Unveiling Nucleon Structure via the Target Remnant

——SIDIS in the TFR

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青岛

Based on:
X.Q. Xi, **KBC**, X.B. Tong, S.Y. Wei, J. Wu, EPJC 85:1458 (2025)
KBC, J.P. Ma, X.B. Tong, JHEP 08 (2024) 227,
KBC, J.P. Ma, X.B. Tong, JHEP 05 (2024) 298,
KBC, J.P. Ma, X.B. Tong, PRD 108 (2023) 094015,
KBC, J.P. Ma, X.B. Tong, JHEP 11 (2021) 038.

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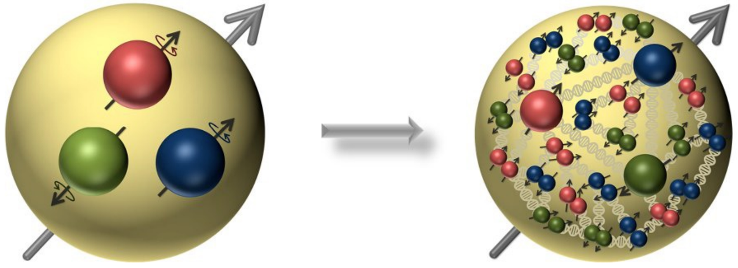
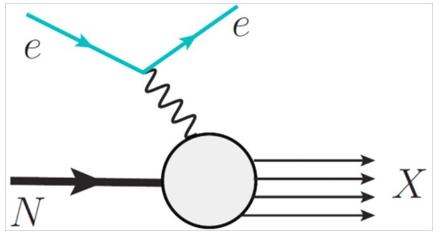
Introduction

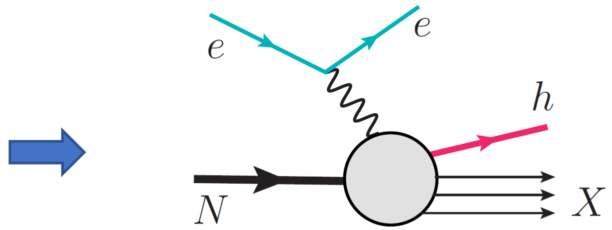
■ Nucleon tomography



Images from Front. Phys. 16(6), 64701

Lepton-nucleon deep inelastic scattering (DIS), the best tool for probing nucleon structure.





Inclusive DIS

Semi-inclusive DIS

Why nucleon structure important?

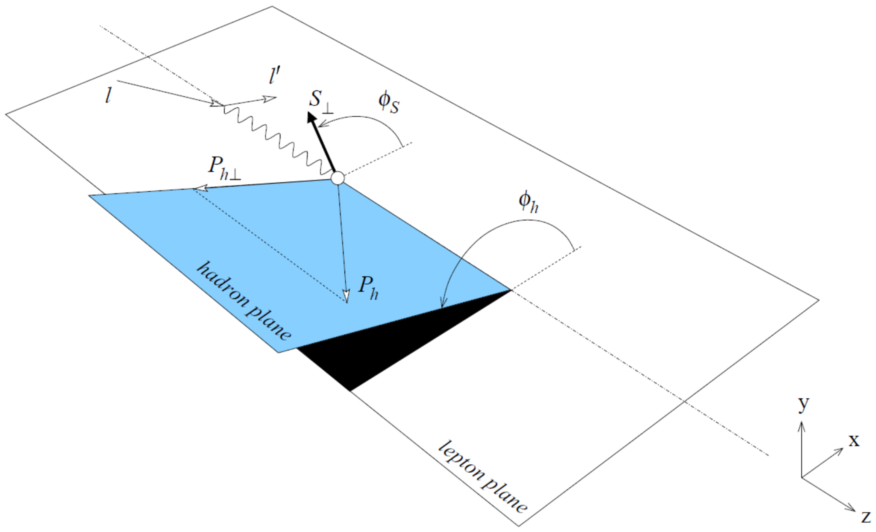
- Fundamental components of matter
- Properties of strong interaction
- Inputs for describing high energy reactions
- Prerequisite for new physics researches
- …… ……

Extra gains from DIS to SIDIS

- Three-dimensional imaging of the nucleon
- Azimuthal and/or spin asymmetries
- Accessing fragmentation functions (FFs)
- Decoding flavor compositions
- …… ……

Introduction

■ SIDIS structure functions



A. Bacchetta et al., JHEP 02, 093 (2007)

18 structure functions for SIDIS with polarized lepton and nucleon beams.

General kinematics, model independent.

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

$$\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h) F_{UL}^{\sin 2\phi_h}\right]$$

$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\sin(\phi_h+\phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S) F_{UT}^{\sin(3\phi_h-\phi_S)}$$

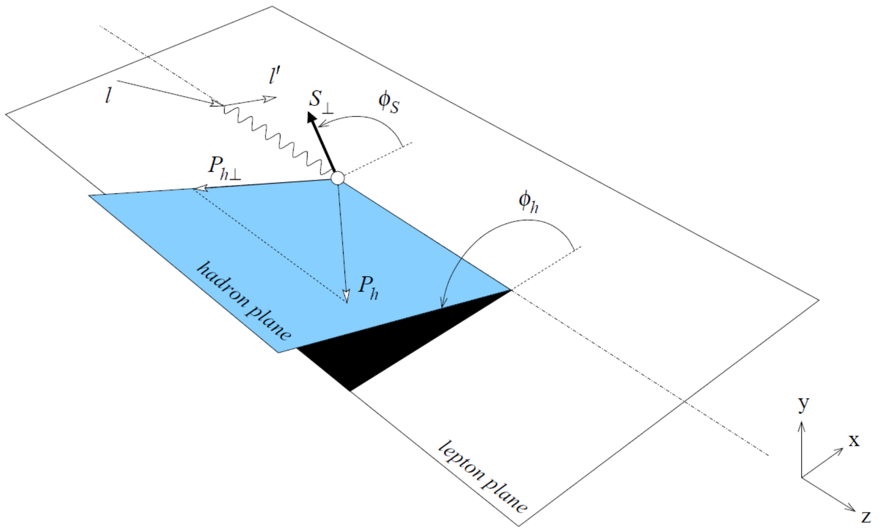
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S) F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S) F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S) F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

Introduction

SIDIS structure functions



A. Bacchetta et al., JHEP 02, 093 (2007)

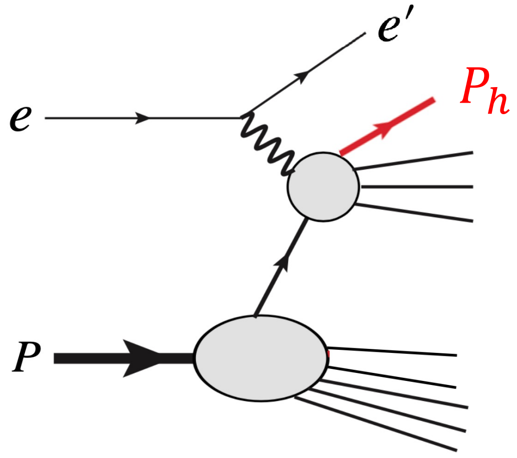
$P_{h\perp} \ll Q$ Transverse-momentum-dependent (TMD) factorization applies.

Structure functions are linked to TMD PDFs and TMD FFs.

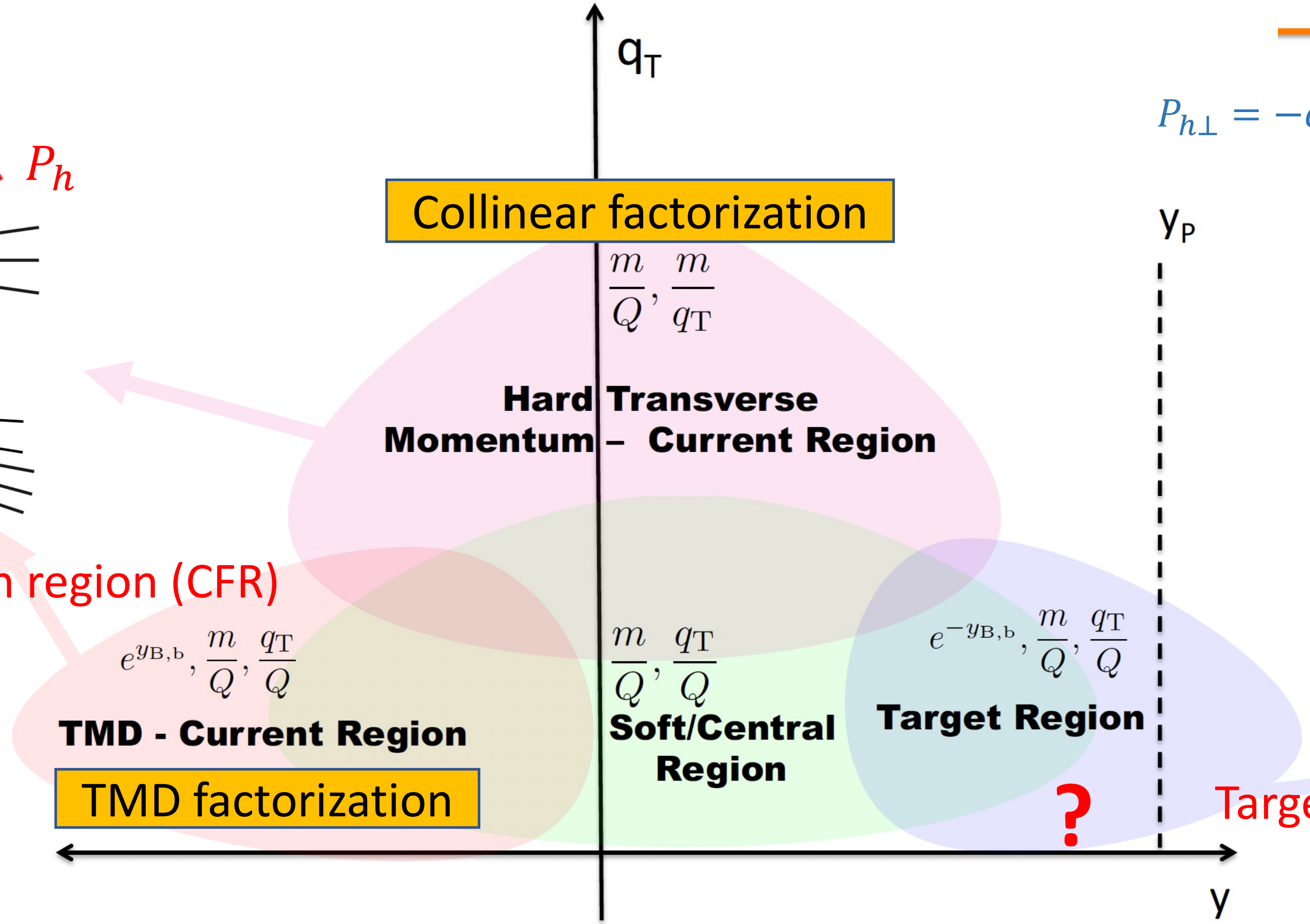
$F_{UU,T} \sim f_1 \otimes D_1$	Number density
$F_{LL} \sim g_{1L} \otimes D_1$	Helicity
$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$	Sivers
$F_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T}^{\perp} \otimes D_1$	Worm-gear L-T
$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$	Transversity
$F_{UU}^{\cos 2\phi_h} \sim h_1^{\perp} \otimes H_1^{\perp}$	Boer-Mulders
$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$	Worm-gear T-L
$F_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^{\perp} \otimes H_1^{\perp}$	Pretzelosity
$\vdots$	

