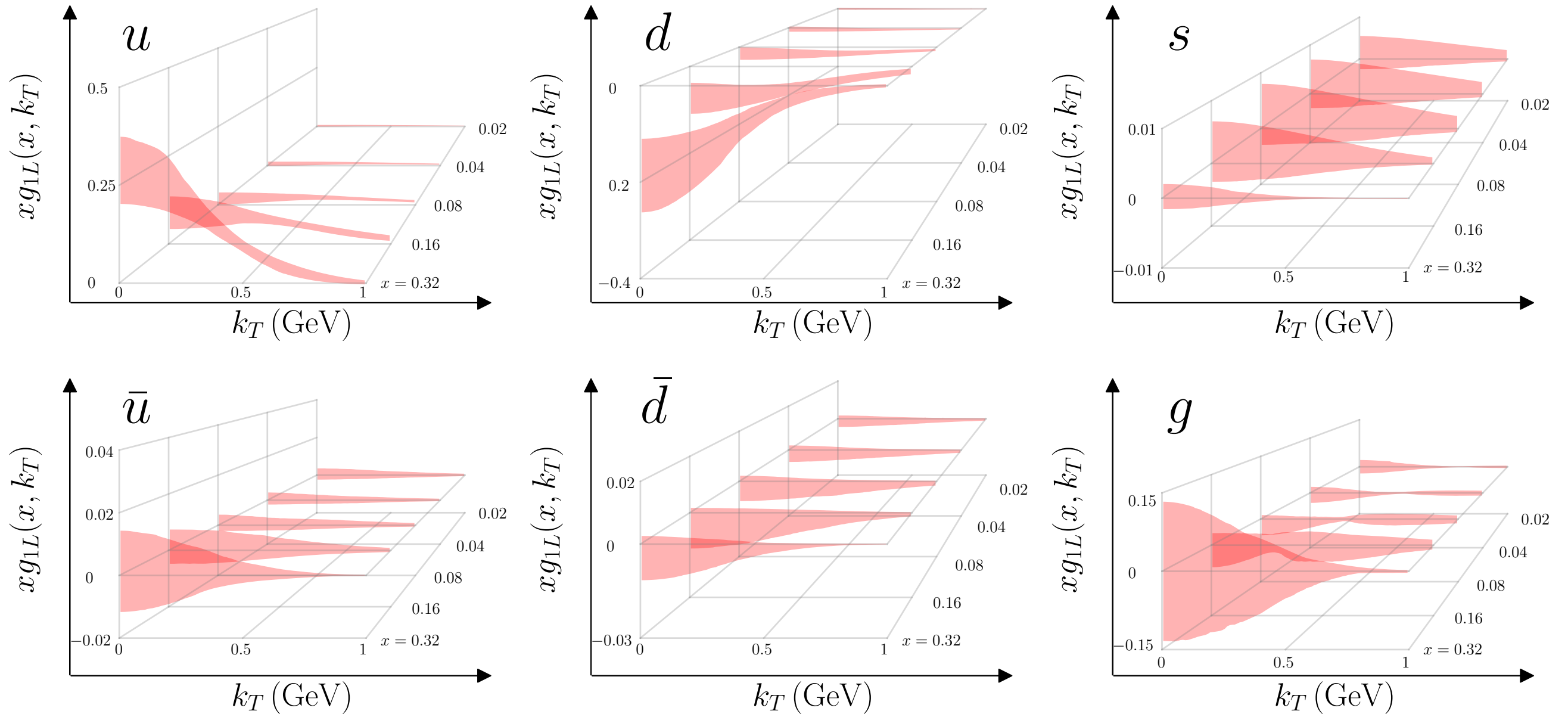
equipotential lines:

$$\begin{aligned} \frac{d \ln \zeta_\mu(\mu, b)}{d \ln \mu^2} &= \frac{\gamma_F(\mu, \zeta_\mu(\mu, b))}{2\mathcal{D}(\mu, b)} \\ \mathcal{D}(\mu_0, b) &= 0, \quad \gamma_F(\mu_0, \zeta_\mu(\mu_0, b)) = 0 \\ F(x, b; Q, Q^2) &= \left(\frac{Q^2}{\zeta_Q(b)} \right)^{-\mathcal{D}(Q, b)} F(x, b), \quad \mu_f^2 = \zeta_f = Q^2 \end{aligned}$$

I. Scimemi, A. Vladimirov, JHEP 06 (2020) 137.

First Extraction of TMD Helicity

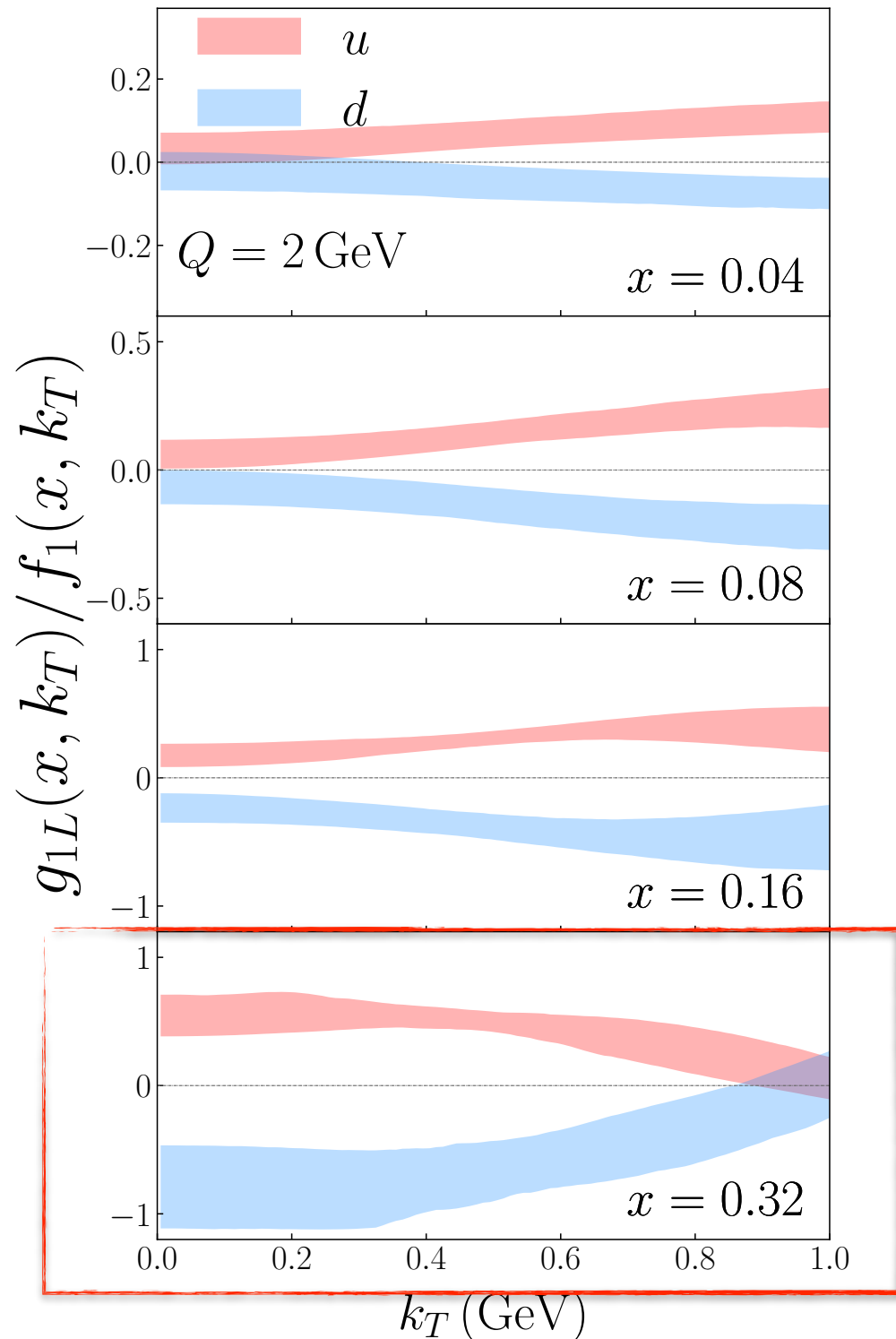
NLO+NNLL analysis results



Nonzero signals for u and d quarks, while sea quarks and gluons are loosely constrained.

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Transverse Momentum Dependent Polarization



$g_{1L}(x, k_T^2)$ gives the absolute number density difference between spin-parallel and spin-antiparallel quarks.

The ratio $g_{1L}(x, k_T^2)/f_1(x, k_T^2)$ measures the polarization rate of quarks.

- At large x , where valence components dominate, the polarization decreases with increasing k_T
Qualitatively consistent with kinetic Wigner rotation effects
- At low x , where the valence component is no longer adequate, distributions are highly driven by complex QCD dynamics
The polarization is found increasing with k_T

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma,
Phys. Rev. Lett. 134 (2025) 121902.

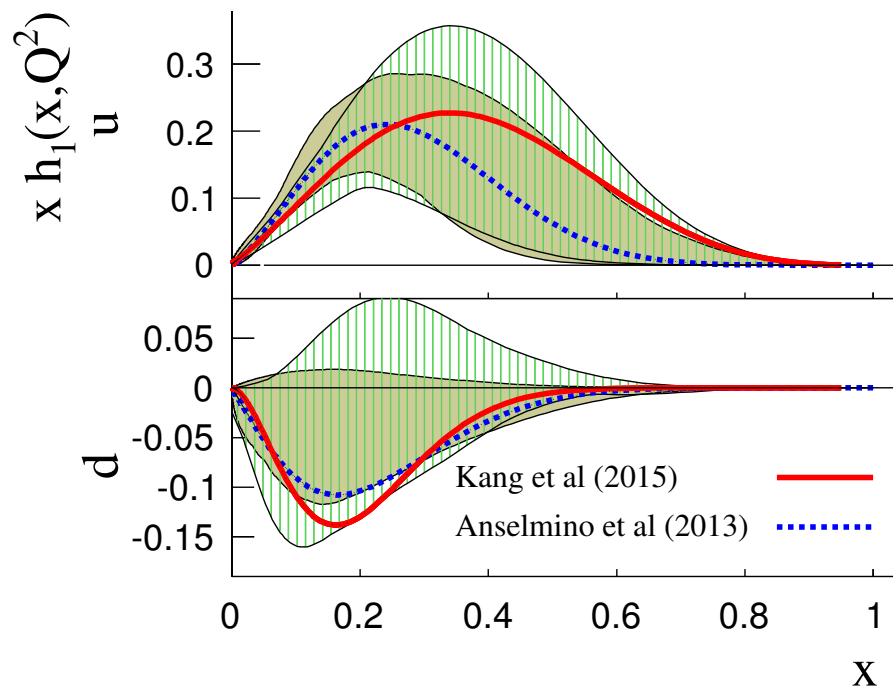
Transversity Distribution

Transversity distribution

$$h_1 \quad \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} - \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} \quad (\text{Collinear \& TMD})$$

A transverse counter part to the longitudinal spin structure: helicity g_{1L} , but NOT the same.

Phenomenological extractions



Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, PRD 93, 014009 (2016).

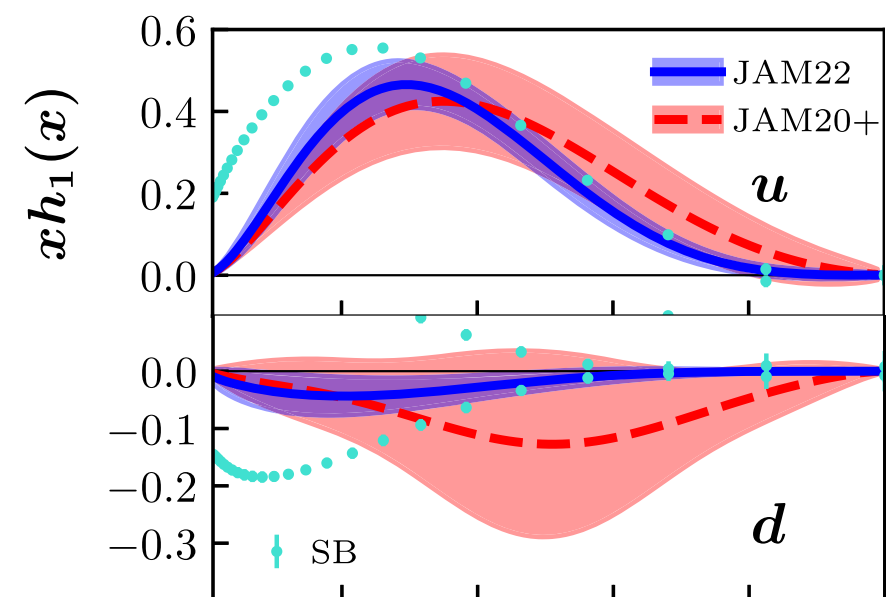
Chiral-odd:

No mixing with gluons
Valence dominant
Couple to another chiral-odd function.

Effect in SIDIS:

transverse single spin asymmetry
(Collins asymmetry)

$$A_{UT}^{\sin(\phi_h + \phi_s)} \sim h_1(x, k_T^2) \otimes H_1^\perp(z, p_T^2)$$

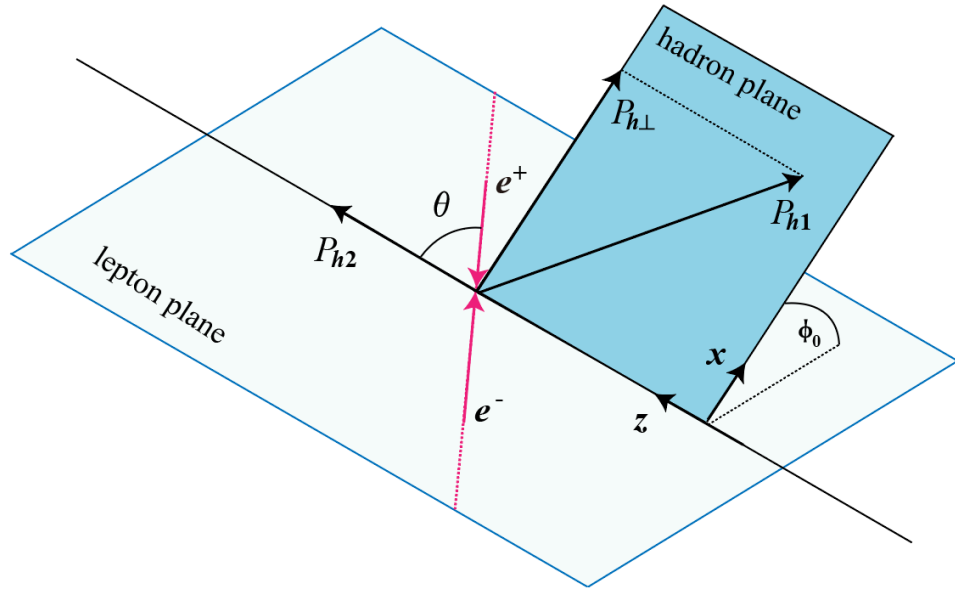


JAM Collaboration, PRD 104, 034014 (2022).

Assuming vanishing transverse polarization of sea quarks!

Complementary Process

Semi-inclusive e^+e^- annihilation: $e^+e^- \rightarrow h_1 h_2 X$



$$\frac{d^5\sigma}{dz_1 dz_2 d^2\mathbf{P}_{h\perp} d\cos\theta} = \frac{3\pi\alpha^2}{2Q^2} z_1^2 z_2^2 \left[(1 + \cos^2\theta) F_{UU}^{h_1 h_2} + \sin^2\theta \cos(2\phi_0) F_{\text{Collins}}^{h_1 h_2} \right]$$

In TMD region: h_1 and h_2 are near back-to-back, $P_{hT} \ll Q$

$$F_{\text{Collins}}^{h_1 h_2} \sim H_1^{\perp h_1} \otimes H_1^{\perp h_2}$$

Experimental measurements:

Belle: $\sqrt{s} = 10.58 \text{ GeV}$

BaBar: $\sqrt{s} = 10.6 \text{ GeV}$

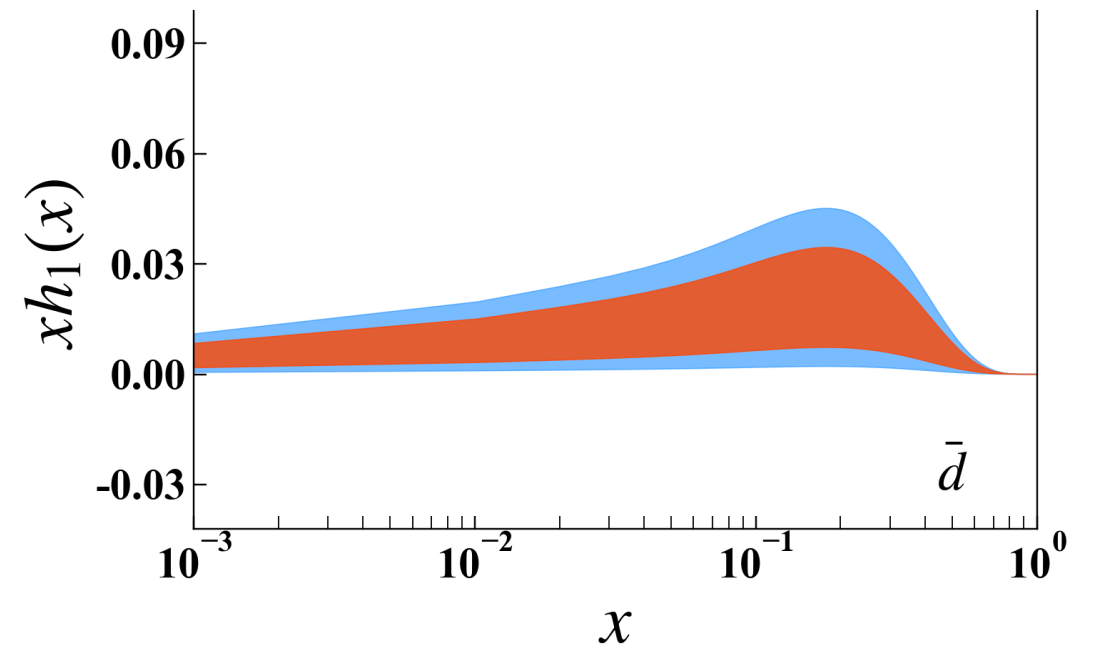
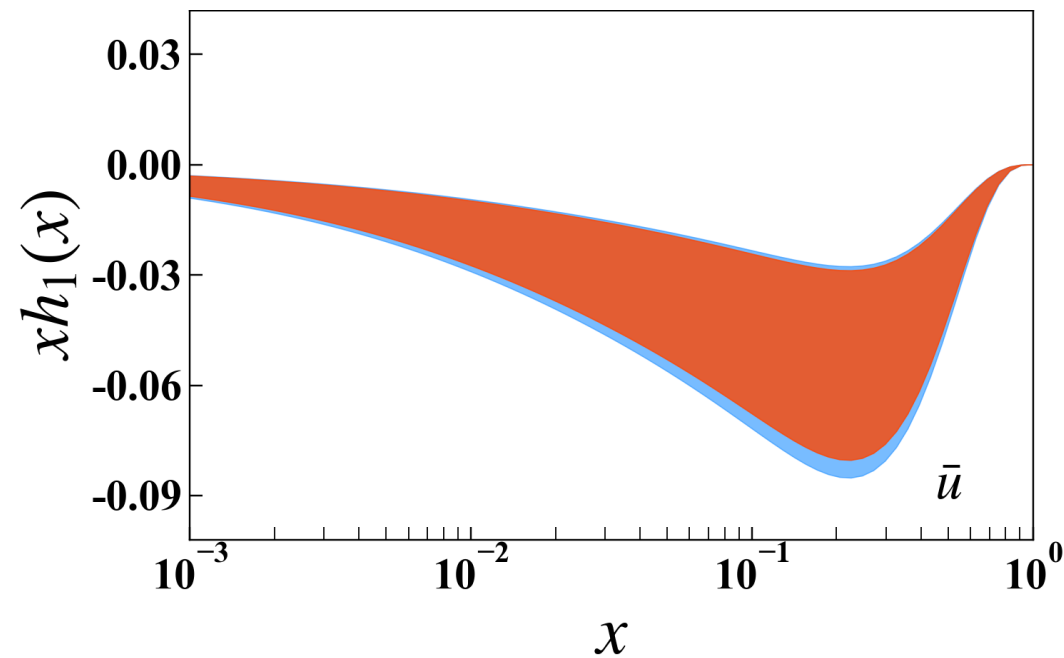
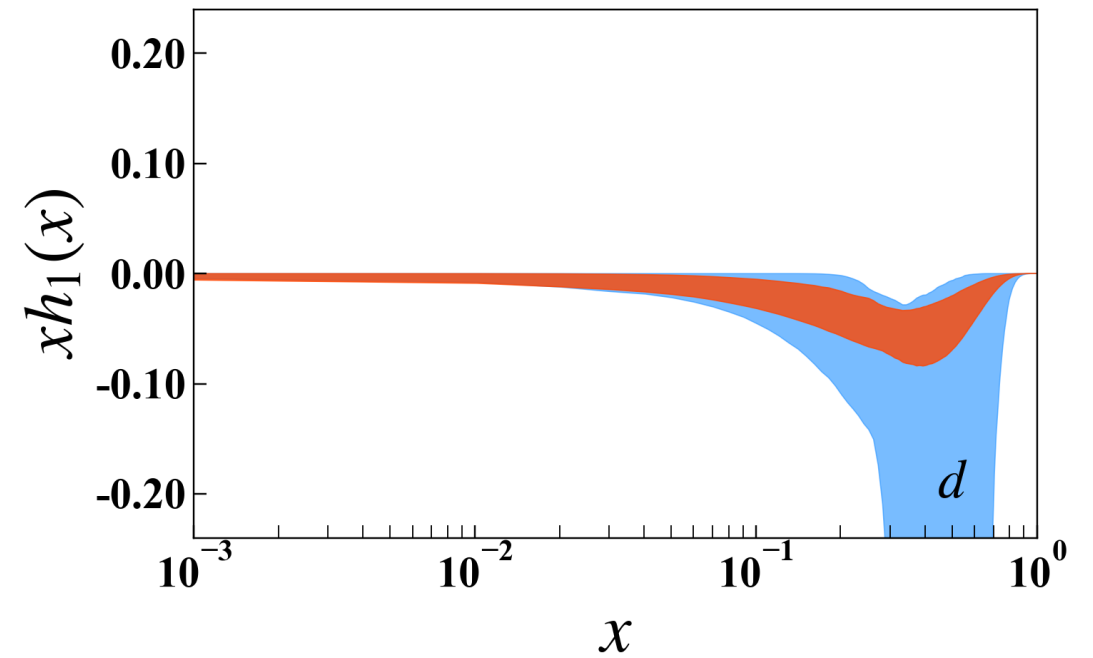
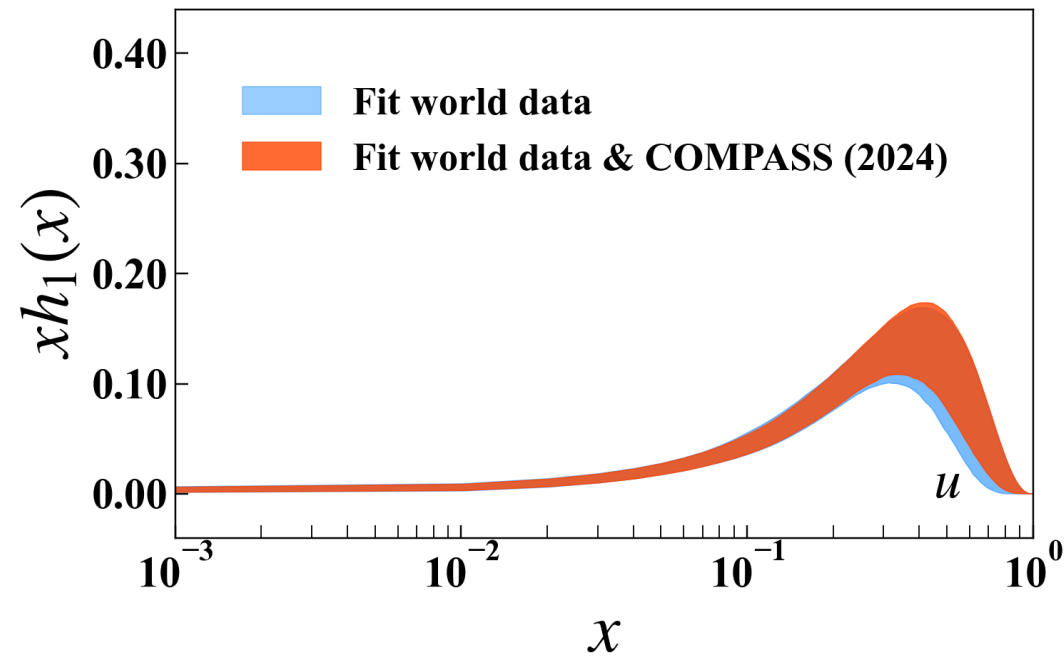
BESIII: $\sqrt{s} = 3.68 \text{ GeV}$

Phys. Rev. D 78 (2008) 032011; 86 (2012) 039905(E).

Phys. Rev. D 90 (2014) 052003; Phys. Rev. D 92 (2015) 111101.

Phys. Rev. Lett. 116 (2016) 042001.

