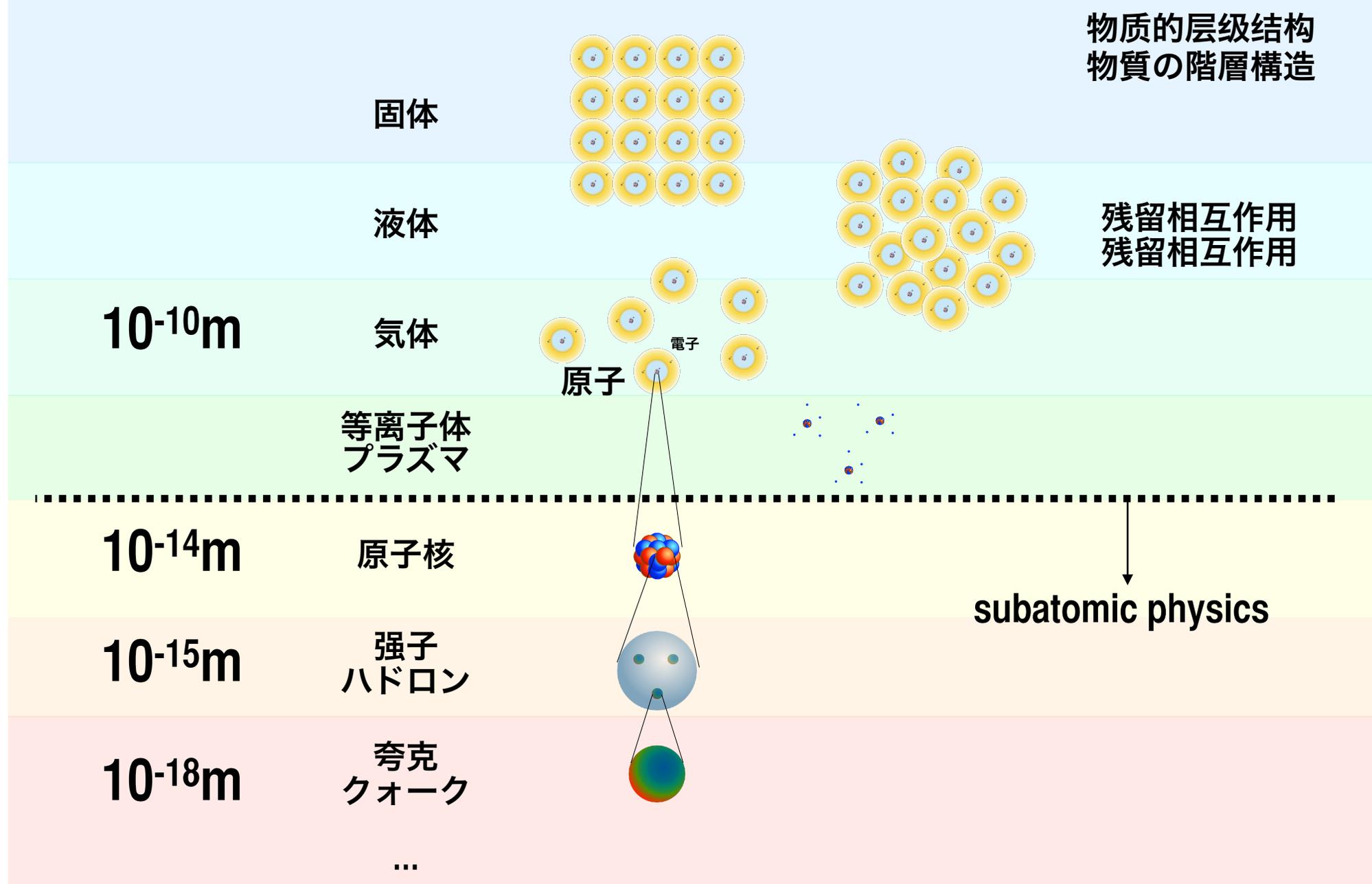
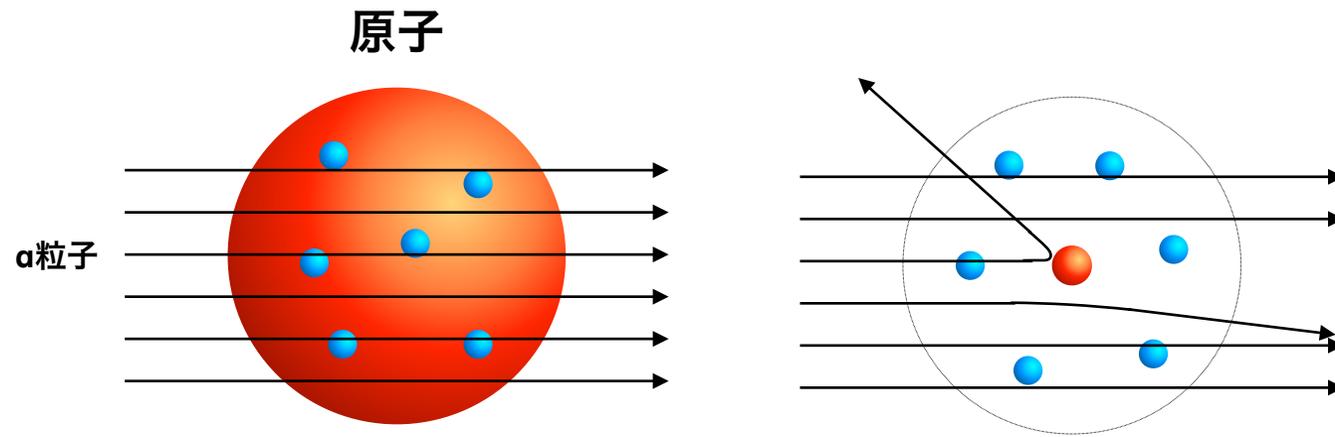


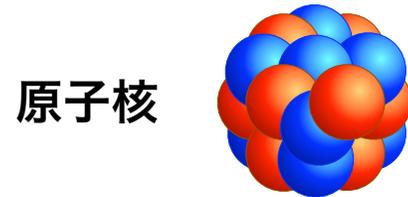
原子核物理

introduction

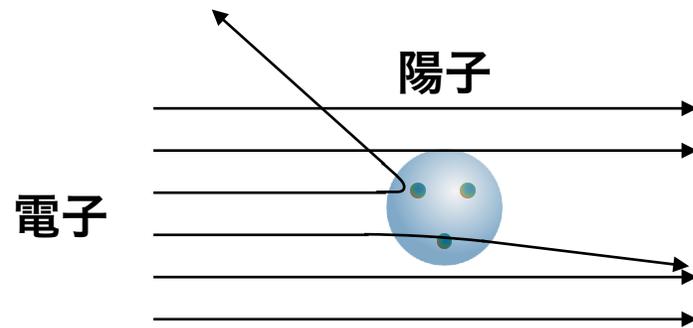




Rutherford散乱



中間子理論



パートンの発見

強い相互作用

量子色力学

QCD (Quantum Chromodynamics)

方法論は変わっていない

原子核物理学で用いる単位

定義値	$\Delta\nu(^{133}\text{Cs})_{\text{hfs}} = 9\,192\,631\,770\text{ s}^{-1}$
光速	$c = 299\,792\,458\text{ m s}^{-1}$
プランク定数	$h = 6.626\,070\,15 \times 10^{-34}\text{ J s}$
素電荷	$e = 1.602\,176\,634 \times 10^{-19}\text{ C}$
アボガドロ数	$N_A = 6.022\,140\,76 \times 10^{23}\text{ mol}^{-1}$
ボルツマン定数	$k_B = 1.380\,649 \times 10^{-23}\text{ J K}^{-1}$
	(1 eV = 1.602 176 634 × 10 ⁻¹⁹ J)

長さ	r	fm	(1 fm = 10 ⁻¹⁵ m)
エネルギー	E	MeV	(1 MeV = 10 ⁶ eV) GeV (1 GeV = 10 ⁹ eV)

質量	m	MeV/c ²	GeV/c ²
運動量	p	MeV/c	GeV/c

$$E^2 - (\underbrace{pc}_{\text{運動量}})^2 = (\underbrace{m_0c^2}_{\text{静止質量}})^2$$

$$E = \underbrace{T}_{\text{運動エネルギー}} + m_0c^2$$

電磁気 CGS $V(r) = \frac{q_1q_2}{r}$

$V(r) = k \frac{q_1q_2}{r}$ 微細構造定数 fine structure constant $\alpha = k \frac{e^2}{\hbar c}$

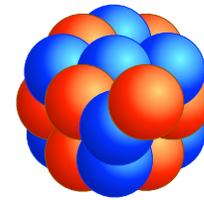
CGS $k = 1$ MKS $k = \frac{1}{4\pi\epsilon_0}$

記憶しておくべき定数

$$\alpha \simeq \frac{1}{137} \quad \alpha = 7.297\,352\,5693(11) \times 10^{-3} = 1/137.035\,999\,084(21)$$

$$\hbar c \simeq 197\text{ MeV fm} \quad \hbar c = 197.326\,980\,4\dots\text{ MeV fm}$$

原子核の構成要素



構成要素

核子 N
nucleon

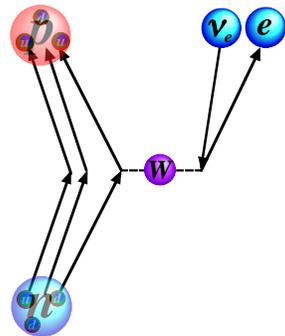
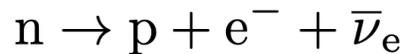
陽子 p 中性子 n
proton neutron



$$m_p = (1.007\,276\,466\,879 \pm 0.000\,000\,000\,091) \text{ u}$$

$$= (938.272\,0813 \pm 0.000\,0058) \text{ MeV}/c^2$$

$$m_n = (939.565\,4133 \pm 0.000\,0058) \text{ MeV}/c^2$$



中間子 $\pi (\pi^+, \pi^0, \pi^-), \dots$
meson



$$m_{\pi^\pm} = (139.57061 \pm 0.00024) \text{ MeV}/c^2$$

$$m_{\pi^0} = (134.9770 \pm 0.0005) \text{ MeV}/c^2$$

電子 e^-
electron



$$m_e = (0.510\,998\,9461 \pm 0.000\,000\,0031) \text{ MeV}/c^2$$

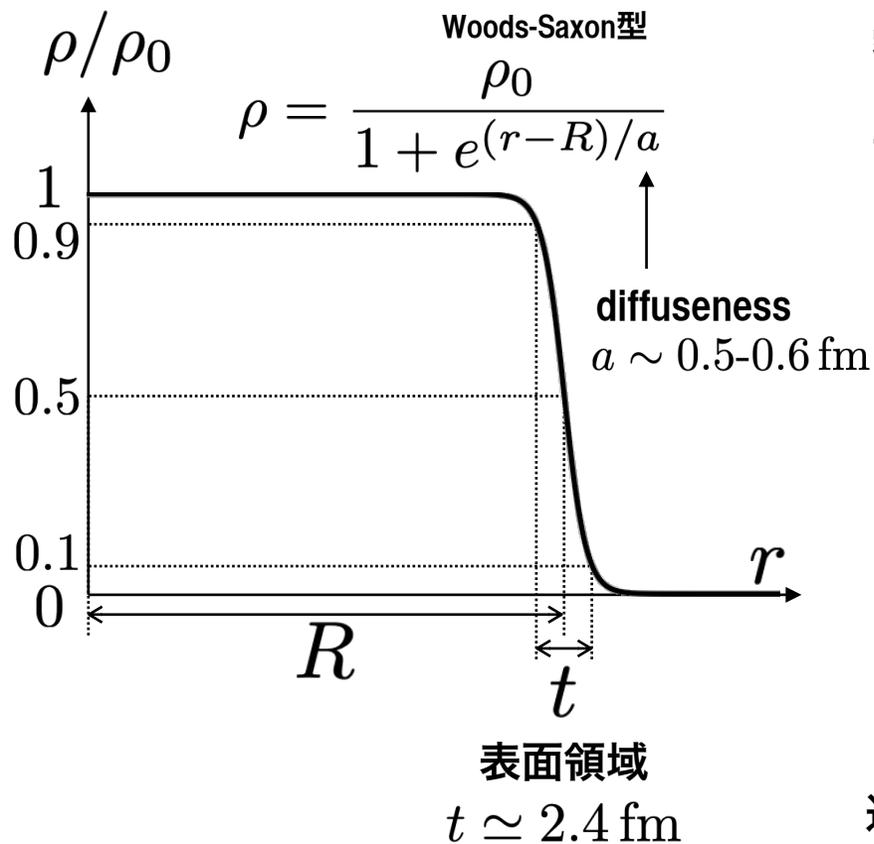
記憶しておくべき定数

$$m_e \sim 511 \text{ keV}/c^2$$

$$m_p \sim m_n \sim 1 \text{ GeV}/c^2$$

原子核の大小

核の大きさ



密度飽和
密度の飽和性

平均密度は $A = N + Z$ に依存しない
質量数 中性子数 陽子数
原子番号

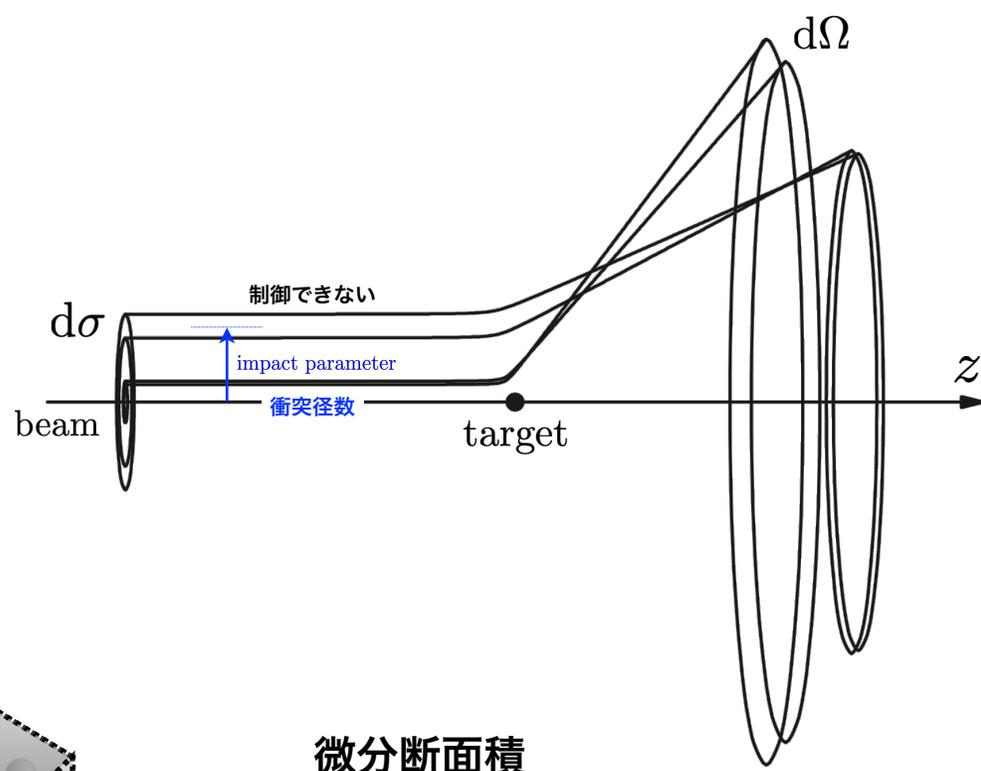
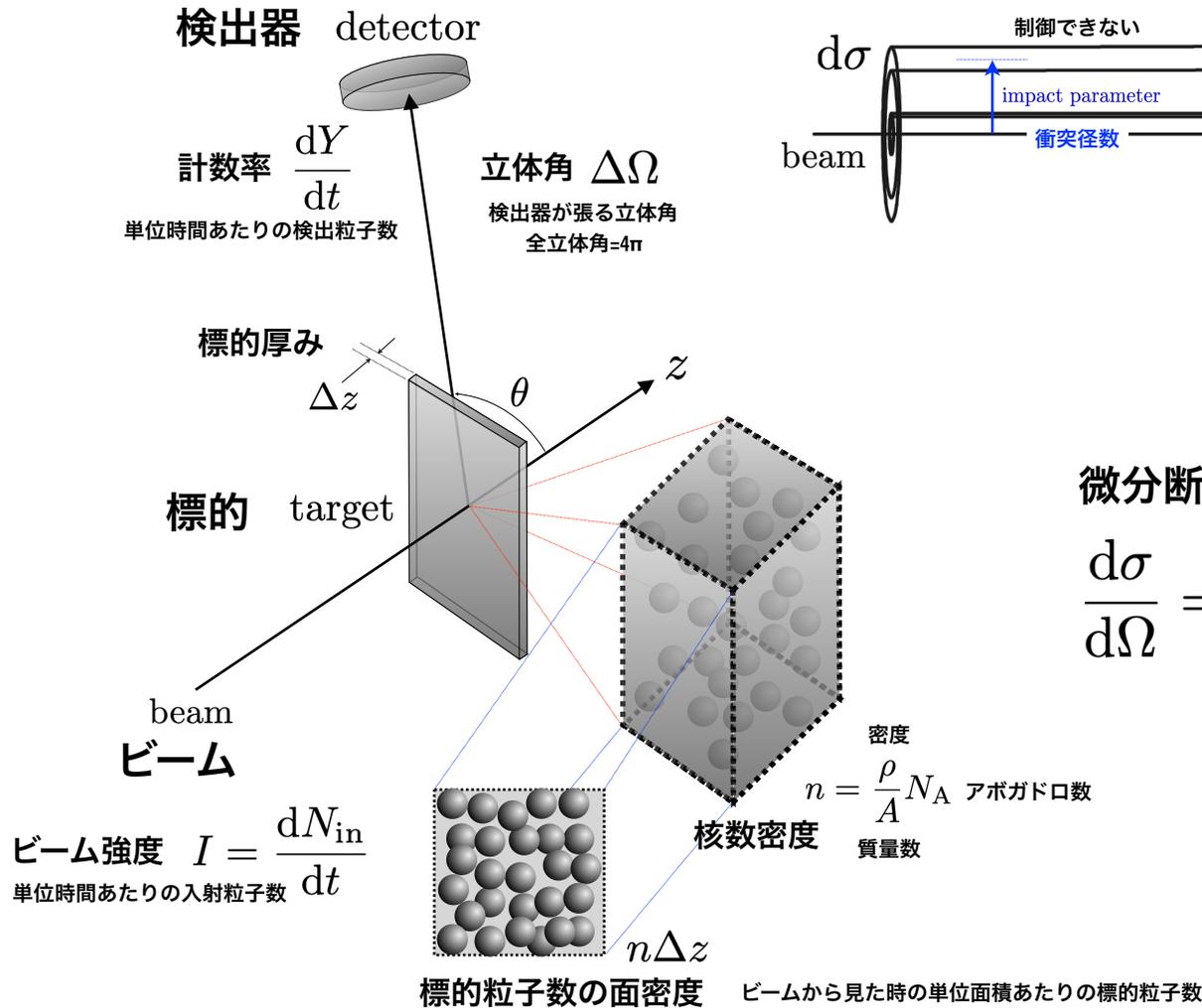
$$\rho_0 = 1.72 \times 10^{38} \text{ nucleon/cm}^3$$

$$R = r_0 A^{1/3}$$

$$r_0 = 1.12 \text{ fm}$$

这是通过粒子束散射实验揭示的
粒子ビームの散乱実験で明らかにされてきた

束流実験 ビーム実験



微分断面積

$$\frac{d\sigma}{d\Omega} = \lim_{\Delta\Omega \rightarrow 0} \frac{dY}{dt} \frac{1}{In\Delta z\Delta\Omega}$$

散乱の量子論

時間に依存しないポテンシャルによる散乱

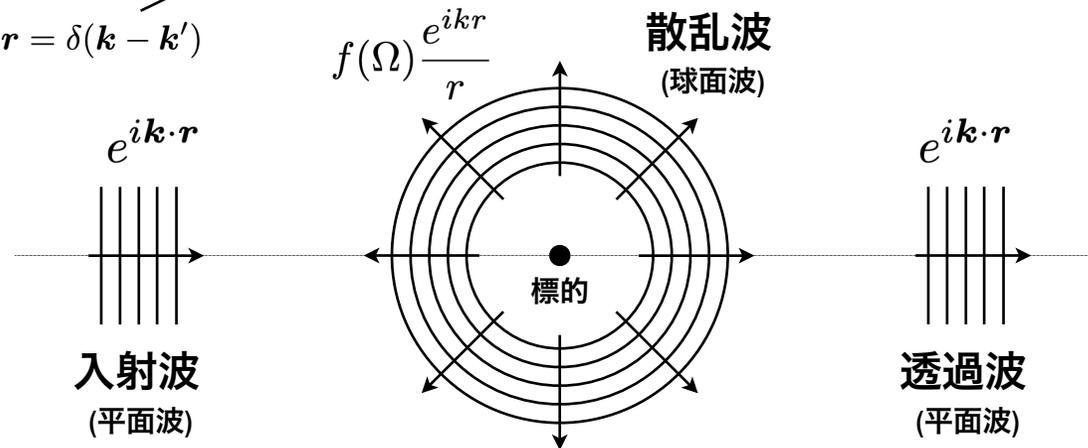
$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

時間を変数分離する

$$\psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt/\hbar} \quad \text{とおくと} \quad \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

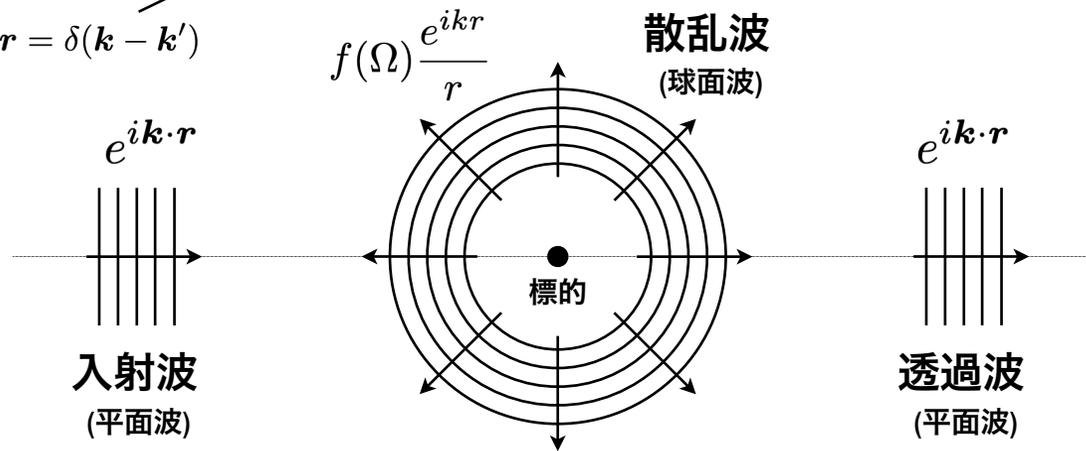
漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$$\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} d\mathbf{r} = \delta(\mathbf{k}-\mathbf{k}')$$



漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$$\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} d\mathbf{r} = \delta(\mathbf{k}-\mathbf{k}')$$



$$\phi = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{(2\pi)^{3/2}}$$

$$j(\phi) = \frac{\hbar}{2mi} \left(\phi^* \frac{\partial \phi}{\partial z} - \phi \frac{\partial \phi^*}{\partial z} \right) = \frac{\hbar k}{(2\pi)^3 m} = \frac{v}{(2\pi)^3}$$

入射粒子の確率流密度：単位時間あたり単位面積あたりの入射粒子数

$$I = \frac{dN_{\text{in}}}{dt} = Cj(\phi)$$

$$\chi = \frac{1}{(2\pi)^{3/2}} \frac{e^{ikr}}{r} f(\Omega)$$

$$j(\chi) = \frac{\hbar}{2mi} \left(\chi^* \frac{\partial \chi}{\partial r} - \chi \frac{\partial \chi^*}{\partial r} \right) = \frac{v}{(2\pi)^3 r^2} |f(\Omega)|^2$$

散乱粒子の確率流密度：単位時間あたり単位面積あたりの散乱粒子数

$$\frac{dY}{dt} = C n \Delta z j(\chi) r^2 d\Omega$$

$$\frac{d\sigma}{d\Omega} = \lim_{\Delta\Omega \rightarrow 0} \frac{dY}{dt} \frac{1}{I n \Delta z \Delta\Omega} = j(\chi) r^2 \frac{1}{j(\phi)} = |f(\Omega)|^2$$

漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$

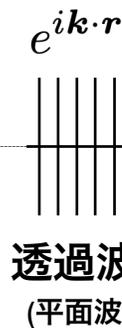
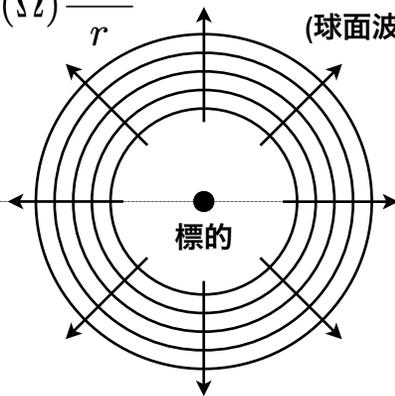
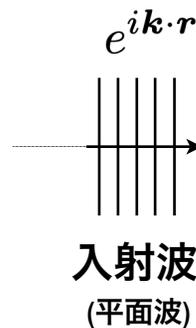
を満たす解を求める

$$\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} d\mathbf{r} = \delta(\mathbf{k}-\mathbf{k}')$$

$$f(\Omega) \frac{e^{ikr}}{r}$$

散乱波
(球面波)

$$\frac{d\sigma}{d\Omega} = |f(\Omega)|^2$$



散乱の量子論

時間に依存しないポテンシャルによる散乱

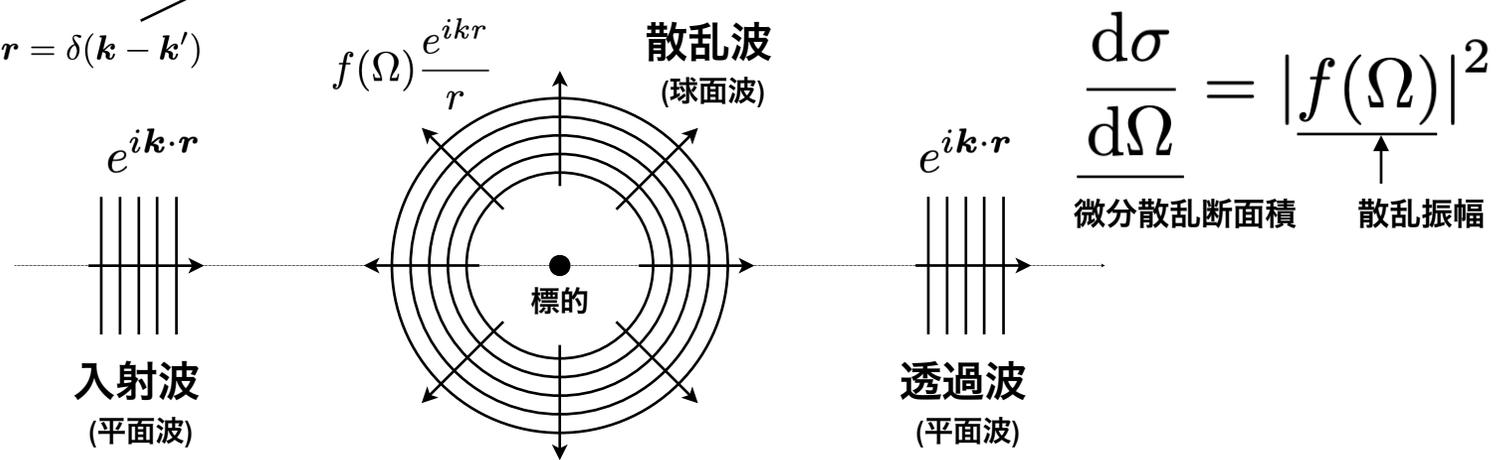
$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

時間を変数分離する

$$\psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt/\hbar} \quad \text{とおくと} \quad \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$$\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} d\mathbf{r} = \delta(\mathbf{k}-\mathbf{k}')$$



散乱の量子論

時間に依存しないポテンシャルによる散乱

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

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漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$k^2 = \frac{2m}{\hbar^2} E$, $U(\mathbf{r}) = \frac{2m}{\hbar^2} V(\mathbf{r})$ とおくと $(\nabla^2 + k^2) \psi(\mathbf{r}) = U(\mathbf{r})\psi(\mathbf{r})$ を解くことになる

$(\nabla^2 + k^2) \psi(\mathbf{r}) = 0$ の一般解と $(\nabla^2 + k^2) \psi(\mathbf{r}) = \delta(\mathbf{r})$ の特解の和で与えられる

$$\downarrow$$
$$\psi(\mathbf{r}) = Ae^{i\mathbf{k}\cdot\mathbf{r}} + Be^{-i\mathbf{k}\cdot\mathbf{r}}$$

散乱の量子論

時間に依存しないポテンシャルによる散乱

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

時間を変数分離する

$$\psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt/\hbar} \quad \text{とおくと} \quad \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$k^2 = \frac{2m}{\hbar^2} E, \quad U(\mathbf{r}) = \frac{2m}{\hbar^2} V(\mathbf{r})$ とおくと $(\nabla^2 + k^2) \psi(\mathbf{r}) = U(\mathbf{r})\psi(\mathbf{r})$ を解くことになる

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$$\psi(\mathbf{r}) = Ae^{i\mathbf{k}\cdot\mathbf{r}} + Be^{-i\mathbf{k}\cdot\mathbf{r}}$$

散乱の量子論

$$(\nabla^2 + k^2) \psi(\mathbf{r}) = U(\mathbf{r})\psi(\mathbf{r})$$

もし $(\nabla^2 + k^2)G(\mathbf{r}) = \delta(\mathbf{r})$ を満たす関数 $G(\mathbf{r})$ があれば

$$v(\mathbf{r}) = \int G(\mathbf{r} - \mathbf{r}')U(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}' \quad \text{という関数は}$$

$$(\nabla^2 + k^2)v(\mathbf{r}) = \int \delta(\mathbf{r} - \mathbf{r}')U(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}' = U(\mathbf{r})\psi(\mathbf{r}) \quad \text{となるので特解である}$$

この $G(\mathbf{r})$ をグリーン関数 (Green's function) と言う 伝播関数 (propagator) とも言う

$$(\nabla^2 + k^2) \psi(\mathbf{r}) = \delta(\mathbf{r}) \quad \text{の特解}$$

グリーン関数を求める

$$(\nabla^2 + k^2)G(\mathbf{r}) = \delta(\mathbf{r})$$

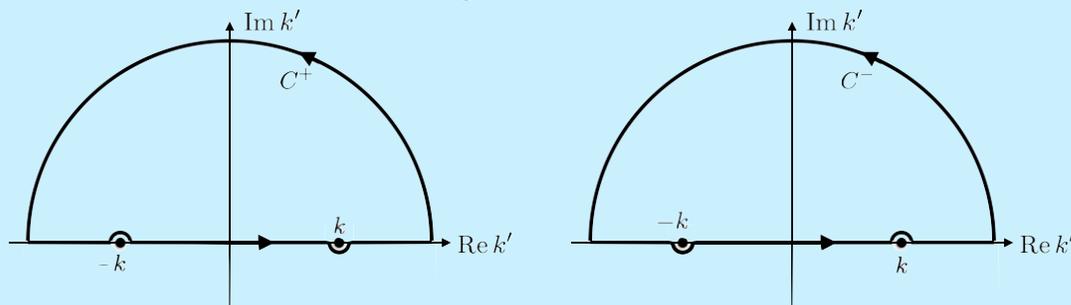
$$G(\mathbf{r}) = \frac{1}{(2\pi)^3} \int \frac{e^{i\mathbf{k}' \cdot \mathbf{r}}}{k^2 - k'^2} d\mathbf{k}' = \frac{1}{(2\pi)^3} \int_0^\infty k'^2 dk' \int_{-1}^1 d \cos \theta' \int_0^{2\pi} d\varphi' \frac{e^{ik'r \cos \theta'}}{k^2 - k'^2} = \frac{1}{4\pi^2 i r} \int_0^\infty \frac{e^{ik'r} - e^{-ik'r}}{k^2 - k'^2} k' dk'$$

$$= \frac{1}{4\pi^2 i r} \int_{-\infty}^\infty \frac{e^{ik'r}}{k^2 - k'^2} k' dk' = -\frac{1}{8\pi^2 i r} \int_{-\infty}^\infty \left(\frac{e^{ik'r}}{k' - k} + \frac{e^{ik'r}}{k' + k} \right) dk'$$

被積分関数は $\text{Im } k' > 0$ の領域で $|k'| \rightarrow \infty$ の時 0 に収束する
極は $k' = k$ と $k' = -k$ にある

留数定理を使って計算する

積分路 C^+ と C^- を使うと



$$G^\pm(\mathbf{r}) = -\lim_{\epsilon \rightarrow +0} \frac{1}{8\pi^2 i r} \int_{-\infty}^\infty \left(\frac{e^{ik'r}}{k' - k \mp i\epsilon} + \frac{e^{ik'r}}{k' + k \pm i\epsilon} \right) dk' = -\frac{1}{4\pi r} e^{\pm ikr}$$

$G^+(\mathbf{r})$ は外向き球面波 $G^-(\mathbf{r})$ は内向き球面波に対応する

$$G(\mathbf{r}) = \int \tilde{G}(\mathbf{k}') e^{i\mathbf{k}' \cdot \mathbf{r}} d\mathbf{k}' \quad \text{と} \quad \text{おいて}$$

$$\delta(\mathbf{r}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{k}' \cdot \mathbf{r}} d\mathbf{k}' \quad \text{を使うと}$$

$$(k^2 - k'^2)\tilde{G}(\mathbf{k}') = \frac{1}{(2\pi)^3} \quad \text{となるので}$$

散乱の量子論

$$(\nabla^2 + k^2) \psi(\mathbf{r}) = U(\mathbf{r})\psi(\mathbf{r})$$

もし $(\nabla^2 + k^2)G(\mathbf{r}) = \delta(\mathbf{r})$ を満たす関数 $G(\mathbf{r})$ があれば

$$v(\mathbf{r}) = \int G(\mathbf{r} - \mathbf{r}')U(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}' \quad \text{という関数は}$$

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この $G(\mathbf{r})$ をグリーン関数 (Green's function) と言う 伝播関数 (propagator) とも言う

$$G^\pm(\mathbf{r}) = - \lim_{\epsilon \rightarrow +0} \frac{1}{8\pi^2 i r} \int_{-\infty}^{\infty} \left(\frac{e^{ik'r}}{k' - k \mp i\epsilon} + \frac{e^{ik'r}}{k' + k \pm i\epsilon} \right) dk' = -\frac{1}{4\pi r} e^{\pm ikr}$$

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} + \int G^+(\mathbf{r} - \mathbf{r}')U(\mathbf{r}')\psi^+(\mathbf{r}')d\mathbf{r}'$$

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi} \int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} U(\mathbf{r}')\psi^+(\mathbf{r}')d\mathbf{r}'$$

散乱の量子論

漸近的境界条件 $(\nabla^2 + k^2)\psi(\mathbf{r}) = U(\mathbf{r})\psi(\mathbf{r})$ を満たす解を求める

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$$

もし $(\nabla^2 + k^2)G(\mathbf{r}) = \delta(\mathbf{r})$ を満たす関数 $G(\mathbf{r})$ があれば

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi} \int \frac{U(\mathbf{r}')\psi^+(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}'$$

$v(\mathbf{r}) = \int G(\mathbf{r} - \mathbf{r}')U(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}'$ という関数は

$$(\nabla^2 + k^2)v(\mathbf{r}) = \int \delta(\mathbf{r} - \mathbf{r}')U(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}' = U(\mathbf{r})\psi(\mathbf{r}) \quad \text{となるので特解である}$$

この $G(\mathbf{r})$ をグリーン関数 (Green's function) と言う 伝播関数 (propagator) とも言う

$$G^\pm(\mathbf{r}) = - \lim_{\epsilon \rightarrow +0} \frac{1}{8\pi^2 i r} \int_{-\infty}^{\infty} \left(\frac{e^{ik'r}}{k' - k \mp i\epsilon} + \frac{e^{ik'r}}{k' + k \pm i\epsilon} \right) dk' = -\frac{1}{4\pi r} e^{\pm ikr}$$

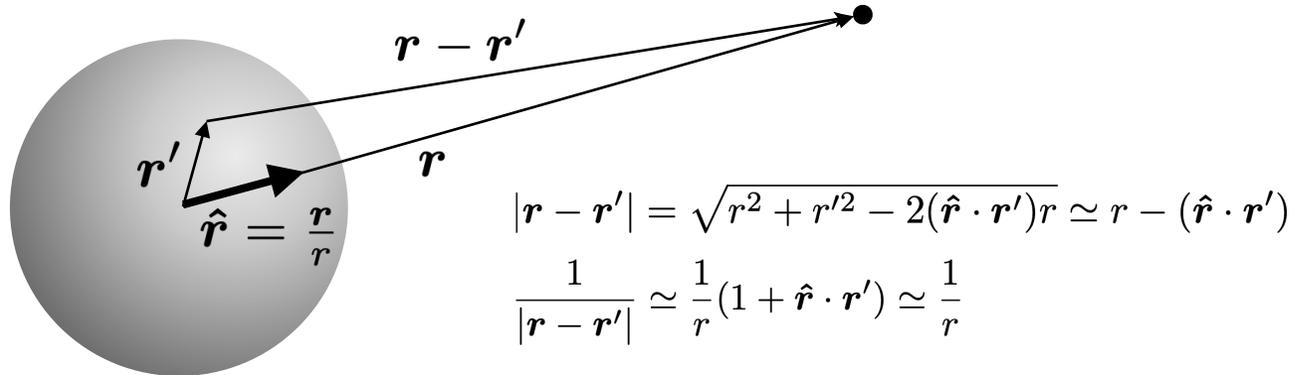
$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} + \int G^+(\mathbf{r} - \mathbf{r}')U(\mathbf{r}')\psi^+(\mathbf{r}')d\mathbf{r}'$$

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi} \int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} U(\mathbf{r}')\psi^+(\mathbf{r}')d\mathbf{r}'$$

散乱の量子論

漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$$\begin{aligned} \psi^+(\mathbf{r}) &= \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi} \int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} U(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \\ &\xrightarrow{r \rightarrow \infty} \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{e^{ikr}}{r} \int \left(-\frac{(2\pi)^{3/2}}{4\pi} e^{-i\mathbf{k}\hat{\mathbf{r}}\cdot\mathbf{r}'} U(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right) \right] \\ &= \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2} m}{2\pi\hbar^2} \frac{e^{ikr}}{r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right] \end{aligned}$$



散乱の量子論

漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$$\begin{aligned}\psi^+(\mathbf{r}) &= \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{1}{4\pi} \int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} U(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \\ &\xrightarrow{r \rightarrow \infty} \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} + \frac{e^{ikr}}{r} \int \left(-\frac{(2\pi)^{3/2}}{4\pi} e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'} U(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right) \right] \\ &= \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2} m}{2\pi\hbar^2} \frac{e^{ikr}}{r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right]\end{aligned}$$

$$f(\Omega) = -\frac{(2\pi)^{3/2} m}{2\pi\hbar^2} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}'$$

散乱の量子論

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散乱の量子論

Lippmann-Schwinger方程式

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2}m}{2\pi\hbar^2} \frac{e^{ikr}}{r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right]$$

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散乱の量子論

Lippmann-Schwinger方程式

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2}m}{2\pi\hbar^2} \frac{e^{i\mathbf{k}\mathbf{r}}}{r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right]$$

書き直し 自由ハミルトニアンと相互作用ハミルトニアンに分ける

自由ハミルトニアンの固有状態

$$H = H_0 + V(\mathbf{r}) = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r})$$

$$\phi(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} \quad H_0\phi(\mathbf{r}) = \frac{\hbar^2 k^2}{2m} \phi(\mathbf{r})$$

$$G^\pm(\mathbf{r} - \mathbf{r}') = \lim_{\epsilon \rightarrow +0} \frac{1}{(2\pi)^3} \int \frac{e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')}}{k^2 - k'^2 \pm i\epsilon} d\mathbf{k}' = \lim_{\epsilon \rightarrow +0} \frac{1}{(2\pi)^3} \frac{\hbar^2}{2m} \int \frac{1}{E - E' \pm i\epsilon} e^{i\mathbf{k}'\cdot\mathbf{r}} e^{-i\mathbf{k}'\cdot\mathbf{r}'} d\mathbf{k}' \quad \text{これを代入する}$$

$$\psi^\pm(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} + \int G^\pm(\mathbf{r} - \mathbf{r}') U(\mathbf{r}') \psi^\pm(\mathbf{r}') d\mathbf{r}' = \phi(\mathbf{r}) + \int d\mathbf{r}' \int d\mathbf{k}' \frac{1}{(2\pi)^3} \frac{\hbar^2}{2m} \frac{e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')}}{E - E' \pm i\epsilon} \frac{2m}{\hbar^2} V(\mathbf{r}') \psi^\pm(\mathbf{r}')$$

$$= \phi(\mathbf{r}) + \frac{1}{(2\pi)^3} \int d\mathbf{r}' \int d\mathbf{k}' \frac{1}{E - E' \pm i\epsilon} e^{i\mathbf{k}'\cdot\mathbf{r}} e^{-i\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}') \psi^\pm(\mathbf{r}')$$

$$= \phi(\mathbf{r}) + \frac{1}{(2\pi)^3} \int d\mathbf{r}' \int d\mathbf{k}' \frac{1}{E - \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \mathbf{r}'^2}\right) \pm i\epsilon} e^{i\mathbf{k}'\cdot\mathbf{r}} e^{-i\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}') \psi^\pm(\mathbf{r}')$$

$$= \phi(\mathbf{r}) + \frac{1}{E - H_0 \pm i\epsilon} \int d\mathbf{r}' \frac{1}{(2\pi)^3} \int d\mathbf{k}' e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')} V(\mathbf{r}') \psi^\pm(\mathbf{r}')$$

$$= \phi(\mathbf{r}) + \frac{1}{E - H_0 \pm i\epsilon} \int d\mathbf{r}' \delta(\mathbf{r} - \mathbf{r}') V(\mathbf{r}') \psi^\pm(\mathbf{r}')$$

$$= \phi(\mathbf{r}) + \frac{1}{E - H_0 \pm i\epsilon} V(\mathbf{r}) \psi^\pm(\mathbf{r})$$

これは表現に依存しないので

Lippmann-Schwinger方程式

$$\psi^\pm = \phi + \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm$$

散乱の量子論

Lippmann-Schwinger方程式

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2}m}{2\pi\hbar^2} \frac{e^{i\mathbf{k}r}}{r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right]$$

書き直し 自由ハミルトニアンと相互作用ハミルトニアンに分ける

$$\psi^\pm = \phi \pm \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm$$

$$H = H_0 + V(\mathbf{r}) \quad H_0 = \frac{\hbar^2 \nabla^2}{2m} + V(\mathbf{r})$$

自由ハミルトニアンの固有状態

$$\phi(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} \quad H_0 \phi(\mathbf{r}) = \frac{\hbar^2 k^2}{2m} \phi(\mathbf{r})$$

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$$\begin{aligned} \psi^\pm(\mathbf{r}) &= \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} + \int G^\pm(\mathbf{r} - \mathbf{r}') U(\mathbf{r}') \psi^\pm(\mathbf{r}') d\mathbf{r}' = \phi(\mathbf{r}) + \int d\mathbf{r}' \int d\mathbf{k}' \frac{1}{(2\pi)^3} \frac{\hbar^2}{2m} \frac{e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')}}{E - E' \pm i\epsilon} \frac{2m}{\hbar^2} V(\mathbf{r}') \psi^\pm(\mathbf{r}') \\ &= \phi(\mathbf{r}) + \frac{1}{(2\pi)^3} \int d\mathbf{r}' \int d\mathbf{k}' \frac{1}{E - E' \pm i\epsilon} e^{i\mathbf{k}'\cdot\mathbf{r}} e^{-i\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}') \psi^\pm(\mathbf{r}') \\ &= \phi(\mathbf{r}) + \frac{1}{(2\pi)^3} \int d\mathbf{r}' \int d\mathbf{k}' \frac{1}{E - \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \mathbf{r}'^2}\right) \pm i\epsilon} e^{i\mathbf{k}'\cdot\mathbf{r}} e^{-i\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}') \psi^\pm(\mathbf{r}') \\ &= \phi(\mathbf{r}) + \frac{1}{E - H_0 \pm i\epsilon} \int d\mathbf{r}' \frac{1}{(2\pi)^3} \int d\mathbf{k}' e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')} V(\mathbf{r}') \psi^\pm(\mathbf{r}') \\ &= \phi(\mathbf{r}) + \frac{1}{E - H_0 \pm i\epsilon} \int d\mathbf{r}' \delta(\mathbf{r} - \mathbf{r}') V(\mathbf{r}') \psi^\pm(\mathbf{r}') \\ &= \phi(\mathbf{r}) + \frac{1}{E - H_0 \pm i\epsilon} V(\mathbf{r}) \psi^\pm(\mathbf{r}) \end{aligned}$$

Lippmann-Schwinger方程式

$$\psi^\pm = \phi + \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm$$

これは表現に依存しないので

散乱の量子論

Lippmann-Schwinger方程式

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2}m}{2\pi\hbar^2} \frac{e^{i\mathbf{k}r}}{r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right]$$

$$\psi^\pm = \phi + \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm \quad G_0^\pm = \frac{1}{E - H_0 \pm i\epsilon} \quad \text{とおくと}$$

$$\psi^\pm = \phi + G_0^\pm V \psi^\pm$$

恒等式 $\frac{1}{A} - \frac{1}{B} = \frac{1}{B}(B - A)\frac{1}{A}$ を使うと

$$\frac{1}{E - H_0 \pm i\epsilon} - \frac{1}{E - H \pm i\epsilon} = \frac{1}{E - H \pm i\epsilon} (H_0 - H) \frac{1}{E - H_0 \pm i\epsilon}$$

$$\frac{1}{E - H_0 \pm i\epsilon} = \frac{1}{E - H \pm i\epsilon} - \frac{1}{E - H \pm i\epsilon} V \frac{1}{E - H_0 \pm i\epsilon}$$

$$\psi^\pm = \phi + \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm$$

$$G_0^\pm = G^\pm - G^\pm V G_0^\pm$$

$$= \phi + \frac{1}{E - H \pm i\epsilon} \left[1 - V \frac{1}{E - H_0 \pm i\epsilon} \right] V \psi^\pm$$

$$= \phi + \frac{1}{E - H \pm i\epsilon} V \left[\psi^\pm - \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm \right]$$

$$= \phi + \frac{1}{E - H \pm i\epsilon} V \phi \quad \text{形式解}$$

散乱の量子論

Lippmann-Schwinger方程式

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2}m}{2\pi\hbar^2} \frac{e^{ikr}}{r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right]$$

$$\psi^\pm = \phi + \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm \quad G_0^\pm = \frac{1}{E - H_0 \pm i\epsilon} \quad \text{とおくと}$$

$$\boxed{\psi^\pm} = \phi + G_0^\pm V \boxed{\psi^\pm}$$

$$\boxed{\psi^\pm} = \phi + G_0^\pm V \phi + G_0^\pm V G_0^\pm V \boxed{\psi^\pm}$$

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$$\psi^\pm = \phi + G_0^\pm V \phi + G_0^\pm V G_0^\pm V \phi + G_0^\pm V G_0^\pm V G_0^\pm V \phi + \dots \quad \text{Born展開}$$

平面波

第一Born項

第二Born項

第三Born項

Born展開

Lippmann-Schwinger方程式 $G_0^\pm V G_0^\pm V \phi + G_0^\pm V G_0^\pm V G_0^\pm V \phi + \dots$

$$\psi^+(r) \stackrel{\text{平面波}}{=} \frac{1}{(2\pi)^{3/2}} \left[e^{ik \cdot r} - \frac{(2\pi)^{3/2} m}{2\pi \hbar^2} \frac{e^{ikr}}{r} \int e^{-ik \cdot r'} V(r') \psi^+(r') dr' \right]$$

第一Born項
第二Born項
第三Born項

$$\psi^\pm = \phi + \frac{1}{E - H_0 \pm i\epsilon} V \psi^\pm \quad G_0^\pm = \frac{1}{E - H_0 \pm i\epsilon} \quad \text{とおくと}$$

$$\boxed{\psi^\pm} = \phi + G_0^\pm V \boxed{\psi^\pm}$$

$$\boxed{\psi^\pm} = \phi + G_0^\pm V \phi + G_0^\pm V G_0^\pm V \boxed{\psi^\pm}$$

$$\boxed{\psi^\pm} = \phi + G_0^\pm V \phi + G_0^\pm V G_0^\pm V \phi + G_0^\pm V G_0^\pm V G_0^\pm V \boxed{\psi^\pm}$$

$$\psi^\pm = \phi + G_0^\pm V \phi + G_0^\pm V G_0^\pm V \phi + G_0^\pm V G_0^\pm V G_0^\pm V \phi + \dots \quad \text{Born展開}$$

平面波

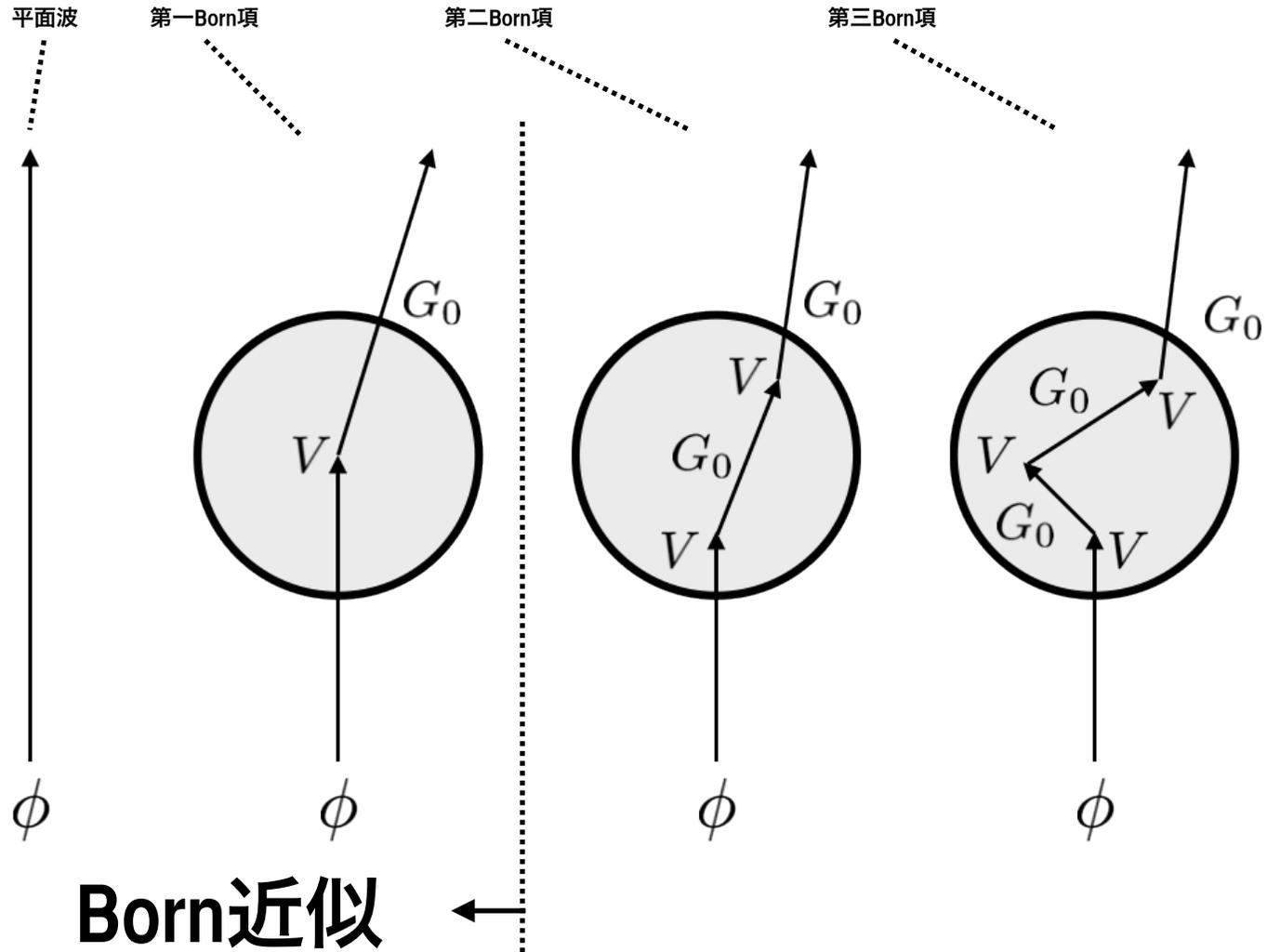
第一Born項

第二Born項

第三Born項

Born展開

$$\psi^\pm = \phi + G_0^\pm V \phi + G_0^\pm V G_0^\pm V \phi + G_0^\pm V G_0^\pm V G_0^\pm V \phi + \dots$$



Born近似

$$\psi^\pm = \phi + G_0^\pm V \phi$$

平面波 第一Born項

座標表現でのLippmann-Schwinger方程式

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2} m e^{ikr}}{2\pi\hbar^2 r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \psi^+(\mathbf{r}') d\mathbf{r}' \right]$$

Born近似

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \frac{(2\pi)^{3/2} m e^{ikr}}{2\pi\hbar^2 r} \int e^{-i\mathbf{k}\cdot\mathbf{r}'} V(\mathbf{r}') \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}'\cdot\mathbf{r}'} d\mathbf{r}' \right]$$

$$\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}} - \underbrace{\left(\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}') e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}'} d\mathbf{r}' \right)}_{f(\Omega)} \frac{e^{ikr}}{r} \right]$$

$f(\Omega)$

ビーム軸について対称な場合、方位角には依存しないので散乱振幅は $f(\theta)$ と書かれる

Born近似

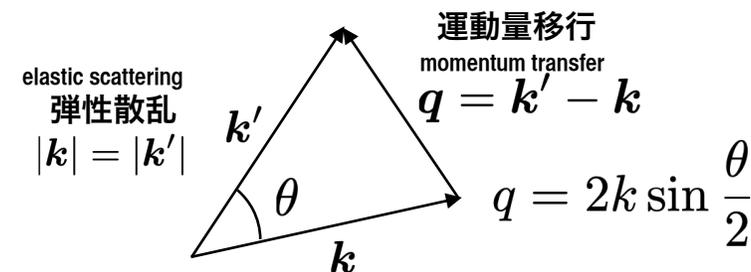
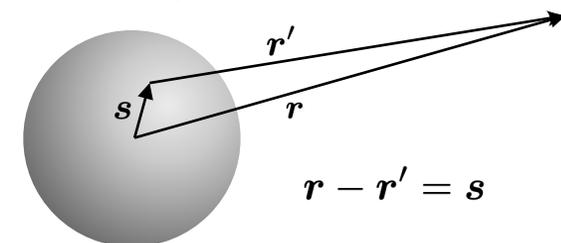
$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad f(\theta) = -\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}) e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} d\mathbf{r}$$

電子散乱

$$V(\mathbf{r}) = -Ze^2 \int \frac{\rho(\mathbf{s})}{|\mathbf{r} - \mathbf{s}|} d\mathbf{s} \quad \int \rho(\mathbf{s}) d\mathbf{s} = 1$$

$$\begin{aligned} f(\theta) &= -\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}) e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}} d\mathbf{r} \\ &= Ze^2 \frac{m}{2\pi\hbar^2} \int \frac{\rho(\mathbf{s})}{|\mathbf{r} - \mathbf{s}|} e^{i\mathbf{q} \cdot (\mathbf{r} - \mathbf{s})} e^{i\mathbf{q} \cdot \mathbf{s}} d\mathbf{r} d\mathbf{s} \\ &= Ze^2 \frac{m}{2\pi\hbar^2} \int \frac{e^{i\mathbf{q} \cdot (\mathbf{r} - \mathbf{s})}}{|\mathbf{r} - \mathbf{s}|} \rho(\mathbf{s}) e^{i\mathbf{q} \cdot \mathbf{s}} d\mathbf{r} d\mathbf{s} \\ &= Ze^2 \frac{m}{2\pi\hbar^2} \int \frac{e^{i\mathbf{q} \cdot \mathbf{r}'}}{r'} d\mathbf{r}' \int \rho(\mathbf{s}) e^{i\mathbf{q} \cdot \mathbf{s}} d\mathbf{s} \end{aligned}$$

$$= \frac{Ze^2}{4E_e} \frac{1}{\sin^2 \frac{\theta}{2}} F(\mathbf{q})$$

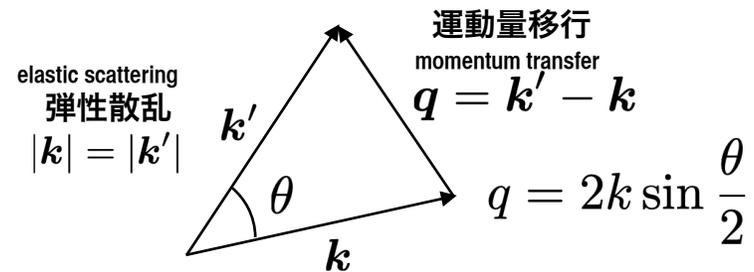


$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4E_e} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} |F(\mathbf{q})|^2$$

ラザフォード散乱 形状因子

Born近似 $\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad f(\theta) = -\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}) e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}} d\mathbf{r}$

電子散乱



$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4E_e} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} |F(\mathbf{q})|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$$

Born近似 $\frac{d\sigma}{d\Omega} = |f(\theta)|^2$ $f(\theta) = -\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}) e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}} d\mathbf{r}$

電子散乱

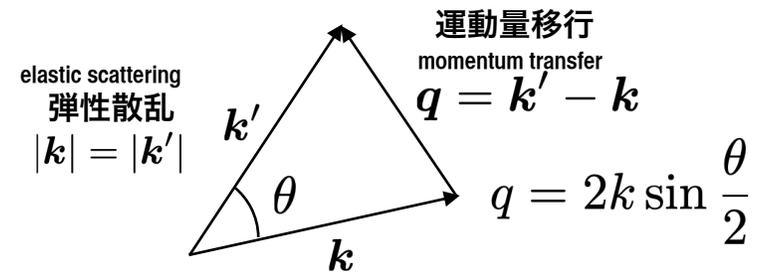
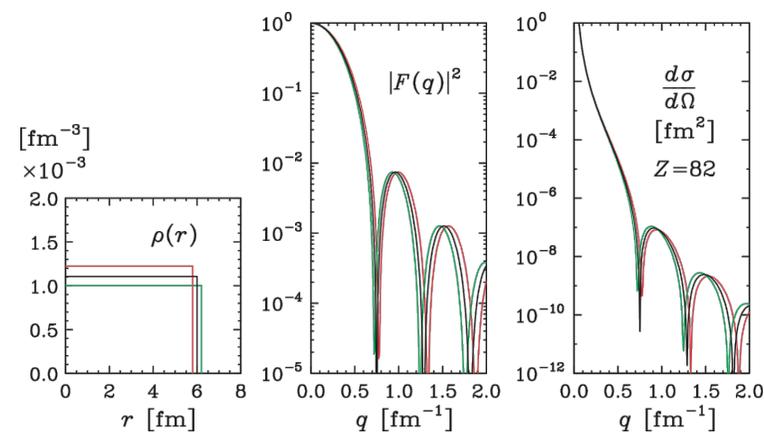
一様な球状電荷分布

$$\rho(\mathbf{r}) = \begin{cases} \rho_0 & (r < R) \\ 0 & (r > R) \end{cases}$$

$$\rho_0 = \frac{3}{4\pi R^3}$$

$$\begin{aligned} F(\mathbf{q}) &= \frac{3}{qR^3} \int_0^R \sin qr \, r dr \\ &= 3 \frac{\sin qR - qR \cos qR}{(qR)^3} \end{aligned}$$

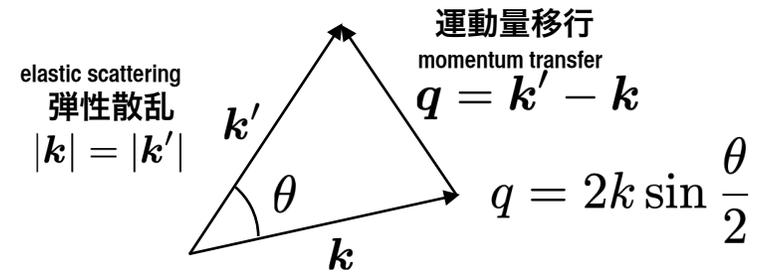
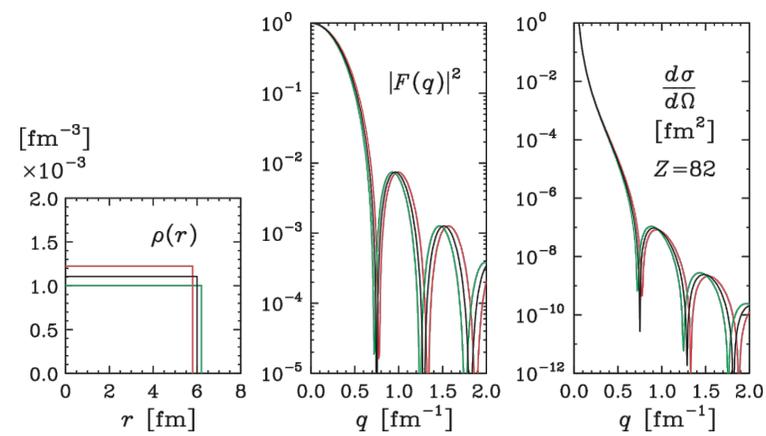
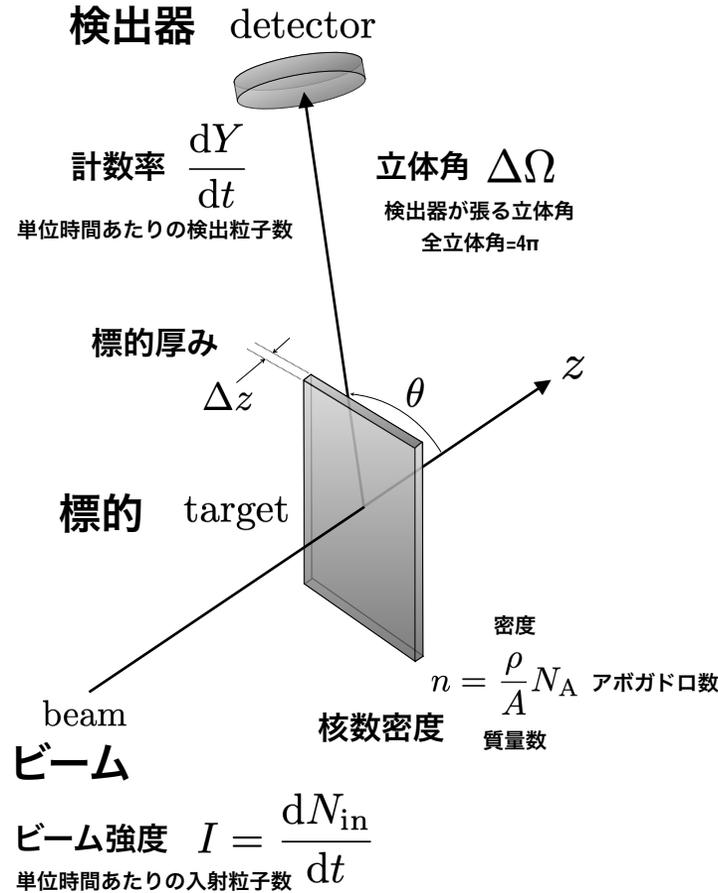
Born近似 $\frac{d\sigma}{d\Omega} = |f(\theta)|^2$ $f(\theta) = -\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}) e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}} d\mathbf{r}$



$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4E_e} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} |F(\mathbf{q})|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$$

電子散乱



$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4E_e} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} |F(\mathbf{q})|^2$$

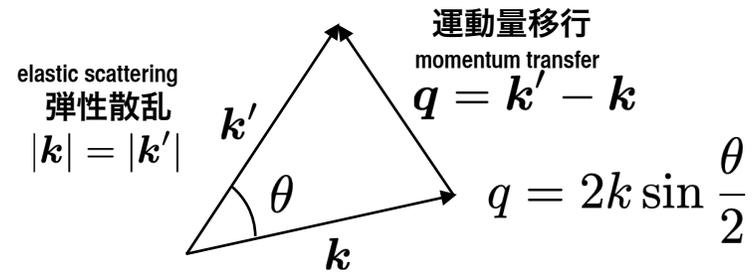
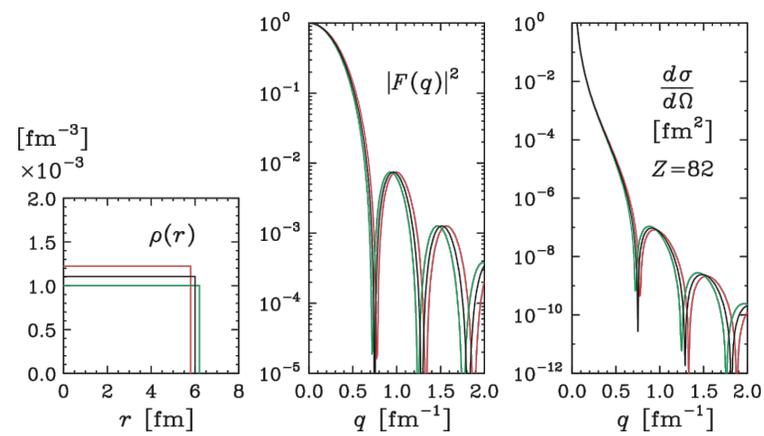
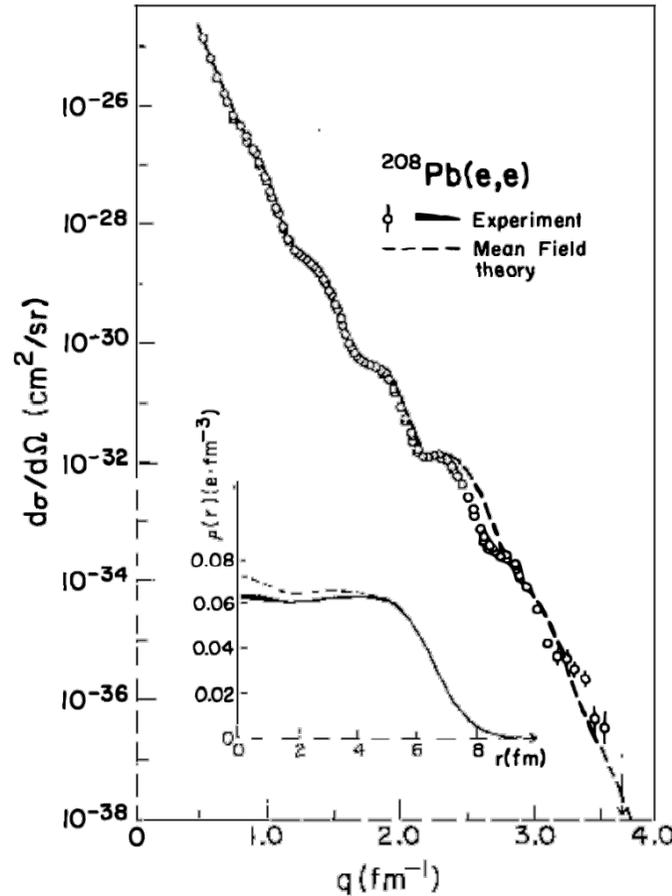
$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$$

Born近似

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad f(\theta) = -\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}) e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}} d\mathbf{r}$$

電子散乱

Ann. Rev. Nucl. Part. Sci. **37**, 133 (1987)



$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4E_e} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} |F(\mathbf{q})|^2$$

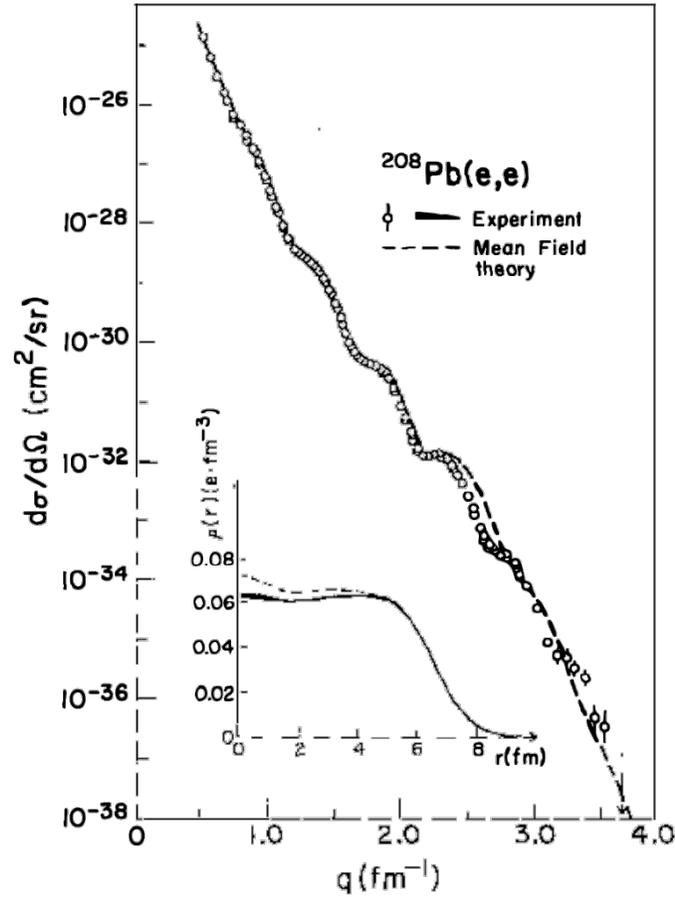
$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$$

Born近似

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \quad f(\theta) = -\frac{m}{2\pi\hbar^2} \int V(\mathbf{r}) e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{r}} d\mathbf{r}$$

電子散乱

Ann. Rev. Nucl. Part. Sci. 37, 133 (1987)



$$E_e = 502 \text{ MeV} \gg m_e c^2 = 0.511 \text{ MeV}$$

Mott散乱

$$\left(\frac{Ze^2}{4E_e} \right)^2 \left(1 - \beta^2 \sin^2 \frac{\theta}{2} \right) \frac{1}{\sin^4 \frac{\theta}{2}}$$

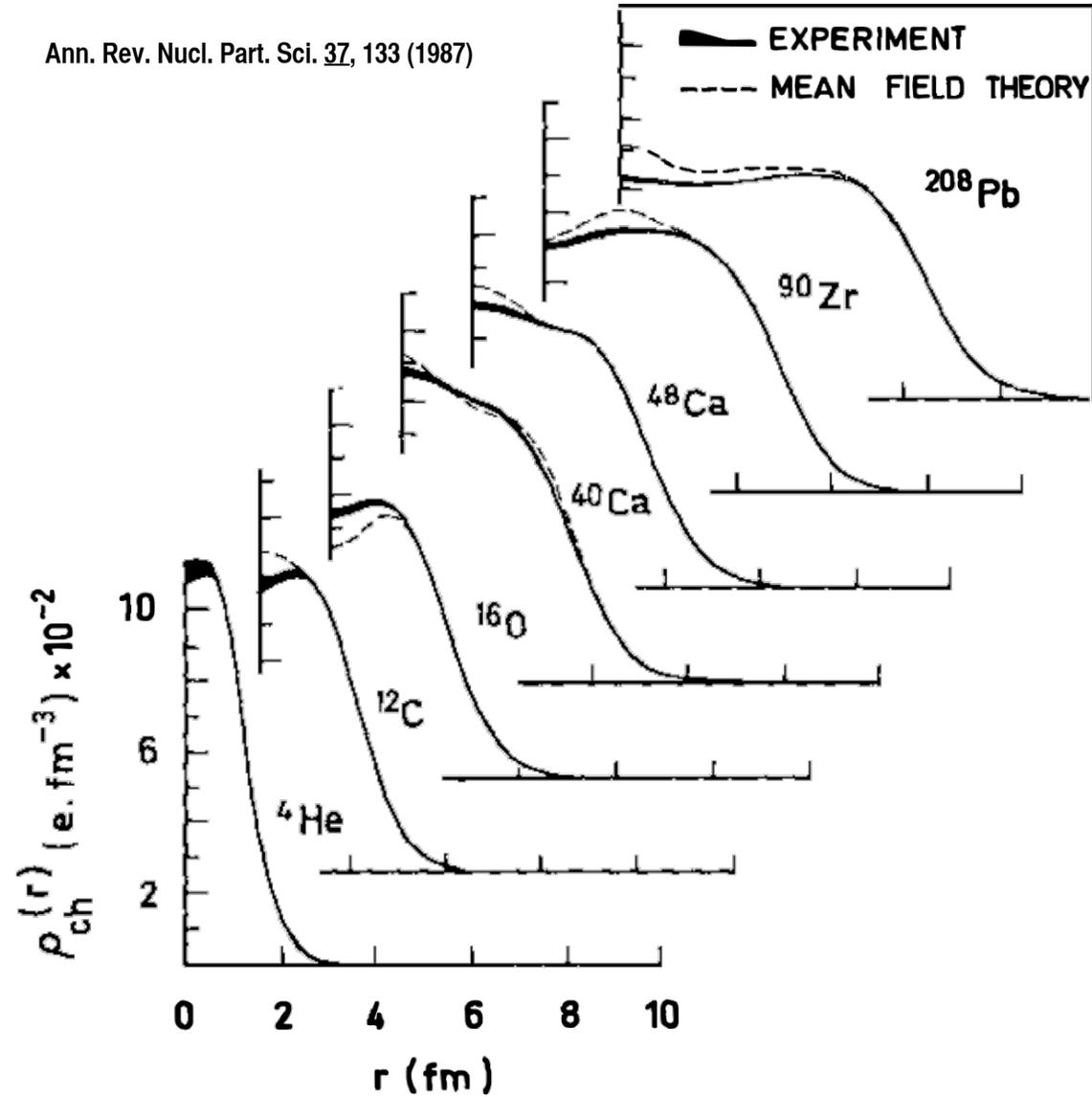
Rutherford散乱

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4E_e} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} |F(\mathbf{q})|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q} \cdot \mathbf{r}} d\mathbf{r}$$

