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Interplay between the chiral and deconfinement transitions from a Curci-Ferrari-based Polyakov loop potential

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the minimal Curci-Ferrari (CF) model adds only one gauge parameter coupled with the NJL model. This framework avoids excessive parameters and systematically incorporates gauge dynamics, enabling us to study QCD' s phase diagram and related observables.

Nambu–Jona-Lasinio model (chiral)+Curci-Ferrari model(confinement)

$$V = V_{quark} + V_{gluon}, \tag{1}$$

$$V_{gluon} \to V_{\mathrm{csCF}}, \tag{2}$$

Polyakov loop model vs Curci-Ferrari model

The most commonly used polynomial PLM effective potential was:

$$\frac{V_{\mathrm{PLM}}(\ell,\bar{\ell})}{T^4} = -\frac{a(T)}{2}\bar{\ell}\ell - \frac{b_3}{6}(\ell^3+\bar{\ell}^3) + \frac{b_4}{4}(\bar{\ell}\ell)^2 \tag{3}$$

where the temperature-dependent coefficient is given by:

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 \tag{4}$$

表: Standard parameters of the polynomial

Parameter	a_0	a_1	a_2	a_3	b_3	b_4
Value	6.75	−1.95	2.625	−7.44	0.75	7.5

$T_0 = 210$–270 MeV

Curci-Ferrari model

Center-symmetric Curci-Ferrari model At one-loop order, $\tilde{V}(\bar{r})$ is given by

$$\tilde{V}(\bar{r}) = \frac{d-1}{2}\sum_{\kappa}\int_{Q}^{T}\ln\left[Q_\kappa^2+m^2\right] - \frac{1}{2}\sum_{\kappa}\int_{Q}^{T}\ln Q_\kappa^2. \quad (5)$$

rewriten as:

$$V_{\mathrm{csCF}}(r_3,r_8) = \frac{m^2}{2g^2}(r-\bar{r})^2+\frac{d-2}{2}\sum_{\kappa}\int_{Q}^{T}\ln\left[Q_\kappa^2+m^2\right]+\frac{1}{2}\sum_{\kappa}\int_{Q}^{T}\ln\left[1+\frac{m^2\bar{Q}_\kappa^2}{(Q_\kappa\cdot\bar{Q}_\kappa)^2}\right]. \quad (6)$$

notations are such that $Q_\kappa = (\omega_n + T(r\cdot\kappa),\vec{q})$ and $\bar{Q}_\kappa = (\omega_n + T(\bar{r}\cdot\kappa),\vec{q})$ with $\omega_n \equiv 2\pi nT$ a bosonic Matsubara frequency and

$$\int_{Q}^{T} f(Q) \equiv T\sum_{n}\int\frac{d^{d-1}q}{(2\pi)^{d-1}}f(\omega_n,\vec{q}) \quad (7)$$

Curci-Ferrari model

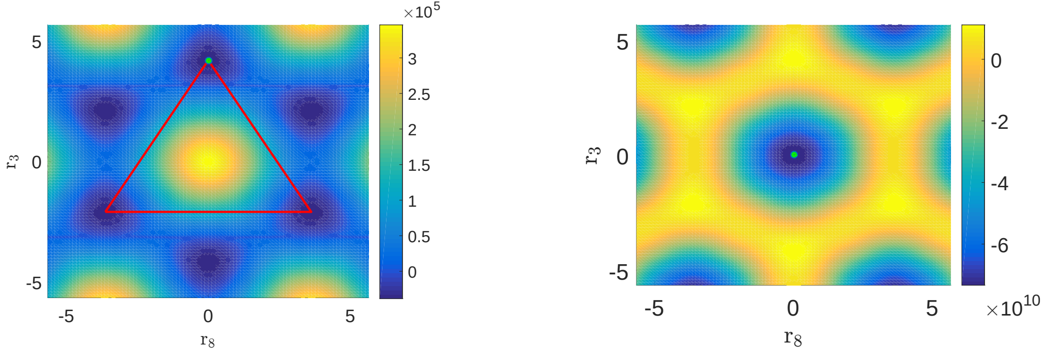


图: SU(3)，左 $T = 10\text{MeV}$, 右 $T = 260\text{MeV}$

$$\begin{aligned}\ell &= \frac{1}{N_c} Tr\left(\exp\left(ig\int_0^\beta d\tau \bar{A}_0\right)\right) = \frac{1}{N_c} Tr\left[\exp\left(ig\beta\bar{A}_0^3 t^3 + ig\beta\bar{A}_0^8 t^8\right)\right] \\ &= \frac{1}{N_c} Tr\left[\exp\left(\frac{ig\bar{A}_0^3}{2T}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix} + \frac{ig\bar{A}_0^8}{2\sqrt{3}T}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}\right)\right] \\ &= \frac{1}{3} Tr\left[\mathsf{diag}\left(e^{\frac{ig\bar{A}_0^3}{2T}+\frac{ig\bar{A}_0^8}{2\sqrt{3}T}}, e^{-\frac{ig\bar{A}_0^3}{2T}+\frac{ig\bar{A}_0^8}{2\sqrt{3}T}}, e^{-\frac{ig\bar{A}_0^8}{\sqrt{3}T}}\right)\right] \\ &= \frac{e^{-i\frac{r_8}{\sqrt{3}}} + 2e^{i\frac{r_8}{2\sqrt{3}}}\cos\left(\frac{r_3}{2}\right)}{3},\end{aligned} \tag{8}$$

the relation between (r_3, r_8) and $(\ell, \bar{\ell})$ is found to be

$$\ell = \frac{e^{-i\frac{r_8}{\sqrt{3}}} + 2e^{i\frac{r_8}{2\sqrt{3}}}\cos\left(\frac{r_3}{2}\right)}{3}, \tag{9}$$

$$\bar{\ell} = \frac{e^{i\frac{r_8}{\sqrt{3}}} + 2e^{-i\frac{r_8}{2\sqrt{3}}}\cos\left(\frac{r_3}{2}\right)}{3}. \tag{10}$$