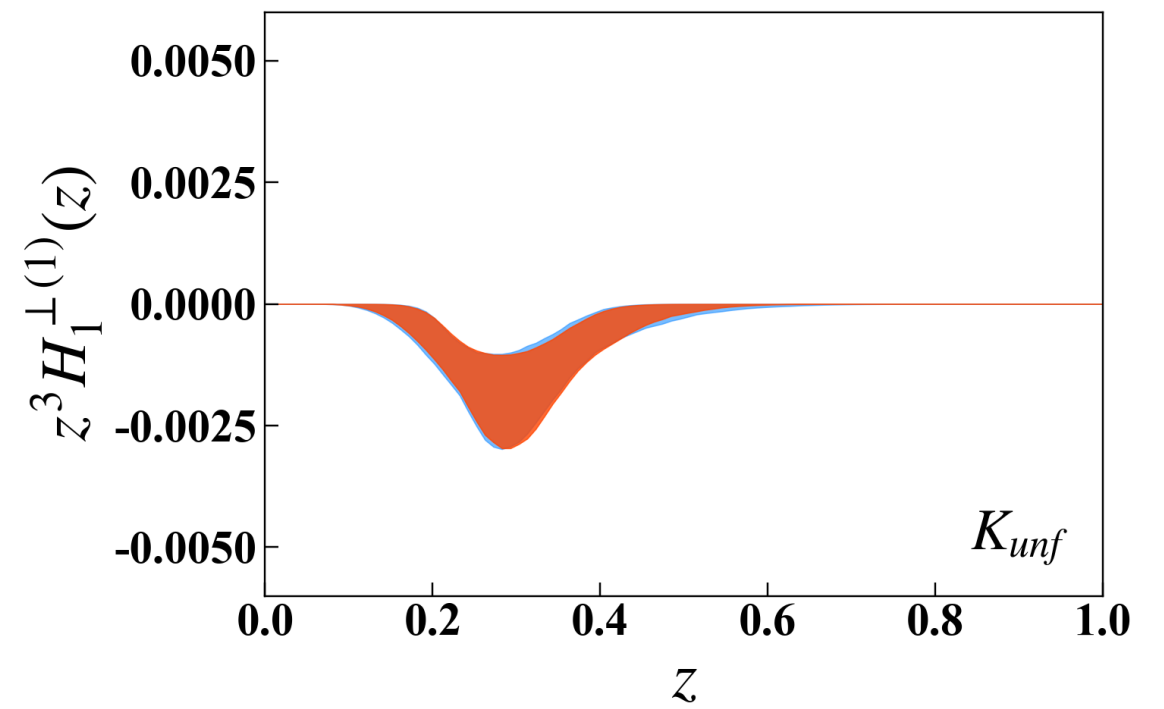
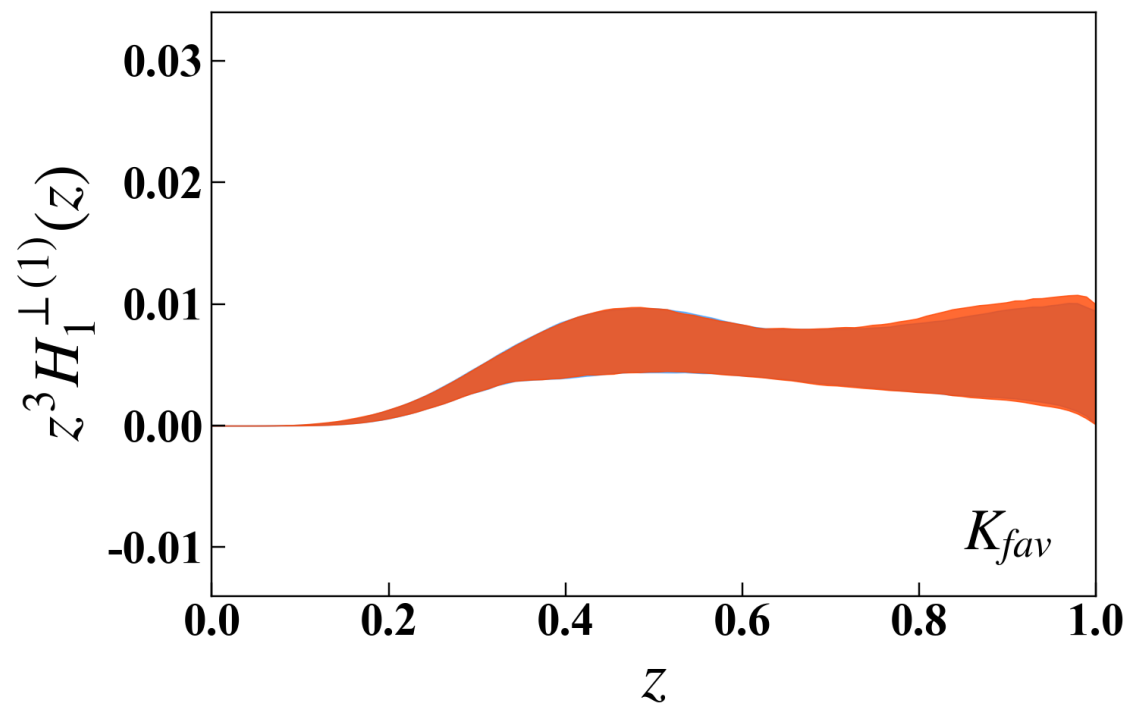
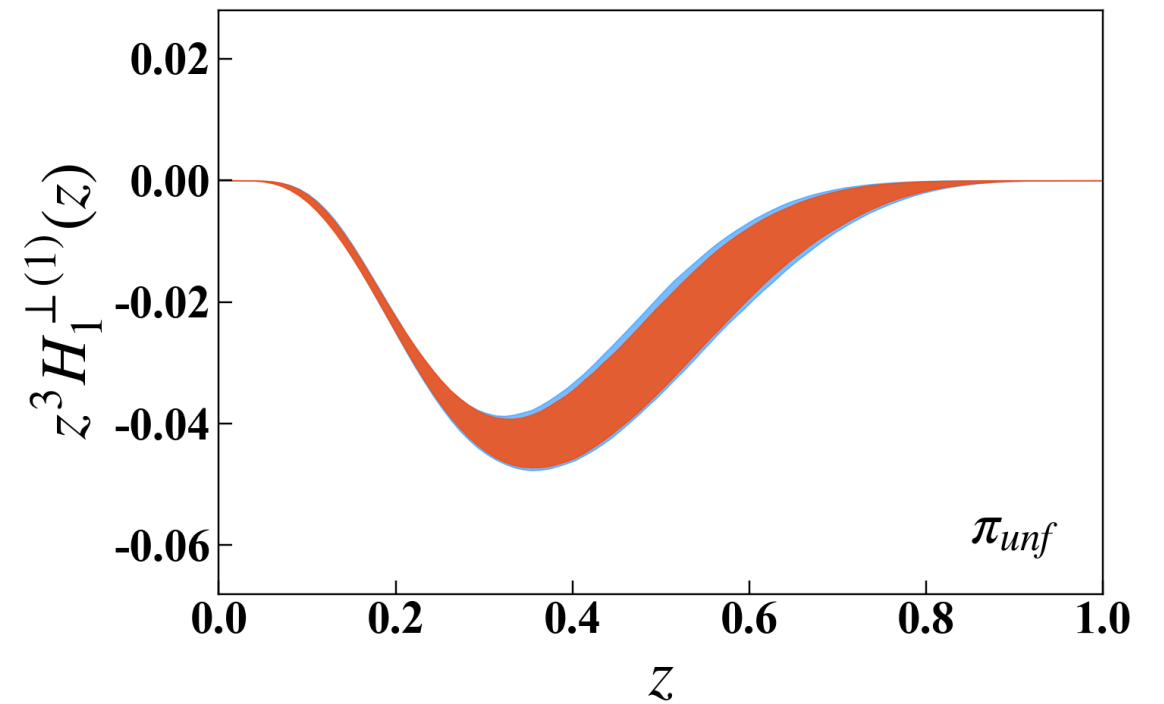
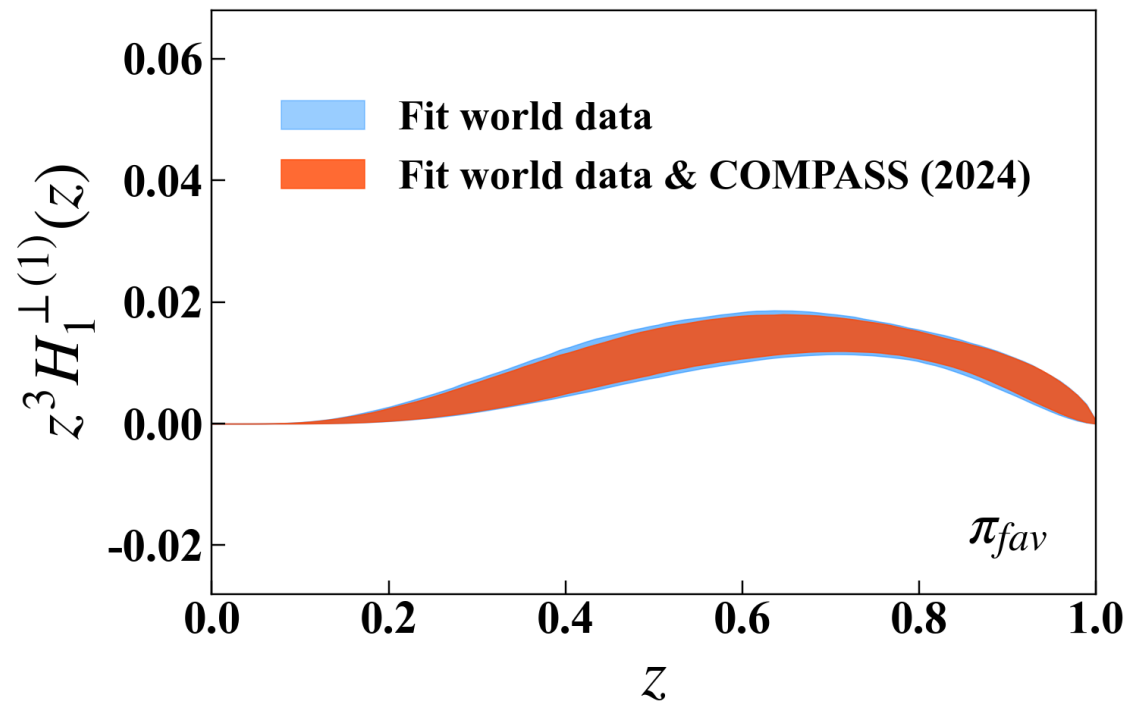
Transversity Distributions



Find $\bar{u}$ favors *negative* transverse polarization

C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, PRD 109 (2024) 056002; arXiv:2412.18324.

Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Collinear Transversity Distributions

Dihadron SIDIS

$$l + N \rightarrow l' + h_1 h_2 + X$$

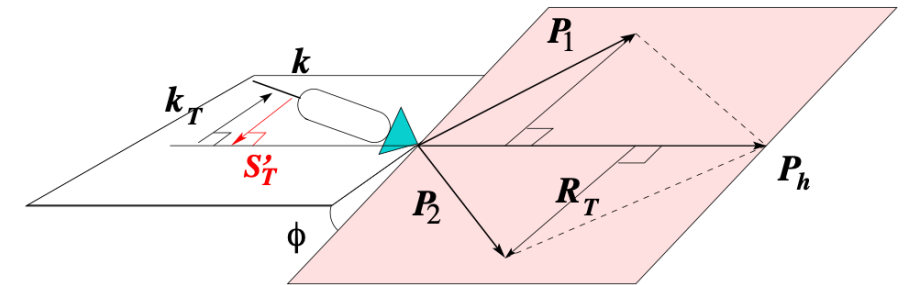
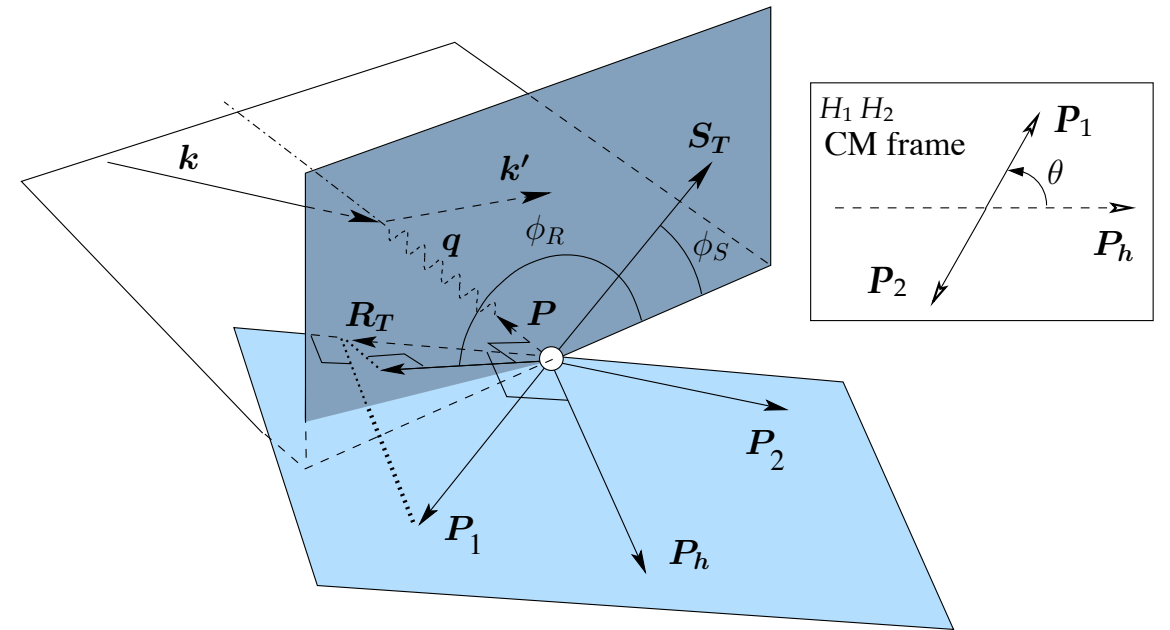
$$\frac{d\sigma}{dx_B dy d\phi_S dz d\phi_R dM_h^2 d\cos\theta} = \frac{\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ F_{UU} + S_T \varepsilon \sin(\phi_R + \phi_S) F_{UT}^{\sin(\phi_R + \phi_S)} \right\}$$

$$F_{UT}^{\sin(\phi_R + \phi_S)} \sim h_1 \otimes H_1^\triangleleft$$

h_1 : collinear transversity distribution function

$H_1^\triangleleft$: a chiral-odd dihadron fragmentation function

describing the correlation between quark transverse polarization and the azimuthal orientation of the hadron pair plane



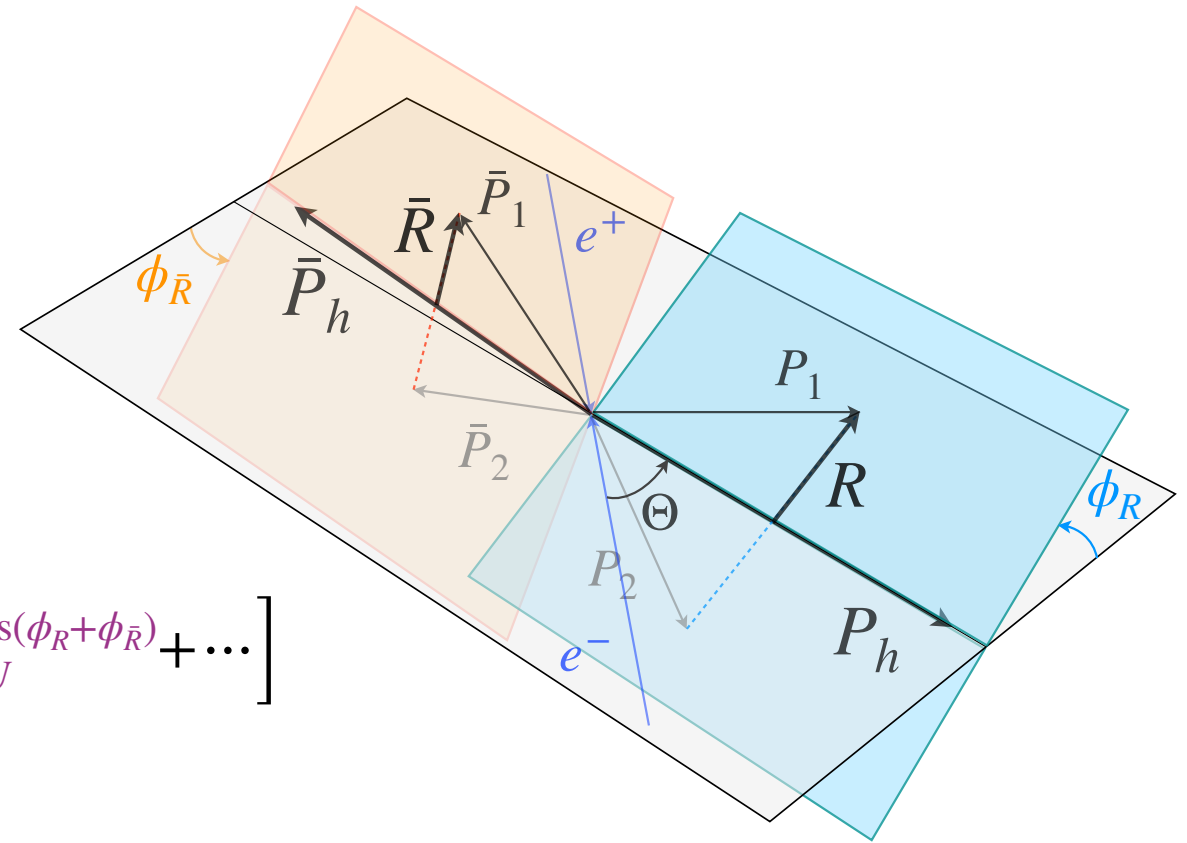
M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. D 65 (2002) 074031;
A. Bacchetta, A. Courtoy, M. Radici, JHEP 03 (2013) 119.

Complimentary Processes

Artru-Collins asymmetry in e^+e^- annihilation

$$e^+ + e^- \rightarrow h_1 h_2 + h'_1 h'_2 + X$$

$$\frac{d\sigma}{dz d\bar{z} dM_h^2 d\bar{M}_h^2 d\cos\Theta d\cos\theta d\cos\bar{\theta} d\phi_R d\phi_{\bar{R}}} = \frac{3\alpha^2}{8\pi^2 Q^2} \left[(1 + \cos^2\Theta) F_{UU}^1 + \sin^2\Theta \cos(\phi_R + \phi_{\bar{R}}) F_{UU}^{\cos(\phi_R + \phi_{\bar{R}})} + \dots \right]$$



$$F_{UU}^1 \sim D_1^{q \rightarrow h_1 h_2} \otimes D_1^{\bar{q} \rightarrow h'_1 h'_2}$$

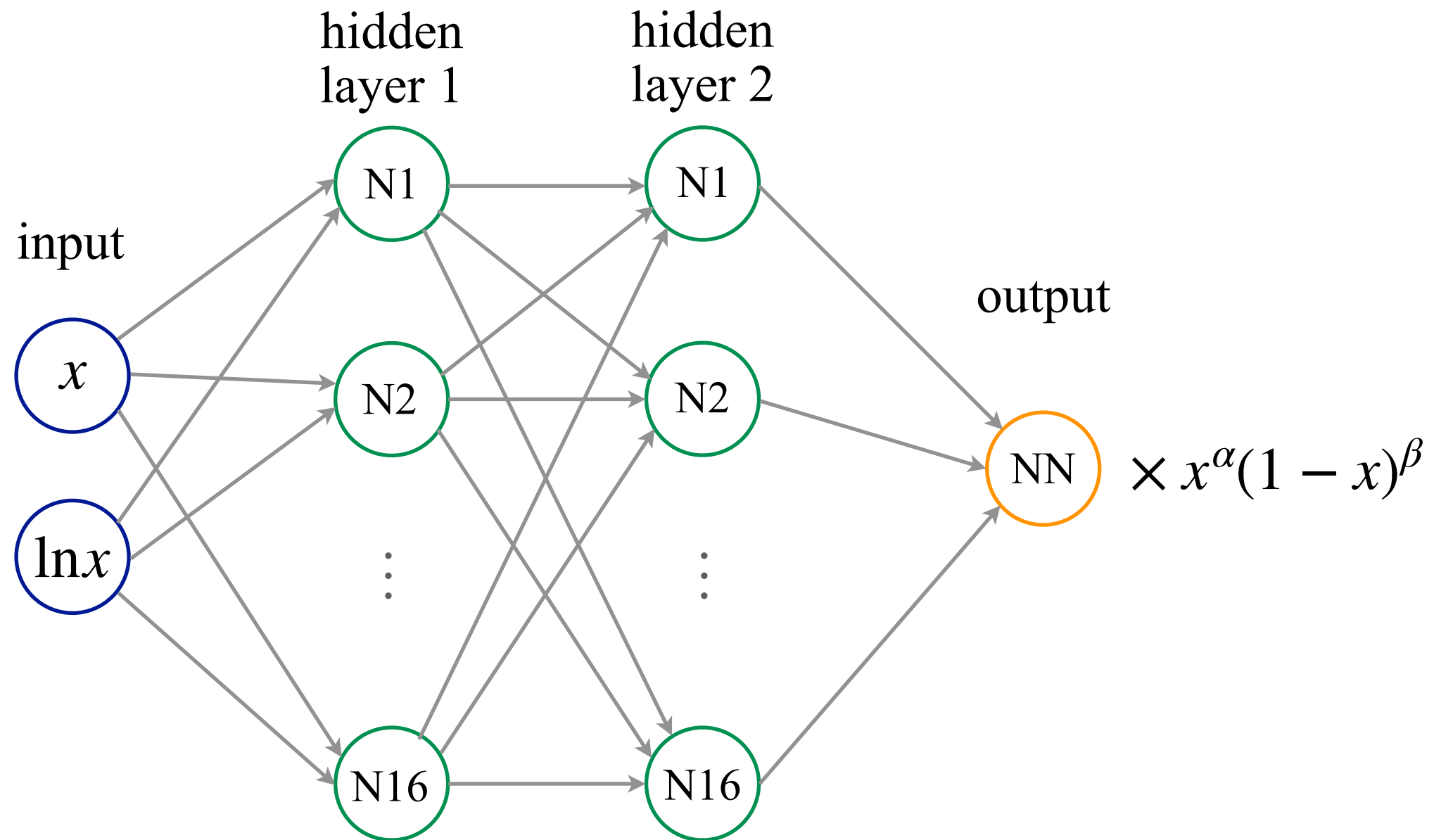
$$F_{UU}^{\cos(\phi_R + \phi_{\bar{R}})} \sim H_1^{\leftarrow q \rightarrow h_1 h_2} \otimes H_1^{\leftarrow \bar{q} \rightarrow h'_1 h'_2}$$

X. Artru, J.C. Collins, Z. Phys. C 69 (1996) 277.

D. Boer, R. Jakob, M. Radici, Phys. Rev. D 67 (2003) 094003.

NN Parametrization

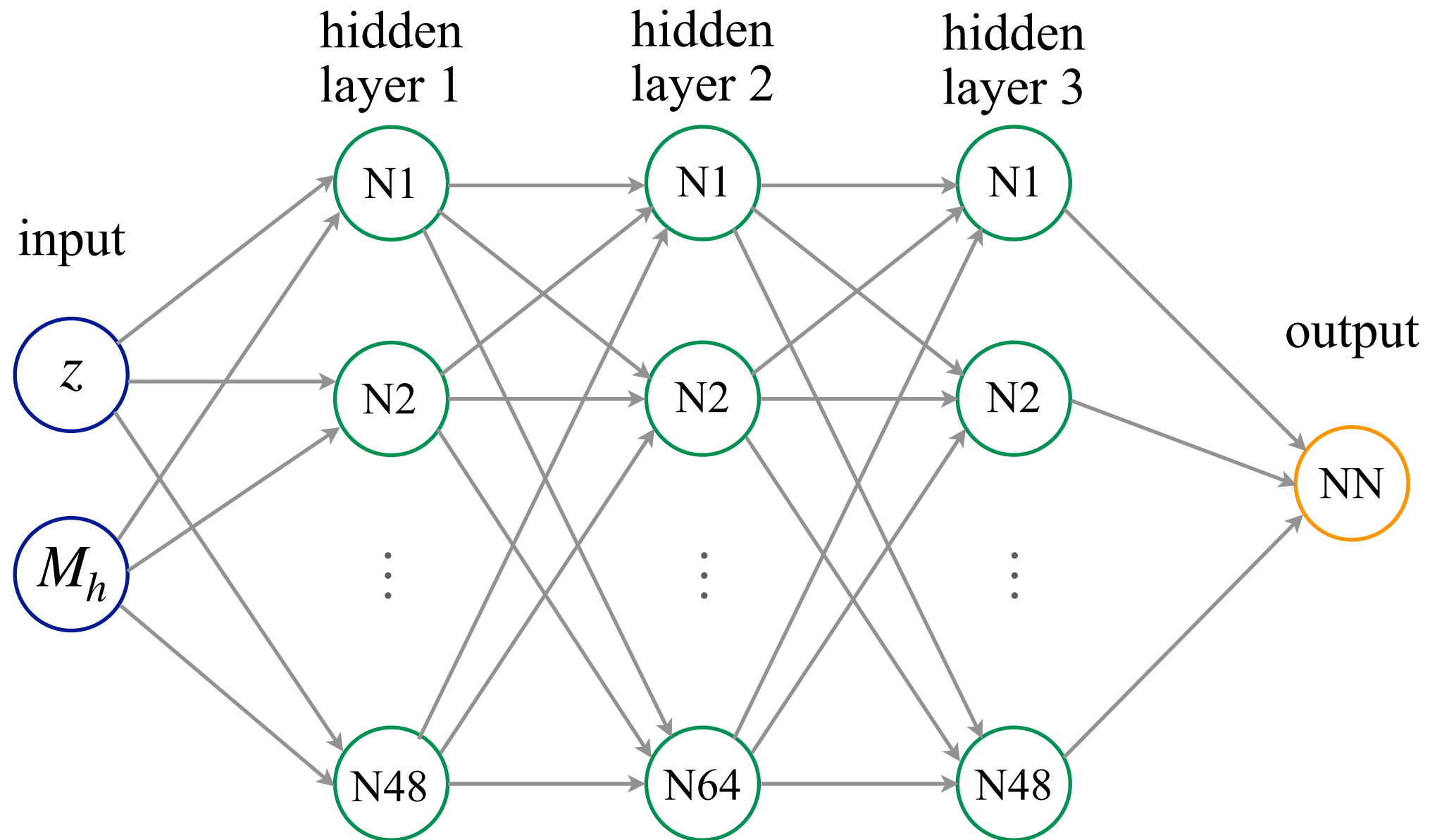
Parametrization of transversity distributions h_1



tested with various architectures: number of layers, number of neurons, activation functions, optimization algorithms

NN Parametrization

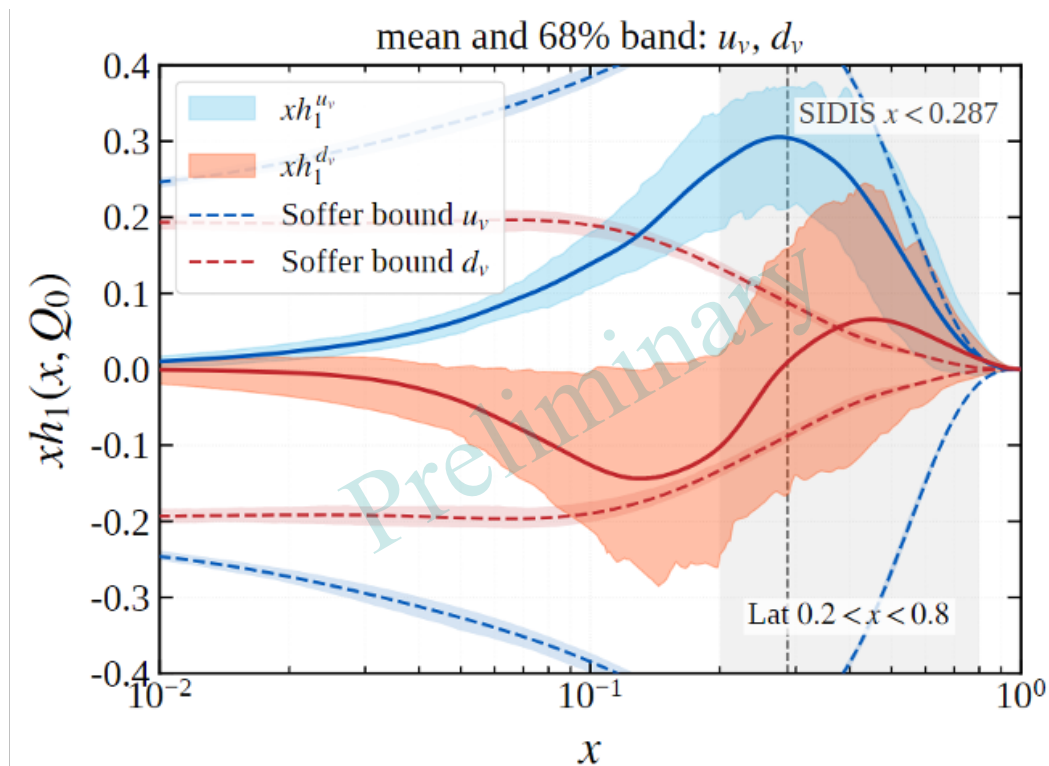
Parametrization of dihadron fragmentation functions D_1 and $H_1^{\perp}$