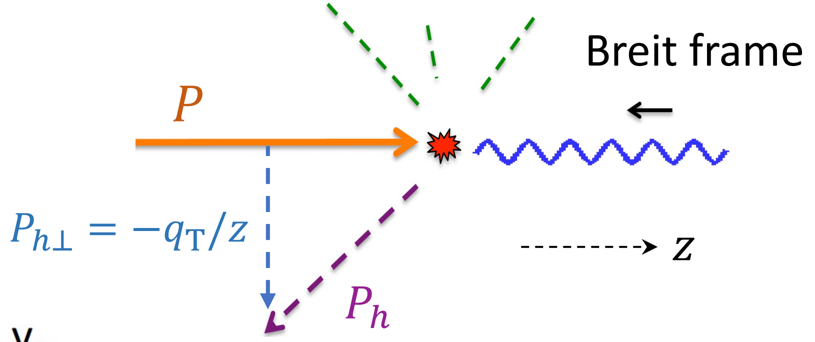
Introduction

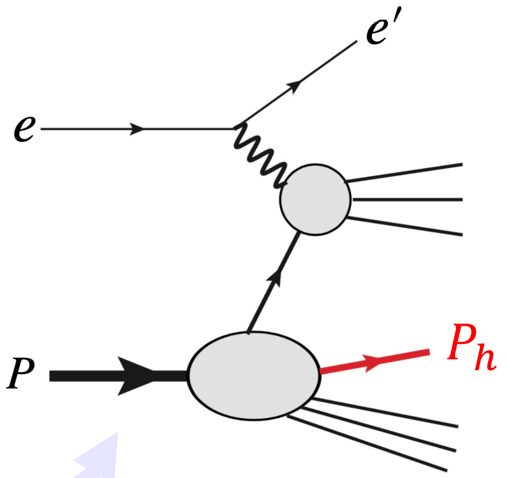
■ Kinematic regions of SIDIS



Current fragmentation region (CFR)







Target fragmentation region (TFR)

A region that has traditionally been overshadowed!

M. Boglione et al. JHEP 10 (2019) 122

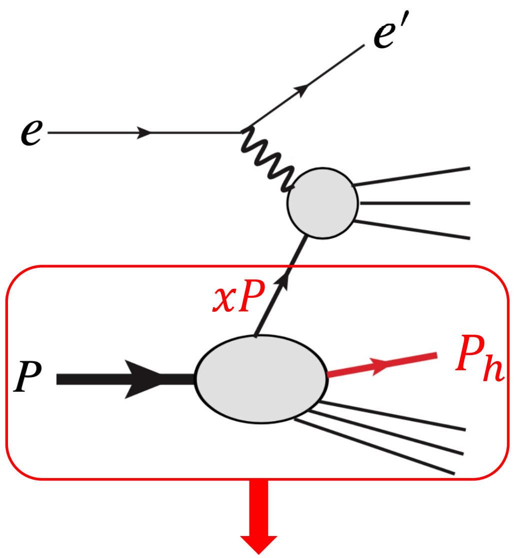
Introduction

■ Fracture functions and SIDIS in the TFR

- Fracture function: $F(x, \xi_h)$

Parton distributions in the presence of an almost collinear particle observed in the final state.
(conditional probability, hybrid of PDFs and FFs)

L. Trentadue and G. Veneziano, Phys. Lett. B323, 201 (1994)



$P_h \approx \xi_h P$

$d\sigma \sim F(x, \xi_h) \otimes H$

- Spin and hadron $P_{h\perp}$ dependent fracture functions

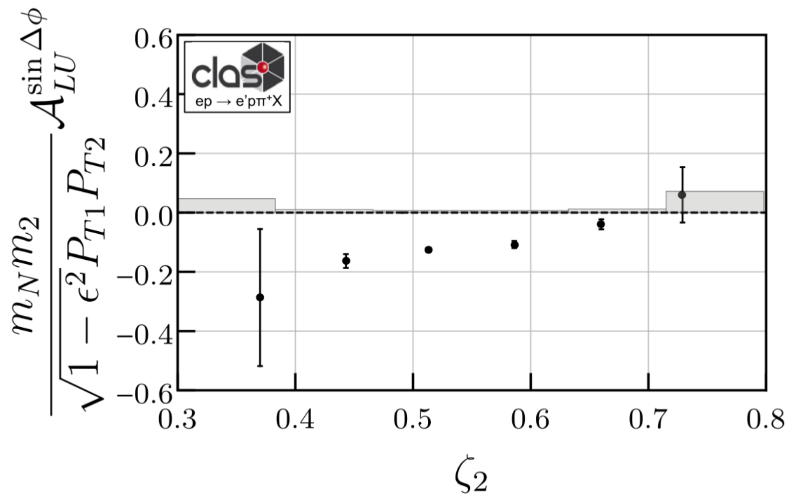
$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\left\{\left(1-y+\frac{y^2}{2}\right)\sum_a e_a^2\left[M(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\sin(\phi_h-\phi_S)\right]\right.$$
$$\left. + \lambda_l y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[S_\parallel \Delta M_L(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) + |\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\Delta M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\cos(\phi_h-\phi_S)\right]\right\}.$$

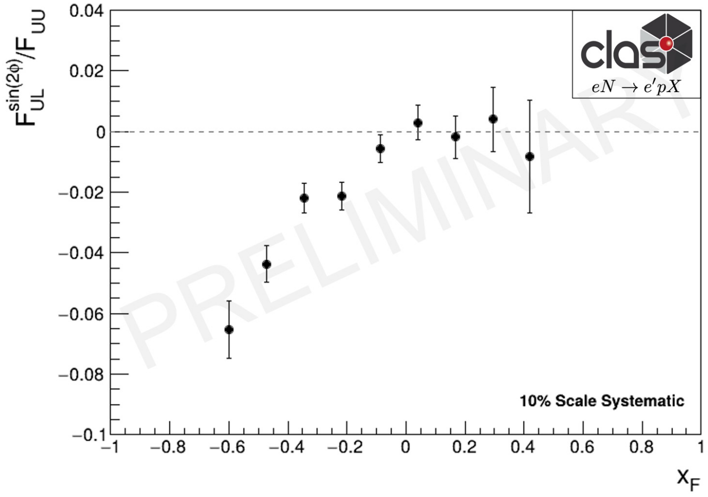
Cross section at twist-2

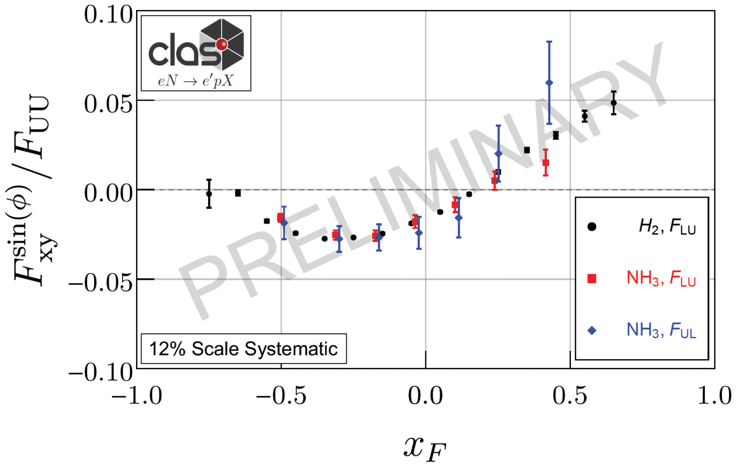
Anselmino, Barone, and Kotzinian, Phys. Lett. B699, 108 (2011); Phys. Lett. B706, 46 (2011).

Introduction

■ Recent experimental progresses







Larger Collins effect in the TFR?

But tiny chiral-odd effect are expected in the TFR.

Beam-spin asymmetry for back2back dihadron (a probe for TMD fracture functions)

H. Avakian et al., PRL 130, 022501 (2023)

Azimuthal dependent structure functions from CFR to TFR

e.g., A. Accardi et al., EPJA 60:173 (2024); Hayward's talk at IWHSS-CPHI-2024

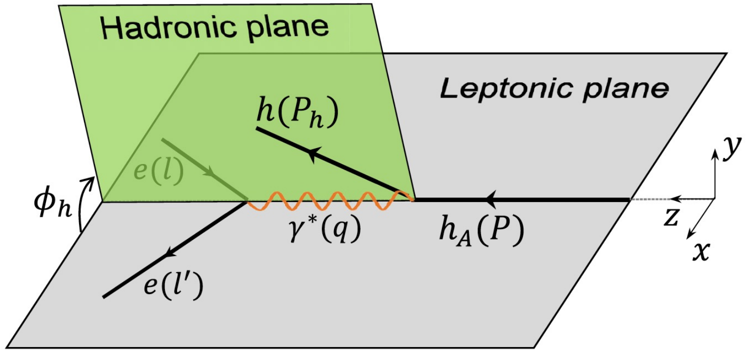
Theoretical point of view

- higher twist effects?
- pQCD corrections?