電子散乱

Ann. Rev. Nucl. Part. Sci. 37, 133 (1987)



核結合能 原子核の結合エネルギー

核の質量

原子の質量単位 静止した基底状態にある自由な 中性原子 ^{12}C を 12 とする

統一原子質量単位 u unified atomic mass unit

ダルトン Da dalton

原子質量単位 amu atomic mass unit
a.m.u.

$$1 \text{ u} = 1.660\,539\,066\,60(50) \times 10^{-27} \text{ kg}$$

核の質量
$$M_{\text{nuclear}} = M_{\text{atom}} - (Zm_e - B_e(Z))$$

電子のクーロン場などの結合エネルギーに伴う質量

$$m_e = (0.510\,998\,9461 \pm 0.000\,000\,0031) \text{ MeV}$$

$$B_e(Z) \simeq 15.73 Z^{7/3} \text{ eV}$$

$$m_p = (1.007\,276\,466\,879 \pm 0.000\,000\,000\,091) \text{ u}$$

$$= (938.272\,0813 \pm 0.000\,0058) \text{ MeV}$$

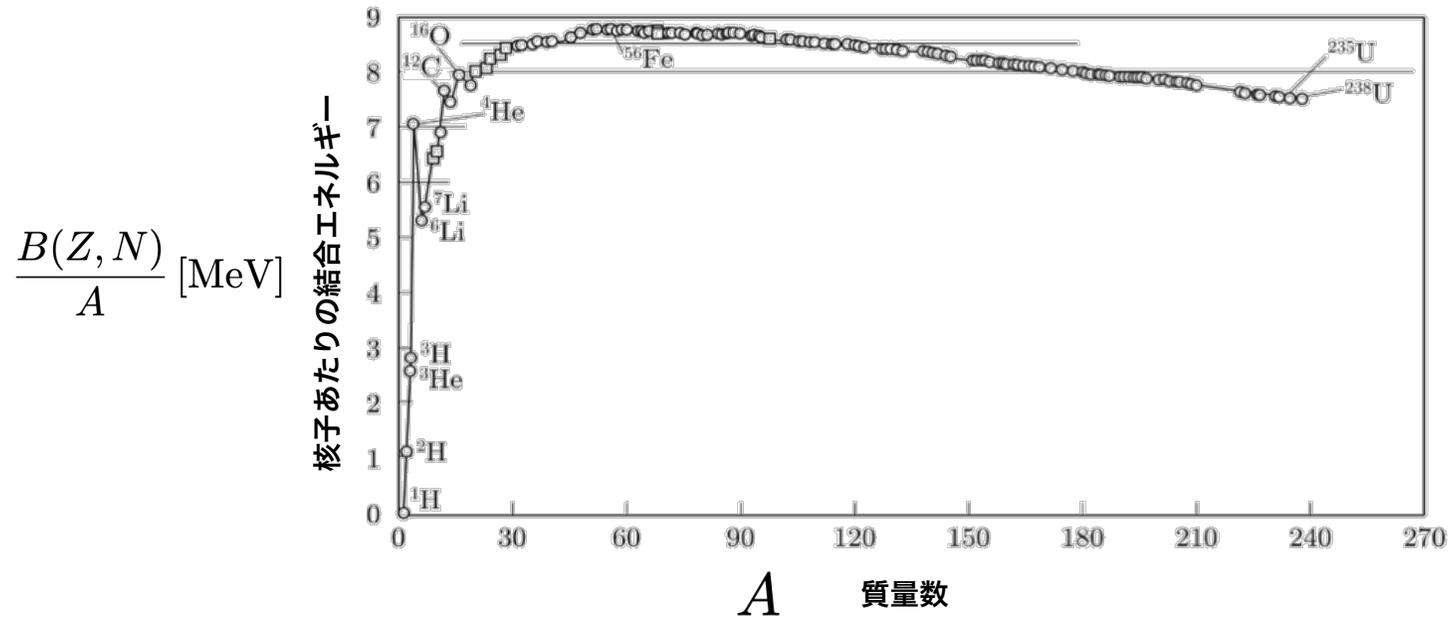
$$m_n = (939.565\,4133 \pm 0.000\,0058) \text{ MeV}$$

核結合エネルギー

定義
$$B(Z, N) = Z \underbrace{M_{\text{H}}}_{\text{水素原子質量}} + N \underbrace{M_{\text{n}}}_{\text{中性子質量}} - \underbrace{M(Z, N)}_{\text{原子質量}} > 0$$

電子質量は相殺される。電子の結合エネルギーは相殺されない ($\sim 10^{-4}$)

実験値から
$$\frac{B(Z, N)}{A} \simeq 8.5 \text{ MeV/nucleon} \quad \text{飽和性 (saturation property)}$$



核結合エネルギー

定義 $B(Z, N) = Z \overbrace{M_H}^{\text{水素原子質量}} + N \overbrace{M_n}^{\text{中性子質量}} - \overbrace{M(Z, N)}^{\text{原子質量}} > 0$

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半経験質量公式 (semi-empirical mass formula) Bethe-Weizsäcker's mass formula

$$M(Z, N) = ZM_p + NM_n - a_1A + a_2A^{2/3} + a_3\frac{Z^2}{A^{1/3}} + a_4\frac{(Z - N)^2}{A} + \delta(A)$$

$$B(Z, N) = \overbrace{a_1A}^{\text{volume energy}} - \overbrace{a_2A^{2/3}}^{\text{lack of surface energy}} - \overbrace{a_3\frac{Z^2}{A^{1/3}}}^{\text{Coulomb energy}} - \overbrace{a_4\frac{(Z - N)^2}{A}}^{\text{symmetry energy}} - \overbrace{\delta(A)}^{\text{pairing energy}}$$

体積エネルギー 表面エネルギーの欠損
 表面では片方しか結合がない

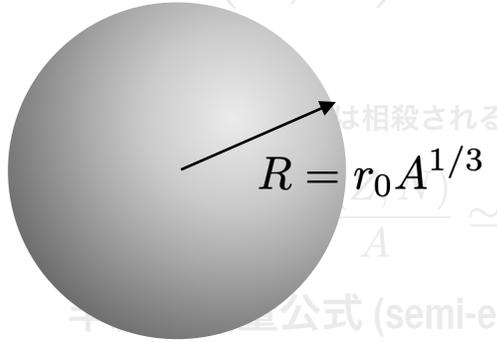
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核結合エネルギー

定義 Z個の陽子が一様分布

$$M_{\text{H}} + \text{Coulomb energy} = \frac{3 Z^2 e^2}{5 R} = \frac{3e^2}{5r_0} \frac{Z^2}{A^{1/3}}$$

↑ 水素原子質量
 ↑ 中性子質量
 ↑ 原子質量



は相殺される。電子の結合エネルギーは相殺されない ($\sim 10^{-4}$)

$$a_3 = \frac{3e^2}{5r_0} = \frac{3}{5} \times \frac{e^2}{\hbar c} \times \hbar c \times \frac{1}{r_0} = 0.69 \text{ MeV}$$

↑ 197 MeV fm
 ↑ 1.25 fm

$$M(Z, N) = ZM_p + NM_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z - N)^2}{A} + \delta(A)$$

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体積エネルギー 表面エネルギーの欠損 クーロンエネルギー 対称エネルギー 対エネルギー

lack of surface energy
表面エネルギーの欠損
表面では片方しか結合がない

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$

核結合エネルギー

定義 $B(Z, N) = Z \overbrace{M_{\text{H}}}^{\text{水素原子質量}} + N \overbrace{M_{\text{n}}}^{\text{中性子質量}} - \overbrace{M(Z, N)}^{\text{原子質量}} > 0$

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体積エネルギー 表面エネルギーの欠損
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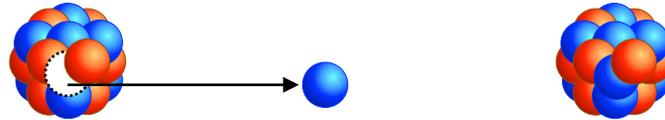
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$$\text{実験値から } \frac{B(Z, N)}{A} \simeq 8.5 \text{ MeV/nucleon} \quad \text{飽和性 (saturation property)}$$

分離エネルギー (separation energy)

一つの中性子の分離エネルギー S_{n}

$$S_{\text{n}} = \{M(Z, N - 1) + M_{\text{n}}\} - M(Z, N)$$



$$= B(Z, N) - B(Z, N - 1) \quad \text{およそ } 8\text{MeV}$$

核分裂

nuclear fission

$$M(Z, N) = ZM_p + NM_n - \underbrace{a_1 A}_{\text{volume energy}} + \underbrace{a_2 A^{2/3}}_{\text{lack of surface energy}} + \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\text{Coulomb energy}} + \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\text{symmetry energy}} + \underbrace{\delta(A)}_{\text{pairing energy}}$$

分裂に伴う質量変化

表面エネルギーとクーロンエネルギー

$$(Z, N) \rightarrow 2 \left(\frac{Z}{2}, \frac{N}{2} \right) \quad \begin{array}{l} a_2 A^{2/3} \rightarrow 2a_2 (A/2)^{2/3} \\ a_3 \frac{Z^2}{A^{1/3}} \rightarrow 2a_3 \frac{(Z/2)^2}{(A/2)^{1/3}} \end{array}$$

自発核分裂 (spontaneous nuclear fission) が起こる条件は

$$Q = 2B \left(\frac{Z}{2}, \frac{N}{2} \right) - B(Z, N) = -a_2(2^{1/3} - 1)A^{2/3} + a_3(1 - 2^{-2/3})\frac{Z^2}{A^{1/3}} > 0$$

$$\frac{Z^2}{A} > \frac{2^{1/3} - 1}{1 - 2^{-2/3}} \frac{a_2}{a_3} = \frac{2 - 2^{2/3}}{2^{2/3} - 1} \cdot \frac{18.56}{0.717} \simeq 18.2$$

$${}_{92}^{238}\text{U}_{146} \quad \left\{ \begin{array}{l} Z = 92 \\ N = 146 \end{array} \right. \rightarrow \frac{Z^2}{A} \simeq 35.6$$

$$M(Z, N) = ZM_p + NM_n - \underbrace{a_1 A}_{\text{volume energy}} + \underbrace{a_2 A^{2/3}}_{\text{lack of surface energy}} + \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\text{Coulomb energy}} + \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\text{symmetry energy}} + \underbrace{\delta(A)}_{\text{pairing energy}}$$

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問題2

^{32}S 原子核が二つの ^{16}O 原子核に分裂したとする。

その際の質量変化が、表面エネルギーとクーロンエネルギーの変化のみから生じているとして、その質量変化を求めよ

問題2 ^{32}S 原子核が二つの ^{16}O 原子核に分裂したとする。

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$$M(Z, N) = ZM_p + NM_n - \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} + \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} + \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} + \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} + \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

$$^{32}\text{S} : Z = 16, N = 16$$

$$^{16}\text{O} : Z = 8, N = 8$$

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$

$$M(^{32}\text{S}) = M(16, 16) = 16M_p + 16M_n - 32a_1 + 32^{2/3}a_2 + a_3 \frac{16^2}{32^{1/3}} + \frac{(16-16)^2}{32}a_4 + \delta(32)$$

$$2M(^{16}\text{O}) = 2M(8, 8) = 2(8M_p + 8M_n - 16a_1 + 16^{2/3}a_2 + a_3 \frac{8^2}{16^{1/3}} + \frac{(8-8)^2}{16}a_4 + \delta(16))$$

$$Q = M(^{32}\text{S}) - 2M(^{16}\text{O}) = \left(32^{2/3} - 2 \cdot 16^{2/3}\right) a_2 + \left(\frac{16^2}{32^{1/3}} - \frac{2 \cdot 8^2}{16^{1/3}}\right) a_3 = -27 \text{ MeV}$$

Q-value (発熱量)

吸熱反応

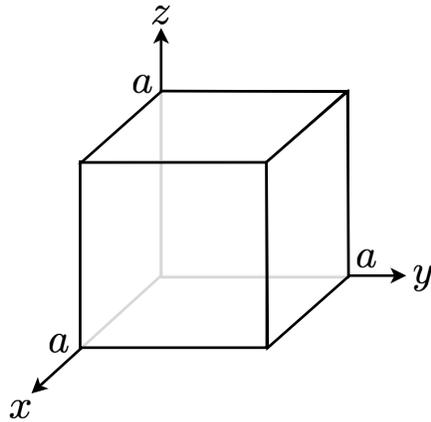
费米气体模型 フェルミガス模型

フェルミガス模型

Fermi gas model

核子が $a \times a \times a$ の立方体に閉じ込められているとする

(一般化したものは bag と呼ばれる)



Schrödinger's equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = E\psi$$

変数分離して解けば

$$\psi = N \sin k_x x \sin k_y y \sin k_z z$$

$$k_i a = n_i \pi \quad n_i \text{ は正の整数として一般性を失わない}$$

↓
波数

$$\epsilon(n_x, n_y, n_z) = \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2) = \frac{\hbar^2}{2m} k^2$$

$\mathbf{n} = (n_x, n_y, n_z)$ が決定されると量子状態が決まる

フェルミオン系として、多粒子系を考える

ある $\mathbf{n} = (n_x, n_y, n_z)$ に対して

スピン上向きの陽子 スピン上向きの中性子

スピン下向きの陽子 スピン下向きの中性子

の自由度が存在し $|\mathbf{k}|$ が小さい方から順に占有されると考える

運動量空間(波数空間)では、フェルミオンの分布は球状になると考えられる

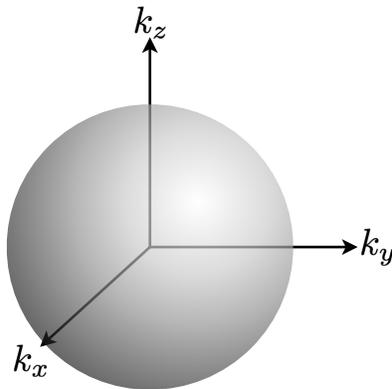
一次元の場合は

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi = E\psi$$

$$\psi(x=a) = 0 \quad \psi(x=0) = 0 \quad \text{境界条件} \quad \psi = N \sin \frac{n\pi x}{a}$$

$$\int_0^a \psi^2 dx = 1 \quad \text{規格化} \quad N = \sqrt{\frac{2}{a}}$$

$$\psi = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$



Volume Energy

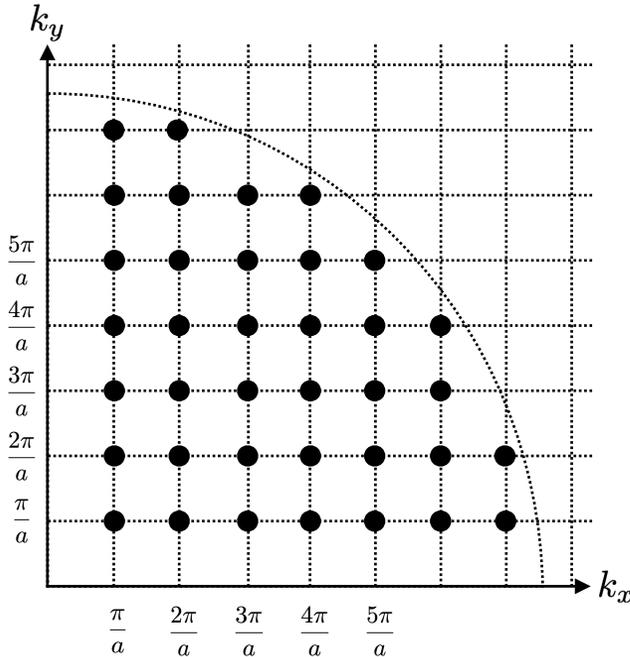
$$B(Z, N) = \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} - \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$

フェルミガス模型

Fermi gas model

k 空間において $|k|$ が k と $k + dk$ の間にある状態の個数を $dn(k)$ とおく



$$dn(k) = \frac{1}{8} \times 4\pi k^2 dk \frac{1}{(\pi/a)^3} \quad \text{なので} \quad n(k) = \frac{1}{8} \frac{4\pi}{3} k^3 \frac{1}{(\pi/a)^3}$$

最大の k を k_F とする

$\hbar k_F$ は最大の運動量で Fermi momentum と呼ばれる

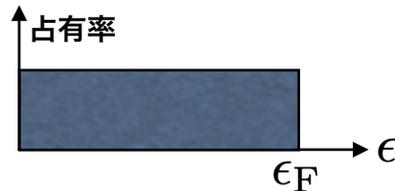
$$\frac{A}{4} = \frac{1}{8} \frac{4\pi}{3} k_F^3 \frac{1}{(\pi/a)^3}$$

$$\Omega = a^3 \quad \text{とおくと} \quad \rho = \frac{A}{\Omega} \quad \text{なので} \quad \rho = \frac{2}{3\pi^2} k_F^3$$

実験値は $\rho_0 = 1.72 \times 10^{38} \text{ nucleon/cm}^3 = 0.17 \text{ nucleon fm}^{-3}$

よって $k_F = 1.36 \text{ fm}^{-1}$ $\epsilon_F = \frac{\hbar^2 k_F^2}{2m} = 38 \text{ MeV}$

フェルミ分布を仮定すれば



$$\langle T \rangle = \frac{1}{A/4} \int_0^{k_F} \frac{\hbar^2 k^2}{2m} \frac{dn}{dk} dk = \frac{3}{5} \epsilon_F \simeq 23 \text{ MeV}$$

$$\frac{4\pi}{3} R^3 \rho = A \quad r_0 = 1.12 \text{ fm} \quad k_F r_0 = 1.52$$

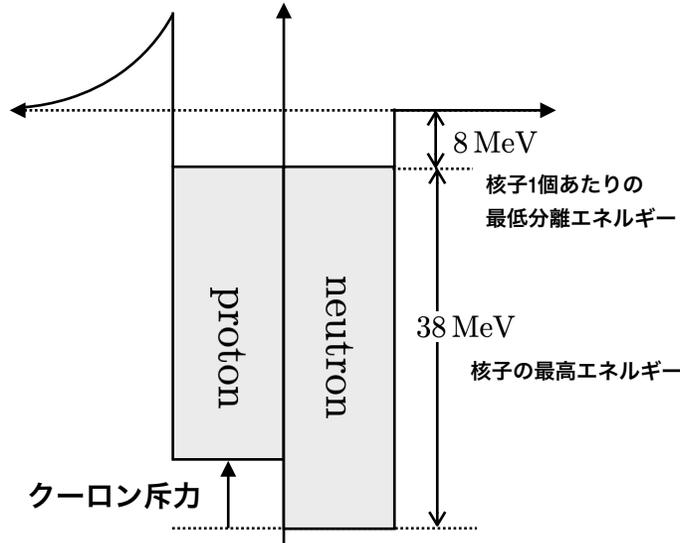
$$R = r_0 A^{1/3}$$

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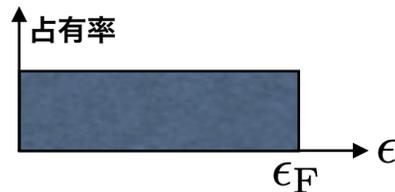
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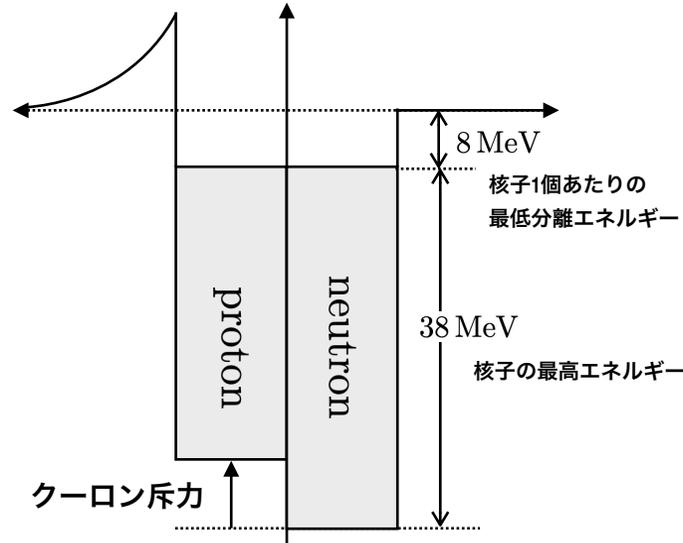
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フェルミガス模型

Fermi gas model

$$B(Z, N) = \underbrace{a_1 A}_{\text{volume energy}} - \underbrace{a_2 A^{2/3}}_{\text{lack of surface energy}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\text{Coulomb energy}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\text{symmetry energy}} - \underbrace{\delta(A)}_{\text{pairing energy}}$$

原子核外では
クーロンポテンシャル



中性子側のフェルミエネルギーが高いとβ崩壊によって高低差分が埋められる

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$

平均ポテンシャルエネルギーを $\langle U \rangle$ とする

$$\text{volume energy} = 15.68 \text{ MeV}$$

$$\langle T \rangle + \langle U \rangle = -15.7 \text{ MeV}$$

$$\langle U \rangle = -39 \text{ MeV}$$

Symmetry Energy

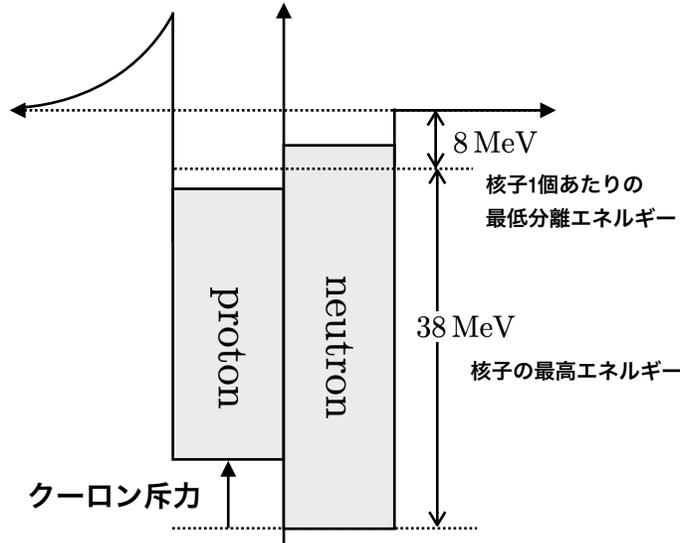
$$B(Z, N) = \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} - \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

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$$A = Z + N \quad \lambda = \frac{N' - Z'}{A}$$

$$\begin{aligned} Z &= \frac{A}{2} & \Rightarrow & \quad Z' = \frac{A}{2}(1 - \lambda) \\ N &= \frac{A}{2} & \Rightarrow & \quad N' = \frac{A}{2}(1 + \lambda) \end{aligned}$$

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$$\begin{aligned} \Delta E &= 2 \int_{\frac{A}{4}}^{\frac{A}{4}(1+\lambda)} \epsilon \, dn - 2 \int_{\frac{A}{4}}^{\frac{A}{4}(1-\lambda)} \epsilon \, dn & \epsilon & \text{核子の運動エネルギー} \\ &= 2 \left\{ F\left(\frac{A}{4}(1+\lambda)\right) - 2F\left(\frac{A}{4}\right) + F\left(\frac{A}{4}(1-\lambda)\right) \right\} \end{aligned}$$

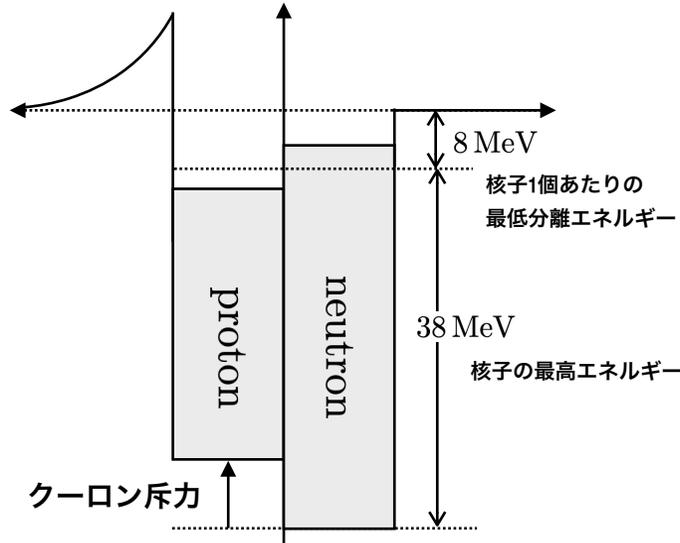
ただし $F(n) \equiv \int_0^n \epsilon \, dn = \int_0^n \epsilon \frac{dn}{d\epsilon} d\epsilon$ と定義した

$$\begin{aligned} &\simeq 2 \left(\frac{A\lambda}{4}\right)^2 \frac{d^2 F(n)}{dn^2} \Big|_{n=\frac{A}{4}} = \frac{1}{8} (A\lambda)^2 \frac{d\epsilon}{dn} \Big|_{n=\frac{A}{4}} \\ &\epsilon = \epsilon_0 n^{2/3} \quad \epsilon_F = \epsilon_0 (A/4)^{2/3} \end{aligned}$$

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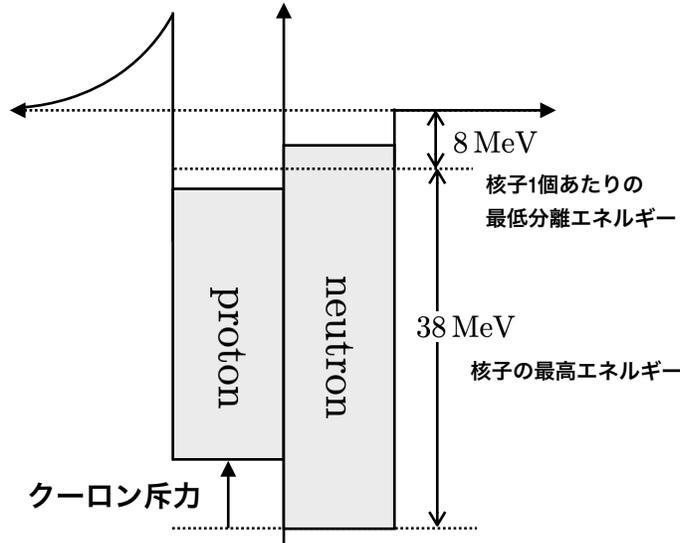
$$\frac{d\epsilon}{dn} = \frac{2}{3} \epsilon_0 n^{-1/3} = \frac{2}{3n} \epsilon$$

$$\frac{d\epsilon}{dn} \Big|_{n=\frac{A}{4}} = \frac{2}{3} \frac{1}{A/4} \epsilon_F = \frac{8}{3A} \epsilon_F$$

フェルミガス模型

Fermi gas model

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$$\boxed{a_4 = \frac{1}{3} \epsilon_F \simeq 13 \text{ MeV}} \quad \text{Pauliの排他律のみで導出}$$

Surface Energy

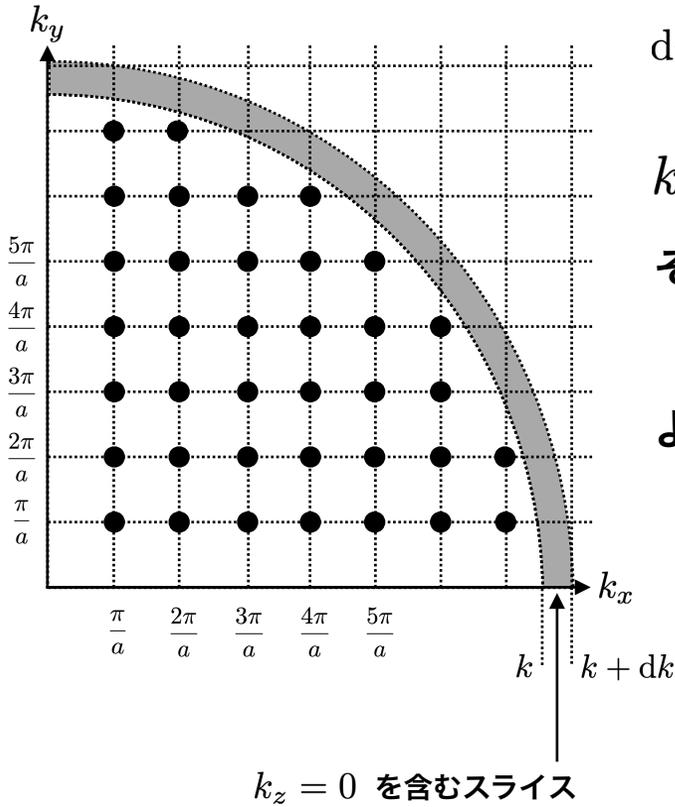
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フェルミガス模型

Fermi gas model

k 空間において $|k|$ が k と $k + dk$ の間にある状態の個数を $dn(k)$ とおく



$$dn(k) = \frac{1}{8} \times 4\pi k^2 dk \frac{1}{(\pi/a)^3} \quad \text{なので} \quad n(k) = \frac{1}{8} \frac{4\pi}{3} k^3 \frac{1}{(\pi/a)^3}$$

$k_i = 0$ は排除する必要がある ($\psi = 0$)

そのような状態数は

$$3 \times \frac{1}{4} \times 2\pi k dk \times \frac{1}{(\pi/a)^2}$$

\uparrow 三平面 \uparrow k_i が正 \uparrow 一平面上のトーラス面積 \uparrow 単位面積

よって

$$dn(k) = \frac{k^2 a^3}{2\pi^2} \left(1 - \frac{3\pi}{ak}\right) dk = \Omega \frac{k^2}{2\pi^2} \left(1 - \frac{\pi S}{2\Omega k}\right) dk$$

$\Omega = a^3 \quad S = 6a^2$

一粒子あたりのエネルギー

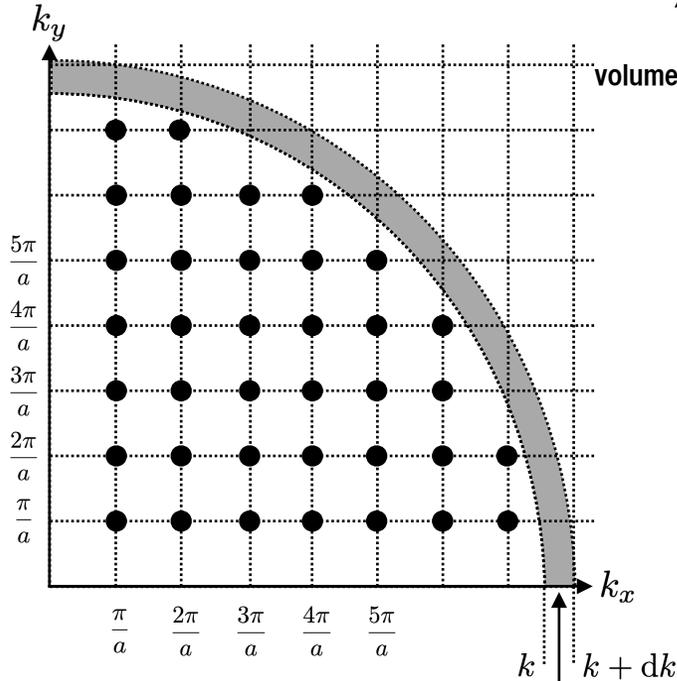
$$\frac{E}{A} = \frac{4 \int_0^{k_F} \epsilon(k) \frac{dn}{dk} dk}{4 \int_0^{k_F} \frac{dn}{dk} dk} = a_0 + b_0 \frac{S}{\Omega} + \dots$$

$a_0 = \frac{3}{5} \epsilon_F \quad b_0 = \frac{\pi a_0}{8 k_F}$
 $\frac{S}{\Omega} A = \frac{3A^{2/3}}{r_0}$

フェルミガス模型

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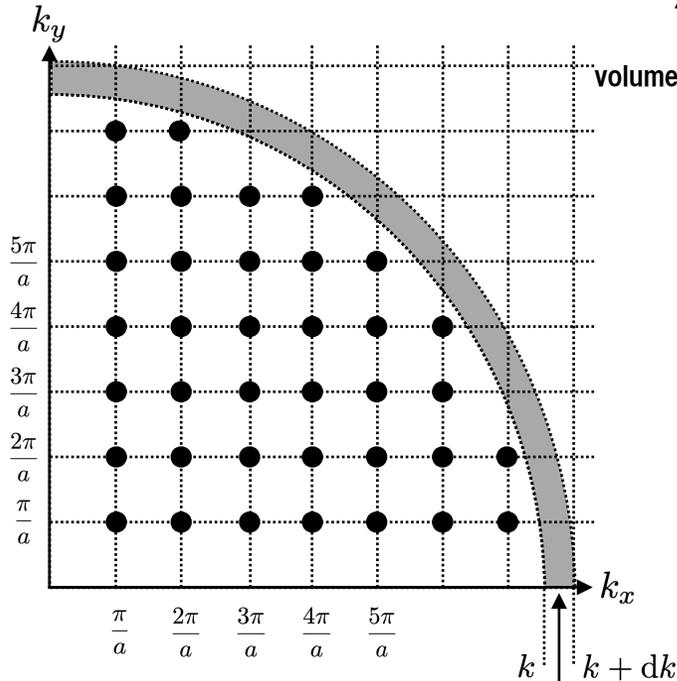
$$\frac{S}{\Omega} A = \frac{3A^{2/3}}{r_0}$$

$$a_2 \leftrightarrow \frac{3b_0}{r_0} = \frac{\pi}{8} \frac{1}{k_F} \frac{3}{5} \epsilon_F \frac{3}{r_0} = 18 \text{ MeV}$$

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一粒子あたりのエネルギー

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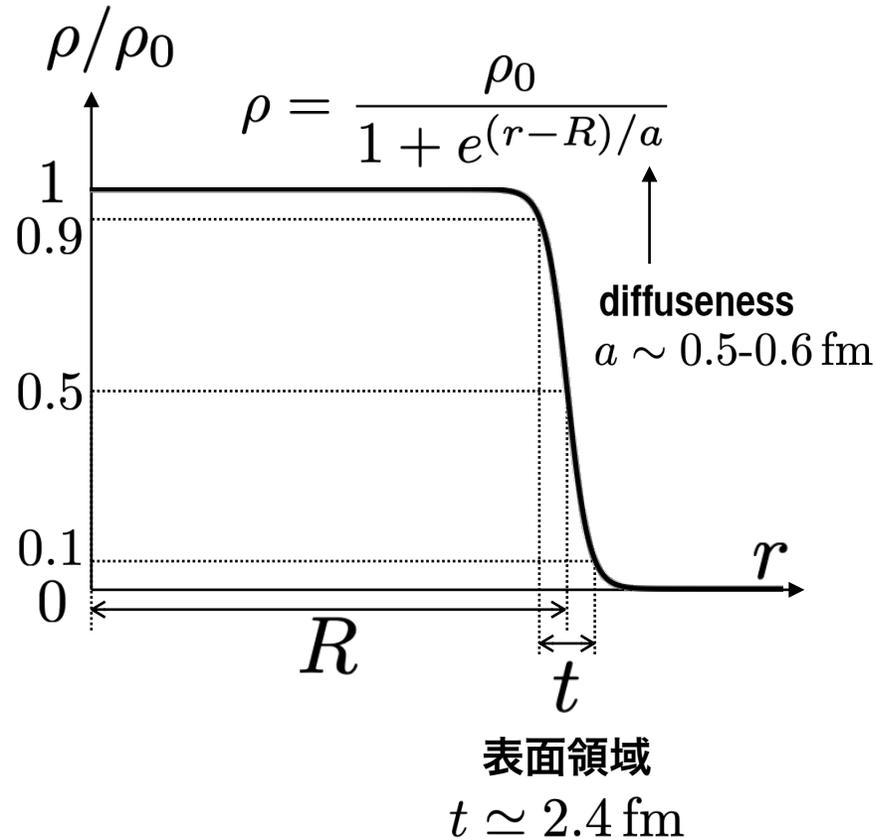
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Surface Thickness

フェルミガス模型

Fermi gas model



密度分布を直線で近似する $\rho(r) = \rho_0 \frac{R+t-r}{t}$

$$A = 4\pi \int_0^\infty \rho(r) r^2 dr = \frac{4\pi}{3} R^3 \rho_0 \left(1 + \frac{1}{2} \frac{S}{\Omega} t\right)$$

一方で

$$\rho_0 = \frac{A}{\Omega} = \frac{4}{\Omega} \int_0^{k_F} \frac{dn(k)}{dk} dk = \frac{4}{3} \frac{k_F^3}{2\pi^2} \left(1 - \frac{3\pi}{4k_F} \frac{S}{\Omega}\right)$$

代入すると

$$A = \frac{4\pi}{3} R^3 \frac{4}{3} \frac{k_F^3}{2\pi^2} \left(1 - \frac{3\pi}{4k_F} \frac{S}{\Omega}\right) \left(1 + \frac{1}{2} \frac{S}{\Omega} t\right)$$

$\left(\frac{S}{\Omega}\right)^2$ は小さいとして無視すると

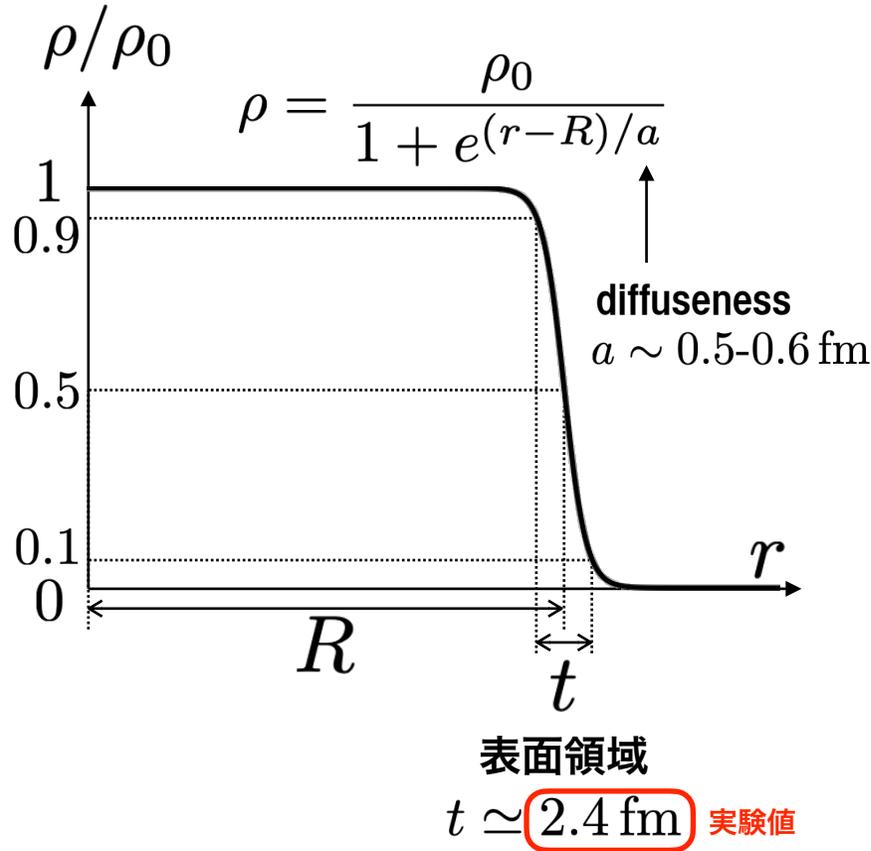
$$A = \frac{4\pi}{3} R^3 \frac{4}{3} \frac{k_F^3}{2\pi^2} \left(1 + \left(\frac{t}{2} - \frac{3\pi}{4k_F}\right) \frac{S}{\Omega}\right)$$

\downarrow
 $0 \quad t \simeq \frac{3\pi}{2k_F} \simeq 3.5 \text{ fm}$

密度が0.1から0.9まで変化する幅は $0.8 \times 3.5 \text{ fm} = 2.8 \text{ fm}$

フェルミガス模型

Fermi gas model



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费米气体模型 フェルミガス模型

Fermi gas model

被限制在特定空间中的非相互作用费米子系统
ある空間に閉じ込められた互いに相互作用しないフェルミオン系

原子核是强关联的费米子系统
原子核は、強い相関を持つフェルミオン系

费米气体模型是有效的
フェルミガス模型が有効

一组在固定势场中独立运动的核子
定まったポテンシャル中を独立に運動する核子の集団

中心力の場合

Schrödinger 方程式 $\left(-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right)\psi(\mathbf{r}) = E\psi(\mathbf{r})$

球対称
角運動量は保存する
角運動量 $\mathbf{l} = \mathbf{r} \times \mathbf{p} = \frac{\hbar}{i} \mathbf{r} \times \nabla$

$\psi(\mathbf{r}) = R(r)Y(\theta, \varphi)$ と変数分離

$$l^2 = -\frac{\hbar^2}{\sin^2\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{\partial^2}{\partial\varphi^2} \right)$$

$$l_z = \frac{\hbar}{i} \frac{\partial}{\partial\varphi}$$

動径方向 $\left(-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2mr^2} + V(r)\right) R_{nl}(r) = ER_{nl}(r)$
遠心ポテンシャル

角度方向 $-\frac{\hbar^2}{\sin^2\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{\partial^2}{\partial\varphi^2} \right) Y_{lm}(\theta, \varphi) = l(l+1)\hbar^2 Y_{lm}(\theta, \varphi)$
 $\frac{\hbar}{i} \frac{\partial}{\partial\varphi} Y_{lm}(\theta, \varphi) = m\hbar Y_{lm}(\theta, \varphi)$

$l^2 Y_{lm}(\theta, \varphi) = l(l+1)\hbar^2 Y_{lm}(\theta, \varphi)$
 $l_z Y_{lm}(\theta, \varphi) = l\hbar Y_{lm}(\theta, \varphi)$

$$\psi(\mathbf{r}) = \sum_{nlm} a_{nlm} R_{nl}(r) Y_{lm}(\theta, \varphi)$$

球調和関数

$$Y_{lm}(\theta, \varphi) = (-1)^{(m+|m|)/2} \sqrt{\frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} P_l^{|m|}(\cos\theta) e^{im\varphi}$$

ルジャンドル陪関数

$$P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

ルジャンドル関数

$$P_l(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

角運動量の復習

$$\mathbf{L} \times \mathbf{L} = i\hbar\mathbf{L}$$

$$[L_x, L_y] = i\hbar L_z$$

$$[L_y, L_z] = i\hbar L_x$$

$$[L_z, L_x] = i\hbar L_y$$

角運動量の復習

$$\mathbf{J} \times \mathbf{J} = i\hbar\mathbf{J}$$

角運動量の復習

$$\mathbf{J} \times \mathbf{J} = i \mathbf{J}$$

角運動量の復習

$$\mathbf{J} \times \mathbf{J} = i\mathbf{J}$$

$$\mathbf{J} = (J_x, J_y, J_z)$$

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$\mathbf{J} \times \mathbf{J} = i\mathbf{J}$$

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{aligned} [J_x, J_y] &= iJ_z \\ [J_y, J_z] &= iJ_x \\ [J_z, J_x] &= iJ_y \end{aligned} \quad \text{と定義する}$$
$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$\mathbf{J} \times \mathbf{J} = i\mathbf{J}$$

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{aligned} [J_x, J_y] &= iJ_z \\ [J_y, J_z] &= iJ_x \\ [J_z, J_x] &= iJ_y \end{aligned} \quad \text{と定義する}$$
$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{aligned} [J_x, J_y] &= iJ_z \\ [J_y, J_z] &= iJ_x \\ [J_z, J_x] &= iJ_y \end{aligned} \quad \text{と定義する}$$
$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$[J^2, J_z] = (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2)$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}
 \quad \text{と定義する}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned}
 [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\
 &= J_x^2J_z + J_y^2J_z - J_zJ_x^2 - J_zJ_y^2
 \end{aligned}$$

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{aligned} [J_x, J_y] &= iJ_z \\ [J_y, J_z] &= iJ_x \\ [J_z, J_x] &= iJ_y \end{aligned} \quad \text{と定義する}$$

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned} [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\ &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\ &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \end{aligned}$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z & J_x J_y - J_y J_x &= iJ_z \\
 & & [J_y, J_z] &= iJ_x & & \text{と定義する} \\
 [J_i, J_j] &= i\epsilon_{ijk} J_k & [J_z, J_x] &= iJ_y & &
 \end{aligned}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned}
 [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\
 &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\
 &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z & J_x J_y &= iJ_z + J_y J_x \\
 & & [J_y, J_z] &= iJ_x & & \text{と定義する} \\
 [J_i, J_j] &= i\epsilon_{ijk} J_k & [J_z, J_x] &= iJ_y
 \end{aligned}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned}
 [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\
 &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\
 &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z & J_x J_y &= iJ_z + J_y J_x \\
 & & [J_y, J_z] &= iJ_x & J_y J_z &= iJ_x + J_z J_y \\
 [J_i, J_j] &= i\epsilon_{ijk} J_k & [J_z, J_x] &= iJ_y & J_z J_x &= iJ_y + J_x J_z
 \end{aligned}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned}
 [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\
 &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\
 &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z & J_x J_y &= iJ_z + J_y J_x \\
 & & [J_y, J_z] &= iJ_x & J_y J_z &= iJ_x + J_z J_y \\
 [J_i, J_j] &= i\epsilon_{ijk} J_k & [J_z, J_x] &= iJ_y & J_z J_x &= iJ_y + J_x J_z
 \end{aligned}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned}
 [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\
 &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\
 &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \\
 &= J_x(-iJ_y + J_z J_x) + J_y(iJ_x + J_z J_y) - (iJ_y + J_x J_z)J_x - (-iJ_x + J_y J_z)J_y
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z & J_x J_y &= iJ_z + J_y J_x \\
 & & [J_y, J_z] &= iJ_x & J_y J_z &= iJ_x + J_z J_y \\
 [J_i, J_j] &= i\epsilon_{ijk} J_k & [J_z, J_x] &= iJ_y & J_z J_x &= iJ_y + J_x J_z
 \end{aligned}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned}
 [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\
 &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\
 &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \\
 &= J_x(-iJ_y + J_z J_x) + J_y(iJ_x + J_z J_y) - (iJ_y + J_x J_z)J_x - (-iJ_x + J_y J_z)J_y \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x && \text{と定義する} \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned}
 [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\
 &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\
 &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \\
 &= J_x(-iJ_y + J_z J_x) + J_y(iJ_x + J_z J_y) - (iJ_y + J_x J_z)J_x - (-iJ_x + J_y J_z)J_y \\
 &= 0
 \end{aligned}$$

ある状態 $|A\rangle$ が J_z の固有状態だとすると $J_z |A\rangle = m |A\rangle$
 $J_z (J^2 |A\rangle) = J_z J^2 |A\rangle = J^2 J_z |A\rangle = m (J^2 |A\rangle)$ なので
 $J^2 |A\rangle$ という状態も J_z の固有状態

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{cases} [J_x, J_y] = iJ_z \\ [J_y, J_z] = iJ_x \\ [J_z, J_x] = iJ_y \end{cases} \quad \text{と定義する}$$

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned} [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\ &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\ &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \\ &= J_x(-iJ_y + J_z J_x) + J_y(iJ_x + J_z J_y) - (iJ_y + J_x J_z)J_x - (-iJ_x + J_y J_z)J_y \\ &= 0 \end{aligned}$$

ある状態 $|A\rangle$ が J_z の固有状態だとすると $J_z |A\rangle = m |A\rangle$

$J_z (J^2 |A\rangle) = J_z J^2 |A\rangle = J^2 J_z |A\rangle = m (J^2 |A\rangle)$ なので

$J^2 |A\rangle$ という状態も J_z の固有状態

ある状態 $|A'\rangle$ が J^2 の固有状態だとすると $J^2 |A'\rangle = \lambda |A'\rangle$

$J^2 (J_z |A'\rangle) = J^2 J_z |A'\rangle = J_z J^2 |A'\rangle = \lambda (J_z |A'\rangle)$ なので

$J_z |A'\rangle$ という状態も J^2 の固有状態

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{cases} [J_x, J_y] = iJ_z \\ [J_y, J_z] = iJ_x \\ [J_z, J_x] = iJ_y \end{cases} \quad \text{と定義する}$$

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned} [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\ &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\ &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \\ &= J_x(-iJ_y + J_z J_x) + J_y(iJ_x + J_z J_y) - (iJ_y + J_x J_z)J_x - (-iJ_x + J_y J_z)J_y \\ &= 0 \end{aligned}$$

ある状態 $|A\rangle$ が J_z の固有状態だとすると $J_z |A\rangle = m |A\rangle$ 同時固有状態

$J_z (J^2 |A\rangle) = J_z J^2 |A\rangle = J^2 J_z |A\rangle = m (J^2 |A\rangle)$ なので

$J^2 |A\rangle$ という状態も J_z の固有状態

ある状態 $|A'\rangle$ が J^2 の固有状態だとすると $J^2 |A'\rangle = \lambda |A'\rangle$

$J^2 (J_z |A'\rangle) = J^2 J_z |A'\rangle = J_z J^2 |A'\rangle = \lambda (J_z |A'\rangle)$ なので

$J_z |A'\rangle$ という状態も J^2 の固有状態

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{cases} [J_x, J_y] = iJ_z \\ [J_y, J_z] = iJ_x \\ [J_z, J_x] = iJ_y \end{cases} \quad \text{と定義する}$$

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned} [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\ &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\ &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \\ &= J_x(-iJ_y + J_z J_x) + J_y(iJ_x + J_z J_y) - (iJ_y + J_x J_z)J_x - (-iJ_x + J_y J_z)J_y \\ &= 0 \end{aligned}$$

ある状態 $|A\rangle$ が J_z の固有状態だとすると $J_z |A\rangle = m |A\rangle$ 同時固有状態

$J_z (J^2 |A\rangle) = J_z J^2 |A\rangle = J^2 J_z |A\rangle = m (J^2 |A\rangle)$ なので

$J^2 |A\rangle$ という状態も J_z の固有状態

$|\lambda m\rangle$

ある状態 $|A'\rangle$ が J^2 の固有状態だとすると $J^2 |A'\rangle = \lambda |A'\rangle$

$J^2 (J_z |A'\rangle) = J^2 J_z |A'\rangle = J_z J^2 |A'\rangle = \lambda (J_z |A'\rangle)$ なので

$J_z |A'\rangle$ という状態も J^2 の固有状態

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{cases} [J_x, J_y] = iJ_z \\ [J_y, J_z] = iJ_x \\ [J_z, J_x] = iJ_y \end{cases} \quad \text{と定義する}$$

$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

$$\begin{aligned} [J^2, J_z] &= (J_x^2 + J_y^2 + J_z^2)J_z - J_z(J_x^2 + J_y^2 + J_z^2) \\ &= J_x^2 J_z + J_y^2 J_z - J_z J_x^2 - J_z J_y^2 \\ &= J_x(J_x J_z) + J_y(J_y J_z) - (J_z J_x)J_x - (J_z J_y)J_y \\ &= J_x(-iJ_y + J_z J_x) + J_y(iJ_x + J_z J_y) - (iJ_y + J_x J_z)J_x - (-iJ_x + J_y J_z)J_y \\ &= 0 \end{aligned}$$

ある状態 $|A\rangle$ が J_z の固有状態だとすると $J_z |A\rangle = m |A\rangle$ 同時固有状態

$J_z (J^2 |A\rangle) = J_z J^2 |A\rangle = J^2 J_z |A\rangle = m (J^2 |A\rangle)$ なので

$J^2 |A\rangle$ という状態も J_z の固有状態

$$|\lambda m\rangle$$

ある状態 $|A'\rangle$ が J^2 の固有状態だとすると $J^2 |A'\rangle = \lambda |A'\rangle$

$J^2 (J_z |A'\rangle) = J^2 J_z |A'\rangle = J_z J^2 |A'\rangle = \lambda (J_z |A'\rangle)$ なので

$J_z |A'\rangle$ という状態も J^2 の固有状態

$$|jm\rangle$$

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{aligned} [J_x, J_y] &= iJ_z \\ [J_y, J_z] &= iJ_x \\ [J_z, J_x] &= iJ_y \end{aligned} \quad \text{と定義する}$$
$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

固有状態は二つの量子数で指定できる

$|jm\rangle$

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{aligned} [J_x, J_y] &= iJ_z \\ [J_y, J_z] &= iJ_x \\ [J_z, J_x] &= iJ_y \end{aligned} \quad \text{と定義する}$$
$$[J_i, J_j] = i\epsilon_{ijk}J_k$$

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}$$

と定義する

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$\begin{aligned}
 J_z |jm\rangle &= m |jm\rangle \\
 \mathbf{J}^2 |jm\rangle &= \lambda |jm\rangle
 \end{aligned}$$

$$\begin{array}{l}
 \mathbf{J} = (J_x, J_y, J_z) \quad [J_x, J_y] = iJ_z \\
 [J_y, J_z] = iJ_x \quad \text{と定義する} \\
 [J_i, J_j] = i\epsilon_{ijk}J_k \quad [J_z, J_x] = iJ_y
 \end{array}$$

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$J_+ = J_x + iJ_y$$

$$J_z |jm\rangle = m |jm\rangle$$

$$\mathbf{J}^2 |jm\rangle = \lambda |jm\rangle$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}$$

と定義する

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$J_z |jm\rangle = m |jm\rangle$$

$$\mathbf{J}^2 |jm\rangle = \lambda |jm\rangle$$

$$J_+ = J_x + iJ_y$$

$J_+ |jm\rangle$ という状態を考えると

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}
 \quad \text{と定義する}$$

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$J_z |jm\rangle = m |jm\rangle$$

$$\mathbf{J}^2 |jm\rangle = \lambda |jm\rangle$$

$$J_+ = J_x + iJ_y$$

$J_+ |jm\rangle$ という状態を考えると

$$\begin{aligned}
 J_z (J_+ |jm\rangle) &= J_z (J_x + iJ_y) |jm\rangle \\
 &= (J_z J_x + iJ_z J_y) |jm\rangle \\
 &= (iJ_y + J_x J_z + J_x + iJ_y J_z) |jm\rangle \\
 &= (J_x + iJ_y + (J_x + iJ_y) J_z) |jm\rangle \\
 &= (J_x + iJ_y) |jm\rangle + (J_x + iJ_y) m |jm\rangle \\
 &= (m + 1) J_+ |jm\rangle
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x && \text{と定義する} \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}$$

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$J_z |jm\rangle = m |jm\rangle$$

$$\mathbf{J}^2 |jm\rangle = \lambda |jm\rangle$$

$$J_+ = J_x + iJ_y$$

$J_+ |jm\rangle$ という状態を考えると

$$\begin{aligned}
 J_z (J_+ |jm\rangle) &= J_z (J_x + iJ_y) |jm\rangle \\
 &= (J_z J_x + iJ_z J_y) |jm\rangle \\
 &= (iJ_y + J_x J_z + J_x + iJ_y J_z) |jm\rangle \\
 &= (J_x + iJ_y + (J_x + iJ_y) J_z) |jm\rangle \\
 &= (J_x + iJ_y) |jm\rangle + (J_x + iJ_y) m |jm\rangle \\
 &= (m + 1) J_+ |jm\rangle
 \end{aligned}$$

$$J_+ |jm\rangle = A |j, m + 1\rangle$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x && \text{と定義する} \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}$$

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$J_z |jm\rangle = m |jm\rangle$$

$$\mathbf{J}^2 |jm\rangle = \lambda |jm\rangle$$

$$J_+ = J_x + iJ_y \quad J_- = J_x - iJ_y$$

$J_+ |jm\rangle$ という状態を考えると

$$\begin{aligned}
 J_z (J_+ |jm\rangle) &= J_z (J_x + iJ_y) |jm\rangle \\
 &= (J_z J_x + iJ_z J_y) |jm\rangle \\
 &= (iJ_y + J_x J_z + J_x + iJ_y J_z) |jm\rangle \\
 &= (J_x J_x + iJ_y J_y + (J_x J_x + iJ_y J_y) J_z) |jm\rangle \\
 &= (J_x J_x + iJ_y J_y) |jm\rangle + (J_x J_x + iJ_y J_y) |j, m+1\rangle \\
 &= (m+1) J_+ |jm\rangle
 \end{aligned}$$

$$J_+ |jm\rangle = A |j, m+1\rangle$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
 \end{aligned}
 \quad \text{と定義する}$$

$|jm\rangle$ 固有状態は二つの量子数で指定できる

$$J_z |jm\rangle = m |jm\rangle$$

$$\mathbf{J}^2 |jm\rangle = \lambda |jm\rangle$$

$$J_+ = J_x + iJ_y \quad J_- = J_x - iJ_y$$

$J_- |jm\rangle$ という状態を考えると

$$\begin{aligned}
 J_z (J_- |jm\rangle) &= J_z (J_x - iJ_y) |jm\rangle \\
 &= (J_z J_x - iJ_z J_y) |jm\rangle \\
 &= (iJ_y + J_x J_z - J_x - iJ_y J_z) |jm\rangle \\
 &= (-J_x + iJ_y + (J_x - iJ_y)J_z) |jm\rangle \\
 &= -(J_x - iJ_y) |jm\rangle + (J_x - iJ_y)m |jm\rangle \\
 &= (m - 1)J_- |jm\rangle
 \end{aligned}$$

$$J_+ |jm\rangle = A |j, m + 1\rangle \quad J_- |jm\rangle = A |j, m - 1\rangle$$

$$\begin{aligned}
 \mathbf{J} &= (J_x, J_y, J_z) & [J_x, J_y] &= iJ_z \\
 & & [J_y, J_z] &= iJ_x && \text{と定義する} \\
 [J_i, J_j] &= i\epsilon_{ijk}J_k & [J_z, J_x] &= iJ_y
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$J_{\pm} |jm\rangle = A |j, m \pm 1\rangle$ という状態を考えると

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 J_z (J_- |jm\rangle) &= J_z (J_x - iJ_y) |jm\rangle \\
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 &= (-J_x + iJ_y + (J_x - iJ_y) J_z) |jm\rangle \\
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$$0 = J_- J_+ |jm_{\max}\rangle = (\mathbf{J}^2 - J_z(J_z + 1)) |jm_{\max}\rangle = (\lambda - m_{\max}(m_{\max} + 1)) |jm_{\max}\rangle$$

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$$J_+ J_- = \mathbf{J}^2 - J_z(J_z - 1)$$

$$0 = J_+ J_- |jm_{\min}\rangle = (\mathbf{J}^2 - J_z(J_z - 1)) |jm_{\min}\rangle = (\lambda - m_{\min}(m_{\min} - 1)) |jm_{\min}\rangle$$

$$\lambda = m_{\max}(m_{\max} + 1) \quad \lambda = m_{\min}(m_{\min} - 1)$$

$$J_+ |jm_{\max}\rangle = 0 \quad J_- |jm_{\min}\rangle = 0$$

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 m_{\max}^2 + m_{\max} &= m_{\min}^2 - m_{\min} \\
 m_{\max}^2 - m_{\min}^2 + m_{\max} + m_{\min} &= 0
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$$m_{\max} = -m_{\min} = j$$

$$\mathbf{J} = (J_x, J_y, J_z) \quad \begin{cases} [J_x, J_y] = iJ_z \\ [J_y, J_z] = iJ_x \\ [J_z, J_x] = iJ_y \end{cases} \quad \text{と定義する}$$

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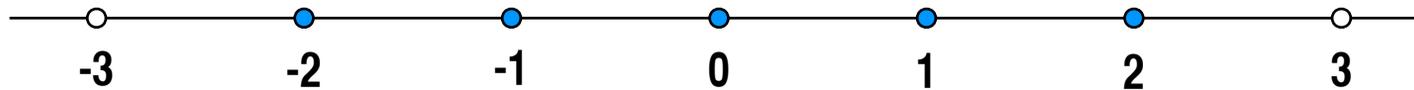
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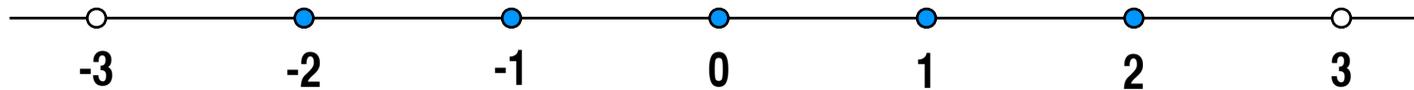
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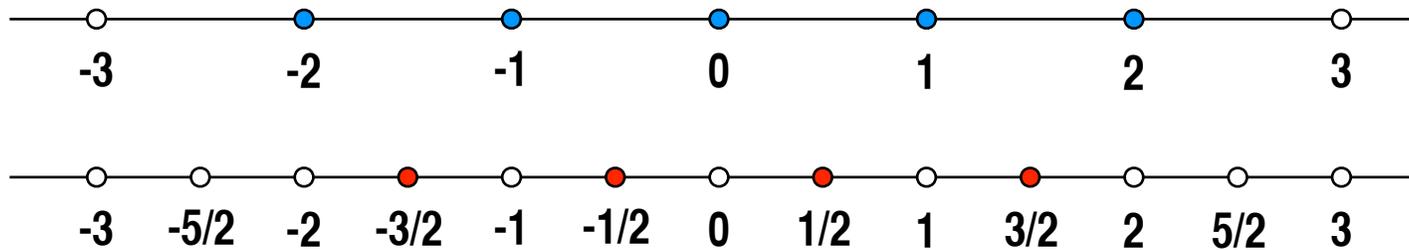
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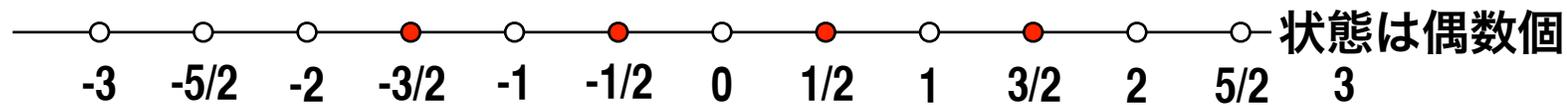
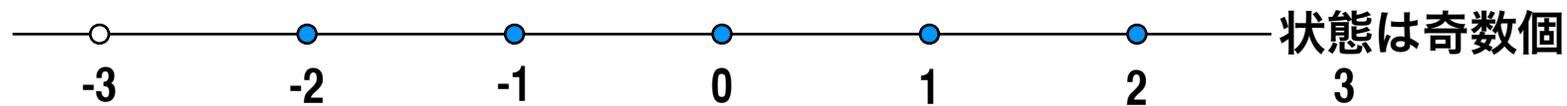
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磁場の多重極展開

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$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}') \times d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} = \frac{\mu_0}{4\pi} \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \omega) \mathbf{j}(\mathbf{r}') \times d\mathbf{r}' \quad \text{多重極展開}$$

$$= \frac{\mu_0}{4\pi} \sum_{lm} \frac{4\pi}{2l+1} \frac{1}{r^{l+1}} Y_{lm}(\theta, \varphi) \int r'^l Y_{lm}(\theta', \varphi') \mathbf{j}(\mathbf{r}') d\mathbf{r}'$$

$$= \frac{\mu_0}{4\pi} \left[\frac{1}{r} \int \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}') (\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' + \dots \right]$$

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{r} \frac{1}{\sqrt{1 - 2\frac{r'}{r} \cos \omega + \left(\frac{r'}{r}\right)^2}} = \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \omega)$$

$$P_l(\cos \omega) = \frac{4\pi}{2l+1} \sum_{m=-l}^l Y_{lm}(\theta_1, \varphi_1) Y_{lm}^*(\theta_2, \varphi_2)$$

$$Y_{lm}(\theta, \varphi) = (-1)^{(m+|m|)/2} \sqrt{\frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} P_l^{|m|}(\cos \theta) e^{im\varphi}$$

$$P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

$$P_l(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

磁場の多重極展開

$$\mathbf{H} = \frac{1}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{r}' \quad \text{ビオ・サバルの法則} \quad \text{ベクトルポテンシャルの定義} \quad \mu_0 \mathbf{H} = \mathbf{B} = \nabla \times \mathbf{A}$$

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$$= \frac{\mu_0}{4\pi} \left[\frac{1}{r} \int \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}') (\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' + \dots \right]$$

$$= \mathbf{A}_0 + \mathbf{A}_1 + \dots$$

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双極子成分

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{r} \frac{1}{\sqrt{1 - 2\frac{r'}{r} \cos \omega + \left(\frac{r'}{r}\right)^2}} = \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \omega)$$

$$P_l(\cos \omega) = \frac{4\pi}{2l+1} \sum_{m=-l}^l Y_{lm}(\theta_1, \varphi_1) Y_{lm}^*(\theta_2, \varphi_2)$$

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磁場の多重極展開

$$\mathbf{H} = \frac{1}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{r}' \quad \text{ビオ・サバルの法則} \quad \text{ベクトルポテンシャルの定義} \quad \mu_0 \mathbf{H} = \mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}') \times d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} = \frac{\mu_0}{4\pi} \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \omega) \mathbf{j}(\mathbf{r}') \times d\mathbf{r}' \quad \text{多重極展開}$$

$$= \frac{\mu_0}{4\pi} \sum_{lm} \frac{4\pi}{2l+1} \frac{1}{r^{l+1}} Y_{lm}(\theta, \varphi) \int r'^l Y_{lm}(\theta', \varphi') \mathbf{j}(\mathbf{r}') d\mathbf{r}'$$

$$= \frac{\mu_0}{4\pi} \left[\frac{1}{r} \int \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}') (\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' + \dots \right]$$

$$= \mathbf{A}_0 + \mathbf{A}_1 + \dots$$

↑ ↑
0 双極子成分

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}') (\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}'$$

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{r} \frac{1}{\sqrt{1 - 2\frac{r'}{r} \cos \omega + \left(\frac{r'}{r}\right)^2}} = \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \omega)$$

$$P_l(\cos \omega) = \frac{4\pi}{2l+1} \sum_{m=-l}^l Y_{lm}(\theta_1, \varphi_1) Y_{lm}^*(\theta_2, \varphi_2)$$

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$$P_l(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

磁場の多重極展開

を求める

$$\mathbf{H} = \frac{1}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{r}' \quad \text{ビオ・サバルの法則} \quad \text{ベクトルポテンシャルの定義} \quad \mu_0 \mathbf{H} = \mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{j}(\mathbf{r}') \times d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} = \frac{\mu_0}{4\pi} \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \omega) \mathbf{j}(\mathbf{r}') \times d\mathbf{r}' \quad \text{多重極展開}$$

$$= \frac{\mu_0}{4\pi} \sum_{lm} \frac{4\pi}{2l+1} \frac{1}{r^{l+1}} Y_{lm}(\theta, \varphi) \int r'^l Y_{lm}(\theta', \varphi') \mathbf{j}(\mathbf{r}') d\mathbf{r}'$$

$$= \frac{\mu_0}{4\pi} \left[\frac{1}{r} \int \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}') (\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' + \dots \right]$$

$$= \mathbf{A}_0 + \mathbf{A}_1 + \dots$$

↑ ↑
0 双極子成分

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}') (\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}'$$

$$\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \frac{1}{r} \frac{1}{\sqrt{1 - 2\frac{r'}{r} \cos \omega + \left(\frac{r'}{r}\right)^2}} = \frac{1}{r} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \omega)$$

$$P_l(\cos \omega) = \frac{4\pi}{2l+1} \sum_{m=-l}^l Y_{lm}(\theta_1, \varphi_1) Y_{lm}^*(\theta_2, \varphi_2)$$

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$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める}$$

$$\begin{aligned} \mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める} \\ &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}') d\mathbf{r}' \right] \end{aligned}$$

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}') (\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) \mathbf{r}') d\mathbf{r}' \right]$$

$$\begin{aligned} \text{さて } \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] &= (\nabla'(\mathbf{r} \cdot \mathbf{r}')) \cdot \mathbf{j}(\mathbf{r}') + (\mathbf{r} \cdot \mathbf{r}') \underbrace{[\nabla' \cdot \mathbf{j}(\mathbf{r}')] }_{\substack{\text{---} \\ \uparrow \\ \nabla' \cdot \mathbf{j}(\mathbf{r}') = 0}} \\ &= ((\mathbf{r} \cdot \mathbf{r}') \mathbf{i} + x' \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}') \\ &= (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') + x' \mathbf{r} \cdot \mathbf{j}(\mathbf{r}') \quad \text{が成り立つ} \end{aligned} \quad \nabla' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'} \right)$$

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求め}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) \mathbf{r}') d\mathbf{r}' \right]$$

$$\begin{aligned} \text{さて } \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] &= (\nabla'(\mathbf{r} \cdot \mathbf{r}')) \cdot \mathbf{j}(\mathbf{r}') + (\mathbf{r} \cdot \mathbf{r}') \underbrace{[\nabla' \cdot \mathbf{j}(\mathbf{r}')] }_{\substack{\text{↑} \\ \nabla' \cdot \mathbf{j}(\mathbf{r}') = 0}} \\ &= ((\mathbf{r} \cdot \mathbf{r}') \mathbf{i} + x' \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}') \\ &= (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') + x' \mathbf{r} \cdot \mathbf{j}(\mathbf{r}') \quad \text{が成り立つ} \end{aligned} \quad \nabla' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'} \right)$$

$$\begin{aligned} \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_V ((\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}')) d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \\ \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_{S=\partial V} [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] \cdot d\mathbf{S}' = 0 \end{aligned} \quad \text{より}$$