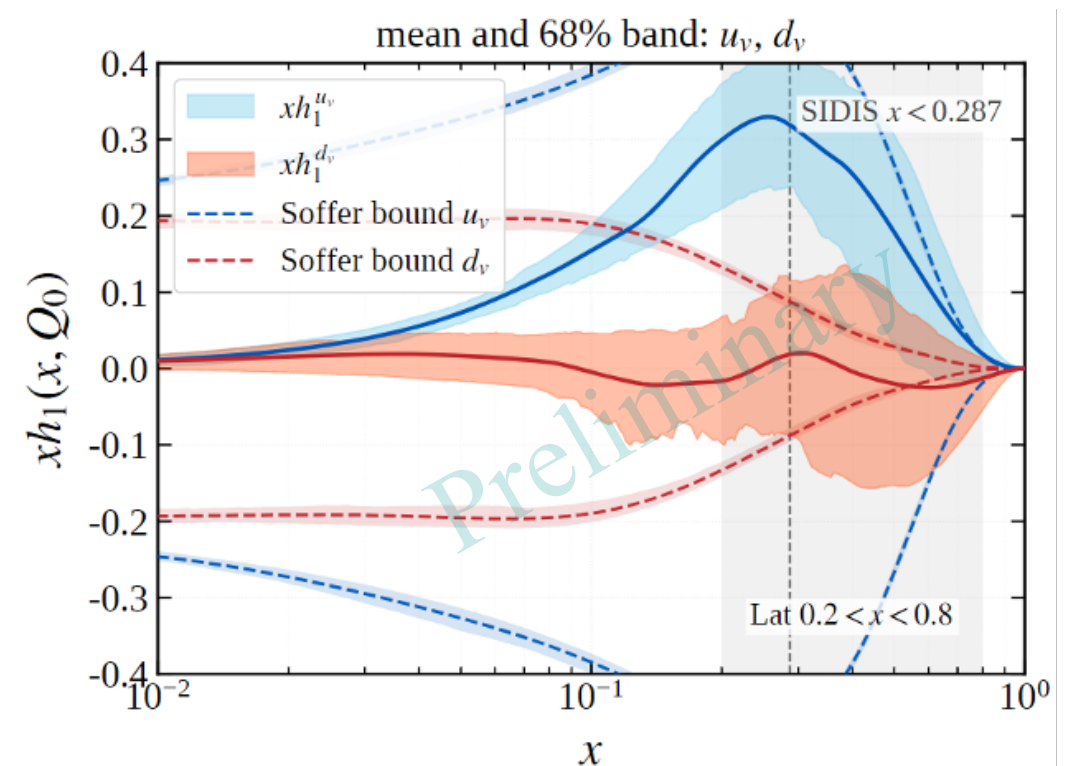
tested with various architectures: number of layers, number of neurons, activation functions, optimization algorithms

A Combined Analysis of Transversity

SIDIS + e^+e^- data

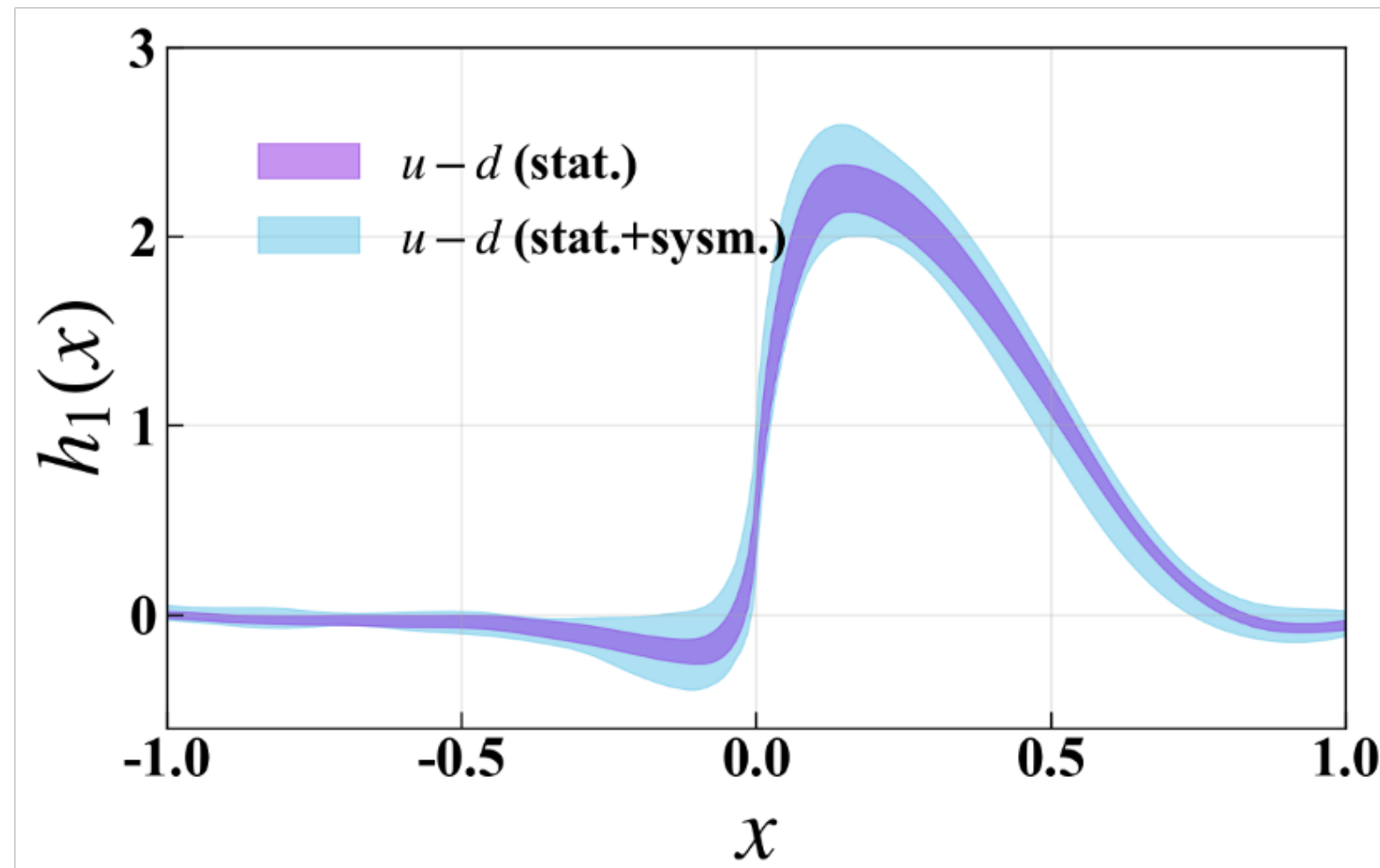


SIDIS + $e^+e^- + p^\uparrow p$ data



H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

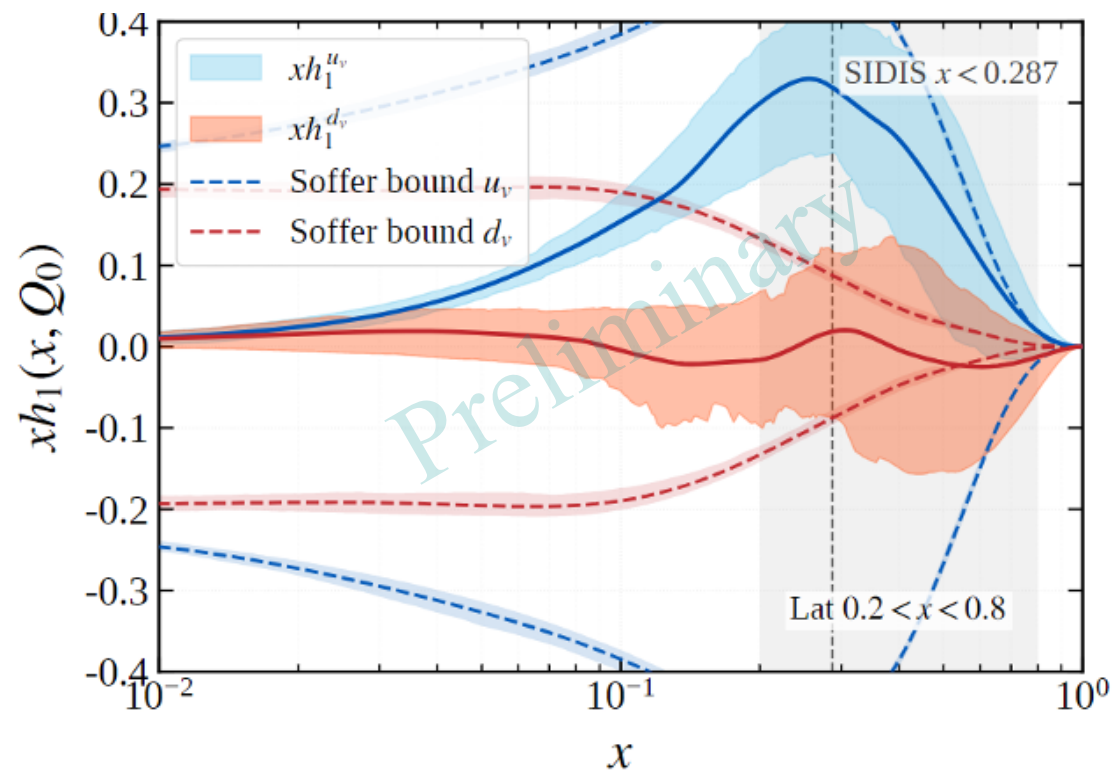
Transversity Distribution from Lattice QCD



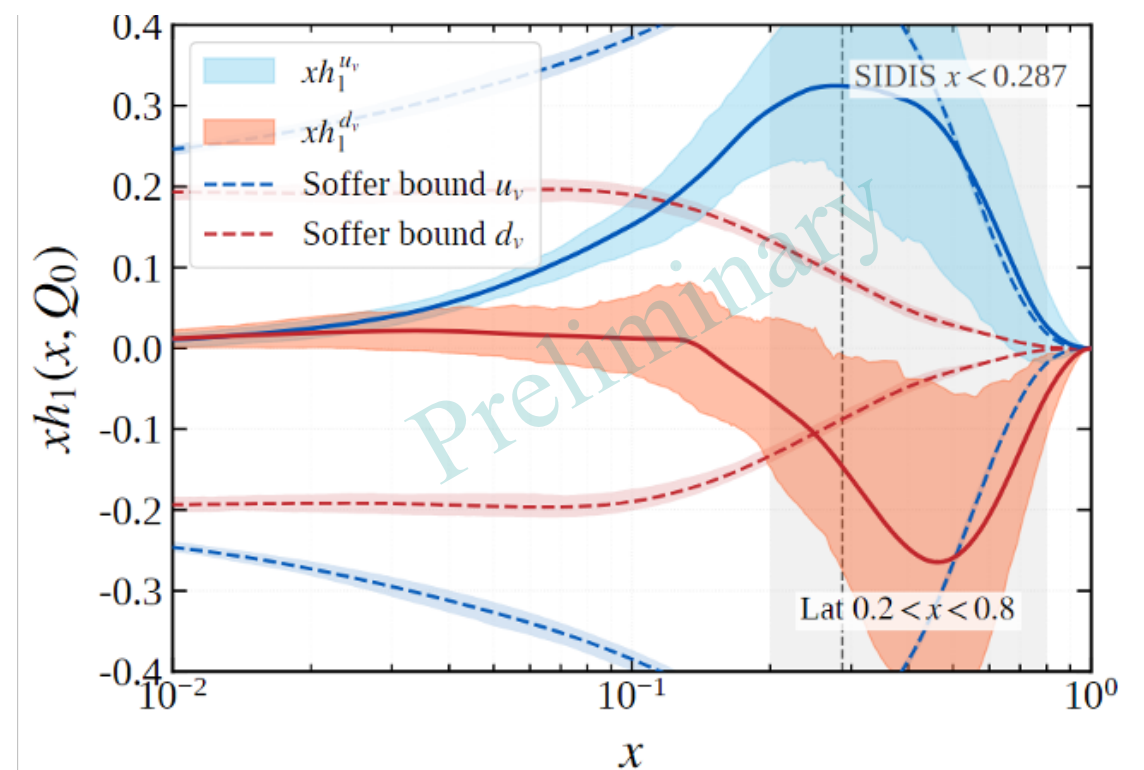
Lattice Parton Collaboration, Phys. Rev. Lett. 131 (2023) 261901.

A Combined Analysis of Transversity

data only



data + lattice

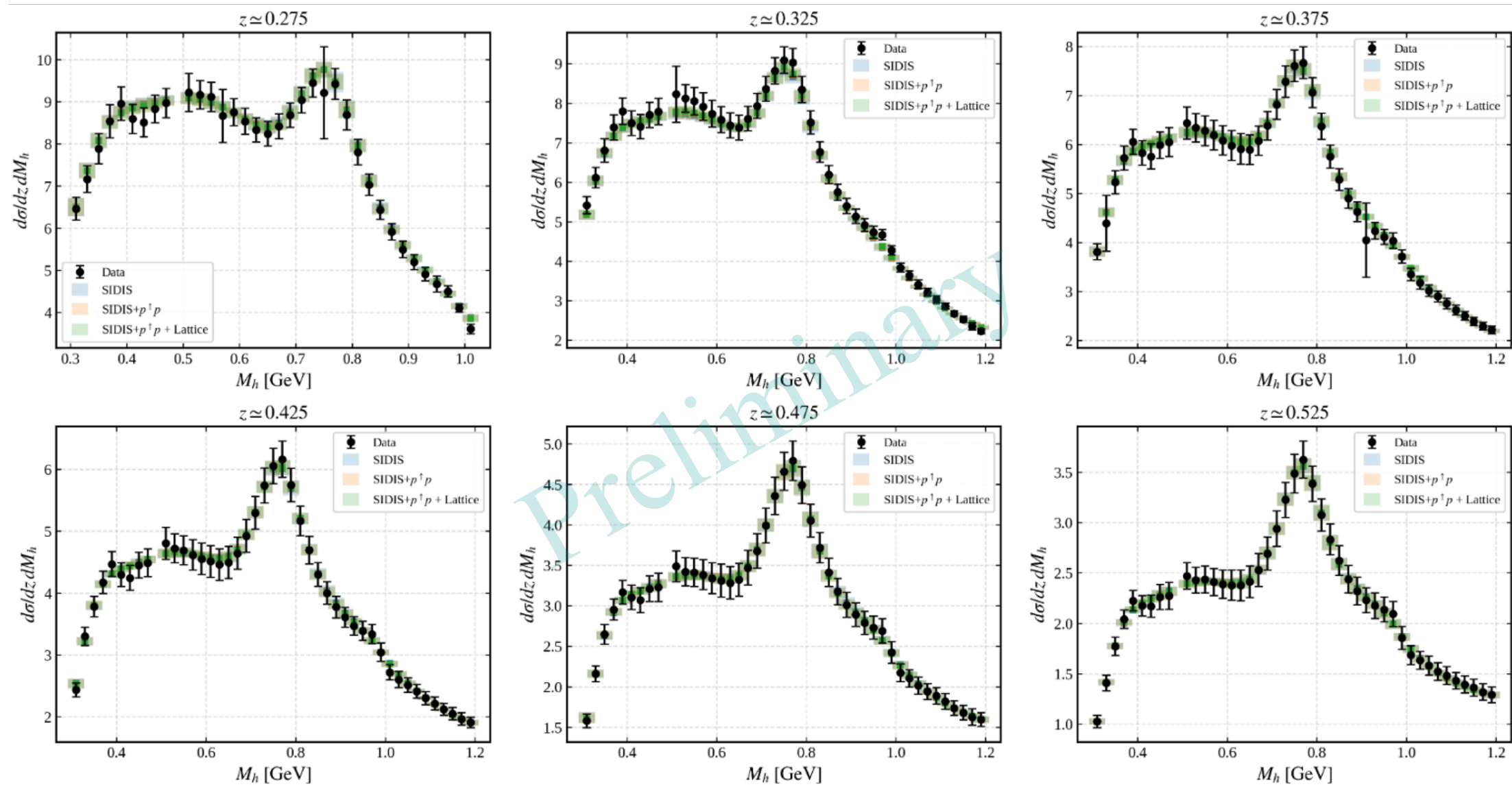


χ^2/N	Belle cross section	Belle asymmetry	HERMES proton	COMPASS proton	COMPASS deuteron	STAR pp
data only	0.277	0.634	0.910	0.868	0.804	1.214
data + lattice	0.262	0.632	0.915	0.980	0.845	1.151

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with Belle cross section data

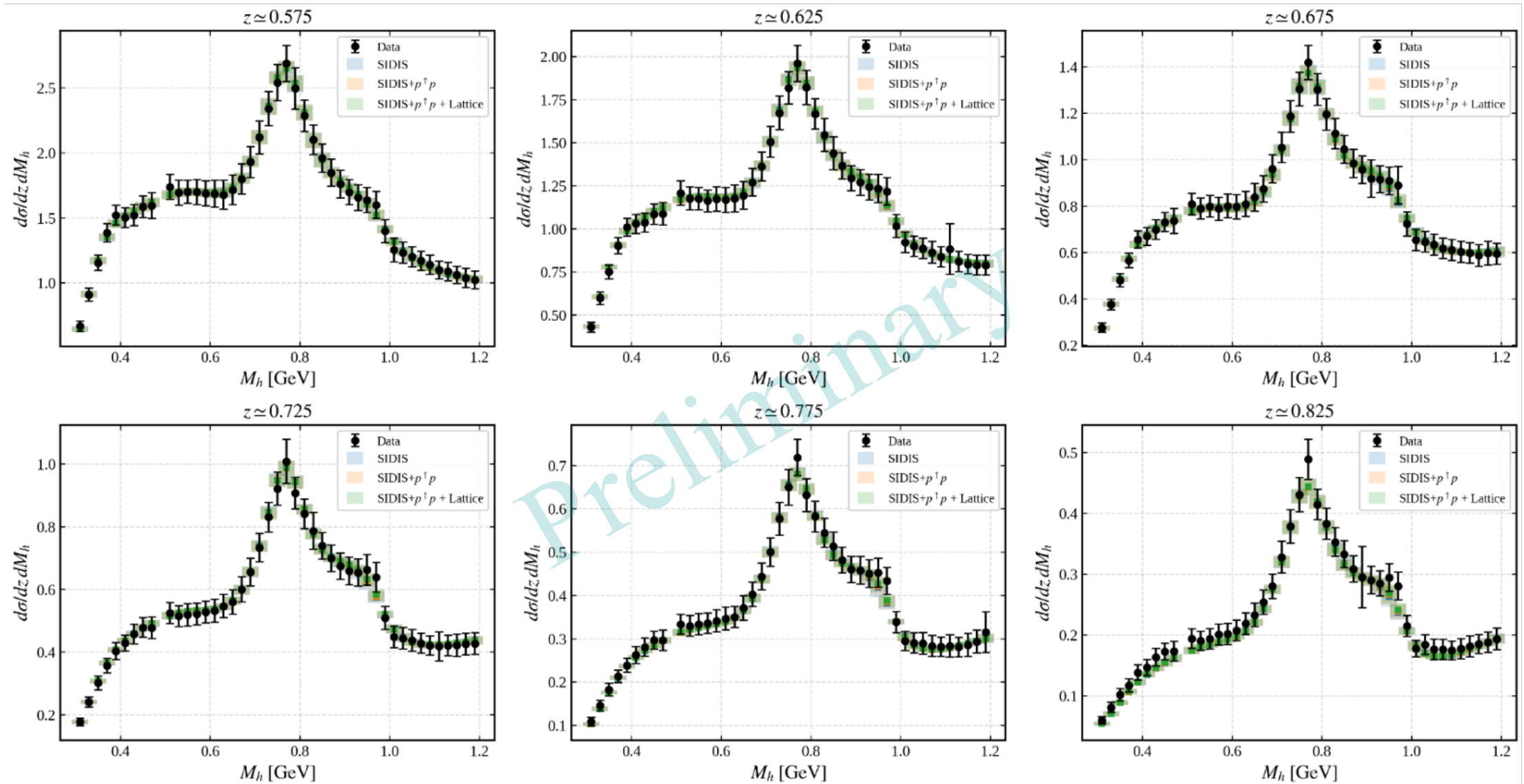


Data: Belle Collaboration, Phys. Rev. D 96 (2017) 032005.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with Belle cross section data

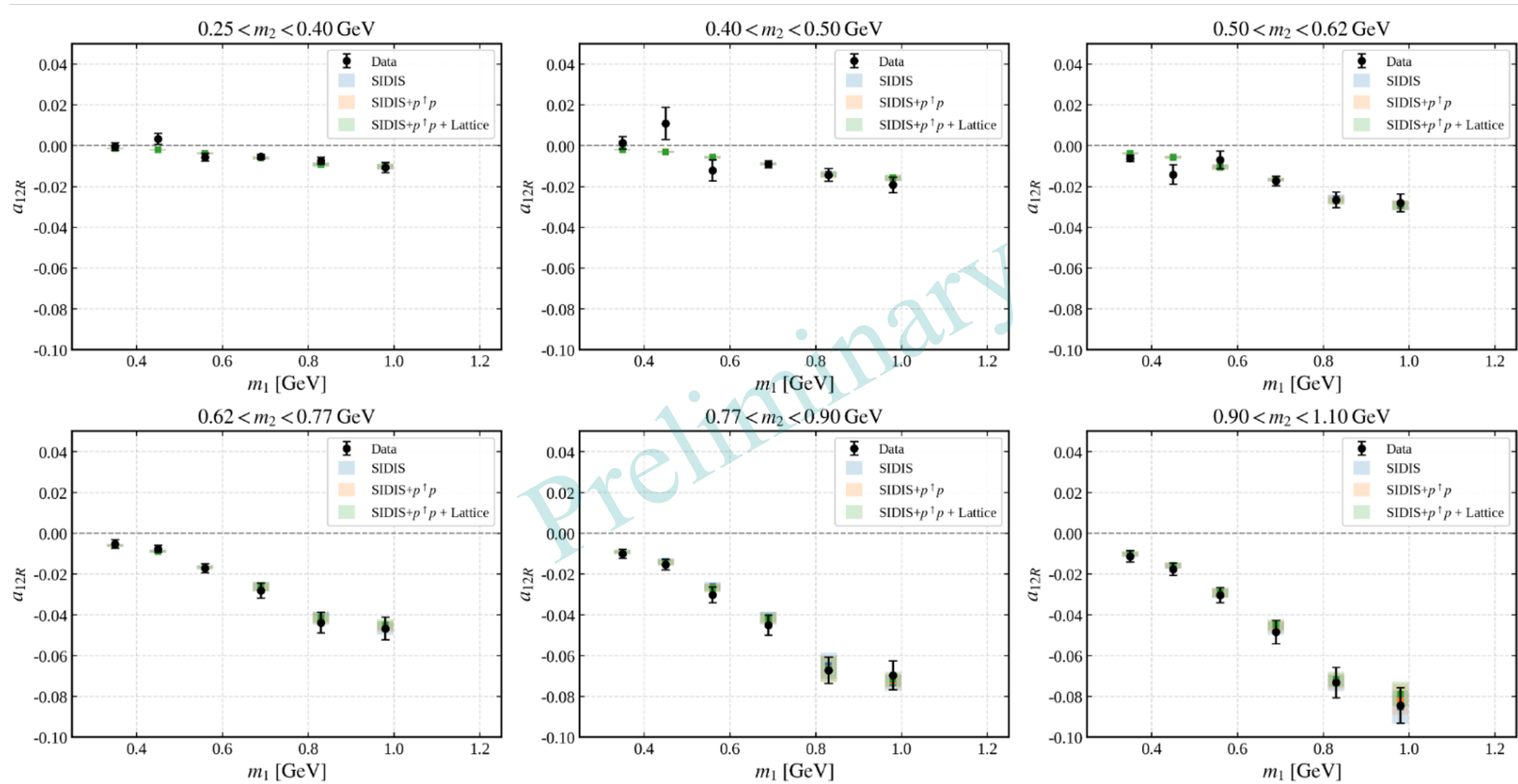


Data: Belle Collaboration, Phys. Rev. D 96 (2017) 032005.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with Belle asymmetry data

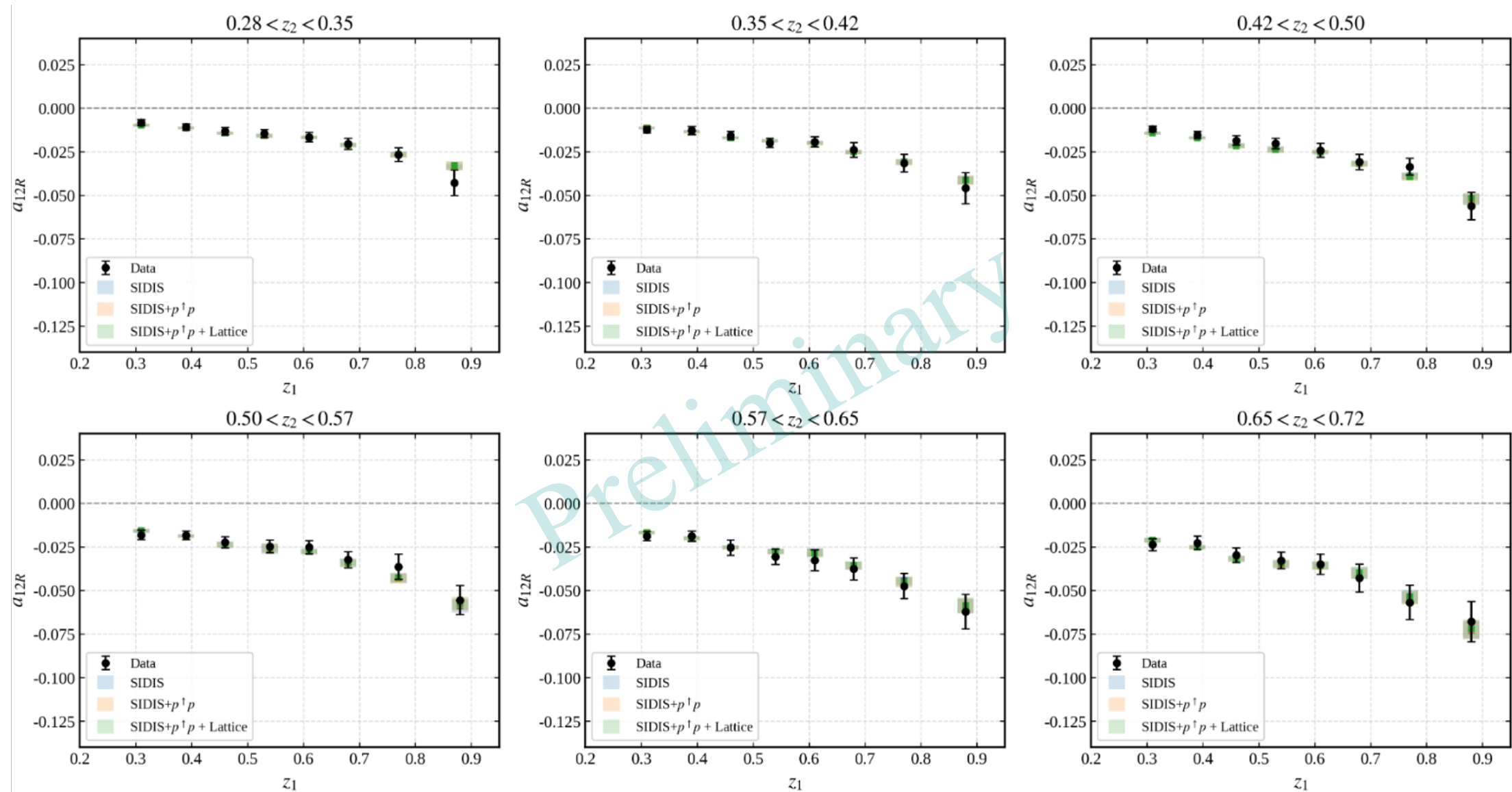


Data: Belle Collaboration, Phys. Rev. Lett. 107 (2011) 072004.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with Belle asymmetry data