Contents

TFR SIDIS at twist-3 or one-loop

■ Kinematics



$$e(l, \lambda_e) + h_A(P, S) \rightarrow e(l') + h(P_h) + X$$

$$\frac{d\sigma}{dx_B dy d\xi_h d\psi d^2 P_{h\perp}} = \frac{\alpha^2 y}{4\xi_h Q^4} L_{\mu\nu}(l, \lambda_e, l') W^{\mu\nu}(q, P, S, P_h)$$

$$L^{\mu\nu}(l, \lambda_e, l') = 2(l^\mu l'^\nu + l^\nu l'^\mu - l \cdot l' g^{\mu\nu}) + 2i\lambda_e \epsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma$$

$$W^{\mu\nu}(q, P, S, P_h) = \sum_X \int \frac{d^4 x}{(2\pi)^4} e^{iq\cdot x} \langle S; h_A | J^\mu(x) | hX \rangle \langle Xh | J^\nu(0) | h_A; S \rangle$$

$$P^\mu \approx (P^+, 0,0,0) \qquad l^\mu = (\frac{1-y}{y} x_B P^+, \frac{Q^2}{2x_B y P^+}, \frac{Q\sqrt{1-y}}{y}, 0)$$

$$P_h^\mu = (P_h^+, P_h^-, \vec{P}_{h\perp}) \qquad q^\mu = (-x_B P^+, \frac{Q^2}{2x_B P^+}, 0,0)$$

$$S^\mu = (\frac{S_L P^+}{M}, -\frac{S_L M}{2P^+}, \vec{S}_\perp)$$

$$Q^2 = -q^2,$$

$$x_B = \frac{Q^2}{2P\cdot q},$$

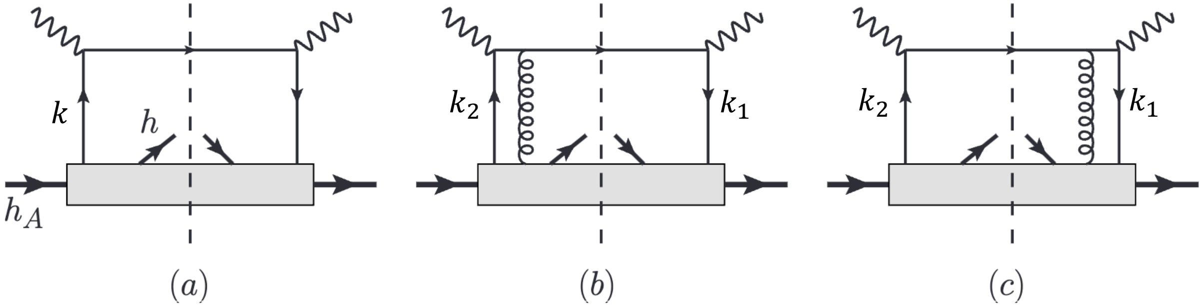
$$y = \frac{P\cdot q}{P\cdot l},$$

$$\cancel{z_h = \frac{P\cdot P_h}{P\cdot q}} \ll 1.$$

$$\xi_h = \frac{P_h \cdot q}{P\cdot q} \approx \frac{P_h^+}{P^+},$$

TFR SIDIS at twist-3 or one-loop

■ Hadronic tensor up to twist-3



……

Multiple gluon scattering in the final state.

$$W^{\mu\nu}\Big|_a = \int \frac{d^3k}{(2\pi)^3}\left[\left(\gamma^\mu(\not{k}+\not{q})\gamma^\nu\right)_{ij} 2\pi\delta\big((k+q)^2\big)\right]\sum_X \int \frac{d^3\eta}{(2\pi)^4} e^{-ik\cdot\eta}\langle h_A|\bar{\psi}_i(\eta)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

$$W^{\mu\nu}\Big|_b = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu(\not{k}_1+\not{q})\gamma_\alpha \frac{i(\not{k}_2+\not{q})}{(k_2+q)^2+i\epsilon}\gamma^\nu\right)_{ij} 2\pi\delta\big((k_1+q)^2\big)\right]$$
$$\times(-ig_s)\sum_X\int\frac{d^3\eta d^3\eta_1}{(2\pi)^4}e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar{\psi}_i(\eta)|hX\rangle\langle Xh|G^\alpha(\eta_1)\psi_j(0)|h_A\rangle,$$

$$W^{\mu\nu}\Big|_c = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu \frac{i(\not{k}_1+\not{q})}{(k_1+q)^2-i\epsilon}\gamma_\alpha(\not{k}_2+\not{q})\gamma^\nu\right)_{ij} 2\pi\delta\big((k_2+q)^2\big)\right]$$
$$\times(-ig_s)\sum_X\int\frac{d^3\eta d^3\eta_1}{(2\pi)^4}e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar{\psi}_i(\eta)G^\alpha(\eta_1)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

$k_i \sim Q(1,\lambda^2,\lambda)$, with $\lambda \sim \frac{\Lambda_{QCD}}{Q}$

Collinear expansion for the hard part, e.g.,

$$H^{(0)}_{\mu\nu}(k,q) = H^{(0)}_{\mu\nu}(\hat{k},q) + k_\perp^\alpha \frac{\partial H^{(0)}_{\mu\nu}(\hat{k},q)}{\partial k_\perp^\alpha} + \cdots$$

TFR SIDIS at twist-3 or one-loop

twist-2

twist-3

$$W^{\mu\nu}=\xi_h(\gamma^\mu\gamma^+\gamma^\nu)_{ij}\mathcal{M}_{ji}(x_B)+\left[\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\mathcal{M}^\alpha_{\partial,ji}(x_B)+(\mu\leftrightarrow\nu)^*\right]$$
$$+\left\{\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\int dx_2\left[\mathrm{P}\frac{1}{x_2-x_B}-i\pi\delta(x_2-x_B)\right]\mathcal{M}^\alpha_{F,ji}(x_B,x_2)+(\mu\leftrightarrow\nu)^*\right\}$$

$$\boxed{\begin{aligned}
\mathcal{M}_{ij}(x)&=\int\frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\mathcal{L}_n(0)\psi_i(0)|h_A\rangle,\\
\mathcal{M}^\alpha_{\partial,ij}(x)&=\int\frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\partial_\perp^\alpha(\mathcal{L}_n\psi_i)(0)]|h_A\rangle,\\
\mathcal{M}^\alpha_{F,ij}(x_1,x_2)&=\int\frac{d\eta^-d\eta_1^-}{4\pi\xi_h(2\pi)^4}e^{-ix_1P^+\eta^--i(x_2-x_1)P^+\eta_1^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)|hX\rangle\langle Xh|g_sF^{+\alpha}(\eta_1^-)\psi_i(0)|h_A\rangle.
\end{aligned}}$$

$$\mathcal{L}_n(x)=\mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\ G^+(\lambda n+x)\right\}$$

Gauge invariant matrix elements!

$$g_sF^{+\alpha}=g_s[\partial^+G^\alpha_\perp-\partial^\alpha_\perp G^+]+\mathcal{O}(g_s^2)$$

TFR SIDIS at twist-3 or one-loop

■ Fracture functions defined via the correlation matrices

$$\mathcal{M}_{ij}(x) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(u_1 - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^h\right) + \frac{1}{P^+}\left(P_{h\perp}^\rho u^h - M\tilde{S}_\perp^\rho u_T - S_L\tilde{P}_{h\perp}^\rho u_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_T^h\right)\right]$$
$$- \frac{(\gamma_\rho\gamma_5)_{ij}}{2N_c}\left[\bar{n}^\rho\left(S_L l_{1L} - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^h\right) + \frac{1}{P^+}\left(\tilde{P}_{h\perp}^\rho l^h + MS_\perp^\rho l_T + S_L P_{h\perp}^\rho l_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_T^h\right)\right] + \cdots,$$

$$\mathcal{M}_{\partial,ij}^\alpha(x) = \frac{(\gamma^-)_{ij}}{2N_c}i\left(-P_{h\perp}^\alpha u_\partial^h + M\tilde{S}_\perp^\alpha u_{\partial T} + S_L\tilde{P}_{h\perp}^\alpha u_{\partial L}^h + \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_{\partial T}^h\right)$$
$$+ \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha l_\partial^h + MS_\perp^\alpha l_{\partial T} + S_L P_{h\perp}^\alpha l_{\partial L}^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_{\partial T}^h\right) + \cdots,$$

$$\mathcal{M}_{F,ij}^\alpha(x_1,x_2) = \frac{(\gamma^-)_{ij}}{2N_c}\left(P_{h\perp}^\alpha w^h - M\tilde{S}_\perp^\alpha w_T - S_L\tilde{P}_{h\perp}^\alpha w_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}w_T^h\right)$$
$$- \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha v^h + MS_\perp^\alpha v_T + S_L P_{h\perp}^\alpha v_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}v_T^h\right) + \cdots,$$

Fracture functions up to twist-3

$$\tilde{a}_\perp^\mu \equiv \varepsilon_\perp^{\mu\nu}a_{\perp\nu}$$

$$P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle} \equiv P_{h\perp}^\alpha P_{h\perp}^\beta + g_\perp^{\alpha\beta}\vec{P}_{h\perp}^2/2$$

- Red: twist-2.
- Green: twist-3

Only chiral-even terms contribute, different from CFR

TFR SIDIS at twist-3 or one-loop

■ Fracture functions defined via the correlation matrices

The twist-3 fracture functions are not independent from each other!

Applying QCD equation of motion $i\gamma \cdot D\psi = 0$, gives:

$$x[u_S^K(x) + il_S^K(x)]$$
$$= u_{\partial S}^K(x) + il_{\partial S}^K(x) + i\int dy \left[\mathrm{P}\frac{1}{y-x} - i\pi\delta(y-x)\right][w_S^K(x,y) - v_S^K(x,y)]$$

Four sets of equations

S	K
null	h
L	h
T	null
T	h

The relations take a unified form!

The hadronic tensor will be expressed by those fracture functions defined only from the quark-quark correlator $\mathcal{M}_{ij}$.

TFR SIDIS at twist-3 or one-loop

Results for structure functions and azimuthal asymmetries

Structure functions

Twist-2

$$F_{UU,T} = x_B u_1, \qquad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B u_{1T}^h,$$

$$F_{LL} = x_B l_{1L}, \qquad F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B l_{1T}^h.$$

Twist-3

$$F_{UU}^{\cos\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u^h, \qquad F_{LU}^{\sin\phi_h} = \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l^h,$$

$$F_{UL}^{\sin\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u_L^h, \qquad F_{LL}^{\cos\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l_L^h,$$

$$F_{UT}^{\sin\phi_S} = -\frac{2M}{Q} x_B^2 u_T, \qquad F_{LT}^{\cos\phi_S} = -\frac{2M}{Q} x_B^2 l_T,$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 u_T^h, \qquad F_{LT}^{\cos(2\phi_h-\phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 l_T^h.$$

Azimuthal/spin asymmetries

Twist-2

$$\langle \sin(\phi_h-\phi_S)\rangle_{UT} = \frac{|\vec{P}_{h\perp}|}{2M}\frac{u_{1T}^h}{u_1},$$

$$\langle \cos(\phi_h-\phi_S)\rangle_{LT} = \frac{|\vec{P}_{h\perp}|C(y)}{2M\,A(y)}\frac{l_{1T}^h}{u_1}.$$

Twist-3

$$\langle\cos\phi_h\rangle_{UU} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_B u^h}{u_1}, \qquad \langle\sin\phi_h\rangle_{LU} = \frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_B l^h}{u_1},$$

$$\langle\sin\phi_h\rangle_{UL} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_B u_L^h}{u_1}, \qquad \langle\cos\phi_h\rangle_{LL} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_B l_L^h}{u_1},$$