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) \mathbf{r}') d\mathbf{r}' \right]$$

$$\begin{aligned} \text{さて } \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] &= (\nabla'(\mathbf{r} \cdot \mathbf{r}')) \cdot \mathbf{j}(\mathbf{r}') + (\mathbf{r} \cdot \mathbf{r}') \underbrace{[\nabla' \cdot \mathbf{j}(\mathbf{r}')] }_{\substack{\text{---} \\ \uparrow \\ \nabla' \cdot \mathbf{j}(\mathbf{r}') = 0}} \\ &= ((\mathbf{r} \cdot \mathbf{r}') \mathbf{i} + x' \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}') \\ &= (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') + x' \mathbf{r} \cdot \mathbf{j}(\mathbf{r}') \quad \text{が成り立つ} \end{aligned} \quad \nabla' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'} \right)$$

$$\begin{aligned} \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_V ((\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}')) d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \\ \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_{S=\partial V} [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] \cdot d\mathbf{S}' = 0 \end{aligned} \quad \text{より}$$

$$\int_V (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求め}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) \mathbf{r}') d\mathbf{r}' \right]$$

$$\begin{aligned} \text{さて } \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] &= (\nabla'(\mathbf{r} \cdot \mathbf{r}')) \cdot \mathbf{j}(\mathbf{r}') + (\mathbf{r} \cdot \mathbf{r}') \underbrace{[\nabla' \cdot \mathbf{j}(\mathbf{r}')] }_{\substack{\text{---} \\ \uparrow \\ \nabla' \cdot \mathbf{j}(\mathbf{r}') = 0}} \\ &= ((\mathbf{r} \cdot \mathbf{r}') \mathbf{i} + x' \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}') \\ &= (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') + x' \mathbf{r} \cdot \mathbf{j}(\mathbf{r}') \quad \text{が成り立つ} \end{aligned} \quad \nabla' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'} \right)$$

$$\begin{aligned} \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_V ((\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}')) d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \\ \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_{S=\partial V} [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] \cdot d\mathbf{S}' = 0 \end{aligned} \quad \text{より}$$

$$\int_V (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\text{同様に } \int_V (\mathbf{r} \cdot \mathbf{r}') j_y(\mathbf{r}') d\mathbf{r}' + \int_V y'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求め}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) \mathbf{r}') d\mathbf{r}' \right]$$

$$\begin{aligned} \text{さて } \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] &= (\nabla'(\mathbf{r} \cdot \mathbf{r}')) \cdot \mathbf{j}(\mathbf{r}') + (\mathbf{r} \cdot \mathbf{r}') \underbrace{[\nabla' \cdot \mathbf{j}(\mathbf{r}')] }_{\substack{\text{↑} \\ \nabla' \cdot \mathbf{j}(\mathbf{r}') = 0}} \\ &= ((\mathbf{r} \cdot \mathbf{r}') \mathbf{i} + x' \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}') \\ &= (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') + x' \mathbf{r} \cdot \mathbf{j}(\mathbf{r}') \quad \text{が成り立つ} \end{aligned} \quad \nabla' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'} \right)$$

$$\begin{aligned} \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_V ((\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}')) d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \\ \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_{S=\partial V} [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] \cdot d\mathbf{S}' = 0 \end{aligned} \quad \text{より}$$

$$\int_V (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\text{同様に } \int_V (\mathbf{r} \cdot \mathbf{r}') j_y(\mathbf{r}') d\mathbf{r}' + \int_V y'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\int_V (\mathbf{r} \cdot \mathbf{r}') j_z(\mathbf{r}') d\mathbf{r}' + \int_V z'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める}$$

$$= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) \mathbf{r}') d\mathbf{r}' \right]$$

$$\begin{aligned} \text{さて } \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] &= (\nabla'(\mathbf{r} \cdot \mathbf{r}')) \cdot \mathbf{j}(\mathbf{r}') + (\mathbf{r} \cdot \mathbf{r}') \underbrace{[\nabla' \cdot \mathbf{j}(\mathbf{r}')] }_{\substack{\text{---} \leftarrow \nabla' \cdot \mathbf{j}(\mathbf{r}') = 0}} \\ &= ((\mathbf{r} \cdot \mathbf{r}') \mathbf{i} + x' \mathbf{r}) \cdot \mathbf{j}(\mathbf{r}') \\ &= (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') + x' \mathbf{r} \cdot \mathbf{j}(\mathbf{r}') \quad \text{が成り立つ} \end{aligned} \quad \nabla' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'} \right)$$

$$\begin{aligned} \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_V ((\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}')) d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \\ \int_V \nabla' \cdot [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] d\mathbf{r}' &= \int_{S=\partial V} [x'(\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')] \cdot d\mathbf{S}' = 0 \end{aligned} \quad \text{より}$$

$$\int_V (\mathbf{r} \cdot \mathbf{r}') j_x(\mathbf{r}') d\mathbf{r}' + \int_V x'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\text{同様に } \int_V (\mathbf{r} \cdot \mathbf{r}') j_y(\mathbf{r}') d\mathbf{r}' + \int_V y'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\int_V (\mathbf{r} \cdot \mathbf{r}') j_z(\mathbf{r}') d\mathbf{r}' + \int_V z'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\text{まとめると } \int_V (\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \int_V \mathbf{r}'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\begin{aligned}
\mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める} \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}') d\mathbf{r}' \right]
\end{aligned}$$

$$\text{まとめると } \int_V (\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \int_V \mathbf{r}' (\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\begin{aligned}
\mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める} \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}') d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \right] \times \mathbf{r} - \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}') d\mathbf{r}' \right]
\end{aligned}$$

$$\text{まとめると } \int_V (\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \int_V \mathbf{r}' (\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\begin{aligned}
\mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}') d\mathbf{r}' \quad \text{を求める} \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r} d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}') d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \right] \times \mathbf{r} - \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}') d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}' \right] \times \mathbf{r} - \mathbf{A}_1
\end{aligned}$$

$$\text{まとめると} \quad \int_V (\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}') d\mathbf{r}' + \int_V \mathbf{r}' (\mathbf{r} \cdot \mathbf{j}(\mathbf{r}')) d\mathbf{r}' = 0$$

$$\begin{aligned}
\mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}')d\mathbf{r}' \quad \text{を求める} \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r}d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r} \cdot \mathbf{r}')\mathbf{j}(\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \mathbf{A}_1 \\
\mathbf{A}_1 &= \frac{1}{2} \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r}
\end{aligned}$$

$$\text{まとめると} \quad \int_V (\mathbf{r} \cdot \mathbf{r}') \mathbf{j}(\mathbf{r}')d\mathbf{r}' + \int_V \mathbf{r}'(\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))d\mathbf{r}' = 0$$

$$\begin{aligned}
\mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}')d\mathbf{r}' \quad \text{を求める} \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r}d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r} \cdot \mathbf{r}')\mathbf{j}(\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \mathbf{A}_1
\end{aligned}$$

$$\mathbf{A}_1 = \frac{1}{2} \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r}$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \quad \text{とおくと} \quad \mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\begin{aligned}
\mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}')d\mathbf{r}' \quad \text{を求める} \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r}d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r} \cdot \mathbf{r}')\mathbf{j}(\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \mathbf{A}_1
\end{aligned}$$

$$\mathbf{A}_1 = \frac{1}{2} \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r}$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \quad \text{とおくと} \quad \mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

磁気双極子能率

$$\begin{aligned}
\mathbf{A}_1 &= \frac{\mu_0}{4\pi} \frac{1}{r^3} \int \mathbf{j}(\mathbf{r}')(\mathbf{r} \cdot \mathbf{r}')d\mathbf{r}' \quad \text{を求める} \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) \times \mathbf{r}d\mathbf{r}' + \int_V ((\mathbf{r} \cdot \mathbf{j}(\mathbf{r}'))\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r} \cdot \mathbf{r}')\mathbf{j}(\mathbf{r}')d\mathbf{r}' \right] \\
&= \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r} - \mathbf{A}_1
\end{aligned}$$

$$\mathbf{A}_1 = \frac{1}{2} \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[\int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \right] \times \mathbf{r}$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}'))d\mathbf{r}' \quad \text{とおくと} \quad \mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

磁気双極子能率

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$\begin{aligned} (B_1)_i &= (\nabla \times \mathbf{A}_1)_i \\ &= \frac{\mu_0}{4\pi} (\nabla \times (\boldsymbol{\mu} \times \frac{\mathbf{r}}{r^3}))_i \\ &= \frac{\mu_0}{4\pi} \sum_{jklm} \epsilon_{ijk} \frac{\partial}{\partial r_j} \epsilon_{klm} \mu_l \frac{r_m}{r^3} \\ &= \frac{\mu_0}{4\pi} \sum_{jlm} (\delta_{il} \delta_{jlm} - \delta_{im} \delta_{jl}) \mu_l \frac{\partial}{\partial r_j} \frac{r_m}{r^3} \\ &= \frac{\mu_0}{4\pi} \sum_j \left(\mu_i \frac{\partial}{\partial r_j} \frac{r_j}{r^3} - \mu_j \frac{\partial}{\partial r_j} \frac{r_i}{r^3} \right) \\ &= \frac{\mu_0}{4\pi} \left(\boldsymbol{\mu} \nabla \cdot \frac{\mathbf{r}}{r^3} - (\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} \right)_i \end{aligned}$$

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$\begin{aligned} (B_1)_i &= (\nabla \times \mathbf{A}_1)_i \\ &= \frac{\mu_0}{4\pi} (\nabla \times (\boldsymbol{\mu} \times \frac{\mathbf{r}}{r^3}))_i \\ &= \frac{\mu_0}{4\pi} \sum_{jklm} \epsilon_{ijk} \frac{\partial}{\partial r_j} \epsilon_{klm} \mu_l \frac{r_m}{r^3} \\ &= \frac{\mu_0}{4\pi} \sum_{jlm} (\delta_{il} \delta_{jlm} - \delta_{im} \delta_{jl}) \mu_l \frac{\partial}{\partial r_j} \frac{r_m}{r^3} \\ &= \frac{\mu_0}{4\pi} \sum_j \left(\mu_i \frac{\partial}{\partial r_j} \frac{r_j}{r^3} - \mu_j \frac{\partial}{\partial r_j} \frac{r_i}{r^3} \right) \\ &= \frac{\mu_0}{4\pi} \left(\boldsymbol{\mu} \nabla \cdot \frac{\mathbf{r}}{r^3} - (\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} \right)_i \end{aligned}$$

$$\frac{\partial}{\partial x} \frac{x}{r^3} = \frac{1}{r^3} - 3 \frac{x^2}{r^5}$$

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$\begin{aligned} (B_1)_i &= (\nabla \times \mathbf{A}_1)_i \\ &= \frac{\mu_0}{4\pi} (\nabla \times (\boldsymbol{\mu} \times \frac{\mathbf{r}}{r^3}))_i \\ &= \frac{\mu_0}{4\pi} \sum_{jklm} \epsilon_{ijk} \frac{\partial}{\partial r_j} \epsilon_{klm} \mu_l \frac{r_m}{r^3} \\ &= \frac{\mu_0}{4\pi} \sum_{jlm} (\delta_{il} \delta_{jlm} - \delta_{im} \delta_{jl}) \mu_l \frac{\partial}{\partial r_j} \frac{r_m}{r^3} \\ &= \frac{\mu_0}{4\pi} \sum_j \left(\mu_i \frac{\partial}{\partial r_j} \frac{r_j}{r^3} - \mu_j \frac{\partial}{\partial r_j} \frac{r_i}{r^3} \right) \\ &= \frac{\mu_0}{4\pi} \left(\boldsymbol{\mu} \nabla \cdot \frac{\mathbf{r}}{r^3} - (\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} \right)_i \end{aligned}$$

$$\frac{\partial}{\partial x} \frac{x}{r^3} = \frac{1}{r^3} - 3 \frac{x^2}{r^5}$$

$$\nabla \cdot \left(\frac{\mathbf{r}}{r^3} \right) = \frac{3}{r^3} - 3 \frac{r^2}{r^5} = 0$$

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$(\mathbf{B}_1)_i = (\nabla \times \mathbf{A}_1)_i$$

$$= \frac{\mu_0}{4\pi} (\nabla \times (\boldsymbol{\mu} \times \frac{\mathbf{r}}{r^3}))_i$$

$$= \frac{\mu_0}{4\pi} \sum_{jklm} \epsilon_{ijk} \frac{\partial}{\partial r_j} \epsilon_{klm} \mu_l \frac{r_m}{r^3}$$

$$= \frac{\mu_0}{4\pi} \sum_{jlm} (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \mu_l \frac{\partial}{\partial r_j} \frac{r_m}{r^3}$$

$$= \frac{\mu_0}{4\pi} \sum_j \left(\mu_i \frac{\partial}{\partial r_j} \frac{r_j}{r^3} - \mu_j \frac{\partial}{\partial r_j} \frac{r_i}{r^3} \right)$$

$$= \frac{\mu_0}{4\pi} \left(\boldsymbol{\mu} \nabla \cdot \frac{\mathbf{r}}{r^3} - (\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} \right)_i$$

$$\frac{\partial}{\partial x} \frac{x}{r^3} = \frac{1}{r^3} - 3 \frac{x^2}{r^5}$$

$$\nabla \cdot \left(\frac{\mathbf{r}}{r^3} \right) = \frac{3}{r^3} - 3 \frac{r^2}{r^5} = 0$$

$$\sum_j \mu_j \frac{\partial}{\partial r_j} \frac{x}{r^3} = \mu_x \frac{\partial}{\partial x} \frac{x}{r^3} + \mu_y \frac{\partial}{\partial y} \frac{x}{r^3} + \mu_z \frac{\partial}{\partial z} \frac{x}{r^3}$$

$$= \mu_x \frac{1}{r^3} - 3\mu_x \frac{x^2}{r^5} - 3\mu_y \frac{xy}{r^5} - 3\mu_z \frac{xz}{r^5}$$

$$= \mu_x \frac{1}{r^3} - 3x \frac{\mu_x x + \mu_y y + \mu_z z}{r^3}$$

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$(\mathbf{B}_1)_i = (\nabla \times \mathbf{A}_1)_i$$

$$= \frac{\mu_0}{4\pi} (\nabla \times (\boldsymbol{\mu} \times \frac{\mathbf{r}}{r^3}))_i$$

$$= \frac{\mu_0}{4\pi} \sum_{jklm} \epsilon_{ijk} \frac{\partial}{\partial r_j} \epsilon_{klm} \mu_l \frac{r_m}{r^3}$$

$$= \frac{\mu_0}{4\pi} \sum_{jlm} (\delta_{il} \delta_{jkm} - \delta_{im} \delta_{jkl}) \mu_l \frac{\partial}{\partial r_j} \frac{r_m}{r^3}$$

$$= \frac{\mu_0}{4\pi} \sum_j \left(\mu_i \frac{\partial}{\partial r_j} \frac{r_j}{r^3} - \mu_j \frac{\partial}{\partial r_j} \frac{r_i}{r^3} \right)$$

$$= \frac{\mu_0}{4\pi} \left(\boldsymbol{\mu} \nabla \cdot \frac{\mathbf{r}}{r^3} - (\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} \right)_i$$

$$\frac{\partial}{\partial x} \frac{x}{r^3} = \frac{1}{r^3} - 3 \frac{x^2}{r^5}$$

$$\nabla \cdot \left(\frac{\mathbf{r}}{r^3} \right) = \frac{3}{r^3} - 3 \frac{r^2}{r^5} = 0$$

$$\sum_j \mu_j \frac{\partial}{\partial r_j} \frac{x}{r^3} = \mu_x \frac{\partial}{\partial x} \frac{x}{r^3} + \mu_y \frac{\partial}{\partial y} \frac{x}{r^3} + \mu_z \frac{\partial}{\partial z} \frac{x}{r^3}$$

$$= \mu_x \frac{1}{r^3} - 3\mu_x \frac{x^2}{r^5} - 3\mu_y \frac{xy}{r^5} - 3\mu_z \frac{xz}{r^5}$$

$$= \mu_x \frac{1}{r^3} - 3x \frac{\mu_x x + \mu_y y + \mu_z z}{r^3}$$

$$(\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} = \boldsymbol{\mu} \frac{1}{r^3} - 3(\boldsymbol{\mu} \cdot \mathbf{r}) \frac{\mathbf{r}}{r^5}$$

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$(B_1)_i = (\nabla \times \mathbf{A}_1)_i$$

$$= \frac{\mu_0}{4\pi} (\nabla \times (\boldsymbol{\mu} \times \frac{\mathbf{r}}{r^3}))_i$$

$$= \frac{\mu_0}{4\pi} \sum_{jklm} \epsilon_{ijk} \frac{\partial}{\partial r_j} \epsilon_{klm} \mu_l \frac{r_m}{r^3}$$

$$= \frac{\mu_0}{4\pi} \sum_{jlm} (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \mu_l \frac{\partial}{\partial r_j} \frac{r_m}{r^3}$$

$$= \frac{\mu_0}{4\pi} \sum_j \left(\mu_i \frac{\partial}{\partial r_j} \frac{r_j}{r^3} - \mu_j \frac{\partial}{\partial r_j} \frac{r_i}{r^3} \right)$$

$$= \frac{\mu_0}{4\pi} \left(\boldsymbol{\mu} \nabla \cdot \frac{\mathbf{r}}{r^3} - (\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} \right)_i$$

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left(-\boldsymbol{\mu} + 3 \frac{\boldsymbol{\mu} \cdot \mathbf{r}}{r^2} \mathbf{r} \right)$$

$$\frac{\partial}{\partial x} \frac{x}{r^3} = \frac{1}{r^3} - 3 \frac{x^2}{r^5}$$

$$\nabla \cdot \left(\frac{\mathbf{r}}{r^3} \right) = \frac{3}{r^3} - 3 \frac{r^2}{r^5} = 0$$

$$\sum_j \mu_j \frac{\partial}{\partial r_j} \frac{x}{r^3} = \mu_x \frac{\partial}{\partial x} \frac{x}{r^3} + \mu_y \frac{\partial}{\partial y} \frac{x}{r^3} + \mu_z \frac{\partial}{\partial z} \frac{x}{r^3}$$

$$= \mu_x \frac{1}{r^3} - 3\mu_x \frac{x^2}{r^5} - 3\mu_y \frac{xy}{r^5} - 3\mu_z \frac{xz}{r^5}$$

$$= \mu_x \frac{1}{r^3} - 3x \frac{\mu_x x + \mu_y y + \mu_z z}{r^3}$$

$$(\boldsymbol{\mu} \cdot \nabla) \frac{\mathbf{r}}{r^3} = \boldsymbol{\mu} \frac{1}{r^3} - 3(\boldsymbol{\mu} \cdot \mathbf{r}) \frac{\mathbf{r}}{r^5}$$

確かに磁気双極子能率

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r}) \quad \boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left(-\boldsymbol{\mu} + 3 \frac{\boldsymbol{\mu} \cdot \mathbf{r}}{r^2} \mathbf{r} \right)$$

$$\mathbf{B} = \frac{1}{4\pi} \frac{1}{r^3} \left(-\mathbf{m} + 3 \frac{\mathbf{m} \cdot \mathbf{r}}{r^2} \mathbf{r} \right)$$

磁気双極子能率

$$\mathbf{A}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} (\boldsymbol{\mu} \times \mathbf{r})$$

$$\boldsymbol{\mu} = \frac{1}{2} \int_V (\mathbf{r}' \times \mathbf{j}(\mathbf{r}')) d\mathbf{r}'$$

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left(-\boldsymbol{\mu} + 3 \frac{\boldsymbol{\mu} \cdot \mathbf{r}}{r^2} \mathbf{r} \right)$$

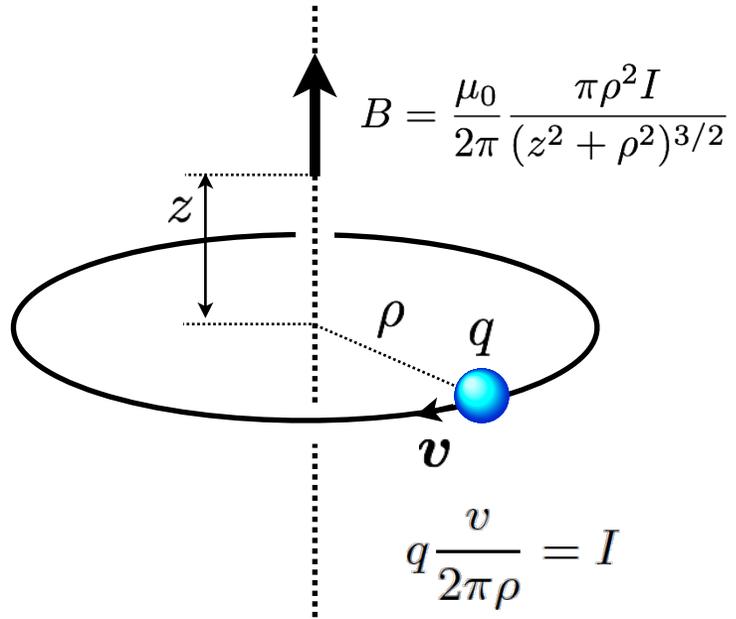
$$\mathbf{B} = \frac{1}{4\pi} \frac{1}{r^3} \left(-\mathbf{m} + 3 \frac{\mathbf{m} \cdot \mathbf{r}}{r^2} \mathbf{r} \right)$$

$$\mathbf{m} = \mu_0 \boldsymbol{\mu}$$

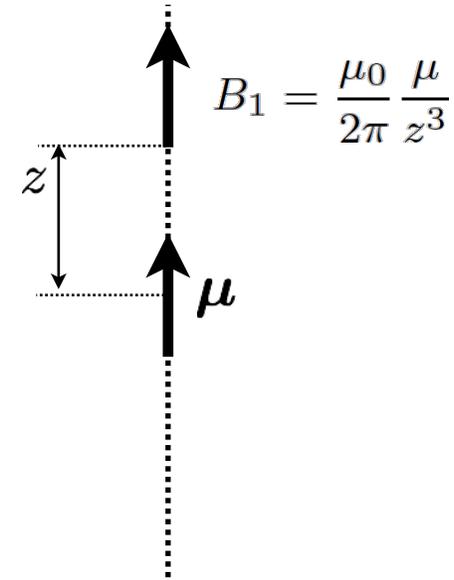
[Wb m]

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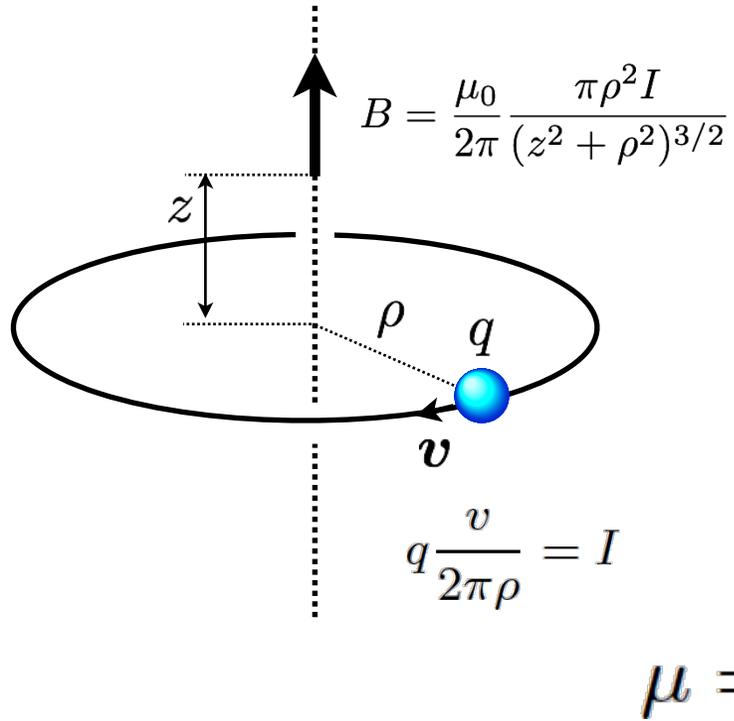
円電流が作る磁場



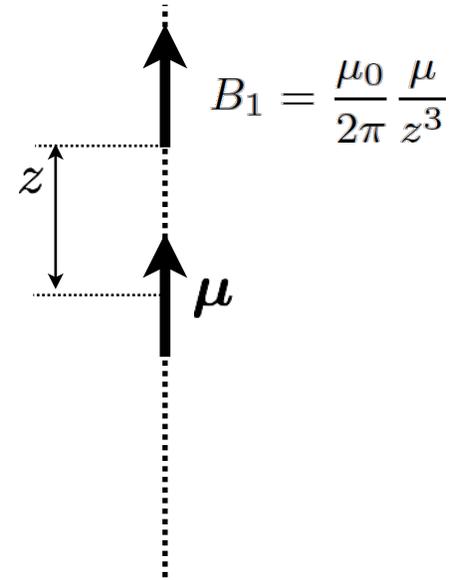
磁気双極子が作る磁場



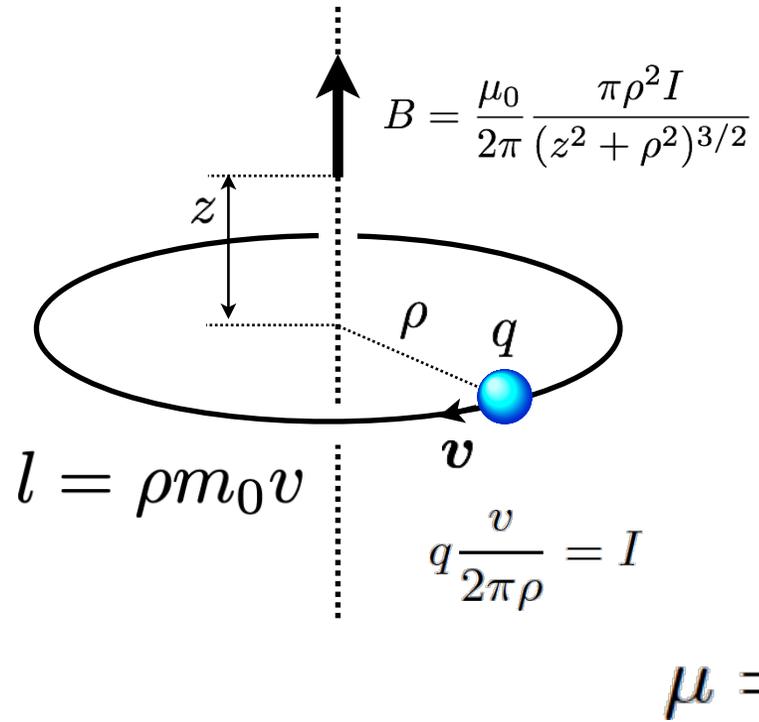
円電流が作る磁場



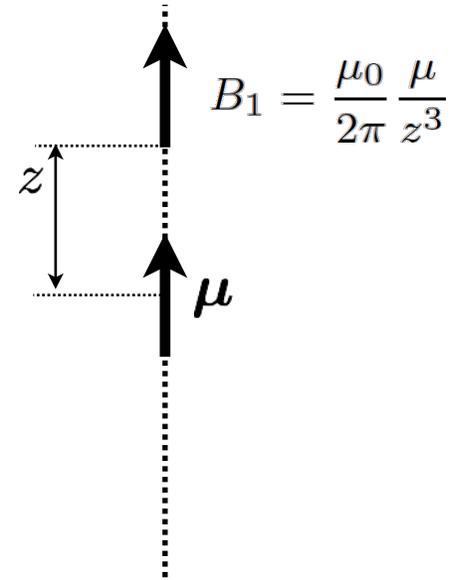
磁気双極子が作る磁場



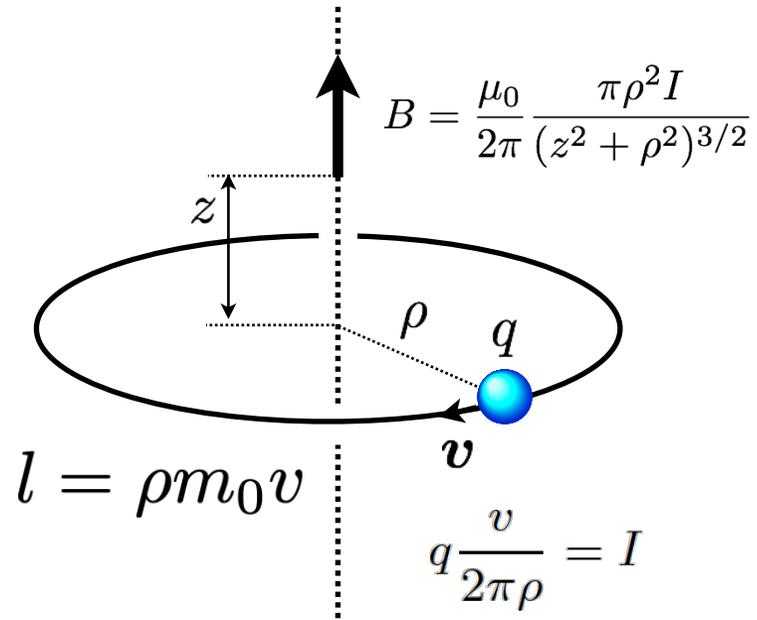
円電流が作る磁場



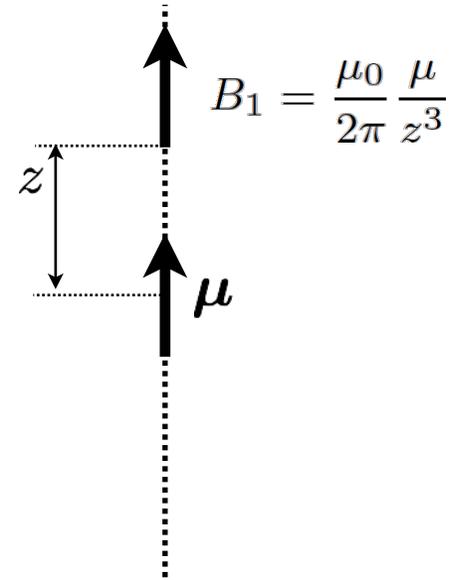
磁気双極子が作る磁場



円電流が作る磁場



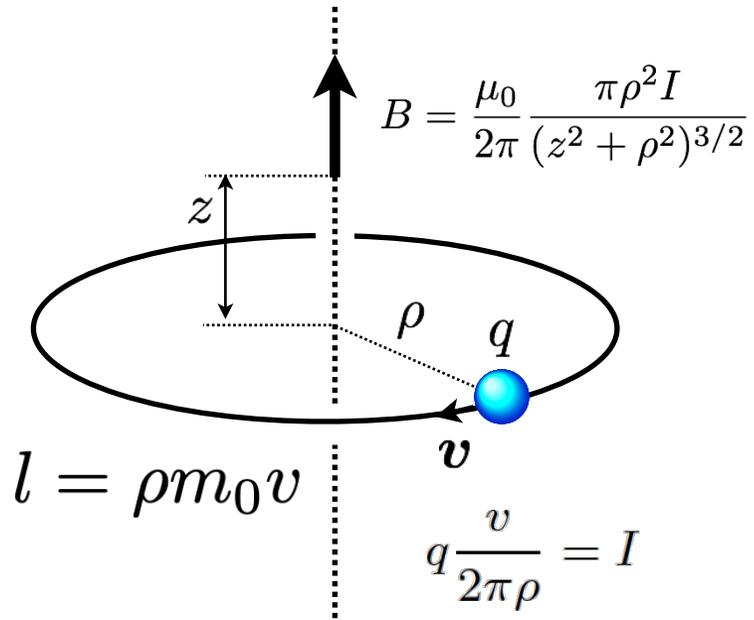
磁気双極子が作る磁場



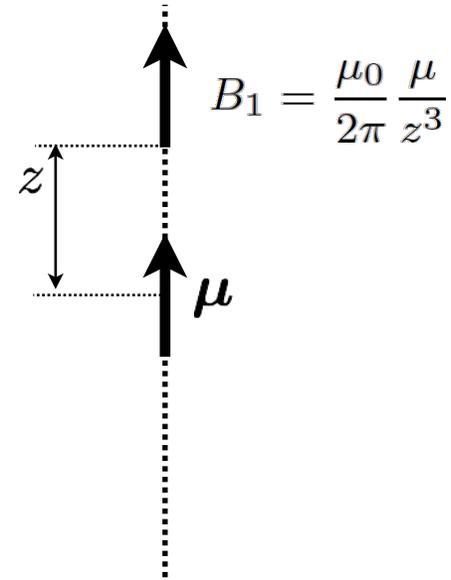
$$\mu = \pi \rho^2 I$$

$$\mu = \frac{q l}{2 m_0}$$

円電流が作る磁場



磁気双極子が作る磁場

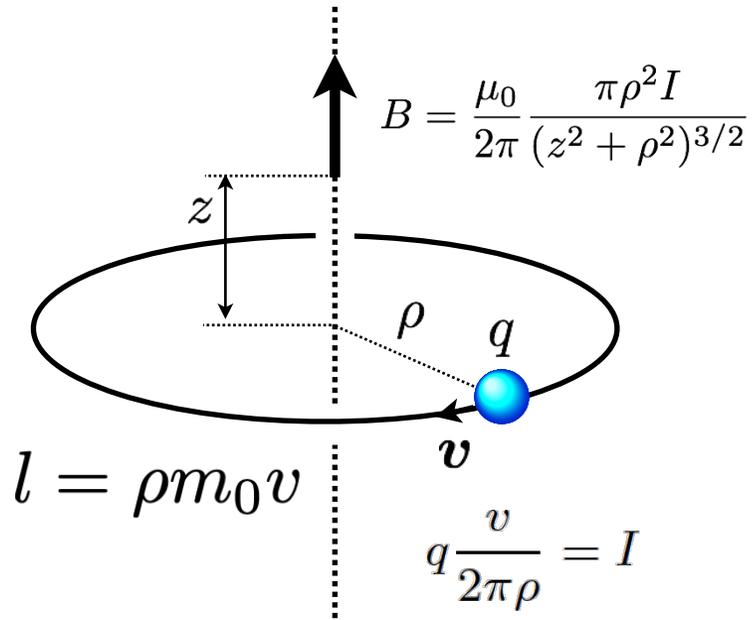


$$\mu = \pi \rho^2 I$$

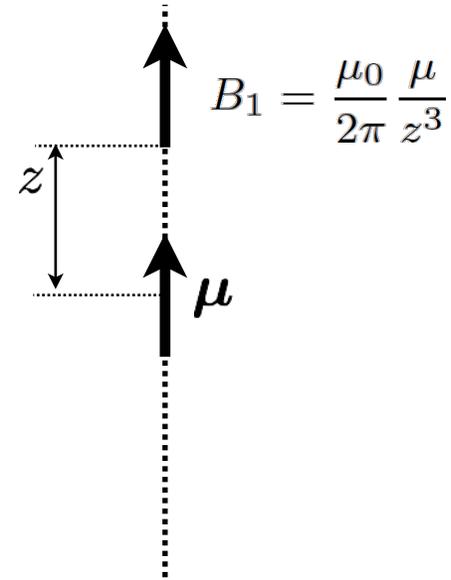
$$\mu = \frac{q l}{2 m_0}$$

$$\mu = \frac{q}{2 m_0} l$$

円電流が作る磁場



磁気双極子が作る磁場

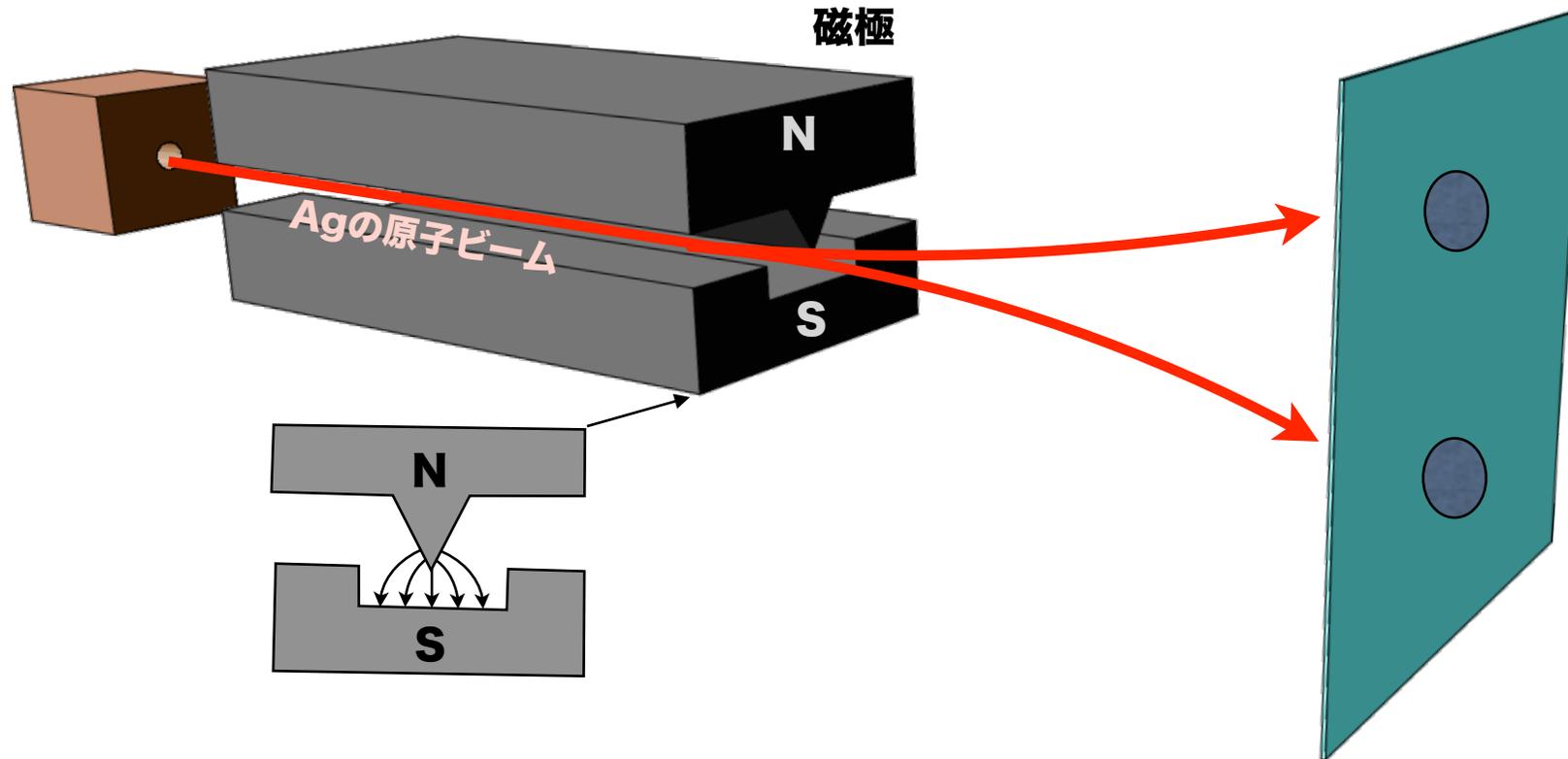


$$\mu = \pi \rho^2 I$$

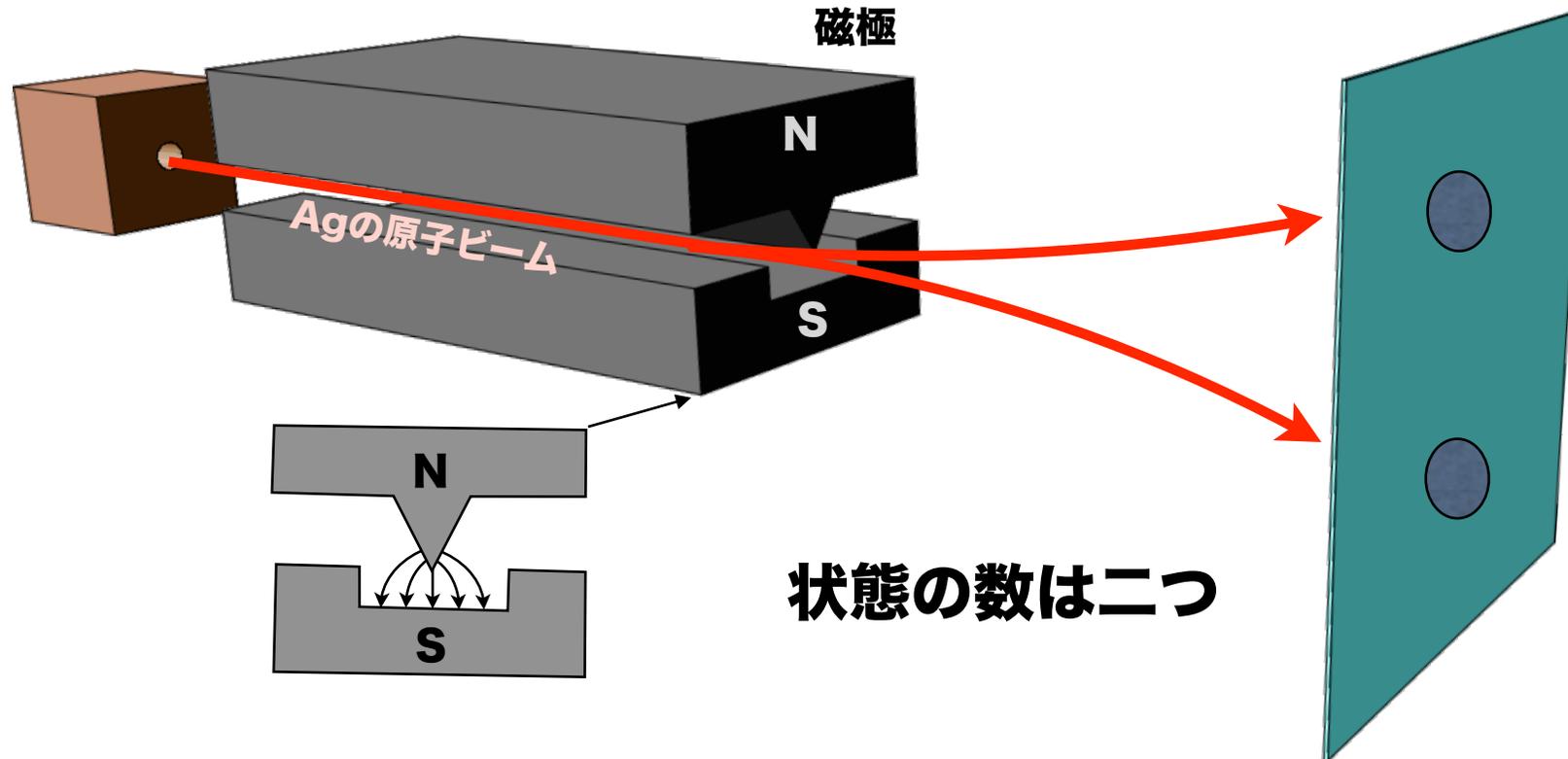
$$\mu = \frac{q l}{2 m_0}$$

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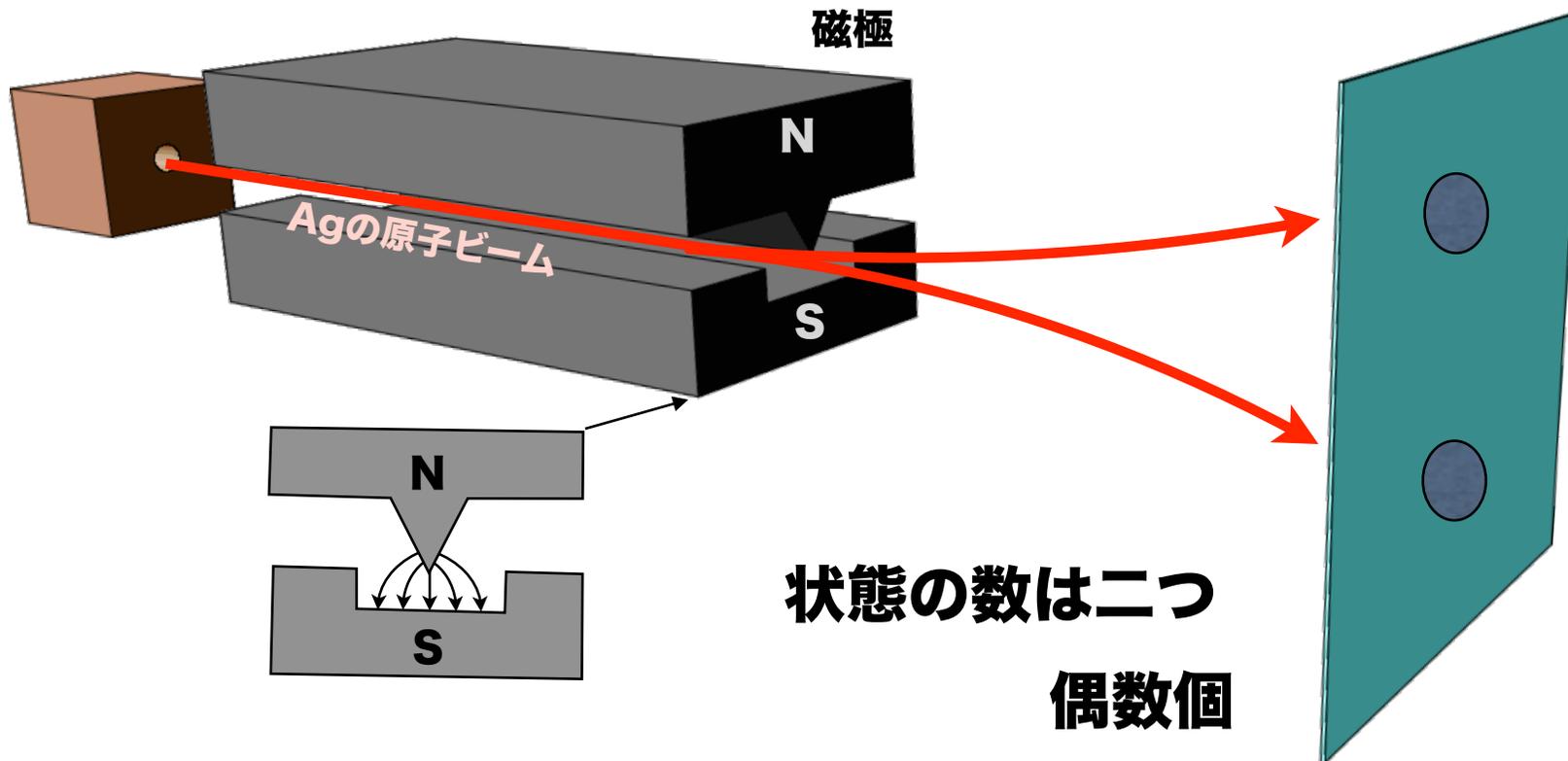
Stern-Gerlachの実験



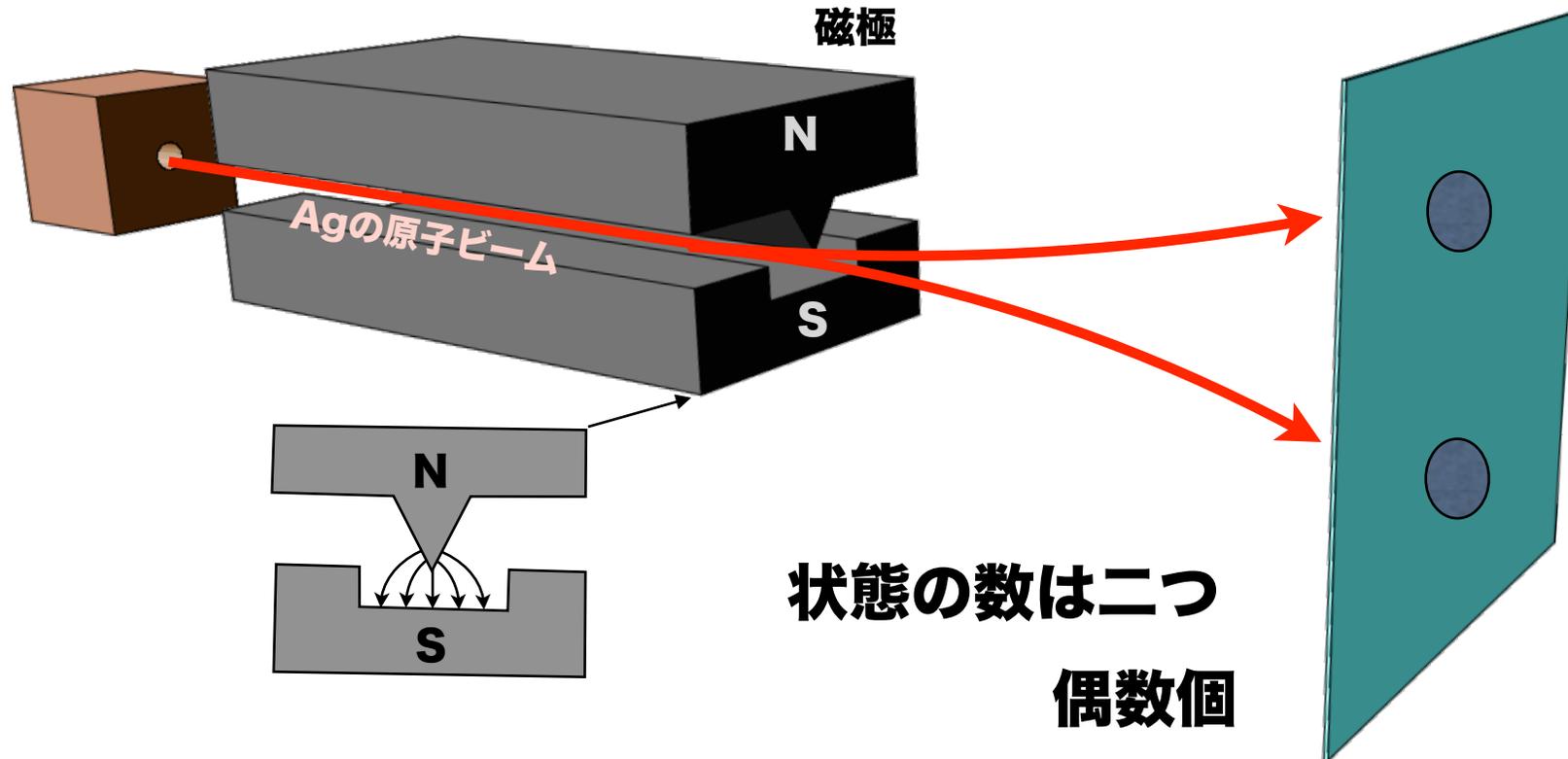
Stern-Gerlachの実験



Stern-Gerlachの実験

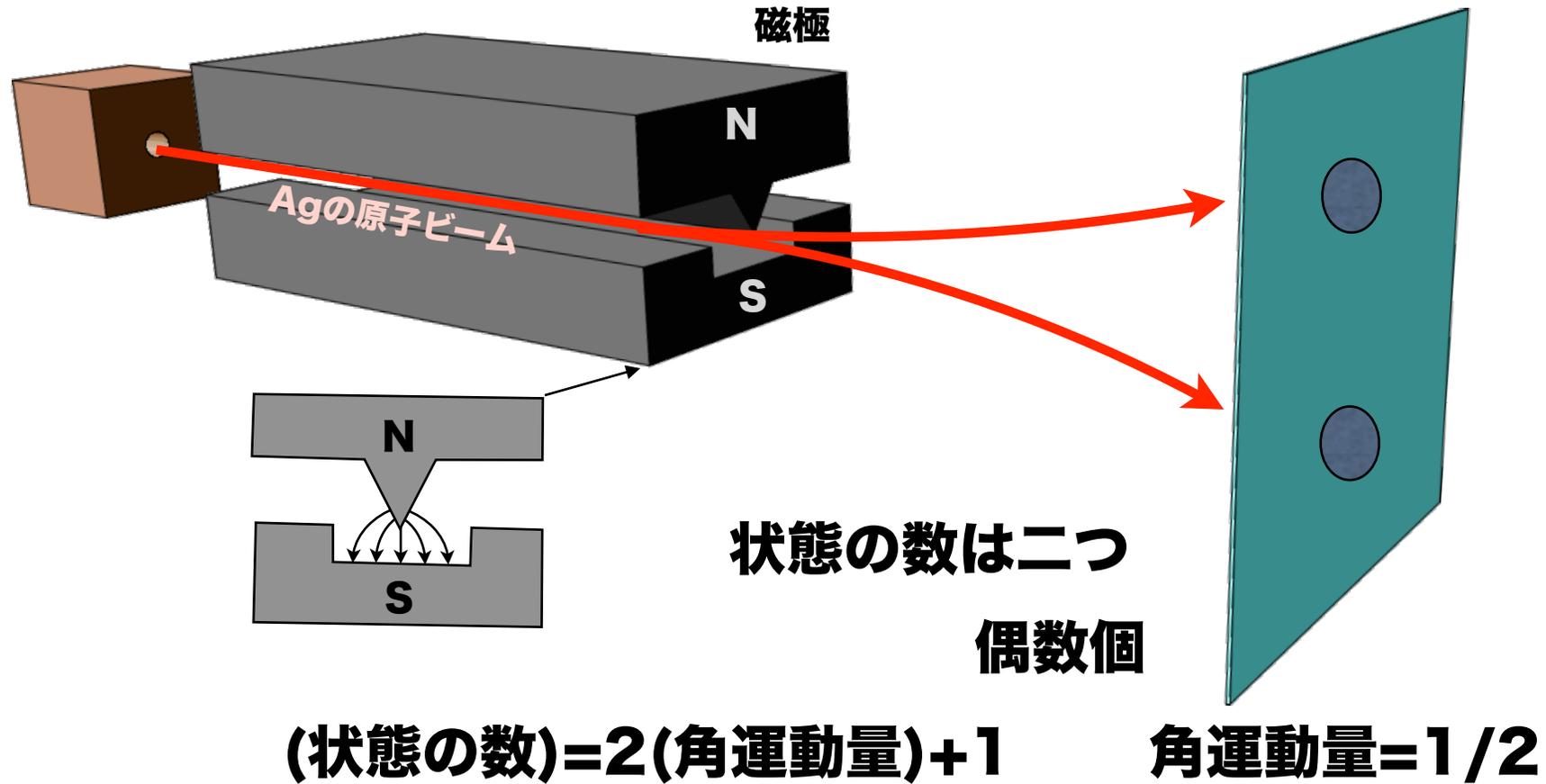


Stern-Gerlachの実験

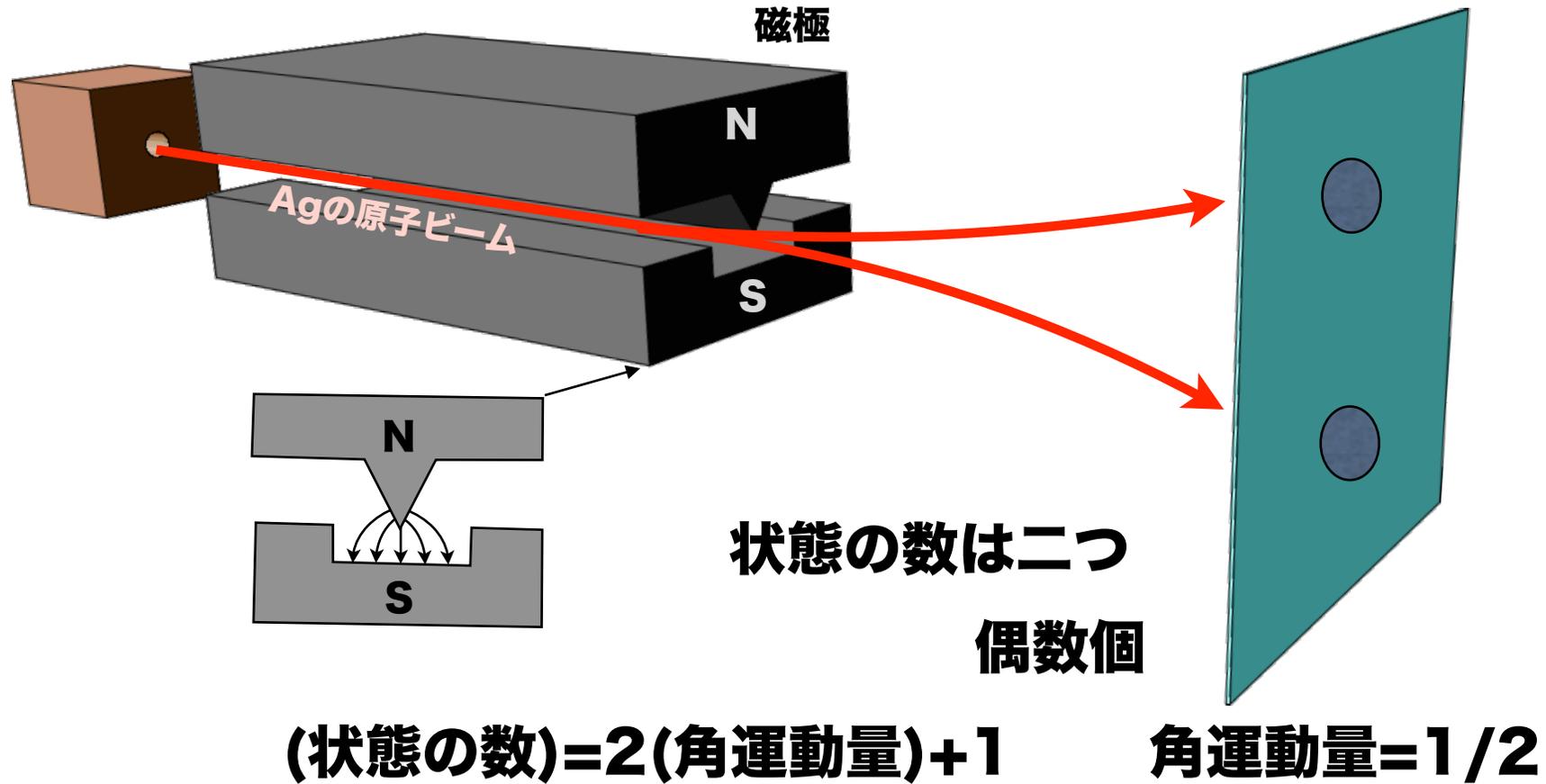


$$(\text{状態の数}) = 2(\text{角運動量}) + 1$$

Stern-Gerlachの実験



Stern-Gerlachの実験



$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$[\sigma_1, \sigma_2]$ を求める

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_1, \sigma_2] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} [\sigma_1, \sigma_2] &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \end{aligned}$$

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} [\sigma_1, \sigma_2] &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \\ &= 2i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} [\sigma_1, \sigma_2] &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \\ &= 2i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= 2i\sigma_3 \end{aligned}$$

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} [\sigma_1, \sigma_2] &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \\ &= 2i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= 2i\sigma_3 \end{aligned}$$

$\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_1, \sigma_2] = 2i\sigma_3$$

$$[\sigma_2, \sigma_3] = 2i\sigma_1$$

$$[\sigma_3, \sigma_1] = 2i\sigma_2$$

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_1, \sigma_2] = 2i\sigma_3$$

$$[\sigma_2, \sigma_3] = 2i\sigma_1$$

$$[\sigma_3, \sigma_1] = 2i\sigma_2$$

$$\left[\frac{\sigma_1}{2}, \frac{\sigma_2}{2} \right] = i \frac{\sigma_3}{2}$$

$$\left[\frac{\sigma_2}{2}, \frac{\sigma_3}{2} \right] = i \frac{\sigma_1}{2}$$

$$\left[\frac{\sigma_3}{2}, \frac{\sigma_1}{2} \right] = i \frac{\sigma_2}{2}$$

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_1, \sigma_2] = 2i\sigma_3$$

$$[\sigma_2, \sigma_3] = 2i\sigma_1$$

$$[\sigma_3, \sigma_1] = 2i\sigma_2$$

$$\left[\frac{\sigma_1}{2}, \frac{\sigma_2}{2} \right] = i \frac{\sigma_3}{2}$$

$$\left[\frac{\sigma_2}{2}, \frac{\sigma_3}{2} \right] = i \frac{\sigma_1}{2}$$

$$\left[\frac{\sigma_3}{2}, \frac{\sigma_1}{2} \right] = i \frac{\sigma_2}{2}$$

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$$

$\sigma = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_1, \sigma_2] = 2i\sigma_3$$

$$[\sigma_2, \sigma_3] = 2i\sigma_1$$

$$[\sigma_3, \sigma_1] = 2i\sigma_2$$

$$\left[\frac{\sigma_1}{2}, \frac{\sigma_2}{2} \right] = i \frac{\sigma_3}{2}$$

$$\left[\frac{\sigma_2}{2}, \frac{\sigma_3}{2} \right] = i \frac{\sigma_1}{2}$$

$$\left[\frac{\sigma_3}{2}, \frac{\sigma_1}{2} \right] = i \frac{\sigma_2}{2}$$

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \quad \left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2} \right] = i\epsilon_{ijk} \frac{\sigma_k}{2}$$

$\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ と定義する

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_1^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_2^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_3^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{s} = \frac{\hbar}{2} \boldsymbol{\sigma}$$

$$s_z \chi_+ = \frac{\hbar}{2} \chi_+ \quad s_z \chi_- = -\frac{\hbar}{2} \chi_-$$

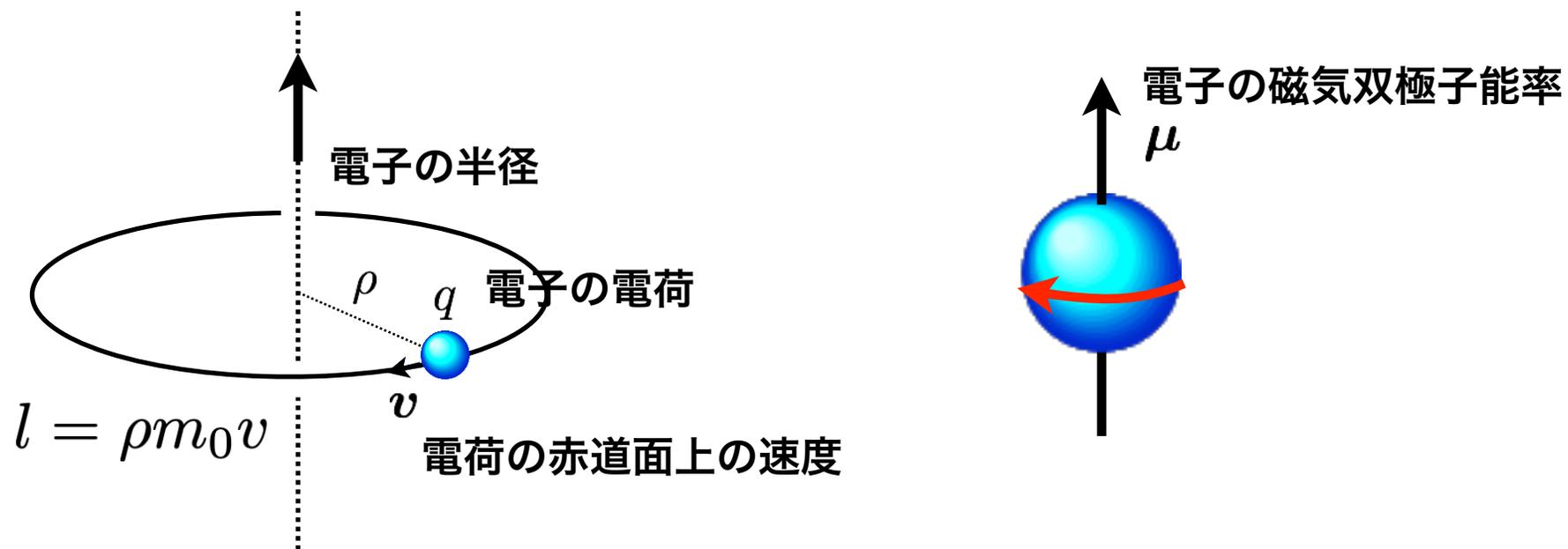
$$\chi_+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \chi_- = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{s}^2 \chi_{\pm} = \frac{\hbar^2}{4} \boldsymbol{\sigma}^2 \chi_{\pm} = \frac{\hbar^2}{4} 3 \chi_{\pm} = \frac{1}{2} \left(\frac{1}{2} + 1 \right) \hbar^2 \chi_{\pm}$$

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \quad \left[\frac{\sigma_i}{2}, \frac{\sigma_j}{2} \right] = i\epsilon_{ijk} \frac{\sigma_k}{2}$$

$$\mathbf{s} \times \mathbf{s} = i\hbar \mathbf{s} \quad [s_i, s_j] = i\hbar \epsilon_{ijk} s_k$$

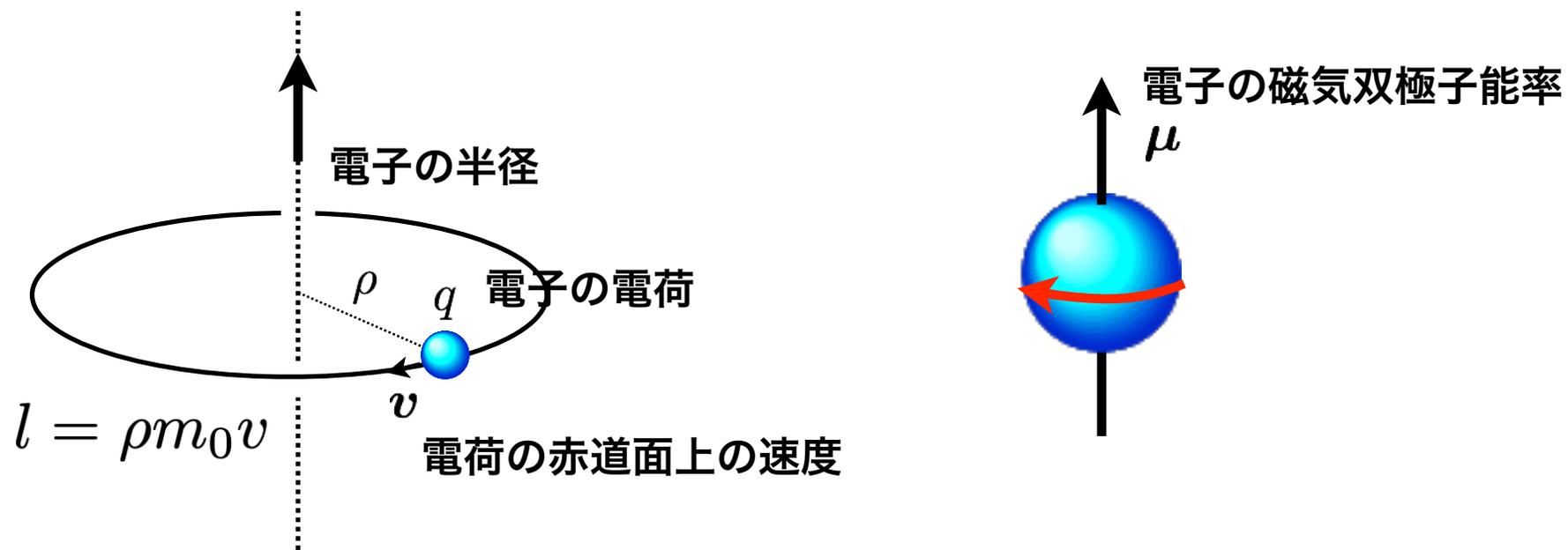
電子の自転で磁気双極子能率が発生しているとするなら



$$\mu = \frac{ql}{2m_0}$$

$$\mu = \frac{q}{2m_0} l$$

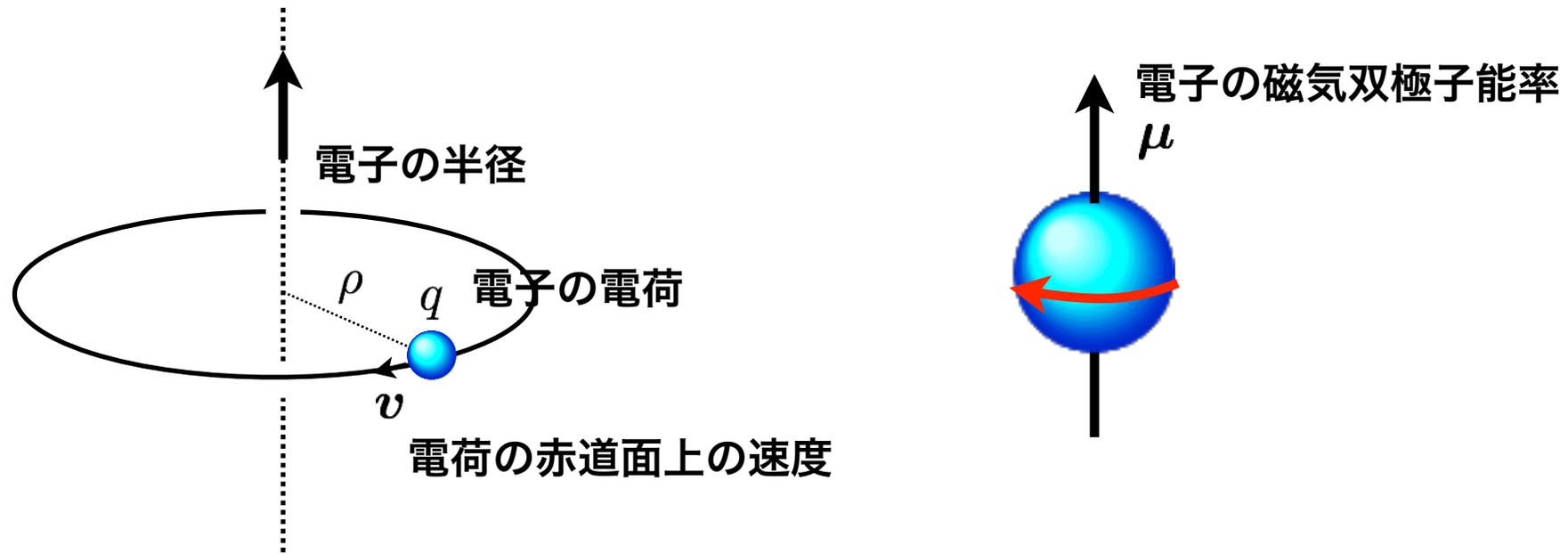
電子の自転で磁気双極子能率が発生しているとするなら



$$\mu = \frac{ql}{2m_0}$$

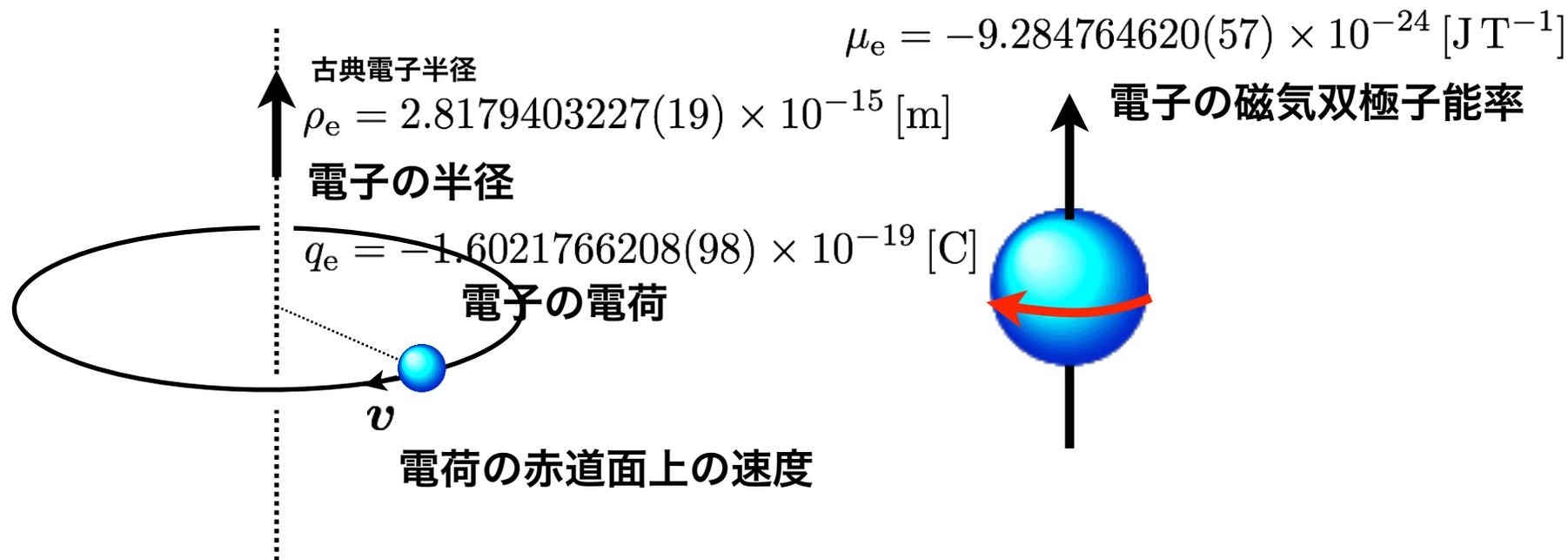
$$\mu_e = \frac{q_e \rho_e m_e v_e}{2m_e}$$