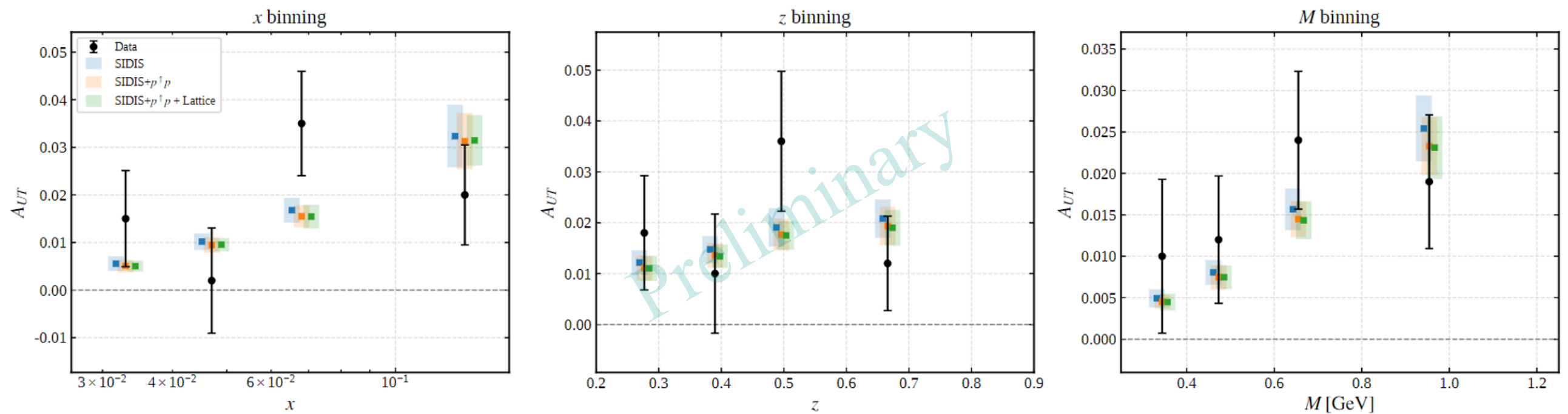


Data: Belle Collaboration, Phys. Rev. Lett. 107 (2011) 072004.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with HERMES proton SIDIS data

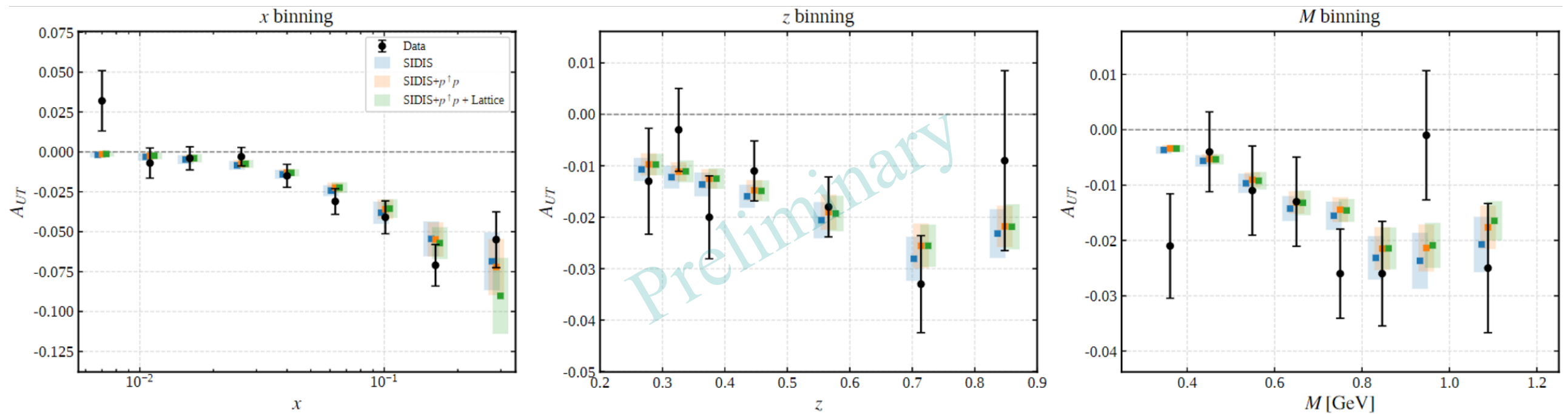


Data: HERMES Collaboration, JHEP 06 (2008) 017.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with COMPASS proton SIDIS data

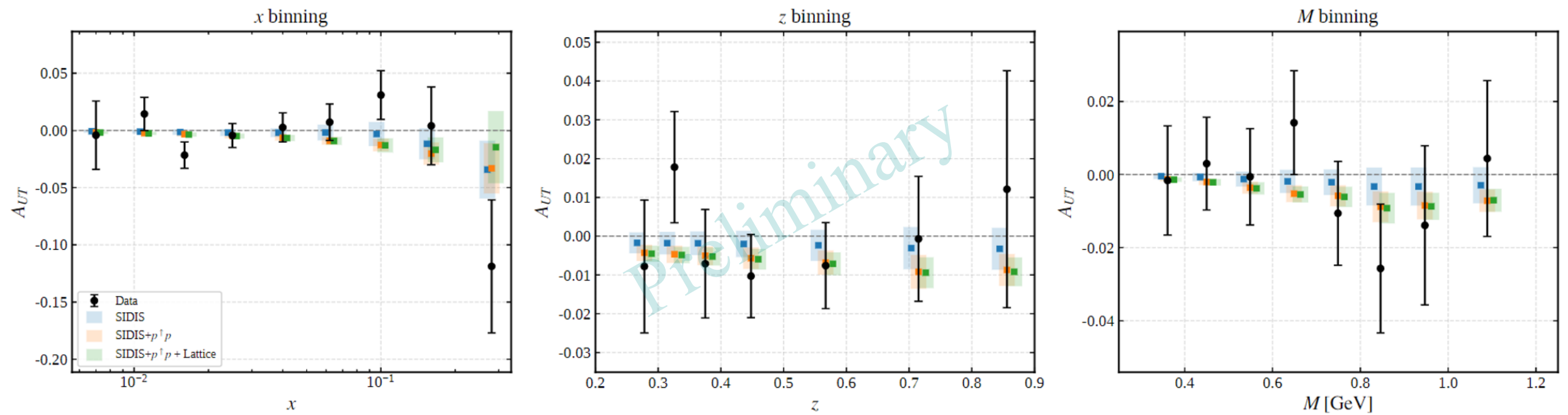


Data: COMPASS Collaboration, Phys. Lett. B 736 (2014) 124.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with COMPASS deuteron SIDIS data

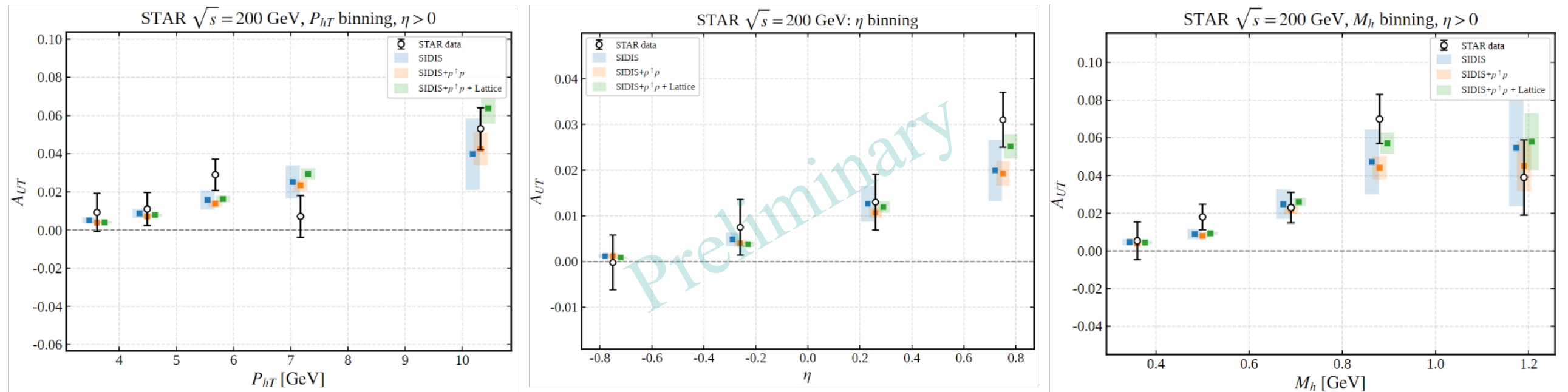


Data: COMPASS Collaboration, Phys. Lett. B 845 (2023) 138155.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with STAR pp data

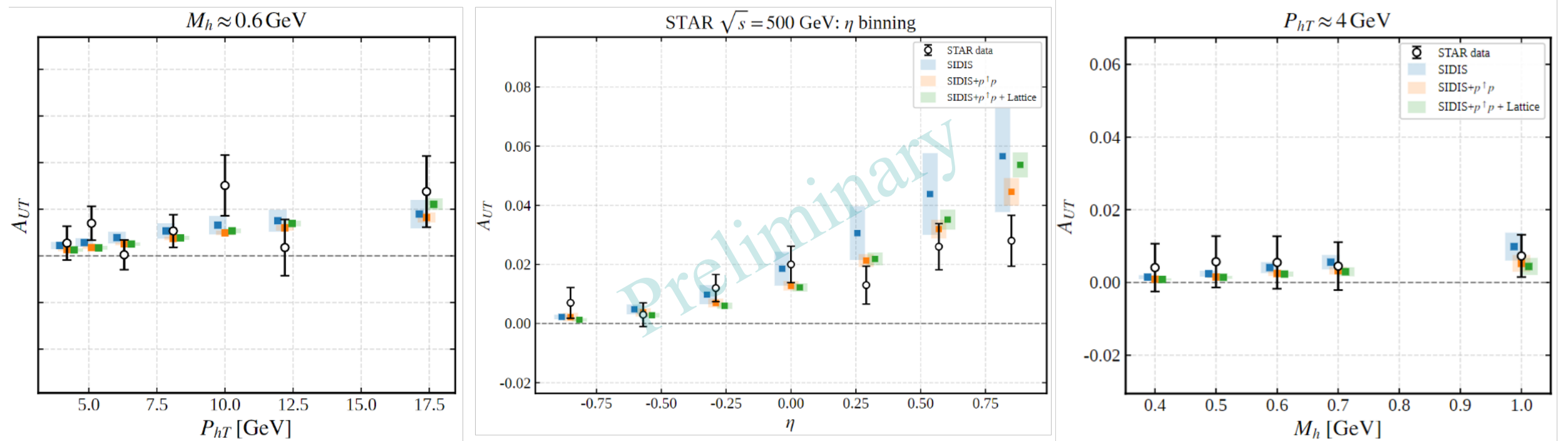


Data: STAR Collaboration, Phys. Rev. Lett. 115 (2015) 242501.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with STAR pp data



Data: STAR Collaboration, Phys. Lett. B 780 (2018) 332.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

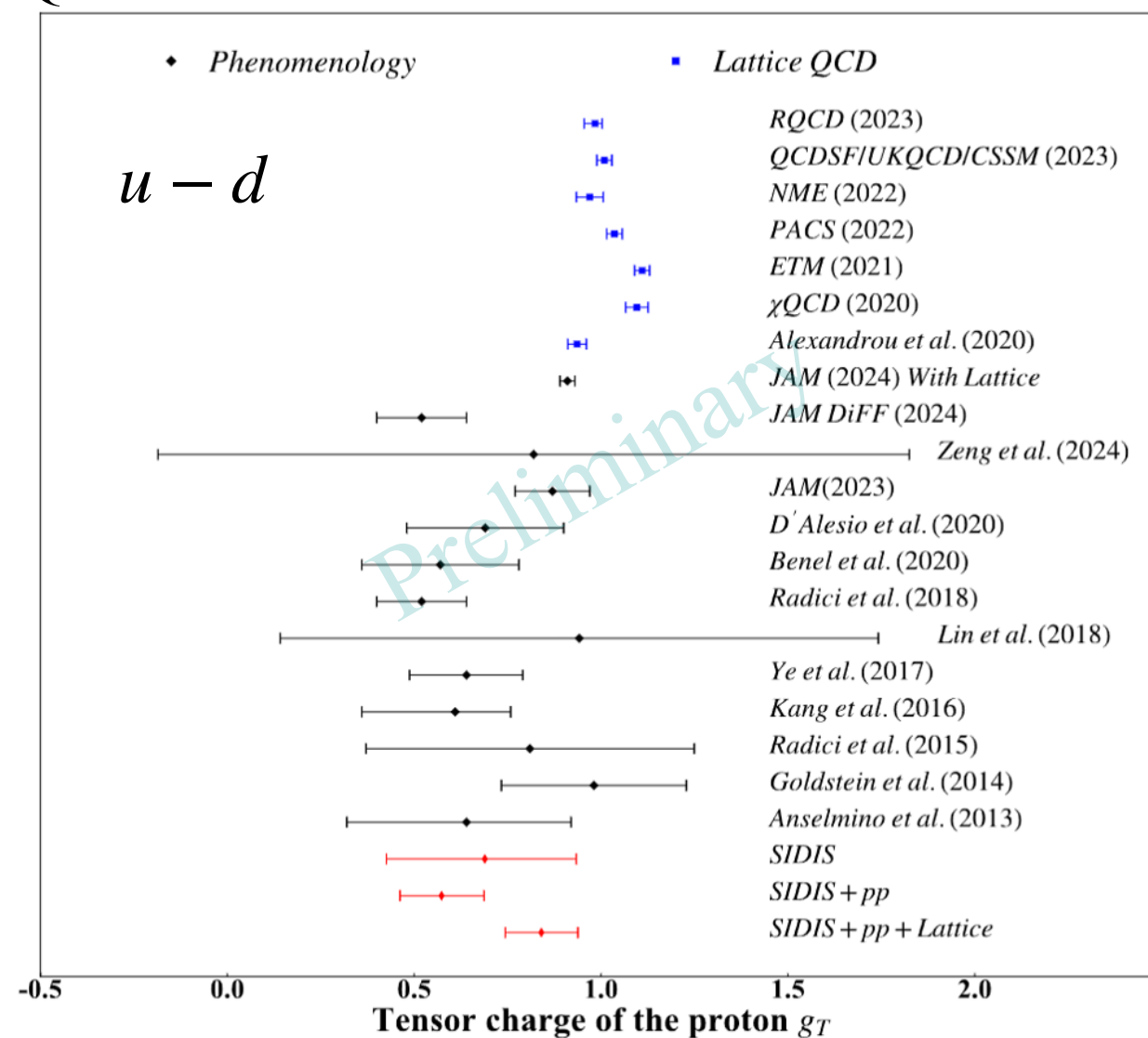
Tensor Charge

Tensor charge

$$\langle P, S | \bar{\psi}^q i\sigma^{\mu\nu} \gamma_5 \psi^q | P, S \rangle = g_T^q \bar{u}(P, S) i\sigma^{\mu\nu} \gamma_5 u(P, S)$$

$$g_T^q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)] dx$$

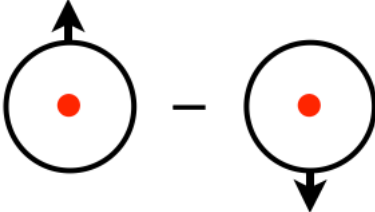
- A fundamental QCD quantity: matrix element of local operators.
- Moment of the transversity distribution: valence quark dominant.
- Calculable in lattice QCD.



H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

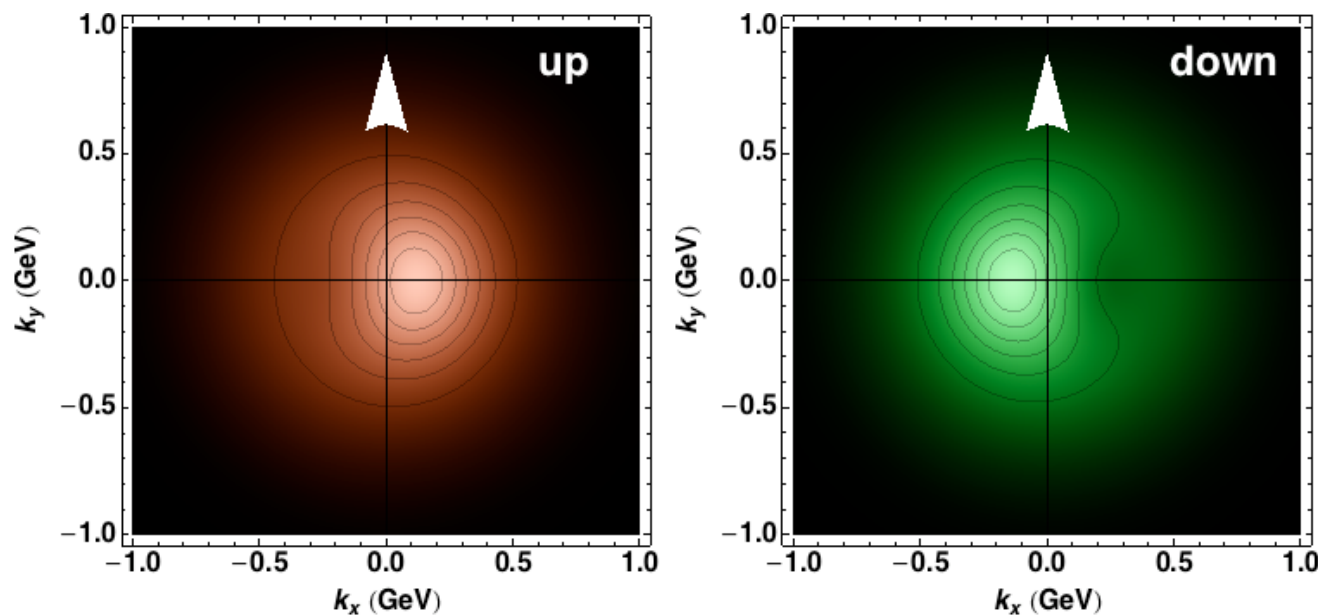
The Sivers Function

Sivers TMD distribution function

$$\frac{\epsilon_{ij} k_T^i S_T^j}{M} f_{1T}^\perp(x, k_T^2)$$


A naive T-odd distribution function

Transverse momentum distribution
distorted by nucleon transverse spin



[Figure from A. Bacchetta]

Effect in SIDIS:

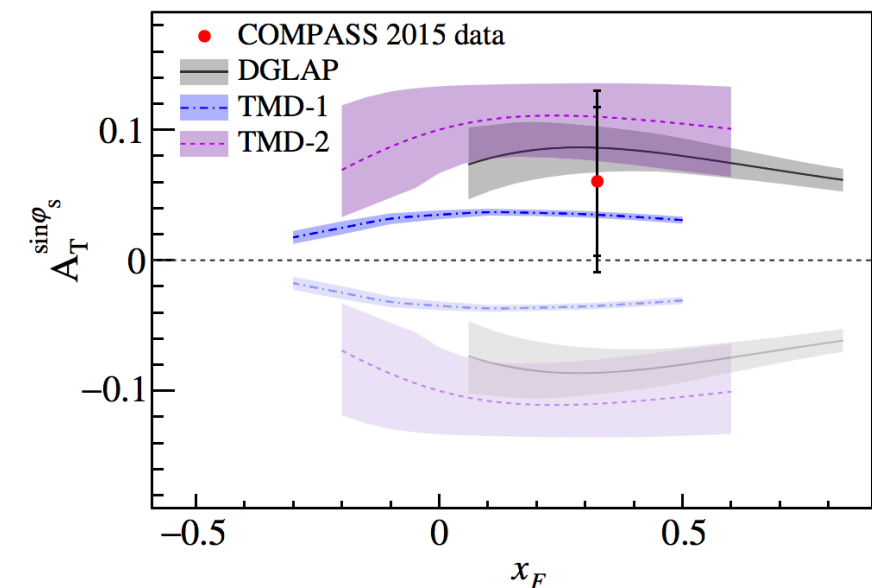
transverse single spin asymmetry
(Sivers asymmetry)

$$A_{UT}^{\sin(\phi_h - \phi_s)} \sim f_{1T}^\perp \otimes D_1$$

sizable Sivers asymmetry observed
by HERMES, COMPASS, JLab

Sign change prediction:

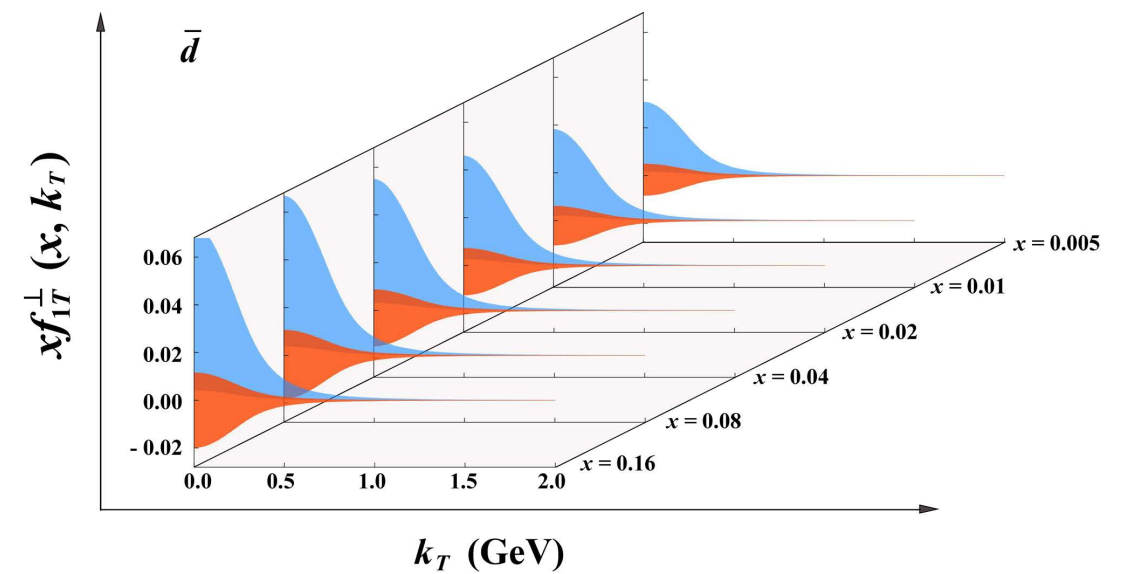
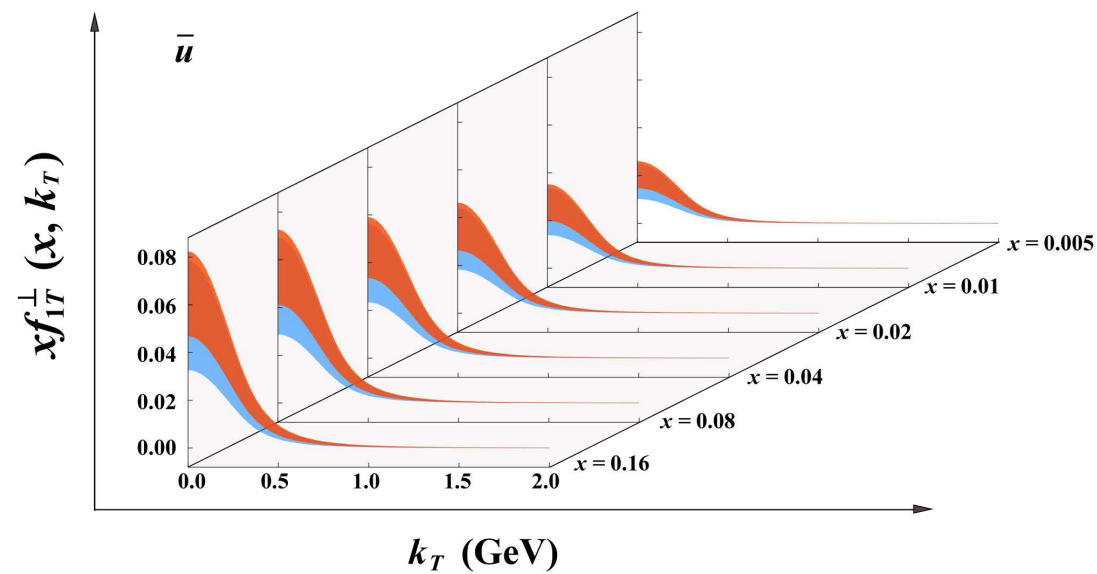
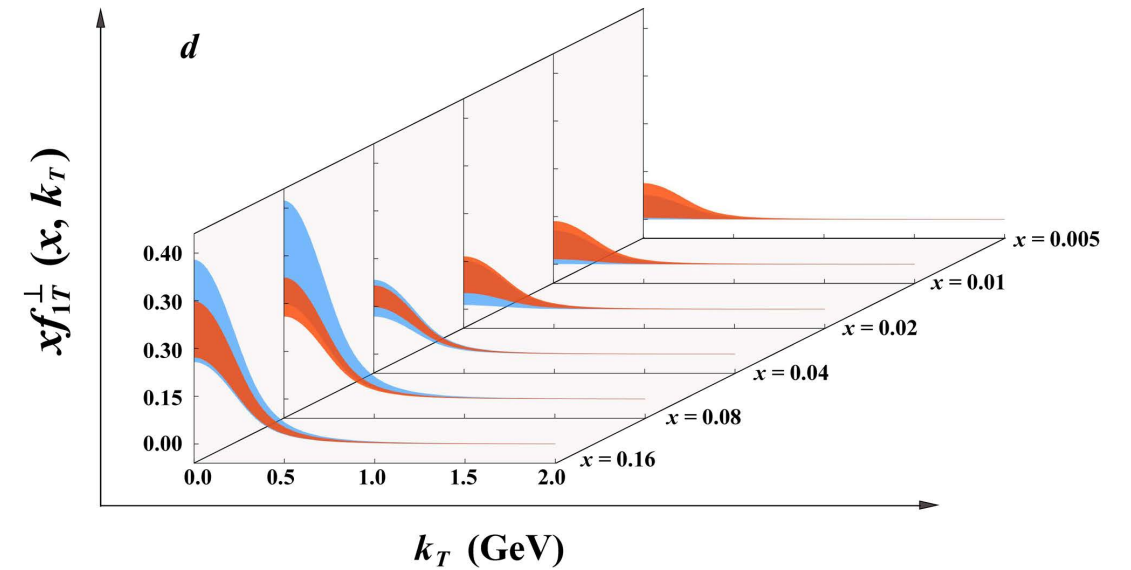
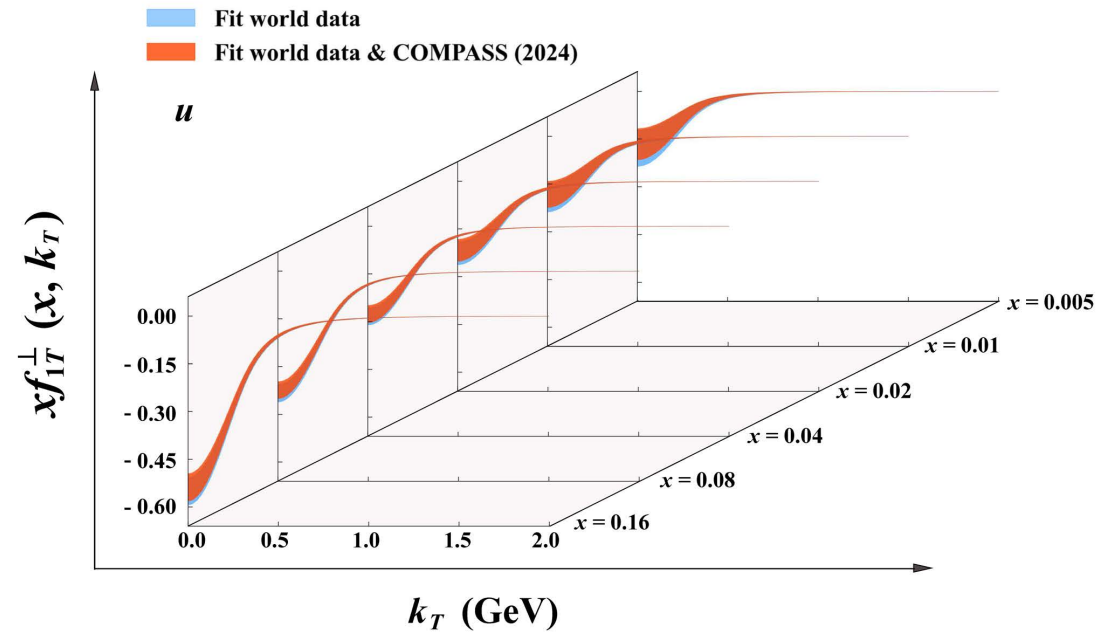
$$f_{1T}^\perp(x, k_T^2)|_{\text{SIDIS}} = -f_{1T}^\perp(x, k_T^2)|_{\text{DY}}$$



COMPASS Collaboration, PRL 119, 112002 (2017).