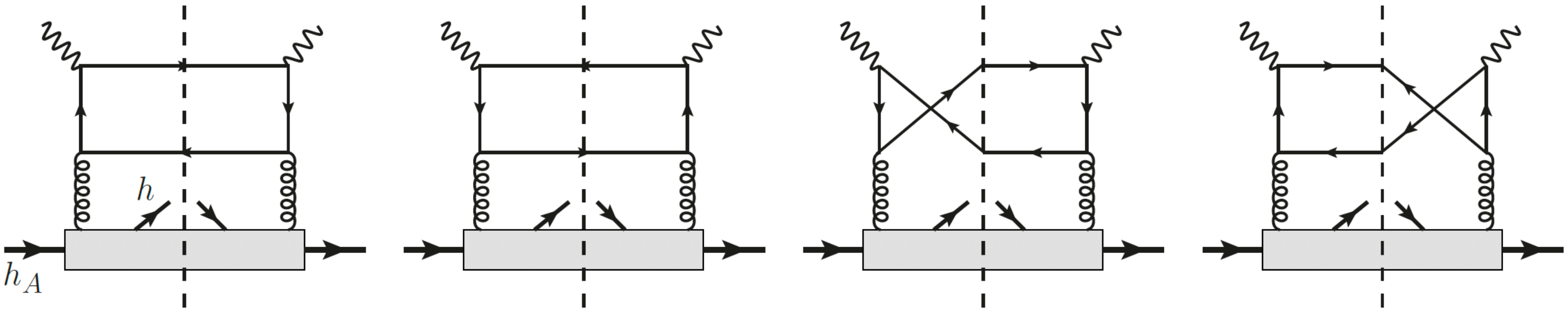
$$\langle\sin\phi_S\rangle_{UT} = -\frac{M}{Q}\frac{B(y)}{A(y)}\frac{x_B u_T}{u_1}, \qquad \langle\cos\phi_S\rangle_{LT} = -\frac{M}{Q}\frac{D(y)}{A(y)}\frac{x_B l_T}{u_1},$$

$$\langle\sin(2\phi_h-\phi_S)\rangle_{UT} = -\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{B(y)}{A(y)}\frac{x_B u_T^h}{u_1}, \qquad \langle\cos(2\phi_h-\phi_S)\rangle_{LT} = -\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{D(y)}{A(y)}\frac{x_B l_T^h}{u_1}.$$

K.B. Chen, J.P. Ma, X.B. Tong, PRD 108 (2023) 094015

TFR SIDIS at twist-3 or one-loop

■ One-loop correction: the gluonic contribution



(a) (b) (c) (d)

$$W^{\mu\nu}(q,P,S,P_h) = \alpha_s T_F \sum_f e_f^2 \int \frac{dx}{x} \int d\Phi_{k_1 k_2} H^{\mu\nu\alpha\beta}(k_g,k_1,k_2)\mathcal{M}_{G,\alpha\beta}(x,\xi_h,P_{h\perp}),$$

$$\mathcal{M}_G^{\alpha\beta}(x,\xi_h,P_{h\perp}) = \frac{1}{2\xi_h(2\pi)^3}\frac{1}{xP^+}\int \frac{d\lambda}{2\pi} e^{-i\lambda x P^+} \sum_X \langle h_A(P)|(G^{+\alpha}(\lambda n)\mathcal{L}_n^\dagger(\lambda n))^a|X h(P_h)\rangle$$
$$\times \langle h(P_h)X|(\mathcal{L}_n(0)G^{+\beta}(0))^a|h_A(P)\rangle,$$

$$\int d\Phi_{k_1 k_2} \equiv \int \frac{d^D k_1}{(2\pi)^D}\int \frac{d^D k_2}{(2\pi)^D}(2\pi)\delta(k_1^2)(2\pi)\delta(k_2^2)(2\pi)^4\delta^{(4)}(q+k_g-k_1-k_2)\theta(q^0+k_g^0)$$

TFR SIDIS at twist-3 or one-loop

Gluonic contribution

Gluon fracture functions

$$\mathcal{M}_G^{\alpha\beta} = -\frac{1}{2-2\epsilon} g_\perp^{\alpha\beta} u_{1g} + \frac{1}{2M^2}\left(P_{h\perp}^\alpha P_{h\perp}^\beta + \frac{1}{2-2\epsilon} g_\perp^{\alpha\beta} P_{h\perp}^2\right) t_{1g}^h + S_L\left[i\frac{\varepsilon_\perp^{\alpha\beta}}{2} l_{1gL} + \frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gL}^h\right]$$
$$+ \frac{g_\perp^{\alpha\beta}}{2-2\epsilon}\frac{P_{h\perp}\cdot\tilde{S}_\perp}{M} u_{1gT}^h + \frac{P_{h\perp}\cdot S_\perp}{M}\left[i\frac{\varepsilon_\perp^{\alpha\beta}}{2} l_{1gT}^h - \frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gT}^{hh}\right] + \frac{\tilde{P}_{h\perp}^{\{\alpha} S_\perp^{\beta\}} + \tilde{S}_\perp^{\{\alpha} P_{h\perp}^{\beta\}}}{8M} t_{1gT}^h + \cdots .$$

The hard parts

$$H_{(a)+(b)}^{\mu\nu\alpha\beta}(k_g,k_1,k_2) = \mathrm{Tr}\left[\not{k}_1\gamma^\nu \frac{(\not{k}_g-\not{k}_2)}{(k_g-k_2)^2}\gamma^\beta \not{k}_2\gamma^\alpha \frac{(\not{k}_g-\not{k}_2)}{(k_g-k_2)^2}\gamma^\mu\right] + (k_1\leftrightarrow k_2),$$

$$H_{(c)+(d)}^{\mu\nu\alpha\beta}(k_g,k_1,k_2) = \mathrm{Tr}\left[\not{k}_1\gamma^\nu \frac{(\not{k}_g-\not{k}_2)}{(k_g-k_2)^2}\gamma^\beta \not{k}_2\gamma^\mu \frac{(\not{k}_1-\not{k}_g)}{(k_1-k_g)^2}\gamma^\alpha\right] + (k_1\leftrightarrow k_2).$$

TFR SIDIS at twist-3 or one-loop

■ Gluonic contribution

$$F_{UU}^{\cos 2\phi_h} = -\frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2 t_{1g}^h(x_B/z,\xi_h,P_{h\perp}),$$

$$F_{UL}^{\sin 2\phi_h} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2 t_{1gL}^h(x_B/z,\xi_h,P_{h\perp}),$$

$$F_{UT}^{\sin(3\phi_h-\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^3}{4M^3}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2 t_{1gT}^{hh}(x_B/z,\xi_h,P_{h\perp}),$$

$$F_{UT}^{\sin(\phi_h+\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}}{2M}x_B\sum_{q,\bar{q}}e_q^2\int_{x_B/\bar{\xi}_h}^1\frac{dz}{z}z^2\left[t_{1gT}^h(x_B/z,\xi_h,P_{h\perp})+\frac{P_{h\perp}^2}{2M^2}t_{1gT}^{hh}(x_B/z,\xi_h,P_{h\perp})\right].$$

Four structure functions generated uniquely by gluon fracture functions!

K.B. Chen, J.P. Ma, X.B. Tong, JHEP 05 (2024) 298

TFR SIDIS at twist-3 or one-loop

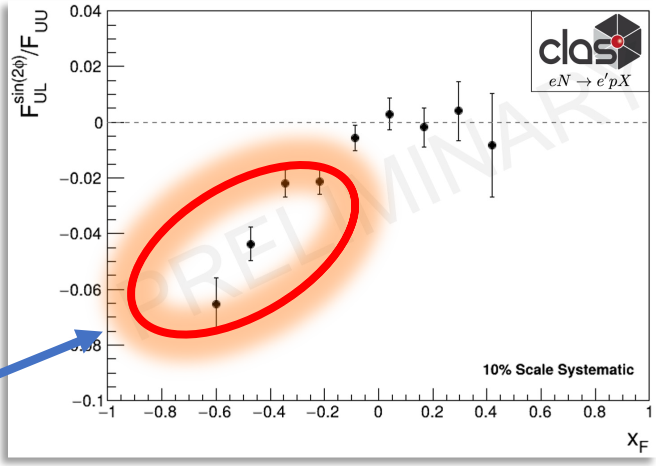
■ Gluonic contribution

$$F_{UU}^{\cos 2\phi_h} = -\frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1g}^h(x_B/z, \xi_h, P_{h\perp}),$$

$$\boxed{F_{UL}^{\sin 2\phi_h} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1gL}^h(x_B/z, \xi_h, P_{h\perp}),}$$

$$F_{UT}^{\sin(3\phi_h-\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^3}{4M^3} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1gT}^{hh}(x_B/z, \xi_h, P_{h\perp}),$$

$$F_{UT}^{\sin(\phi_h+\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}}{2M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 \left[t_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \frac{P_{h\perp}^2}{2M^2} t_{1gT}^{hh}(x_B/z, \xi_h, P_{h\perp})\right].$$



Four structure functions generated uniquely by gluon fracture functions!

K.B. Chen, J.P. Ma, X.B. Tong, JHEP 05 (2024) 298

TFR SIDIS at twist-3 or one-loop

■ Quark and gluon contribution

$$F_{UU,L} = \frac{\alpha_s}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \Big[4T_F z\bar{z} u_{1g}(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_1(x_B/z, \xi_h, P_{h\perp}) \Big],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \frac{\alpha_s}{2\pi} \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \Big[4T_F z\bar{z} u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \Big].$$

NLO,
twist-4 if at tree level

$$F_{UU,T} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \Big[\mathcal{H}_g(z) u_{1g}(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_1(x_B/z, \xi_h, P_{h\perp}) \Big],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \Big[\mathcal{H}_g(z) u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \Big],$$

$$F_{LL} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \Big[\Delta\mathcal{H}_g(z) l_{1gL}(x_B/z, \xi_h, P_{h\perp}) + \Delta\mathcal{H}_q(z) l_{1L}(x_B/z, \xi_h, P_{h\perp}) \Big],$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \Big[\Delta\mathcal{H}_g(z) l_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \Delta\mathcal{H}_q(z) l_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \Big],$$

LO + NLO

$$\mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ P_{qq}(z) \ln\frac{Q^2}{\mu^2} + C_F \left[2\left(\frac{\ln\bar{z}}{\bar{z}}\right)_+ - \frac{3}{2}\left(\frac{1}{\bar{z}}\right)_+ - (1+z)\ln\bar{z} - \frac{1+z^2}{\bar{z}}\ln z + 3 - \left(\frac{\pi^2}{3} + \frac{9}{2}\right)\delta(\bar{z}) \right] \right\},$$

$$\mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[P_{qg}(z) \ln\frac{Q^2\bar{z}}{\mu^2 z} - T_F(1-2z)^2 \right],$$

$$\Delta\mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ \Delta P_{qq}(z) \ln\frac{Q^2}{\mu^2} + C_F \left[(1+z^2)\left(\frac{\ln\bar{z}}{\bar{z}}\right)_+ - \frac{3}{2}\left(\frac{1}{\bar{z}}\right)_+ - \frac{1+z^2}{\bar{z}}\ln z + 2 + z - \left(\frac{\pi^2}{3} + \frac{9}{2}\right)\delta(\bar{z}) \right] \right\},$$

$$\Delta\mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[\Delta P_{qg}(z) \left(\ln\frac{Q^2\bar{z}}{\mu^2 z} - 1\right) + 2T_F\bar{z} \right].$$