電子の自転で磁気双極子能率が発生しているとするなら



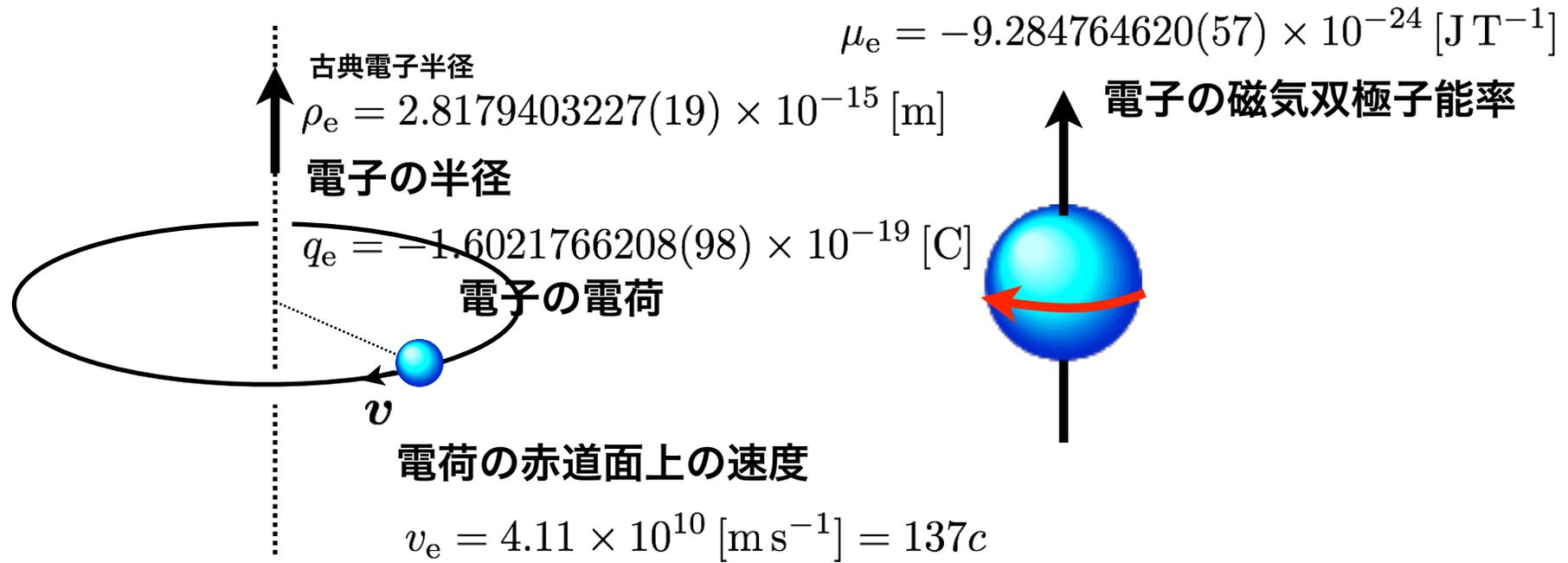
$$\mu_e = \frac{q_e \rho_e v_e}{2}$$

電子の自転で磁気双極子能率が発生しているとするなら



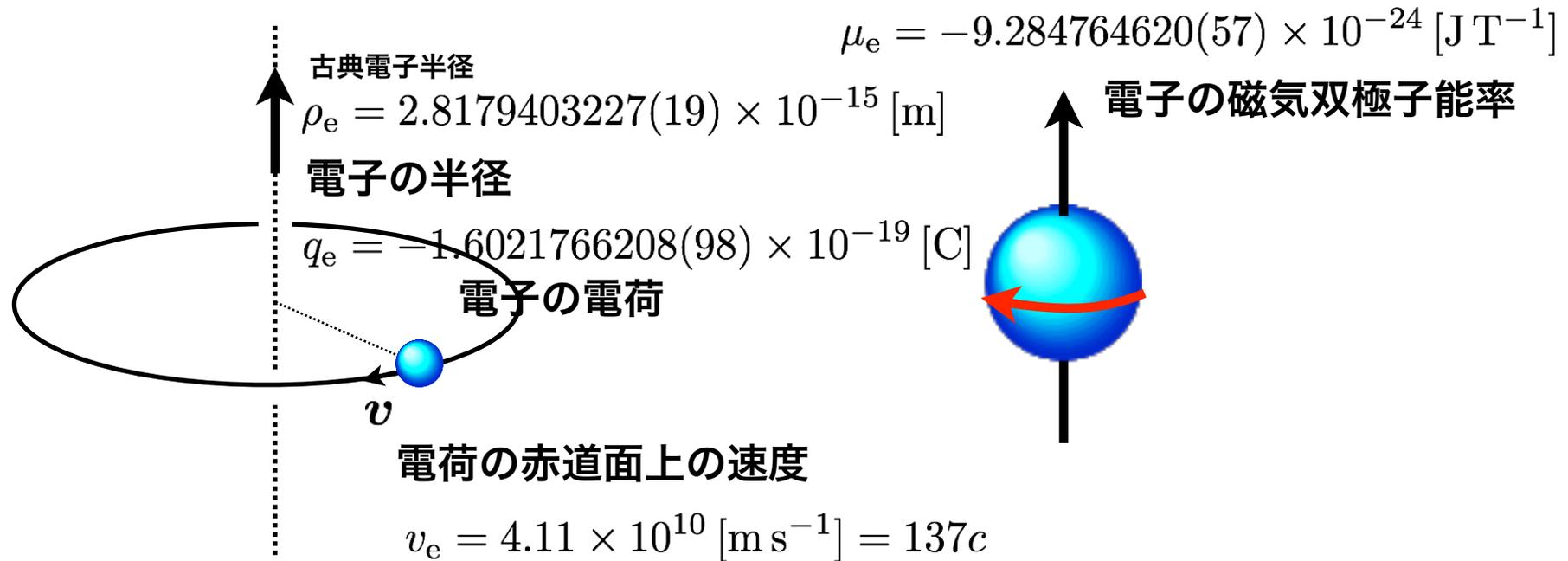
$$\mu_e = \frac{q_e \rho_e v_e}{2}$$

電子の自転で磁気双極子能率が発生しているとするなら



$$\mu_e = \frac{q_e \rho_e v_e}{2}$$

電子の自転で磁気双極子能率が発生しているとするなら



スピン (spin)

$$\mu_e = \frac{q_e \rho_e v_e}{2}$$

$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

全角運動量

スピン角運動量

軌道角運動量

$$\boldsymbol{\mu} = \frac{q}{2m_0} \mathbf{l} = \frac{q\hbar}{2m_0} \frac{\mathbf{l}}{\hbar} \rightarrow \frac{q\hbar}{2m_0} \frac{\mathbf{j}}{\hbar} = \frac{q\hbar}{2m_0} \frac{\mathbf{l} + \mathbf{s}}{\hbar}$$

$$|\Psi\rangle = |\psi\rangle \otimes |\chi\rangle$$

$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

全角運動量

スピン角運動量

軌道角運動量

$$\boldsymbol{\mu} = \frac{q}{2m_0} \mathbf{l} = \frac{q\hbar}{2m_0} \frac{\mathbf{l}}{\hbar} \rightarrow \frac{q\hbar}{2m_0} \frac{\mathbf{j}}{\hbar} = \frac{q\hbar}{2m_0} \frac{\mathbf{l} + \mathbf{s}}{\hbar}$$

磁気双極子能率

$$\boldsymbol{\mu} = \frac{q\hbar}{2m} g \frac{\mathbf{j}}{\hbar}$$

角運動量

磁子 (magneton)

g因子 (g-factor)

軌道角運動量に起因する場合は g=1

Dirac粒子のスピンの場合は g=2

電子 $\mu_e = -\frac{e\hbar}{2m_e} g_e \mathbf{s} \quad \mu_B = \frac{e\hbar}{2m_e} \quad g_e = 2.002319$

ボーア磁子 (Bohr magneton)
 $\mu_B = 5.788\,381\,8060(17) \times 10^{-11} \text{ MeV T}^{-1}$

陽子 $\mu_p = \frac{e\hbar}{2m_p} g_p \mathbf{s} \quad \mu_N = \frac{e\hbar}{2m_p} \quad g_p = 5.585$

核磁子 (nuclear magneton)
 $\mu_N = 3.152\,451\,258\,44(96) \times 10^{-14} \text{ MeV T}^{-1}$

中性子 $g_n = -3.826$

磁気双極子能率 $\mu = \frac{q\hbar}{2m} g \frac{j}{\hbar}$ 角運動量

磁子 (magneton) g因子 (g-factor)

軌道角運動量に起因する場合は $g=1$

Dirac粒子のスピンの場合は $g=2$

$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

全角運動量

スピン角運動量

軌道角運動量

$$\boldsymbol{\mu} = \frac{q}{2m_0} \mathbf{l} = \frac{q\hbar}{2m_0} \frac{\mathbf{l}}{\hbar} \rightarrow \frac{q\hbar}{2m_0} \frac{\mathbf{j}}{\hbar} = \frac{q\hbar}{2m_0} \frac{\mathbf{l} + \mathbf{s}}{\hbar}$$

$$|\Psi\rangle = |\psi\rangle \otimes |\chi\rangle$$

$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

全角運動量

スピン角運動量

軌道角運動量

$$\boldsymbol{\mu} = \frac{q}{2m_0} \mathbf{l} = \frac{q\hbar}{2m_0} \frac{\mathbf{l}}{\hbar} \rightarrow \frac{q\hbar}{2m_0} \frac{\mathbf{j}}{\hbar} = \frac{q\hbar}{2m_0} \frac{\mathbf{l} + \mathbf{s}}{\hbar}$$

磁気双極子能率

$$\boldsymbol{\mu} = \frac{q\hbar}{2m} g \frac{\mathbf{j}}{\hbar}$$

角運動量

磁子 (magneton)

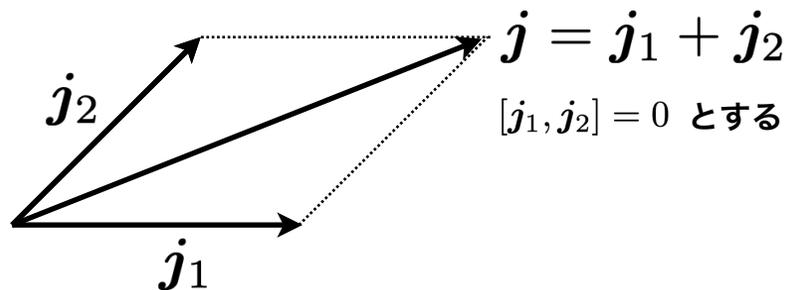
g因子 (g-factor)

軌道角運動量に起因する場合は g=1

Dirac粒子のスピンの場合は g=2

角運動量の合成

角運動量の合成



$$[j_{1i}, j_{1j}] = i\hbar \sum_k \epsilon_{ijk} j_{1k} \quad \mathbf{j}_1^2 \text{ と } j_{1z} \text{ の同時固有状態を } |j_1 m_1\rangle \quad \begin{aligned} \mathbf{j}_1^2 |j_1 m_1\rangle &= j_1(j_1 + 1) |j_1 m_1\rangle \\ j_{1z} |j_1 m_1\rangle &= m_1 |j_1 m_1\rangle \end{aligned}$$

$$[j_{2i}, j_{2j}] = i\hbar \sum_k \epsilon_{ijk} j_{2k} \quad \mathbf{j}_2^2 \text{ と } j_{2z} \text{ の同時固有状態を } |j_2 m_2\rangle \quad \begin{aligned} \mathbf{j}_2^2 |j_2 m_2\rangle &= j_2(j_2 + 1) |j_2 m_2\rangle \\ j_{2z} |j_2 m_2\rangle &= m_2 |j_2 m_2\rangle \end{aligned}$$

$\mathbf{j}_1^2, j_{1z}, \mathbf{j}_2^2, j_{2z}$ の同時固有状態 $|j_1 m_1 j_2 m_2\rangle = |j_1 m_1\rangle \otimes |j_2 m_2\rangle$ は完全系をなす

よって $\sum_{m_1 m_2} |j_1 m_1 j_2 m_2\rangle \langle j_1 m_1 j_2 m_2| = 1$

$$[j_i, j_k] = i\hbar \sum_l \epsilon_{ikl} j_l \quad \mathbf{j}^2 \text{ と } j_z \text{ の同時固有状態を } |(j_1 j_2) j m\rangle \quad \begin{aligned} \mathbf{j}^2 |(j_1 j_2) j m\rangle &= j(j + 1) |(j_1 j_2) j m\rangle \\ j_z |(j_1 j_2) j m\rangle &= m |(j_1 j_2) j m\rangle \end{aligned}$$

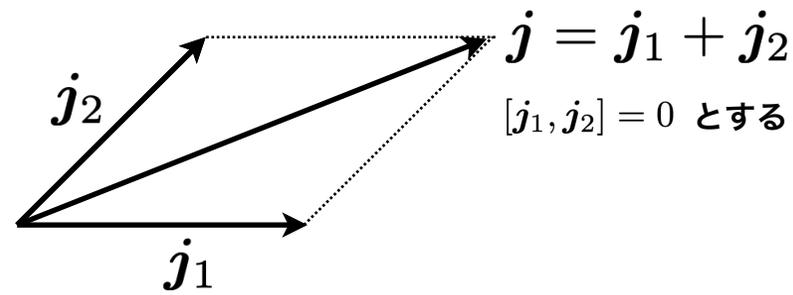
$\mathbf{j}_1^2, \mathbf{j}_2^2, \mathbf{j}^2, j_z$ の同時固有状態 $|(j_1 j_2) j m\rangle$ も完全系をなす

ここで $|(j_1 j_2) j m\rangle = \sum_{m_1 m_2} |j_1 m_1 j_2 m_2\rangle \langle j_1 m_1 j_2 m_2 | (j_1 j_2) j m\rangle$ と両基底間の変換を表したとき

↓
この展開係数を $\langle j_1 m_1 j_2 m_2 | j m\rangle$ と書いて Clebsch-Gordan係数 と呼ぶ

通常これを $|j m\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | j m\rangle |j_1 m_1\rangle |j_2 m_2\rangle$ と書く

角運動量の合成



$$\langle j_1 m_1 j_2 m_2 | j_z | j m \rangle = m \langle j_1 m_1 j_2 m_2 | j m \rangle$$

$$\parallel$$

$$\langle j_1 m_1 j_2 m_2 | j_{1z} + j_{2z} | j m \rangle = (m_1 + m_2) \langle j_1 m_1 j_2 m_2 | j m \rangle \quad \therefore (m - m_1 - m_2) \langle j_1 m_1 j_2 m_2 | j m \rangle = 0$$

$m = m_1 + m_2$ でなければ $\langle j_1 m_1 j_2 m_2 | j m \rangle = 0$

$$|j_1 - j_2| \leq j \leq j_1 + j_2$$

$$(2j_1 + 1) \otimes (2j_2 + 1) = (2|j_1 - j_2| + 1) \oplus (2|j_1 - j_2| + 3) \oplus \cdots \oplus (2(j_1 + j_2) - 1) \oplus (2(j_1 + j_2) + 1)$$

$$j_1 = 2, j_2 = 3 \rightarrow 5 \otimes 7 = \underset{j=1}{3} \oplus \underset{j=2}{5} \oplus \underset{j=3}{7} \oplus \underset{j=4}{9} \oplus \underset{j=5}{11}$$

$$j_1 = \frac{3}{2}, j_2 = 2 \rightarrow 4 \otimes 5 = \underset{j=\frac{1}{2}}{2} \oplus \underset{j=\frac{3}{2}}{4} \oplus \underset{j=\frac{5}{2}}{6} \oplus \underset{j=\frac{7}{2}}{8}$$

Condon-Shortleyの位相規約 (Condon-Shortley's phase convention) $\langle j_1 j_1 j_2 j_2 | j_1 + j_2 j_1 + j_2 \rangle = 1$

$\langle j_1 m_1 j_2 m_2 | j m \rangle$ は実数

$$Y_{lm}(\theta, \varphi) = (-1)^{(m+|m|)/2} \sqrt{\frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} P_l^{|m|}(\cos \theta) e^{im\varphi}$$

$\langle j m | j_1 m_1 j_2 m_2 \rangle = \langle j_1 m_1 j_2 m_2 | j m \rangle$ と書くこともある

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 \quad s_1 = \frac{1}{2} \quad s_2 = \frac{1}{2} \quad \text{の場合}$$

$$|s_1 m_1 s_2 m_2\rangle = |s_1 m_1\rangle \otimes |s_2 m_2\rangle \quad \begin{aligned} \left| \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right\rangle &= \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle = |\uparrow\uparrow\rangle & \left| \frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2} \right\rangle &= \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle = |\uparrow\downarrow\rangle \\ \left| \frac{1}{2} -\frac{1}{2} \frac{1}{2} \frac{1}{2} \right\rangle &= \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle = |\downarrow\uparrow\rangle & \left| \frac{1}{2} -\frac{1}{2} \frac{1}{2} -\frac{1}{2} \right\rangle &= \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle = |\downarrow\downarrow\rangle \end{aligned}$$

$$|sm\rangle = \sum_{m_1 m_2} \langle s_1 m_1 s_2 m_2 | sm \rangle |s_1 m_1 s_2 m_2\rangle$$

$$|11\rangle = |\uparrow\uparrow\rangle \quad s_- |11\rangle = \sqrt{1 \cdot 2 - 1 \cdot 0} |10\rangle = \sqrt{2} |10\rangle$$

$$s_- = s_{1-} + s_{2-} \quad (s_{1-} + s_{2-}) |\uparrow\uparrow\rangle = \sqrt{\frac{1}{2} \frac{3}{2} - \frac{1}{2} \left(-\frac{1}{2}\right)} |\downarrow\uparrow\rangle + \sqrt{\frac{1}{2} \frac{3}{2} - \frac{1}{2} \left(-\frac{1}{2}\right)} |\uparrow\downarrow\rangle = |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$$

$$|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$|1-1\rangle = |\downarrow\downarrow\rangle$$

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

問題4.

$$|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \text{ の両辺に}$$

$s_- = s_{1-} + s_{2-}$ を作用させて

$$|1-1\rangle = |\downarrow\downarrow\rangle \text{ となることを示せ}$$

$$J_{\pm} |jm\rangle = \sqrt{j(j+1) - m(m \pm 1)} |j, m \pm 1\rangle$$

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 \quad s_1 = \frac{1}{2} \quad s_2 = \frac{1}{2} \quad \text{の場合}$$

$$|s_1 m_1 s_2 m_2\rangle = |s_1 m_1\rangle \otimes |s_2 m_2\rangle$$

$$\left| \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right\rangle = \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle = |\uparrow\uparrow\rangle \quad \left| \frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2} \right\rangle = \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle = |\uparrow\downarrow\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \frac{1}{2} \frac{1}{2} \right\rangle = \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle = |\downarrow\uparrow\rangle \quad \left| \frac{1}{2} -\frac{1}{2} \frac{1}{2} -\frac{1}{2} \right\rangle = \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle = |\downarrow\downarrow\rangle$$

$$|sm\rangle = \sum_{m_1 m_2} \langle s_1 m_1 s_2 m_2 | s m \rangle |s_1 m_1 s_2 m_2\rangle$$

$$|11\rangle = |\uparrow\uparrow\rangle \quad s_- |11\rangle = \sqrt{1 \cdot 2 - 1 \cdot 0} |10\rangle = \sqrt{2} |10\rangle$$

$$s_- = s_{1-} + s_{2-} \quad (s_{1-} + s_{2-}) |\uparrow\uparrow\rangle = \sqrt{\frac{1}{2} \frac{3}{2} - \frac{1}{2} \left(-\frac{1}{2}\right)} |\downarrow\uparrow\rangle + \sqrt{\frac{1}{2} \frac{3}{2} - \frac{1}{2} \left(-\frac{1}{2}\right)} |\uparrow\downarrow\rangle = |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$$

$$|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$|11\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left| 11 \right. \right\rangle |\uparrow\uparrow\rangle$$

$$|1-1\rangle = |\downarrow\downarrow\rangle$$

$$|00\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2} \left| 00 \right. \right\rangle |\uparrow\downarrow\rangle + \left\langle \frac{1}{2} -\frac{1}{2} \frac{1}{2} \frac{1}{2} \left| 00 \right. \right\rangle |\downarrow\uparrow\rangle$$

$$\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}}$$

$$|10\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2} \left| 10 \right. \right\rangle |\uparrow\downarrow\rangle + \left\langle \frac{1}{2} -\frac{1}{2} \frac{1}{2} \frac{1}{2} \left| 10 \right. \right\rangle |\downarrow\uparrow\rangle$$

$$\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}$$

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|1-1\rangle = \left\langle \frac{1}{2} -\frac{1}{2} \frac{1}{2} -\frac{1}{2} \left| 1-1 \right. \right\rangle |\downarrow\downarrow\rangle$$

$$1$$

$$1/2 \times 1/2$$

		1		
	+1	1	0	
+1/2 + 1/2	1	0	0	
+1/2 - 1/2	1/2	1/2	1	
-1/2 + 1/2	1/2	-1/2	-1	
		-1/2 - 1/2	1	

<http://pdg.lbl.gov/2019/reviews/rpp2019-rev-clebsch-gordan-coefs.pdf>

$$j = l + s \quad s = \frac{1}{2} \quad \text{の場合} \quad j = l \pm \frac{1}{2} \quad \text{である}$$

$$|jm\rangle = \sum_{m_l m_s} \langle l m_l s m_s | j m \rangle |l m_l\rangle \otimes |s m_s\rangle$$

$$j = l + \frac{1}{2} \quad |jm\rangle = \sqrt{\frac{l+m+\frac{1}{2}}{2l+1}} \left| l, m - \frac{1}{2} \right\rangle \otimes |\uparrow\rangle + \sqrt{\frac{l-m+\frac{1}{2}}{2l+1}} \left| l, m + \frac{1}{2} \right\rangle \otimes |\downarrow\rangle$$

$$j = l - \frac{1}{2} \quad |jm\rangle = -\sqrt{\frac{l-m+\frac{1}{2}}{2l+1}} \left| l, m - \frac{1}{2} \right\rangle \otimes |\uparrow\rangle + \sqrt{\frac{l+m+\frac{1}{2}}{2l+1}} \left| l, m + \frac{1}{2} \right\rangle \otimes |\downarrow\rangle$$

角運動量の状態を l_j と表す

$l = 0$	s	$l = 1$	$j = \frac{1}{2}$	$p_{\frac{1}{2}}$
$l = 1$	p		$j = \frac{3}{2}$	$p_{\frac{3}{2}}$
$l = 2$	d			
$l = 3$	f			
	\vdots			

3j記号 3j symbol

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \frac{(-1)^{j_1-j_2-m_3}}{\sqrt{2j_3+1}} \langle j_1 m_1 j_2 m_2 | j_3 -m_3 \rangle$$

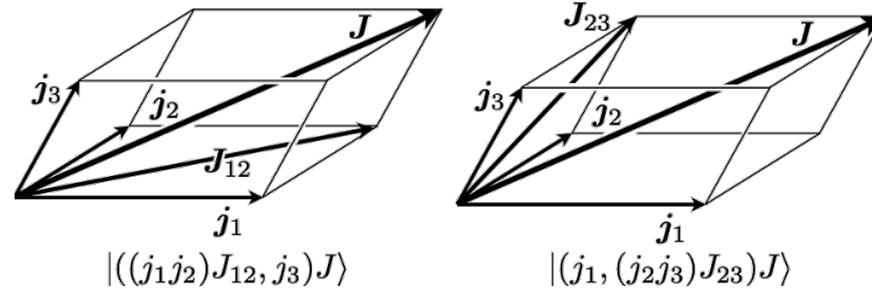
$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} j_2 & j_3 & j_1 \\ m_2 & m_3 & m_1 \end{pmatrix} = \begin{pmatrix} j_3 & j_1 & j_2 \\ m_3 & m_1 & m_2 \end{pmatrix}$$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_2 & j_1 & j_3 \\ m_2 & m_1 & m_3 \end{pmatrix} = (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_3 & j_2 \\ m_1 & m_3 & m_2 \end{pmatrix}$$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ -m_1 & -m_2 & -m_3 \end{pmatrix} = (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

角運動量の組み替え

Racah係数



$$\langle (j_1, (j_2 j_3) J_{23}) J | ((j_1 j_2) J_{12}, j_3) J \rangle = \sqrt{(2J_{12} + 1)(2J_{23} + 1)} W(j_1 j_2 J j_3; J_{12} J_{23})$$

$$W(abcd; ef) = \Delta(a, b, e) \Delta(c, d, e) \Delta(a, c, f) \Delta(b, d, f) \sum_{z=\max(\alpha_1, \alpha_2, \alpha_3, \alpha_4)}^{\min(\beta_1, \beta_2, \beta_3)} \frac{(-1)^{z+\beta_1} (z+1)!}{(z-\alpha_1)!(z-\alpha_2)!(z-\alpha_3)!(z-\alpha_4)!(\beta_1-z)!(\beta_2-z)!(\beta_3-z)!}$$

$$\Delta(a, b, c) = \sqrt{\frac{(a+b-c)!(b+c-a)!(c+a-b)!}{(a+b+c+1)!}}$$

$$\alpha_1 = a+b+e, \quad \alpha_2 = c+d+e, \quad \alpha_3 = a+c+f, \quad \alpha_4 = b+d+f, \\ \beta_1 = a+b+c+d, \quad \beta_2 = a+d+e+f, \quad \beta_3 = b+c+e+f$$

6j記号 6j symbol

$$\left\{ \begin{matrix} j_1 & j_2 & J_{12} \\ j_3 & J & J_{23} \end{matrix} \right\} = (-1)^{j_1+j_2+j_3+J} W(j_1 j_2 J j_3; J_{12} J_{23})$$

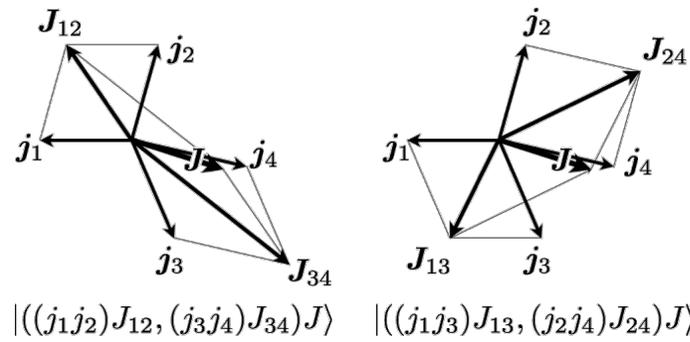
$$\left\{ \begin{matrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{matrix} \right\} = \sum_{m_1 m_2 m_3 m_4 m_5 m_6} (-1)^{\sum_{k=1}^6 (j_k - m_k)} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -m_1 & m_5 & m_6 \end{pmatrix} \begin{pmatrix} j_4 & j_5 & j_3 \\ m_4 & -m_5 & m_3 \end{pmatrix} \begin{pmatrix} j_4 & j_2 & j_6 \\ -m_4 & -m_2 & -m_6 \end{pmatrix}$$

$$\left\{ \begin{matrix} e & a & f \\ b & d & c \end{matrix} \right\} = \frac{(-1)^{b+c+e+f} \Delta(abc) \Delta(aef) \Delta(cde) \Delta(bdf) (a+b+c+1)! (b+d+f+1)!}{(a+b-c)!(c-d+e)!(c+d-e)!(a-e+f)!(-a+e+f)!(b+d-f)!} \sum_z \frac{(-1)^z (2b-z)!(b+c-e+f-z)!(b+c+e+f+1-z)!}{z!(-a+b+c-z)!(b-d+f-z)!(a+b+c+1-z)!(b+d+f+1-z)!}$$

$$0 \leq z \leq \min(2b, b+c-e, -a+b+c, b-d+f)$$

角運動量の組み替え

9j記号 9j symbol



$$\langle (j_1 j_2) J_{12}, (j_3 j_4) J_{34}; J | (j_1 j_3) J_{13}, (j_2 j_4) J_{24}; J \rangle = \sqrt{(2J_{12} + 1)(2J_{34} + 1)(2J_{13} + 1)(2J_{24} + 1)} \begin{Bmatrix} j_1 & j_2 & J_{12} \\ j_3 & j_4 & J_{34} \\ J_{13} & J_{24} & J \end{Bmatrix}$$

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & j_9 \end{Bmatrix} = \sum_x (-1)^{2x} (2x + 1) \begin{Bmatrix} j_1 & j_4 & j_7 \\ j_8 & j_9 & x \end{Bmatrix} \begin{Bmatrix} j_2 & j_5 & j_8 \\ j_4 & x & j_6 \end{Bmatrix} \begin{Bmatrix} j_3 & j_6 & j_9 \\ x & j_1 & j_2 \end{Bmatrix}$$

$$\begin{Bmatrix} j_{11} & j_{21} & j_{31} \\ j_{12} & j_{22} & j_{32} \\ j_{13} & j_{23} & j_{33} \end{Bmatrix} = (-1)^{j_{13}+j_{23}-j_{33}} \frac{\nabla(j_{21}j_{11}j_{31})\nabla(j_{12}j_{22}j_{32})\nabla(j_{33}j_{31}j_{32})}{\nabla(j_{21}j_{22}j_{23})\nabla(j_{12}j_{11}j_{13})\nabla(j_{33}j_{13}j_{23})}$$

$$\times \sum_{x,y,z} \frac{(-1)^{x+y+z}}{x!y!z!}$$

$$\times \frac{(2j_{23} - x)!(j_{21} + j_{22} - j_{23} + x)!(j_{13} - j_{23} + j_{33} + x)!}{(j_{22} - j_{21} + j_{23} - x)!(j_{13} + j_{23} - j_{33} - x)!}$$

$$\times \frac{(j_{31} + j_{32} - j_{33} + y)!(j_{22} - j_{12} + j_{32} + y)!}{(2j_{32} + 1 + y)!(j_{31} - j_{32} + j_{33} - y)!(j_{12} + j_{22} - j_{32} - y)!}$$

$$\times \frac{(2j_{11} - z)!(j_{12} - j_{11} + j_{13} + z)!}{(j_{11} + j_{21} + j_{31} + 1 - z)!(j_{11} - j_{12} + j_{13} - z)!(j_{11} + j_{21} - j_{31} - z)!}$$

$$\times \frac{(j_{11} + j_{21} - j_{32} + j_{33} - y - z)!}{(j_{21} - j_{12} + j_{32} - j_{23} + x + y)!(j_{12} - j_{11} - j_{23} + j_{33} + x + z)!}$$

$$\nabla(abc) = \sqrt{\frac{(a-b+c)!(a+b-c)!(a+b+c+1)!}{(b+c-a)!}}$$

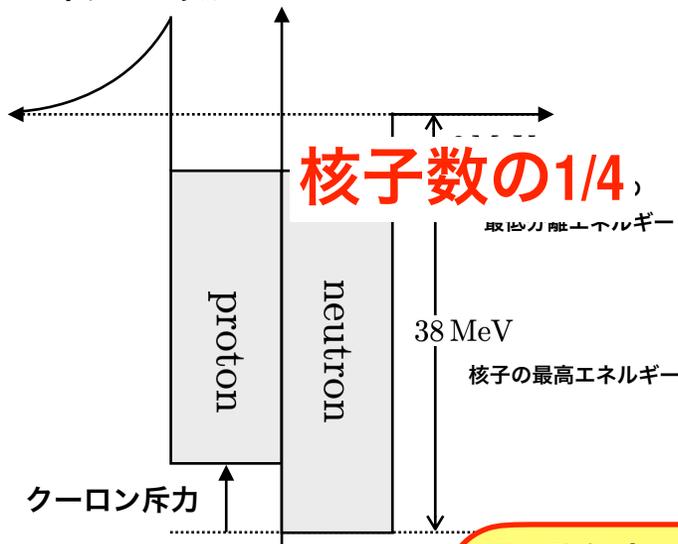
重水素

フェルミガス模型

Fermi gas model

k 空間において $|k|$ が k と $k + dk$ の間にある状態の個数を $dn(k)$ とおく

原子核外では
クーロンポテンシャル



$$dn(k) = \frac{1}{8} \times 4\pi k^2 dk \frac{1}{(\pi/a)^3} \quad \text{なので} \quad n(k) = \frac{1}{8} \frac{4\pi}{3} k^3 \frac{1}{(\pi/a)^3}$$

最大の k を k_F とする

$\hbar k_F$ は最大の運動量で Fermi momentum と呼ばれる

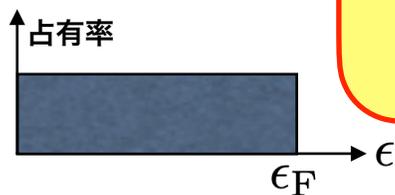
$$\frac{A}{4} = \frac{1}{8} \frac{4\pi}{3} k_F^3 \frac{1}{(\pi/a)^3} \quad \text{状態数}$$

$$\Omega = a^3 \quad \text{とおくと} \quad \rho = \frac{A}{\Omega} \quad \text{なので} \quad \rho = \frac{2}{3\pi^2} k_F^3$$

実験値は $\rho_0 = 1.72 \times 10^{38} \text{ nucleon/cm}^3 = 0.17 \text{ nucleon fm}^{-3}$

よって $k_F = 1.36 \text{ fm}^{-1}$ $\epsilon_F = \frac{\hbar^2 k_F^2}{2m} = 38 \text{ MeV}$

フェルミ分布を仮定



フェルミオン系として、多粒子系を考える

ある $n = (n_x, n_y, n_z)$ に対して

スピン上向きの陽子 スピン上向きの中性子

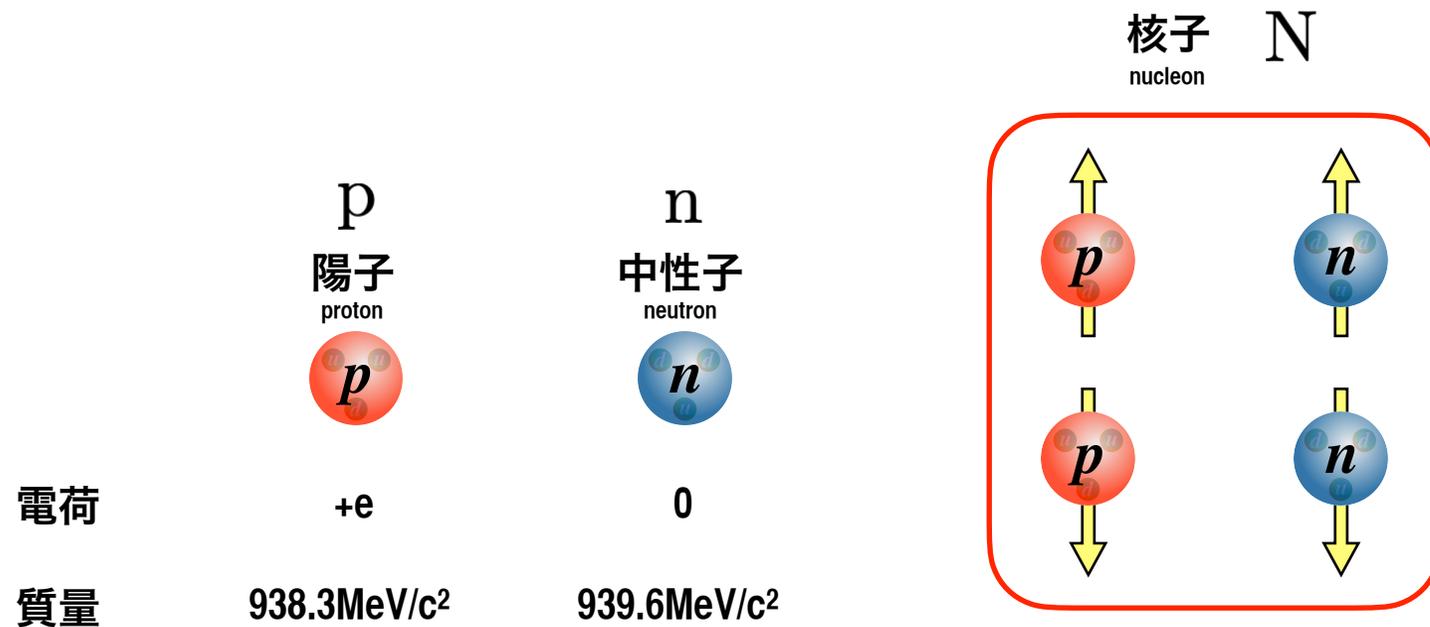
スピン下向きの陽子 スピン下向きの中性子

の自由度が存在し $|k|$ が小さい方から順に占有されると考える

$$\simeq 23 \text{ MeV}$$

$$R = r_0 A^{1/3}$$

$$r_0 = 1.12 \text{ fm} \quad r_F r_0 = 1.52$$



フェルミオン系として、多粒子系を考える

ある $\mathbf{n} = (n_x, n_y, n_z)$ に対して

スピン上向きの陽子 スピン上向きの中性子

スピン下向きの陽子 スピン下向きの中性子

の自由度が存在し $|k|$ が小さい方から順に占有されると考える

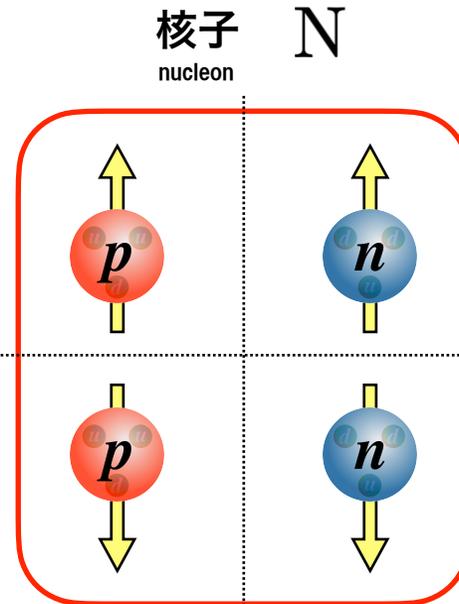
スピンの状態 $s = \frac{1}{2}$ $|s m_s\rangle$

$$|p\rangle = |\psi\rangle \otimes |s m_s\rangle$$

$$|n\rangle = |\psi\rangle \otimes |s m_s\rangle$$

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$



$$\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3) \quad \tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$t = \frac{1}{2} \boldsymbol{\tau} \quad \text{アイソスピン} \quad t = \frac{1}{2} \quad |t m_t\rangle$$

荷電スピン

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle \quad \left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

逆に定義される場合もあるので注意

原子核物理では陽子を $m_t = -1/2$ 、中性子を $m_t = 1/2$ とする

$$|N\rangle = |\psi\rangle \otimes |s m_s\rangle \otimes |t m_t\rangle$$

核子 空間 スピン アイソスピン

$$q = m_t + \frac{1}{2}$$

電荷

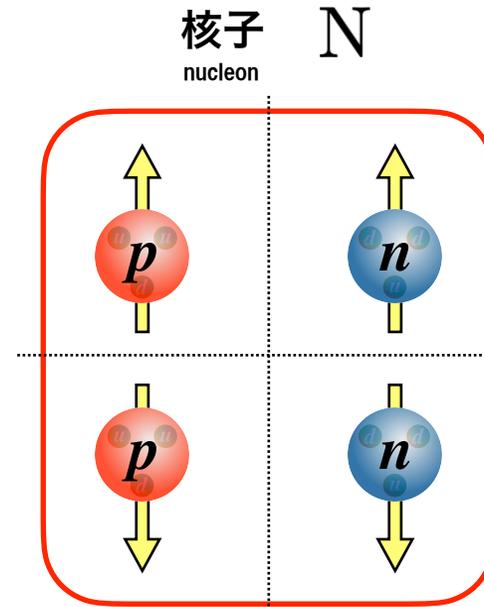
アイソスピンの合成の定義

代数的構造がスピンと同じなので

$t = t_1 + t_2$ とするとき、対応する状態ベクトルはスピンと全く同様に

$$|t m_t\rangle = \sum_{m_{t_1} m_{t_2}} \langle t_1 m_{t_1} t_2 m_{t_2} | t m_t\rangle |t_1 m_{t_1}\rangle |t_2 m_{t_2}\rangle$$

で与えられる



$$t = \frac{1}{2}\tau \quad \text{アイソスピン} \quad t = \frac{1}{2}$$

荷電スピン

$$|t m_t\rangle \quad \left| \frac{1}{2} \frac{1}{2} \right\rangle \quad \left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

逆に定義される場合もあるので注意

原子核物理では陽子を $m_t = -1/2$ 、中性子を $m_t = 1/2$ とする

$$q = m_t + \frac{1}{2}$$

電荷

$$|N\rangle = |\psi\rangle \otimes |s m_s\rangle \otimes |t m_t\rangle$$

核子 空間 スピン アイソスピン

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

p

$$I(J^P) = \frac{1}{2}(1^+)$$

n

$$I(J^P) = \frac{1}{2}(1^+)$$

Isospin **Spin**

We have omitted some data superseded by later experiments. See our **Isospin** **Spin**

The mass is known much more precisely in u (atomic mass units) than in MeV. See the next data block.

Anyone interested in the neutron should look at these two review articles: D. Dubbers and M.G. Schmidt, "The neutron and its role in cosmology and particle physics," *Reviews of Modern Physics* **83**

VALUE (u)	DOCUMENT ID	TECN	COMMENT
1.007276466879 ± 0.000000000091	MOHR	16	RVUE
• • • We do not use the following data for averages, fits, limits, etc.			
1.007276466583 ± 0.000000000032	¹ HEISSE	17	SPEC
1.007276466812 ± 0.000000000015	MOHR	93	RVUE
1.00727646677 ± 0.000000000015	MOHR	93	RVUE
1.00727646688 ± 0.000000000013	MOHR	99	RVUE
1.007276470 ± 0.0000000012	COHEN	87	RVUE

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

83 1173 (2011).

π[±]

$$I^G(J^P) = 1^-(0^-)$$

アイソスピン三重項

Isospin **Spin**

π⁰

$$I^G(J^{PC}) = 1^-(0^{-+})$$

Isospin **Spin**

¹ The statistical and systematic errors are 15 and 29 in the last two places of the value. The value disagrees with the MOHR 16 value by over 3 standard deviations.

We have omitted some data superseded by later experiments. See our **Isospin** **Spin**

強い相互作用はアイソスピン空間における回転について対称

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
938.2720813 ± 0.0000058	MOHR	16	RVUE
• • • We do not use the following data for averages, fits, limits, etc.			
938.272046 ± 0.000021	MOHR	12	RVUE
938.272013 ± 0.000023	MOHR	08	RVUE
938.272029 ± 0.000080	MOHR	05	RVUE
938.271998 ± 0.000038	MOHR	99	RVUE

VALUE (MeV)	DOCUMENT ID	TECN	CHG	COMMENT
134.9770 ± 0.0005 OUR FIT	AGAN 96			Earlier mass determinations with the relatively poorly known $m_{\pi^{\pm}}$ have included a scale factor of 1.1.
Measurements with an error of > 0.005 MeV have been omitted from this Listing.				
939.5654133 ± 0.0000058	MOHR	16		2014 CODATA value
Measurements with an error > 0.01 MeV have been omitted.				
139.57061 ± 0.00024 OUR FIT				Error includes scale factor of 1.6.
139.57061 ± 0.00023 OUR AVERAGE				Error includes scale factor of 1.5. See the ideogram below.
4.5936 ± 0.0005 OUR FIT				
4.5936 ± 0.0005 OUR AVERAGE				
139.57077 ± 0.00010	TRASSINELLI 16	CNTR		X-ray transitions in pionic $N^2 p \rightarrow \pi^0 n, n$ TOF
4.59364 ± 0.00048	CRAWFORD 91	CNTR		
139.57071 ± 0.00053	LENZ 08	CNTR		pionic N^2 atoms, gas target
4.5930 ± 0.0019	CRAWFORD 86	CNTR		pionic N^2 atoms, gas target
139.56995 ± 0.00035	JECKELMANN 94	CNTR		π^+ atom, Soln. B
4.59366 ± 0.00048	CRAWFORD 88b	CNTR		See CRAWFORD 91

原子核を核子の集まりと考えるとき

$$|N\rangle = |\psi\rangle \otimes |s m_s\rangle \otimes |t m_t\rangle$$

核子 空間 スピン アイソスピン

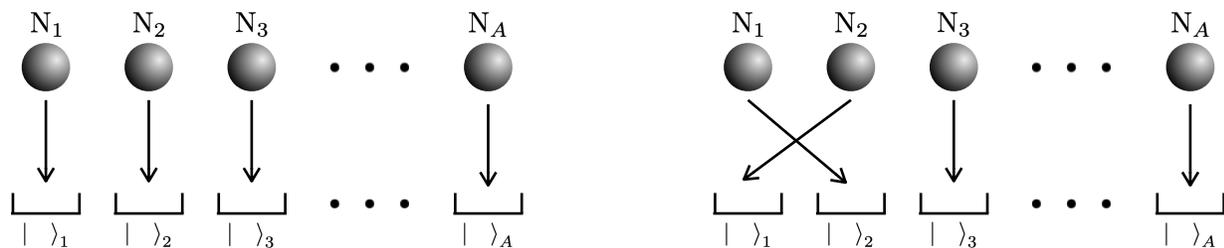
A個の核子波動関数 $\Psi(N_1, \dots, N_i, \dots, N_j, \dots, N_A)$

Boson $\Psi(N_1, \dots, N_j, \dots, N_i, \dots, N_A) = \Psi(N_1, \dots, N_i, \dots, N_j, \dots, N_A)$

Fermion $\Psi(N_1, \dots, N_j, \dots, N_i, \dots, N_A) = -\Psi(N_1, \dots, N_i, \dots, N_j, \dots, N_A)$

$N_i = N_j$ の場合 $\Psi(N_1, \dots, N_i, \dots, N_i, \dots, N_A) = -\Psi(N_1, \dots, N_i, \dots, N_i, \dots, N_A)$

$\rightarrow \Psi(N_1, \dots, N_i, \dots, N_i, \dots, N_A) = 0$ **Pauliの排他律**



$$|N_1\rangle_1 \quad |N_2\rangle_2 \quad |N_3\rangle_3 \quad \dots \quad |N_A\rangle_A \quad - \quad |N_2\rangle_1 \quad |N_1\rangle_2 \quad |N_3\rangle_3 \quad \dots \quad |N_A\rangle_A$$

Slater行列式

$$\Psi(N_1, N_2, N_3, \dots, N_A) = \frac{1}{\sqrt{A!}} \sum_{\sigma} (-1)^{\sigma(i_1 i_2 i_3 \dots i_A)} |N_{i_1}\rangle_1 |N_{i_2}\rangle_2 |N_{i_3}\rangle_3 \dots |N_{i_A}\rangle_A = \frac{1}{A!} \begin{vmatrix} |N_1\rangle_1 & |N_1\rangle_2 & |N_1\rangle_3 & \dots & |N_1\rangle_A \\ |N_2\rangle_1 & |N_2\rangle_2 & |N_2\rangle_3 & \dots & |N_2\rangle_A \\ |N_3\rangle_1 & |N_3\rangle_2 & |N_3\rangle_3 & \dots & |N_3\rangle_A \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ |N_A\rangle_1 & |N_A\rangle_2 & |N_A\rangle_3 & \dots & |N_A\rangle_A \end{vmatrix}$$

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 \quad s_1 = \frac{1}{2} \quad s_2 = \frac{1}{2} \quad \text{の場合}$$

$$|s_1 m_1 s_2 m_2\rangle = |s_1 m_1\rangle \otimes |s_2 m_2\rangle$$

$$\left| \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \right\rangle = \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle = |\uparrow\uparrow\rangle \quad \left| \frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2} \right\rangle = \left| \frac{1}{2} \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle = |\uparrow\downarrow\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \frac{1}{2} \frac{1}{2} \right\rangle = \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \frac{1}{2} \right\rangle = |\downarrow\uparrow\rangle \quad \left| \frac{1}{2} -\frac{1}{2} \frac{1}{2} -\frac{1}{2} \right\rangle = \left| \frac{1}{2} -\frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} -\frac{1}{2} \right\rangle = |\downarrow\downarrow\rangle$$

$$|sm\rangle = \sum_{m_1 m_2} \langle s_1 m_1 s_2 m_2 | s m \rangle |s_1 m_1 s_2 m_2\rangle$$

$$|11\rangle = |\uparrow\uparrow\rangle \quad s_- |11\rangle = \sqrt{1 \cdot 2 - 1 \cdot 0} |10\rangle = \sqrt{2} |10\rangle$$

$$s_- = s_{1-} + s_{2-} \quad (s_{1-} + s_{2-}) |\uparrow\uparrow\rangle = \sqrt{\frac{1}{2} \frac{3}{2} - \frac{1}{2} \left(-\frac{1}{2}\right)} |\downarrow\uparrow\rangle + \sqrt{\frac{1}{2} \frac{3}{2} - \frac{1}{2} \left(-\frac{1}{2}\right)} |\uparrow\downarrow\rangle = |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$$

$$|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$|11\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \middle| 11 \right\rangle |\uparrow\uparrow\rangle$$

$$|1-1\rangle = |\downarrow\downarrow\rangle$$

$$|00\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2} \middle| 00 \right\rangle |\uparrow\downarrow\rangle + \left\langle \frac{1}{2} -\frac{1}{2} \frac{1}{2} \frac{1}{2} \middle| 00 \right\rangle |\downarrow\uparrow\rangle \quad |10\rangle = \left\langle \frac{1}{2} \frac{1}{2} \frac{1}{2} -\frac{1}{2} \middle| 10 \right\rangle |\uparrow\downarrow\rangle + \left\langle \frac{1}{2} -\frac{1}{2} \frac{1}{2} \frac{1}{2} \middle| 10 \right\rangle |\downarrow\uparrow\rangle$$

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|1-1\rangle = \left\langle \frac{1}{2} -\frac{1}{2} \frac{1}{2} -\frac{1}{2} \middle| 1-1 \right\rangle |\downarrow\downarrow\rangle$$

入れ替えに対して 反対称

1/2 x 1/2		1	0	対称
		+1	0	
1/2	+1/2	1/2	1/2	対称
	-1/2	1/2	-1/2	
1/2	+1/2	1/2	-1/2	対称
	-1/2	-1/2	-1/2	

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$|1-1\rangle = |\downarrow\downarrow\rangle$$

<http://pdg.lbl.gov/2019/reviews/rpp2019-rev-clebsch-gordan-coefs.pdf>

同じ空間状態に核子二つを入れたとすると

陽子二つ

反対称

$$|p_1\rangle = |\psi\rangle \begin{pmatrix} \frac{1}{2} m_{s_{p1}} \\ \frac{1}{2} m_{s_{p2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} \frac{1}{2} \end{pmatrix}$$

$$|p_2\rangle = |\psi\rangle \begin{pmatrix} \frac{1}{2} m_{s_{p1}} \\ \frac{1}{2} m_{s_{p2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} \frac{1}{2} \end{pmatrix}$$

↓ ↓

$S = 0, T = 1$ $|00\rangle$ $|11\rangle$

反対称 対称

陽子と中性子

反対称

$$|p\rangle = |\psi\rangle \begin{pmatrix} \frac{1}{2} m_{s_p} \\ \frac{1}{2} m_{s_n} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} \frac{1}{2} \end{pmatrix}$$

$$|n\rangle = |\psi\rangle \begin{pmatrix} \frac{1}{2} m_{s_p} \\ \frac{1}{2} m_{s_n} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} -\frac{1}{2} \end{pmatrix}$$

↓ ↓ ↓ ↓

$S = 1, T = 0$ $|10\rangle$ $|00\rangle$

対称 反対称

$S = 0, T = 1$ $|00\rangle$ $|10\rangle$

反対称 対称

中性子二つ

反対称

$$|n_1\rangle = |\psi\rangle \begin{pmatrix} \frac{1}{2} m_{s_{n1}} \\ \frac{1}{2} m_{s_{n2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2} -\frac{1}{2} \\ \frac{1}{2} -\frac{1}{2} \end{pmatrix}$$

$$|n_2\rangle = |\psi\rangle \begin{pmatrix} \frac{1}{2} m_{s_{n1}} \\ \frac{1}{2} m_{s_{n2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2} -\frac{1}{2} \\ \frac{1}{2} -\frac{1}{2} \end{pmatrix}$$

↓ ↓

$S = 0, T = 1$ $|00\rangle$ $|1-1\rangle$

反対称 対称

入れ替えに対して 反対称

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

対称

$$|11\rangle = |\uparrow\uparrow\rangle$$

対称

$$|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

対称

$$|1-1\rangle = |\downarrow\downarrow\rangle$$

同じ空間状態に核子二つを入れたとすると

陽子二つ

反対称

$$|p_1\rangle = |\psi\rangle \left[\begin{array}{c} \frac{1}{2} m_{s_{p1}} \\ \frac{1}{2} m_{s_{p2}} \end{array} \right] \left[\begin{array}{c} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} \frac{1}{2} \end{array} \right]$$

$$|p_2\rangle = |\psi\rangle \left[\begin{array}{c} \frac{1}{2} m_{s_{p1}} \\ \frac{1}{2} m_{s_{p2}} \end{array} \right] \left[\begin{array}{c} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} \frac{1}{2} \end{array} \right]$$

$S=0$ $T=1$
 反対称 対称

陽子と中性子

反対称

$$|p\rangle = |\psi\rangle \left[\begin{array}{c} \frac{1}{2} m_{s_p} \\ \frac{1}{2} m_{s_n} \end{array} \right] \left[\begin{array}{c} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} \frac{1}{2} \end{array} \right]$$

$$|n\rangle = |\psi\rangle \left[\begin{array}{c} \frac{1}{2} m_{s_p} \\ \frac{1}{2} m_{s_n} \end{array} \right] \left[\begin{array}{c} \frac{1}{2} \frac{1}{2} \\ \frac{1}{2} -\frac{1}{2} \end{array} \right]$$

$S=1$ $T=0$
 対称 反対称

$S=0$ $T=1$
 反対称 対称

中性子二つ

反対称

$$|n_1\rangle = |\psi\rangle \left[\begin{array}{c} \frac{1}{2} m_{s_{n1}} \\ \frac{1}{2} m_{s_{n2}} \end{array} \right] \left[\begin{array}{c} \frac{1}{2} -\frac{1}{2} \\ \frac{1}{2} -\frac{1}{2} \end{array} \right]$$

$$|n_2\rangle = |\psi\rangle \left[\begin{array}{c} \frac{1}{2} m_{s_{n1}} \\ \frac{1}{2} m_{s_{n2}} \end{array} \right] \left[\begin{array}{c} \frac{1}{2} -\frac{1}{2} \\ \frac{1}{2} -\frac{1}{2} \end{array} \right]$$

$S=0$ $T=1$
 反対称 対称

入れ替えに対して 反対称

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

対称

$$|11\rangle = |\uparrow\uparrow\rangle$$

対称

$$|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

対称

$$|1-1\rangle = |\downarrow\downarrow\rangle$$

核子二つで原子核を作る場合は

陽子二つ 反対称

$$\begin{aligned}
 |p_1\rangle &= |\psi_{p_1}\rangle \left| \frac{1}{2} m_{s_{p_1}} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle \\
 |p_2\rangle &= |\psi_{p_2}\rangle \left| \frac{1}{2} m_{s_{p_2}} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle
 \end{aligned}$$

$l=0$ $S=0$ $T=1$
 $l=1$ $S=1$ 対称

中性子二つ 反対称

$$\begin{aligned}
 |n_1\rangle &= |\psi_{n_1}\rangle \left| \frac{1}{2} m_{s_{n_1}} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle \\
 |n_2\rangle &= |\psi_{n_2}\rangle \left| \frac{1}{2} m_{s_{n_2}} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle
 \end{aligned}$$

$l=0$ $S=0$ $T=1$
 $l=1$ $S=1$ 対称

陽子と中性子 反対称

$$\begin{aligned}
 |p\rangle &= |\psi_p\rangle \left| \frac{1}{2} m_{s_p} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle \\
 |n\rangle &= |\psi_n\rangle \left| \frac{1}{2} m_{s_n} \right\rangle \left| \frac{1}{2} -\frac{1}{2} \right\rangle
 \end{aligned}$$

$l=0$ $S=1$ $T=0$
 $l=1$ $S=0$ 反対称
 $l=0$ $S=0$ $T=1$
 $l=1$ $S=1$ 対称

$$\psi(\mathbf{r}) = \sum_{nlm} a_{nlm} R_{nl}(r) Y_{lm}(\theta, \varphi)$$

$$Y_{lm}(\theta, \varphi) \rightarrow Y_{lm}(\pi - \theta, \pi + \varphi) = (-1)^l Y_{lm}(\theta, \varphi)$$

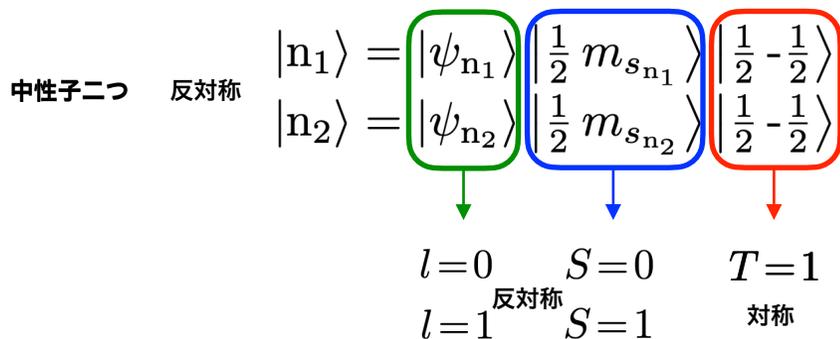
入れ替えに対して 反対称

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

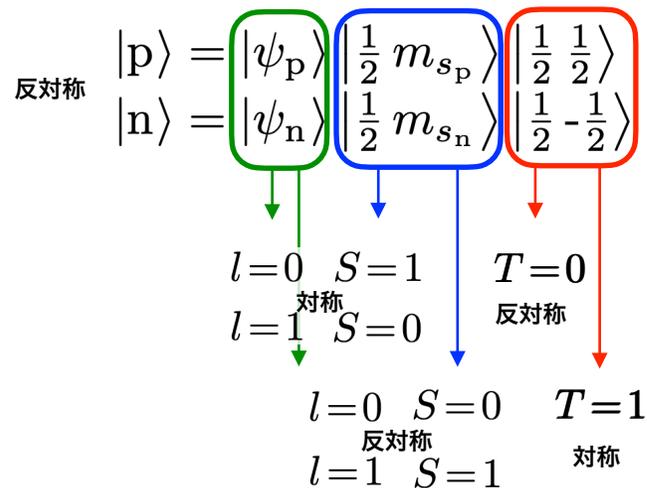
対称 $|11\rangle = |\uparrow\uparrow\rangle$
 対称 $|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$
 対称 $|1-1\rangle = |\downarrow\downarrow\rangle$

核子二つで原子核を作る場合は

陽子二つ	$l=1$	$S=1$	$T=1$
中性子二つ	$l=0$	$S=0$	$T=1$
陽子と中性子	$l=1$	$S=1$	$T=0$
陽子と中性子	$l=1$	$S=0$	$T=1$
陽子と中性子	$l=0$	$S=0$	$T=1$
陽子と中性子	$l=1$	$S=1$	$T=0$



陽子と中性子



$$\psi(\mathbf{r}) = \sum_{nlm} a_{nlm} R_{nl}(r) Y_{lm}(\theta, \varphi)$$

$$Y_{lm}(\theta, \varphi) \rightarrow Y_{lm}(\pi - \theta, \pi + \varphi) = (-1)^l Y_{lm}(\theta, \varphi)$$

入れ替えに対して 反対称

$$|00\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

対称 $|11\rangle = |\uparrow\uparrow\rangle$

対称 $|10\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

対称 $|1-1\rangle = |\downarrow\downarrow\rangle$

陽子二つ	$l=0$	$S=0$	$T=1$
陽子二つ	$l=1$	$S=1$	$T=1$
中性子二つ	$l=0$	$S=0$	$T=1$
中性子二つ	$l=1$	$S=1$	$T=1$
陽子と中性子	$l=0$	$S=1$	$T=0$
陽子と中性子	$l=1$	$S=0$	$T=0$
陽子と中性子	$l=0$	$S=0$	$T=1$
陽子と中性子	$l=1$	$S=1$	$T=1$

$l=0$ を考えれば良い

重陽子 (deuteron)

結合エネルギー = 2.23 MeV

半径 $r_d \equiv \langle r \rangle = 4.2 \text{ fm}$

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

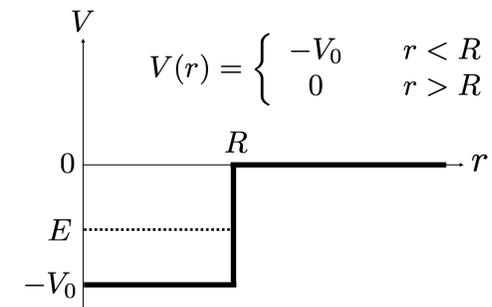
$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

重心運動を分離すると動径方向の方程式は

$$-\frac{\hbar^2}{2\mu} \frac{d^2 u}{dr^2} + \left(V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} \right) u = Eu$$

$\underbrace{\hspace{10em}}_{\text{遠心ポテンシャル}}$
 (有効ポテンシャル) (effective potential)

井戸型ポテンシャル



ただし $\mu \simeq \frac{m}{2}$ 換算質量
 $u(r) = r\psi(r)$

陽子二つ	$l=0$	$S=0$	$T=1$
陽子二つ	$l=1$	$S=1$	$T=1$
中性子二つ	$l=0$	$S=0$	$T=1$
中性子二つ	$l=1$	$S=1$	$T=1$
陽子と中性子	$l=0$	$S=1$	$T=0$
陽子と中性子	$l=1$	$S=0$	$T=0$
陽子と中性子	$l=0$	$S=0$	$T=1$
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重陽子 (deuteron)

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$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

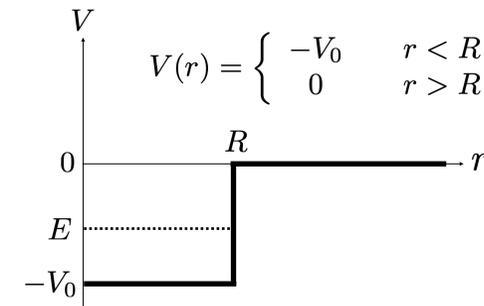
重心運動を分離すると動径方向の方程式は

$$-\frac{\hbar^2}{2\mu} \frac{d^2 u}{dr^2} + \left(V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} \right) u = Eu$$

$\xrightarrow{\text{遠心ポテンシャル}}$
 (有効ポテンシャル) (effective potential)

ただし $\mu \simeq \frac{m}{2}$ 換算質量
 $u(r) = r\psi(r)$

井戸型ポテンシャル



$l=0$ を考えれば良い

$$k = \frac{\sqrt{2\mu(E + V_0)}}{\hbar}$$

$$\kappa = \frac{\sqrt{-2\mu E}}{\hbar}$$

とにおいて $u = \begin{cases} A \sin kr & r < R \\ B e^{-\kappa r} & r > R \end{cases}$

$r = R$ における接続条件から

$$\begin{cases} \cot kR = -\kappa/k \\ B = A e^{\kappa R} \sin kR \end{cases}$$

$$r_d^2 = \langle r^2 \rangle = \int r^2 \psi(r)^2 dr = A^2 \int_0^R r^2 \sin^2 kr dr + B^2 \int_R^\infty r^2 e^{-2\kappa r} dr = (4.2 \text{ fm})^2$$

波動関数の規格条件から $1 = A^2 \int_0^R \sin^2 kr dr + B^2 \int_R^\infty e^{-2\kappa r} dr$

$R = 2.4 \text{ fm}$

$V_0 = 27 \text{ MeV}$ $V_{\text{Coulomb}} = 0.6 \text{ MeV}$

$S_d = 1$
重陽子のスピンは1

$S=1$	結合エネルギー大	束縛
$S=0$	結合エネルギー小	非束縛

pp	nn	は非束縛
陽子同士	中性子同士	

磁気双極子能率

(magnetic dipole moment)

$$\mu_d = 0.8574\mu_N$$

$$\mu_p = 2.7925\mu_N$$

$$\mu_n = -1.9128\mu_N$$

$$\mu_p + \mu_n = 0.8797\mu_N$$

電気四重極能率

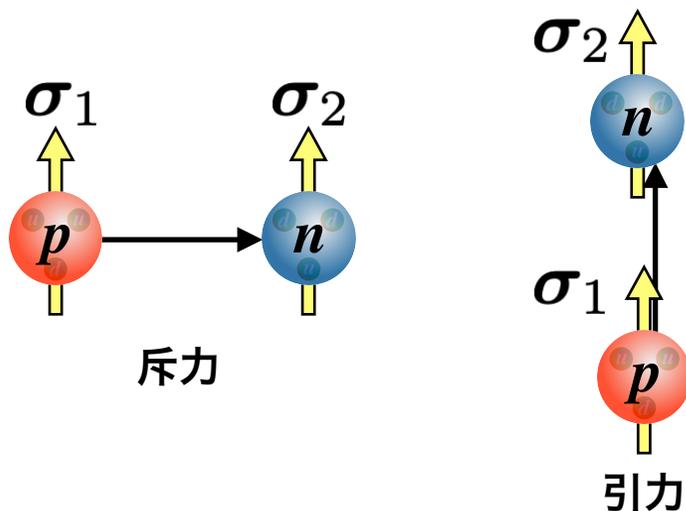
(electric quadrupole moment)

$$Q_d = 0.0028 [e \times 10^{-24} \text{ cm}^2]$$

$l \neq 0$ の成分を持つと考えざるを得ない

$$\psi = a_0\psi_{l=0} + a_2\psi_{l=2} \quad |a_2|^2 = 0.04$$

角運動量を保存しない



磁気双極子能率

(magnetic dipole moment)

$$\mu_d = 0.8574\mu_N$$

$$\mu_p = 2.7925\mu_N$$

$$\mu_n = -1.9128\mu_N$$

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電気四重極能率

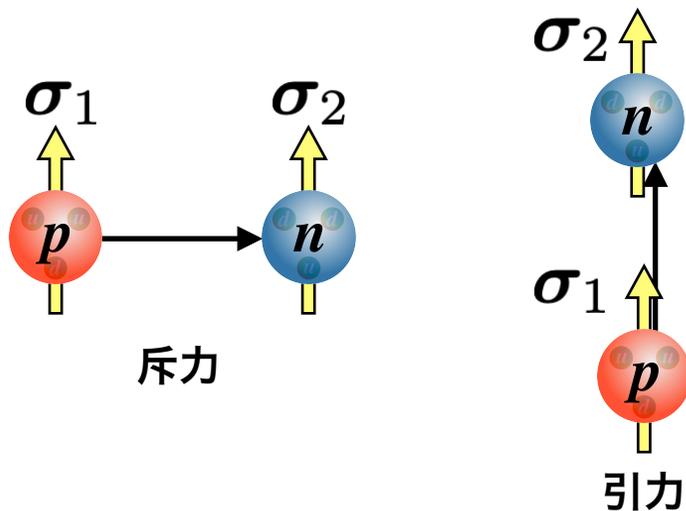
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$l \neq 0$ の成分を持つと考えざるを得ない

$$\psi = a_0\psi_{l=0} + a_2\psi_{l=2} \quad |a_2|^2 = 0.04$$

角運動量を保存しない



$$S_{12} = 3 \frac{(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^2} - (\sigma_1 \cdot \sigma_2)$$

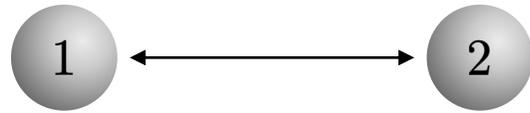
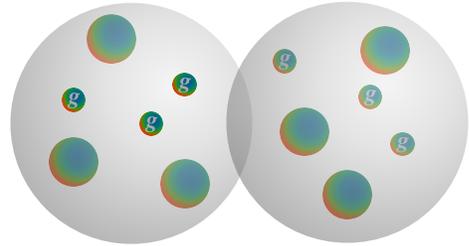
テンソル力

磁気双極子間のポテンシャル

$$U(r) = -\frac{1}{4\pi\mu} \frac{1}{r^3} \left[\frac{3(r \cdot m_1)(r \cdot m_2)}{r^2} - m_1 \cdot m_2 \right]$$

核力

核力 二つの核子の間に働く力



$$H = -\frac{\hbar^2}{2m_1} \left(\frac{\partial}{\partial \mathbf{r}_1} \right)^2 - \frac{\hbar^2}{2m_2} \left(\frac{\partial}{\partial \mathbf{r}_2} \right)^2 + V(|\mathbf{r}_1 - \mathbf{r}_2|)$$

重心運動の分離

$$\mathbf{R} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$M = m_1 + m_2 \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

重心を原点にとって

$$\mathbf{r}_1 = \frac{\mathbf{r}}{2} \quad \mathbf{r}_2 = -\frac{\mathbf{r}}{2}$$

$$\mathbf{p}_1 = \mathbf{p} \quad \mathbf{p}_2 = -\mathbf{p}$$

$$l_1 = \frac{l}{2} \quad l_2 = \frac{l}{2}$$

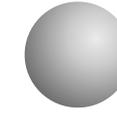
$$H = -\frac{\hbar^2}{2M} \left(\frac{\partial}{\partial \mathbf{R}} \right)^2 - \frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial \mathbf{r}} \right)^2 + V(r)$$

$$\left(-\frac{\hbar^2}{2M} \left(\frac{\partial}{\partial \mathbf{R}} \right)^2 - \frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial \mathbf{r}} \right)^2 + V(r) \right) \Psi(\mathbf{R}, \mathbf{r}) = E \Psi(\mathbf{R}, \mathbf{r})$$

$$\Psi(\mathbf{R}, \mathbf{r}) = \psi_R(\mathbf{R}) \psi_r(\mathbf{r}) \longrightarrow \begin{cases} \left(-\frac{\hbar^2}{2M} \left(\frac{\partial}{\partial \mathbf{R}} \right)^2 \right) \psi_R(\mathbf{R}) = E_R \psi_R(\mathbf{R}) \\ \left(-\frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial \mathbf{r}} \right)^2 + V(r) \right) \psi_r(\mathbf{r}) = E_r \psi_r(\mathbf{r}) \end{cases} \quad E = E_R + E_r$$

$$|N\rangle = |\psi\rangle \otimes |s m_s\rangle \otimes |t m_t\rangle$$

核子 空間 スピン アイソスピン



核子 N
nucleon

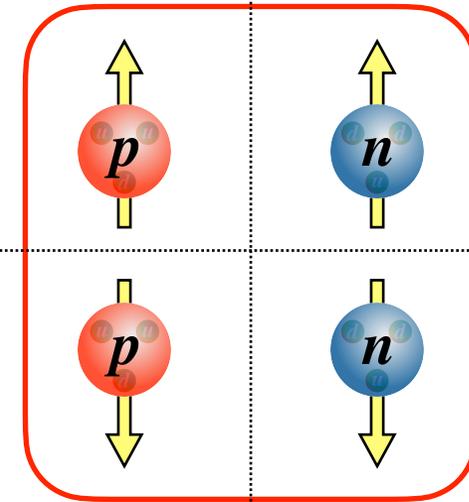
スピン

$$|s m_s\rangle$$

$$s = \frac{1}{2}$$

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$



$$|t m_t\rangle$$

$$t = \frac{1}{2}$$

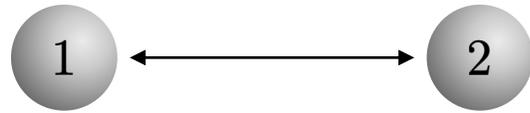
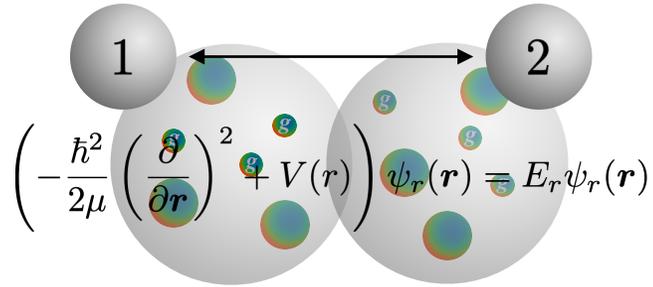
$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

アイソスピン

荷電スピン

核力 二つの核子の間に働く力



$$H = -\frac{\hbar^2}{2m_1} \left(\frac{\partial}{\partial \mathbf{r}_1}\right)^2 - \frac{\hbar^2}{2m_2} \left(\frac{\partial}{\partial \mathbf{r}_2}\right)^2 + V(|\mathbf{r}_1 - \mathbf{r}_2|)$$

重心運動の分離

$$\mathbf{R} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$M = m_1 + m_2 \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

重心を原点にとって

$$\mathbf{r}_1 = \frac{\mathbf{r}}{2} \quad \mathbf{r}_2 = -\frac{\mathbf{r}}{2}$$

$$\mathbf{p}_1 = \mathbf{p} \quad \mathbf{p}_2 = -\mathbf{p}$$

$$l_1 = \frac{l}{2} \quad l_2 = \frac{l}{2}$$

$$H = -\frac{\hbar^2}{2M} \left(\frac{\partial}{\partial \mathbf{R}}\right)^2 - \frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial \mathbf{r}}\right)^2 + V(r)$$

$$\left(-\frac{\hbar^2}{2M} \left(\frac{\partial}{\partial \mathbf{R}}\right)^2 - \frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial \mathbf{r}}\right)^2 + V(r)\right) \Psi(\mathbf{R}, \mathbf{r}) = E \Psi(\mathbf{R}, \mathbf{r})$$

$$\Psi(\mathbf{R}, \mathbf{r}) = \psi_R(\mathbf{R}) \psi_r(\mathbf{r}) \longrightarrow \begin{cases} \left(-\frac{\hbar^2}{2M} \left(\frac{\partial}{\partial \mathbf{R}}\right)^2\right) \psi_R(\mathbf{R}) = E_R \psi_R(\mathbf{R}) \\ \left(-\frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial \mathbf{r}}\right)^2 + V(r)\right) \psi_r(\mathbf{r}) = E_r \psi_r(\mathbf{r}) \end{cases} \quad E = E_R + E_r$$

$$|N\rangle = |\psi\rangle \otimes |s m_s\rangle \otimes |t m_t\rangle$$

核子 空間 スピン アイソスピン

スピン

$$|s m_s\rangle$$

$$s = \frac{1}{2}$$

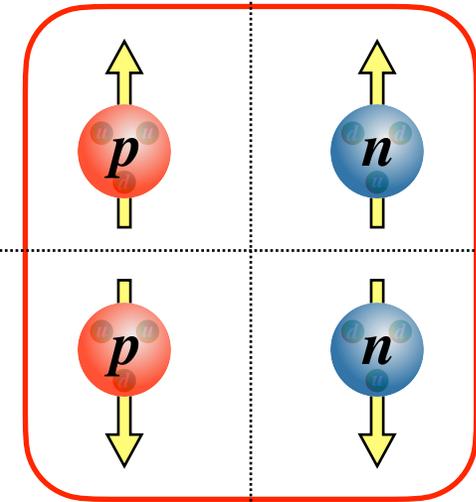
$$\left|\frac{1}{2} \frac{1}{2}\right\rangle$$

$$\left|\frac{1}{2} -\frac{1}{2}\right\rangle$$

$$|t m_t\rangle$$

$$t = \frac{1}{2}$$

核子 N
nucleon

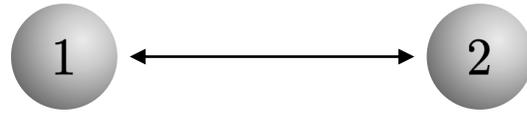


$$\left|\frac{1}{2} \frac{1}{2}\right\rangle \quad \left|\frac{1}{2} -\frac{1}{2}\right\rangle$$

アイソスピン

荷電スピン

核力 二つの核子の間に働く力



$$\left(-\frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial \mathbf{r}} \right)^2 + V(r) \right) \psi_r(\mathbf{r}) = E_r \psi_r(\mathbf{r})$$

変数分離 (動径方向と角度方向)

$$\psi_r(\mathbf{r}) = R_{nl}(r) Y_{lm}(\theta, \varphi)$$

$$\left(-\frac{\hbar^2}{2\mu} \frac{1}{r} \frac{d^2}{dr^2} r + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) \right) R_{nl}(r) = E_{nl} R_{nl}(r)$$

$u_{nl}(r) = r R_{nl}(r)$ とおくのが常套手段

$$\left(-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) \right) u_{nl}(r) = E_{nl} u_{nl}(r)$$

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 \quad \mathbf{t} = \mathbf{t}_1 + \mathbf{t}_2$$

$$\psi_{nljm}^{stm_t}(\mathbf{r}) = R_{nlj}(r) \underbrace{\mathcal{Y}_{jm}^{(ls)}(\theta, \varphi)}_{\langle \theta, \varphi | (ls) jm \rangle} |tm_t\rangle$$