Sivers Functions

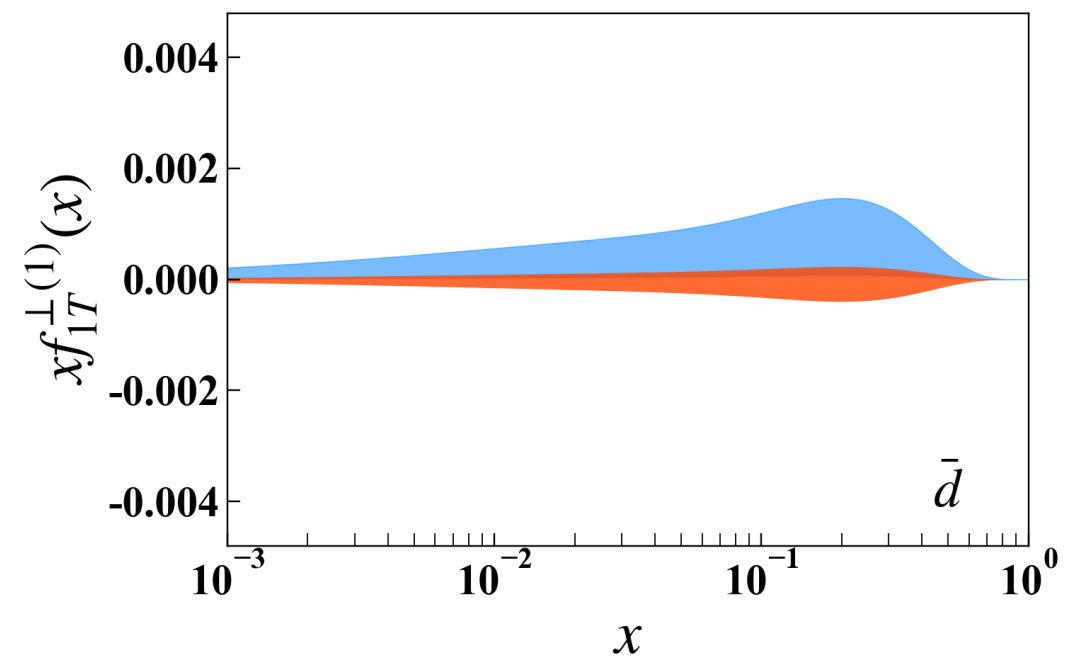
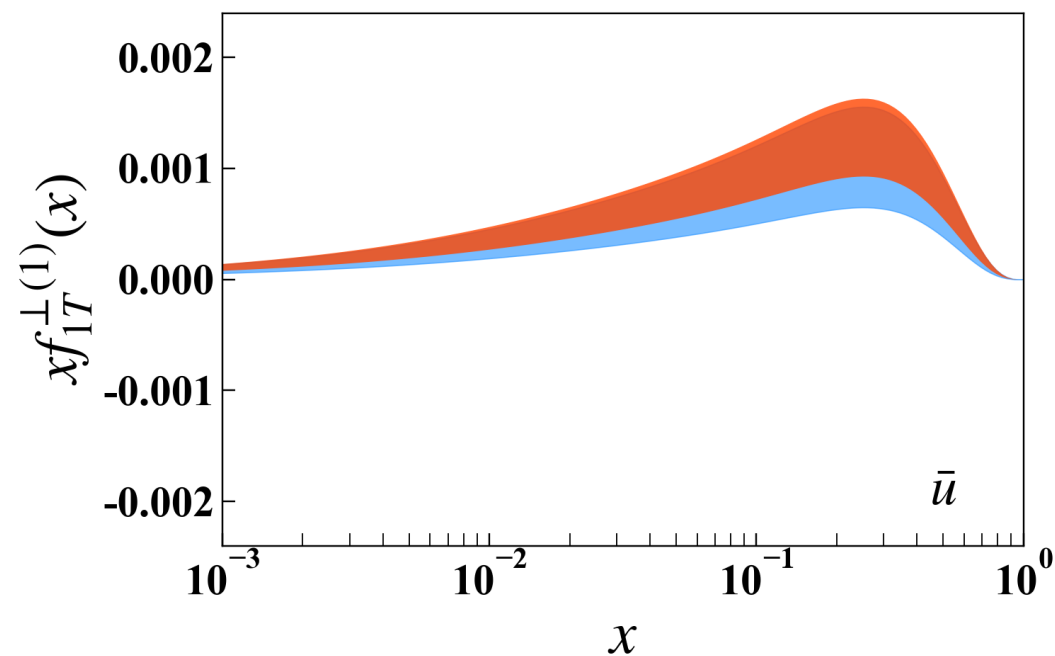
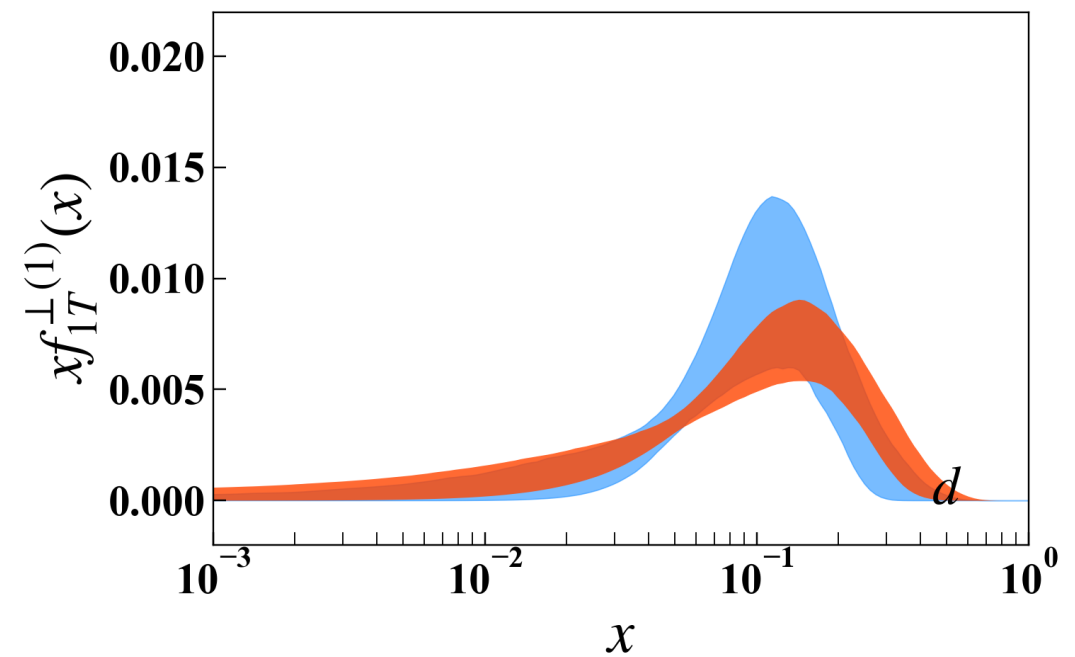
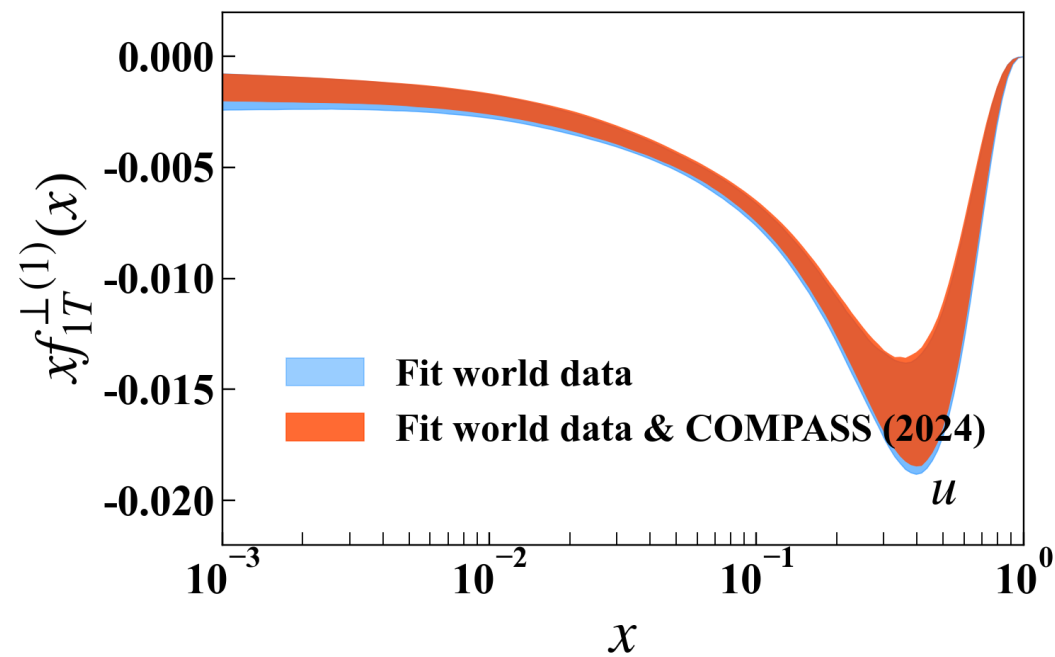
Global analysis of SIDIS, Drell-Yan, $W^\pm/Z^0$ production data



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Sivers Functions

Global analysis of SIDIS, Drell-Yan, $W^\pm/Z^0$ production data

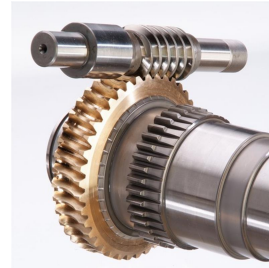
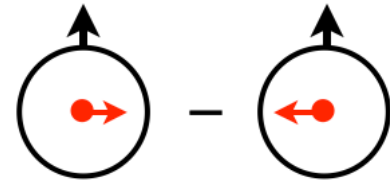


C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Double Spin Asymmetry and Worm-gear

Trans-helicity worm-gear distribution

$$\frac{k_T \cdot S_T}{M} g_{1T}^\perp(x, k_T^2)$$



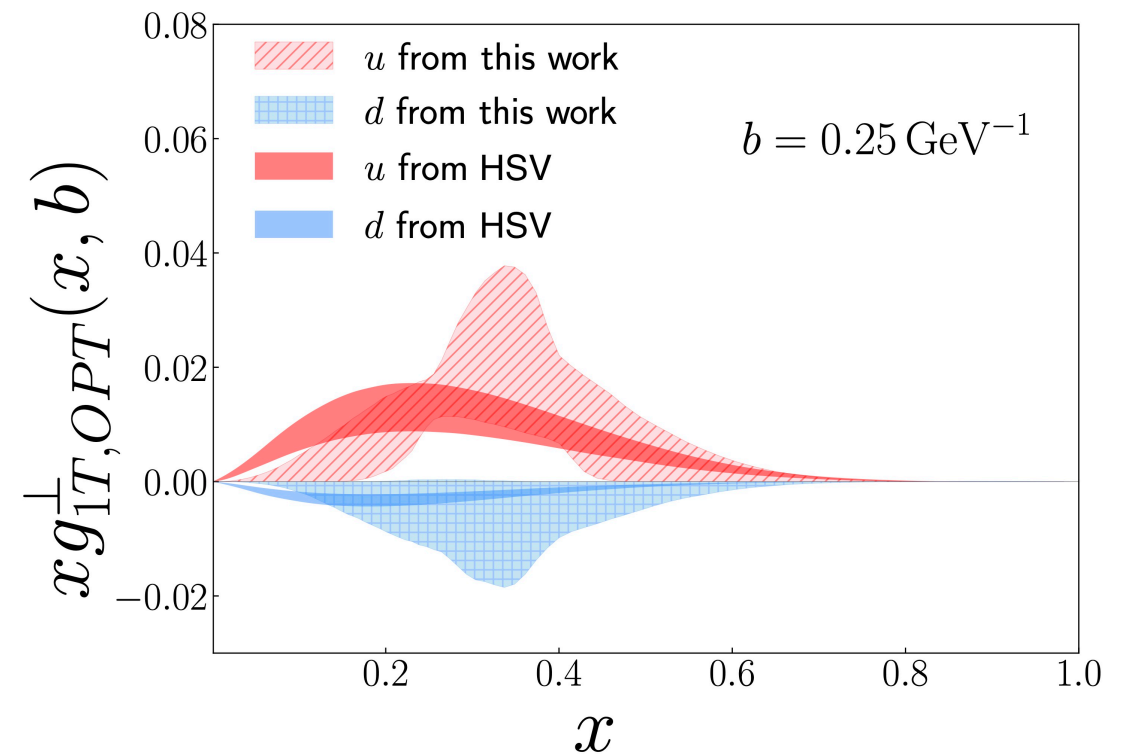
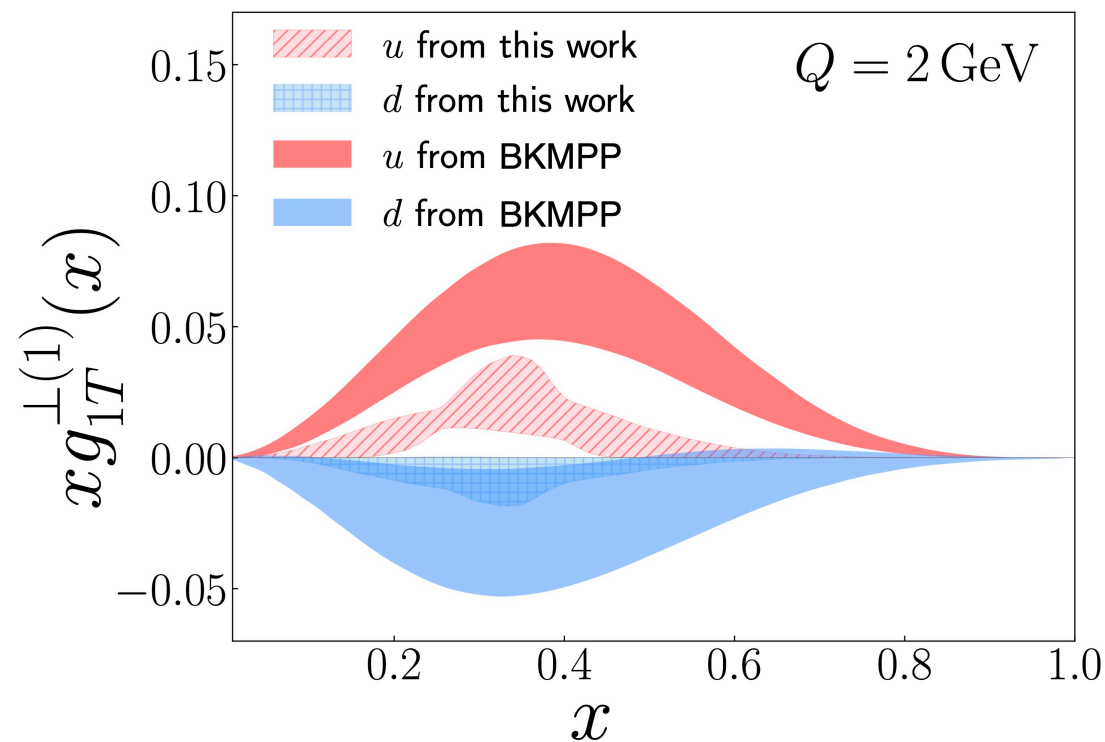
Effect in SIDIS:

A longitudinal-transverse double spin asymmetry

- Longitudinally polarized quark density in a transversely polarized nucleon
- Overlap between wave functions differing by one unit of orbital angular momentum

$$A_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T}^\perp \otimes D_1$$

Phenomenological extraction

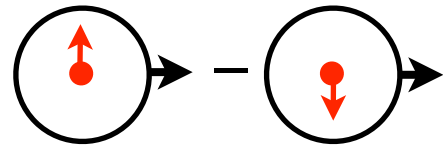


K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. D 110 (2024) 034036.

Longitudinal SSA and Worm-gear

Longi-transversity worm-gear distribution

$$\frac{S_L k_T^\alpha}{M} h_{1L}^\perp(x, k_T^2)$$



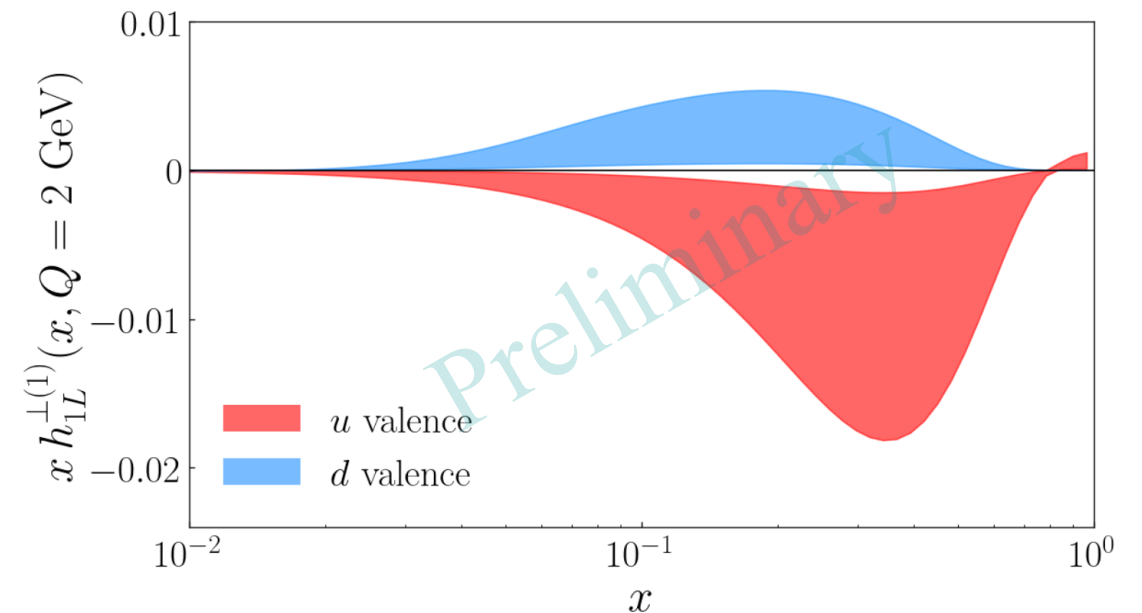
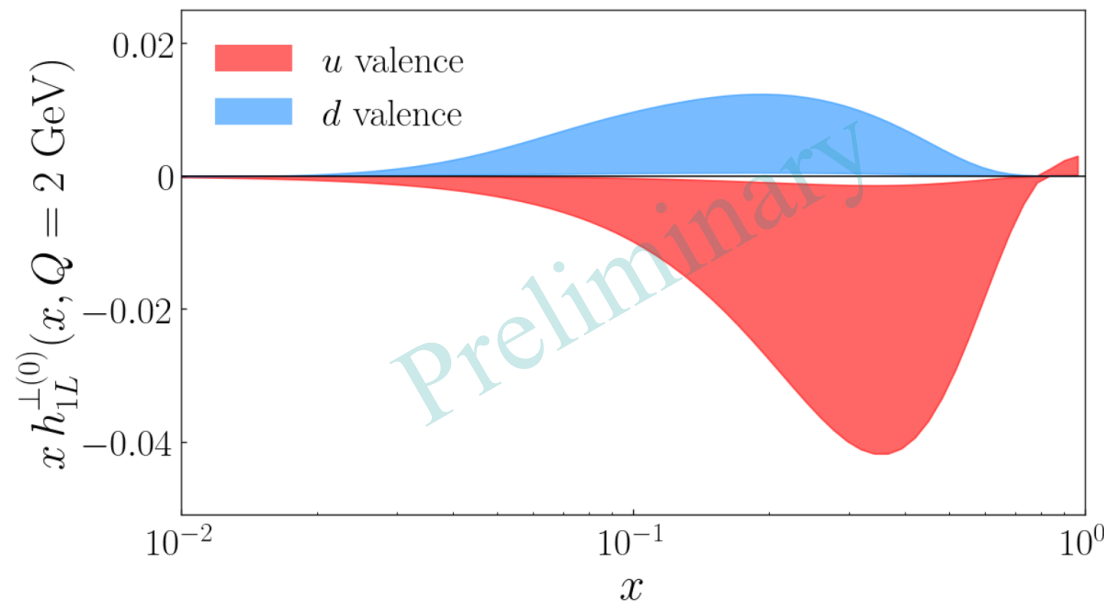
Effect in SIDIS:

A longitudinal target single spin asymmetry

$$A_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$$

- Transversely polarized quark density in a longitudinally polarized nucleon
- Overlap between wave functions differing by one unit of orbital angular momentum

First extraction



W. Chen, K. Yang, TL, P. Sun, Y. Zhao, 2026

Summary

- Nucleon spin structure is still not well understood
- Rich information is contained in TMDs
 - helicity: quark polarization has nontrivial dependence on transverse momentum;
 - transversity: sea quarks may have nonzero transverse polarization, suggest intrinsic sea;
 - Sivers: quark transverse momentum is distorted by the nucleon transverse spin;
 - worm-gears: correlation between quark and nucleon longitudinal/transverse and transverse/longitudinal polarizations
 - ...
- SIDIS with polarized beam and target is a main process to study polarized TMDs
- Electron-positron annihilation is important to constrain nucleon spin structures and to understand the role of spin in hadronization process
- Lattice QCD can provide complementary information to determine the distributions, especially when lacking precise data
- Opportunities from existing experiments at JLab12, BESIII, BelleII, and future facilities, EIC, EicC, STCF to understand nucleon spin structures and fragmentation functions.

Thank you!

Backup

Imposing a Bound?

Question:

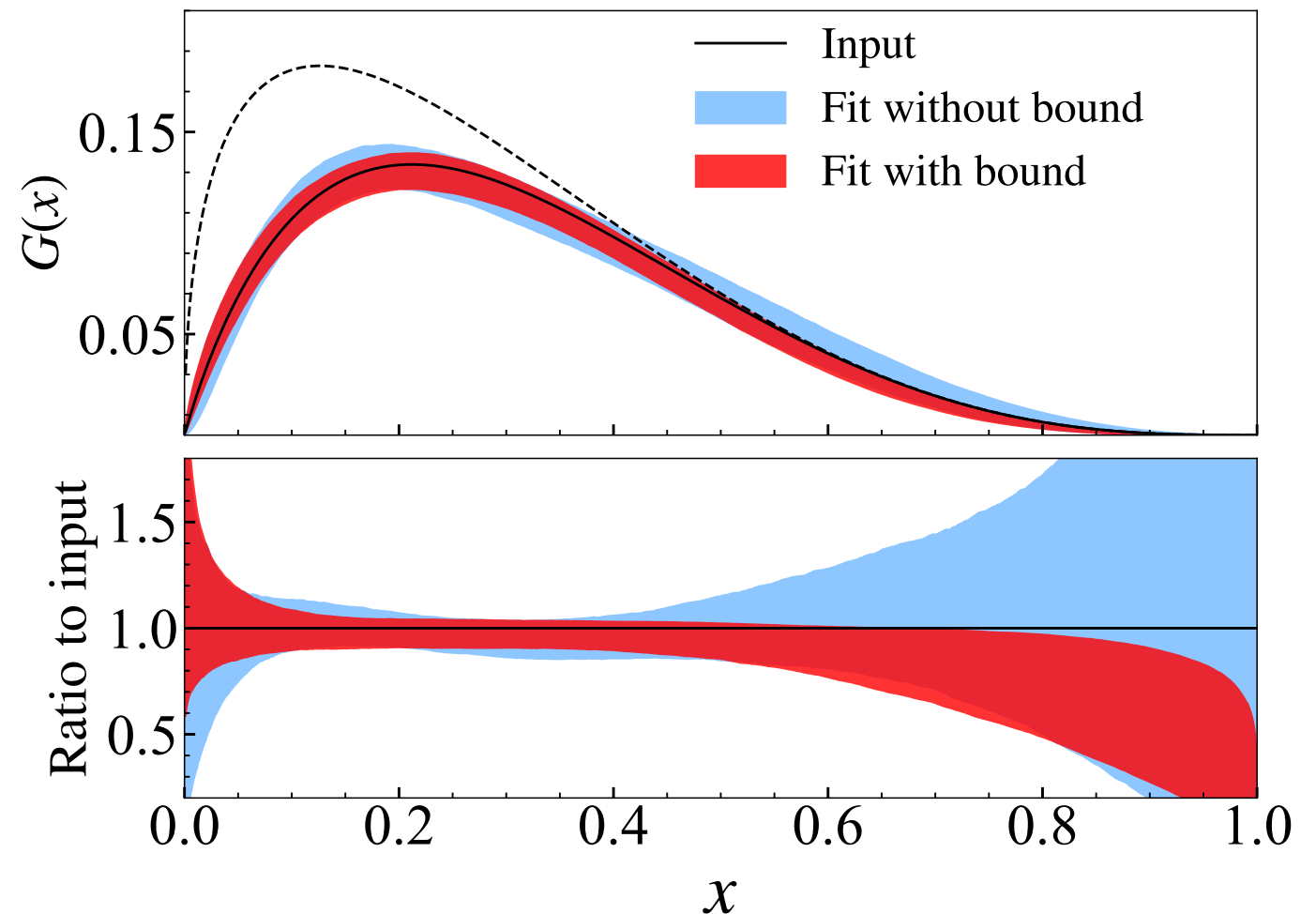
Whether we should impose the bound $|g_1(x)| \leq f_1(x)$ in the fit?
Similar situations in many other quantities.