TFR SIDIS at twist-3 or one-loop

Structure functions	Twist	Order (α_S)
$F_{UU,T}$	2	0
$F_{UU,L}$		1
$F_{UU}^{\cos\phi_h}$	3	
$F_{UU}^{\cos 2\phi_h}$		1
$F_{LU}^{\sin\phi_h}$	3	
$F_{UL}^{\sin\phi_h}$	3	
$F_{UL}^{\sin 2\phi_h}$		1
F_{LL}	2	0
$F_{LL}^{\cos\phi_h}$	3	

Structure functions	Twist	Order (α_S)
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	0
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$		1
$F_{UT}^{\sin(\phi_h+\phi_S)}$		1
$F_{UT}^{\sin\phi_S}$	3	
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	
$F_{UT}^{\sin(3\phi_h-\phi_S)}$		1
$F_{LT}^{\cos\phi_S}$	3	
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	0
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

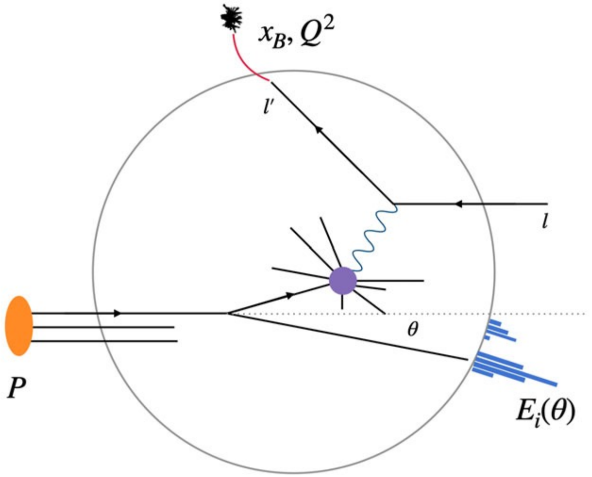
Beyond twist-3 or $\mathcal{O}(\alpha_S)$

All structure functions are non-zero up to twist-3 or one-loop.

Contents

Connecting NEEC with fracture function

■ **Nucleon energy-energy correlator (NEEC)**



NEEC: $f_{EEC}(z,\theta) = \int \frac{dy^-}{4\pi} e^{-izP^+y^-} \left\langle P \middle| \bar{\psi}(y^-) \frac{\gamma^+}{2} \hat{\mathcal{E}}(\theta) \psi(0) \middle| P \right\rangle$

The energy flow operator: $\hat{\mathcal{E}}(\theta)|X\rangle \equiv \sum_{i\in X} \delta(\theta^2 - \theta_i^2) \frac{E_i}{E_P} |X\rangle$

$$\Sigma(x_B, Q^2, \theta) \equiv \sum_i \int \frac{E_i}{E_P} d\sigma(x_B, Q^2, p_i) \delta(\theta - \theta_i)$$

$$\xrightarrow{\theta \ll 1} = \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}, Q^2, \mu\right) f_{EEC}(z, \theta, \mu)$$

X.H. Liu and H.X. Zhu, PRL 130, 091901 (2023)
H.T. Cao, X.H. Liu and H.X. Zhu PRD 107, 114008 (2023)

Hadrons with $\theta \ll 1$ are almost collinear with the target nucleon. They are from the **TFR**!
A sum rule should exist between NEEC and fracture function.

Connecting NEEC with fracture function

■ Relation between NEEC and fracture function

Matrix elements for fracture functions and NEECs:

$$\mathcal{M}^q_{ij,\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h\perp}) = \int \frac{d\eta^-}{2\xi_h(2\pi)^4} e^{-ixP^+\eta^-} \sum_X \int \frac{d^3\boldsymbol{P}_X}{2E_X(2\pi)^3} \times \langle PS|\bar{\psi}_j(\eta^-)\mathcal{L}^\dagger_n(\eta^-)|P_h X\rangle\langle X P_h|\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

$$\sum_X \int \frac{d^3\boldsymbol{P}_X}{2E_X(2\pi)^3}|P_h X\rangle\langle X P_h| = a^\dagger_h a_h$$

$$\mathcal{M}^q_{ij,\mathrm{EEC}}(x,\theta,\phi) = \int \frac{d\eta^-}{2\pi} e^{-ixP^+\eta^-} \langle PS|\bar{\psi}_j(\eta^-)\mathcal{L}^\dagger_n(\eta^-)\mathcal{E}(\theta,\phi)\mathcal{L}_n(0)\psi_i(0)|PS\rangle$$

$$\mathcal{E}(\theta,\phi) = \sum_h \int \frac{d^3\boldsymbol{P}_h}{2E_h(2\pi)^3}\frac{E_h}{E_N}\delta(\theta^2-\theta_h^2)\delta(\phi-\phi_h)a^\dagger_h a_h$$

$$\Rightarrow \boxed{\mathcal{M}^q_{ij,\mathrm{EEC}}(x,\theta,\phi) = \sum_h \int_0^{1-x} d\xi_h\ \xi_h \frac{\boldsymbol{P}^2_{h\perp}}{2\theta^2}\mathcal{M}^q_{ij,\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h\perp})\Big|_{\boldsymbol{P}_{h\perp}=\frac{\xi_h\theta P^+}{\sqrt{2}}\boldsymbol{n}_t}}$$

$$\boldsymbol{n}_t \equiv (\cos\phi, \sin\phi)$$

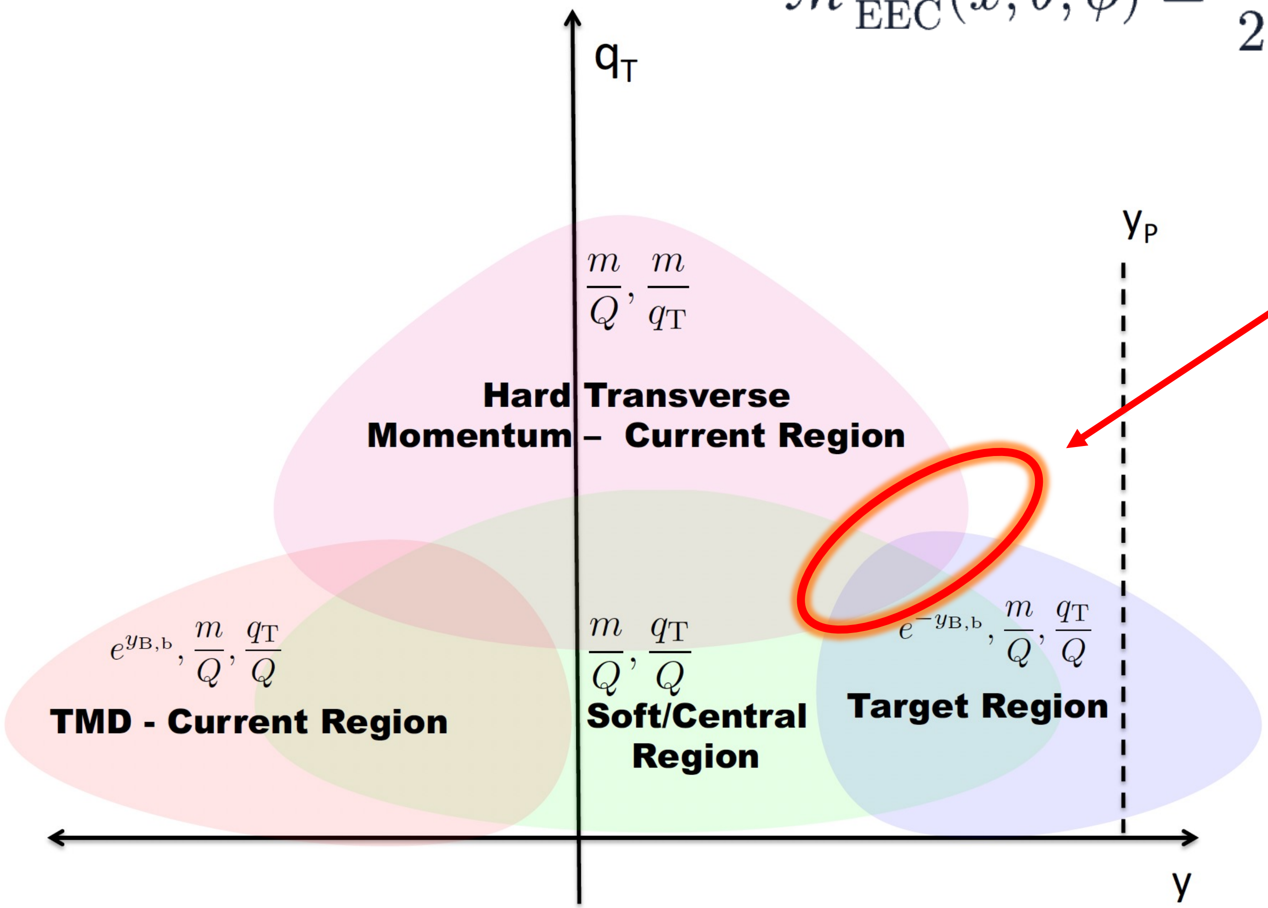
- ✓ Fracture functions act as generating functions of NEECs.
- ✓ Shortcut for accessing NEECs properties through fracture functions.

Connecting NEEC with fracture function

Example: collinear matching of Sivers-type NEEC at $\Lambda_{QCD} \ll \theta Q \ll Q$

$$\mathcal{M}^{q}_{\rm EEC}(x,\theta,\phi) = \frac{\gamma^+}{2N_c}\left[\frac{1}{2\pi} f_1^q(x,\theta) - n_t \cdot \tilde{S}_\perp f_{1T}^{t,q}(x,\theta)\right] + \cdots$$

Sivers-type NEEC



Overlap region $\Lambda_{QCD} \ll P_{h\perp} \ll Q$,

$P_{h\perp}$ is generated perturbatively.

$$f_{1T}^{t,q}(x,\theta) = \sum \frac{|\boldsymbol{P}_{h\perp}|}{M} u_{1T}^{h,q}(x,\xi_h,\boldsymbol{P}_{h\perp}^2)$$

Re-factorized and matched to collinear PDFs/FFs in the overlap region.