$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

$$\mathcal{Y}_{jm}^{(ls)}(\theta, \varphi) = \sum_{m_l m_s} \langle l m_l s m_s | j m \rangle \underbrace{Y_{lm}(\theta, \varphi)}_{\langle \theta, \varphi | l m_l \rangle} |s m_s\rangle$$

直交関係 $\int \left(\mathcal{Y}_{jm}^{(ls)}(\theta, \varphi) \right)^* \left(\mathcal{Y}_{j'm'}^{(l's')}(\theta, \varphi) \right) d\Omega = \delta_{ll'} \delta_{ss'} \delta_{jj'} \delta_{mm'}$

$$|N\rangle = |\psi\rangle \otimes |s m_s\rangle \otimes |t m_t\rangle$$

核子 空間 スピン アイソスピン

スピン

$$|s m_s\rangle$$

$$s = \frac{1}{2}$$

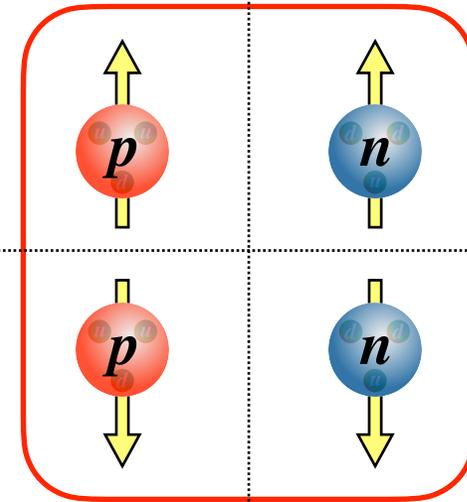
$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

$$|t m_t\rangle$$

$$t = \frac{1}{2}$$

核子 N
nucleon



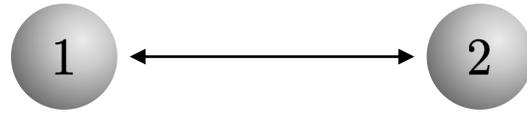
$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

アイソスピン

荷電スピン

核力 二つの核子の間に働く力



$$\psi_{nljm}^{stm_t}(\mathbf{r}) = \frac{\hbar^2}{2\mu} \left(\frac{\partial}{\partial r} R_{nlj}(r) \right) \mathcal{Y}_{jm}^{(ls)}(\theta, \varphi) \psi_t(m_t)$$

変数分離 (動径方向と角度方向)

$$\psi_r(\mathbf{r}) = R_{nl}(r) Y_{lm}(\theta, \varphi)$$

$$\left(-\frac{\hbar^2}{2\mu} \frac{1}{r} \frac{d^2}{dr^2} r + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) \right) R_{nl}(r) = E_{nl} R_{nl}(r)$$

$u_{nl}(r) = r R_{nl}(r)$ とおくのが常套手段

$$\left(-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V(r) \right) u_{nl}(r) = E_{nl} u_{nl}(r)$$

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2 \quad \mathbf{t} = \mathbf{t}_1 + \mathbf{t}_2$$

$$\psi_{nljm}^{stm_t}(\mathbf{r}) = R_{nlj}(r) \underbrace{\mathcal{Y}_{jm}^{(ls)}(\theta, \varphi)}_{\langle \theta, \varphi | (ls) jm \rangle} |tm_t\rangle$$

$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

$$\mathcal{Y}_{jm}^{(ls)}(\theta, \varphi) = \sum_{m_l m_s} \langle l m_l s m_s | j m \rangle \underbrace{Y_{lm}(\theta, \varphi)}_{\langle \theta, \varphi | l m_l \rangle} |s m_s\rangle$$

直交関係 $\int \left(\mathcal{Y}_{jm}^{(ls)}(\theta, \varphi) \right)^* \left(\mathcal{Y}_{j'm'}^{(l's')}(\theta, \varphi) \right) d\Omega = \delta_{ll'} \delta_{ss'} \delta_{jj'} \delta_{mm'}$

$$|N\rangle = |\psi\rangle \otimes |s m_s\rangle \otimes |t m_t\rangle$$

核子 空間 スピン アイソスピン

スピン

$$|s m_s\rangle$$

$$s = \frac{1}{2}$$

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

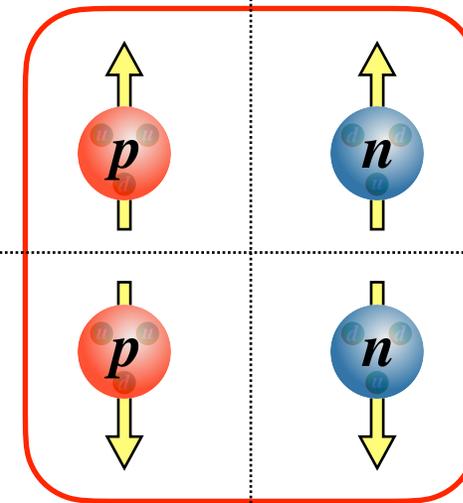
$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

$$|t m_t\rangle$$

$$t = \frac{1}{2}$$



核子 N
nucleon



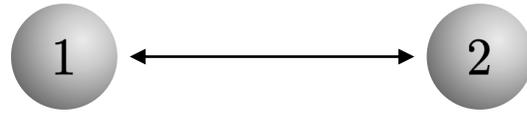
$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

アイソスピン

荷電スピン

核力 二つの核子の間に働く力



$$\psi_{nljm}^{stm_t}(\mathbf{r}) = R_{nlj}(r) \mathcal{Y}_{jm}^{(ls)}(\theta, \varphi) |tm_t\rangle$$

$$P = P_r P_s P_t \quad P_r Y_{lm} = \frac{(-1)^l Y_{lm}}{\pi} \quad \text{空間反転と同じ}$$

$$P_s |sm_s\rangle = (-1)^{s+1} |sm_s\rangle$$

$$P_t |tm_t\rangle = (-1)^{t+1} |tm_t\rangle$$

$$P \psi_{nljm}^{stm_t}(\mathbf{r}) = (-1)^{l+s+1+t+1} \psi_{nljm}^{stm_t}(\mathbf{r}) = -\psi_{nljm}^{stm_t}(\mathbf{r})$$

反対称化

$$l + s + t = \text{odd}$$

$$2s+1 l_j$$

t	s	l	π
0	0(singlet)	1, 3, ...	-1
0	1(triplet)	0, 2, ...	+1
1	0(singlet)	0, 2, ...	+1
1	1(triplet)	1, 3, ...	-1

$${}^1P_1, {}^1F_3, \dots \quad J^\pi = 1^+, T=0$$

$${}^3S_1, {}^3D_1, {}^3D_2, {}^3D_3, \dots$$

$${}^1S_0, {}^1D_2, \dots$$

$${}^3P_0, {}^3P_1, {}^3P_2, {}^3F_2, {}^3F_3, {}^3F_4, \dots$$

$$|N\rangle = |\psi\rangle \otimes |sm_s\rangle \otimes |tm_t\rangle$$

核子 空間 スピン アイソスピン

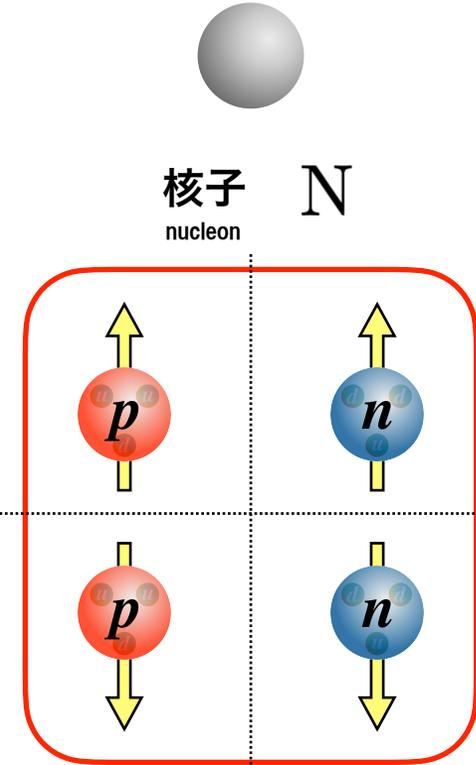
スピン

$$|sm_s\rangle$$

$$s = \frac{1}{2}$$

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$



$$|tm_t\rangle$$

$$\left| \frac{1}{2} \frac{1}{2} \right\rangle$$

$$\left| \frac{1}{2} -\frac{1}{2} \right\rangle$$

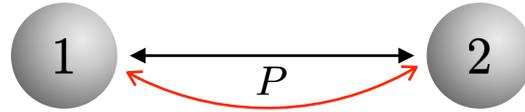
$$t = \frac{1}{2}$$

アイソスピン

荷電スピン

核力

二つの核子の間に働く力



局所的なポテンシャル
非相対論的の点粒子

粒子交換対称
並進対称性

回転対称性
空間反転対称性
時間反転対称性
荷電対称性

粒子交換対称	並進対称性	回転対称性	空間反転	時間反転	
	$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$		奇関数 (P-odd)	偶関数 (T-even)	$\mathbf{r} \cdot \mathbf{p}$ (T-odd)
最低次を考慮→	$\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$		奇関数 (P-odd)	奇関数 (T-odd)	$\mathbf{p} \cdot \mathbf{s}$ (P-odd)
	$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2$		偶関数 (P-even)	奇関数 (T-odd)	$\mathbf{s} \cdot \mathbf{r}$ (P-odd) (T-odd)
	のスカラール演算子の関数				

スピンの演算子は一次のみ ← $s_1^2 = \frac{3}{4}$ $s_2^2 = \frac{3}{4}$

$$S_{12} = 3 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \quad \mathbf{l} = \mathbf{r} \times \mathbf{p} = \frac{\hbar}{2i} (\mathbf{r}_1 - \mathbf{r}_2) \times \left(\frac{\partial}{\partial \mathbf{r}_1} - \frac{\partial}{\partial \mathbf{r}_2} \right)$$

$$V_{NN} = \boxed{V_c(r)} + \boxed{V_t(r) S_{12}} + \boxed{V_{LS}(r) \mathbf{l} \cdot \mathbf{s}} + \mathcal{O}(p^2)$$

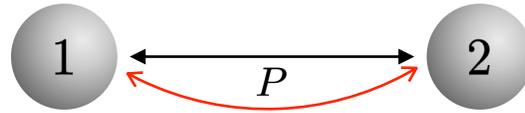
中心力
central force

テンソル力
tensor force

スピン軌道力
spin-orbit force

核力

二つの核子の間に働く力



局所的なポテンシャル
非相対論的の点粒子

粒子交換対称
並進対称性

回転対称性
空間反転対称性
時間反転対称性
荷電対称性

$$V_{NN} = V_C(r) + V_t(r)S_{12} + V_{LS}(r)l \cdot s + \mathcal{O}(p^2)$$

粒子交換対称 | 並進対称性 | 回転対称性 | 空間反転 | 時間反転

最低次を考慮 →

$r = r_1 - r_2$	奇関数 (P-odd)	偶関数 (T-even)	$r \cdot p$ (T-odd)
$p = \frac{1}{2}(p_1 - p_2)$	奇関数 (P-odd)	奇関数 (T-odd)	$p \cdot s$ (P-odd)
$s = s_1 + s_2$	偶関数 (P-even)	奇関数 (T-odd)	$s \cdot r$ (P-odd) (T-odd)

のスカラ-演算子の関数

スピン演算子は一次のみ ← $s_1^2 = \frac{3}{4}$ $s_2^2 = \frac{3}{4}$

$$S_{12} = 3 \frac{(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^2} - (\sigma_1 \cdot \sigma_2) \quad l = r \times p = \frac{\hbar}{2i} (r_1 - r_2) \times \left(\frac{\partial}{\partial r_1} - \frac{\partial}{\partial r_2} \right)$$

$$V_{NN} = V_C(r) + V_t(r)S_{12} + V_{LS}(r)l \cdot s + \mathcal{O}(p^2)$$

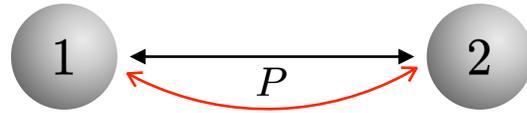
中心力
central force

テンソル力
tensor force

スピン軌道力
spin-orbit force

核力

二つの核子の間に働く力



局所的なポテンシャル
非相対論的の点粒子

粒子交換対称
並進対称性

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$$V_{NN} = V_c(r) + V_t(r)S_{12} + V_{LS}(r)l \cdot s + \mathcal{O}(p^2)$$

中心力

中心力
central force

テンソル力
tensor force

スピン軌道力
spin-orbit force

空間反転対称性

→ スカラー

$$\mathbf{s}_1 \cdot \mathbf{s}_2 = \frac{1}{2}(\mathbf{s}^2 - \mathbf{s}_1^2 - \mathbf{s}_2^2)$$

荷電対称性

→ アイソスカラー

$$\mathbf{t}_1 \cdot \mathbf{t}_2 = \frac{1}{2}(\mathbf{t}^2 - \mathbf{t}_1^2 - \mathbf{t}_2^2)$$

のみ

$$\mathbf{s}_1 = \frac{1}{2}\boldsymbol{\sigma}_1, \mathbf{s}_2 = \frac{1}{2}\boldsymbol{\sigma}_2 \quad \mathbf{t}_1 = \frac{1}{2}\boldsymbol{\tau}_1, \mathbf{t}_2 = \frac{1}{2}\boldsymbol{\tau}_2$$

$$V_c(r) = V_0(r) + V_\sigma(r)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + V_\tau(r)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + V_{\sigma\tau}(r)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

交換演算子 $P_s = \frac{1}{2}(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$ $P_t = \frac{1}{2}(1 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$ $P_s P_r P_t = -1$

$$V_c(r) = V_W(r) + V_B(r)P_s + V_M(r)P_r + V_H(r)P_s P_r$$

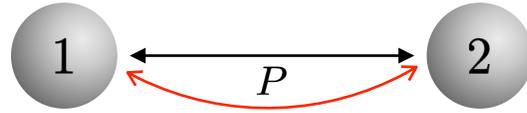
Wigner Bartlet Majorana Heisenberg

射影演算子 $P_{st} = \frac{1}{2}(1 - (-1)^s P_s) \frac{1}{2}(1 - (-1)^t P_t)$

$$V_c(r) = V_{SE}(r)P_{01} + V_{SO}(r)P_{00} + V_{TE}(r)P_{10} + V_{TO}(r)P_{11}$$

核力

二つの核子の間に働く力



局所的なポテンシャル
非相対論的の点粒子

粒子交換対称
並進対称性

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荷電対称性

$$V_{NN} = V_c(r) + V_t(r)S_{12} + V_{LS}(r)l \cdot s + \mathcal{O}(p^2)$$

中心力
central force

テンソル力
tensor force

スピン軌道力
spin-orbit force

テンソル力

$$S_{12} = 3 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = 2 \left(3 \left(\frac{\mathbf{s} \cdot \mathbf{r}}{r} \right)^2 - s^2 \right)$$

$s = 0$ に対しては働かない

$$V_t = V_{t0}(r) + V_{t\tau}(r)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

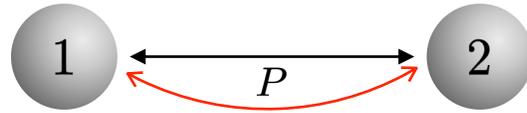
射影演算子 $\mathcal{P}_t = \frac{1}{2}(1 - (-1)^t)P_t$

$$V_t(r) = V_{te}(r)\mathcal{P}_0 + V_{to}(r)\mathcal{P}_1$$

tensor even tensor odd

核力

二つの核子の間に働く力



局所的なポテンシャル
非相対論的の点粒子

粒子交換対称
並進対称性

回転対称性
空間反転対称性
時間反転対称性
荷電対称性

$$V_{NN} = V_c(r) + V_t(r)S_{12} + V_{LS}(r)l \cdot s + \mathcal{O}(p^2)$$

中心力
central force

テンソル力
tensor force

スピン軌道力
spin-orbit force

テンソル力

$$\psi_d = \psi_{dS} + \psi_{dD}$$

$$\psi_{dS} = \frac{u_S(r)}{r} \mathcal{Y}_{1m}^{01}(\theta, \varphi) \quad \int (u_S^2(r) + u_D^2(r)) dr = 1$$

$$\psi_{dD} = \frac{u_D(r)}{r} \mathcal{Y}_{1m}^{21}(\theta, \varphi) \quad p_d = \int u_D^2(r) dr$$

$$\boldsymbol{\mu}_d = \left[g_p \mathbf{s}_p + g_n \mathbf{s}_n + \frac{1}{2} \mathbf{l} \right] \mu_N = \left[\frac{1}{2} (g_p + g_n) \mathbf{j} + \frac{1}{2} (1 - g_p - g_n) \mathbf{l} + \frac{1}{2} (g_p - g_n) (\mathbf{s}_p - \mathbf{s}_n) \right] \mu_N$$

$$(\boldsymbol{\mu}_d)_z = \frac{j_z}{j^2} \boldsymbol{\mu}_d \cdot \mathbf{j} \mu_N \quad \langle j^2 \rangle_{11}^{01} = 2u_S^2, \langle s^2 \rangle_{11}^{01} = 2u_S^2, \langle j^2 \rangle_{11}^{21} = 2u_D^2, \langle s^2 \rangle_{11}^{21} = 2u_D^2, \langle l^2 \rangle_{11}^{01} = 0, \langle l^2 \rangle_{11}^{21} = 6u_D^2$$

$$= \frac{j_z}{j^2} \left[\frac{1}{2} (g_p + g_n) j^2 + \frac{1}{2} (1 - g_p - g_n) \mathbf{l} \cdot \mathbf{j} \right] = \frac{j_z}{j^2} \left[\frac{1}{4} (j^2 - s^2) + \frac{1}{4} (g_p + g_n) (j^2 + s^2) + \frac{1}{4} (1 - g_p - g_n) l^2 \right]$$

$$(\boldsymbol{\mu}_d)_z = \left[\frac{1}{2} (g_p + g_n) + \frac{3}{4} (1 - g_p - g_n) p_d \right] \mu_N$$

$$\mu_d = 0.8574 \mu_N$$

$$\mu_p + \mu_n = 0.8797 \mu_N$$

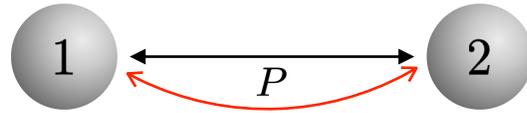
$$\mu_p = 2.7925 \mu_N$$

$$\mu_n = -1.9128 \mu_N$$

$$p_d = 0.04$$

核力

二つの核子の間に働く力



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$$V_{NN} = V_c(r) + V_t(r)S_{12} + V_{LS}(r)l \cdot s + \mathcal{O}(p^2)$$

中心力
central force

テンソル力
tensor force

スピン軌道力
spin-orbit force

テンソル力

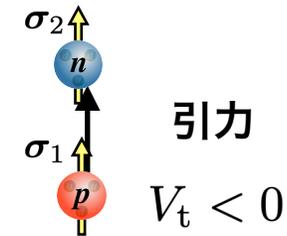
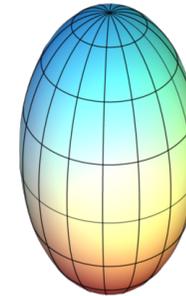
$$S_{12} = 3 \frac{(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^2} - (\sigma_1 \cdot \sigma_2) = 2 \left(3 \left(\frac{s \cdot r}{r} \right)^2 - s^2 \right)$$

$$Q_d \propto 3z^2 - r^2$$

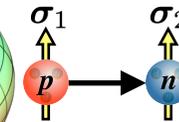
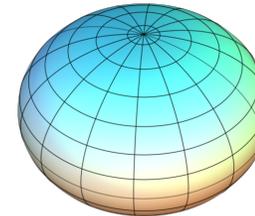
$$\langle Q_d \rangle = 2 \langle S | Q_d | D \rangle + \langle D | Q_d | D \rangle = 0.286 \text{ fm}^2$$

$$R_d^2 \simeq 10 \text{ fm}^2$$

prolate
 $Q > 0$

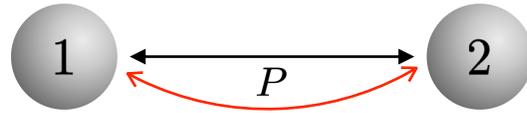


oblate
 $Q < 0$



核力

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tensor force

スピン軌道力
spin-orbit force

スピン軌道力 $s = 0$ に対しては働かない

$$V_{ls}(r) = V_{ls0}(r) + V_{ls\tau}(r)(\tau_1 \cdot \tau_2)$$

射影演算子 $\mathcal{P}_t = \frac{1}{2}(1 - (-1)^t)P_t$

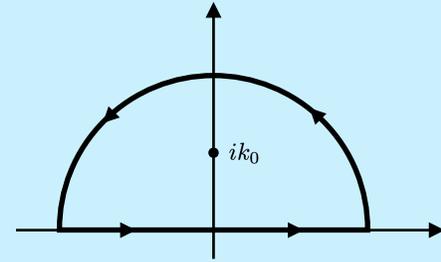
$$V_{ls}(r) = \underset{\text{LS even}}{V_{lse}(r)}\mathcal{P}_0 + \underset{\text{LS odd}}{V_{lso}(r)}\mathcal{P}_1$$

$$\left(\nabla^2 - \frac{m^2 c^2}{\hbar^2}\right) V(\mathbf{r}) = -g\delta(\mathbf{r})$$

$$V(\mathbf{r}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{k}\cdot\mathbf{r}} V(\mathbf{k}) d\mathbf{k}$$

$$\delta(\mathbf{r}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{k}$$

$$(\mathbf{k}^2 + k_0^2) V(\mathbf{k}) = g \quad k_0 = \frac{mc}{\hbar} \quad \lambda = \frac{1}{k_0} = \frac{\hbar}{mc} \text{ はコンプトン波長}$$



$$\begin{aligned} V(\mathbf{r}) &= \frac{1}{(2\pi)^3} \int e^{i\mathbf{k}\cdot\mathbf{r}} \frac{g}{k^2 + k_0^2} d\mathbf{k} = \frac{2\pi g}{(2\pi)^3} \int \frac{e^{ikr \cos \theta}}{k^2 + k_0^2} k^2 \sin \theta dk d\theta = \frac{2\pi g}{(2\pi)^3} \int_0^\infty \frac{e^{ikr} - e^{-ikr}}{k^2 + k_0^2} k^2 \frac{1}{ikr} dk = \frac{2\pi g}{(2\pi)^3} \frac{1}{ir} \int_{-\infty}^\infty \frac{e^{ikr}}{k^2 + k_0^2} k dk \\ &= -\frac{g}{(2\pi)^2 r} \frac{d}{dr} \int_{-\infty}^\infty \frac{e^{ikr}}{k^2 + k_0^2} dk = -\frac{g}{(2\pi)^2 r} \frac{d}{dr} \int_{-\infty}^\infty \frac{e^{ikr}}{(k + ik_0)(k - ik_0)} dk = -\frac{g}{(2\pi)^2 r} \frac{d}{dr} \times 2\pi i \frac{e^{-k_0 r}}{2ik_0} = \frac{g}{4\pi} \frac{e^{-k_0 r}}{r} = \frac{g}{4\pi} \frac{e^{-r/\lambda}}{r} \end{aligned}$$

$$\nabla^2 V(\mathbf{r}) = -e\delta(\mathbf{r})$$

$$\left(\nabla^2 - \frac{m^2 c^2}{\hbar^2}\right) V(\mathbf{r}) = -g\delta(\mathbf{r})$$

$$(\mathbf{pc})^2 V(\mathbf{r}) = (\hbar c)^2 e\delta(\mathbf{r})$$

$$((\mathbf{pc})^2 + (mc^2)^2) V(\mathbf{r}) = (\hbar c)^2 g\delta(\mathbf{r})$$

$$V(\mathbf{r}) = \frac{g}{4\pi} \frac{e^{-r/\lambda}}{r}$$

$$\lambda = \frac{\hbar}{mc} \simeq 1.4 \text{ fm}$$

$$\frac{g^2}{\hbar c} \sim 0.28$$

$$\frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$V_{\text{NN}} = V_c(r) + V_t(r)S_{12} + V_{\text{LS}}(r)\mathbf{l} \cdot \mathbf{s} + \mathcal{O}(p^2)$$

中心力
central force

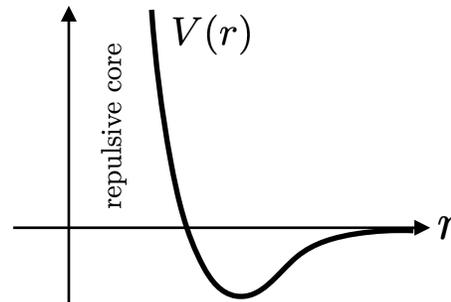
テンソル力
tensor force

スピン軌道力
spin-orbit force

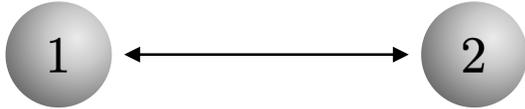
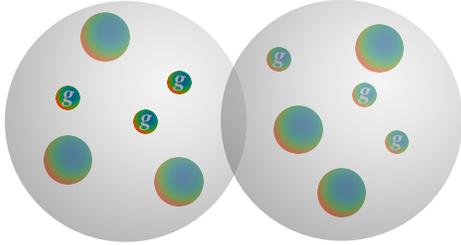
$$S_{12} = 3 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

$$\begin{aligned} V_{\text{NN}} &= V_0 + V_\sigma(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + V_\tau(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + V_{\sigma\tau}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ &+ V_{t0}S_{12} + V_{t\tau}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)S_{12} \\ &+ V_{\text{LS}0}(\mathbf{l} \cdot \mathbf{s}) + V_{\text{LS}\tau}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\mathbf{l} \cdot \mathbf{s}) \\ &+ \dots \end{aligned}$$

実際には中心部に反発力があるはず



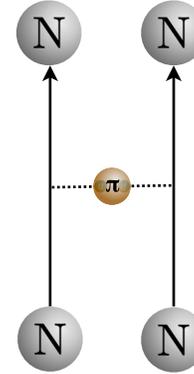
核力 二つの核子の間に働く力



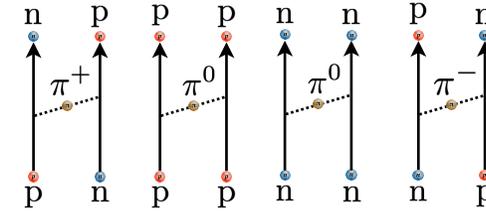
$$V_{NN} = \underbrace{V_c(r)}_{\text{中心力}} + \underbrace{V_t(r)S_{12}}_{\text{テンソル力}} + \underbrace{V_{LS}(r)\mathbf{l} \cdot \mathbf{s}}_{\text{スピン軌道力}} + \mathcal{O}(p^2)$$

$$S_{12} = 3 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

$$V_{NN} = V_0 + V_\sigma(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + V_\tau(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + V_{\sigma\tau}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ + V_{t0}S_{12} + V_{t\tau}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)S_{12} \\ + V_{LS0}(\mathbf{l} \cdot \mathbf{s}) + V_{LS\tau}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\mathbf{l} \cdot \mathbf{s}) \\ + \dots$$



$$m_p = (1.007\,276\,466\,879 \pm 0.000\,000\,000\,091) \text{ u} \\ = (938.272\,0813 \pm 0.000\,0058) \text{ MeV}/c^2 \\ m_n = (939.565\,4133 \pm 0.000\,0058) \text{ MeV}/c^2 \\ m_{\pi^\pm} = (139.57061 \pm 0.00024) \text{ MeV}/c^2 \\ m_{\pi^0} = (134.9770 \pm 0.0005) \text{ MeV}/c^2$$

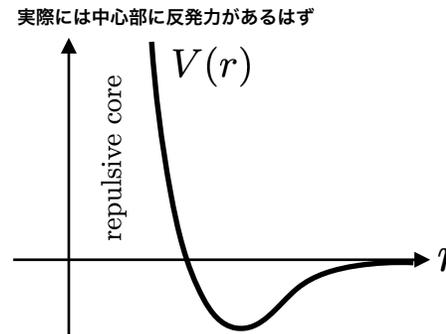


$$\left(\nabla^2 - \frac{m^2 c^2}{\hbar^2}\right) V(\mathbf{r}) = -g\delta(\mathbf{r}) \quad \begin{cases} \nabla^2 V(\mathbf{r}) = -e\delta(\mathbf{r}) \\ p = \frac{\hbar}{i} \nabla \end{cases} \begin{cases} (pc)^2 V(\mathbf{r}) = (\hbar c)^2 e\delta(\mathbf{r}) \\ ((pc)^2 + (mc^2)^2) V(\mathbf{r}) = (\hbar c)^2 g\delta(\mathbf{r}) \end{cases}$$

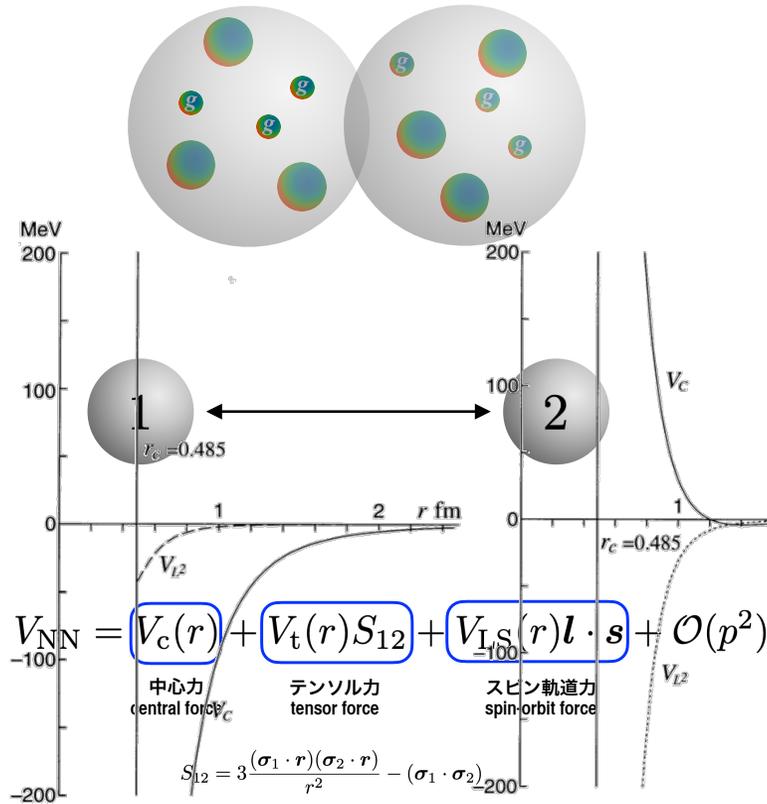
$$V(\mathbf{r}) = \frac{g}{4\pi} \frac{e^{-r/\lambda}}{r}$$

$$\lambda = \frac{\hbar}{mc} \simeq 1.4 \text{ fm}$$

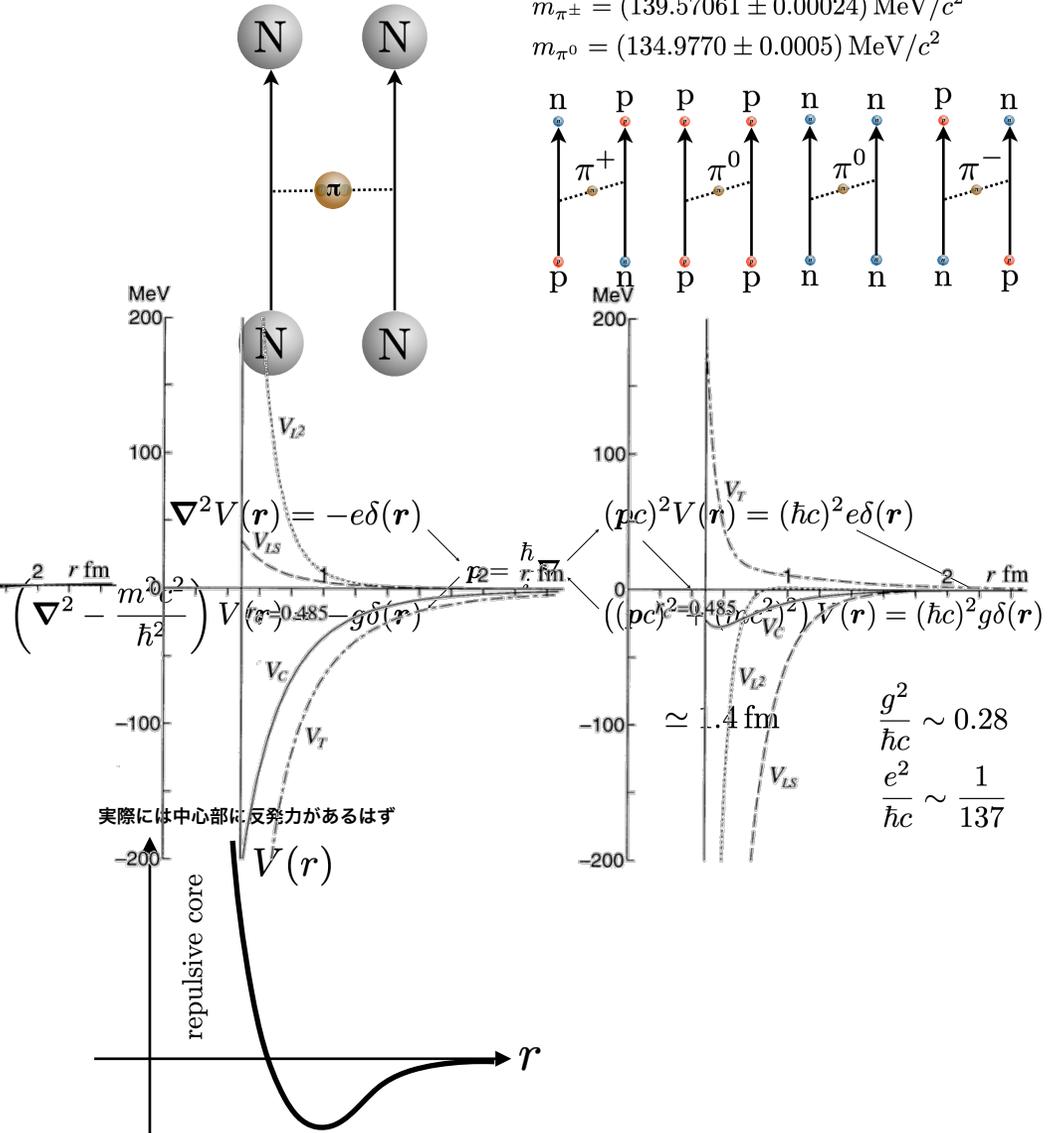
$$\frac{g^2}{\hbar c} \sim 0.28 \\ \frac{e^2}{\hbar c} \sim \frac{1}{137}$$



核力 二つの核子の間に働く力



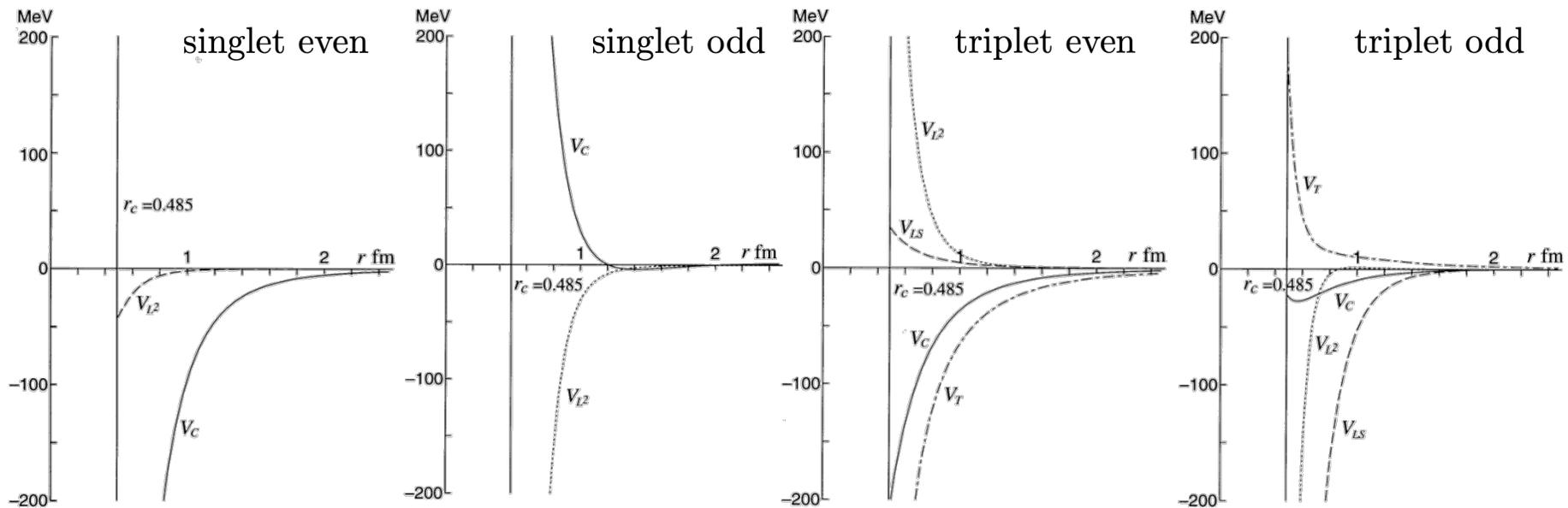
$$\begin{aligned}
 V_{NN} = & V_0 + V_\sigma(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + V_\tau(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + V_{\sigma\tau}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\
 & + V_{t0}S_{12} + V_{t\tau}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)S_{12} \\
 & + V_{LS0}(\mathbf{l} \cdot \mathbf{s}) + V_{LS\tau}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\mathbf{l} \cdot \mathbf{s}) \\
 & + \dots
 \end{aligned}$$



$$\begin{aligned}
 m_p &= (1.007276466879 \pm 0.000000000091) u \\
 &= (938.2720813 \pm 0.0000058) \text{ MeV}/c^2 \\
 m_n &= (939.5654133 \pm 0.0000058) \text{ MeV}/c^2 \\
 m_{\pi^\pm} &= (139.57061 \pm 0.00024) \text{ MeV}/c^2 \\
 m_{\pi^0} &= (134.9770 \pm 0.0005) \text{ MeV}/c^2
 \end{aligned}$$

核力 二つの核子の間に働く力

Hamada-Johnston potential



質量公式

定義 $B(Z, N) = Z \underbrace{M_{\text{H}}}_{\text{水素原子質量}} + N \underbrace{M_{\text{n}}}_{\text{中性子質量}} - \underbrace{M(Z, N)}_{\text{原子質量}} > 0$

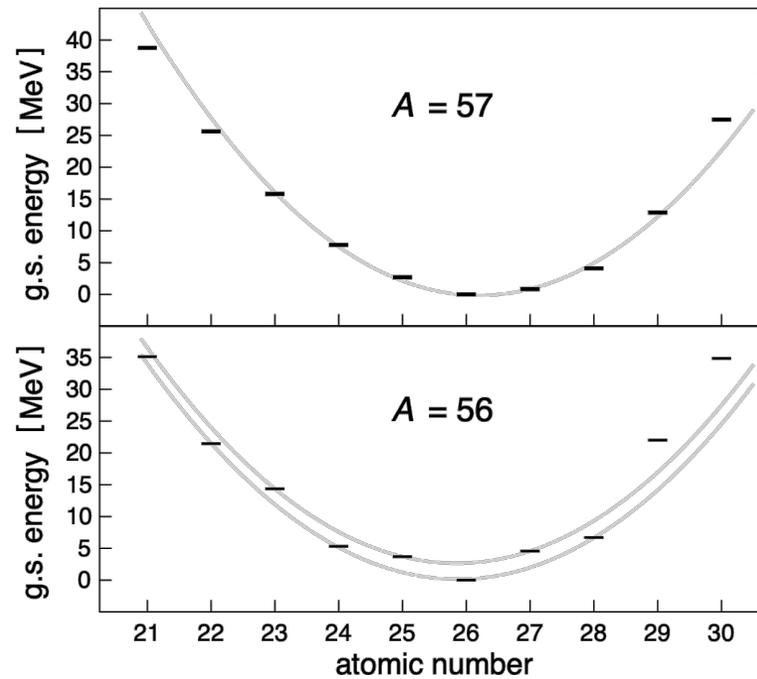
半経験質量公式 (semi-empirical mass formula)

Weizäcker's mass formula

$$M(Z, N) = ZM_{\text{p}} + NM_{\text{n}} - a_1A + a_2A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(Z - N)^2}{A} + \delta(A)$$

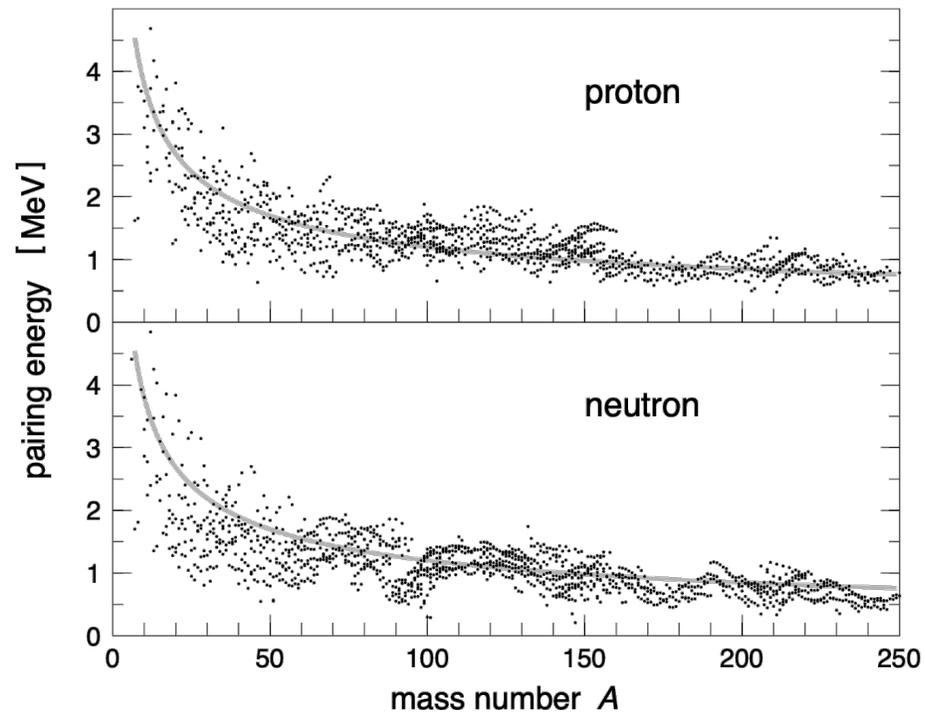
$$B(Z, N) = \underbrace{a_1A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} - \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$



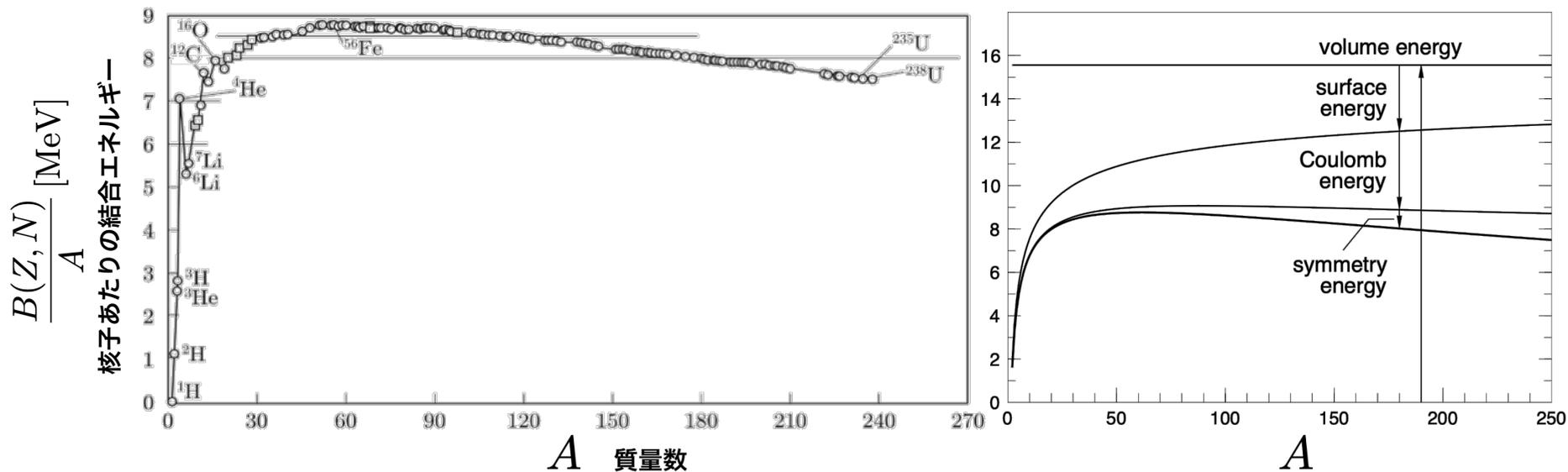
$$B(Z, N) = \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} - \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$



$$B(Z, N) = \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} - \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$

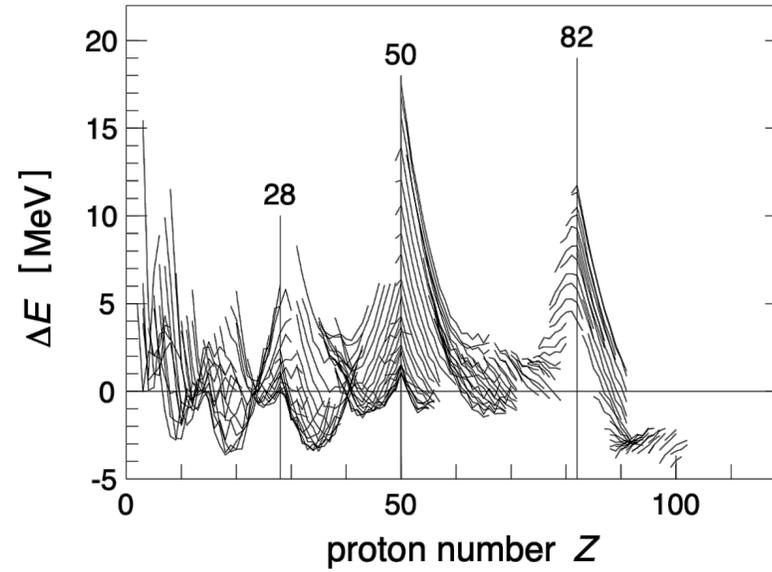


$$B(Z, N) = \underbrace{a_1 A}_{\text{volume energy}} - \underbrace{a_2 A^{2/3}}_{\text{lack of surface energy}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\text{Coulomb energy}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\text{symmetry energy}} - \underbrace{\delta(A)}_{\text{pairing energy}}$$

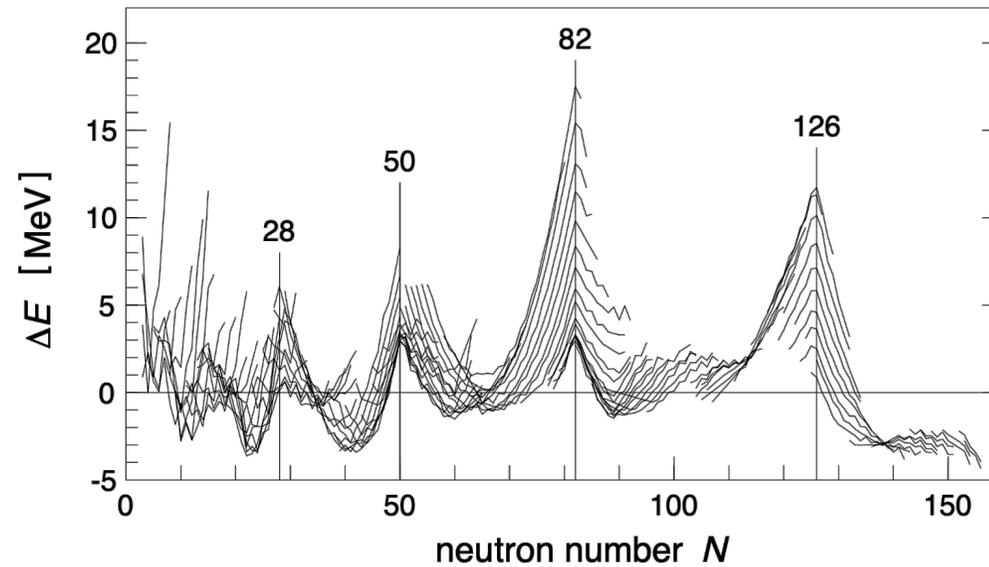
lack of surface energy
表面エネルギーの欠損
表面では片方しか結合がない

$$\begin{cases} a_1 = 15.68 \text{ MeV} \\ a_2 = 18.56 \text{ MeV} \\ a_3 = 0.717 \text{ MeV} \\ a_4 = 28.1 \text{ MeV} \end{cases} \quad \delta(A) = \begin{cases} 34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34 A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$

$$\Delta E(A, Z) = B(A, Z)_{\text{measured}} - B(A, Z)_{\text{mass formula}}$$



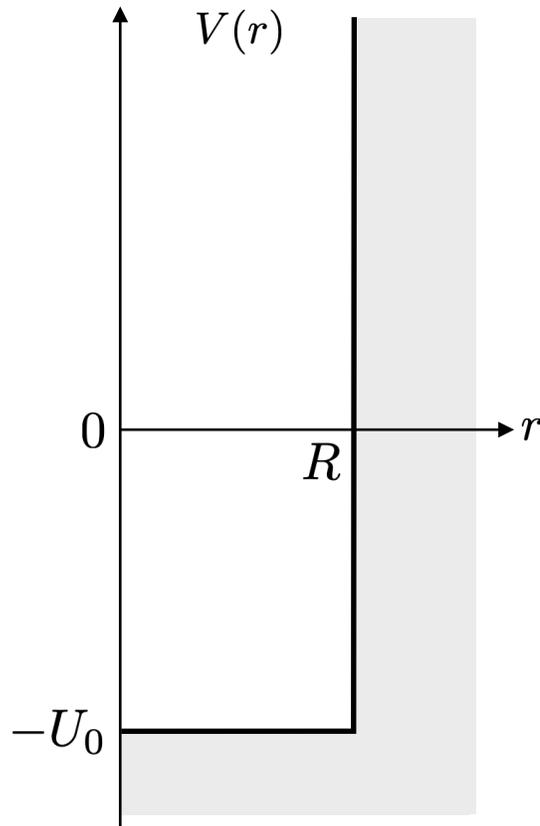
magic number
魔法数
2, 8, 20, 28, 50, 82, 126



殻模型

nuclear shell model

magic number = 2, 8, 20, 28, 50, 82, 126



Schrödinger's equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = E \psi$$

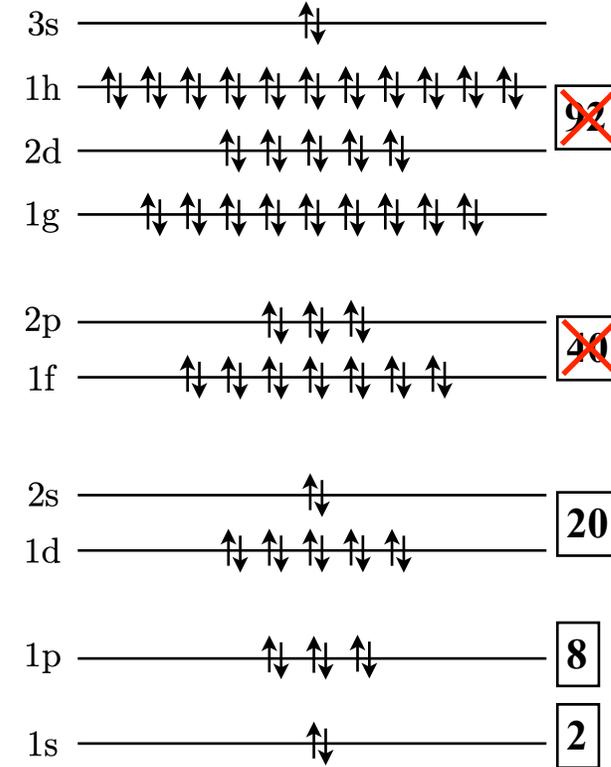
$$\psi_{nlm}(\mathbf{r}) = a_l j_l(k_n r) Y_{lm}(\theta, \varphi)$$

球Bessel関数 球調和関数

$r \geq R$ において $\psi(\mathbf{r}) = 0$ なので
 $j_l(k_n R) = 0$ でなければならない

球Bessel関数の n 番目の零点を x_{nl} とおくと

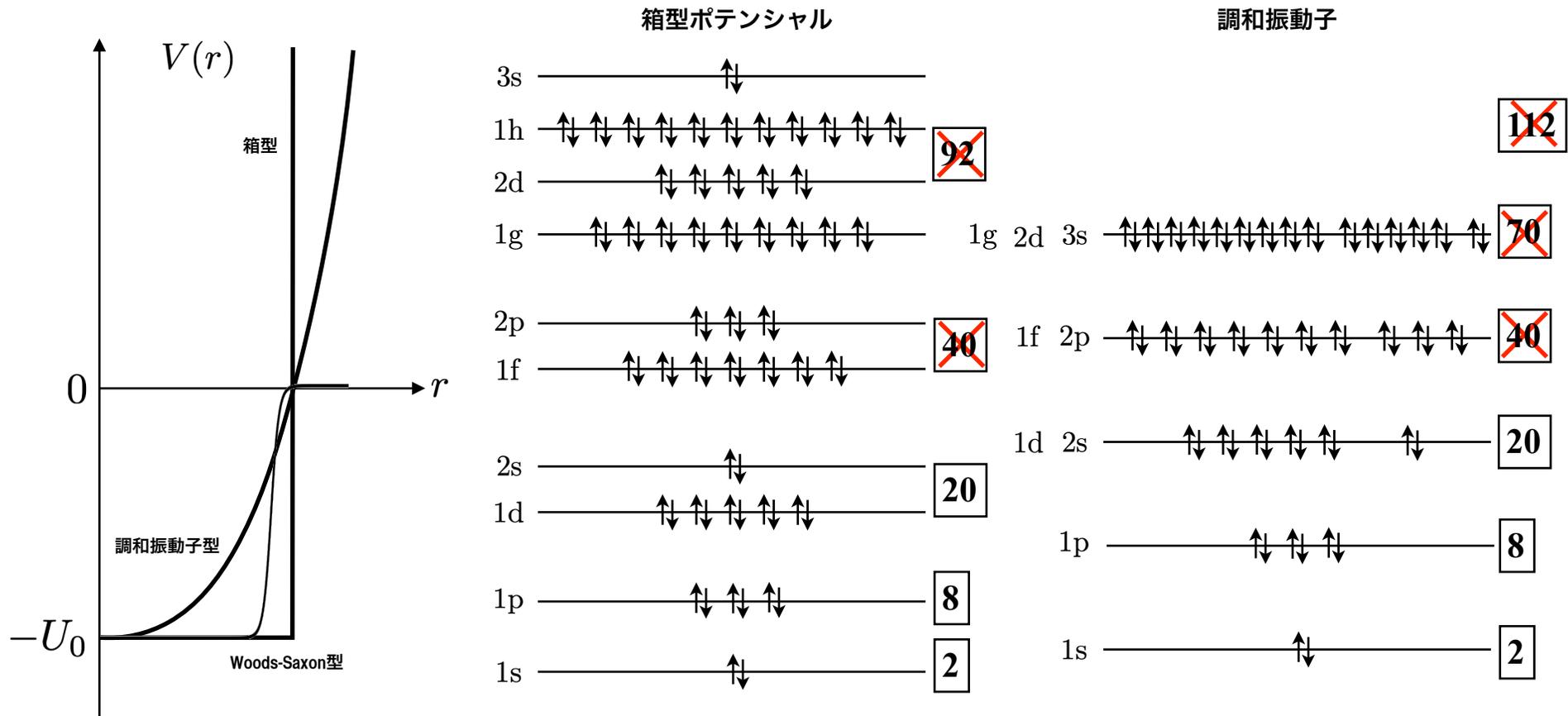
$$E_{nl} = -U_0 + \frac{\hbar^2}{2mR^2} x_{nl}^2$$



殻模型

nuclear shell model

magic number = 2, 8, 20, 28, 50, 82, 126



$$V(r) \rightarrow V_0(r) + V_{LS}(\mathbf{l} \cdot \mathbf{s})$$

$$2\mathbf{l} \cdot \mathbf{s} = j^2 - l^2 - s^2 = j(j+1) - l(l+1) - s(s+1) = \begin{cases} l & (j = l + \frac{1}{2}) \\ -l - 1 & (j = l - \frac{1}{2}) \end{cases}$$

殼模型

nuclear shell model

jj-coupling shell model (1949) M.G.Mayer, J.H.D.Jensen

magic number = 2, 8, 20, 28, 50, 82, 126

$$A = 142$$

$$nl_j \quad s,p,d,f,g,h,i$$

$$\frac{n = 6}{1i, 2g, 3d, 4s} \quad 168$$

$$\frac{n = 5}{1h, 2f, 3p} \quad 112$$

$$\frac{n = 4}{1g, 2d, 3s} \quad 70$$

$$\frac{n = 3}{1f, 2p} \quad 40$$

$$\frac{n = 2}{1d, 2s} \quad 20$$

HO

←
←
←
intruder orbit

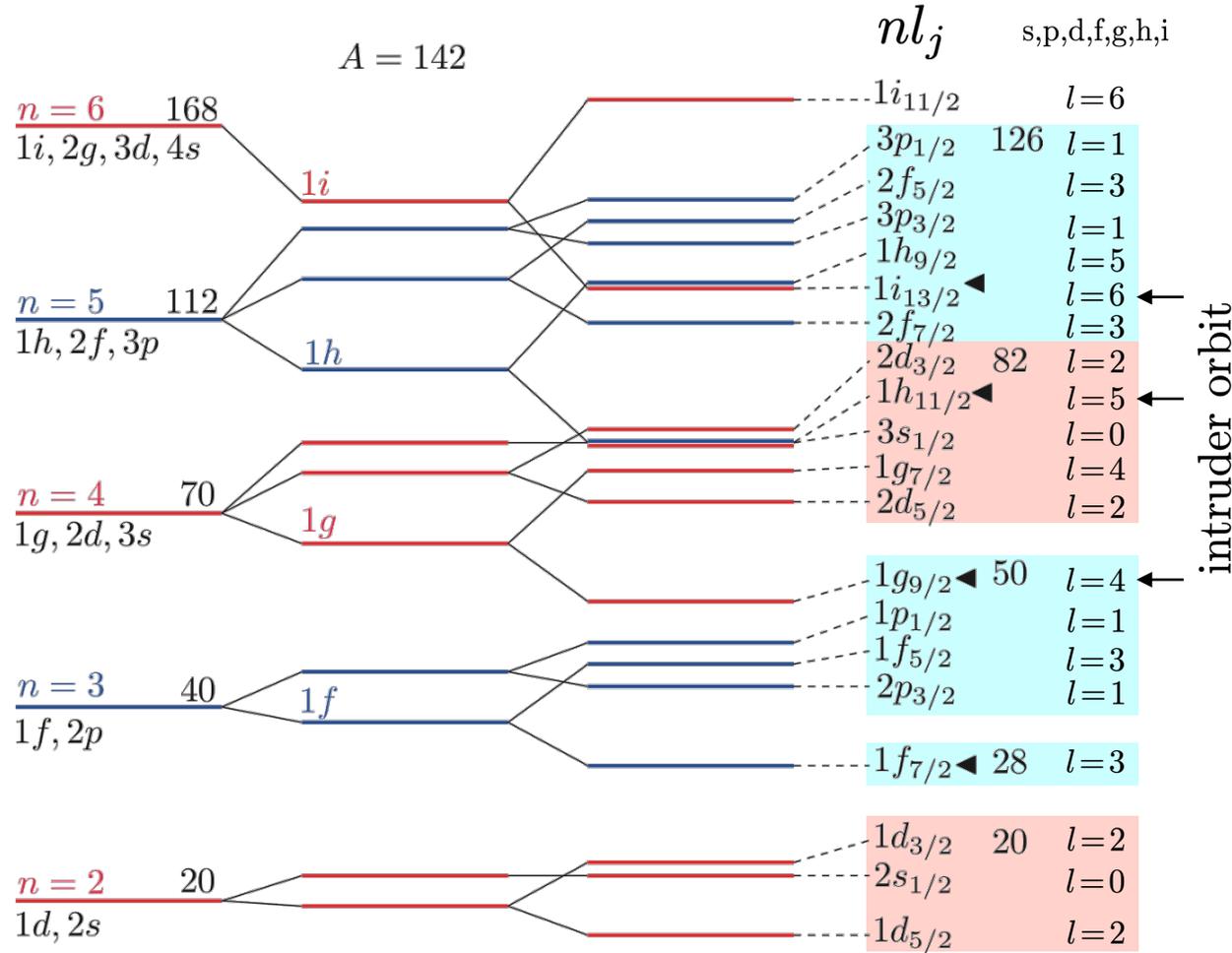
$$= \sum_{m_l m_s} \langle l m_l s m_s | j m \rangle Y_{lm}(\theta, \varphi) | s m_s \rangle$$

殼模型

nuclear shell model

jj-coupling shell model (1949) M.G.Mayer, J.H.D.Jensen

magic number = 2, 8, 20, 28, 50, 82, 126



HO

$$a_l(l^2 - \langle l^2 \rangle_N)$$

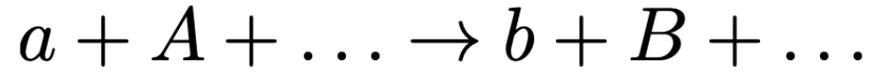
$$a_{ls} \mathbf{l} \cdot \mathbf{s}$$

$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

$$\mathcal{Y}_{jm}^{(ls)}(\theta, \varphi) = \sum_{m_l m_s} \langle l m_l s m_s | j m \rangle Y_{lm}(\theta, \varphi) | s m_s \rangle$$

原子核反応

原子核反応



始状態

終状態

チャンネル

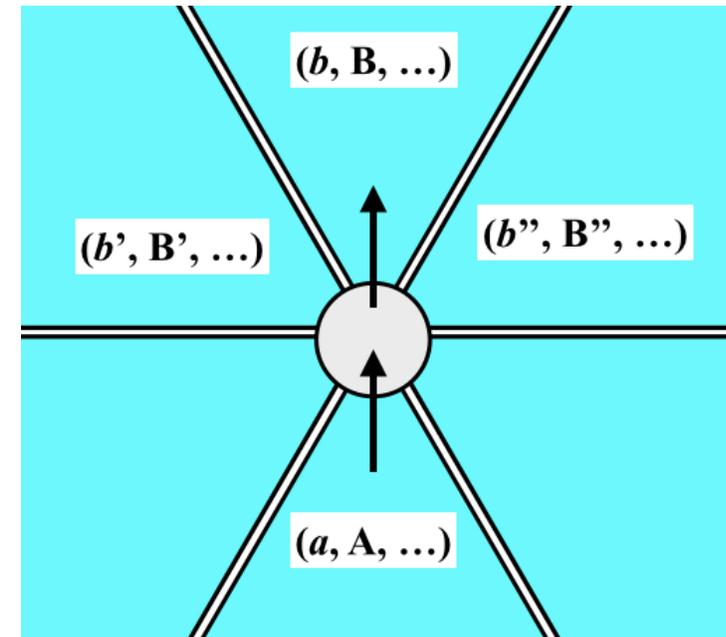
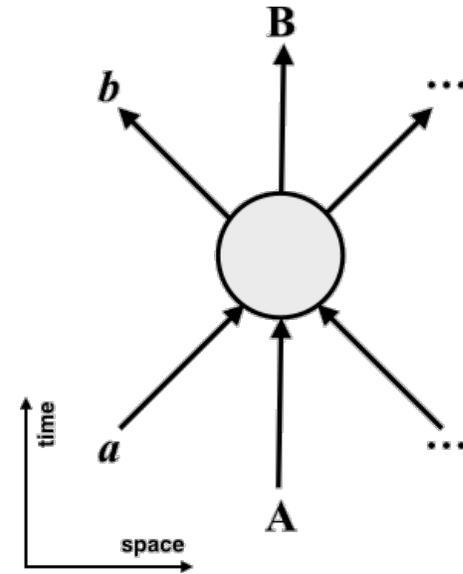
(a, A, \dots)

入口チャンネル

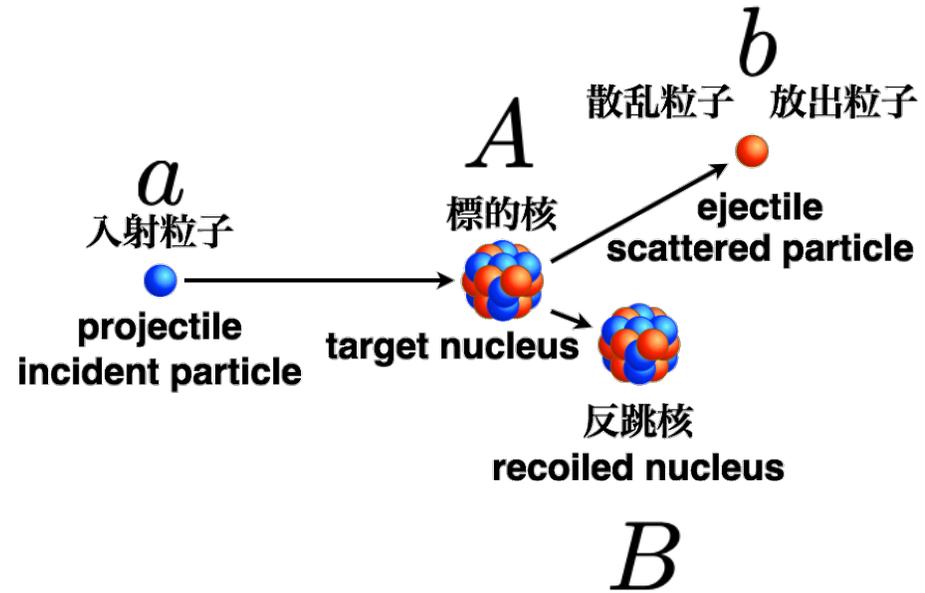
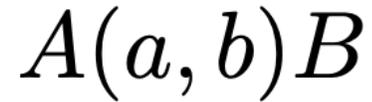
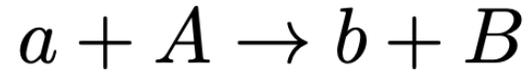
(b, B, \dots)

出口チャンネル

(核子の) 配位空間



原子核反應



發熱量 (Q-value)

$$Q = M_a c^2 + M_A c^2 - M_b c^2 - M_B c^2$$

$$Q > 0 \quad \text{發熱反應}$$

$$Q < 0 \quad \text{吸熱反應}$$

崩壊

放射線を発生する能力

放射能の単位：単位時間あたりの崩壊数

$$1 \text{ Bq} = 1 \text{ s}^{-1}$$

ベクレル

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

キュリー
1グラムの ^{226}Ra の単位時間あたりの崩壊数



単位時間あたり一定確率で崩壊する

$$dN_A = -\lambda N_A dt \longrightarrow N_A(t) = N_A(0)e^{-\lambda t}$$

崩壊定数、崩壊率

平均寿命 $\tau \equiv \langle t \rangle = \frac{\int_0^\infty t N(0) e^{-\lambda t} dt}{\int_0^\infty N(0) e^{-\lambda t} dt} = \frac{1}{\lambda^2} = \frac{1}{\lambda}$ $N(t) = N(0) e^{-\frac{t}{\tau}}$

半減期 $t_{\frac{1}{2}}$ $N(t) = N(0) \left(\frac{1}{2}\right)^{t/t_{\frac{1}{2}}}$ $t_{\frac{1}{2}} = \tau \ln 2$

崩壊

α 崩壊

α 線を放出する

${}^4\text{He}$ 原子核

β 崩壊

β 線を放出する

e^- 電子

β^- 崩壊

e^+ 陽電子

β^+ 崩壊

内部電子捕獲

γ 崩壊

γ 線を放出する

高エネルギーの電磁波

核子崩壊

核子を放出する

陽子崩壊

陽子が崩壊することではない

陽子を放出する

中性子崩壊

中性子を放出する

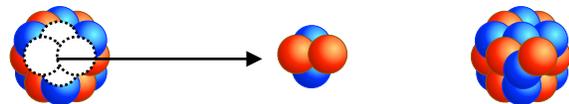
核分裂

(原子核を放出する)

α崩壊

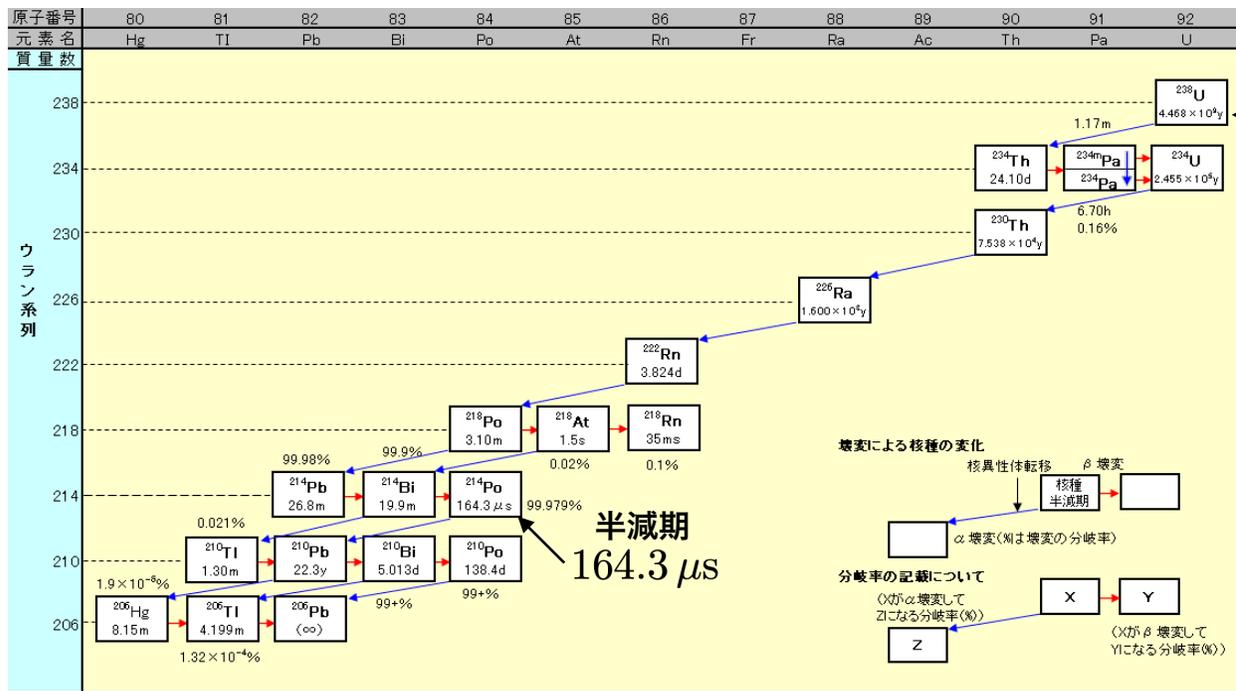
$$(Z, N) \rightarrow (Z-2, N-2) + \alpha$$

$$Q_\alpha = B(Z-2, N-2) + B(\alpha) - B(Z, N)$$



28 MeV

崩壊系列 (decay chain)



半減期の記号:s(秒), ms(10⁻³秒), μs(10⁻⁶秒), m(分), h(時), d(日), y(年)

図1-1 天然放射性核種の壊変系列図(ウラン系列)(1/4)

【出典】 国立天文台(編):理科年表 2010年版、丸善(2009年10月)、p.468-469

崩壊系列 (decay chain)

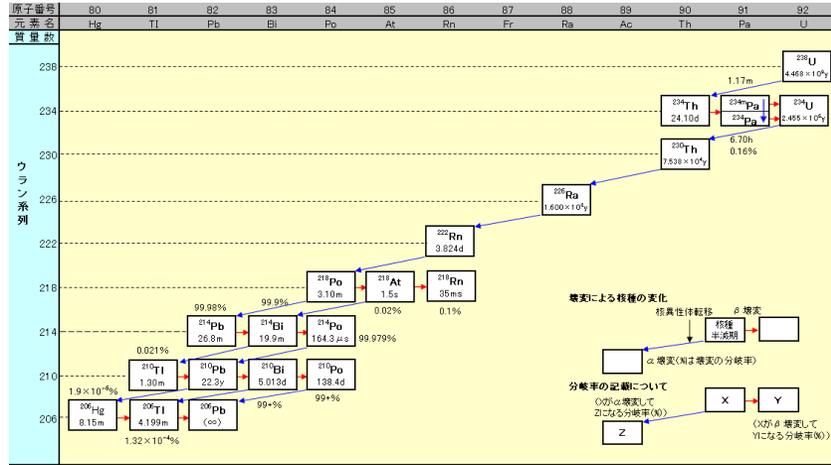


図1-1 天然放射性核種の壊変系列図(ウラン系列)(1/4)