A toy model test:

Input some “pol” function $G(x)$
and “unpol” function $F(x)$
Generate asymmetry “data”

Fit the “data” with or without
imposing the bound $G(x) \leq F(x)$

*Imposing a bound in the fit may
bias the fit result!*

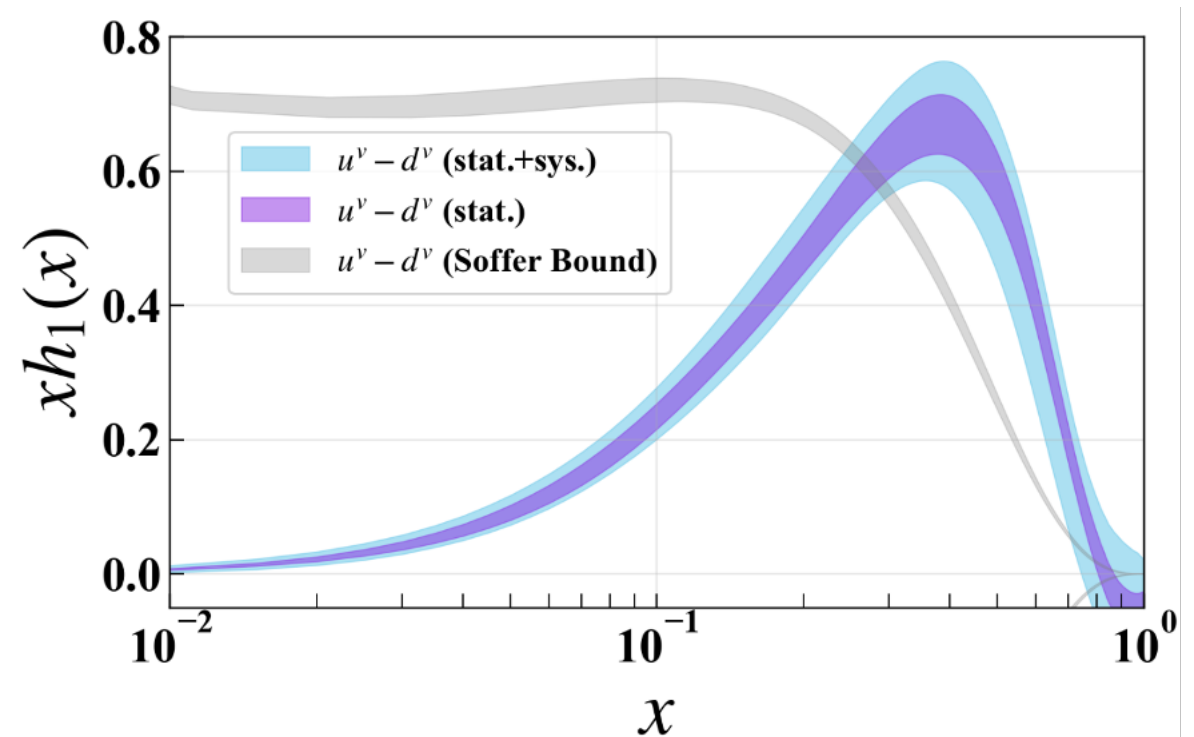
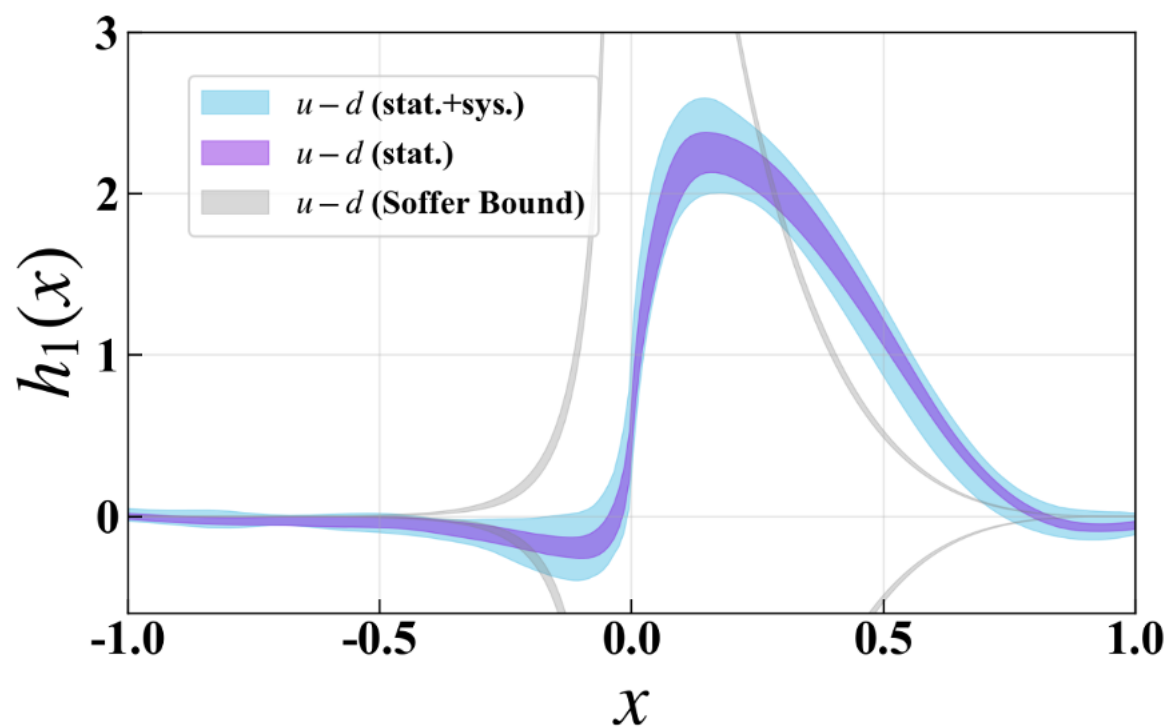


About Soffer's Bound

Soffer's bound on transversity distributions

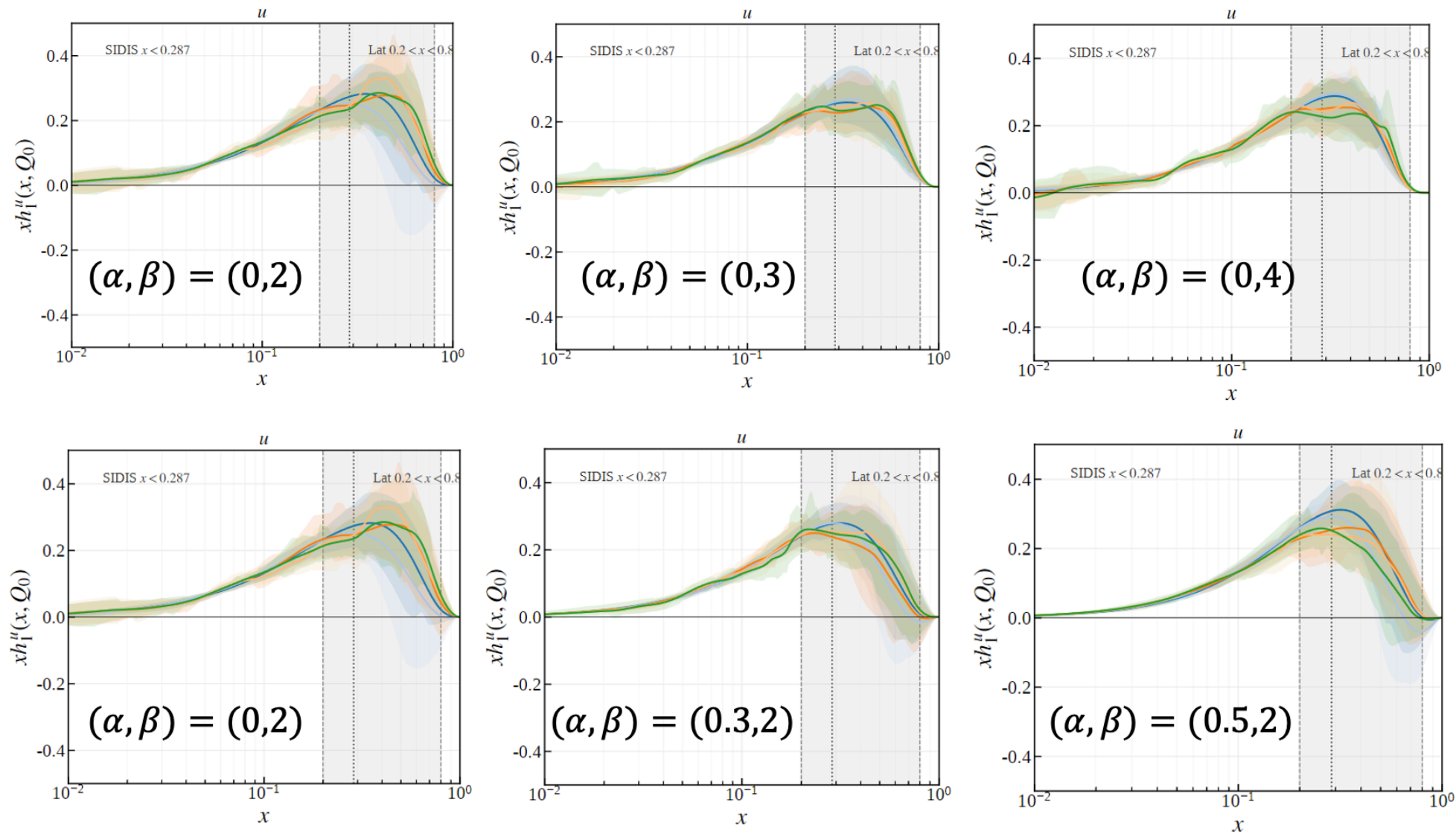
$$|h_1(x)| \leq \frac{1}{2}[f_1(x) + g_1(x)]$$

It is broken in existing lattice QCD calculations

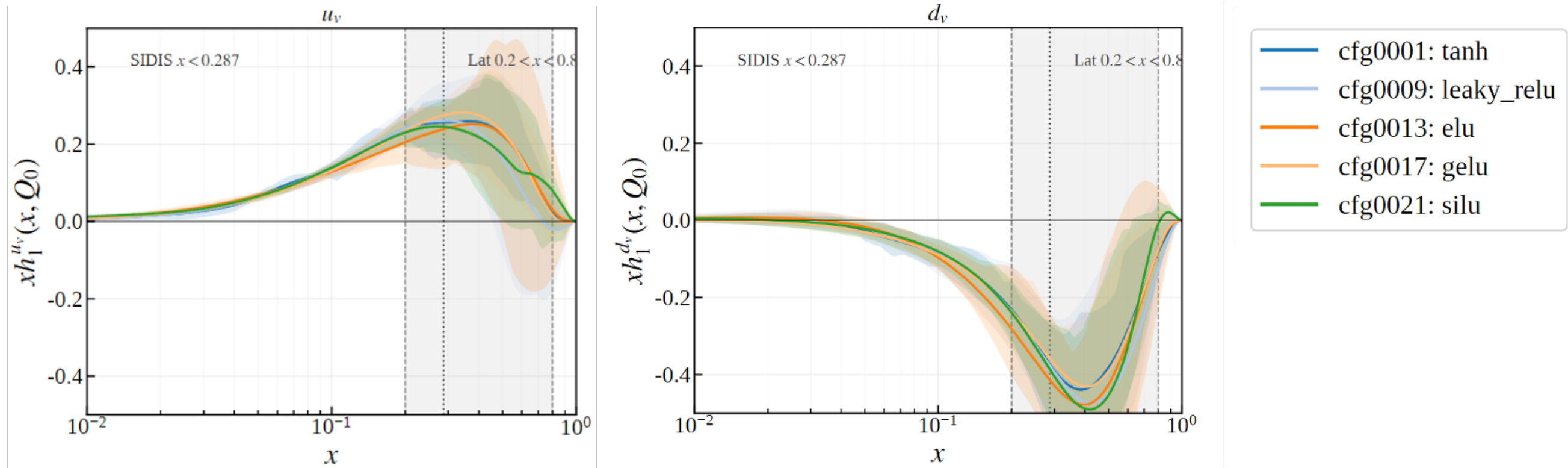


Lattice results from: LPC Collaboration, Phys. Rev. Lett. 131 (2023) 261901.

NN Hyperparameters

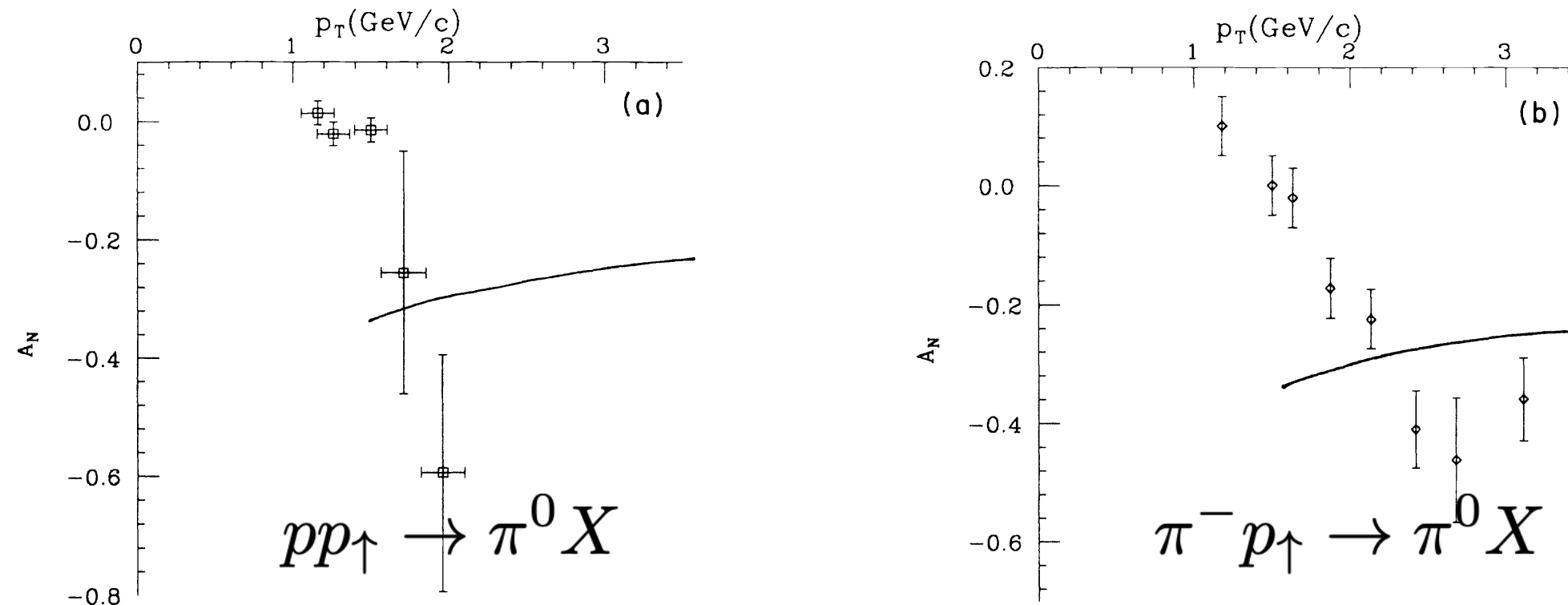


NN Hyperparameters



The Sivers Function: Early Story

Transverse single spin asymmetry observed in experiments



Data: J. Antille *et al.*, Phys. Lett B94 (1980) 523.

Data: 7th Symposium on High Energy Spin Physics (1986).

D. Sivers proposed to explain such SSA a new distribution function

Sivers function $\Delta^N G_{a/p(\uparrow)}(x, \mathbf{k}_T; \mu^2)$

D. Sivers, Phys. Rev. D 41 (1990) 83.

However it was soon shown this function was T-odd and prohibited by QCD

J. Collins, Nucl. Phys. B 396 (1993) 161.

For the next decade, the “Sivers effect” was thought to vanish.

The Siverson Function: Early Story

Until an explicit model calculation showing ...

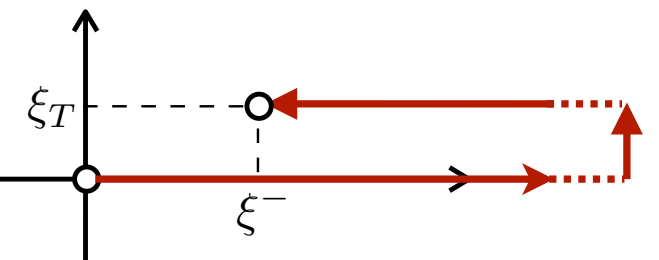
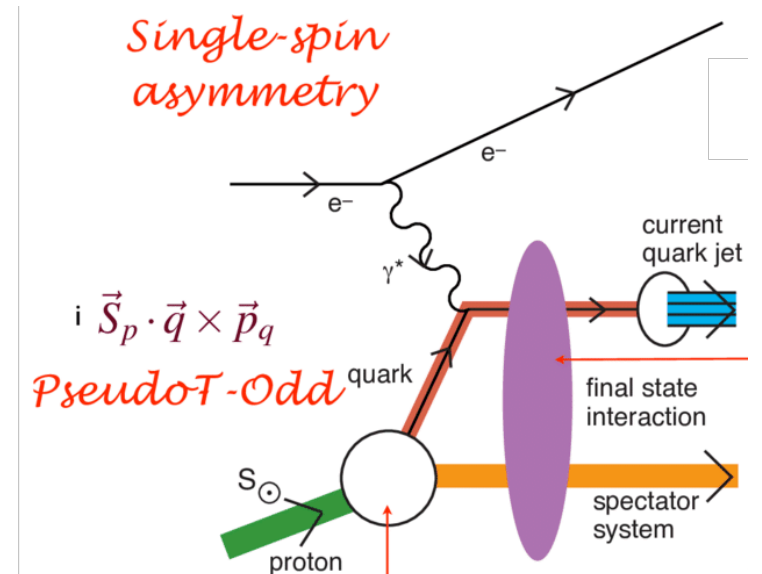
*nonzero Siverson effects exist at leading twist
due to final-state interactions*

S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99.

Siverson function can exist due to nontrivial gauge link

$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) \mathcal{U}_{(0,+\infty)}^{n-} \mathcal{U}_{(+\infty,\xi)}^{n-} \psi_i(\xi) | P \rangle \Big|_{\xi^+=0}$$

J.C. Collins, Phys. Lett. B 536 (2002) 43.



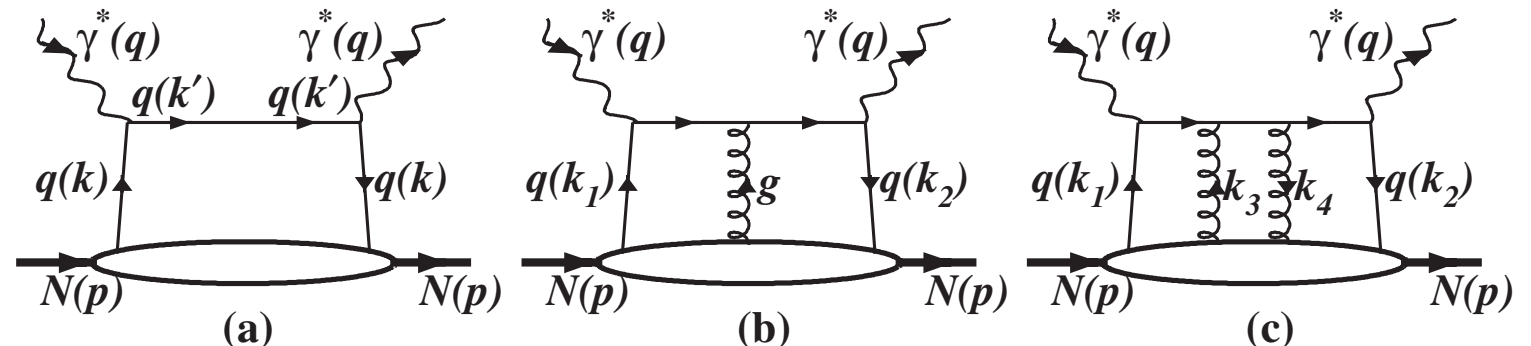
[Figure from A. Bacchetta]

This gauge link effect cannot be removed by choosing light-cone gauge $A^+ = 0$

X. Ji and F. Yuan, Phys. Lett. B 543 (2002) 66.

Collinear expansion

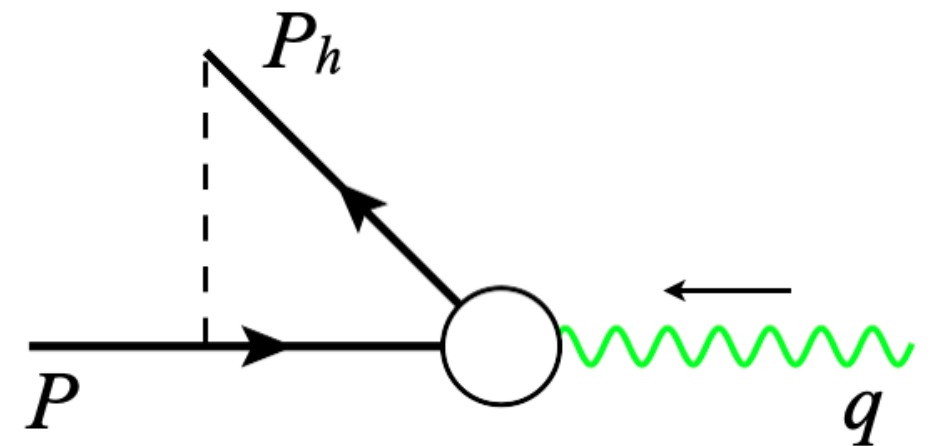
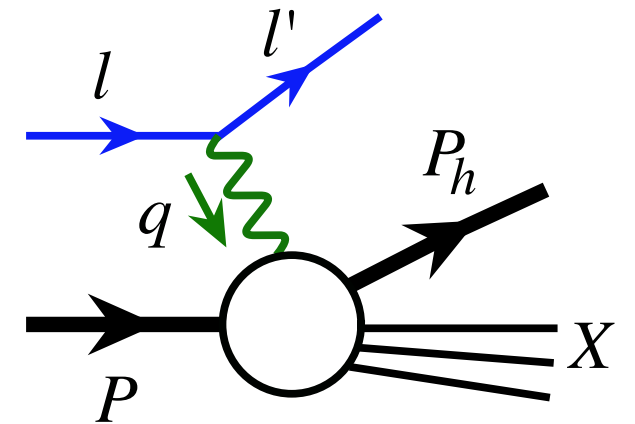
Z.T. Liang and X.N. Wang,
Phys. Rev. D 75 (2007) 094002.



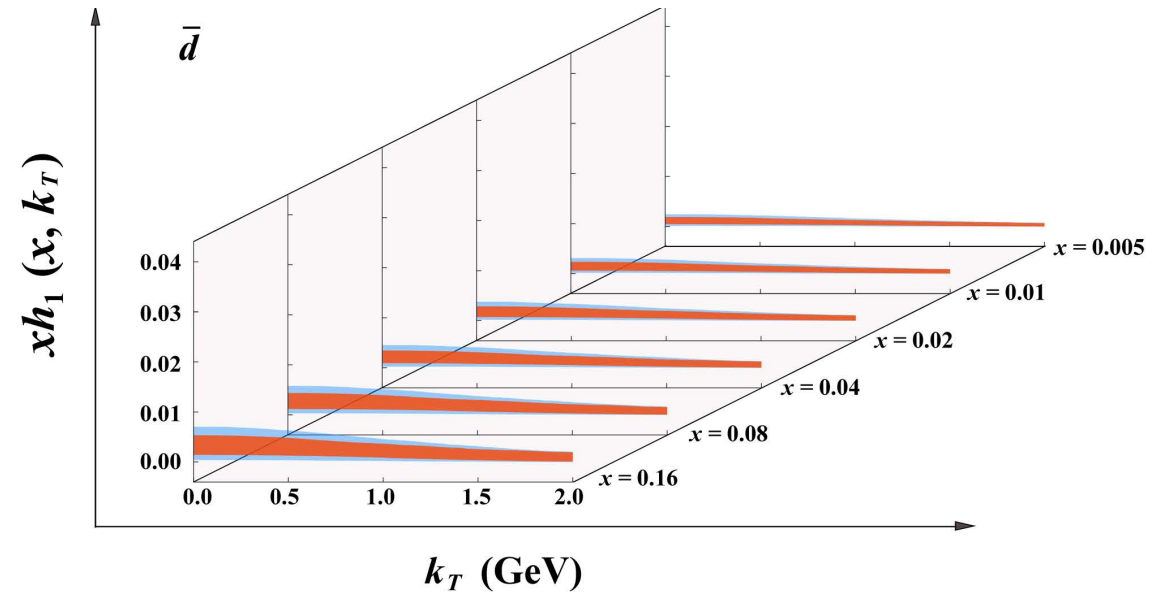
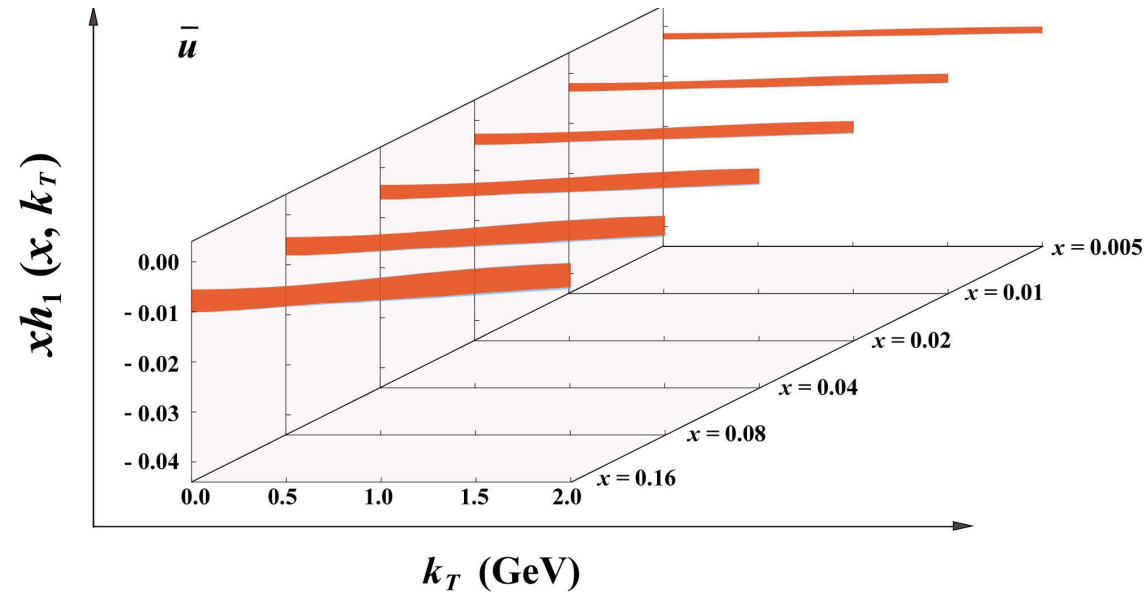
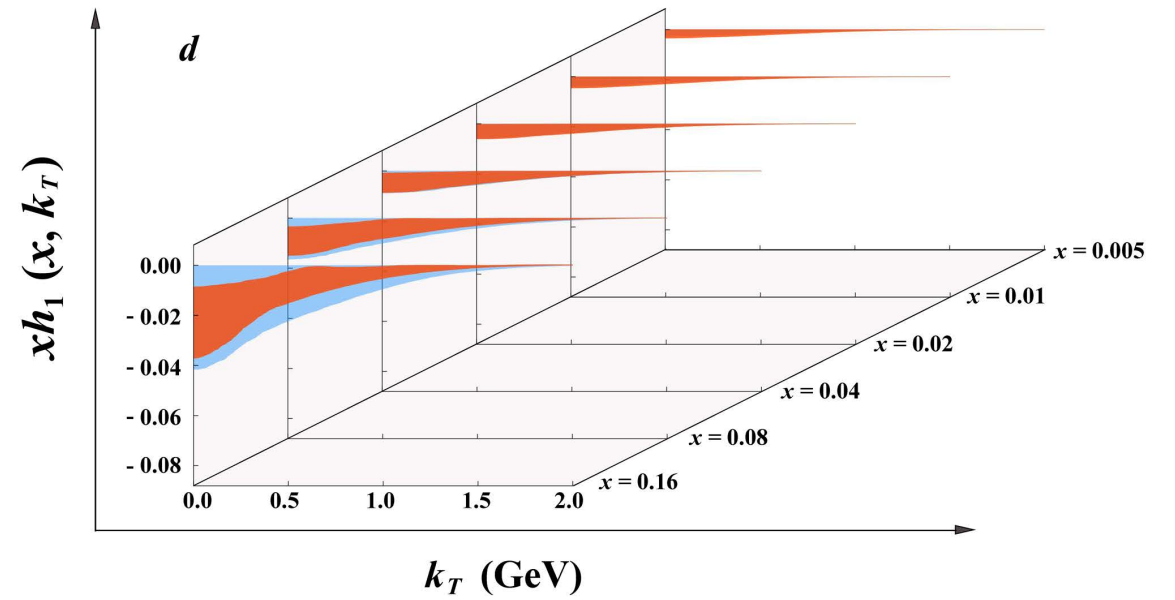
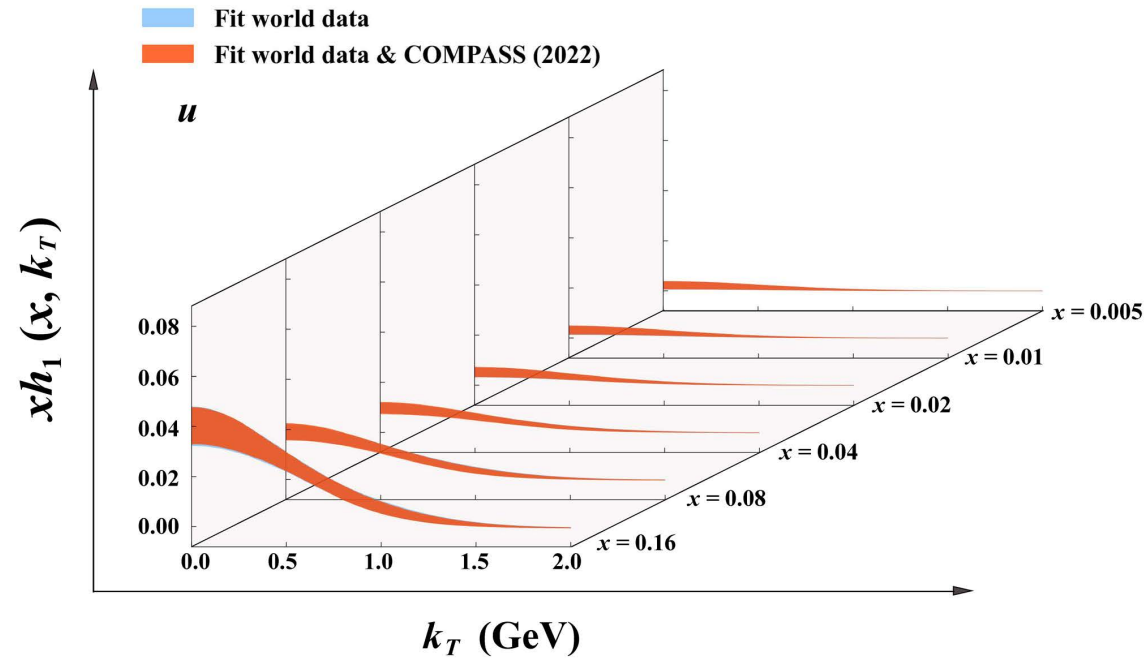
Semi-inclusive Deep Inelastic Scattering