Connecting NEEC with fracture function

Example: collinear matching of Sivers-type NEEC at $\Lambda_{QCD} \ll \theta Q \ll Q$

$$f_{1T}^{t,q}(x,\theta) = f_{1T}^{t,q}(x,\theta)\Big|_{\rm HP} + f_{1T}^{t,q}(x,\theta)\Big|_{\rm SFP} + f_{1T}^{t,q}(x,\theta)\Big|_{{\rm SGP},qG\bar q} + f_{1T}^{t,q}(x,\theta)\Big|_{{\rm SGP},3G}$$

$$f_{1T}^{t,q}(x,\theta)\Big|_{\rm HP} = \frac{\alpha_s N_c}{2(2\pi)^2\theta^3 E_N}\int_x^1 \frac{dy}{y}\Big[T_\Delta(y,x) - (1+2x/(y-x))T_F(y,x)\Big],$$

$$f_{1T}^{t,q}(x,\theta)\Big|_{\rm SFP} = \frac{\alpha_s}{2(2\pi)^2 N_c\theta^3 E_N}\int_x^1 dy\frac{x}{y^3}\left[(2x-y)T_F(y,0) - yT_\Delta(y,0)\right],$$

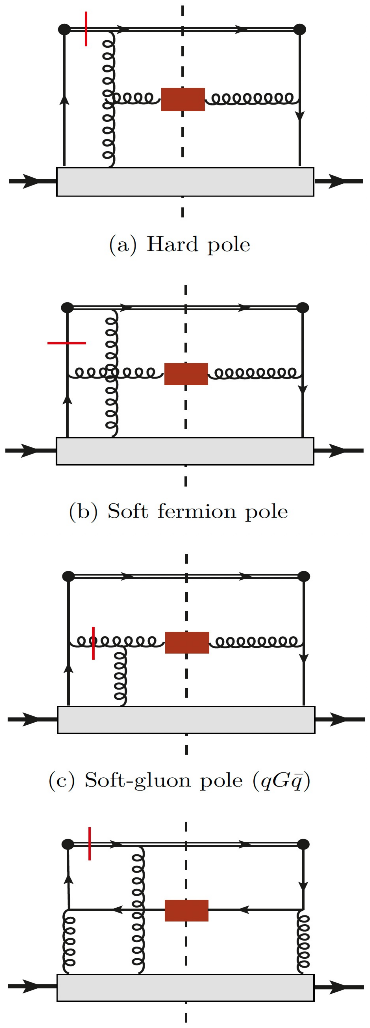
$$f_{1T}^{t,q}(x,\theta)\Big|_{{\rm SGP},qG\bar q} = \frac{\alpha_s N_c}{2(2\pi)^2\theta^3 E_N}\int_x^1 \frac{dy}{y^3}\left[\frac{1}{y-x}(y^3+3x^2y-2x^3)T_F(y,y) - y(y^2+x^2)\frac{dT_F(y,y)}{dy}\right],$$

$$\begin{aligned} f_{1T}^{t,q}(x,\theta)\Big|_{{\rm SGP},3G} = \frac{\alpha_s}{2\pi\theta^3 E_N}\int_x^1 dy\frac{y-x}{y^5}\Big\{&2(4x^2-3xy+y^2)[N(y,y)-O(y,y)] \\ &-2(8x^2-5xy+y^2)[N(y,0)+O(y,0)] + y(2x-y)^2\frac{d}{dy}[N(y,0)+O(y,0)] \\ &-y(2x^2+y^2-2xy)\frac{d}{dy}[N(y,y)-O(y,y)]\Big\}. \end{aligned}$$

Sivers-type NEEC will match to collinear twist-3 PDFs.

KBC, J.P. Ma, X.B. Tong, JHEP 08 (2024) 227; JHEP 11 (2021) 038

Example diagrams



(a) Hard pole

(b) Soft fermion pole

(c) Soft-gluon pole ($qG\bar q$)

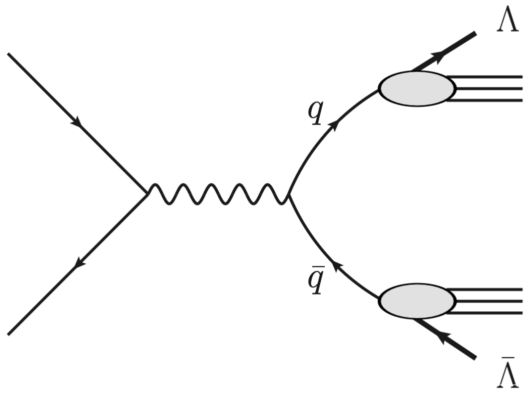
(d) Soft-gluon pole ($3G$)

Contents

Helicity correlation in CFR+TFR SIDIS

S.Y. Wei's talk

- **Accessing polarized FFs in unpolarized process via spin correlation**



$e^+e^- \to \Lambda + \bar{\Lambda} + X$

Pair produced $q\bar{q}$, **opposite helicities**

Spin transfer in fragmentation

Correlated helicities of $\Lambda\bar{\Lambda}$

$$d\sigma^{\lambda_\Lambda \lambda_{\bar{\Lambda}}} \sim \sigma_0 \left[D_{1q}^{\Lambda}(z_1) D_{1\bar{q}}^{\bar{\Lambda}}(z_2) - \lambda_\Lambda \lambda_{\bar{\Lambda}} G_{1Lq}^{\Lambda}(z_1) G_{1L\bar{q}}^{\bar{\Lambda}}(z_2) \right]$$

Helicity correlation of dihadron:

$$\mathcal{C}_{LL} \equiv \frac{d\sigma^{++} + d\sigma^{++} - d\sigma^{+-} - d\sigma^{-+}}{d\sigma^{++} + d\sigma^{++} + d\sigma^{+-} + d\sigma^{-+}}$$

$$= \frac{\sum_q \sigma_0 G_{1Lq}^{\Lambda}(z_1) G_{1L\bar{q}}^{\bar{\Lambda}}(z_2)}{\sum_q \sigma_0 D_{1q}^{\Lambda}(z_1) D_{1\bar{q}}^{\bar{\Lambda}}(z_2)}$$

A novel way to probe $G_{1Lq}(z)$ in unpolarized collision processes

Applied to various processes: e^+e^-, pp, AA, SIDIS……

H.C. Zhang, S.Y. Wei, PLB 839 (2023) 137821
X. Li, Z.X. Chen, S. Cao, S.Y. Wei, PRD 109, 014035 (2024)
Z.X. Chen, H. Dong, S.Y. Wei, PRD 110, 056040 (2024)
L. Yang, Y.K. Song, S.Y. Wei, PRD 111, 054035 (2025)
F. Huang, T.B. Liu, Y.K. Song, S.Y. Wei, PLB 862 (2025) 139346
… …

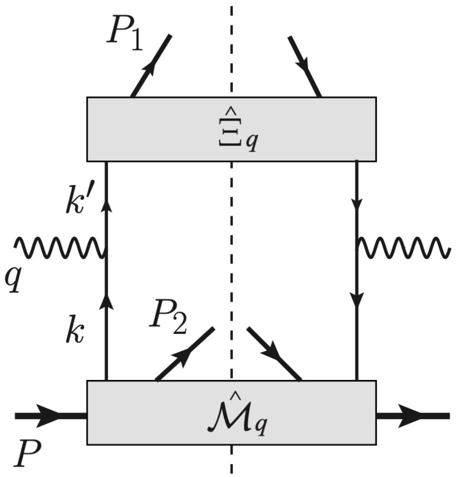
Helicity correlation in CFR+TFR SIDIS

SIDIS with CFR+TFR hadron production

The process: $e(l) + N(P) \to e(l') + h_1(P_1, S_1) + h_2(P_2, S_2) + X$

CFR hadron

TFR hadron



The hadronic tensor

The hadronic tensor and the cross section:

$$\frac{d\sigma}{dx_B dy dz_1 d\xi_2 d^2\boldsymbol{P}_{2\perp} d\psi} = \frac{\alpha_{\rm em}^2 y z_1}{4Q^4} L_{\mu\nu}(l,l') W^{\mu\nu}(x_B; z_1, S_1; \xi_2, P_{2\perp}, S_2)$$

$$W^{\mu\nu}(x_B; z_1, S_1; \xi_2, P_{2\perp}, S_2) = \sum_q e_q^2 \mathrm{Tr}\left[\hat{\mathcal{M}}_q(x_B; \xi_2, P_{2\perp}, S_2)\gamma^\mu \hat{\Xi}_q(z_1, S_1)\gamma^\nu\right] + \{q \leftrightarrow \bar{q}\}$$

longitudinal spin correlation between the initial struck quark and the TFR hadron

$$z_1 \hat{\Xi}_{q,ij}(z_1, S_1) = (\gamma^+)_{ij} D_{1q}(z_1) + (\gamma_5\gamma^+)_{ij} \lambda_1 G_{1Lq}(z_1) + \cdots \ ,$$

$$\hat{\mathcal{M}}_{q,ij}(x_B; \xi_2, P_{2\perp}, S_2) = \frac{1}{2}\left[(\gamma^-)_{ij} u_{1q}(x_B, \xi_2, P_{2\perp}) + (\gamma_5\gamma^-)_{ij} \lambda_2 \underline{l_{1q}^L(x_B, \xi_2, P_{2\perp})}\right] + \cdots \ .$$

Helicity correlation in CFR+TFR SIDIS

■ Dihadron helicity correlation

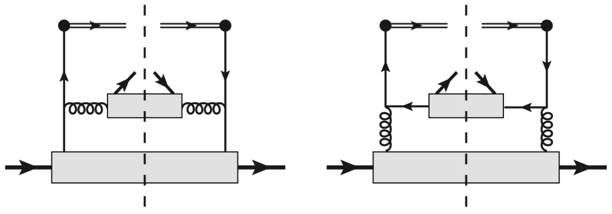
$$\frac{d\sigma}{dx_B dy dz_1 d\xi_2 d^2\boldsymbol{P}_{2\perp} d\psi} = \frac{\alpha_{\rm em}^2}{yQ^2}\sum_q e_q^2 A(y)\left[u_{1q}(x_B,\xi_2,P_{2\perp})D_{1q}(z_1) + \lambda_1\lambda_2 l_{1q}^L(x_B,\xi_2,P_{2\perp})G_{1Lq}(z_1)\right] + \{q\leftrightarrow\bar{q}\}$$

The helicity correlation: $\boxed{\mathcal{C}_{LL}(x_B,\xi_2,P_{2\perp},z_1) = \frac{\sum_q e_q^2 l_{1q}^L(x_B,\xi_2,P_{2\perp})G_{1Lq}(z_1) + \{q\leftrightarrow\bar{q}\}}{\sum_q e_q^2 u_{1q}(x_B,\xi_2,P_{2\perp})D_{1q}(z_1) + \{q\leftrightarrow\bar{q}\}}}$

Numerical estimation?

No available parameterization for u_{1q}, l_{1q}^L !

Match fracture functions to PDF&FFs in the region of $\Lambda_{\rm QCD} \ll P_{2\perp} \ll Q$.