【出典】国立天文台(編):理科年表 2010年版, 丸善(2009年10月), p.468-469

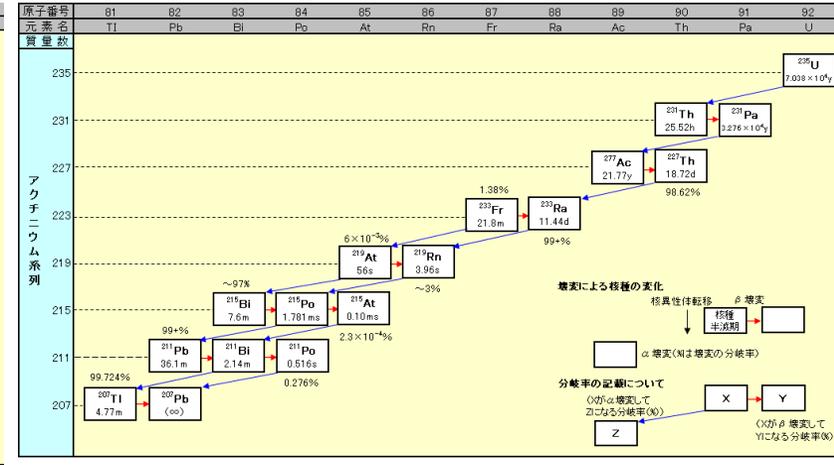


図1-2 天然放射性核種の壊変系列図(アクチニウム系列)(2/4)

【出典】国立天文台(編):理科年表 2010年版, 丸善(2009年10月), p.468-469

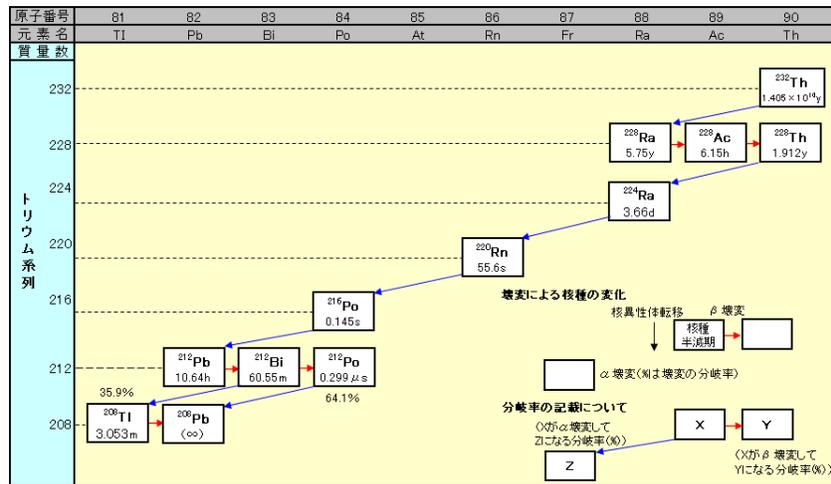


図1-3 天然放射性核種の壊変系列図(トリウム系列)(3/4)

【出典】国立天文台(編):理科年表 2010年版, 丸善(2009年10月), p.468-469

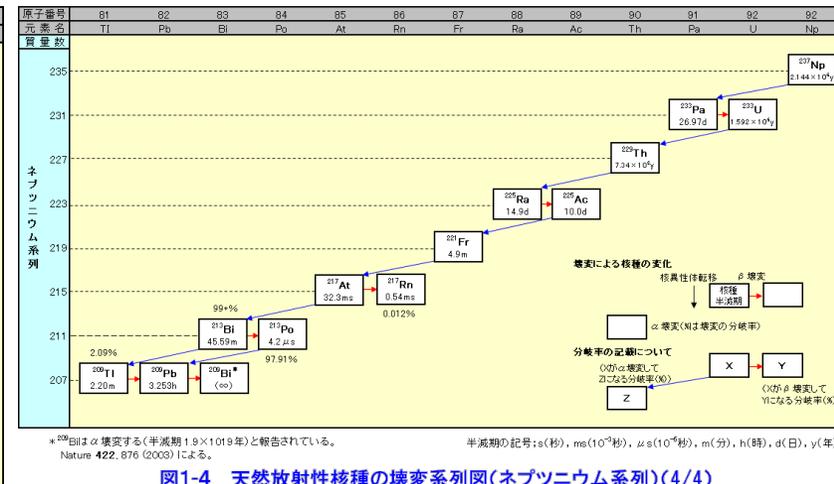
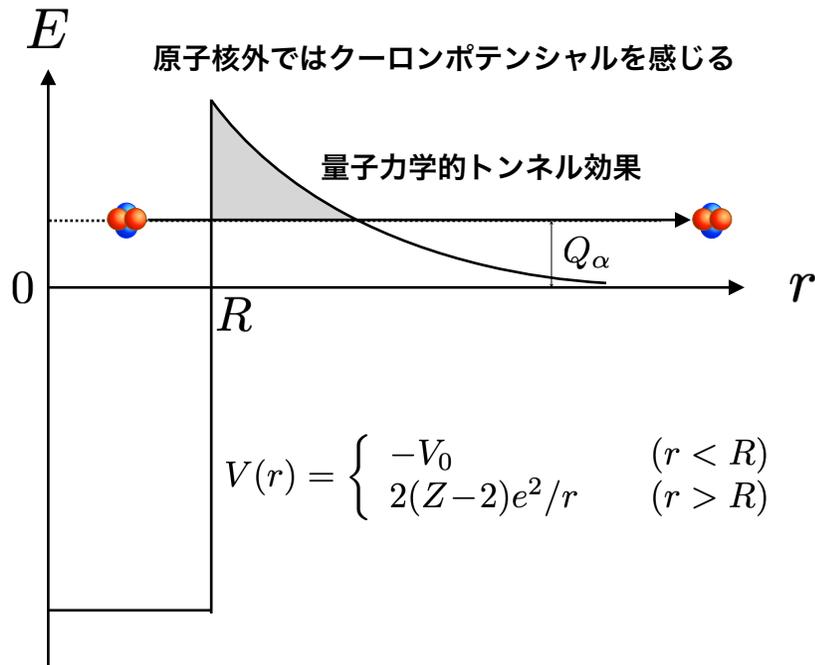


図1-4 天然放射性核種の壊変系列図(ネプツニウム系列)(4/4)

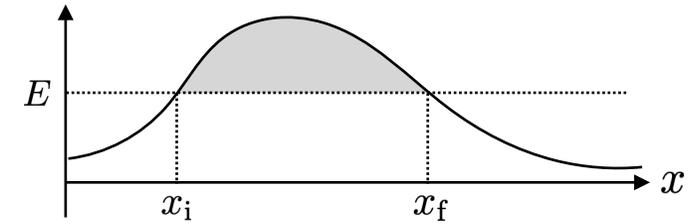
【出典】国立天文台(編):理科年表 2010年版, 丸善(2009年10月), p.468-469

α 崩壊



ポテンシャル障壁の透過確率

WKB近似 Wentzel, Kramers, Brillouin



$$P(E) = e^{-2W(E)/\hbar}$$

$$W(E) = \int_{x_i(E)}^{x_f(E)} \sqrt{2M(V(x) - E)} dx$$

崩壊定数のエネルギー依存性 $\lambda(E) = \lambda_0 P(E)$

1次元Schrödinger方程式

$$\left[\frac{1}{2m} \left(\frac{\hbar}{i} \frac{d}{dx} \right)^2 + V(r) - E \right] \psi(x) = 0 \quad \text{に対して}$$

$$\psi(x) = e^{\frac{i}{\hbar} S(x)} \quad \text{とおくと}$$

$$\frac{1}{2m} \left[\frac{\hbar}{i} \frac{d^2 S}{dx^2} + \left(\frac{dS}{dx} \right)^2 \right] + V(x) - E = 0$$

$$S(x) = S_0(x) + \frac{\hbar}{i} S_1(x) + \left(\frac{\hbar}{i} \right)^2 S_2(x) + \dots \quad \text{と展開して代入すると}$$

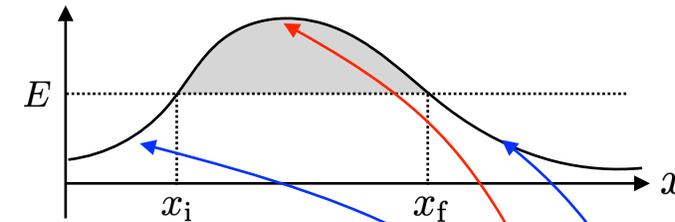
$$\frac{\hbar}{i} \frac{d^2 S_0}{dx^2} + \left(\frac{\hbar}{i} \right)^2 \frac{d^2 S_1}{dx^2} + \left(\frac{\hbar}{i} \right)^3 \frac{d^2 S_2}{dx^2} + \dots + \left(\frac{dS_0}{dx} + \frac{\hbar}{i} \frac{dS_1}{dx} + \left(\frac{\hbar}{i} \right)^2 \frac{dS_2}{dx} + \dots \right)^2 + 2m(V(r) - E) = 0$$

$$\hbar\text{の0次から} \quad \left(\frac{dS_0}{dx} \right)^2 + 2m(V(r) - E) = 0 \quad S_0 = \pm \int^x \sqrt{2m(E - V(x'))} dx'$$

$$\hbar\text{の1次から} \quad \frac{d^2 S_0}{dx^2} + 2 \frac{dS_1}{dx} = 0 \quad S_1 = \log \frac{1}{\sqrt{2m(E - V(x'))}} + C$$

$$\psi_{\downarrow}(x) = A_{\downarrow} \frac{1}{\sqrt{2m(E - V(x))}} e^{\frac{i}{\hbar} \int \sqrt{2m(E - V(x'))} dx'} + B_{\downarrow} \frac{1}{\sqrt{2m(E - V(x))}} e^{-\frac{i}{\hbar} \int \sqrt{2m(E - V(x'))} dx'}$$

$$\psi_{\uparrow}(x) = A_{\uparrow} \frac{1}{\sqrt{2m(V(x) - E)}} e^{\frac{i}{\hbar} \int \sqrt{2m(V(x') - E)} dx'} + B_{\uparrow} \frac{1}{\sqrt{2m(V(x) - E)}} e^{-\frac{i}{\hbar} \int \sqrt{2m(V(x') - E)} dx'}$$



$$P(E) = e^{-2W(E)/\hbar}$$

$$W(E) = \int_{x_i(E)}^{x_f(E)} \sqrt{2M(V(x) - E)} dx$$

前項定数のエネルギー依存性 $\lambda(E) = \lambda_0 P(E)$

$$V(x) = V(x_1) + V'(x_1)(x - x_1), \quad V(x_1) = E$$

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V'(x_1)(x - x_1) \right] \psi(x) = 0$$

Airyの微分方程式

$$\left[\frac{d^2}{dy^2} - y \right] \psi = 0 \quad y = \sqrt[3]{\frac{2m}{\hbar^2} V'(x_1)(x - x_1)}$$

$$\psi = C_1 \text{Ai}(y) + C_2 \text{Bi}(y) \quad \text{Ai, Bi : Airy関数}$$

$y \gg 1$ における漸近形

$$\text{Ai}(+y) \xrightarrow{y \gg 1} \frac{1}{2\sqrt{\pi}} y^{-\frac{1}{4}} e^{-\frac{2}{3}y^{3/2}}, \quad \text{Bi}(+y) \xrightarrow{y \gg 1} \frac{1}{\sqrt{\pi}} y^{-\frac{1}{4}} e^{\frac{2}{3}y^{3/2}}$$

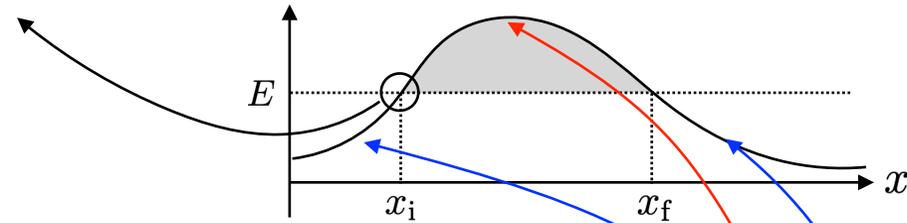
$$\text{Ai}(-y) \xrightarrow{y \gg 1} \frac{1}{\sqrt{\pi}} y^{-\frac{1}{4}} \sin\left(\frac{2}{3}y^{3/2} + \frac{\pi}{4}\right), \quad \text{Bi}(-y) \xrightarrow{y \gg 1} \frac{1}{\sqrt{\pi}} y^{-\frac{1}{4}} \cos\left(\frac{2}{3}y^{3/2} + \frac{\pi}{4}\right)$$

$$\psi_{\downarrow}(x) = A_{\downarrow} \frac{1}{\sqrt{2m(E - V(x))}} e^{\frac{i}{\hbar} \int \sqrt{2m(E - V(x'))} dx'} + B_{\downarrow} \frac{1}{\sqrt{2m(E - V(x))}} e^{-\frac{i}{\hbar} \int \sqrt{2m(E - V(x'))} dx'}$$

$$\psi_{\uparrow}(x) = A_{\uparrow} \frac{1}{\sqrt{2m(V(x) - E)}} e^{\frac{1}{\hbar} \int \sqrt{2m(V(x') - E)} dx'} + B_{\uparrow} \frac{1}{\sqrt{2m(V(x) - E)}} e^{-\frac{1}{\hbar} \int \sqrt{2m(V(x') - E)} dx'}$$

ポテンシャル障壁の透過確率

WKB近似 Wentzel, Kramers, Brillouin



$$P(E) = e^{-2W(E)/\hbar}$$

$$W(E) = \int_{x_i(E)}^{x_f(E)} \sqrt{2M(V(x) - E)} dx$$

崩壊定数のエネルギー依存性 $\lambda(E) = \lambda_0 P(E)$

を使って波動関数を接続

$$V(x) = V(x_1) + V'(x_1)(x - x_1), \quad V(x_1) = E$$

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V'(x_1)(x - x_1) \right] \psi(x) = 0$$

Airyの微分方程式

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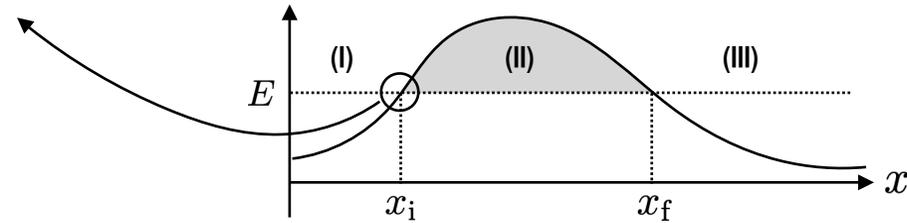
$y \gg 1$ における漸近形

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$$\text{Ai}(-y) \xrightarrow{y \gg 1} \frac{1}{\sqrt{\pi}} y^{-\frac{1}{4}} \sin\left(\frac{2}{3}y^{3/2} + \frac{\pi}{4}\right), \quad \text{Bi}(-y) \xrightarrow{y \gg 1} \frac{1}{\sqrt{\pi}} y^{-\frac{1}{4}} \cos\left(\frac{2}{3}y^{3/2} + \frac{\pi}{4}\right)$$

ポテンシャル障壁の透過確率

WKB近似 Wentzel, Kramers, Brillouin



$$P(E) = e^{-2W(E)/\hbar}$$

$$W(E) = \int_{x_i(E)}^{x_f(E)} \sqrt{2M(V(x) - E)} dx$$

崩壊定数のエネルギー依存性

$$\lambda(E) = \lambda_0 P(E)$$

を使って波動関数を接続

$$V(x) = V(x_1) + V'(x_1)(x - x_1), \quad V(x_1) = E$$

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Airyの微分方程式

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$$\text{Ai}(-y) \xrightarrow{y \gg 1} \frac{1}{\sqrt{\pi}} y^{-\frac{1}{4}} \sin\left(\frac{2}{3}y^{3/2} + \frac{\pi}{4}\right), \quad \text{Bi}(-y) \xrightarrow{y \gg 1} \frac{1}{\sqrt{\pi}} y^{-\frac{1}{4}} \cos\left(\frac{2}{3}y^{3/2} + \frac{\pi}{4}\right)$$

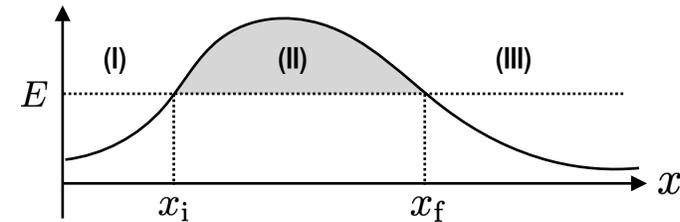
$$\psi_{\text{(III)}}(x) = + \frac{iC\hbar}{\sqrt{2m(V(x) - E)}} \left[\cos\left(\int_{x_2}^x \frac{\sqrt{2m(V(x') - E)}}{\hbar} dx' - \frac{\pi}{4}\right) + i \sin\left(\int_{x_2}^x \frac{\sqrt{2m(V(x') - E)}}{\hbar} dx' - \frac{\pi}{4}\right) \right]$$

$$\psi_{\text{(II)}}(x) = + \frac{C\hbar}{\sqrt{2m(E - V(x))}} \exp \left[\int_x^{x_1} \frac{\sqrt{2m(E - V(x'))}}{\hbar} dx' - \int_{x_2}^x \frac{\sqrt{2m(E - V(x'))}}{\hbar} dx' \right]$$

$$\psi_{\text{(I)}}(x) = \frac{iC\hbar}{\sqrt{2m(V(x) - E)}} e^{\int_{x_1}^{x_2} \frac{\sqrt{2m(V(x') - E)}}{\hbar} dx'} \left[e^{i \int_x^{x_1} \frac{\sqrt{2m(V(x') - E)}}{\hbar} dx' - i\frac{\pi}{4}} + e^{-i \int_x^{x_1} \frac{\sqrt{2m(V(x') - E)}}{\hbar} dx' + i\frac{\pi}{4}} \right]$$

ポテンシャル障壁の透過確率

WKB近似 Wentzel, Kramers, Brillouin



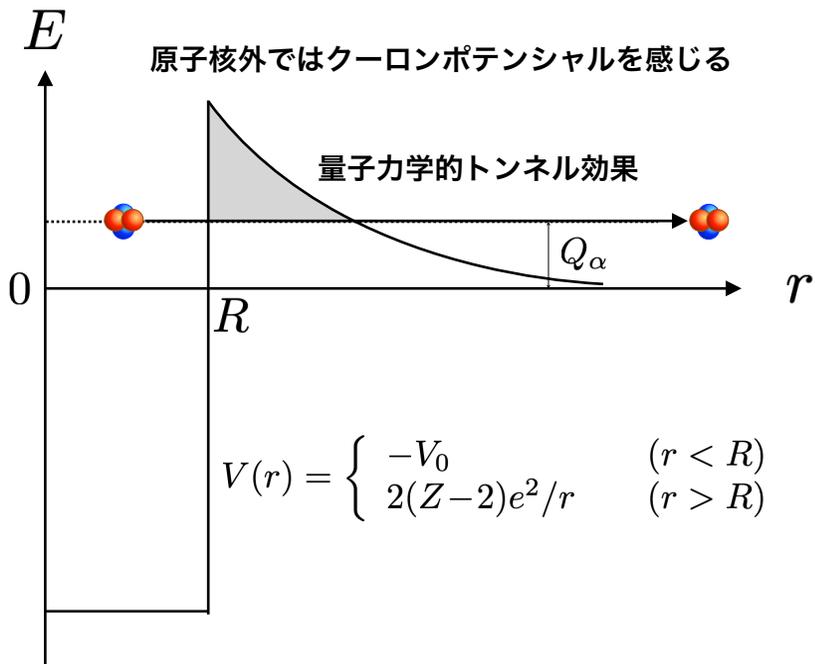
$$P(E) = e^{-2W(E)/\hbar}$$

$$W(E) = \int_{x_i(E)}^{x_f(E)} \sqrt{2M(V(x) - E)} dx$$

前項定数のエネルギー依存性 $\lambda(E) = \lambda_0 P(E)$

を使って波動関数を接続

WKB近似



$$V(r) = \begin{cases} -V_0 & (r < R) \\ 2(Z-2)e^2/r & (r > R) \end{cases}$$

s波の場合を考えて計算すると

$$\ln \tau \sim \text{const.} + \frac{2\pi e^2}{\hbar} \sqrt{2m_\alpha} \frac{Z}{\sqrt{E_\alpha}}$$

詳しくは

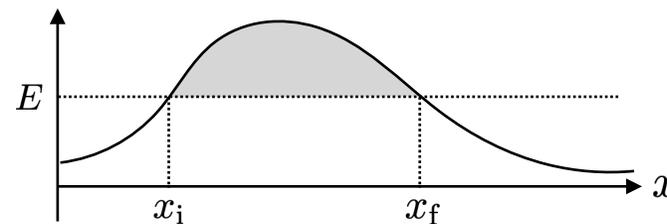
$$t_{1/2} \simeq \frac{F_0}{\sqrt{E}} \exp \left(\frac{\pi\sqrt{2mc^2 \cdot 2Ze^2}}{\sqrt{E}} - 4\sqrt{2mc^2 \cdot 2Ze^2} \frac{R}{\hbar c} + \frac{2}{3} \sqrt{\frac{2mc^2}{2Ze^2}} \left(\frac{R}{\hbar c} \right)^3 E \right)$$

Geiger-Nuttall の法則

$$t_{1/2} \simeq \frac{F_0}{\sqrt{E}} \exp \left(\frac{\pi\sqrt{2mc^2 \cdot 2Ze^2}}{\sqrt{E}} - 4\sqrt{2mc^2 \cdot 2Ze^2} \frac{R}{\hbar c} \right)$$

ポテンシャル障壁の透過確率

WKB近似 Wentzel, Kramers, Brillouin

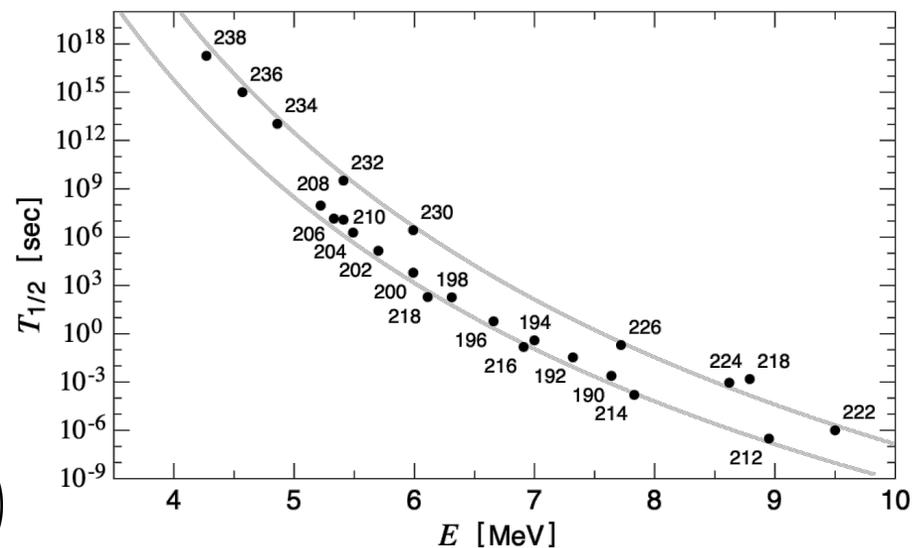


$$P(E) = e^{-2W(E)/\hbar}$$

$$W(E) = \int_{x_i(E)}^{x_f(E)} \sqrt{2M(V(x) - E)} dx$$

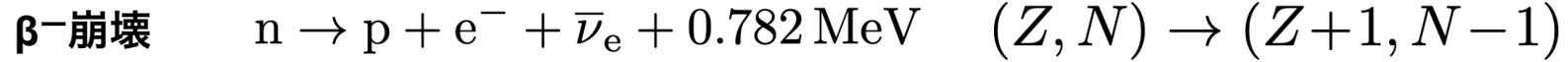
崩壊定数のエネルギー依存性

$$\lambda(E) = \lambda_0 P(E)$$

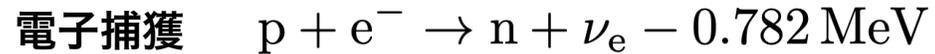


β 崩壊

$$\tau = 10^{-3} - 10^8 \text{ s}$$



$$Q_{\beta^-} = B(Z+1, N-1) - B(Z, N) \sim -\frac{\partial B}{\partial N} = \frac{\partial B}{\partial Z}$$



$$Q_{\beta^+} = B(Z-1, N+1) - B(Z, N) \sim -\frac{\partial B}{\partial Z} = \frac{\partial B}{\partial N}$$

$$\frac{\partial B}{\partial Z} = -a_3 \frac{2Z}{(Z+N)^{1/3}} - 2a_4 \frac{Z-N}{Z+N} = 0$$

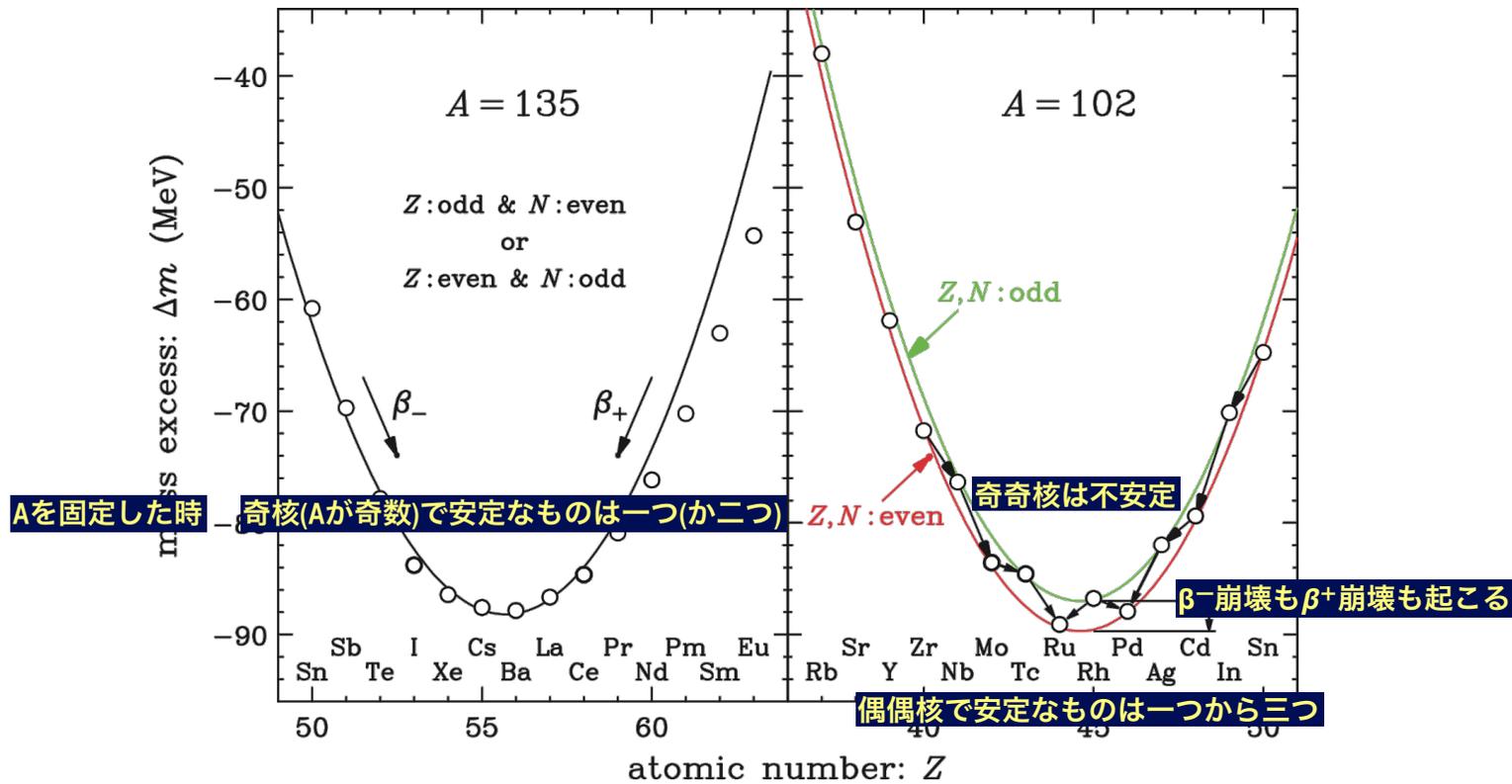
$$Z = \frac{A}{2 + (a_3/a_4)A^{2/3}}$$

β -安定線

β崩壊

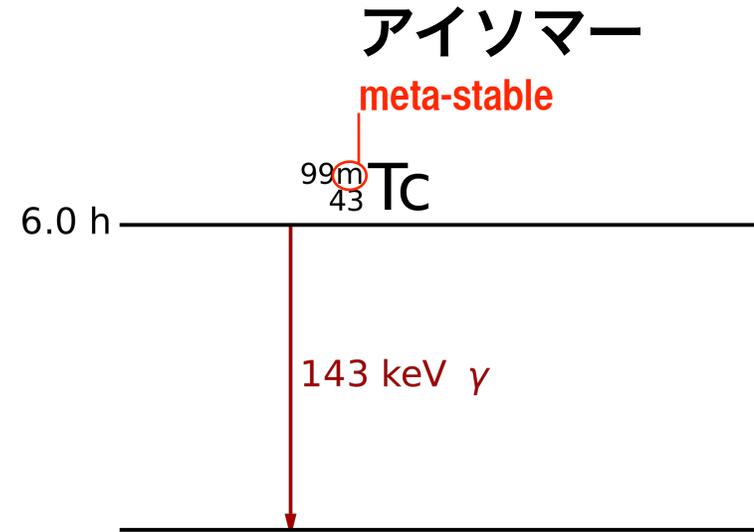
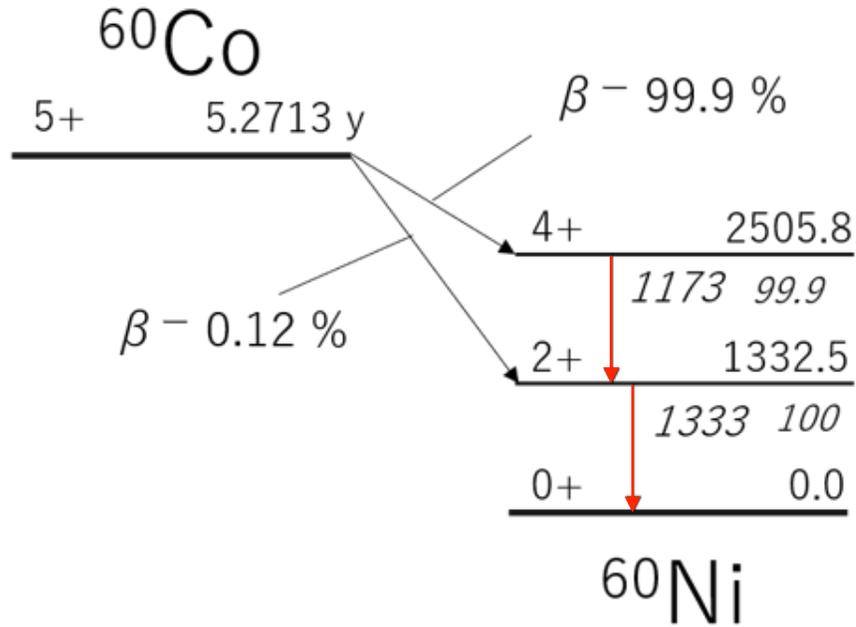
$$B(Z, N) = \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} + \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

$$\delta(A) = \begin{cases} 34A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$



γ崩壊

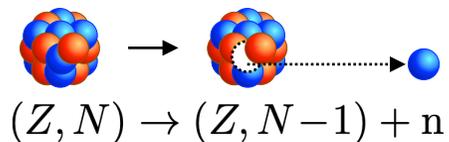
$$\tau = 10^{-10} - 10^{-15} \text{ s}$$



原子核の核子崩壊 (核子放出)

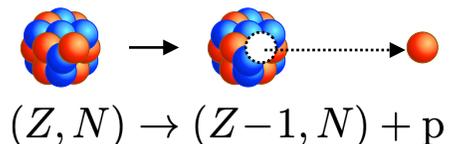
$$\tau = 10^{-22} - 10^{-20} \text{ s} \quad (\text{drip lineの外では})$$

原子核の中性子崩壊 (中性子放出)



$$Q = M(Z, N) - M(Z, N-1) - M_n = -S_n$$

原子核の陽子崩壊 (陽子放出)

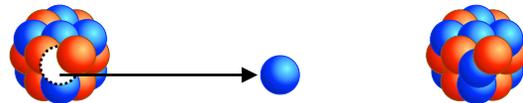


$$Q = M(Z, N) - M(Z-1, N) - M_p = -S_p$$

分離エネルギー (separation energy)

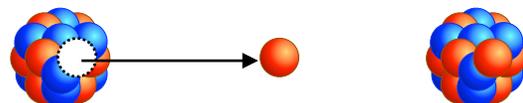
一つの中性子的分離エネルギー

$$S_n = \{M(Z, N-1) + M_n\} - M(Z, N) = B(Z, N) - B(Z, N-1)$$



一つの陽子の分離エネルギー

$$S_p = \{M(Z-1, N) + M_p\} - M(Z, N) = B(Z, N) - B(Z-1, N)$$



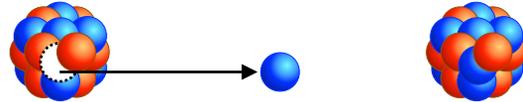
原子核の核子崩壊 (核子放出)

$$B(Z, N) = \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} + \underbrace{\delta}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

分離エネルギー (separation energy)

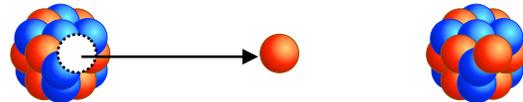
一つの中性子の分離エネルギー

$$S_n = \{M(Z, N - 1) + M_n\} - M(Z, N) = B(Z, N) - B(Z, N - 1) \sim \frac{dB}{dN}$$



一つの陽子の分離エネルギー

$$S_p = \{M(Z - 1, N) + M_p\} - M(Z, N) = B(Z, N) - B(Z - 1, N) \sim \frac{dB}{dZ}$$



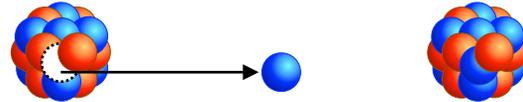
原子核の核子崩壊 (核子放出)

$$B(Z, N) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z - N)^2}{A}$$

分離エネルギー (separation energy)

一つの中性子の分離エネルギー

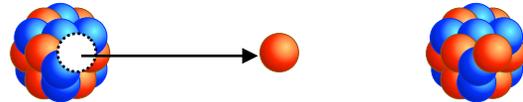
$$S_n = \{M(Z, N - 1) + M_n\} - M(Z, N) = B(Z, N) - B(Z, N - 1) \sim \frac{dB}{dN}$$



$$= a_1 - a_2 \frac{2}{3} (Z + N)^{1/3} + a_3 \frac{1}{3} \frac{Z^2}{(Z + N)^{4/3}} + a_4 \frac{1}{2} \frac{(Z - N)(3Z + N)}{(Z + N)^2}$$

一つの陽子の分離エネルギー

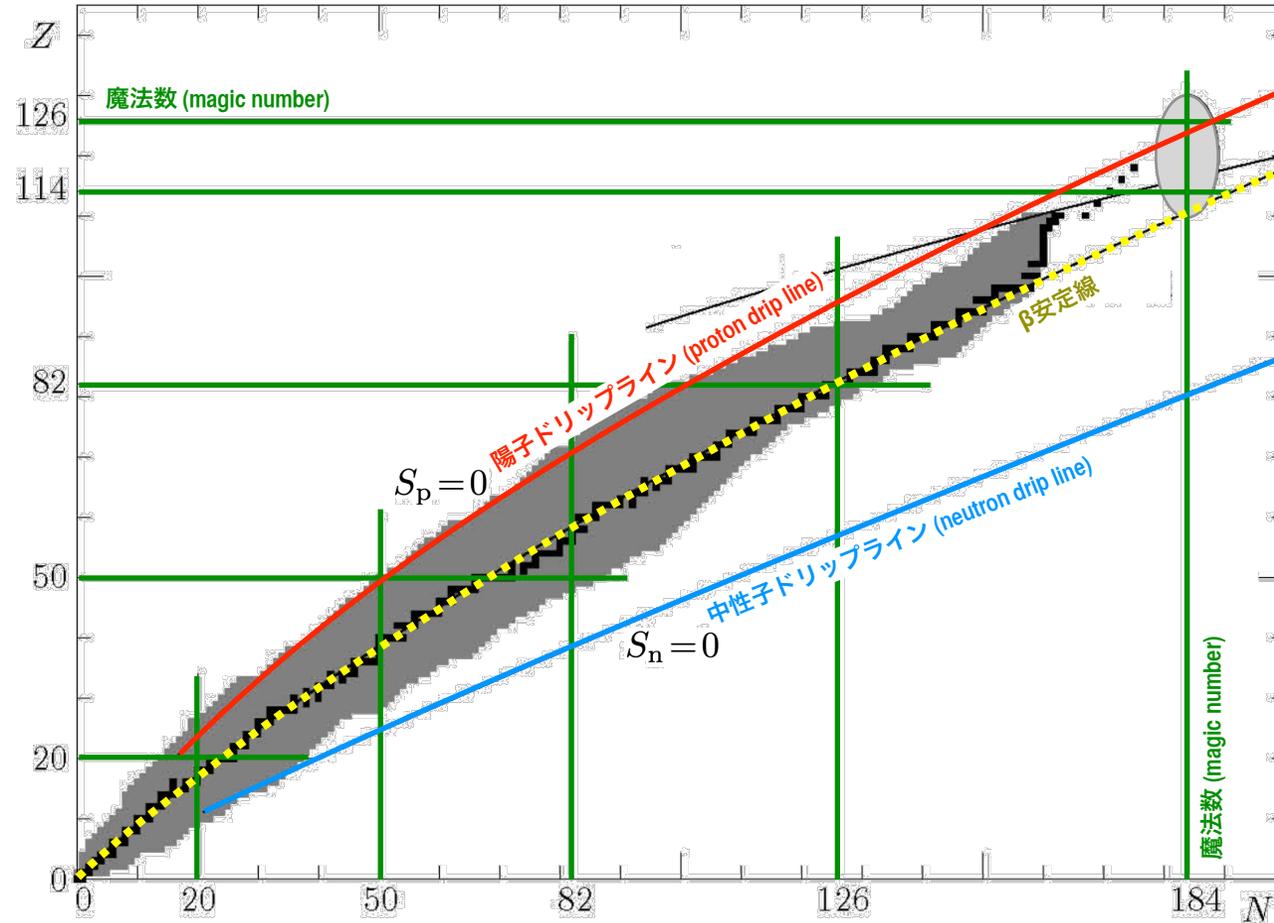
$$S_p = \{M(Z - 1, N) + M_p\} - M(Z, N) = B(Z, N) - B(Z - 1, N) \sim \frac{dB}{dZ}$$



$$= a_1 - a_2 \frac{2}{3} (Z + N)^{1/3} - a_3 \frac{1}{3} \frac{Z(5Z + 6N)}{(Z + N)^{4/3}} - a_4 \frac{1}{2} \frac{(Z - N)(Z + 3N)}{(Z + N)^2}$$

原子核の核子崩壊 (核子放出)

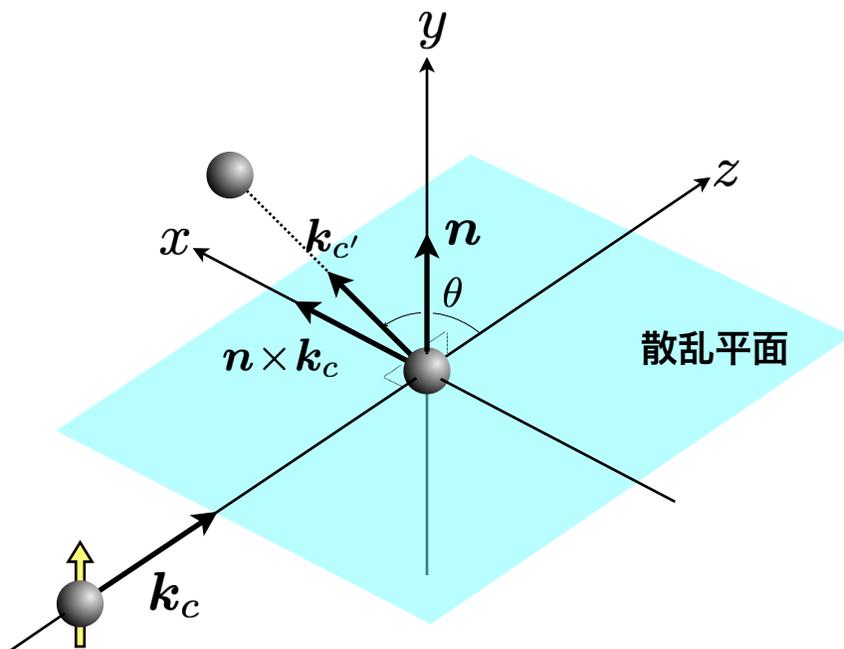
核図表 (nuclear chart)



問題 7

Analyzing power または polarization を測定する方法を考えよ。

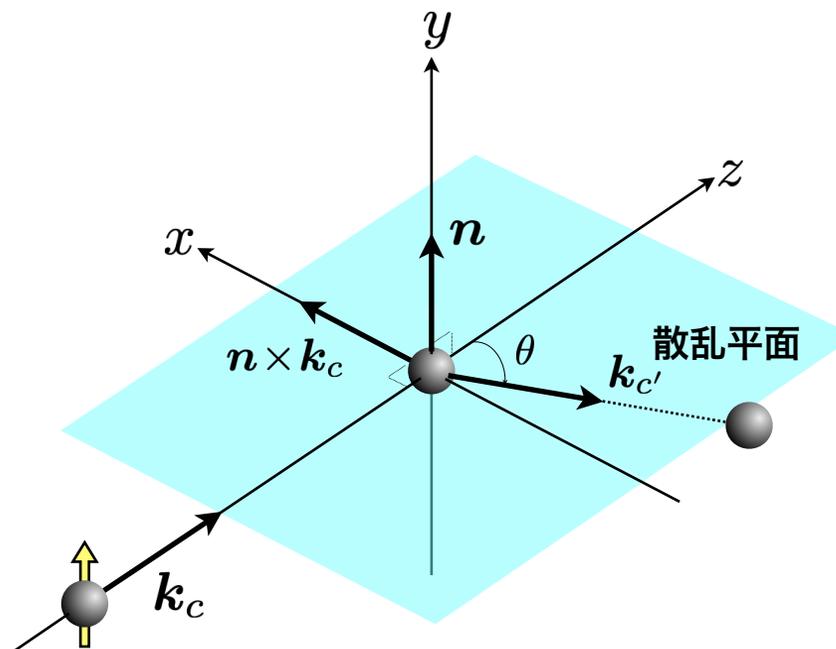
Analyzing Power



入射粒子のスピン偏極

$$\sigma_L(\theta) = \frac{d\sigma_{c',c1/2}(\theta, 0)}{d\Omega_{c'}}$$

Polarization (偏極度) $P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$

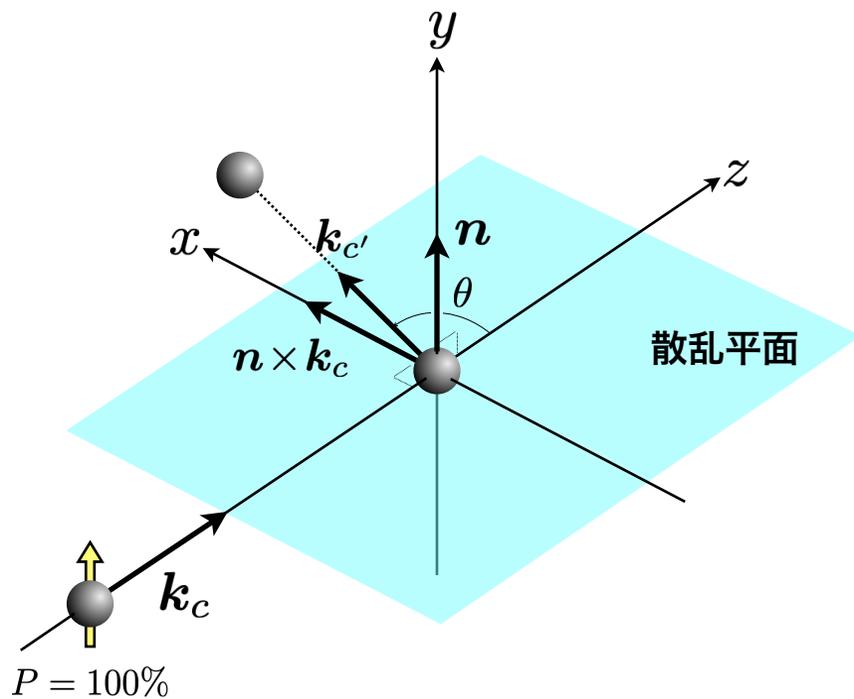


入射粒子のスピン偏極

$$\sigma_R(\theta) = \frac{d\sigma_{c',c1/2}(\theta, \pi)}{d\Omega_{c'}}$$

Analyzing Power (偏極分解能) $A(\theta) = \frac{\sigma_L(\theta) - \sigma_R(\theta)}{\sigma_L(\theta) + \sigma_R(\theta)}$

Analyzing Power

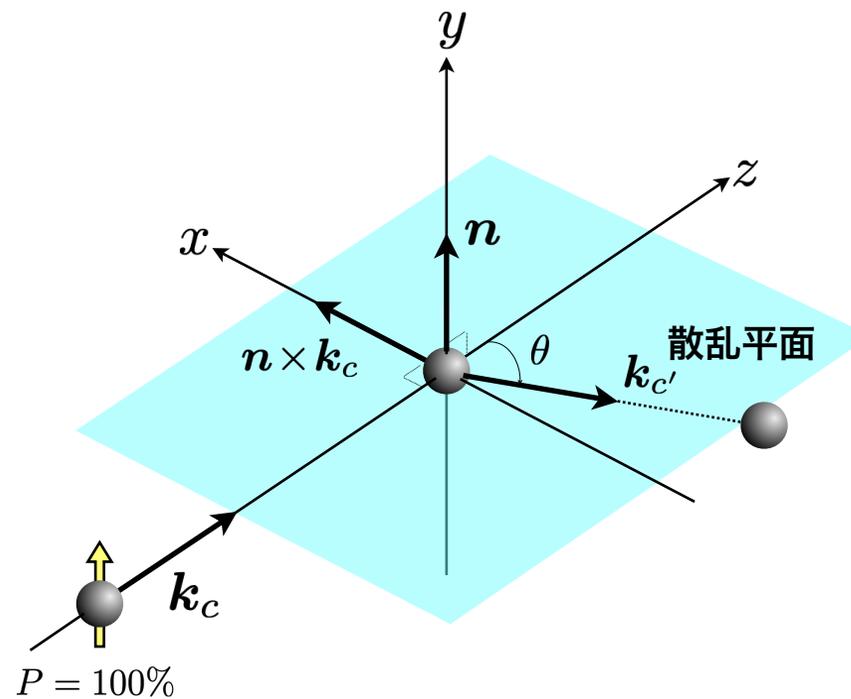


$$\sigma_L(\theta) = \frac{d\sigma_{c',c1/2}(\theta, 0)}{d\Omega_{c'}}$$

Polarization
(偏極度)

$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

$$N_{\uparrow} = \frac{1+P}{2} N_0 \quad N_{\downarrow} = \frac{1-P}{2} N_0$$



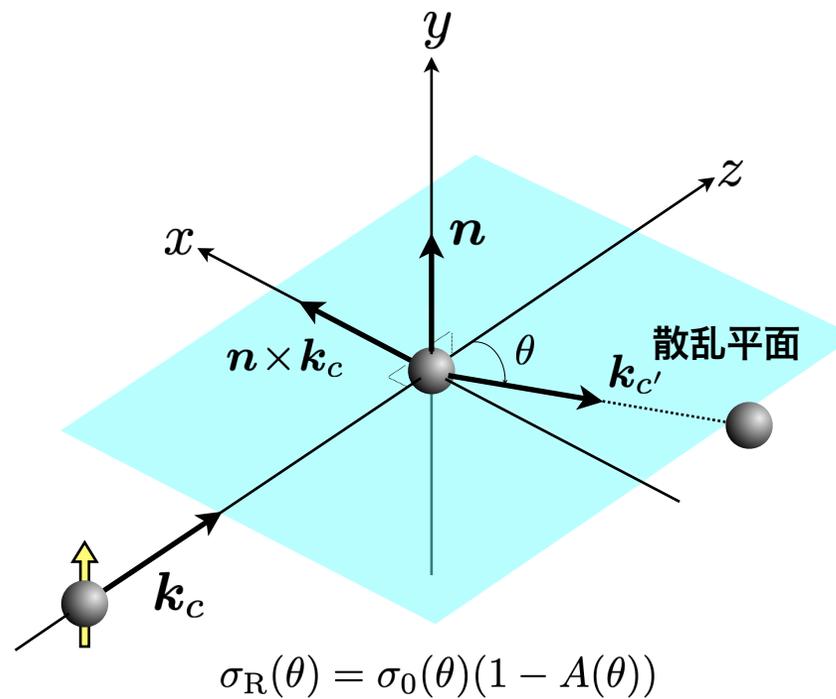
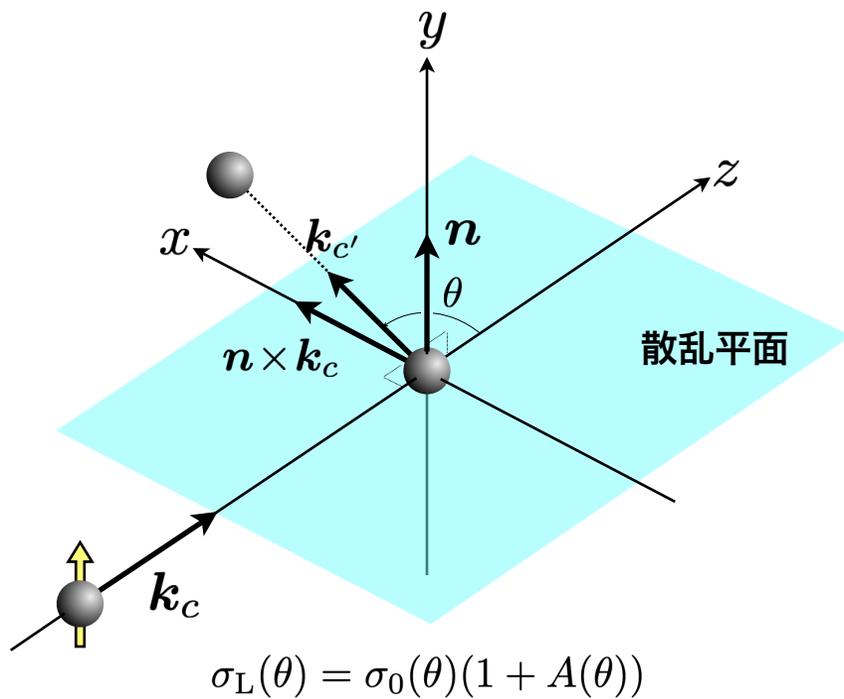
$$\sigma_R(\theta) = \frac{d\sigma_{c',c1/2}(\theta, \pi)}{d\Omega_{c'}}$$

Analyzing Power
(偏極分解能)

$$A(\theta) = \frac{\sigma_L(\theta) - \sigma_R(\theta)}{\sigma_L(\theta) + \sigma_R(\theta)}$$

$$\sigma_L(\theta) = \sigma_0(\theta)(1 + A(\theta)) \quad \sigma_R(\theta) = \sigma_0(\theta)(1 - A(\theta))$$

Analyzing Power



Polarization
(偏極度)

$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

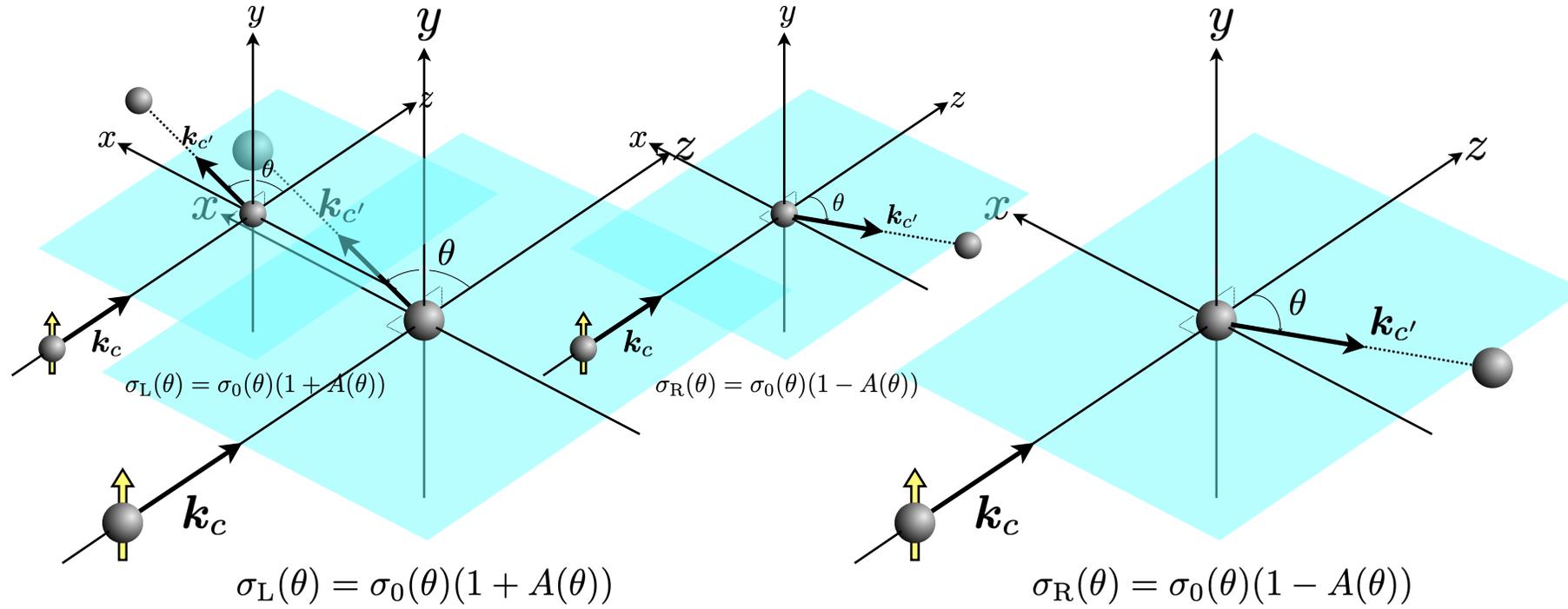
$$N_{\uparrow} = \frac{1 + P}{2} N_0 \quad N_{\downarrow} = \frac{1 - P}{2} N_0$$

Analyzing Power
(偏極分解能)

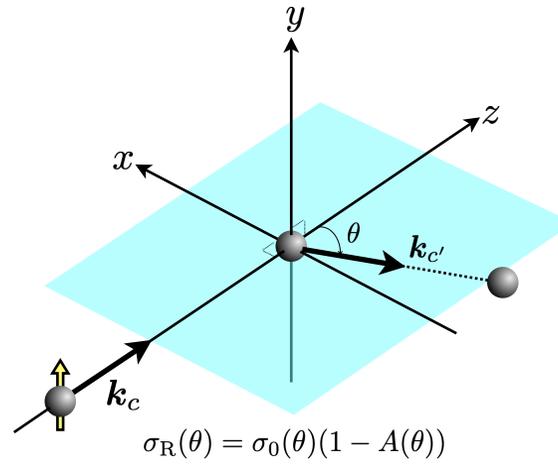
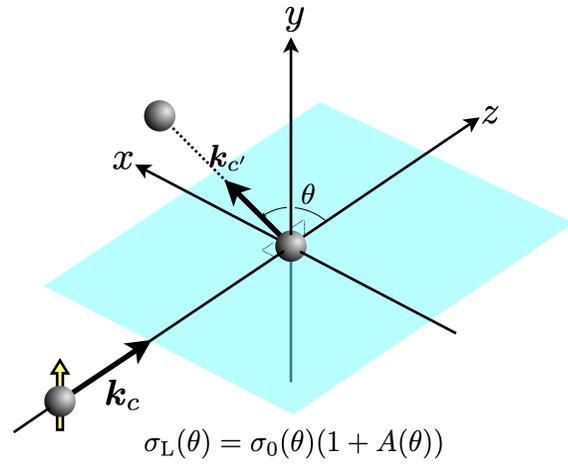
$$A(\theta) = \frac{\sigma_L(\theta) - \sigma_R(\theta)}{\sigma_L(\theta) + \sigma_R(\theta)}$$

$$\sigma_L(\theta) = \sigma_0(\theta)(1 + A(\theta)) \quad \sigma_R(\theta) = \sigma_0(\theta)(1 - A(\theta))$$

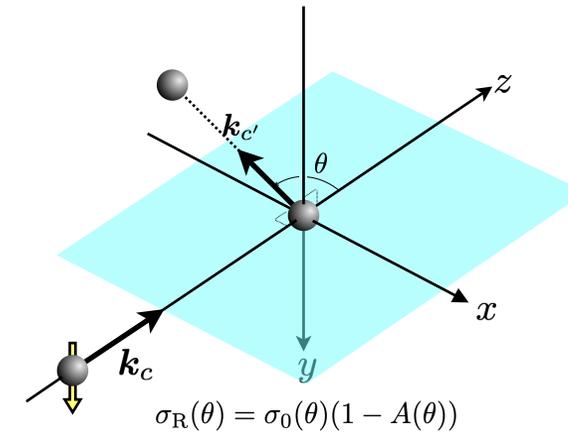
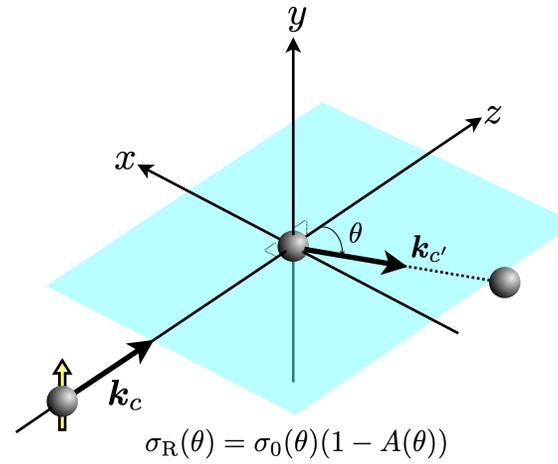
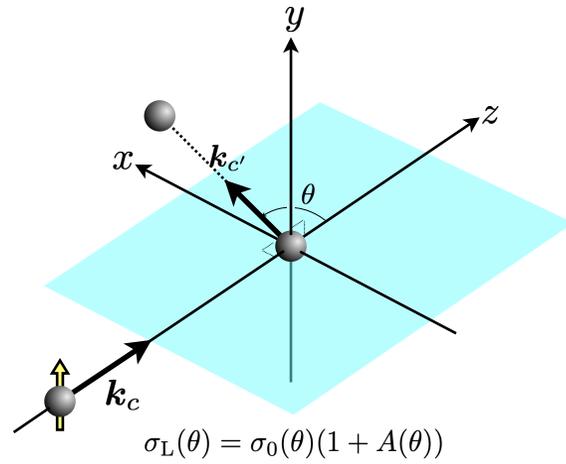
Analyzing Power



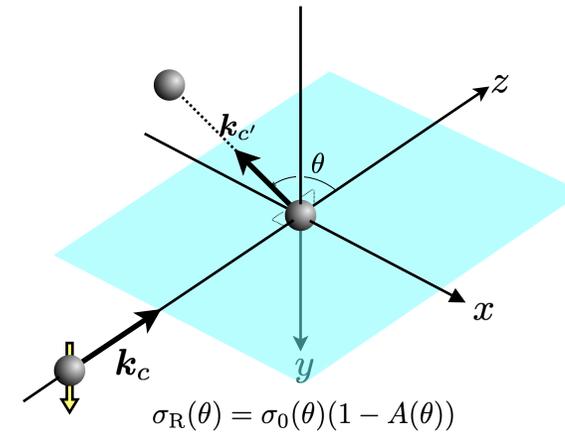
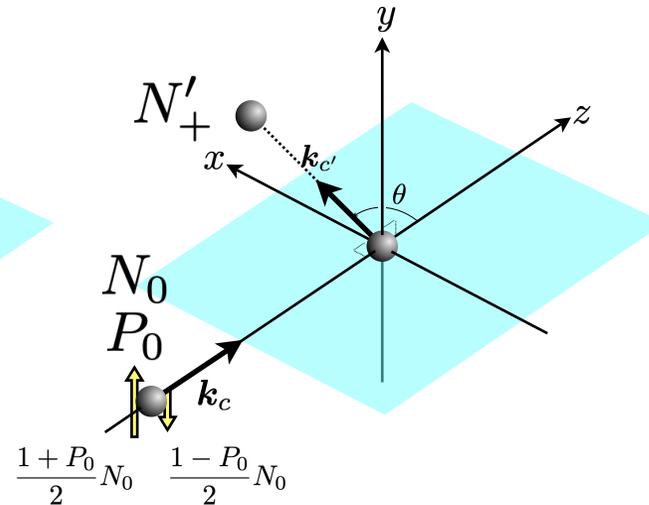
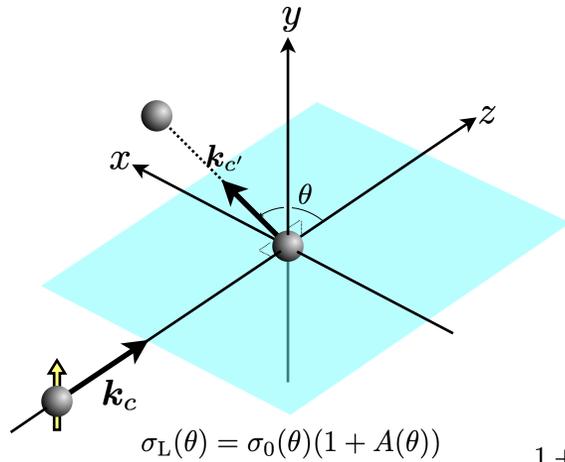
Analyzing Power



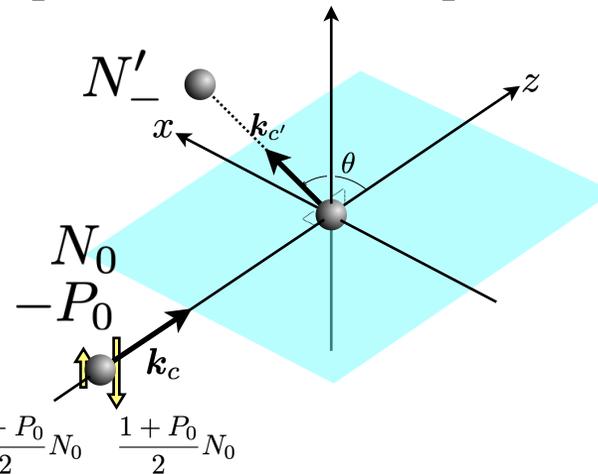
Analyzing Power



Analyzing Power



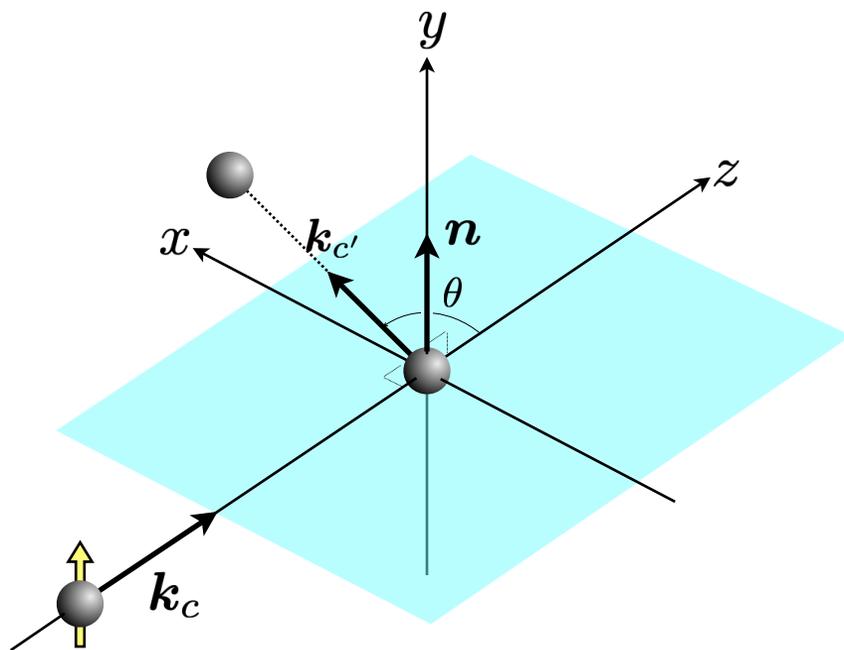
$$N'_+ = \frac{1 + P_0}{2} N_0 n \sigma_0 (1 + A) dz d\Omega + \frac{1 - P_0}{2} N_0 n \sigma_0 (1 - A) dz d\Omega = N_0 n \sigma_0 dz d\Omega (1 + P_0 A)$$



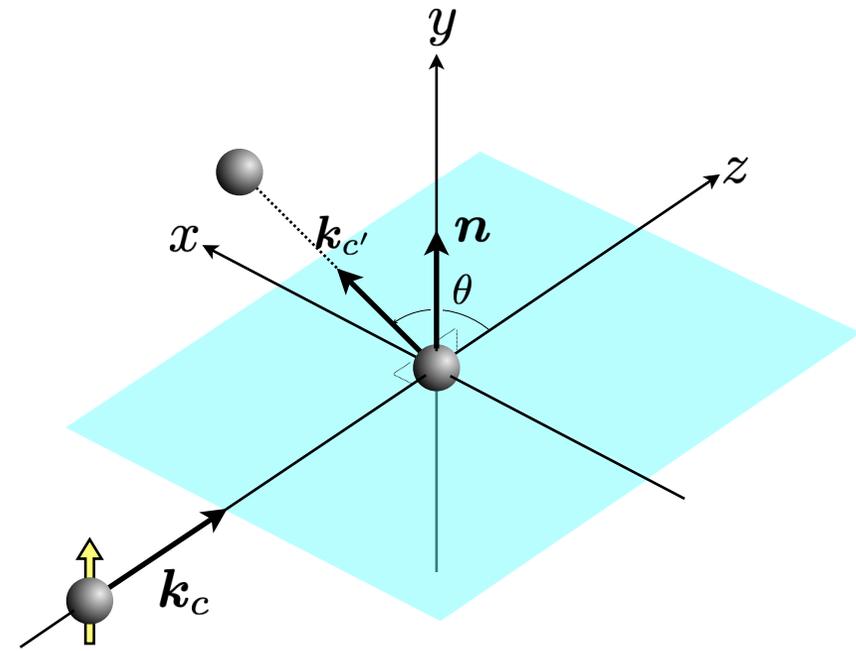
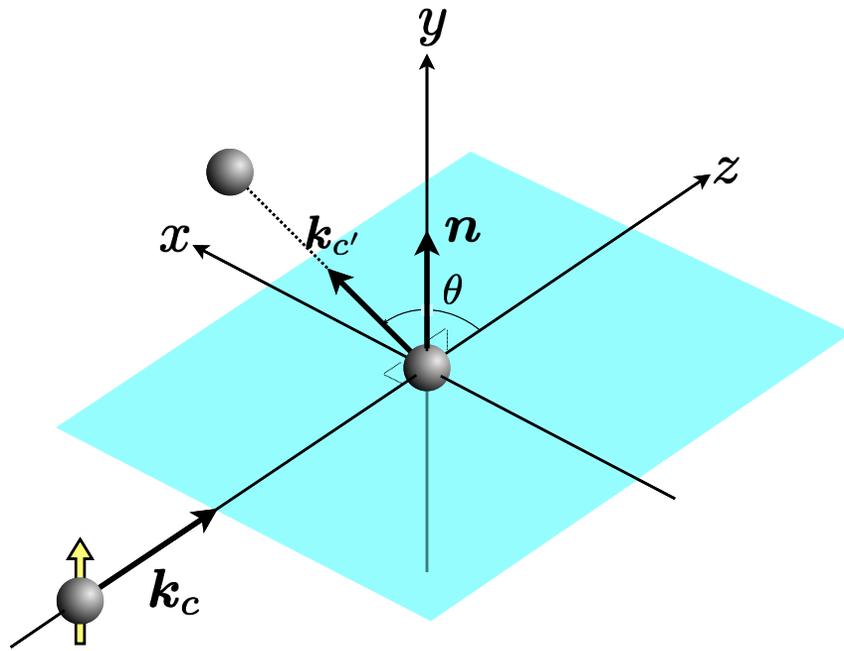
$$\frac{1}{P_0} \frac{N'_+ - N'_-}{N'_+ + N'_-} = A$$

$$N'_- = \frac{1 - P_0}{2} N_0 n \sigma_0 (1 + A) dz d\Omega + \frac{1 + P_0}{2} N_0 n \sigma_0 (1 - A) dz d\Omega = N_0 n \sigma_0 dz d\Omega (1 - P_0 A)$$

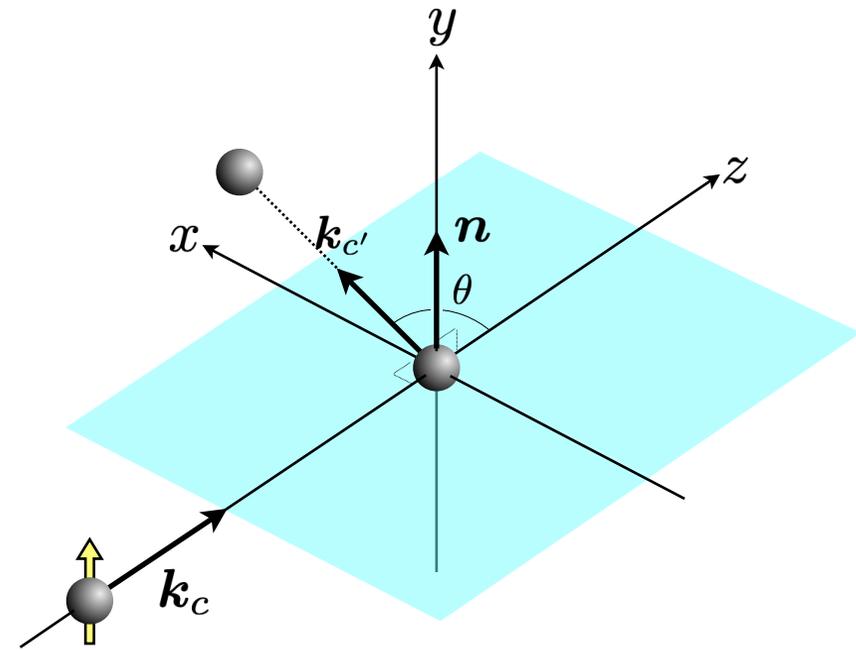
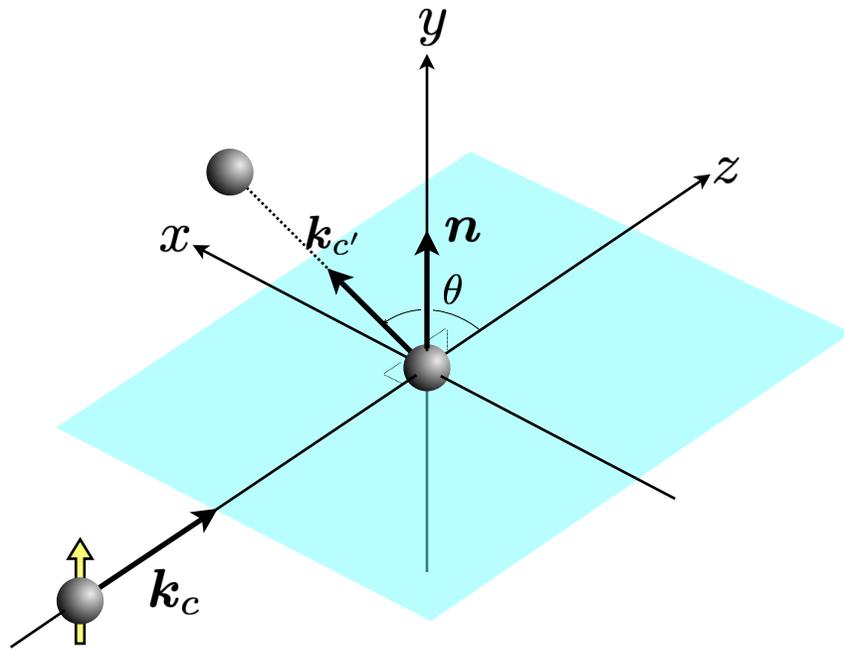
時間反転