Semi-inclusive DIS: a final state hadron (P_h) is identified

- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, ...
- a large momentum transfer Q provides a short-distance probe
- an additional and adjustable momentum scale P_{hT}
- multidimensional imaging of the nucleon

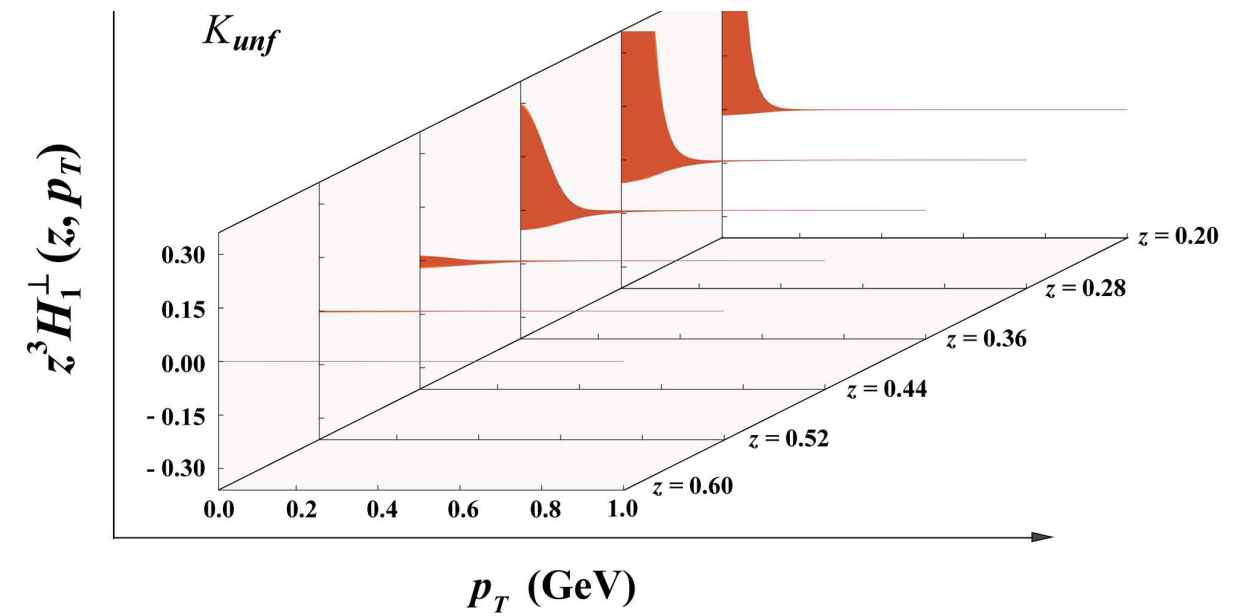
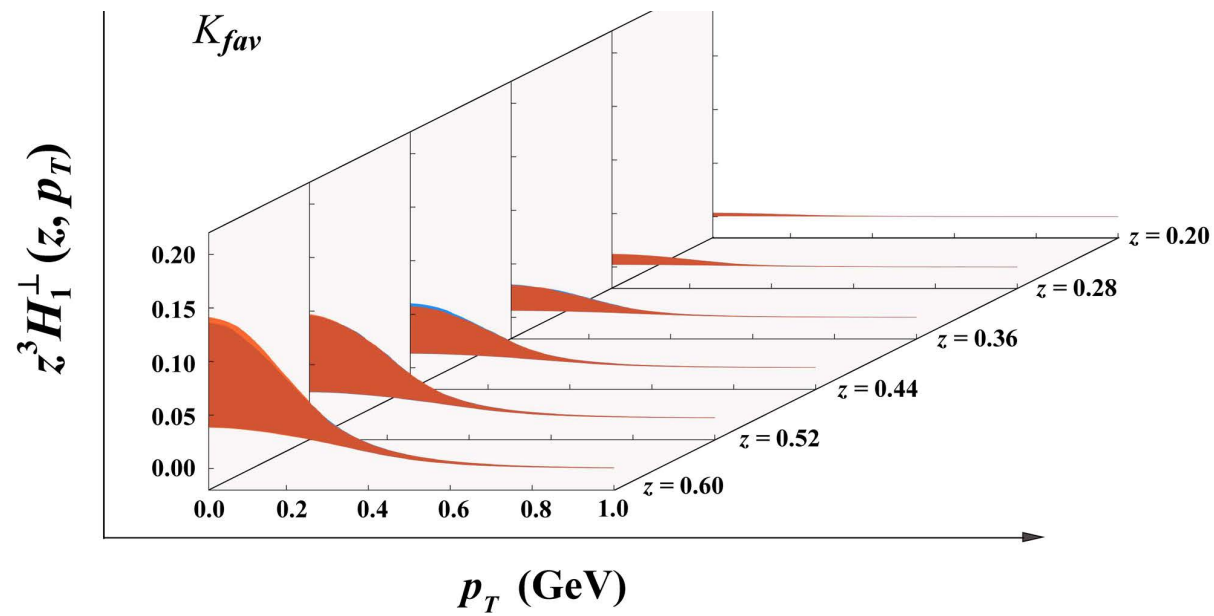
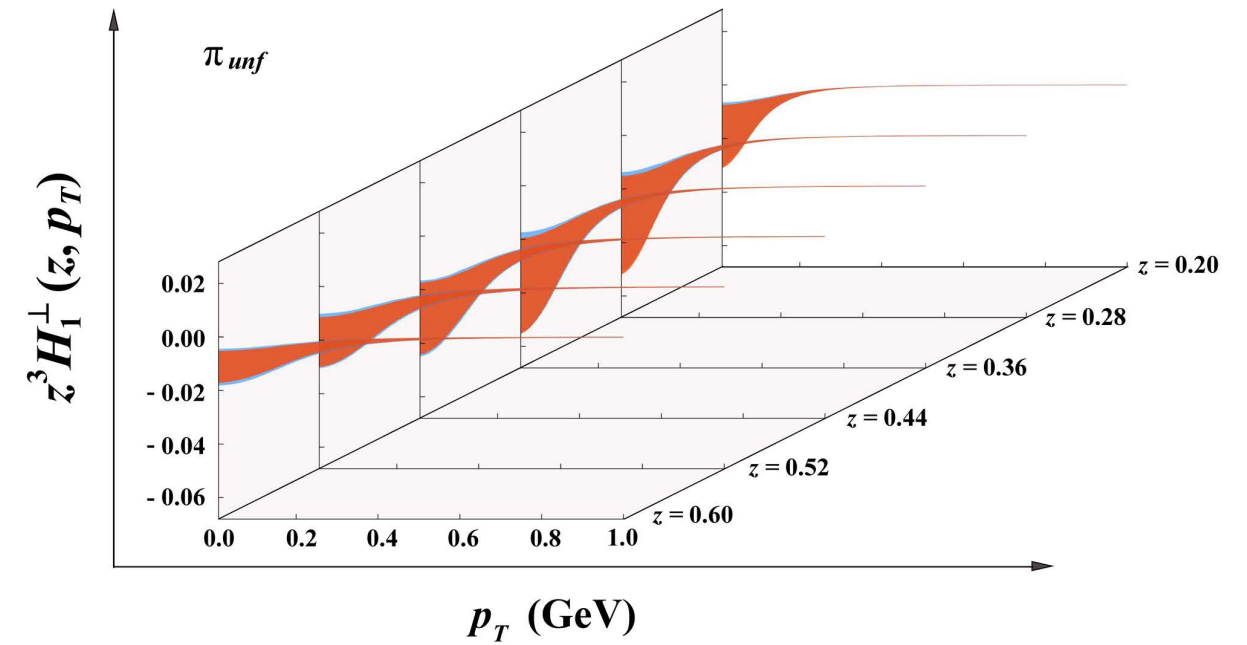
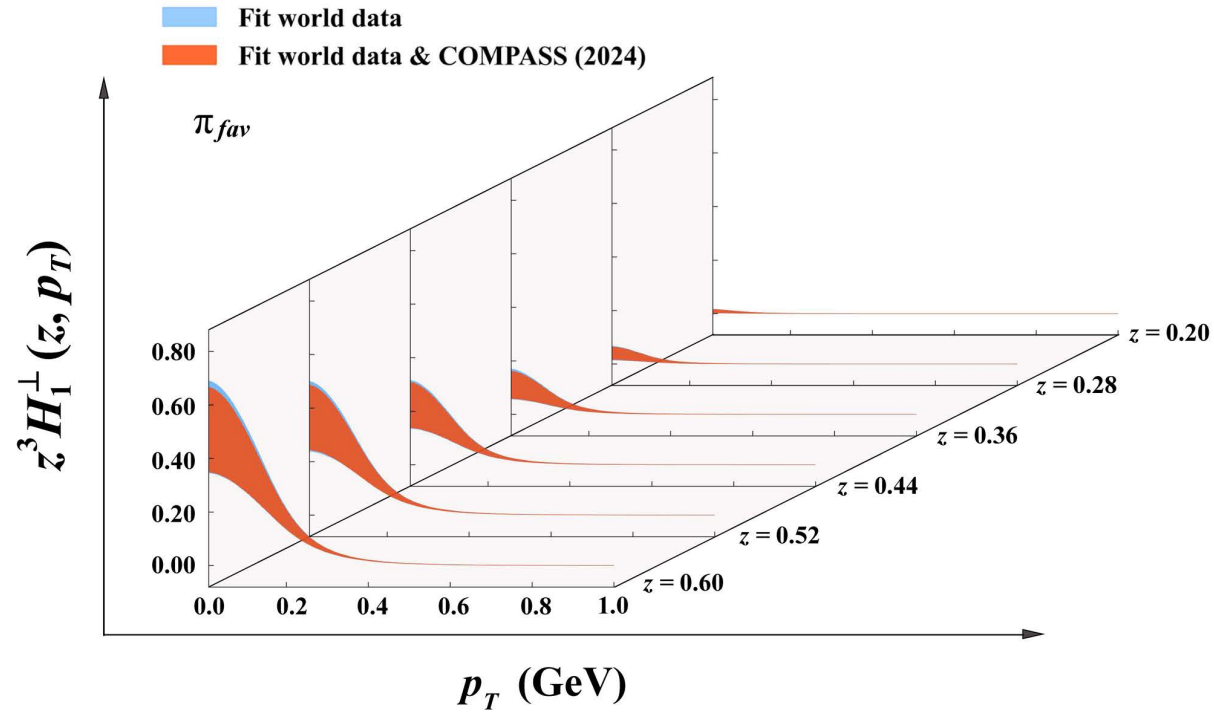


Transversity Distributions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

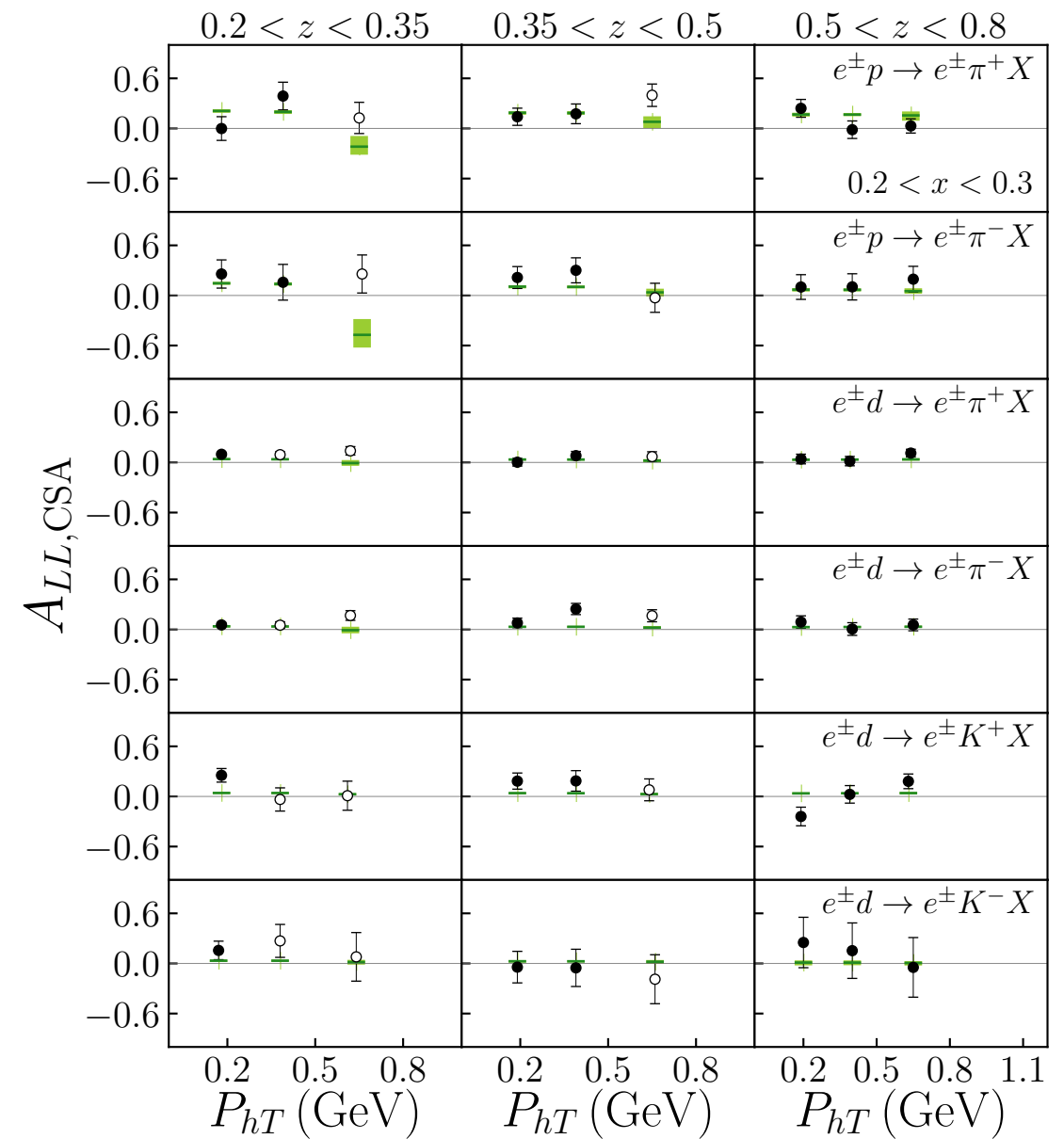
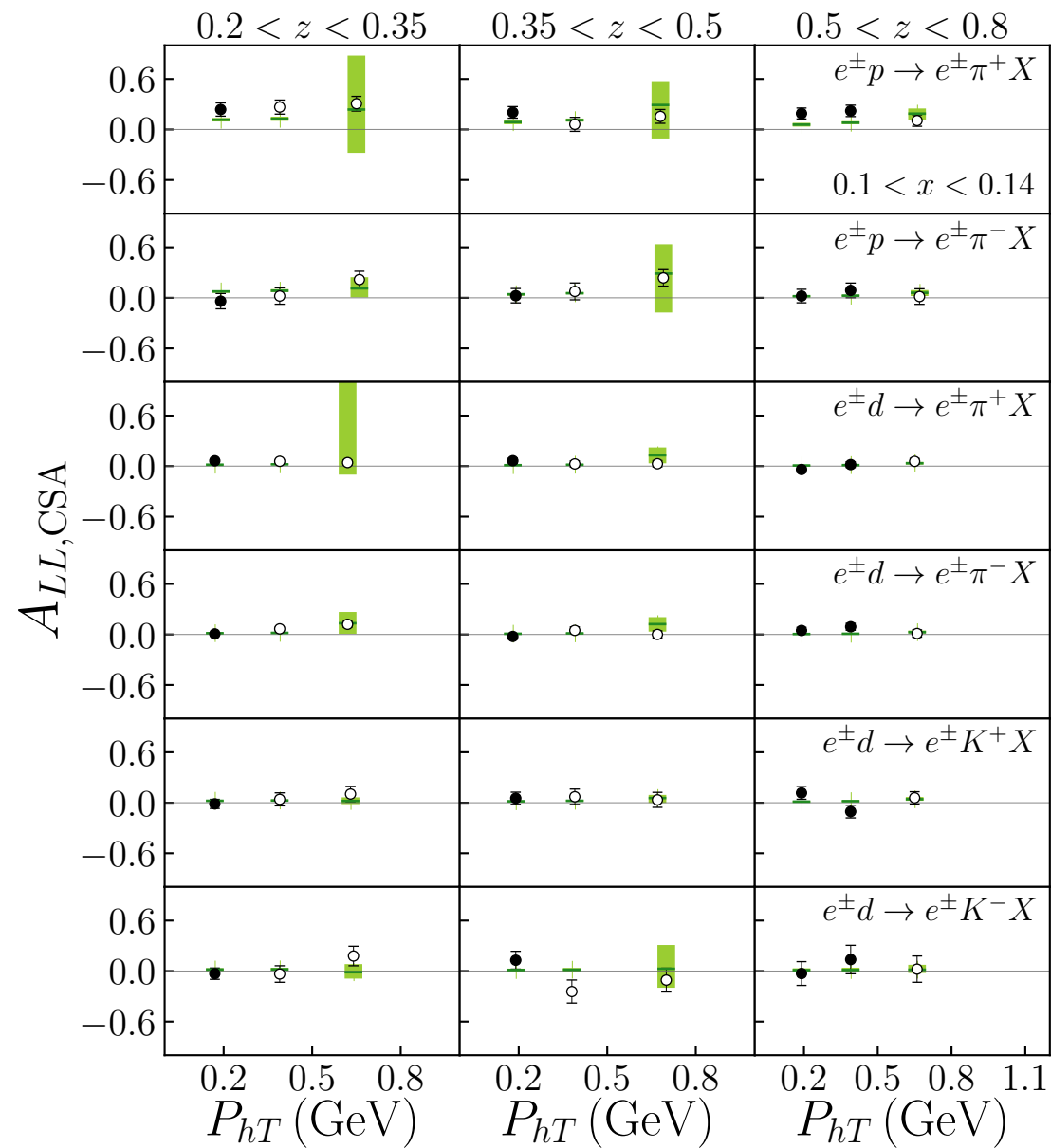
Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

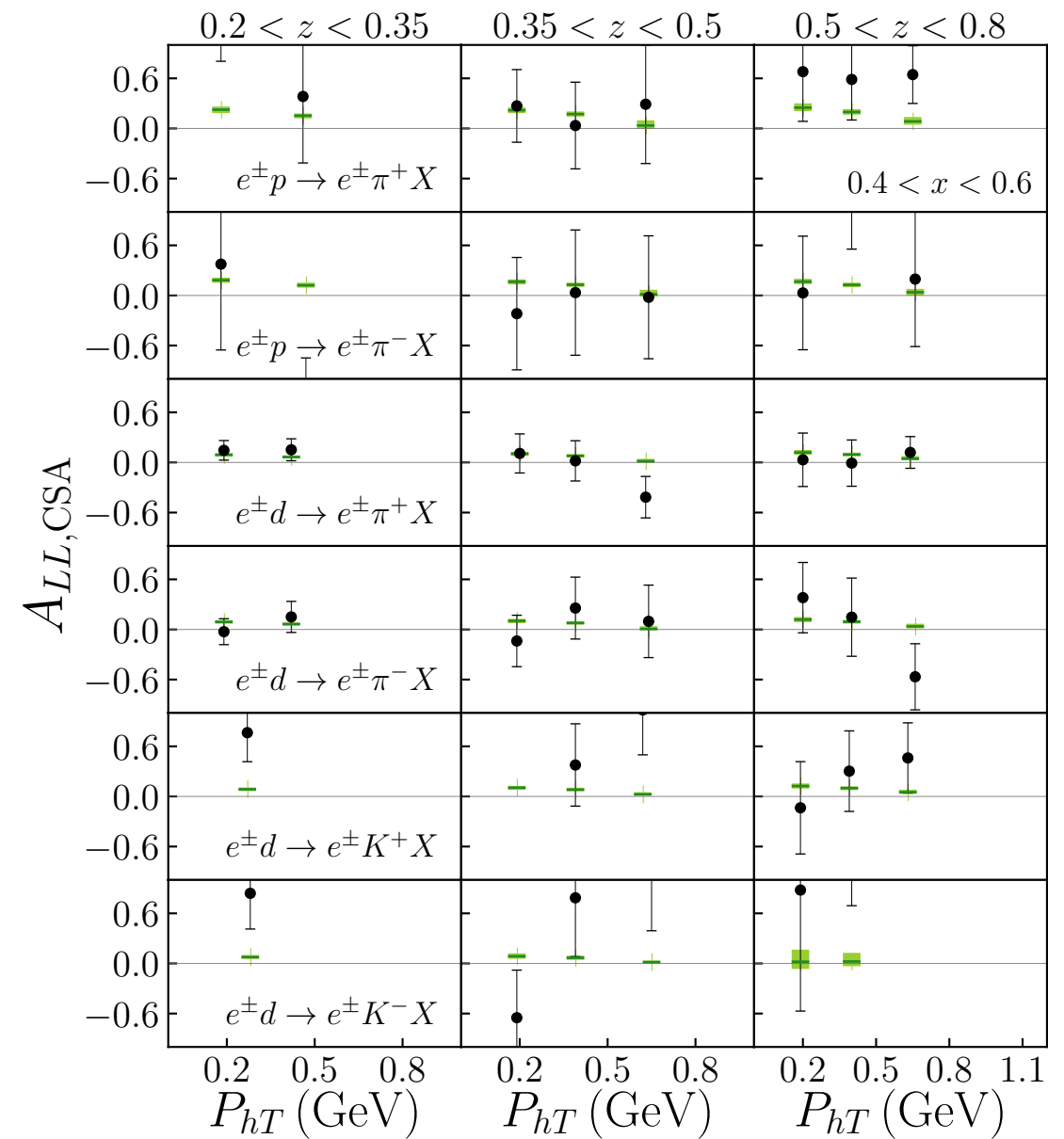
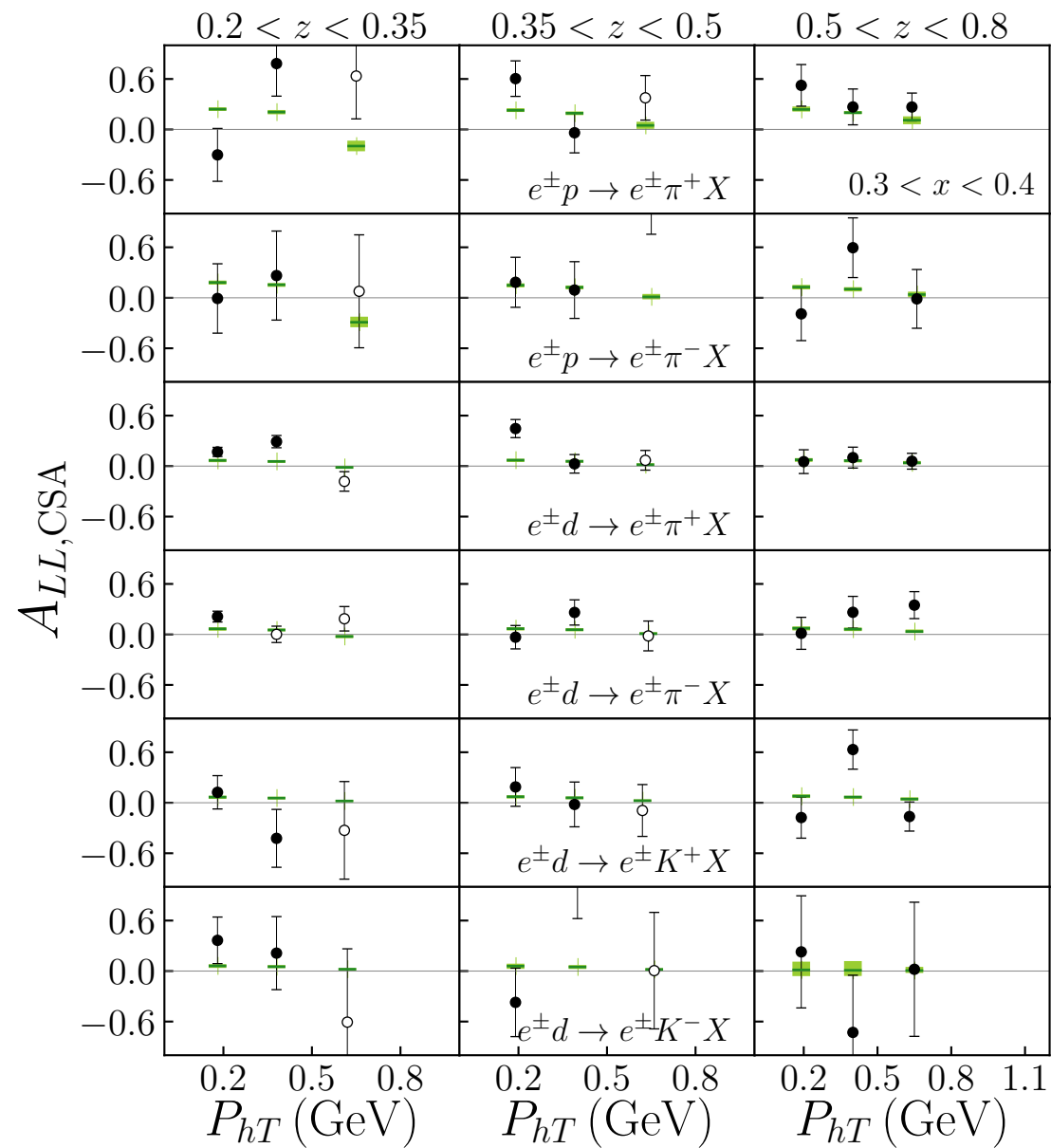
Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