(a) (b)

$$u_{1q}(x_B,\xi_2,P_{2\perp}) = \frac{\alpha_s}{2\pi^2\xi_2\boldsymbol{P}_{2\perp}^2}\int_{x_B+\xi_2}^1 \frac{dy}{y} z\left[P_{gq}\left(\frac{\xi_2}{yz}\right) f_{1q}(y)D_{1g}(z) + \frac{T_F}{2}P_{qg}\left(\frac{\xi_2}{yz}\right) f_{1g}(y)D_{1\bar{q}}(z)\right],$$

$$l_{1q}^L(x_B,\xi_2,P_{2\perp}) = \frac{\alpha_s}{2\pi^2\xi_2\boldsymbol{P}_{2\perp}^2}\int_{x_B+\xi_2}^1 \frac{dy}{y} z\left[\tilde{P}_{gq}\left(\frac{\xi_2}{yz}\right) f_{1q}(y)G_{1Lg}(z) + \frac{T_F}{2}\tilde{P}_{qg}\left(\frac{\xi_2}{yz}\right) f_{1g}(y)G_{1L\bar{q}}(z)\right],$$

Helicity correlation in CFR+TFR SIDIS

■ Numerical estimation

$G_{1Lq}^{\Lambda}(z)$ from DSV:

scenario-1: only s quark contribute;
scenario-2: u, d have negative contributions;
scenario-3: u, d, s contribute equally.

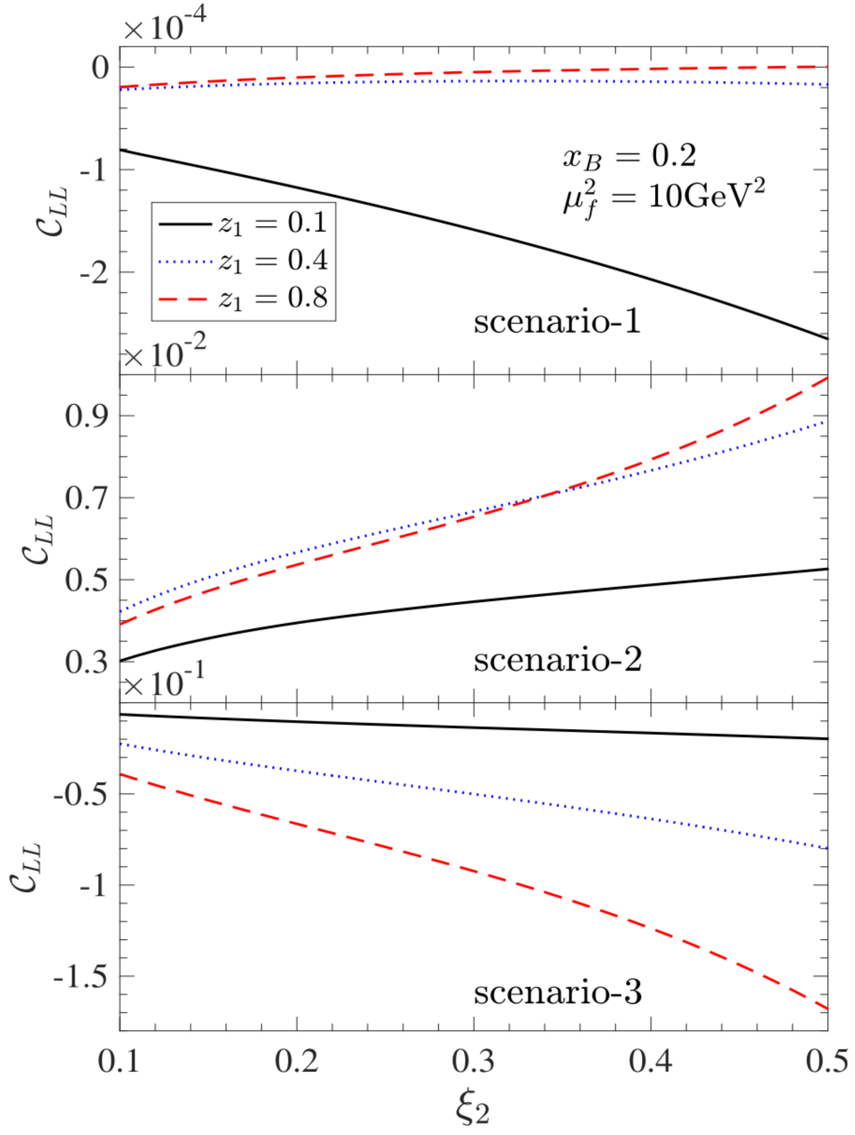
➢ Flavor sensitivity:

- $\mathcal{C}_{LL}$ differs drastically across three scenarios.
- $\mathcal{C}_{LL}$ is positive for scenario-2.

➢ Kinematical dependence:

- $\mathcal{C}_{LL}$ grows with ξ_2 for all scenarios.
- $\mathcal{C}_{LL}$ also grows with z_1 for scenario-2&3 (large rapidity separation)

A clear discriminator for polarized FFs scenarios.



X.Q. Xi, KBC, X.B. Tong, S.Y. Wei and J. Wu, EPJC 85:1458 (2025)

Contents

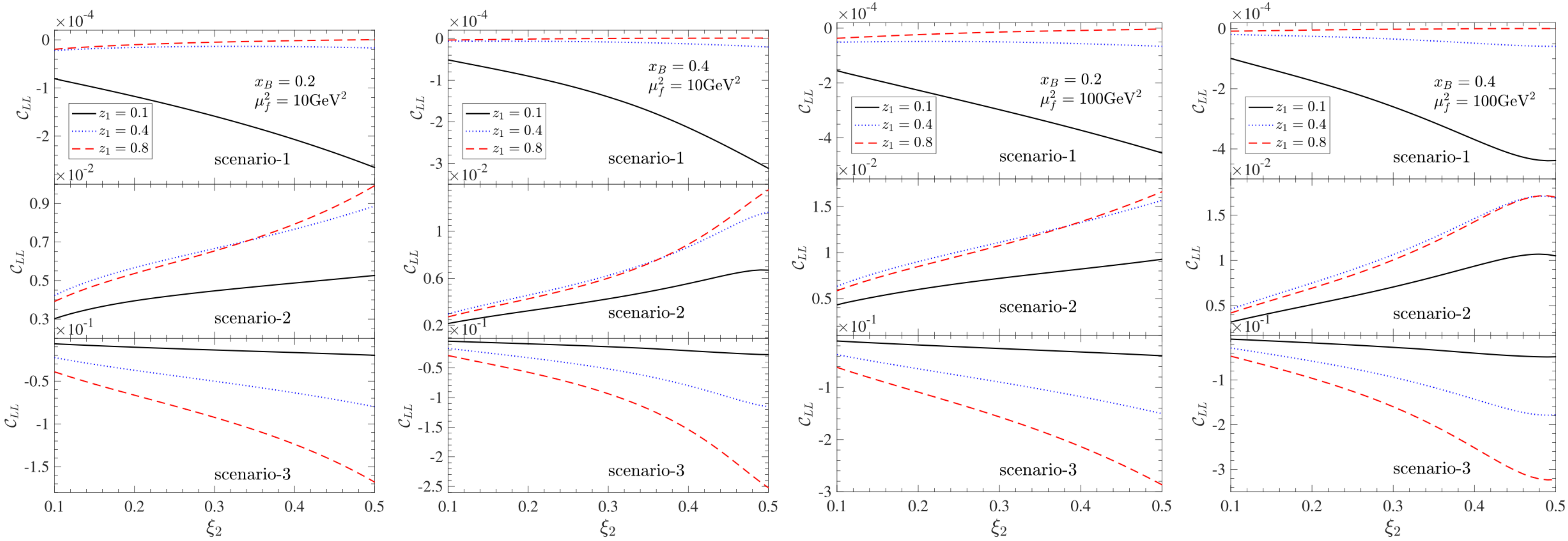
Summary

- SIDIS in the TFR is factorized with fracture functions and is complementary to CFR for studying the nucleon structure.
- We calculated the TFR SIDIS up to twist-3 at the tree level and to one-loop at twist-2. All 18 structure functions are non-zero, including 8 at twist-3 and 4 generated uniquely by gluon fracture functions.
- Connection between NEEC and fracture functions is established, which will facilitate the study of NEEC through fracture functions.
- Dihadron helicity correlation in CFR+TFR SIDIS can serve as a clear way to distinguish different polarized FFs scenarios.

谢谢！

Back up

Helicity correlation in CFR+TFR SIDIS



Connecting NEEC with fracture function

■ Relation between NEECs and fracture functions

$$\mathcal{M}^q_{ij,\mathrm{FrF}}(x,\xi_h,\boldsymbol{P}_{h\perp}) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(u_1^q - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^{h,q}\right) + \frac{1}{P^+}\left(P^\rho_{h\perp}u^{h,q} - M\tilde{S}^\rho_\perp u_T^q - S_L\tilde{P}^\rho_{h\perp}u_L^{h,q} - \frac{P^{\langle\rho}_{h\perp}P^{\beta\rangle}_{h\perp}}{M}\tilde{S}_{\perp\beta}u_T^{h,q}\right)\right]$$
$$+ \frac{(\gamma_5\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(S_L l_{1L}^q - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^{h,q}\right) + \frac{1}{P^+}\left(\tilde{P}^\rho_{h\perp}l^{h,q} + MS^\rho_\perp l_T^q + S_L P^\rho_{h\perp}l_L^{h,q} - \frac{P^{\langle\rho}_{h\perp}P^{\beta\rangle}_{h\perp}}{M}S_{\perp\beta}l_T^{h,q}\right)\right] + \cdots$$

$$\mathcal{M}^q_{ij,\mathrm{EEC}}(x,\theta,\phi) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(\frac{1}{2\pi}f_1^q - n_t\cdot\tilde{S}_\perp\; f_{1T}^{t,q}\right) + \frac{M}{P^+}\left(n_t^\rho\; f^{t,q} - \frac{1}{2\pi}\tilde{S}^\rho_\perp\; f_T^q - S_L\tilde{n}_t^\rho\; f_L^{t,q} - n_t^{\langle\rho}n_t^{\beta\rangle}\tilde{S}_{\perp\beta}\; f_T^{t,q}\right)\right]$$
$$+ \frac{(\gamma_5\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(\frac{1}{2\pi}S_L\; g_{1L}^q - n_t\cdot S_\perp\; g_{1T}^{t,q}\right) + \frac{M}{P^+}(\tilde{n}_t^\rho\; g^{t,q} + \frac{1}{2\pi}S^\rho_\perp\; g_T^q + S_L n_t^\rho\; g_L^{t,q} - n_t^{\langle\rho}n_t^{\beta\rangle}S_{\perp\beta}\; g_T^{t,q})\right]$$

$$f_1^q(x,\theta) = 2\pi\sum\!\!\!\!\!\!\int u_1^q(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad f_{1T}^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}u_{1T}^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad f_T^q(x,\theta) = 2\pi\sum\!\!\!\!\!\!\int u_T^q(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad f_T^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{\boldsymbol{P}^2_{h\perp}}{M^2}u_T^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}),$$
$$g_{1L}^q(x,\theta) = 2\pi\sum\!\!\!\!\!\!\int l_{1L}^q(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad g_{1T}^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}l_{1T}^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad g^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}l^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad g_L^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}l_L^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}),$$
$$f^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}u^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad f_L^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{|\boldsymbol{P}_{h\perp}|}{M}u_L^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad g_T^q(x,\theta) = 2\pi\sum\!\!\!\!\!\!\int l_T^q(x,\xi_h,\boldsymbol{P}^2_{h\perp}),\quad g_T^{t,q}(x,\theta) = \sum\!\!\!\!\!\!\int \frac{\boldsymbol{P}^2_{h\perp}}{M^2}l_T^{h,q}(x,\xi_h,\boldsymbol{P}^2_{h\perp}).$$