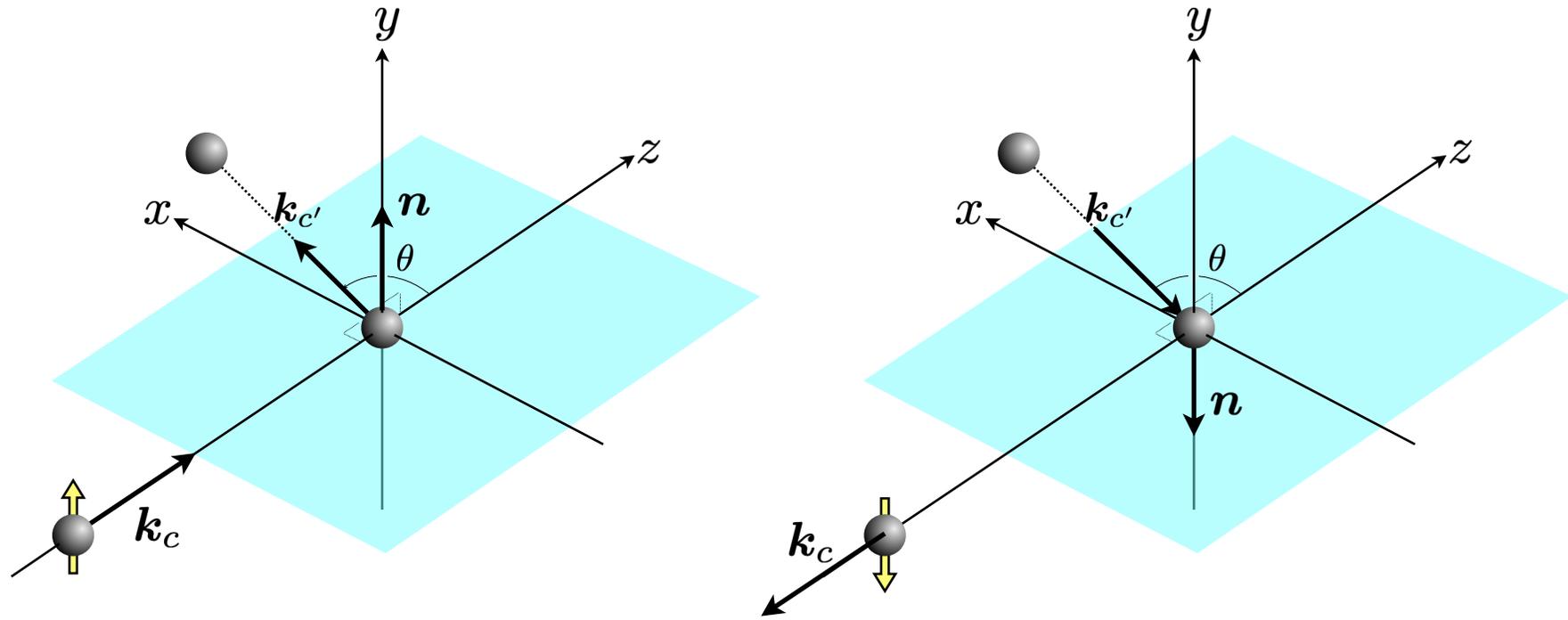
時間反転



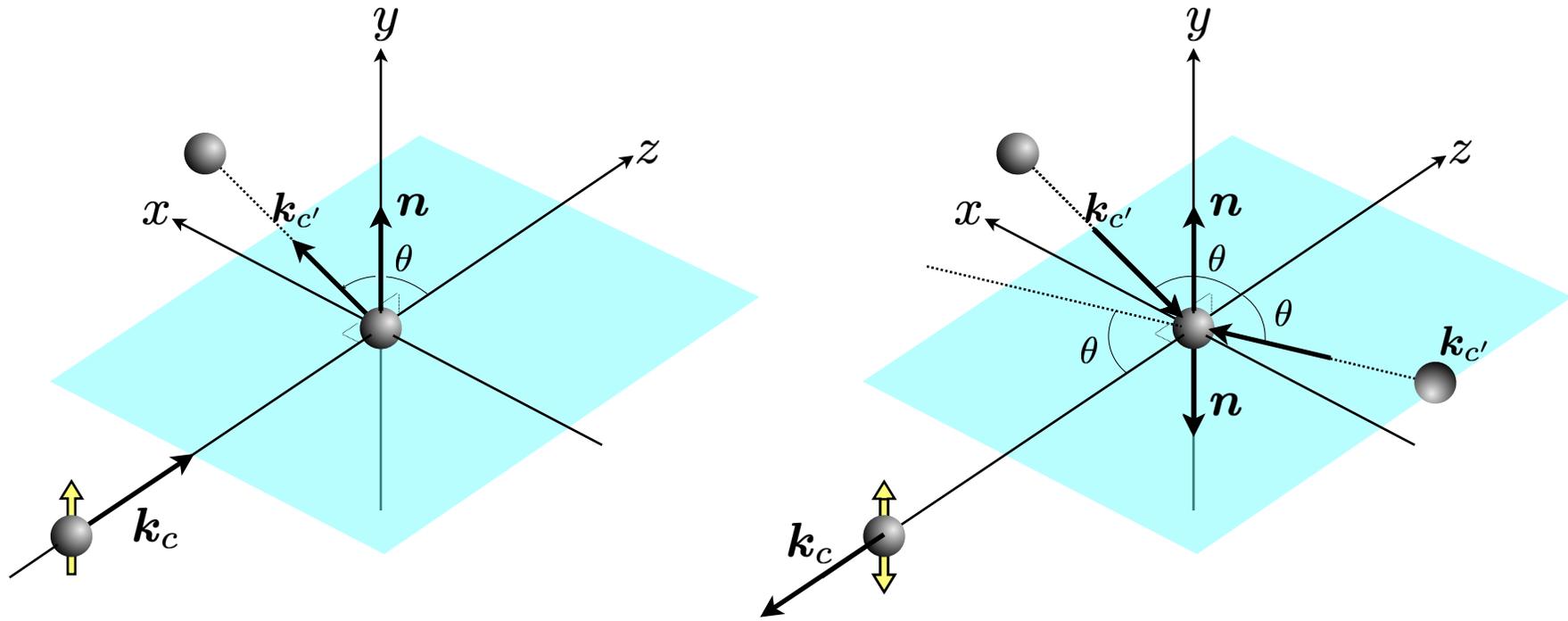
時間反転



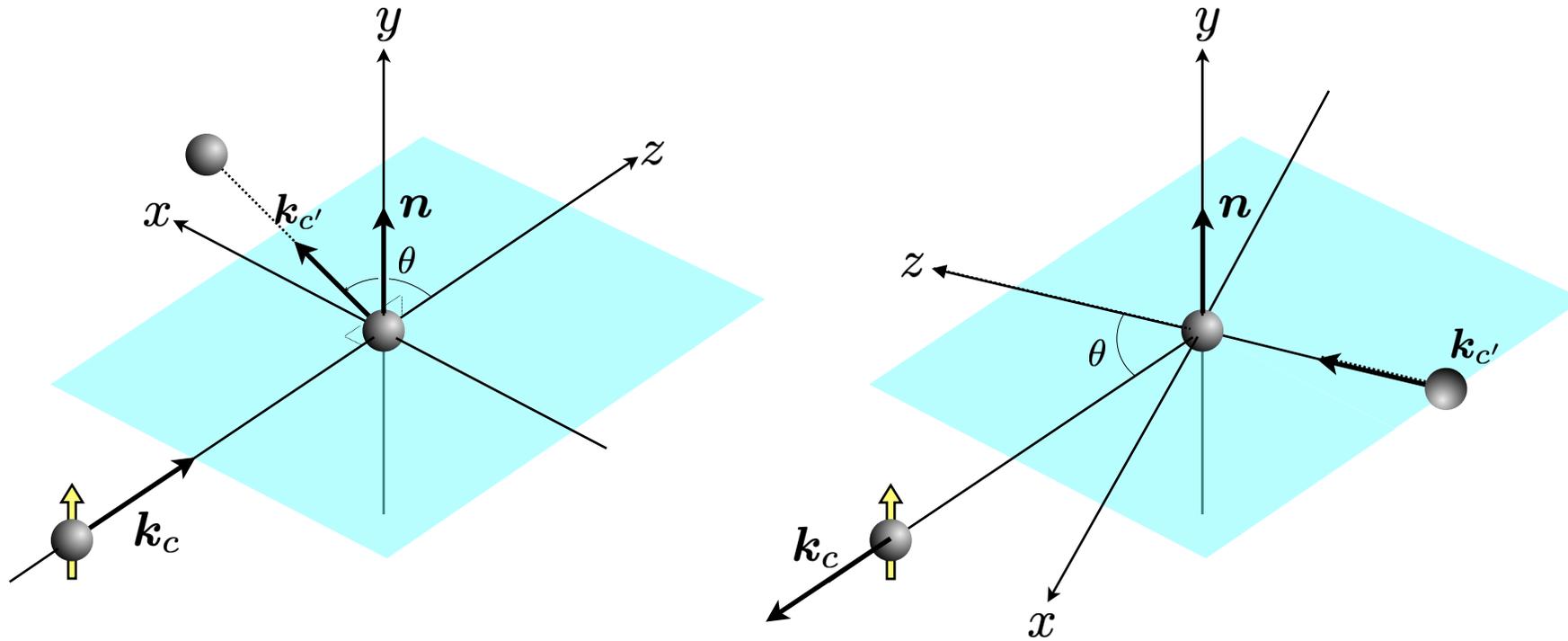
時間反転



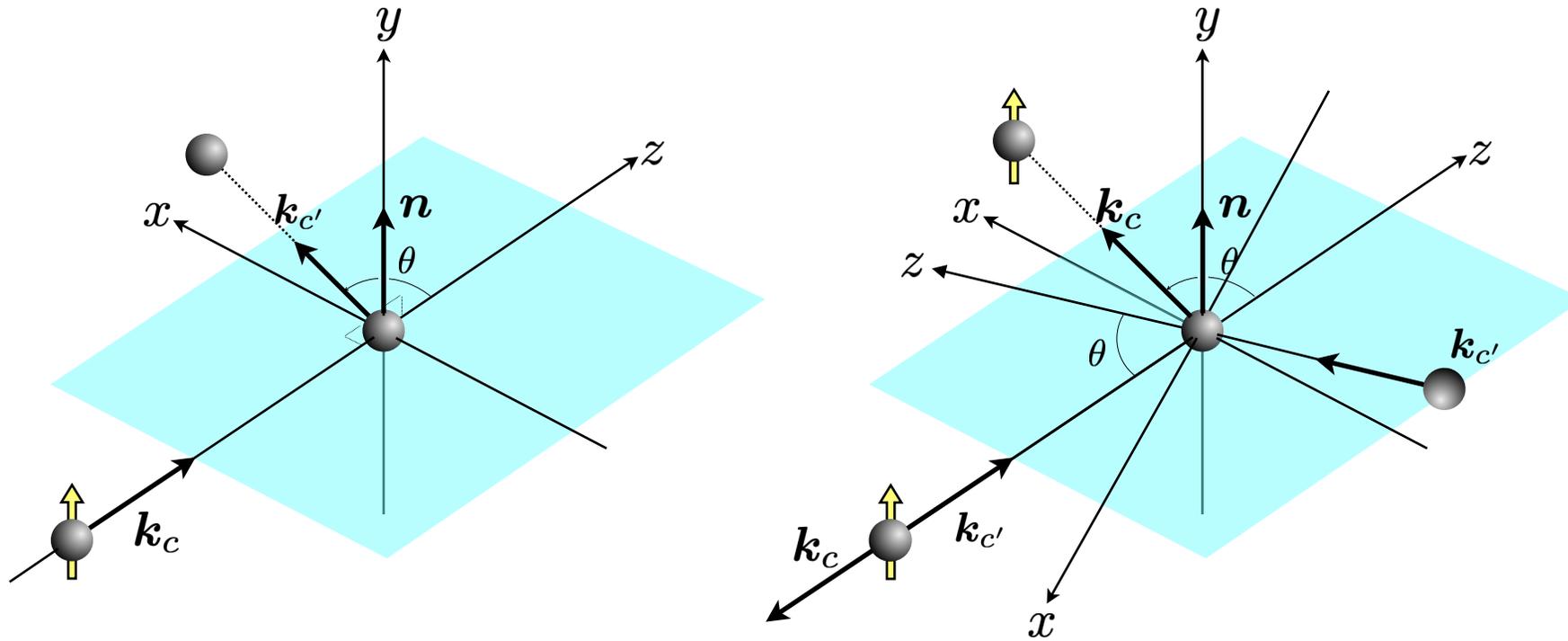
時間反転



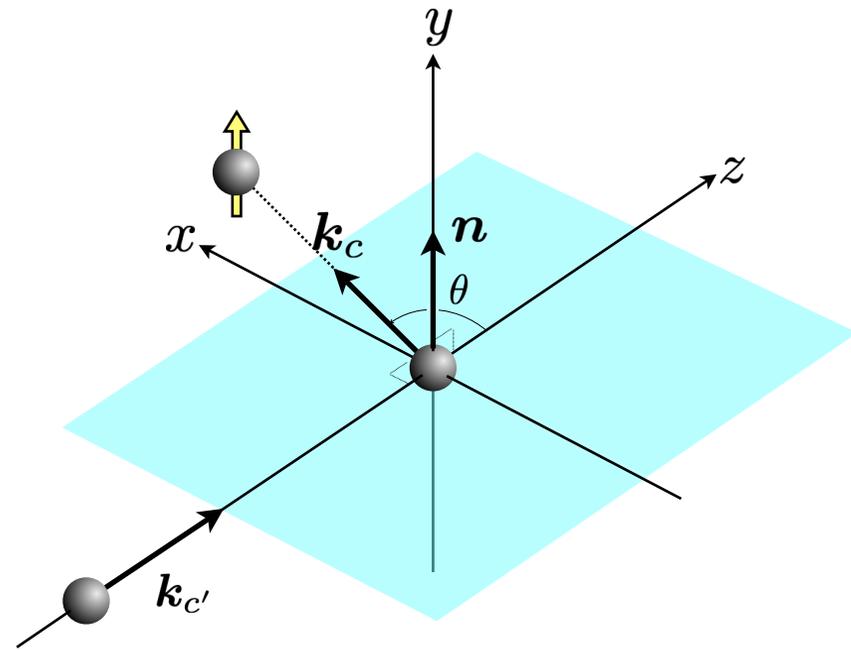
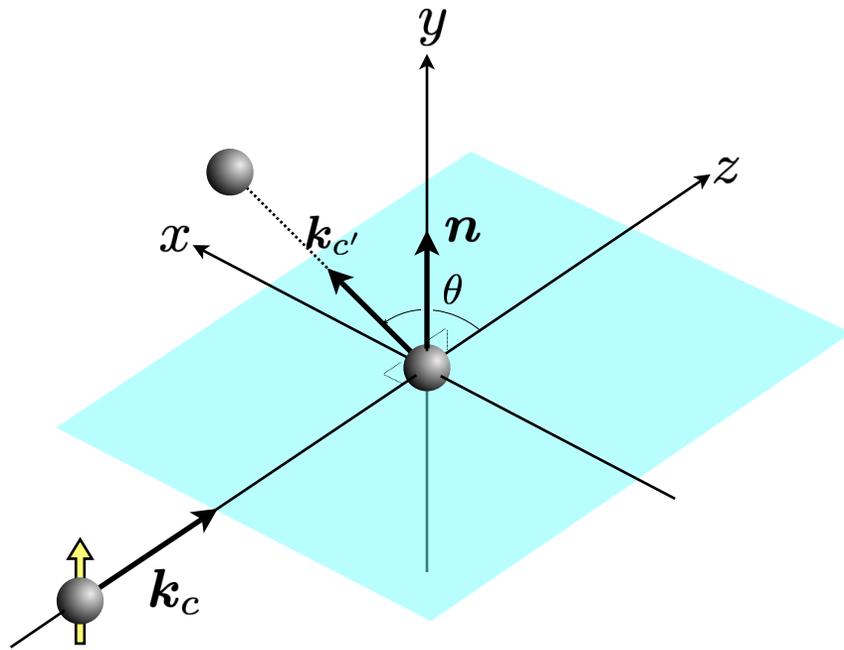
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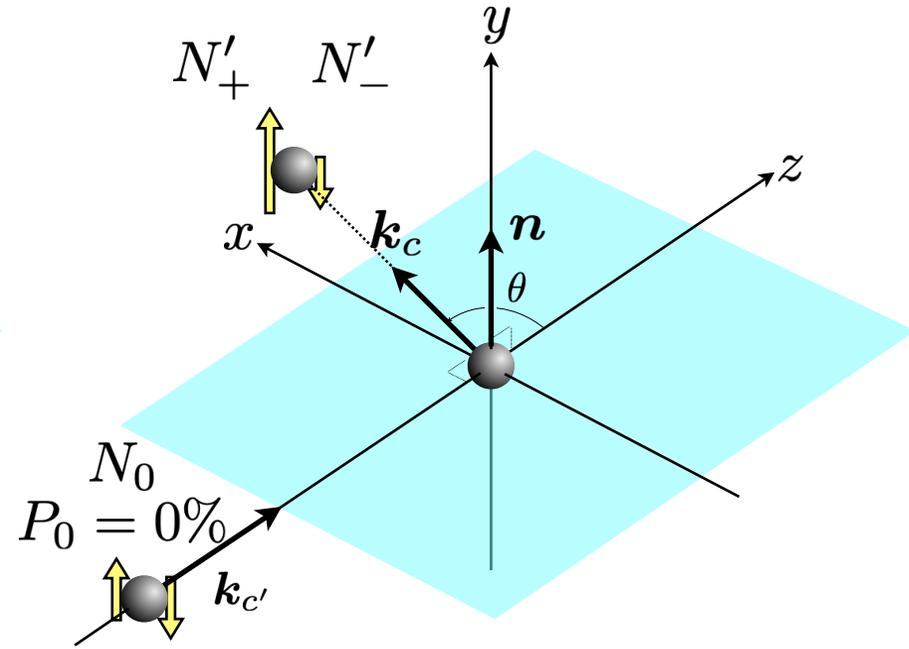
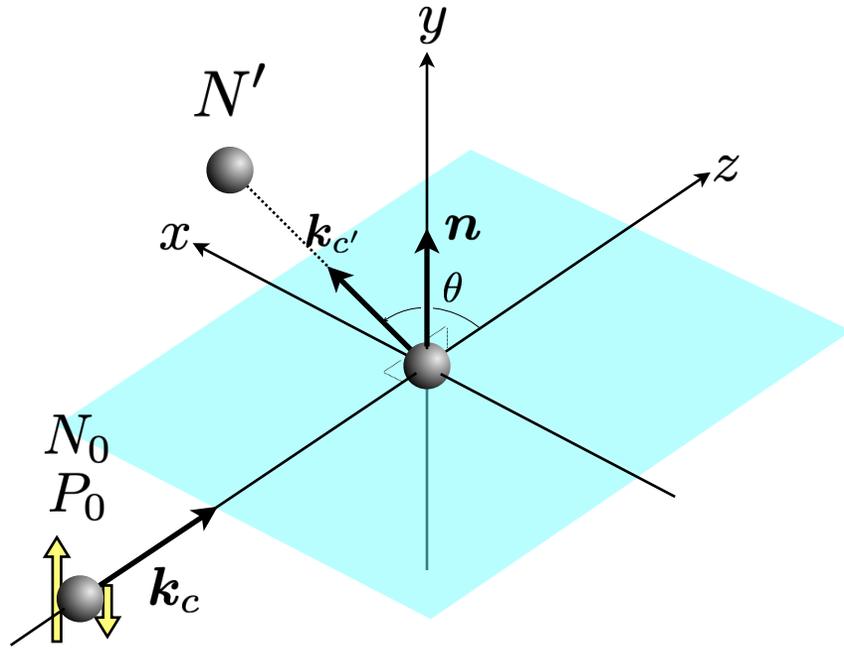
時間反転



時間反転



時間反転

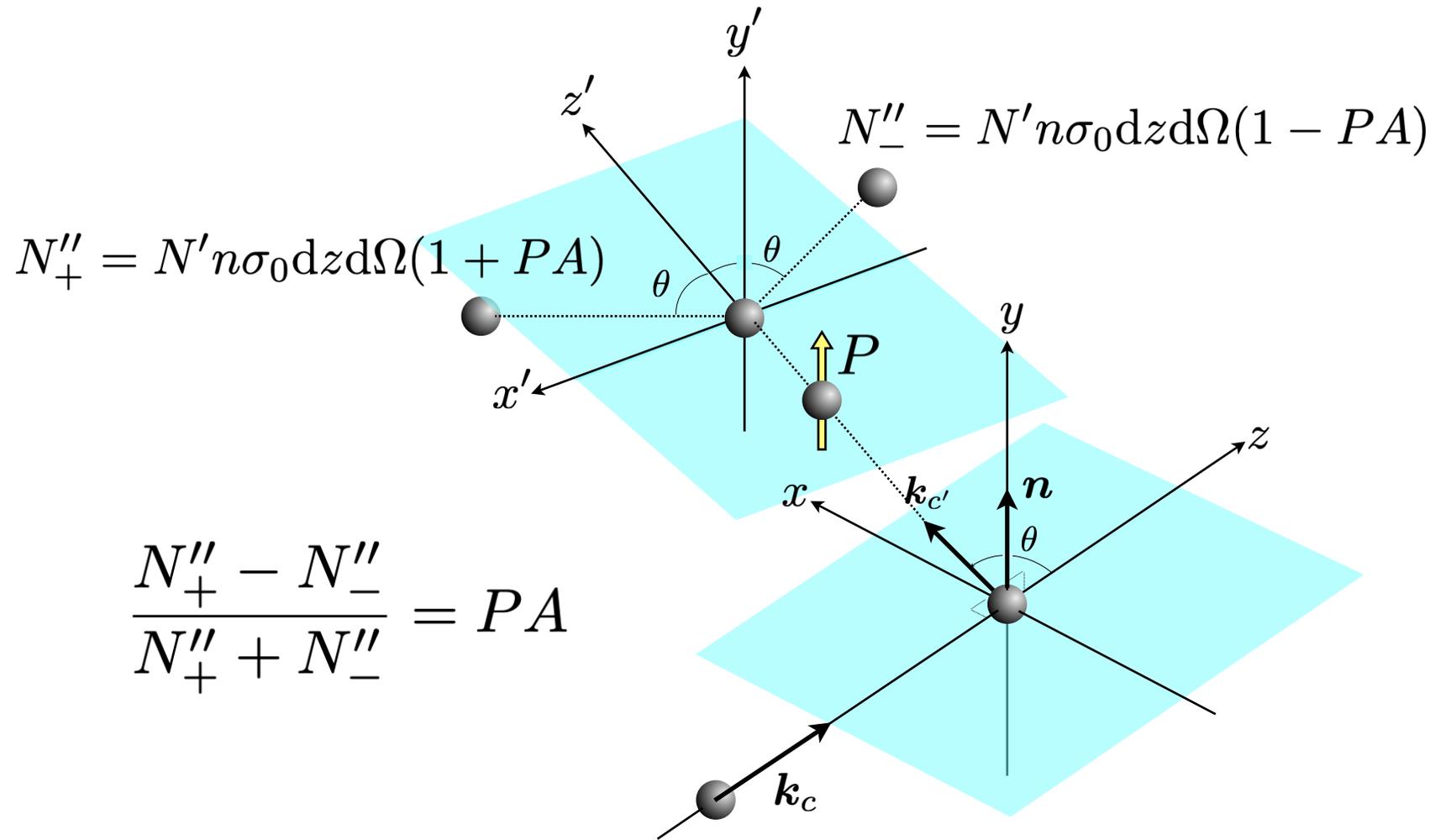


$$N'_+ = \frac{N_0}{2} n \sigma dz d\Omega (1 + A)$$

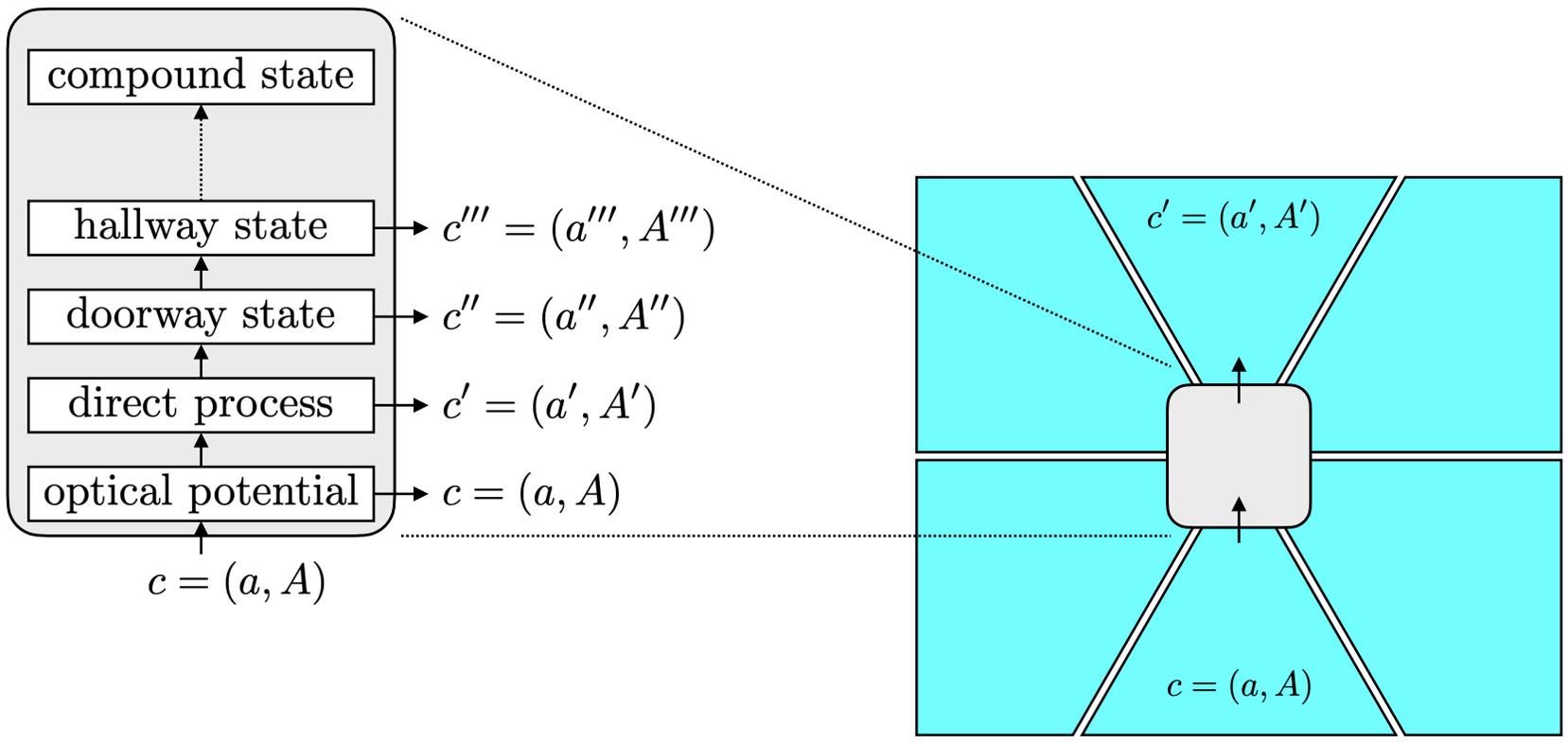
$$N'_- = \frac{N_0}{2} n \sigma dz d\Omega (1 - A)$$

$$P = \frac{N'_+ - N'_-}{N'_+ + N'_-} = A$$

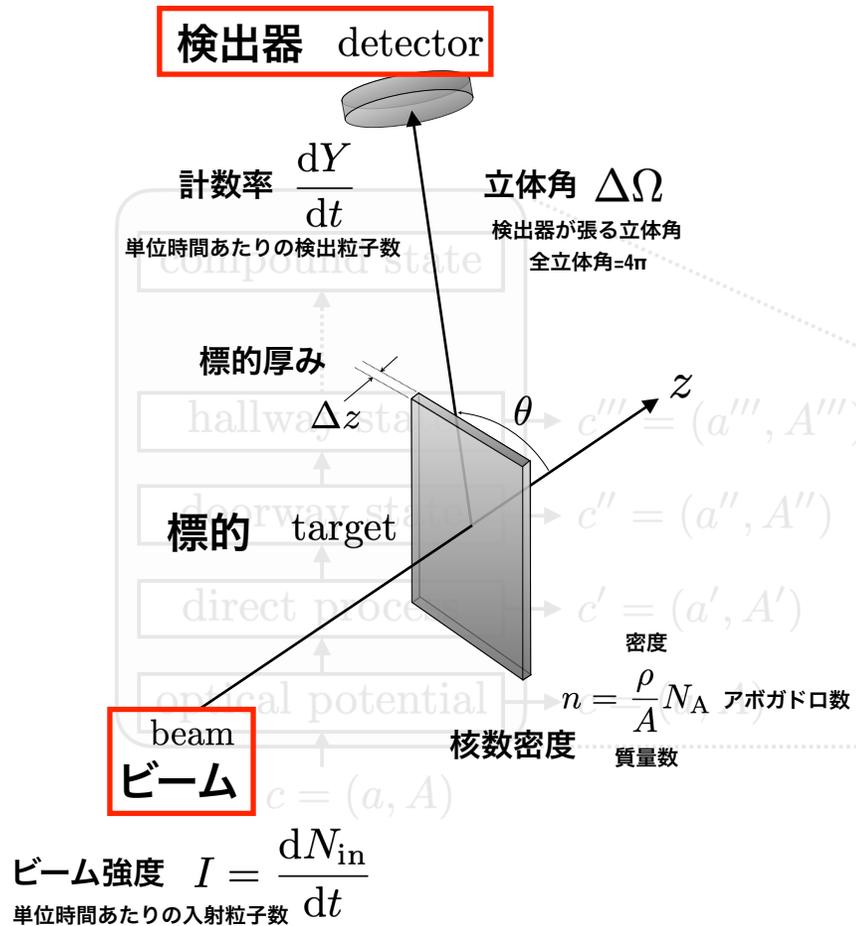
二回散乱



原子核反応



ビーム実験



原子核反応論

$$H = H_c + V_c = H_{c'} + V_{c'}$$

$$H_c \phi_c = E \phi_c \quad \phi_{c\mathbf{k}} = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}_c} \chi_c(i_c)$$

$$H_{c'} \phi_{c'} = E \phi_{c'} \quad \phi_{c'\mathbf{k}'} = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{k}'\cdot\mathbf{r}_{c'}} \chi_{c'}(i_{c'})$$

$$\Psi_{c'}^{(\pm)} = \phi_{c'} + \frac{1}{E - H_{c'} \pm i\epsilon} V_{c'} \Psi_{c'}^{(\pm)}$$

$$\Psi_{c\mathbf{k}_c}^{\pm} \rightarrow \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{k}\cdot\mathbf{r}_c} \chi_c(i_c) + \sum_{c'} \frac{e^{i\mathbf{k}_{c'}\cdot\mathbf{r}_{c'}}}{r_{c'}} f_{c'c}(\Omega_{c'}) \chi_{c'}(i_{c'}) \right]$$

$$f_{c'c}(\Omega_{c'}) = -\frac{(2\pi)^2 \mu_{c'}}{\hbar^2} T_{c'c}$$

$$T_{c'\mathbf{k}_{c'}, c\mathbf{k}_c} = \langle \phi_{c'\mathbf{k}_{c'}} | V_{c'} | \Psi_{c\mathbf{k}_c}^{(+)} \rangle$$

$$U_{c'l'm', clm} = 2\pi i \frac{\sqrt{\mu_c \mu_{c'} k_c k_{c'}}}{\hbar^2} T_{c'l'm', clm}$$

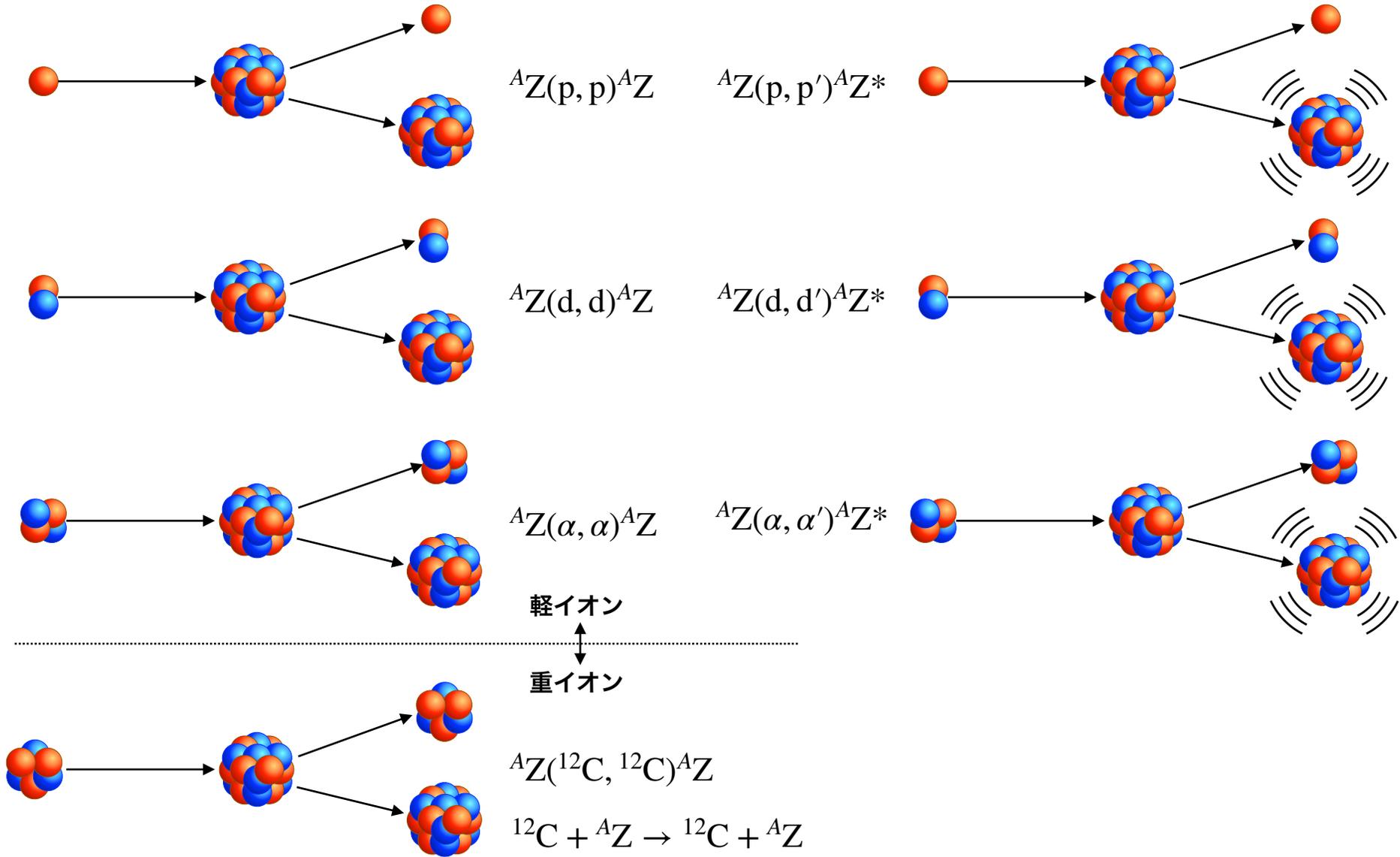
$$f_{c'c}(\Omega_{\mathbf{k}_{c'}}) = \sqrt{\frac{v_c}{v_{c'}}} \frac{4\pi i}{2k_c} \sum_{lm'l'm'} Y_{l'm'}(\Omega_{\mathbf{k}_{c'}}) U_{c'l'm', clm} Y_{lm}^*(\Omega_{\mathbf{k}_c})$$

$$\frac{d\sigma}{d\Omega} = \lim_{\Delta\Omega \rightarrow 0} \frac{dY}{dt} \frac{1}{In\Delta z \Delta\Omega}$$

微分断面積

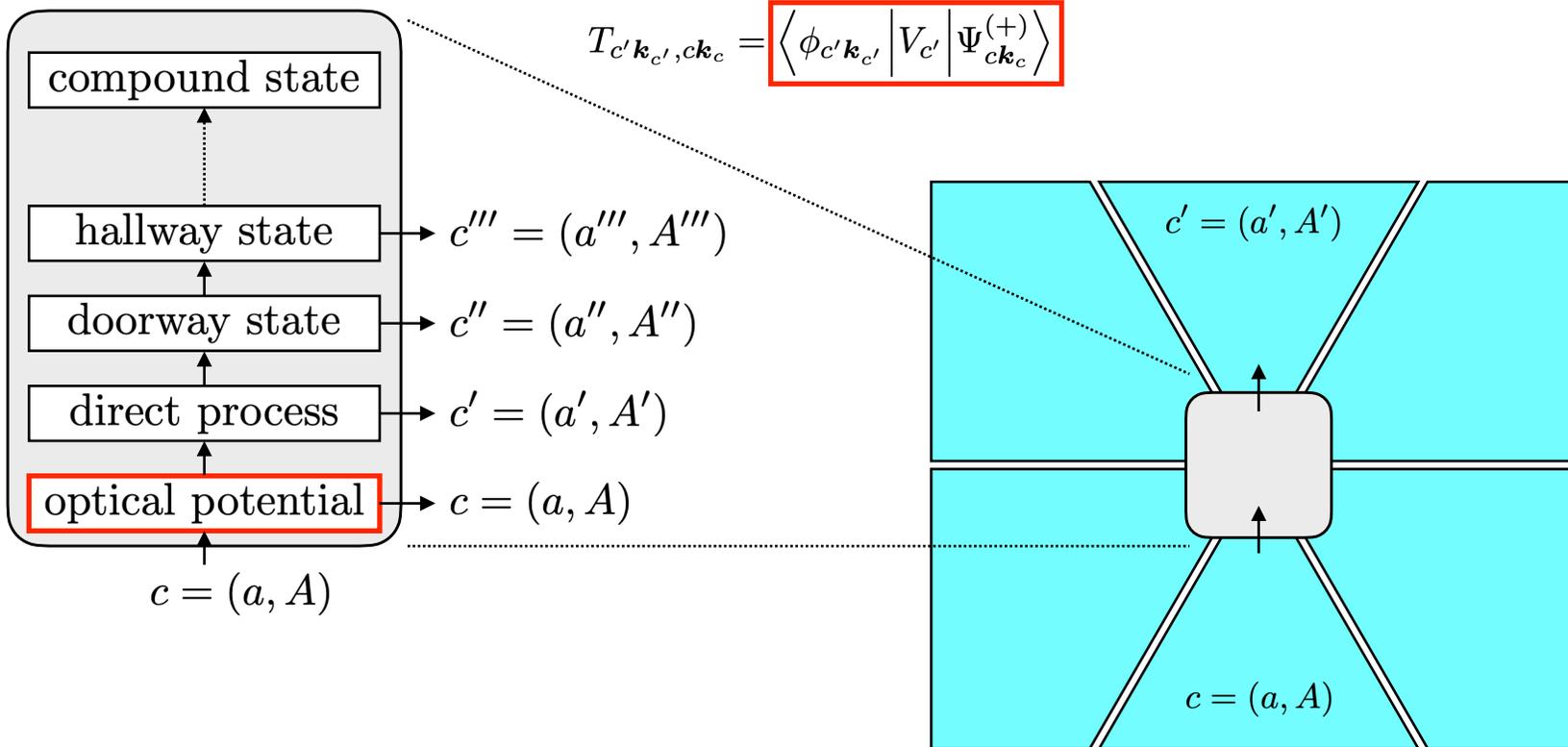
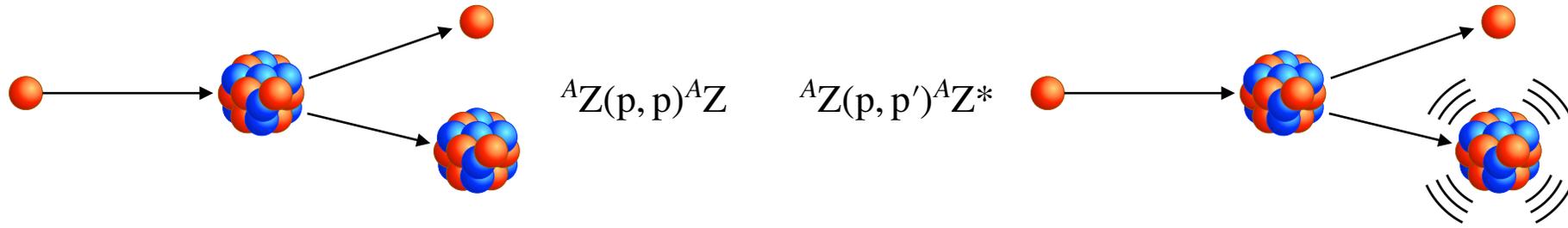
$$\frac{d\sigma_{c'c}}{d\Omega_{c'}} = \frac{v_{c'}}{v_c} |f_{c'c}(\Omega_{c'})|^2$$

ポテンシャル散乱 (potential scattering)

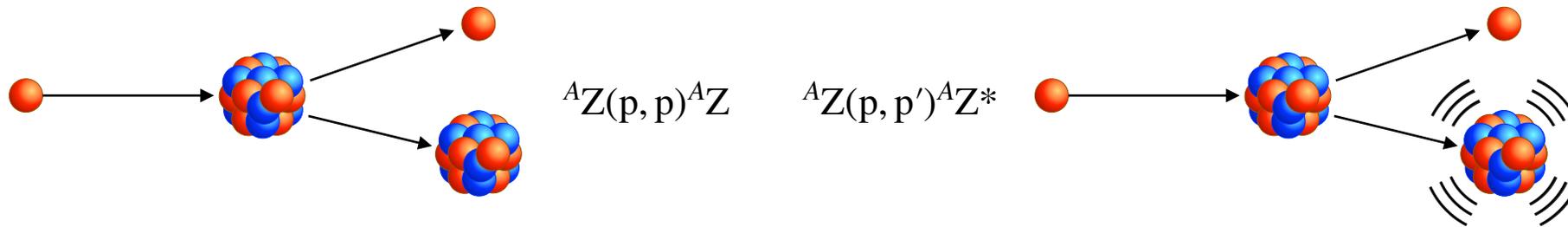


ポテンシャル散乱

(potential scattering)



ポテンシャル散乱 (potential scattering)



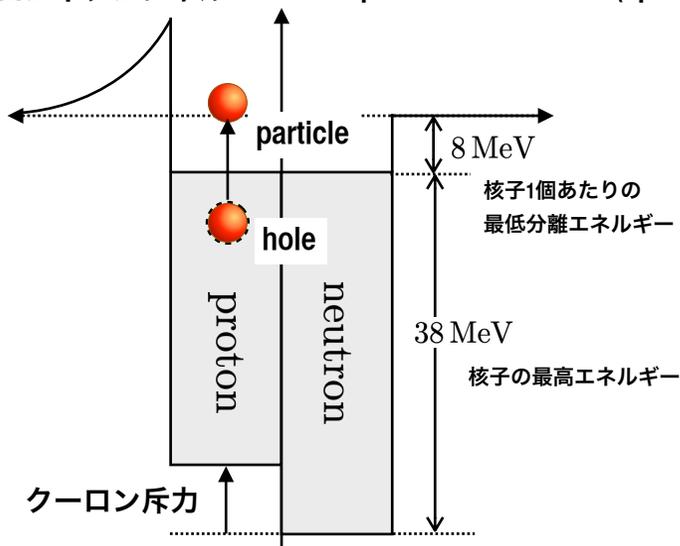
$$T_{c'k_{c'}, ck_c} = \langle \phi_{c'k_{c'}} | V_{c'} | \Psi_{ck_c}^{(+)} \rangle$$

ボルン近似

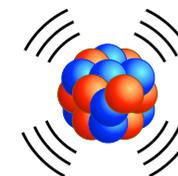
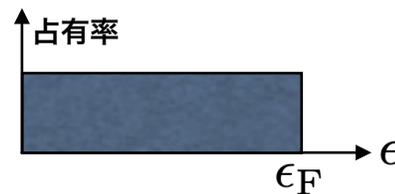
$$\langle \phi | V | \phi \rangle \quad \text{平面波ボルン近似} \\ \text{PWBA (Plane Wave Born Approximation)}$$

原子核外では
クーロンポテンシャル

1-particle 1-hole state (1p1h), ...

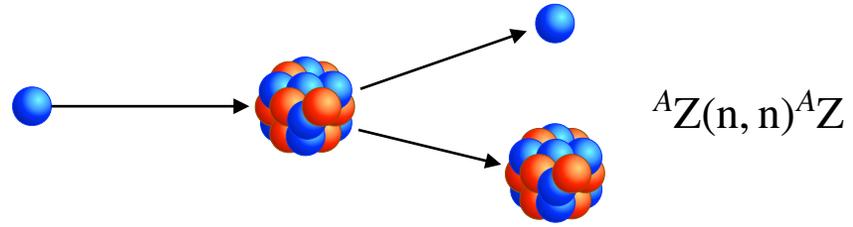


フェルミ分布を仮定



ポテンシャル散乱

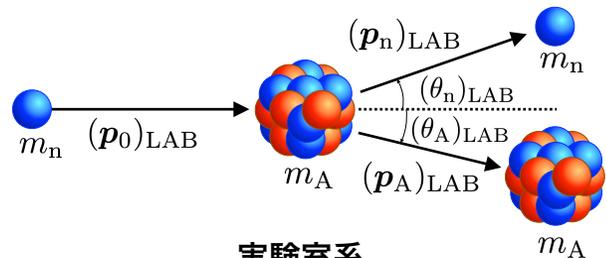
(potential scattering)



ポテンシャル散乱

(potential scattering)

$$H_{\text{rel}} = \frac{p^2}{2\mu} + V(r)$$



実験室系
laboratory frame

$${}^A Z(n, n){}^A Z$$

$$H = \frac{p_n^2}{2m_n} + \frac{p_A^2}{2m_A} + U(r_n - r_A)$$

運動学

(kinematics)

$$\tan(\theta_n)_{\text{LAB}} = \frac{m_A \sin \theta}{m_n + m_A \cos \theta}$$

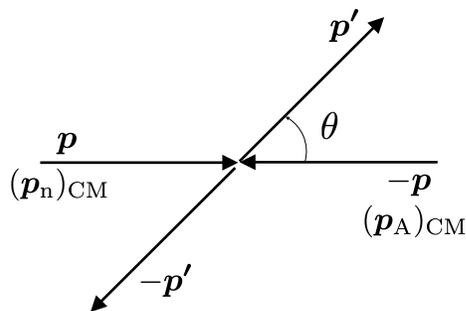
$$(\theta_A)_{\text{LAB}} = \frac{\pi - \theta}{2}$$

重心 $R = \frac{m_n(r_n)_{\text{LAB}} + m_A(r_A)_{\text{LAB}}}{m_n + m_A}$

相対位置 $r = (r_n)_{\text{LAB}} - (r_A)_{\text{LAB}} \quad P = (m_n + m_A)\dot{R} = (p_n)_{\text{LAB}} + (p_A)_{\text{LAB}}$

ポテンシャルが相対位置のみの関数であるとき

重心系
center-of-mass frame



$$H = \frac{P^2}{2M} + \frac{p^2}{2\mu} + U(r)$$

$$H_{\text{CM}} = \frac{P^2}{2M} \quad H_{\text{rel}} = \frac{p^2}{2\mu} + U(r)$$

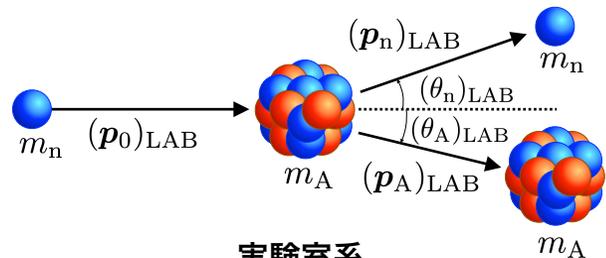
$$M = m_n + m_A \quad \mu = \frac{m_n m_A}{m_n + m_A}$$

換算質量

ポテンシャル散乱

(potential scattering)

$$H_{\text{rel}} = \frac{p^2}{2\mu} + U(r)$$



実験室系
laboratory frame

$${}^A Z(n, n){}^A Z$$

$$H = \frac{p_n^2}{2m_n} + \frac{p_A^2}{2m_A} + U(r_n - r_A)$$

運動学

(kinematics)

$$\tan(\theta_n)_{\text{LAB}} = \frac{m_A \sin \theta}{m_n + m_A \cos \theta}$$

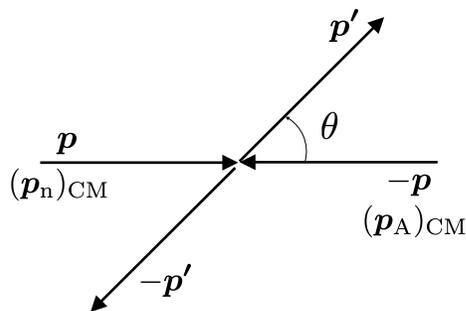
$$(\theta_A)_{\text{LAB}} = \frac{\pi - \theta}{2}$$

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相対位置 $r = (r_n)_{\text{LAB}} - (r_A)_{\text{LAB}} \quad P = (m_n + m_A)\dot{R} = (p_n)_{\text{LAB}} + (p_A)_{\text{LAB}}$

ポテンシャルが相対位置のみの関数であるとき

重心系
center-of-mass frame



$$H = \frac{P^2}{2M} + \frac{p^2}{2\mu} + U(r)$$

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換算質量

ポテンシャル散乱 (potential scattering)

$$H_{\text{rel}} = \frac{p^2}{2\mu} + U(\mathbf{r})$$

時間に依存しないポテンシャルによる散乱

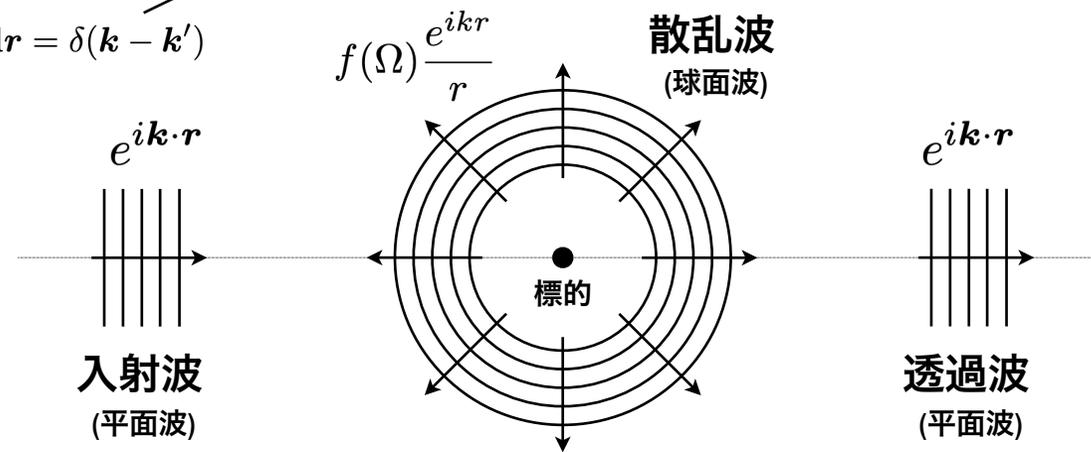
$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2\mu} \nabla^2 + U(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

時間を変数分離する

$$\psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt/\hbar} \quad \text{とおくと} \quad \left[-\frac{\hbar^2}{2\mu} \nabla^2 + U(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k}\cdot\mathbf{r}} + f(\Omega) \frac{e^{ikr}}{r} \right)$ を満たす解を求める

$$\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}} d\mathbf{r} = \delta(\mathbf{k}-\mathbf{k}')$$



ポテンシャル散乱 (potential scattering)

$$H_{\text{rel}} = \frac{p^2}{2\mu} + U(\mathbf{r})$$

時間に依存しないポテンシャルによる散乱

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2\mu} \nabla^2 + U(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

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球面座標で表して $\left[-\frac{\hbar^2}{2\mu} \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right\} + U(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$

整えると $\frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 \psi}{\partial \varphi^2} + \frac{2\mu r^2}{\hbar^2} (E - U(\mathbf{r})) \psi = 0$

ポテンシャルが角度に依存しない時 $\psi(\mathbf{r}) = R(r)Y(\theta, \varphi)$ において変数分離

$$\frac{1}{R} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{2\mu r^2}{\hbar^2} (E - U(r)) + \frac{1}{Y} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial Y}{\partial \theta} \right) + \frac{1}{Y} \frac{1}{\sin^2 \theta} \frac{\partial^2 Y}{\partial \varphi^2} = 0$$

$$= l(l+1) \quad = -l(l+1) \quad \text{角度方向の解は球面調和関数}$$

動径方向には遠心力ポテンシャルが加わる

$$\frac{\hbar^2}{2\mu} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left(E - U(r) - \frac{\hbar^2}{2\mu} \frac{l(l+1)}{r^2} \right) R = 0$$

低エネルギーの場合 $l=0$ (s波)のみを考えれば十分

$$Y_{lm}(\theta, \varphi) = (-1)^{(m+|m|)/2} \sqrt{\frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} P_l^{|m|}(\cos \theta) e^{im\varphi}$$

$$P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x) \quad P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2-1)^l$$

ポテンシャル散乱 (potential scattering)

等方ポテンシャルによる散乱

$$\frac{\hbar^2}{2\mu} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left(E - U(r) - \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) R = 0$$

時間に依存しないポテンシャルによる散乱

低エネルギーの場合 $l=0$ (s波)のみを考えれば十分

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2\mu} \nabla^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t) + \frac{2\mu r^2}{\hbar^2} (E - U(r)) R = 0$$

時間を変数分離する

$$\psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt/\hbar} \quad \text{とおくと} \quad \left[-\frac{\hbar^2}{2\mu} \nabla^2 + U(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

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ポテンシャル散乱 (potential scattering)

等方ポテンシャルによる散乱

$$\frac{\hbar^2}{2\mu} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left(E - U(r) - \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) R = 0$$

低エネルギーの場合 $l=0$ (s波)のみを考えれば十分

$$\frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{2\mu r^2}{\hbar^2} (E - U(r)) R = 0$$

$u(r) = rR(r)$ とおくと

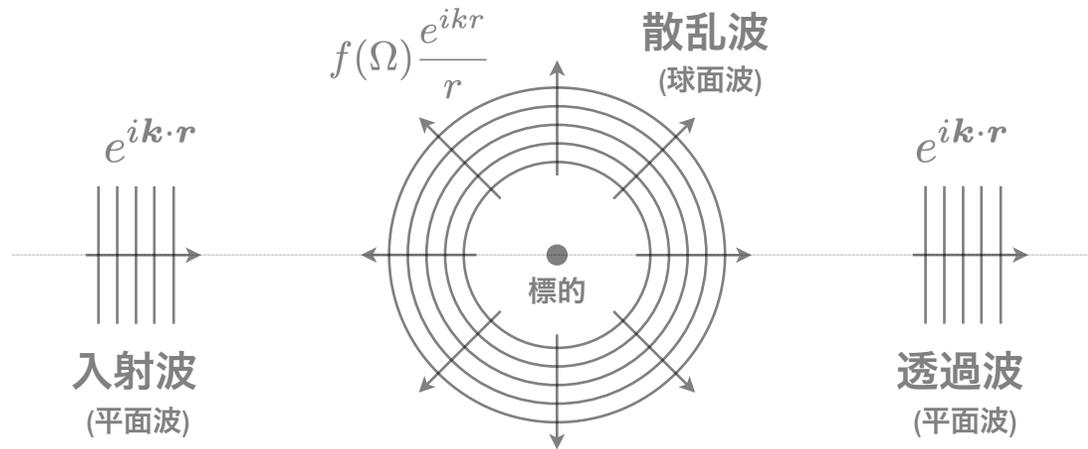
$$\frac{d^2 u}{dr^2} + k^2 u = 0 \quad k^2 = \frac{2\mu}{\hbar^2} (E - U(r))$$

$u(r)$ が原点で有界であるためには $u(0) = 0$ であることが必要 $u(r) \xrightarrow{r \rightarrow 0} C \sin kr$

$U(r)$ が遠方で十分にはやく0に近づくのであれば $u(r) \xrightarrow{r \rightarrow \infty} C \sin(k_0 r + \delta)$, $k_0^2 = \frac{2\mu}{\hbar^2} E$

散乱振幅は $\frac{1}{k_0} \frac{\sin \delta}{\cos \delta}$ ↑ 位相のずれ (phase shift)

漸近的境界条件 $\psi^+(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \left(e^{i\mathbf{k} \cdot \mathbf{r}} + \underbrace{f(\Omega) \frac{e^{ikr}}{r}}_{\text{散乱振幅}} \right)$



ポテンシャル散乱 (potential scattering)

$u(r) = rR(r)$ とおくと

$$\frac{d^2 u}{dr^2} + k^2 u = 0 \quad k^2 = \frac{2\mu}{\hbar^2} (E - U(r))$$

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$$\frac{\tan kR_{\text{nuc}}}{kR_{\text{nuc}}} = \frac{\tan(k_0 R_{\text{nuc}} + \delta)}{k_0 R_{\text{nuc}}} \quad \text{散乱振幅は } \frac{1}{k_0} \frac{\sin \delta}{\cos \delta} \quad \text{位相のずれ (phase shift)}$$

$$k_0 \cot \delta = -\frac{1}{a} + \frac{r_0}{2} k_0^2 - pr_0^3 k_0^4 + \dots \quad \begin{array}{l} a : \text{scattering length} \quad \text{散乱長} \\ r_0 : \text{effective range} \quad \text{有効距離} \end{array}$$

等方ポテンシャルによる散乱

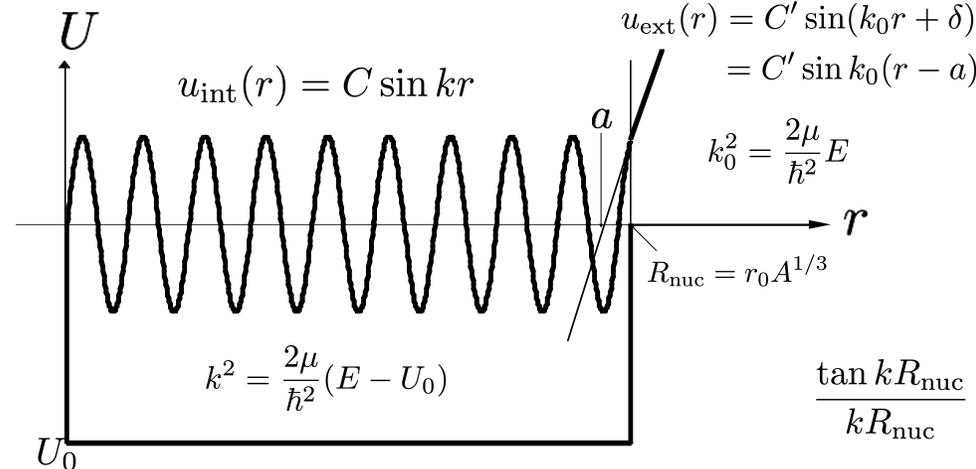
$$\frac{\hbar^2}{2\mu} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left(E - U(r) - \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) R = 0$$

低エネルギーの場合 $l=0$ (s波)のみを考えれば十分

$$\frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{2\mu r^2}{\hbar^2} (E - U(r)) R = 0$$

箱型ポテンシャル

$$U(r) = \begin{cases} U_0 & (r < R_{\text{nuc}}) \\ 0 & (r > R_{\text{nuc}}) \end{cases}$$



$$\frac{\tan kR_{\text{nuc}}}{kR_{\text{nuc}}} = \frac{\tan k_0(R_{\text{nuc}} - a)}{k_0 R_{\text{nuc}}}$$

ポテンシャル散乱 (potential scattering)

$$a = R \left(1 - \frac{\tan \kappa R_{\text{nuc}}}{\kappa R_{\text{nuc}}} \right)$$

$$\kappa^2 = k^2 - k_0^2$$

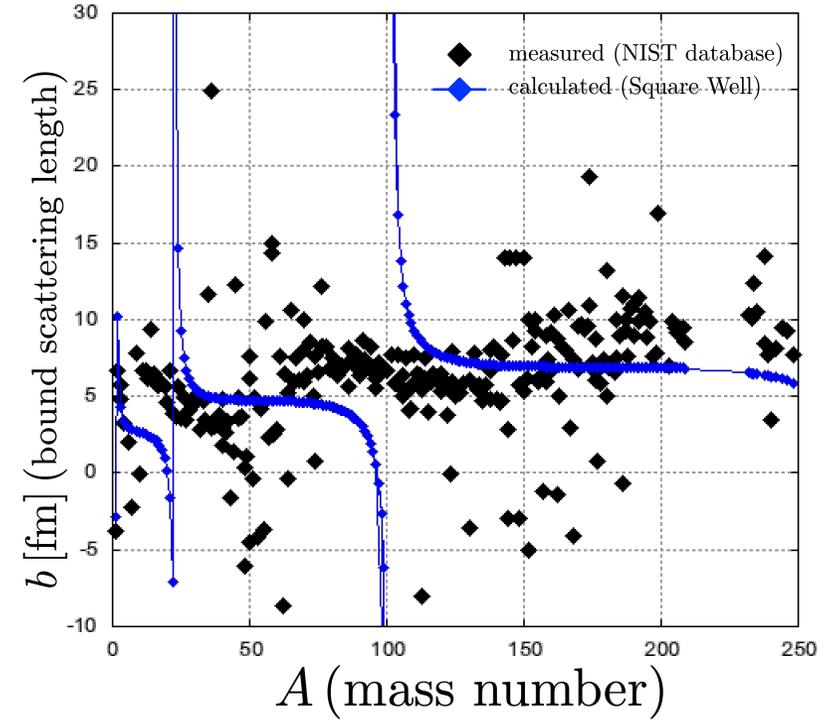
$$y = x \left(1 - \frac{\tan x}{x} \right)$$

$$U_0 = -38 \text{ MeV}$$

$$R_{\text{nuc}} = 1.25 A^{1/3} \text{ fm}$$

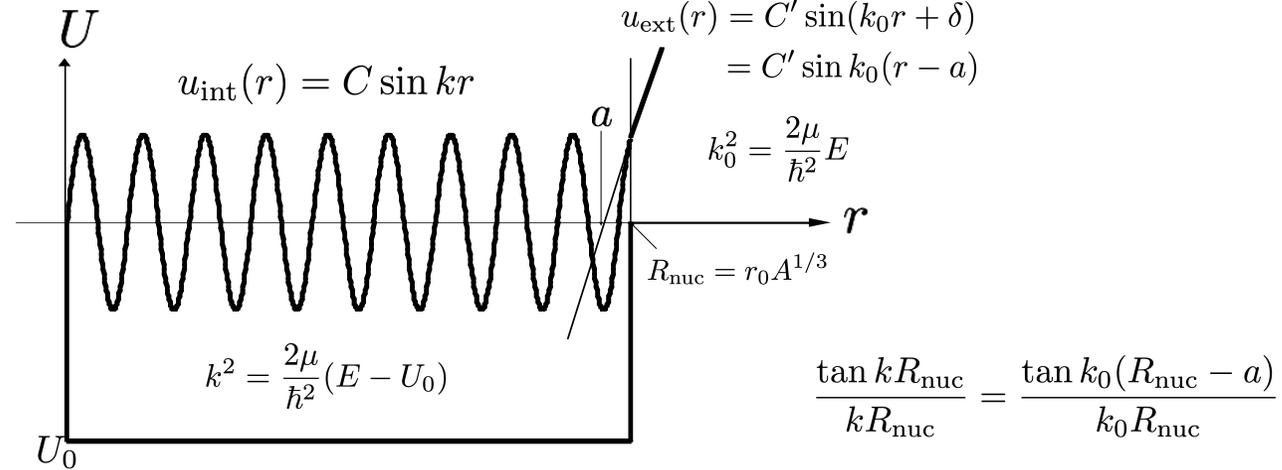


測定値との比較



箱型ポテンシャル

$$U(r) = \begin{cases} U_0 & (r < R_{\text{nuc}}) \\ 0 & (r > R_{\text{nuc}}) \end{cases}$$



$$\frac{\tan k R_{\text{nuc}}}{k R_{\text{nuc}}} = \frac{\tan k_0 (R_{\text{nuc}} - a)}{k_0 R_{\text{nuc}}}$$

ポテンシャル散乱 (potential scattering)

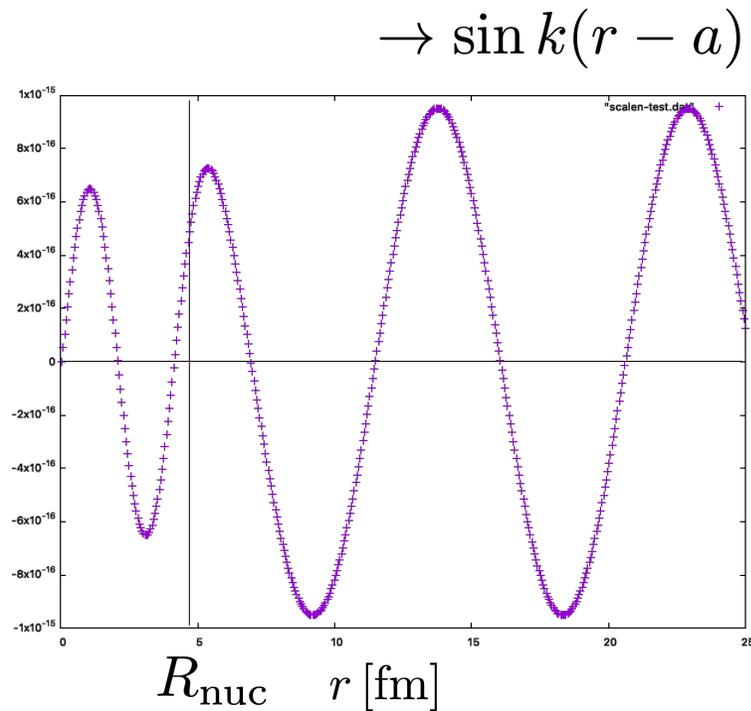
Woods-Saxon ポテンシャル

$$U(r) = \frac{U_0}{1 + e^{(r-R_{\text{nuc}})/a_{\text{dif}}}}$$

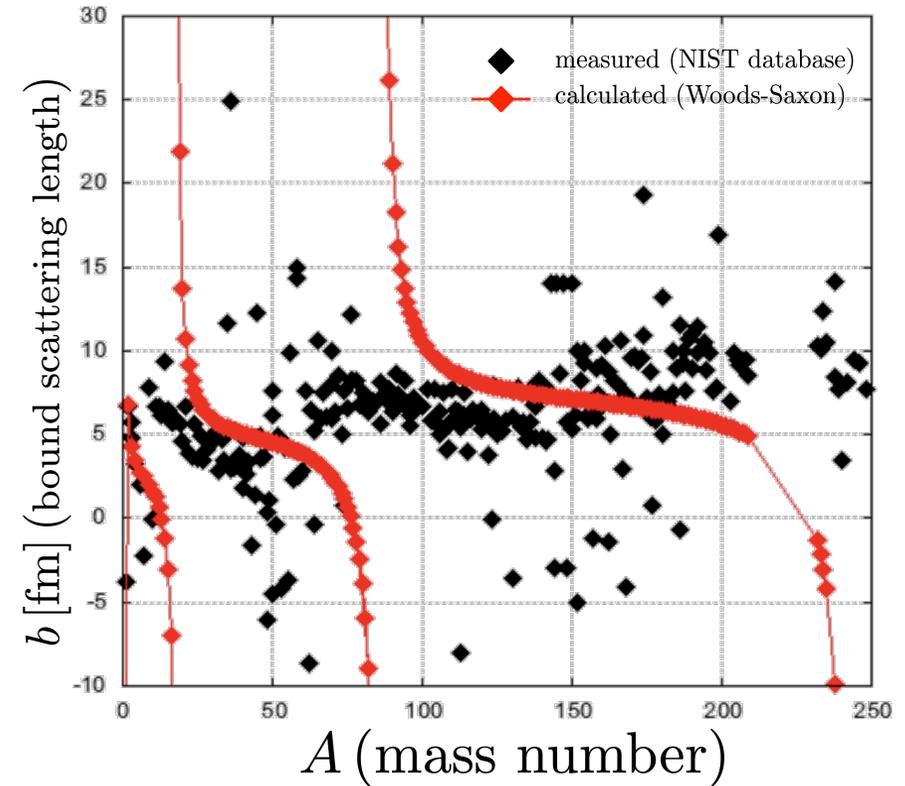
$$U_0 = -38 \text{ MeV}$$

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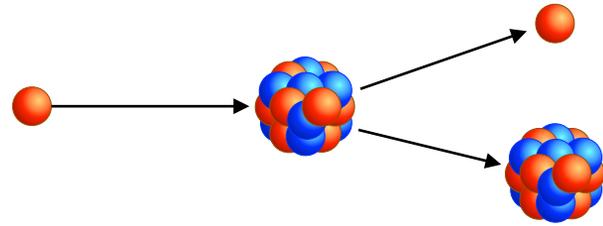
$$a_{\text{dif}} = 0.55 \text{ fm}$$



測定値との比較



ポテンシャル散乱



(potential scattering)

$${}^A Z(p, p) {}^A Z$$

等方ポテンシャルによる散乱

$$\frac{\hbar^2}{2\mu} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left(E - U(r) - \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) R = 0$$

クーロン相互作用が存在する場合

$$\frac{\hbar^2}{2\mu} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left(E - U_{\text{nucl}}(r) - \frac{Ze^2}{r} - \frac{\hbar^2 l(l+1)}{2\mu r^2} \right) R = 0$$

$u^C(r) = rR(r)$ とおいたときに満たされる方程式は

$$\frac{d^2 u_l^C}{dr^2} + \left(k^2 - \frac{l(l+1)}{r^2} - \frac{2k\eta}{r} \right) u_l^C = 0$$

無限遠点で三角関数に漸近しない

クーロン波動関数

$$u_l^{C(+)}(k, r) = (iF_l^C(k, r) + G_l^C(k, r))e^{-i\sigma_l}$$

$$u_l^{C(-)}(k, r) = (-iF_l^C(k, r) + G_l^C(k, r))e^{i\sigma_l}$$

ただし

$$F_l^C(k, r) = \frac{1}{2i} \left(u_l^{C(+)}(k, r)e^{i\sigma_l} - u_l^{C(-)}(k, r)e^{-i\sigma_l} \right)$$

$$G_l^C(k, r) = \frac{1}{2} \left(u_l^{C(+)}(k, r)e^{i\sigma_l} + u_l^{C(-)}(k, r)e^{-i\sigma_l} \right)$$

無限遠点での漸近形は

$$F_l^C(k, r) = kr \frac{e^{-i\frac{\eta\pi}{2}} |\Gamma(l+1+i\eta)|}{(2l+1)!} (2kr)^l e^{ikr} F(l+1+i\eta, 2l+2, -2ikr)$$

$$\rightarrow \sin \left(kr - \eta \ln 2kr - \frac{l\pi}{2} + \sigma_l \right)$$

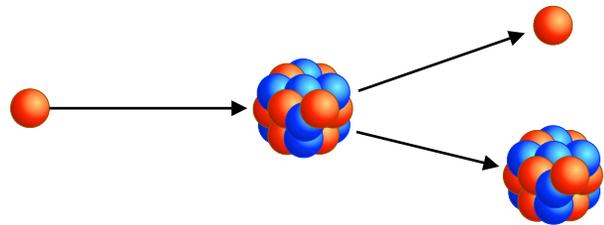
(位相のずれがrに依存する)

$$F(a, b, z) = 1 + \frac{a}{b \cdot 1} z + \frac{a(a+1)}{b(b+1) \cdot 2!} z^2 + \dots$$

合流型超幾何関数 (confluent hypergeometric function)

ポテンシャル散乱

(potential scattering)



$${}^A Z(p, p) {}^A Z$$

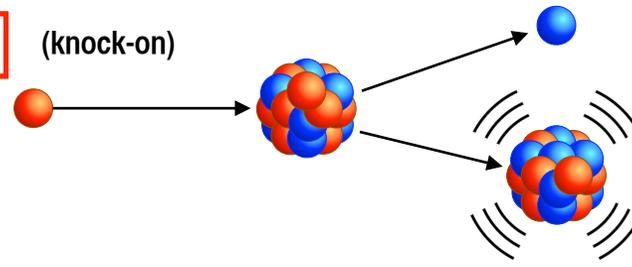
組み替え反応

(rearrangement)

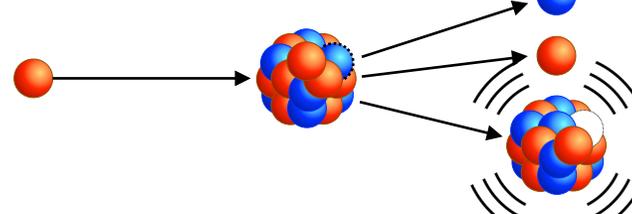
ノックオン反応

(knock-on)

$${}^A Z(p, n) {}^{A(Z+1)}(^*)$$



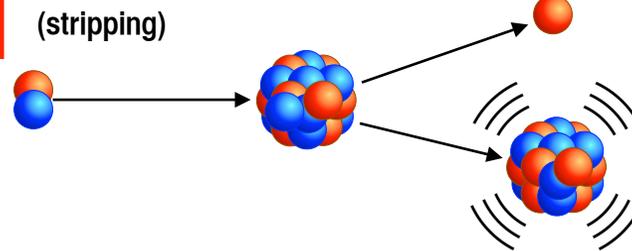
$${}^A Z(p, p'n) {}^{A-1} Z(^*)$$



ストリッピング反応

(stripping)