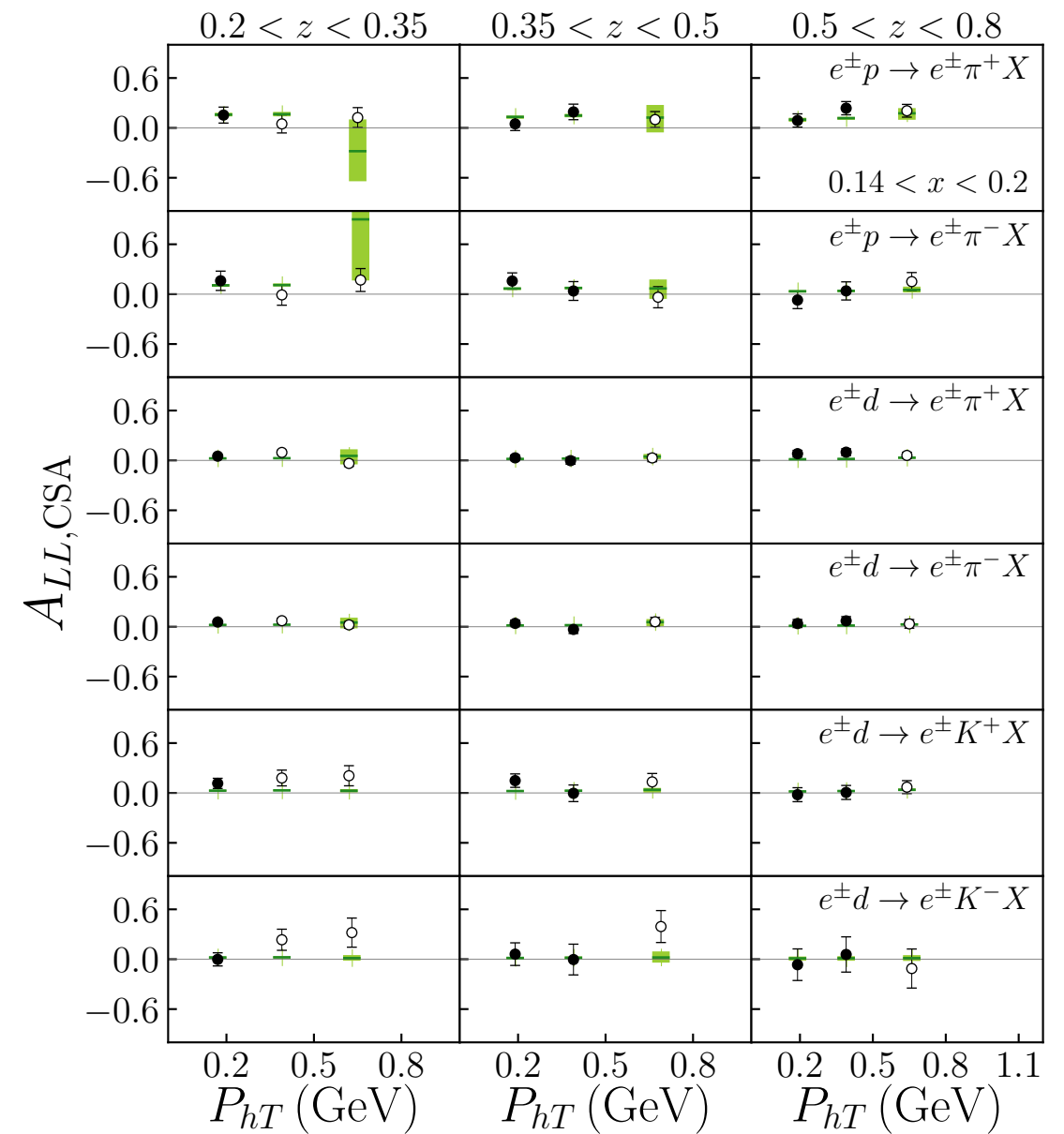
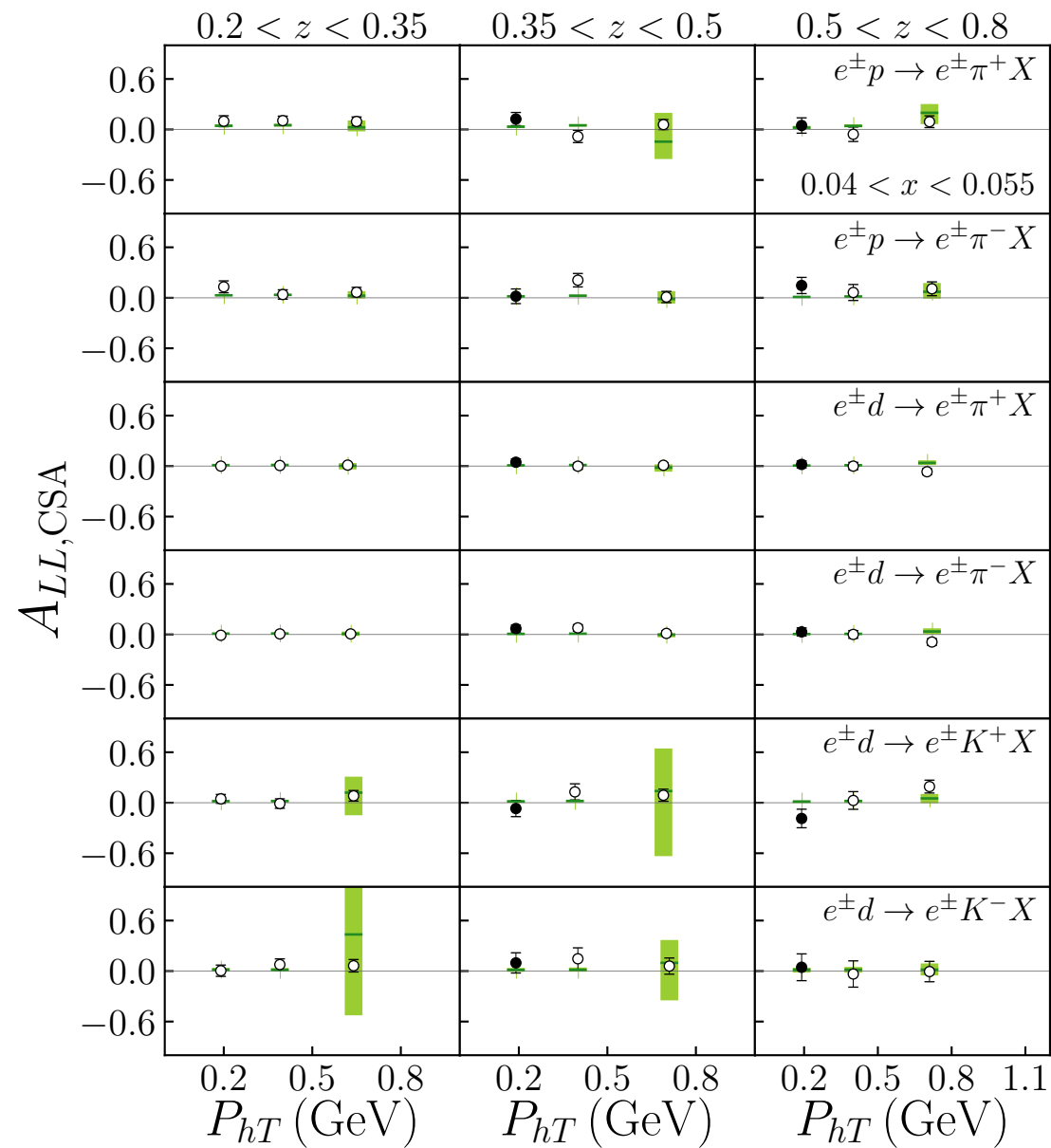
Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

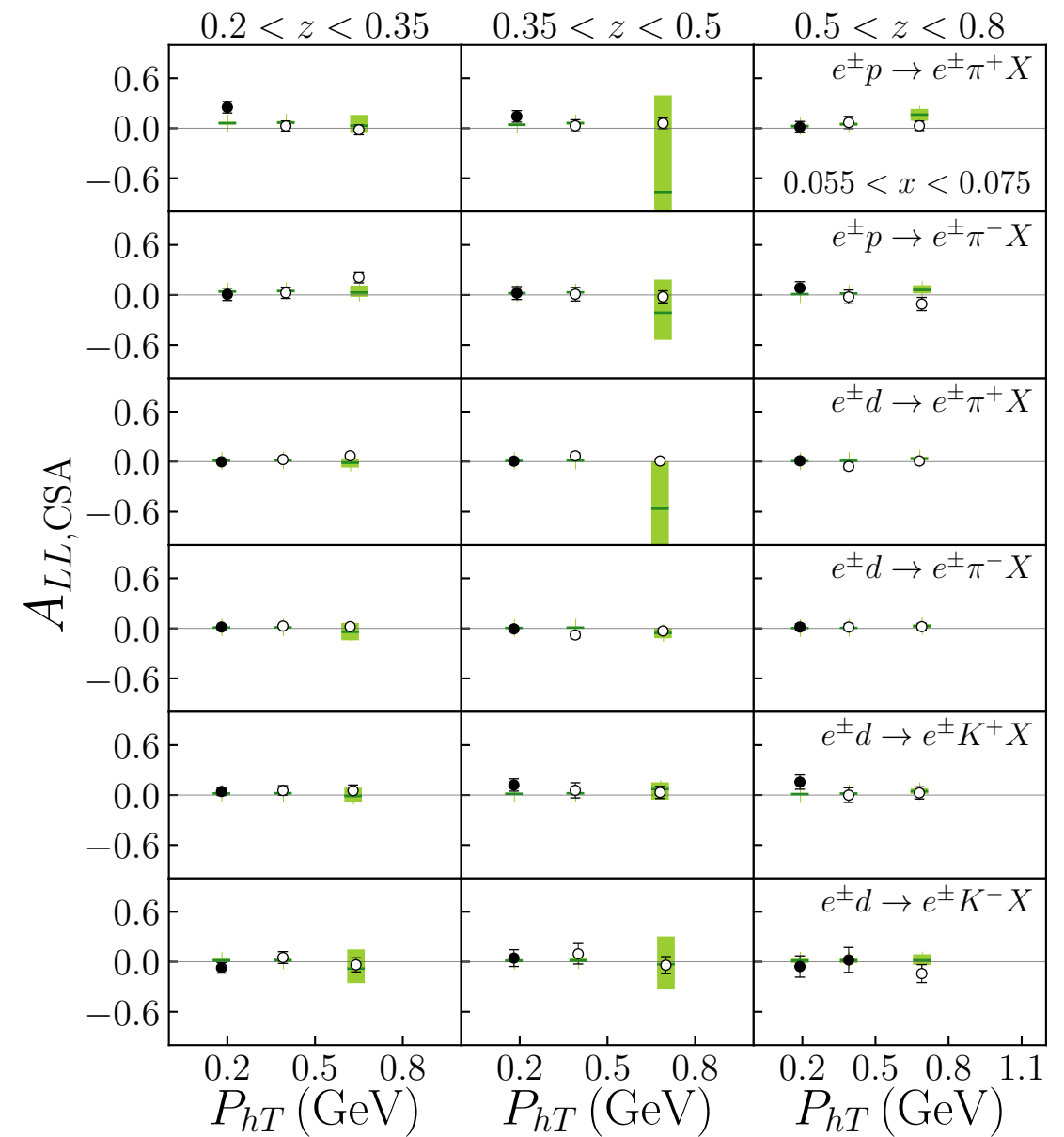
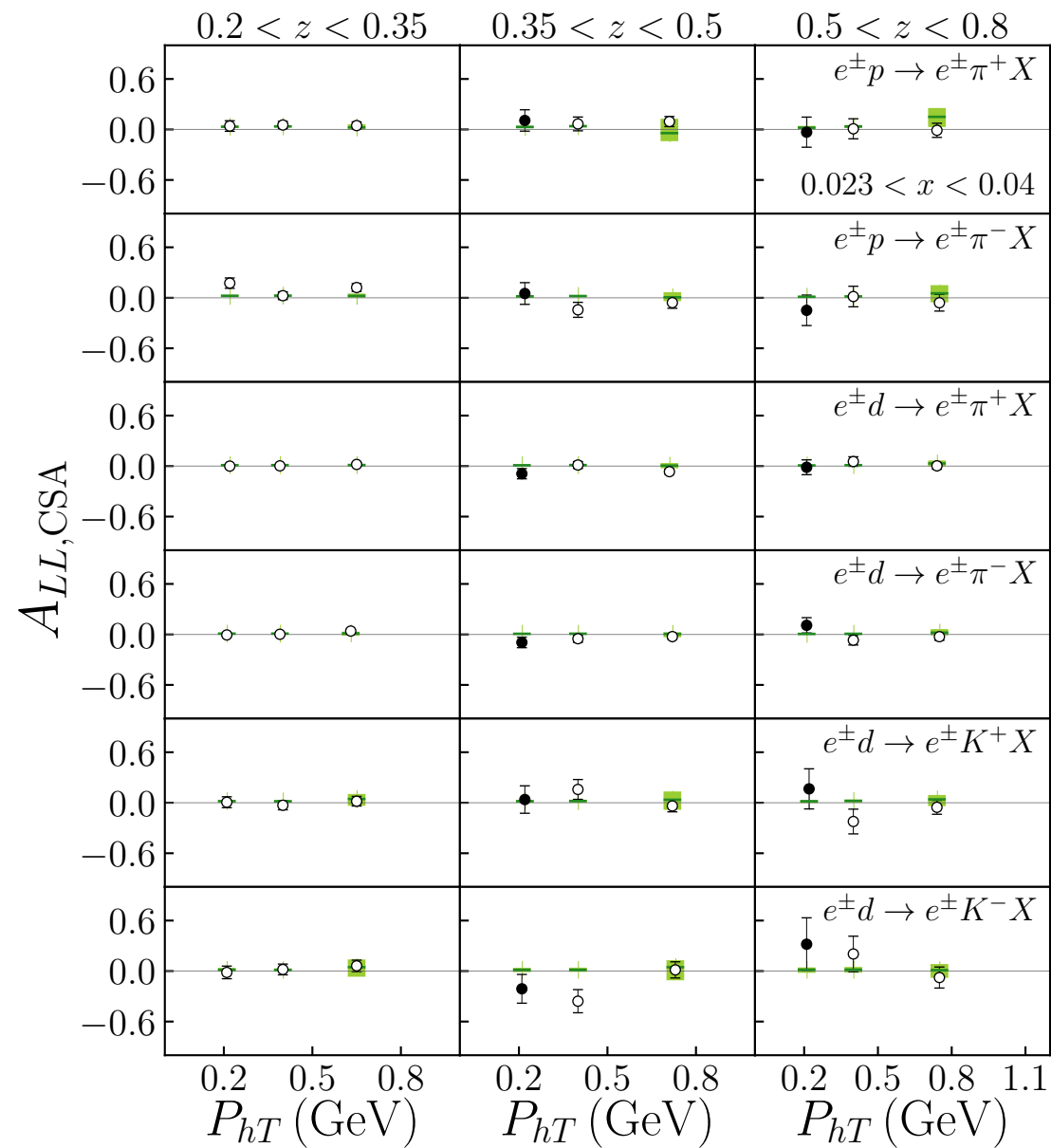
Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

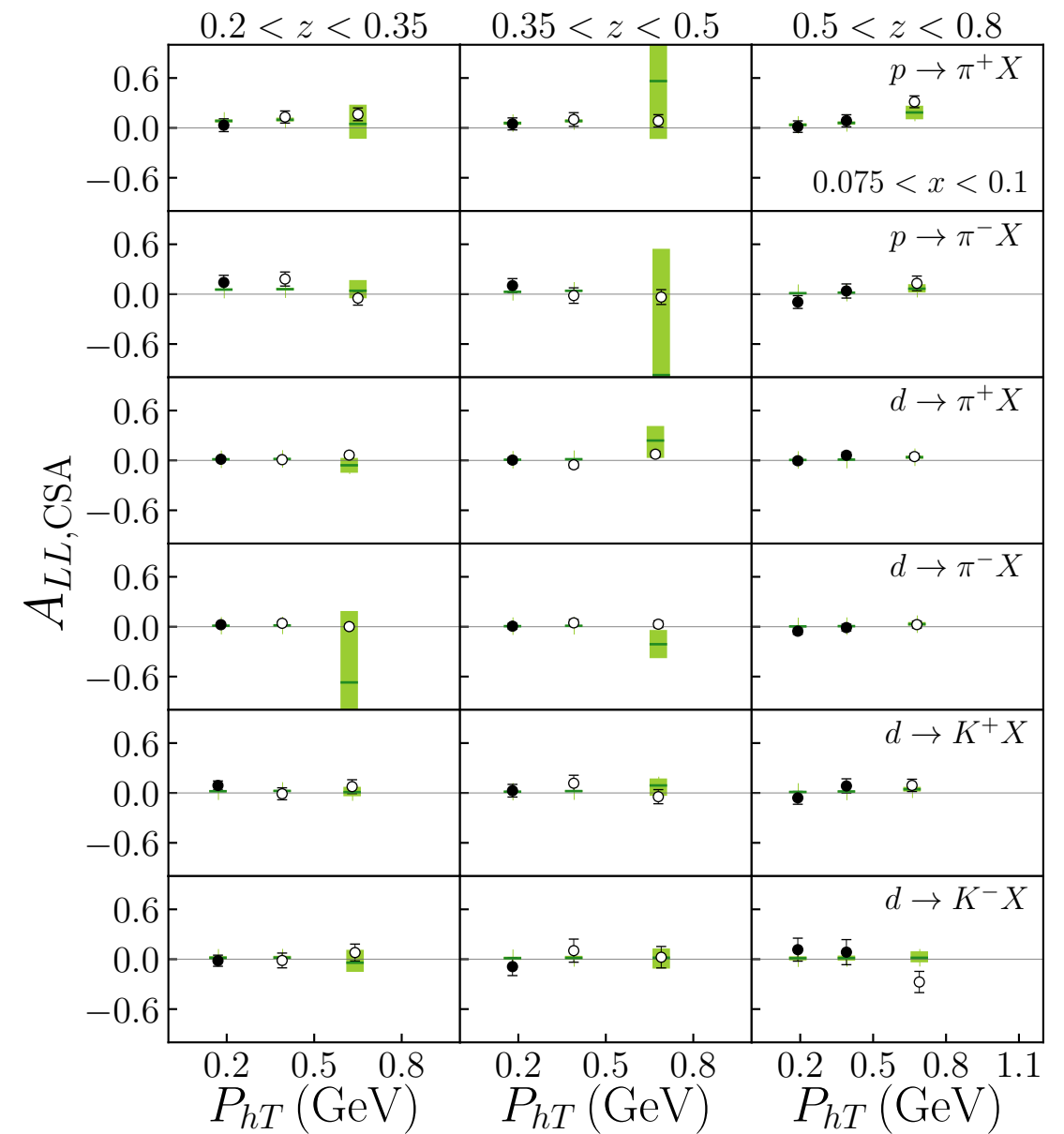
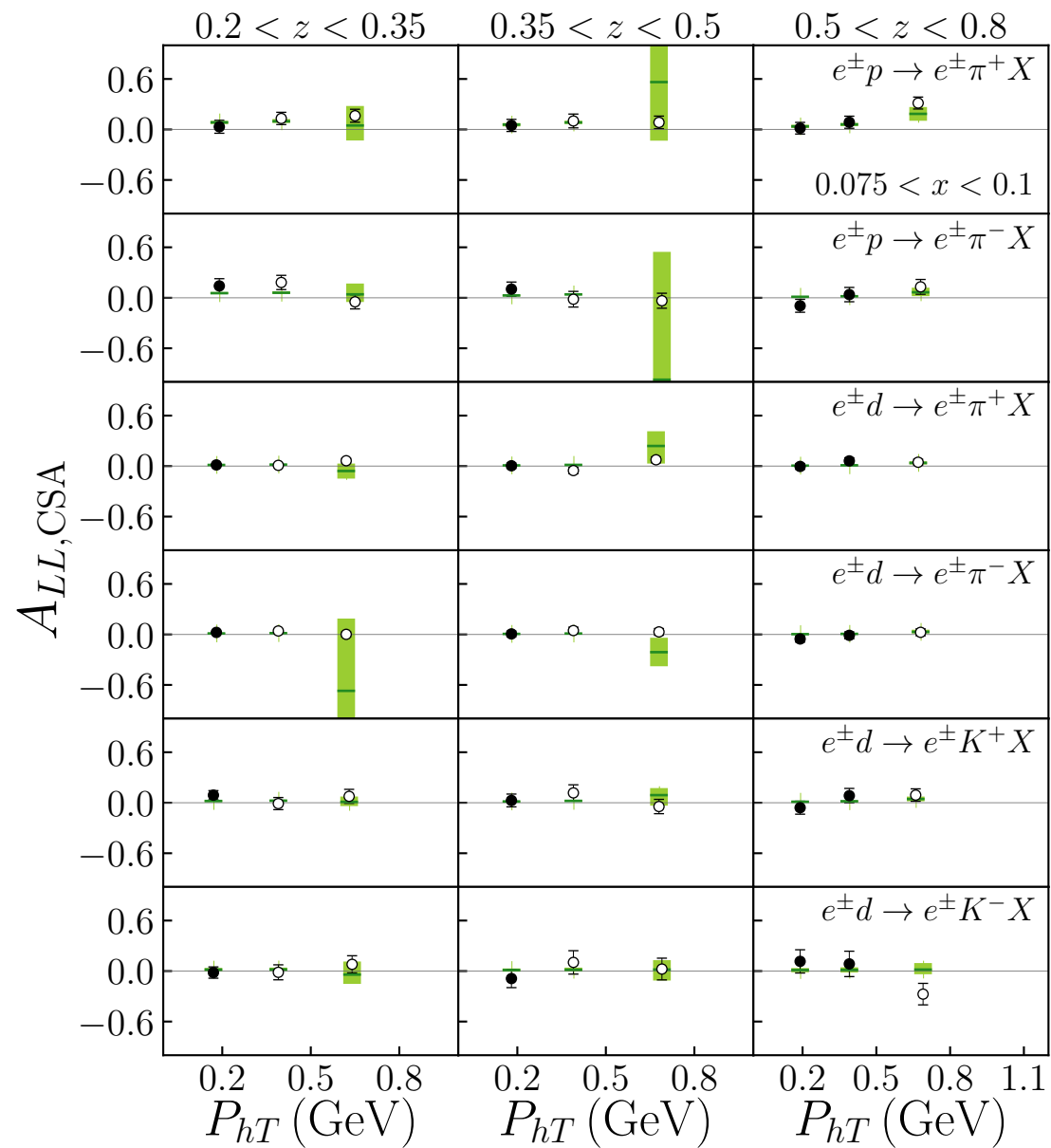
Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

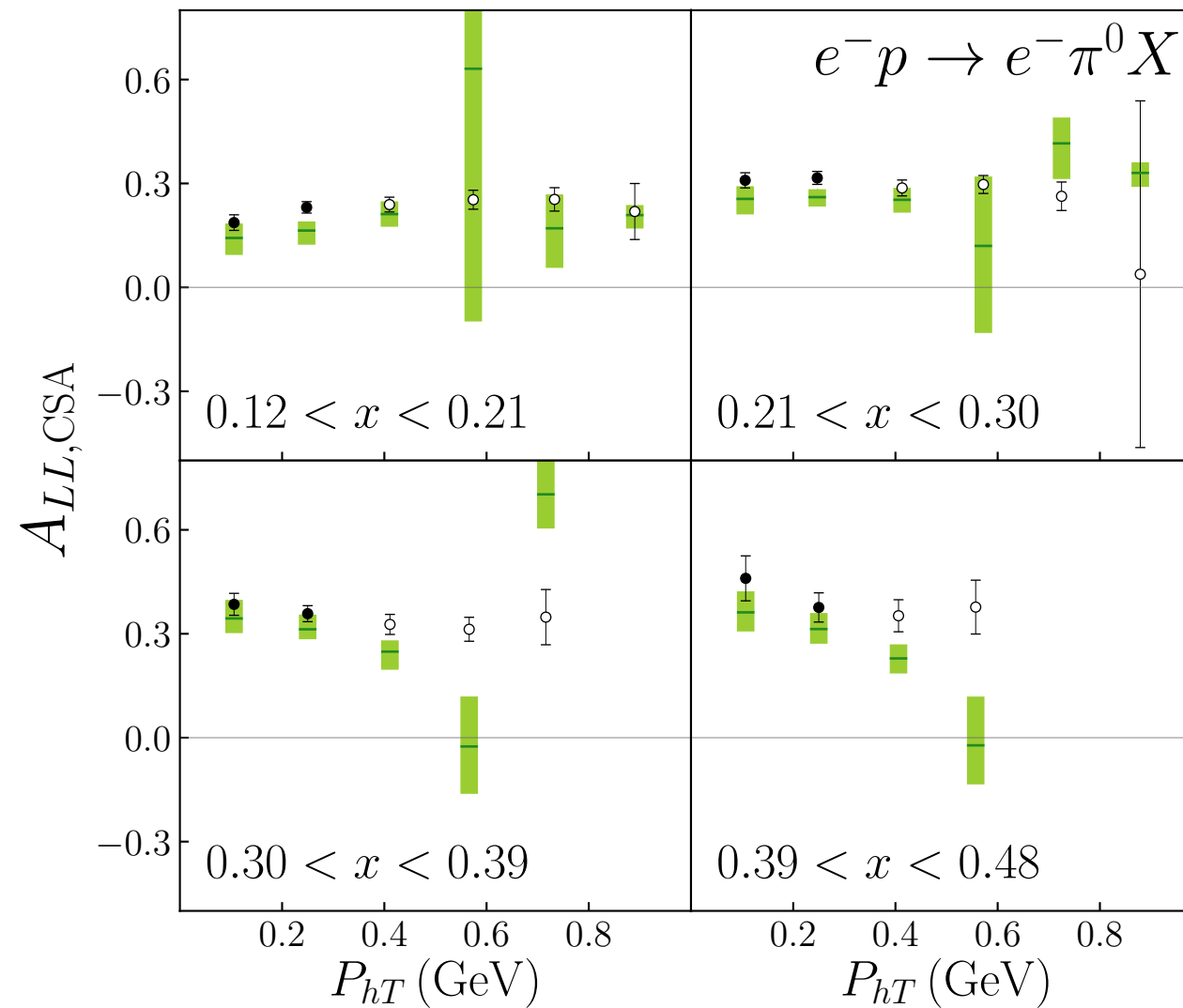
Compare with HERMES data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.

Comparison with Data

Compare with CLAS data



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902.