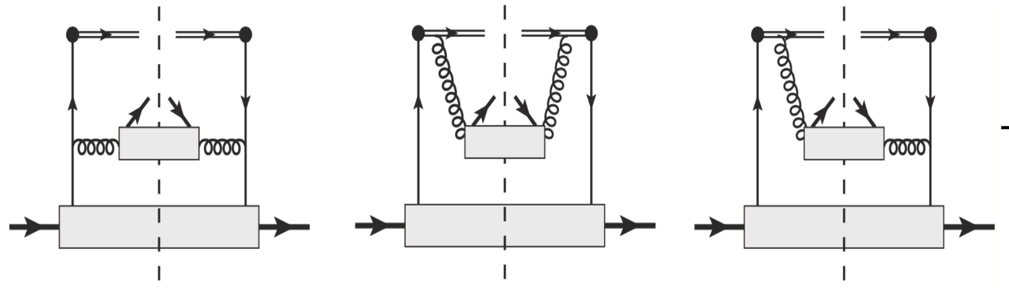
$$\sum\!\!\!\!\!\!\int \mathcal{A} \equiv \sum_h \int_0^{1-x} d\xi_h\;\xi_h\frac{\boldsymbol{P}^2_{h\perp}}{2\theta^2}\;\mathcal{A}\Big|_{|\boldsymbol{P}_{h\perp}|=\frac{\theta\xi_h P^+}{\sqrt{2}}}$$

Shortcut for accessing NEEC properties through fracture function

Collinear matching of fracture functions

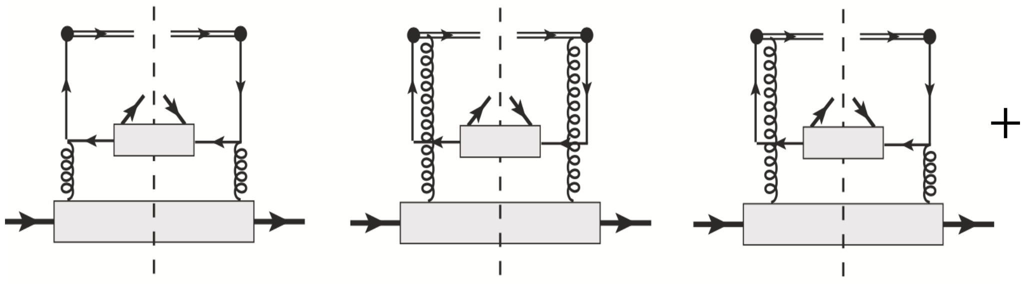
■ Matching of unpolarized/longitudinal polarized fracture function

$$\mathcal{M}_{ij} = \frac{(\gamma^-)_{ij}}{2N_c}\left[\boxed{u_1(x,\xi_h,P_{h\perp})} - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M} u_{1T}^h(x,\xi_h,P_{h\perp})\right] + \frac{(\gamma_5\gamma^-)_{ij}}{2N_c}\left[S_L \boxed{l_{1L}(x,\xi_h,P_{h\perp})} - \frac{P_{h\perp}\cdot S_\perp}{M} l_{1T}^h(x,\xi_h,P_{h\perp})\right] + \cdots$$



$+h.c.$

quark PDF ⊗ gluon FF



$+h.c.$

gluon PDF ⊗ antiquark FF

Twist-2 matching

$$u_1(x,\xi_h,P_{h\perp}) = g_s^2 \frac{1}{\vec{P}_{h\perp}^2}\int \frac{dz}{z^2}\left[2C_F \underline{d_g(z)q(y)}\frac{z^2}{y^2}(x^2+y^2) + \underline{d_{\bar q}(z)g(y)}\frac{\xi_h}{zy^3}(z^2x^2+\xi_h^2)\right]$$

$$l_{1L}(x,\xi_h,P_{h\perp}) = g_s^2 \frac{1}{\vec{P}_{h\perp}^2}\int \frac{dz}{z^2}\left[2C_F \underline{d_g(z)\Delta q(y)}\frac{z^2}{y^2}(x^2+y^2) + \underline{d_{\bar q}(z)\Delta g(y)}\frac{\xi_h}{y^2}(xz-\xi_h)\right]$$

$$y = x + \xi_h/z$$

Scales as $1/P_{h\perp}^2$

Accessing the twist-2 unpolarized/longitudinal polarized PDF in TFR

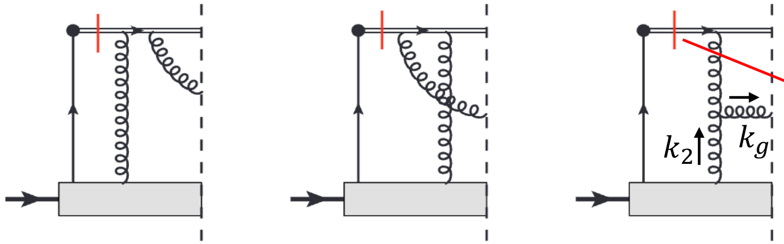
Collinear matching of fracture functions

■ Matching of transverse polarized fracture function

$$\mathcal{M}_{ij} = \frac{(\gamma^-)_{ij}}{2N_c}\left[u_1(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}\boxed{u_{1T}^h(x,\xi_h,P_{h\perp})}\right] + \frac{(\gamma_5\gamma^-)_{ij}}{2N_c}\left[S_L l_{1L}(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot S_\perp}{M} l_{1T}^h(x,\xi_h,P_{h\perp})\right] + \cdots$$

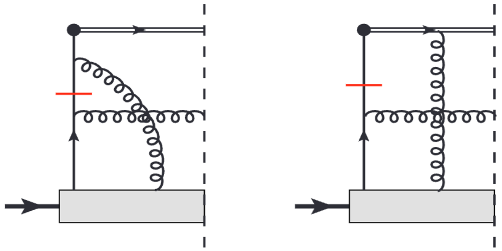
T-odd

Absorptive parts required! (single-spin asymmetry)

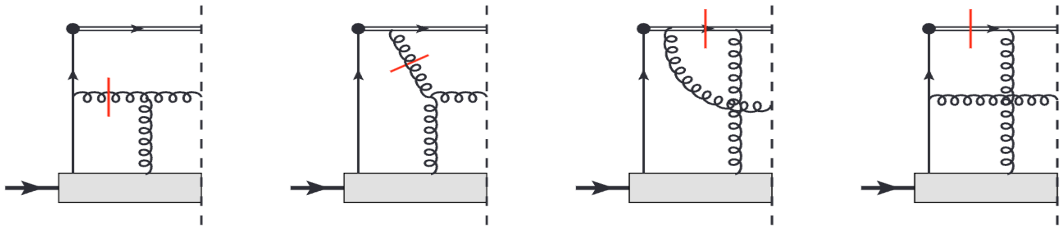


$$\boxed{\frac{1}{k_g^+ - k_2^+ + i\epsilon} \to -i\pi\delta\left(k_g^+ - k_2^+\right)}$$

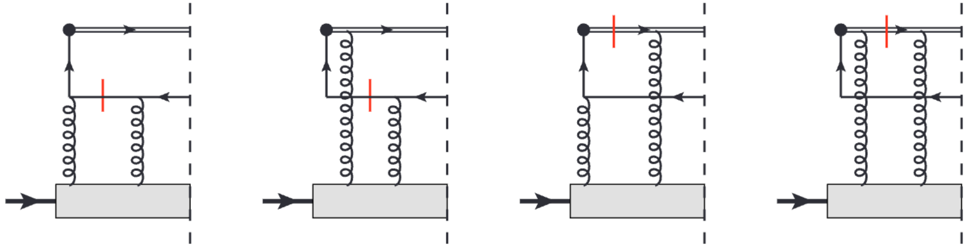
Hard-pole



Soft-fermion-pole



Soft-gluon-pole



Soft pole from 3-gluon correlations

Collinear matching of fracture functions

Matching results after collinear expansion:

Gauge invariant!

$$\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{HP} = g_s^2\frac{N_c}{P_{h\perp}^4}\int\frac{dz}{z^2}d_g(z)\frac{z^2}{y}\Big[\xi_h T_\Delta(y,x)-(\xi_h+2xz)T_F(y,x)\Big]$$

$$\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{SFP} = g_s^2\frac{1}{N_c}\frac{1}{P_{h\perp}^4}\int\frac{dz}{z^2}d_g(z)\frac{x\xi_h z}{y^3}\Big[(xz-\xi_h)T_F(y,0)-(\xi_h+xz)T_\Delta(y,0)\Big]$$

$$\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{SGP} = \frac{g_s^2N_c}{P_{h\perp}^4}\int\frac{dz}{z^2}d_g(z)\frac{1}{y^3}\Big[z^3(y^3+3x^2y-2x^3)T_F(y,y)-y\xi_h z^2(y^2+x^2)\frac{\partial T_F(y,y)}{\partial y}\Big]$$

Twist-3 PDFs
⊗ twist-2 FFs

$$\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{3G} = \frac{4\pi g_s^2\xi_h^2}{P_{h\perp}^4}\int\frac{dz}{z^2}d_{\bar q}(z)\frac{1}{zy^5}\Big\{2(\xi_h^2+2x^2z^2-\xi_h xz)\big[N(y,y)-O(y,y)\big]$$
$$-2(\xi_h^2+4x^2z^2-3\xi_h xz)\big[N(y,0)+O(y,0)\big]+y(\xi_h-xz)^2\frac{d}{dy}\big[N(y,0)+O(y,0)\big]$$
$$-y(\xi_h^2+x^2z^2)\frac{d}{dy}\big[N(y,y)-O(y,y)\big]\Big\}$$

Scales as $1/P_{h\perp}^4$

$y\equiv x+\xi_h/z$

$T_F(x_1,x_2)$, $T_\Delta(x_1,x_2)$, $O(x_1,x_2)$, $N(x_1,x_2)$: gauge invariant twist-3 collinear PDFs.

Efremov and Teryaev, PLB 150, 383. Qiu and Sterman, PRL 67, 2264. Ji, PLB 289, 137. Beppu et al., PRD 82, 034005.