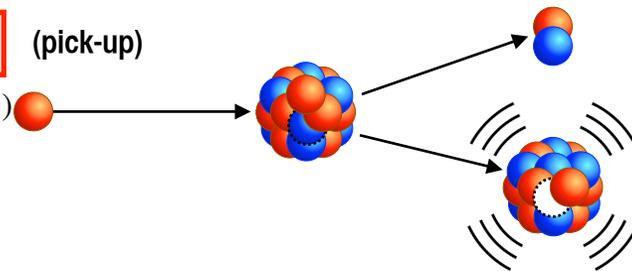
$${}^A Z(d, d) {}^A Z \quad {}^A Z(d, p) {}^{A+1} Z(^*)$$



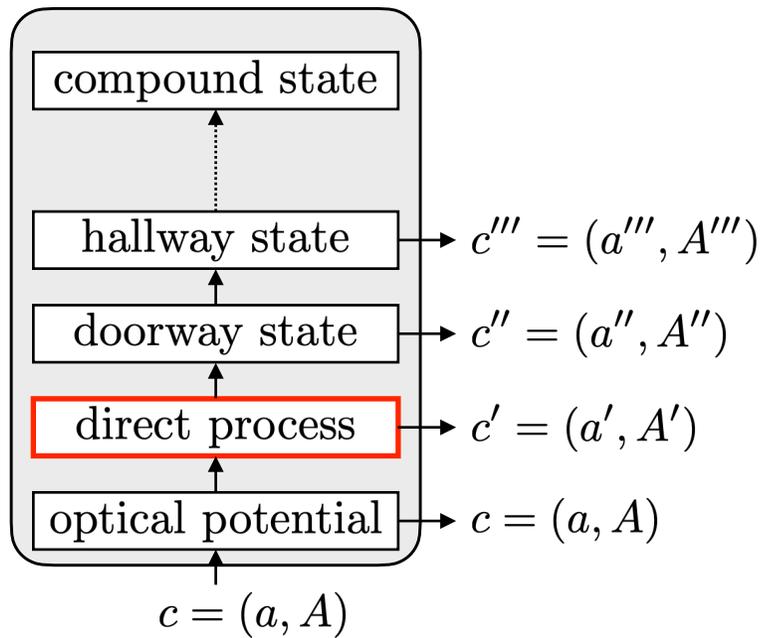
ピックアップ反応

(pick-up)

$${}^A Z(p, d) {}^{A-1} (Z-1) (^*)$$



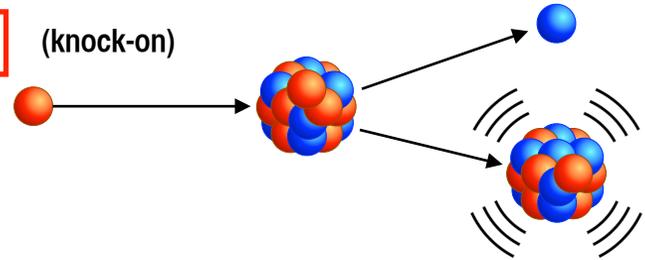
$$T_{c'k_{c'},ck_c} = \langle \phi_{c'k_{c'}} | V_{c'} | \Psi_{ck_c}^{(+)} \rangle$$



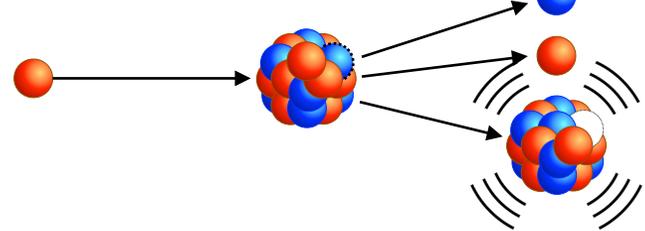
組み替え反応 (rearrangement)

ノックオン反応 (knock-on)

$${}^A Z(p, n) {}^A (Z+1)^{(*)}$$

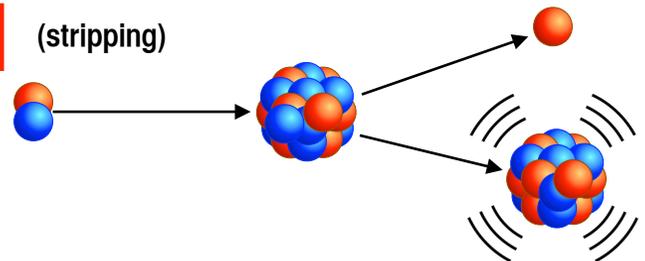


$${}^A Z(p, p'n) {}^{A-1} Z^{(*)}$$



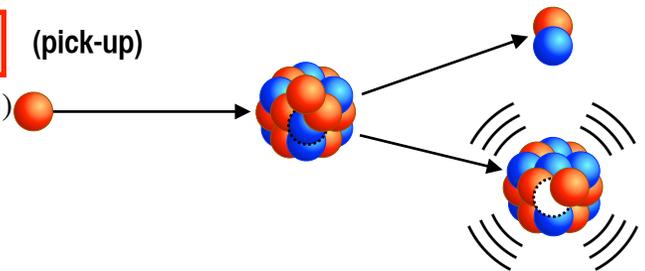
ストリッピング反応 (stripping)

$${}^A Z(d, p) {}^{A+1} Z^{(*)}$$



ピックアップ反応 (pick-up)

$${}^A Z(p, d) {}^{A-1} (Z-1)^{(*)}$$



$$T_{c'k_{c'},ck_c} = \langle \phi_{c'k_{c'}} | V_{c'} | \Psi_{ck_c}^{(+)} \rangle$$

ボルン近似

$$\langle \phi | V | \phi \rangle$$

平面波ボルン近似
PWBA (Plane Wave Born Approximation)

$$\langle \tilde{\phi} | V | \tilde{\phi} \rangle$$

歪曲波ボルン近似
DWBA (Distorted Wave Born Approximation)

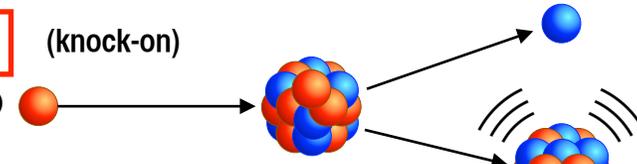
組み替え反応

(rearrangement)

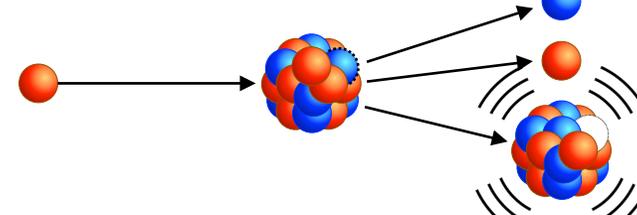
ノックオン反応

(knock-on)

$${}^A Z(p, n) {}^A (Z+1)^{(*)}$$



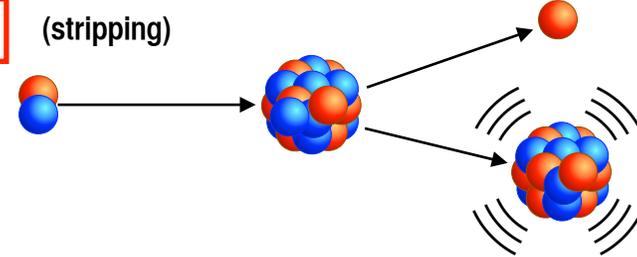
$${}^A Z(p, p'n) {}^{A-1} Z^{(*)}$$



ストリッピング反応

(stripping)

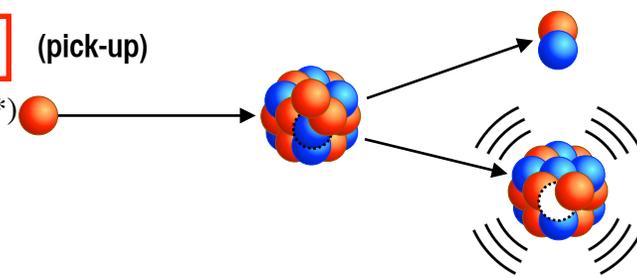
$${}^A Z(d, p) {}^{A+1} Z^{(*)}$$



ピックアップ反応

(pick-up)

$${}^A Z(p, d) {}^{A-1} (Z-1)^{(*)}$$



$$T_{c'k_{c'},ck_c} = \langle \phi_{c'k_{c'}} | V_{c'} | \Psi_{ck_c}^{(+)} \rangle$$

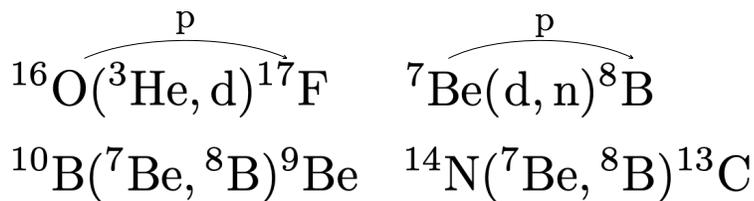
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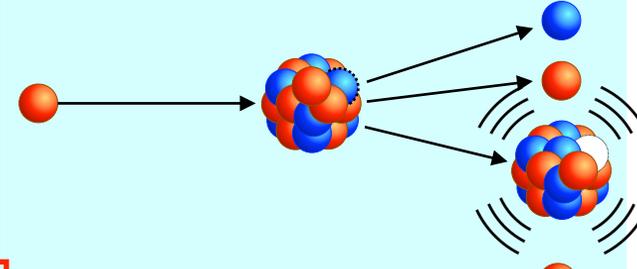
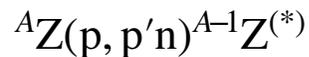
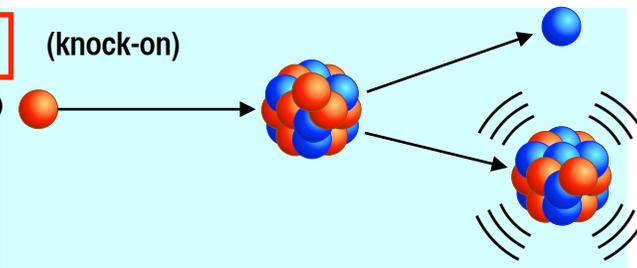
歪曲波ボルン近似
DWBA (Distorted Wave Born Approximation)



組み替え反応 (rearrangement)

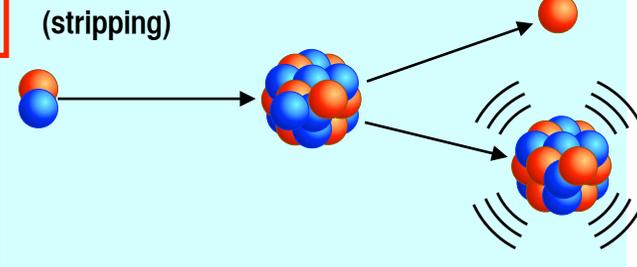
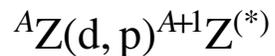
ノックオン反応

(knock-on)



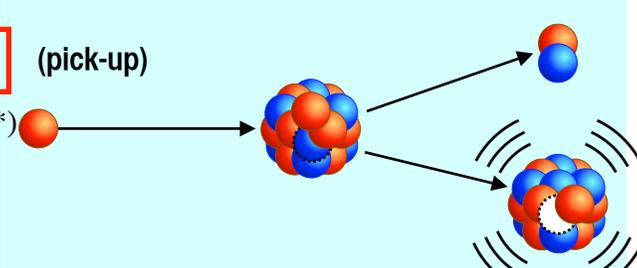
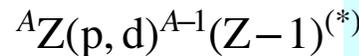
ストリッピング反応

(stripping)



ピックアップ反応

(pick-up)



← 二核子移行反応

← 一核子移行反応

$$T_{c'k_{c'},ck_c} = \langle \phi_{c'k_{c'}} | V_{c'} | \Psi_{ck_c}^{(+)} \rangle$$

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$$\langle \tilde{\phi} | V | \tilde{\phi} \rangle$$

歪曲波ボルン近似
DWBA (Distorted Wave Born Approximation)

$$\alpha \rightarrow \beta$$

$$\alpha \rightarrow \{\gamma\} \rightarrow \beta \quad \text{二段階過程}$$

$$\alpha \rightarrow \{\gamma\} \rightarrow \{\gamma'\} \rightarrow \beta$$

...

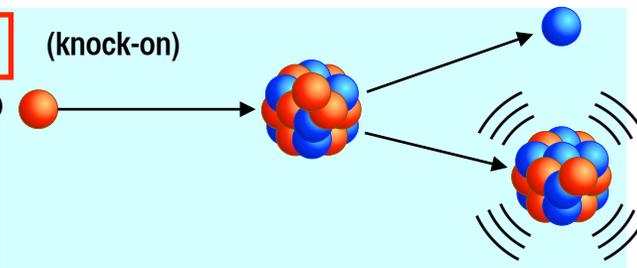
多段階過程

組み替え反応 (rearrangement)

ノックオン反応 (knock-on)

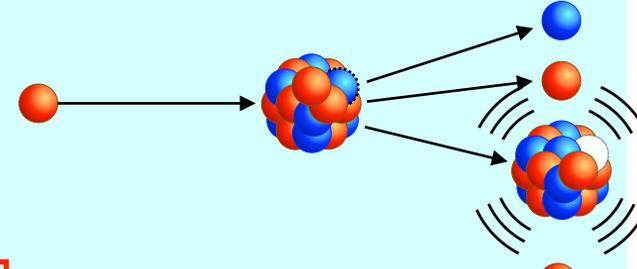
$${}^A Z(p, n) {}^A (Z+1)^{(*)}$$

(knock-on)



$${}^A Z(p, p'n) {}^{A-1} Z^{(*)}$$

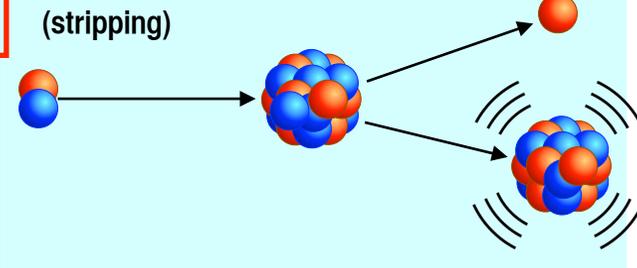
(stripping)



ストリッピング反応 (stripping)

$${}^A Z(d, p) {}^{A+1} Z^{(*)}$$

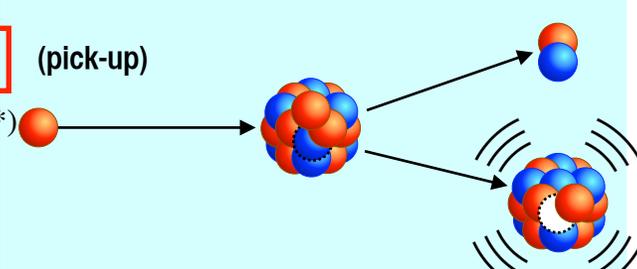
(stripping)



ピックアップ反応 (pick-up)

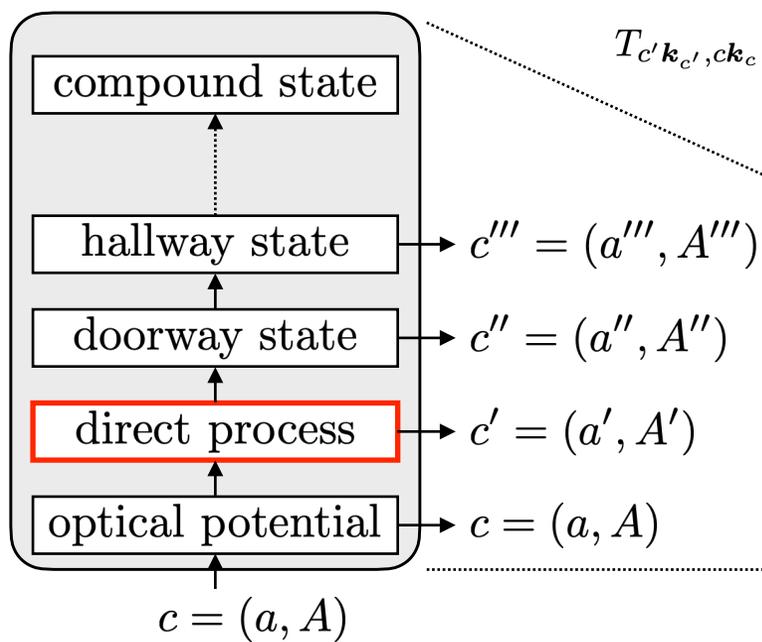
$${}^A Z(p, d) {}^{A-1} (Z-1)^{(*)}$$

(pick-up)

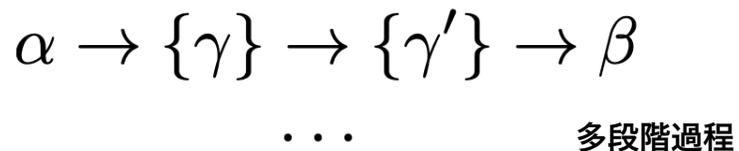
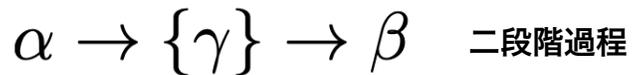
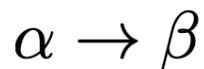
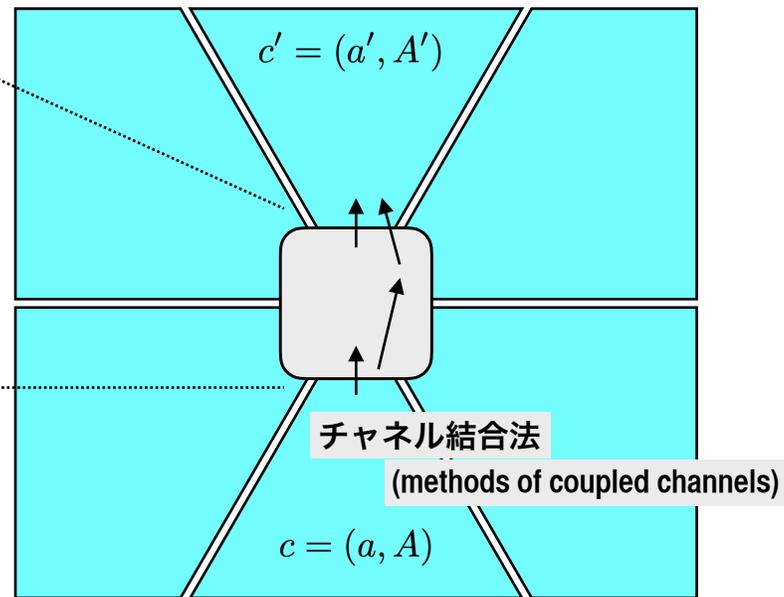


← 二核子移行反応 ←

← 一核子移行反応 ←



$$T_{c'k_{c'},ck_c} = \langle \phi_{c'k_{c'}} | V_{c'} | \Psi_{ck_c}^{(+)} \rangle$$



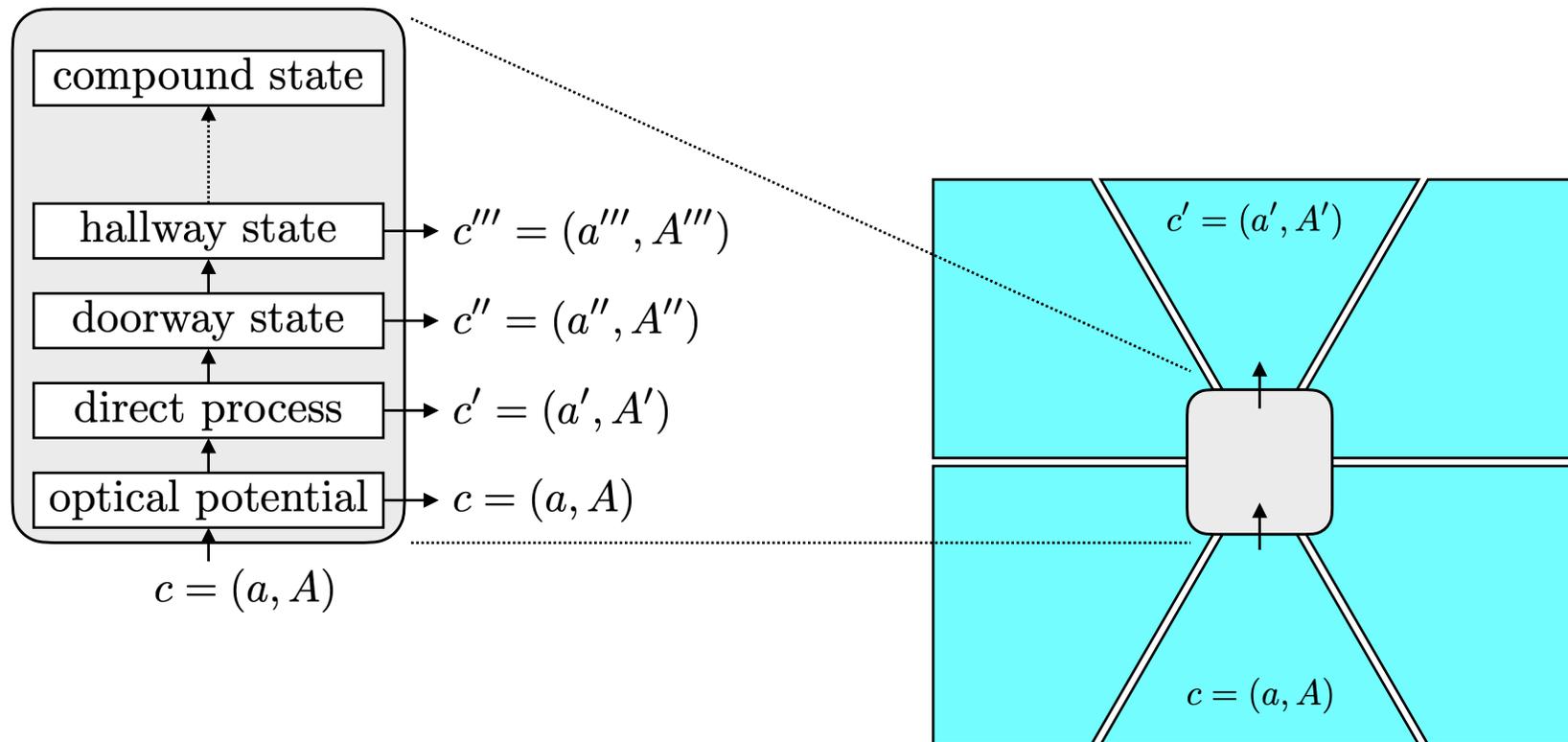
核内カスケード模型 (intra-nuclear cascade)

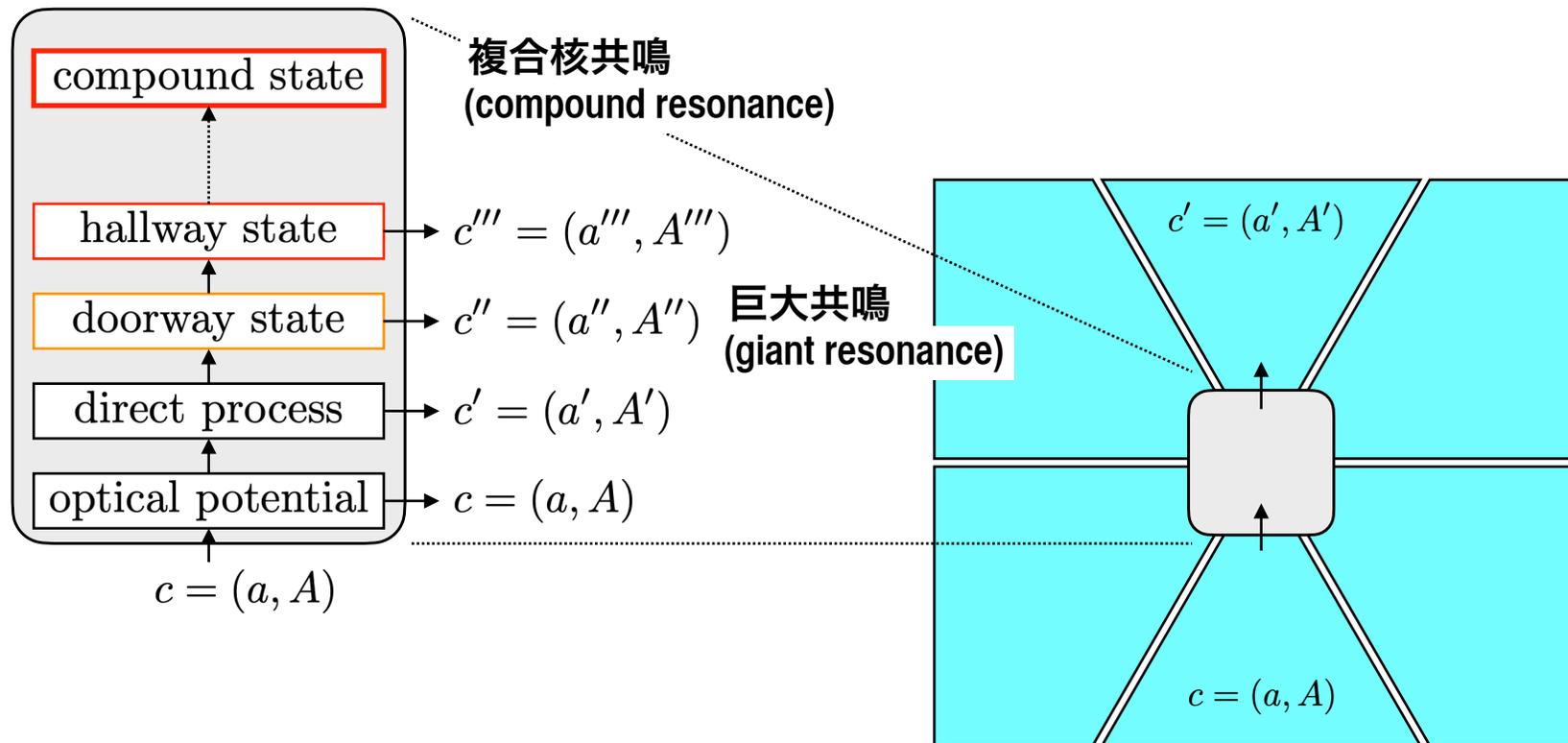
量子分子動力学 (quantum molecular dynamics)

反対称化分子動力学

(anti-symmetrized quantum molecular dynamics)

複合核反応





複合核過程

重心系
$$H = - \sum_i \frac{\hbar^2}{2\mu} \nabla_i^2 + \frac{1}{2} \sum_{ij} V_{ij} \quad \left(-\frac{\hbar^2}{2\mu} \nabla^2 + \frac{Z_1 Z_2 e^2}{r} \right) \psi = E \psi$$

部分波展開
$$\left(-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + U(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - E \right) u(r) = 0 \quad k = \frac{\sqrt{2\mu E}}{\hbar}$$

$$u(r) = u_l^{(-)}(k, r) - S_{cc} u_l^{(+)}(k, r)$$

$$u_l^{(+)}(k, r) = u_l^{(-)*}(k, r) = (G_l(k, r) + iF_l(k, r)) e^{-i\sigma_l}$$

$$\simeq e^{i(kr - \eta \ln 2kr - \frac{l\pi}{2})} \quad \eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

Sommerfeld parameter

クーロン波動関数
$$\frac{d^2 u_l^C}{dr^2} + \left(k^2 - \frac{l(l+1)}{r^2} - \frac{2k\eta}{r} \right) u_l^C = 0$$

$$u_l^{C(+)}(k, r) = (iF_l^C(k, r) + G_l^C(k, r)) e^{-i\sigma_l} \quad F_l^C(k, r) = \frac{1}{2i} (u_l^{C(+)}(k, r) e^{i\sigma_l} - u_l^{C(-)}(k, r) e^{-i\sigma_l})$$

$$u_l^{C(-)}(k, r) = (-iF_l^C(k, r) + G_l^C(k, r)) e^{i\sigma_l} \quad G_l^C(k, r) = \frac{1}{2} (u_l^{C(+)}(k, r) e^{i\sigma_l} + u_l^{C(-)}(k, r) e^{-i\sigma_l})$$

$$F_l^C(k, r) = kr \frac{e^{-i\frac{\eta\pi}{2}} |\Gamma(l+1+i\eta)|}{(2l+1)!} (2kr)^l e^{ikr} F(l+1+i\eta, 2l+2, -2ikr)$$

$$\rightarrow \sin \left(kr - \eta \ln 2kr - \frac{l\pi}{2} + \sigma_l \right)$$

$$F(a, b, z) = 1 + \frac{a}{b \cdot 1} z + \frac{a(a+1)}{b(b+1) \cdot 2!} z^2 + \dots$$

合流型超幾何関数 (confluent hypergeometric function)

複合核過程

重心系 $H = - \sum_i \frac{\hbar^2}{2\mu} \nabla_i^2 + \frac{1}{2} \sum_{ij} V_{ij} \quad \left(-\frac{\hbar^2}{2\mu} \nabla^2 + \frac{Z_1 Z_2 e^2}{r} \right) \psi = E\psi$

部分波展開 $\left(-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + U(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - E \right) u(r) = 0 \quad k = \frac{\sqrt{2\mu E}}{\hbar}$

$$u(r) = u_l^{(-)}(k, r) - S_{cc} u_l^{(+)}(k, r)$$

$$S_{cc} = e^{-2i\Omega_c} \frac{f_c - \Delta_c + iP_c}{f_c - \Delta_c - iP_c} = e^{-i\Omega_c} \left(1 + \frac{2iP_c}{f_c - \Delta_c - iP_c} \right) e^{i\Omega_c}$$

$$\Delta_c = \frac{r(F_c F'_c + G_c G'_c)}{F_c^2 + G_c^2} \Big|_{r=a} \quad P_c = kav_c = \frac{kr}{F_c^2 + G_c^2} \Big|_{r=a}$$

貫通因子 (penetrability)

$$|1 - S_{cc}|^2 = \left| 2ie^{i\Omega_c} \sin \Omega_c - \frac{2iP_c}{f_c - \Delta_c - iP_c} \right|^2$$

剛体球散乱項

共鳴項

複合核過程

$$|1 - S_{cc}|^2 = \left| \frac{H_c - i\Omega_c}{2ie^{i\Omega_c} \sin \Omega_c} \sum_i \frac{\hbar^2}{2\mu} \nabla_i^2 + \frac{1}{2} \frac{\sum_{ij} V_{ij}}{f_c - \Delta_c - iP_c} \right|^2 \left(-\frac{\hbar^2}{2\mu} \nabla^2 + \frac{Z_1 Z_2 e^2}{r} \right) \psi = E\psi$$

部分波展開 $\left(\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + U(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2 l(l+1)}{2\mu r^2} - E \right) u(r) = 0 \quad k = \frac{\sqrt{2\mu E}}{\hbar}$

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貫通因子 (penetrability)

$$|1 - S_{cc}|^2 = \left| \underbrace{2ie^{i\Omega_c} \sin \Omega_c}_{\text{剛体球散乱項}} - \underbrace{\frac{2iP_c}{f_c - \Delta_c - iP_c}}_{\text{共鳴項}} \right|^2$$

複合核過程

$$|1 - S_{cc}|^2 = \left| \underbrace{2ie^{i\Omega_c} \sin \Omega_c}_{\text{剛体球散乱項}} - \underbrace{\frac{2iP_c}{f_c - \Delta_c - iP_c}}_{\text{共鳴項}} \right|^2$$

$$S_{c'c} = e^{-i\Omega_{c'}} \left(\delta_{c'c} + 2i \sum_{\lambda} \frac{\gamma_{\lambda c'} \gamma_{\lambda c}}{E_{\lambda} - E} \right) e^{-i\Omega_c}$$

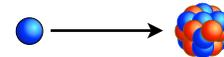
$$\begin{aligned} \gamma_{\lambda c} &= \sqrt{P_c} g_{\lambda c} \\ P_l &= \rho v_l \simeq \frac{(2l+1)^2}{((2l+1)!!)^2} \rho^{2l+1} \propto k^{2l+1} \\ \Gamma_{\lambda c} &= 2 |\gamma_{\lambda c}|^2 \end{aligned}$$

Breit-Wignerの一準位共鳴公式

$$\sigma_{c'c} = \frac{\pi}{k_c^2} \frac{2J+1}{(2I_{c1}+1)(2I_{c2}+1)} \frac{\Gamma_{\lambda c} \Gamma_{\lambda c'}}{(E - E_{\lambda})^2 + \Gamma_{\lambda}^2/4}$$

不完全性

- 共鳴パラメータが境界条件に依存
- 共鳴パラメータが入射エネルギーに依存
- ...

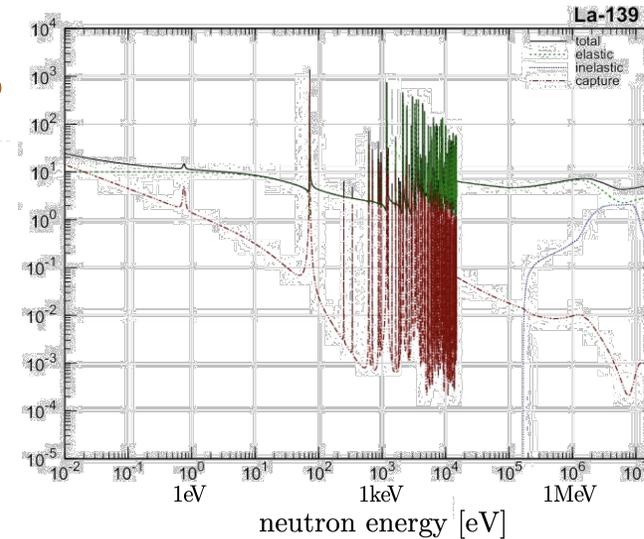


統計理論

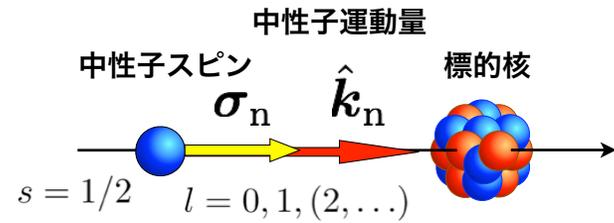
共鳴幅、準位間隔などの分布を統計的に理解

ランダム行列理論 \longrightarrow 巨大なパリティ非保存過程

- GOE (Gaussian orthogonal ensemble)
- GUE (Gaussian unitary ensemble)
- GSE (Gaussian symplectic ensemble)



複合核における空間反転非対称の増幅



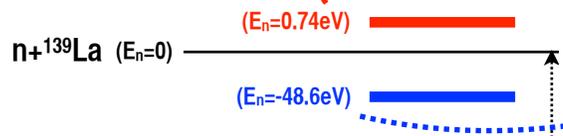
$$\sigma = \sigma_0 + \Delta\sigma(\sigma_n \cdot \hat{k}_n)$$

断面積のヘリシティ依存性は、空間反転非対称を意味する

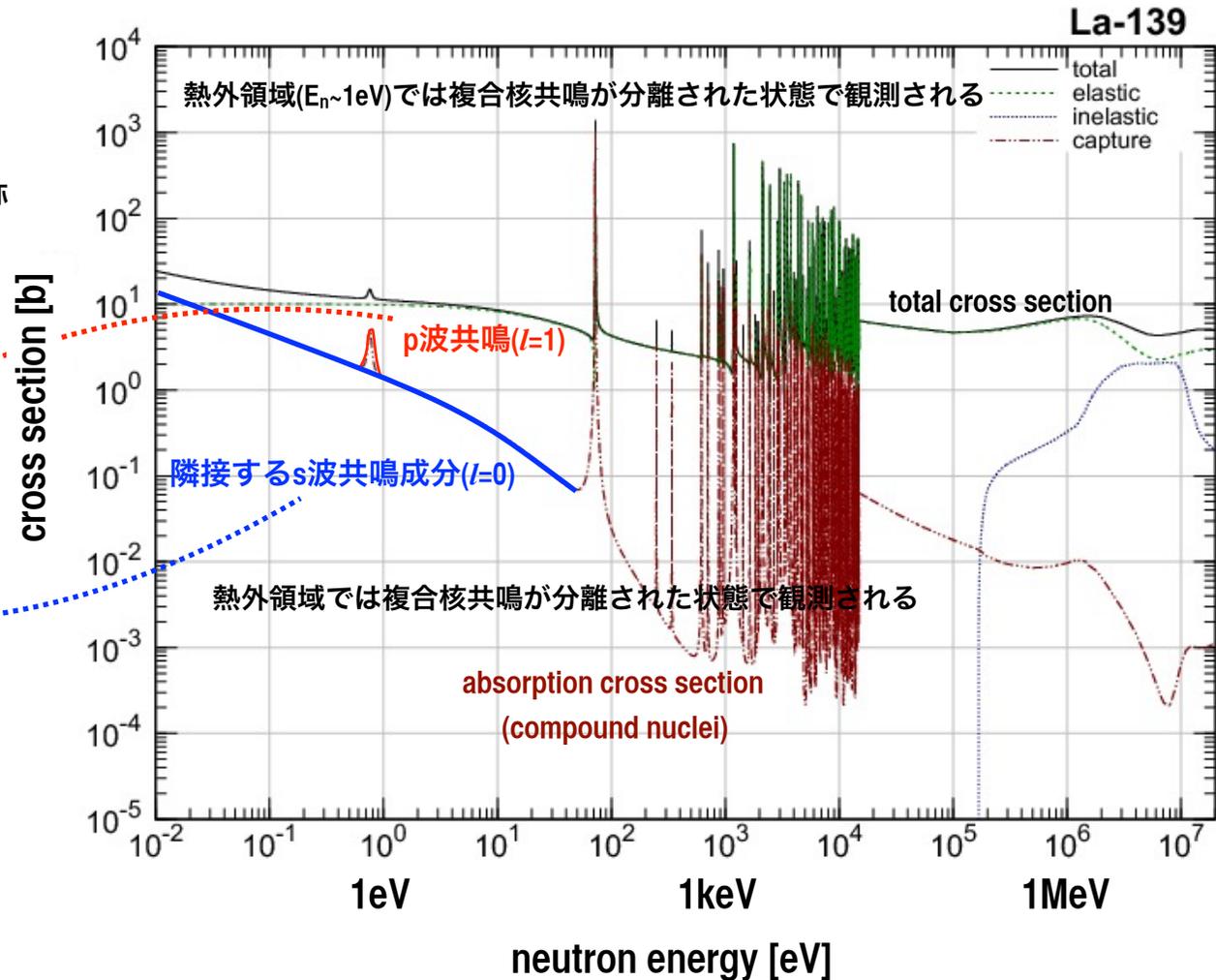
$$A_L = \frac{\Delta\sigma}{\sigma_0} \times \frac{\sigma_0}{\sigma_p}$$

核子核子相互作用では、核力にわずかに含まれる弱い相互作用によって空間反転非対称が起こるが、その大きさは $A_L \sim 10^{-7}$ 程度

p波とs波が隣接している場合には、桁違いに大きな A_L が観測される場合が多数知られている



$n+^{139}\text{La}$ ($E_n=0.74\text{eV}$) の場合は $A_L=0.1$ (10^6 倍の増幅)



^{140}La

対称性の破れ \longleftrightarrow 直結 \longleftrightarrow 根源的な物理法則

誕生直後の宇宙における対称性の破れ



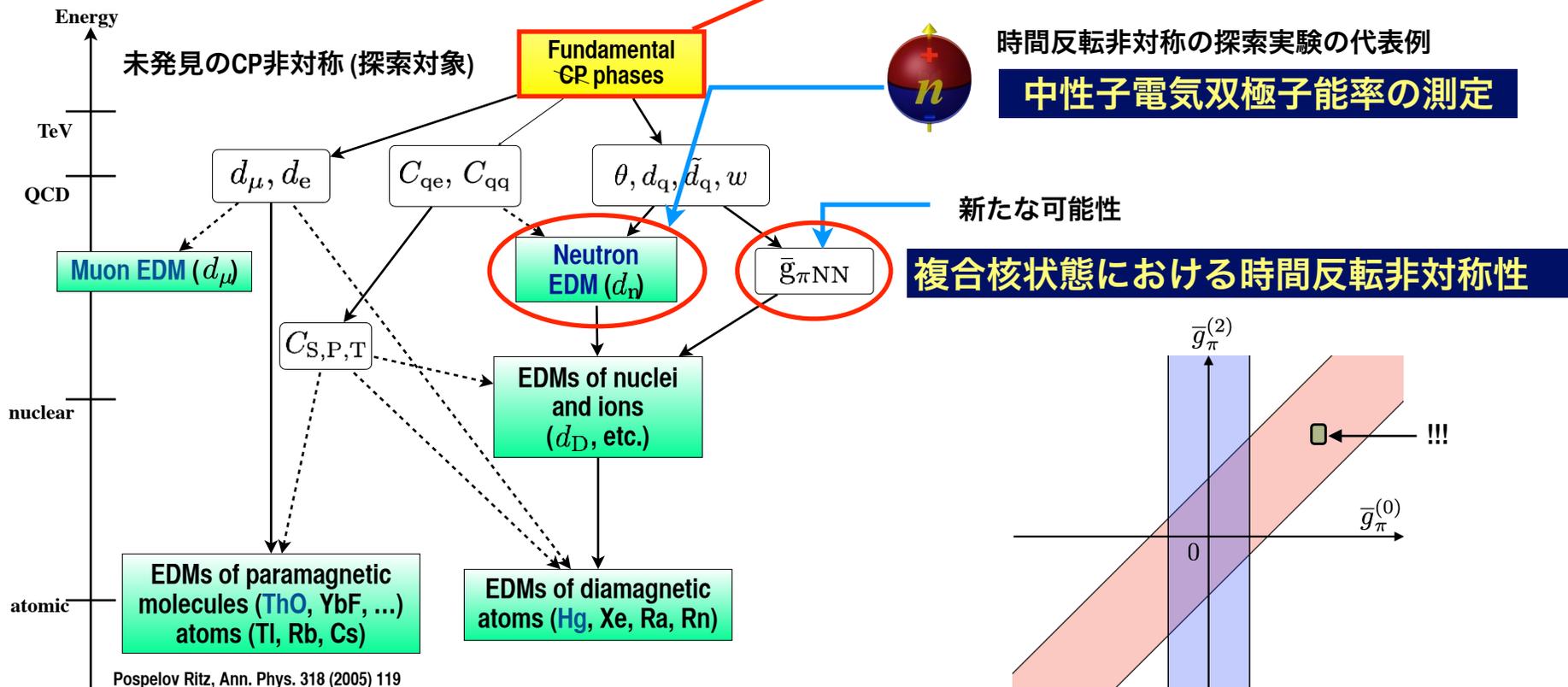
現在の宇宙に存在する物質の起源

未発見のCP非対称の存在

時間反転非対称と等価

新物理

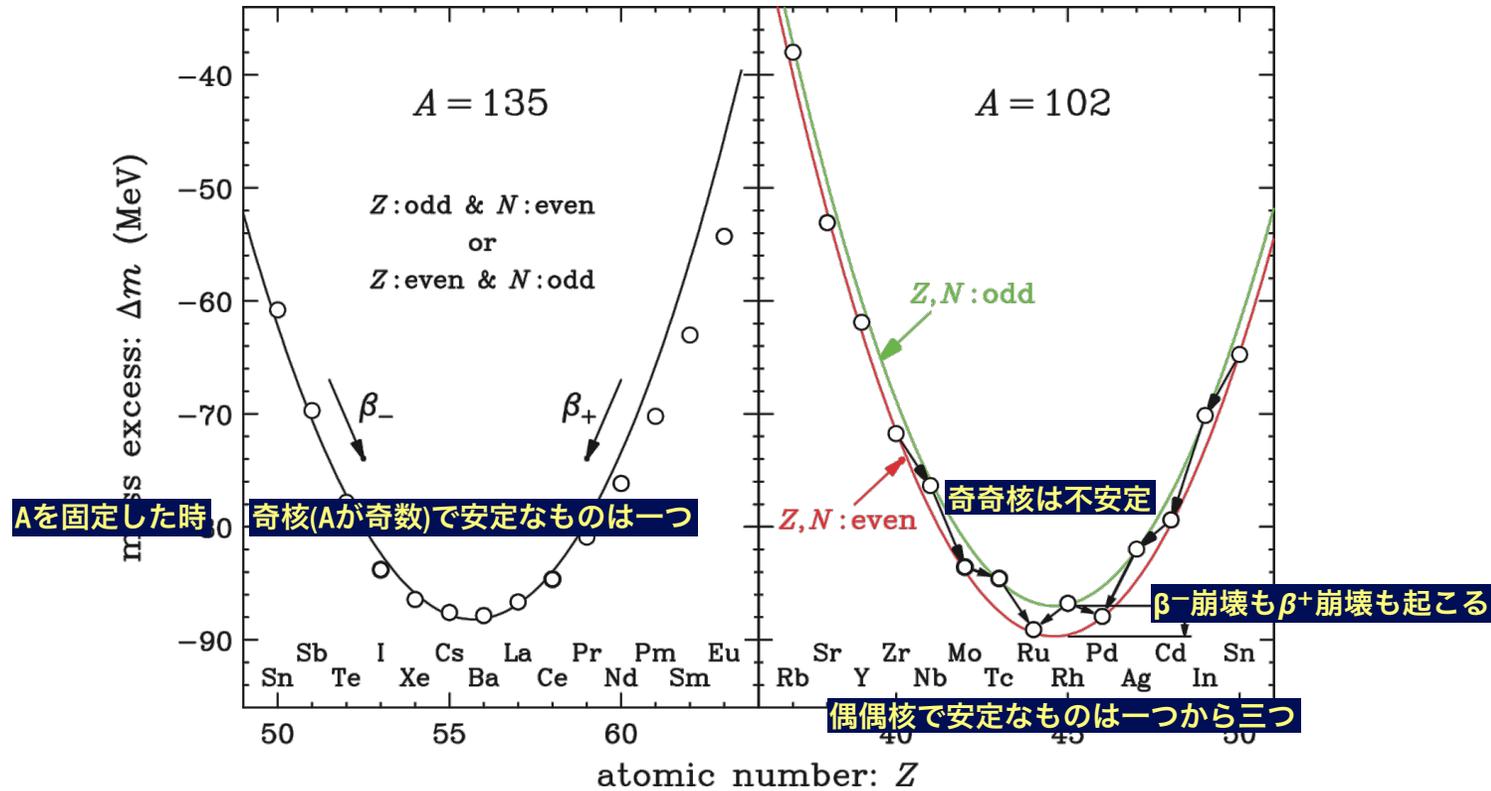
未発見のCP非対称 (探索対象) が低エネルギー現象に伝播する様子



β崩壊

$$B(Z, N) = \underbrace{a_1 A}_{\substack{\text{volume energy} \\ \text{体積エネルギー}}} - \underbrace{a_2 A^{2/3}}_{\substack{\text{lack of surface energy} \\ \text{表面エネルギーの欠損} \\ \text{表面では片方しか結合がない}}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\substack{\text{Coulomb energy} \\ \text{クーロンエネルギー}}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\substack{\text{symmetry energy} \\ \text{対称エネルギー}}} + \underbrace{\delta(A)}_{\substack{\text{pairing energy} \\ \text{対エネルギー}}}$$

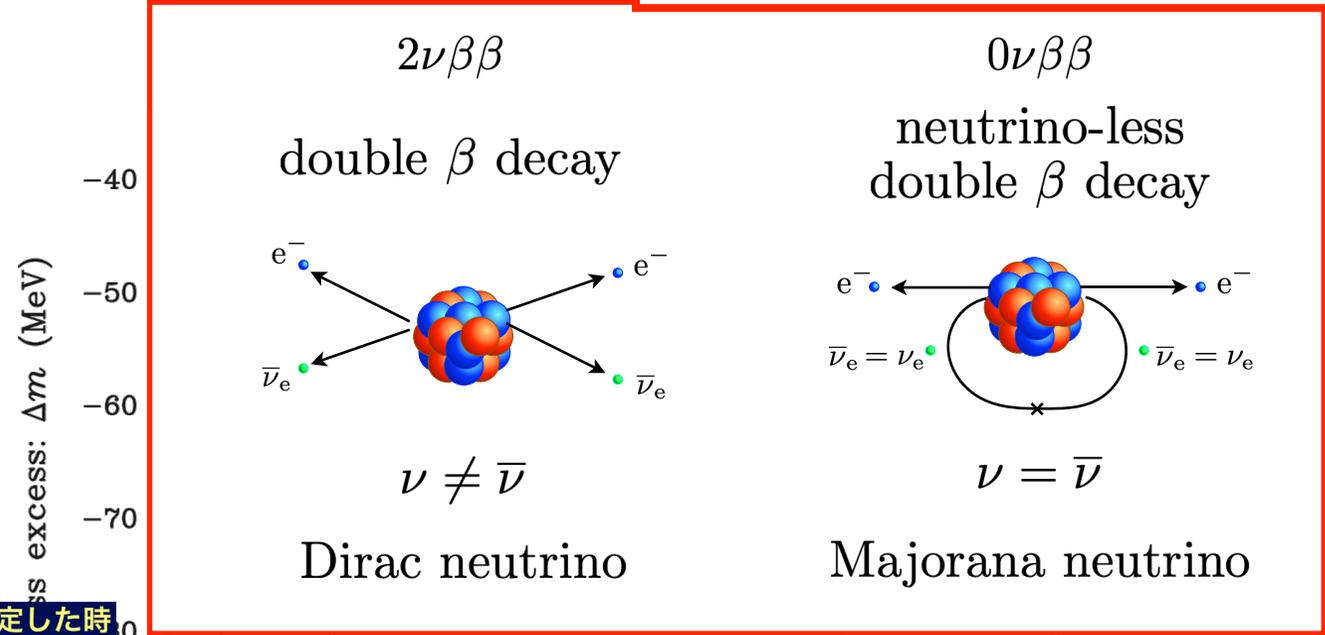
$$\delta(A) = \begin{cases} 34A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{odd} \\ 0 \text{ MeV} & \text{for } A=\text{odd} \\ -34A^{-3/4} \text{ MeV} & \text{for } Z=N=\text{even} \end{cases}$$



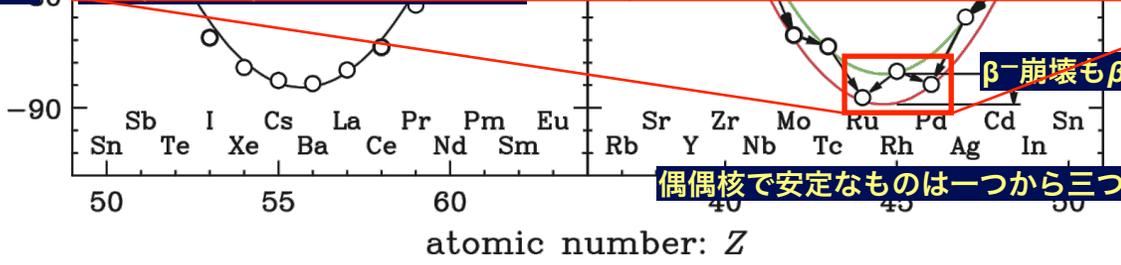
β崩壊

$$B(Z, N) = \underbrace{a_1 A}_{\text{volume energy / 体積エネルギー}} - \underbrace{a_2 A^{2/3}}_{\text{lack of surface energy / 表面エネルギーの欠損}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\text{Coulomb energy / クーロンエネルギー}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\text{symmetry energy / 対称エネルギー}} - \underbrace{\delta(A)}_{\text{pairing energy / 対エネルギー}}$$

Lepton number non-conservation



Aを固定した時



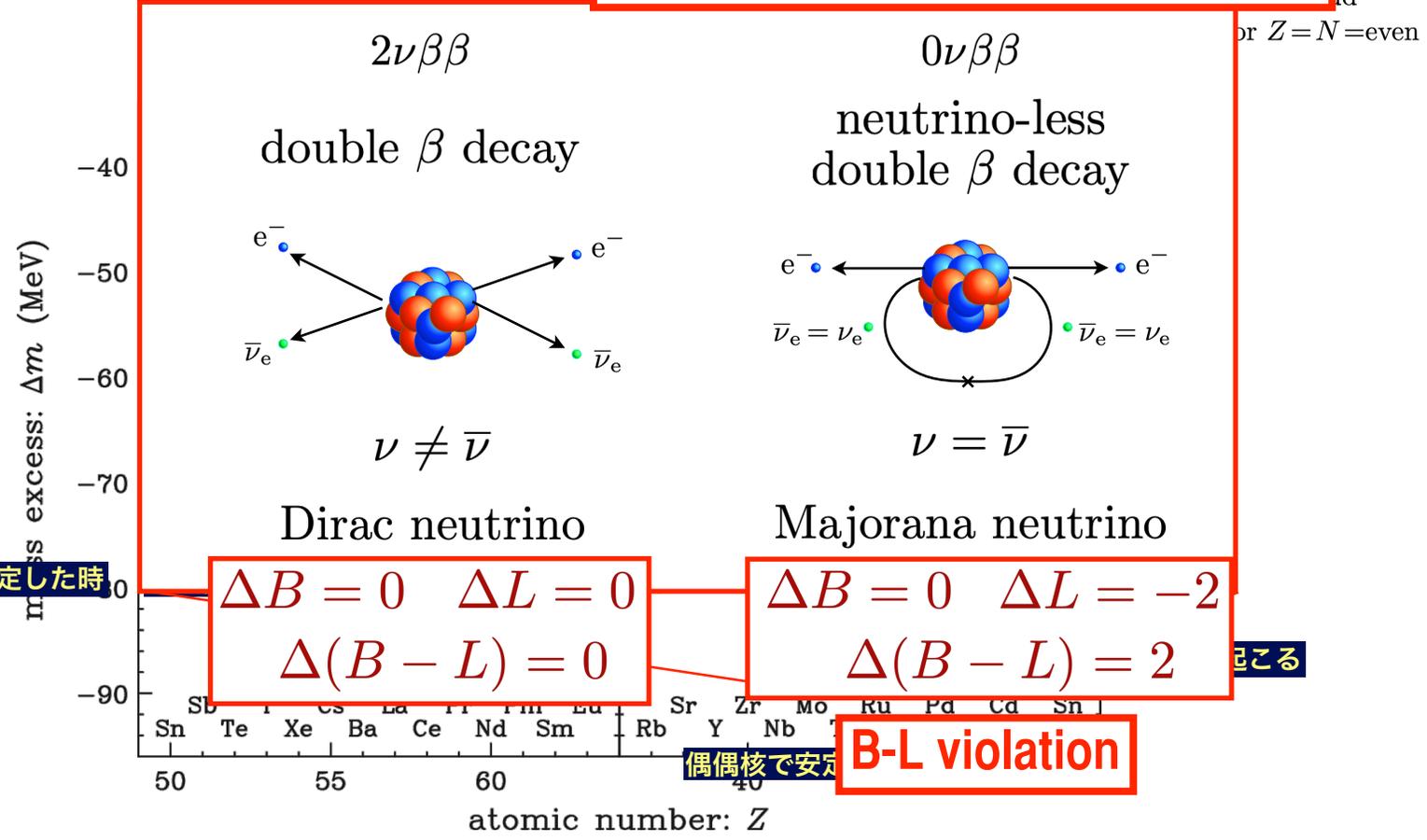
偶偶核で安定なもの一つから三つ

β-崩壊もβ+崩壊も起こる

β崩壊

$$B(Z, N) = \underbrace{a_1 A}_{\text{volume energy / 体積エネルギー}} - \underbrace{a_2 A^{2/3}}_{\text{lack of surface energy / 表面エネルギーの欠損}} - \underbrace{a_3 \frac{Z^2}{A^{1/3}}}_{\text{Coulomb energy / クーロンエネルギー}} - \underbrace{a_4 \frac{(Z - N)^2}{A}}_{\text{symmetry energy / 対称エネルギー}} + \text{pairing energy / 対エネルギー}$$

Lepton number non-conservation



Aを固定した時

偶偶核で安定

起こる

**Standard ModelにおけるB非保存
(Chiral Anomaly)**

$$\Delta(B - L) = \int d^4x \partial_\mu (j_B^\mu - j_L^\mu) = 0$$

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = N_g \left(\frac{g_2^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a + \frac{g_1^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)$$

**Standard ModelにおけるB非保存
(Chiral Anomaly)**

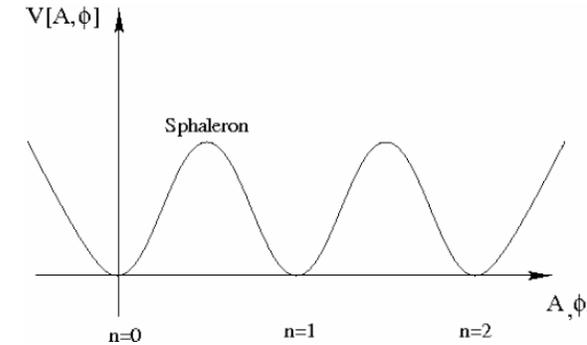
$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = N_g \left(\frac{g_2^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a + \frac{g_1^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)$$

$$\Delta(B - L) = \int d^4x \partial_\mu (j_B^\mu - j_L^\mu) = 0$$

$$\Delta(B + L) = \int d^4x \partial_\mu (j_B^\mu + j_L^\mu) = 2N_g \frac{g_2^2}{64\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a = 2N_g n$$

Sphaleron (n=0 ⇌ n=1)

$$E_{\text{sph}} = \mathcal{O}(8\pi m_W / g_2^2) = (8 - 14) \text{TeV}$$



Sakharovの3条件

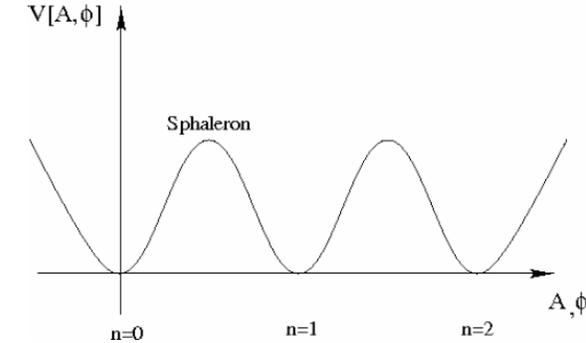
1. B非保存
2. C, CP非保存
3. 熱平衡からのズレ

Standard ModelにおけるB非保存 (Chiral Anomaly)

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = N_g \left(\frac{g_2^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a + \frac{g_1^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)$$

$$\Delta(B - L) = \int d^4x \partial_\mu (j_B^\mu - j_L^\mu) = 0$$

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Sphaleron ($n=0 \rightleftharpoons n=1$)

$$E_{\text{sph}} = \mathcal{O}(8\pi m_W / g_2^2) = (8 - 14) \text{TeV}$$

$$B_f = \frac{8N_g + 4}{22N_g + 13} (B - L)_i \simeq 0.35(B - L)_i \quad \text{for SM and MSSM}$$

$$B_i \neq 0, (B - L)_i = 0 \rightarrow B_f = 0$$

$$B_i = 0, L_i \neq 0, (B - L)_i \neq 0 \rightarrow B_f \neq 0$$

$$(B - L)_i = 0, (B + L)_i \neq 0 \rightarrow B_f \neq 0$$

~~SU(5) GUT Baryogenesis~~

Leptogenesis

Electroweak Baryogenesis

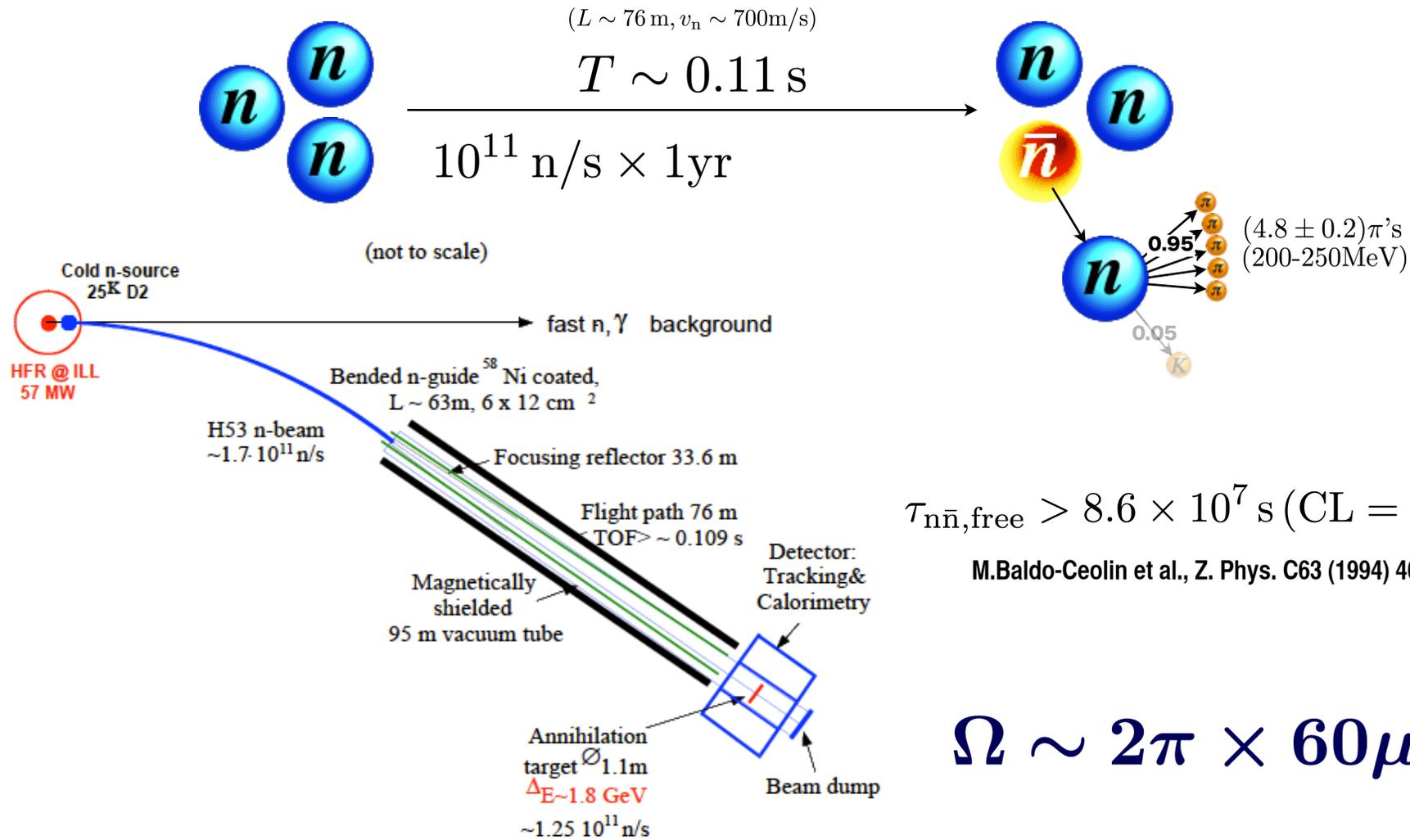
中性子反中性子振動 neutron anti-neutron oscillation ($n\bar{n}$)

$$\begin{array}{ccc}
 \textcircled{n} & \xrightarrow{\begin{array}{l} \Delta B = -2 \quad \Delta L = 0 \\ \Delta(B - L) = -2 \end{array}} & \textcircled{\bar{n}}
 \end{array}$$

$$L = \bar{\psi} M \psi \quad \psi = \begin{bmatrix} n \\ \bar{n} \end{bmatrix} \quad M = \begin{bmatrix} E_n & c^2 \delta m \\ c^2 \delta m & E_{\bar{n}} \end{bmatrix} \quad |n_{1,2}\rangle = \frac{1}{\sqrt{2}} (|n\rangle \pm |\bar{n}\rangle) \quad m_{1,2} = m_n \pm \delta m$$

$$I(t) = I(0) \sin^2 \frac{c^2 \delta m}{h} t$$

中性子反中性子振動 neutron anti-neutron oscillation ($n\bar{n}$)



$$\tau_{n\bar{n}, \text{free}} > 8.6 \times 10^7 \text{ s (CL = 90\%)}$$

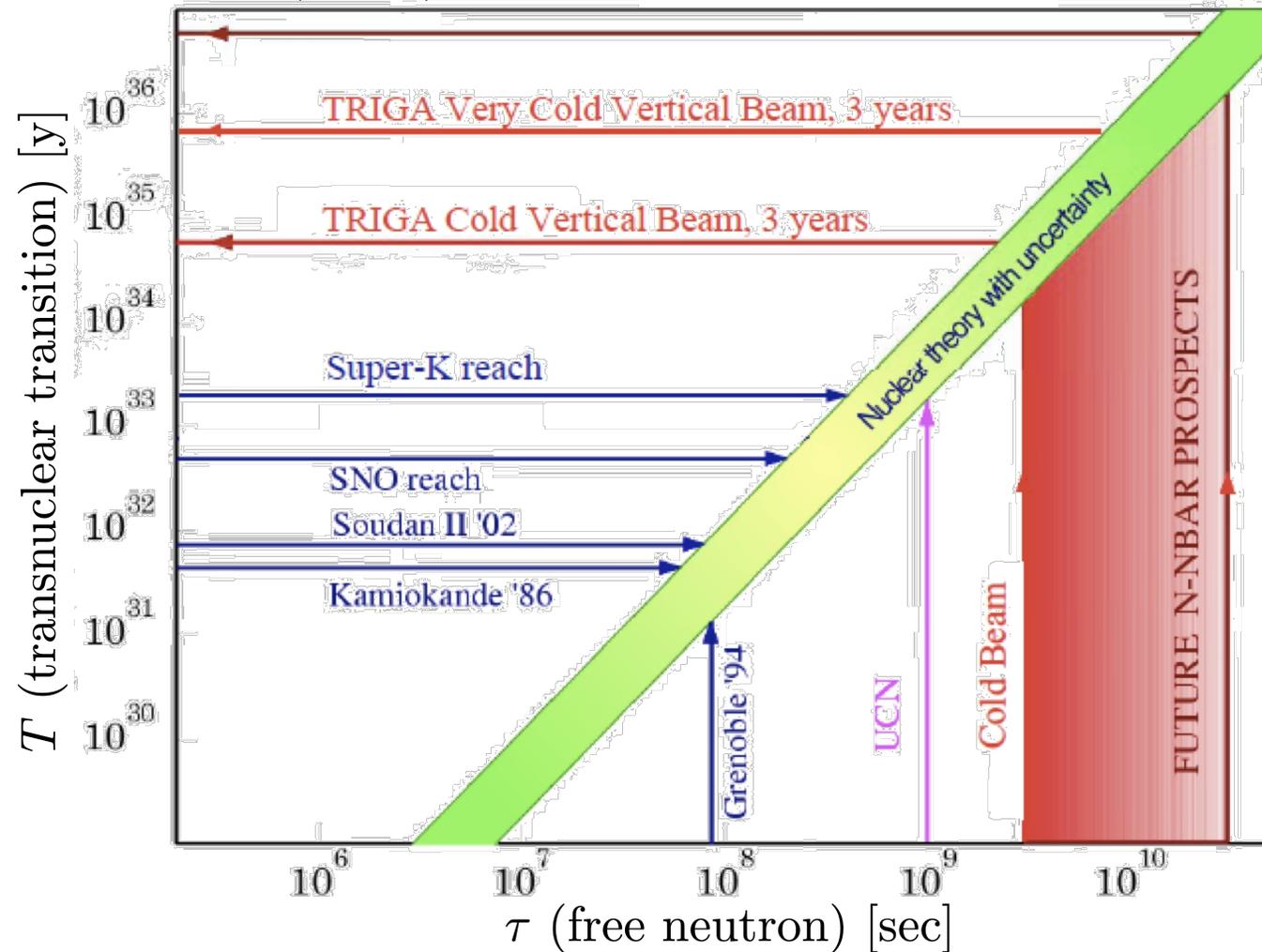
M. Baldo-Ceolin et al., Z. Phys. C63 (1994) 409.

$$\Omega \sim 2\pi \times 60 \mu\text{sr}$$

中性子反中性子振動 neutron anti-neutron oscillation ($n\bar{n}$)

stability of matter from neutron-antineutron transition search

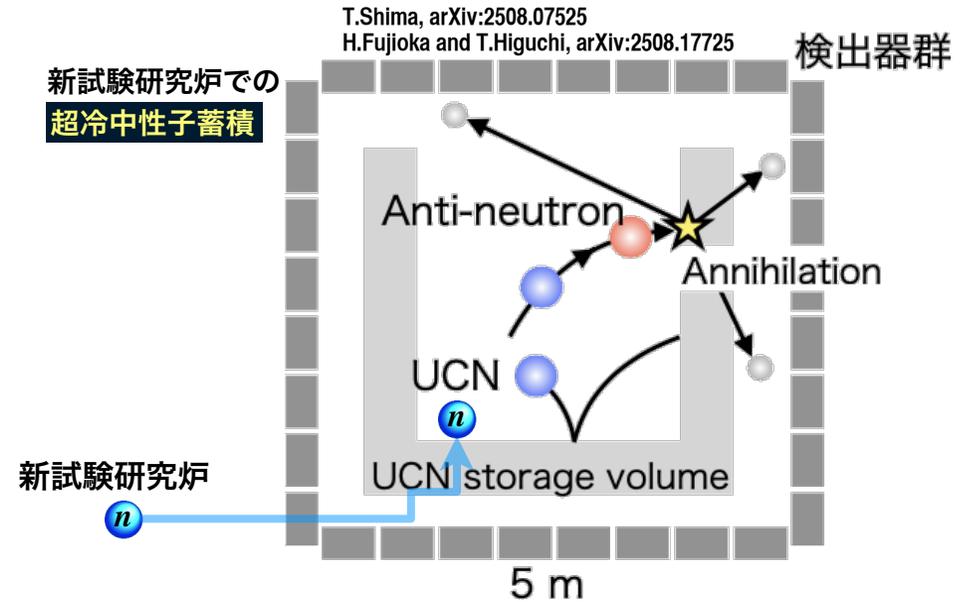
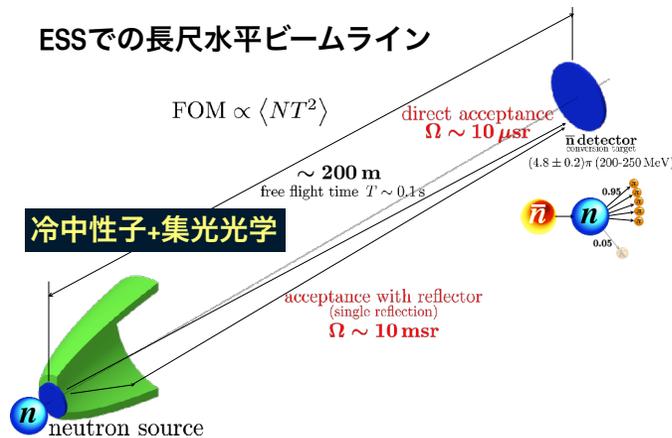
$$T = R(\tau_{\text{free}})^2 \quad R \text{ nuclear suppression factor in intranuclear transition}$$



中性子反中性子振動 neutron anti-neutron oscillation ($n\bar{n}$)

post-sphaleron baryogenesis K.S.Babu et al., Phys. Rev. D87, 115019 (2013)

$$\begin{aligned} \text{SO}(10) &\supset \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R \\ &\supset \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{R-L} \end{aligned}$$



中性子散乱長 a_n 中性子ミラー

反中性子散乱長 $a_{\bar{n}}$ 反中性子ミラー

感度最大 $\text{Re } a_n = \text{Re } a_{\bar{n}}$

QCD
quantum chromo-dynamics

NN
nucleon-nucleon interaction

nA
neutron-nucleus

中性子

反中性子

$$\mathcal{L}_{\text{QCD}} = -g_3 \bar{\psi} \not{D} \psi - \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$

$$\mathcal{L}_{\text{QCD}} = -g_3 \bar{\psi} \not{D} \psi - \frac{1}{4} G^{\mu\nu} G_{\mu\nu}$$



$$V_{\text{NN}}$$



$$\text{Re } a_n = -\text{Re } f_0$$



$$V_{\text{N}\bar{\text{N}}}$$



$$\text{Re } a_{\bar{n}}$$

$$\text{Re } f_0 = \frac{1}{2k_0} [(1 - \Sigma_2) \sin 2\delta_0 - \Sigma_1 \cos 2\delta_0]$$

$$\text{Im } f_0 = \frac{1}{2k_0} [1 - (1 - \Sigma_2) \cos 2\delta_0 - \Sigma_1 \sin 2\delta_0]$$

interferometry
reflectometry

σ_{sca}

$$\text{Re } b_{\text{exp}} = -\frac{m_n + m_A}{m_A} \text{Re } f_{0,\text{exp}}$$

$$\Sigma_1 = \sum_i \frac{k}{k_i} \frac{\Gamma_{ni}(E - E_i)}{(E - E_i)^2 + \Gamma_i^2/4}$$

$$\Sigma_2 = \sum_i \frac{k}{k_i} \frac{\Gamma_{ni}\Gamma_i/2}{(E - E_i)^2 + \Gamma_i^2/4}$$

$\leftarrow \sigma_{n\gamma}$