Quark sector

The Nambu–Jona-Lasinio (NJL) model It starts from the free quark part of the QCD Lagrangian density and adds a contact four-quark interaction term:

$$\mathcal{L} = \bar{\psi}\left(\gamma^{\mu}\partial_{\mu} + \hat{m}_0 - \mu\gamma_0\right)\psi + \mathcal{L}_{\text{int}} \,. \tag{11}$$

In practice, this means replacing the normal derivative $\gamma^{\mu}\partial_{\mu}$ in this equation by a covariant derivative $\gamma^{\mu}D_{\mu} = \gamma^{\mu}\partial_{\mu} - ig\langle A_{\mu}^{a}\rangle t^{a}$ in the presence of the background $\langle A_{\mu}^{a}\rangle$.

$$\langle gA_{\mu}^{a}(x)\rangle = T\left(r_3\frac{\lambda_3}{2} + r_8\frac{\lambda_8}{2}\right), \tag{12}$$

In this formula, the labels ρ denote the defining weights of SU(N_c), corresponding to a sophisticated but very useful way to label the colors in the defining representation.

For $N_c = 3$,

$$\boldsymbol{\rho}_1 = \frac{1}{2}\left(1, \frac{1}{\sqrt{3}}\right), \; \boldsymbol{\rho}_2 = \frac{1}{2}\left(-1, \frac{1}{\sqrt{3}}\right), \; \boldsymbol{\rho}_3 = \left(0, -\frac{1}{\sqrt{3}}\right)$$

with $\hat{Q}_\rho = (\hat{\omega}_n + T(r \cdot \rho) - i\mu)$. The variable r is also a vector $r \equiv (r_3, r_8)$ and we have introduced the short-hand notation $r \cdot \rho \equiv r_3\rho_3 + r_8\rho_8$.

$$V_{\mathrm{PNJL}}(\sigma, r_3, r_8) = \frac{\sigma^2}{2G} - 2N_f \sum_\rho \int_Q^T \ln\left(\hat{Q}_\rho^2 + M^2\right), \tag{13}$$

σ is scalar quark condensate

$$\ell = \frac{1}{3}\sum_{\rho} e^{iT\boldsymbol{r}\cdot\boldsymbol{\rho}}, \quad \bar{\ell} = \frac{1}{3}\sum_{\rho} e^{-iT\boldsymbol{r}\cdot\boldsymbol{\rho}}$$

$$2N_f \sum_{\rho} \int_Q^T \ln\left(\hat{Q}_\rho^2 + M^2\right) = \frac{2N_f}{\pi^2}\int_0^\Lambda dq\ q^2 \sum_{\rho}\left[\varepsilon_q + T\ln\left(1 + e^{-\beta(\varepsilon_q - \mu - iT\boldsymbol{r}\cdot\rho)}\right)\right]$$

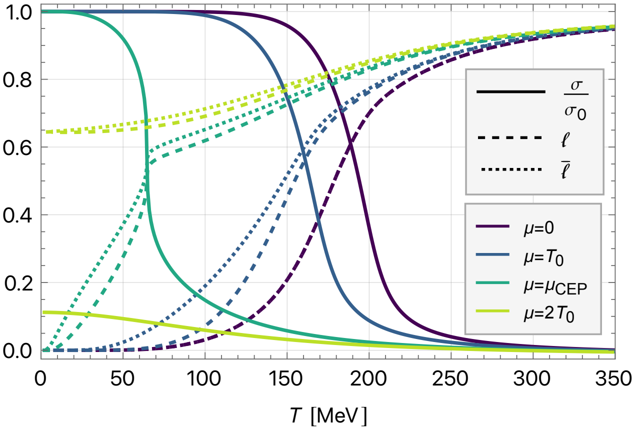
$$\sum_{\rho} \ln(1 + e^{-\beta(\varepsilon_q - \mu - iT\boldsymbol{r}\cdot\boldsymbol{\rho})}) = \ln\left[1 + 3\ell e^{-\beta(\varepsilon_q - \mu)} + 3\bar{\ell} e^{-2\beta(\varepsilon_q - \mu)} + e^{-3\beta(\varepsilon_q - \mu)}\right]$$

$$\begin{aligned} V_{\mathrm{PNJL}}(M, \ell, \bar{\ell}) = \frac{(M - m_l)^2}{2G} - \frac{N_f}{\pi^2}\int dq q^2 \{3\varepsilon_q \\ + Tln\left[1 + 3\ell e^{-\beta(\varepsilon_q - \mu)} + 3\bar{\ell} e^{-2\beta(\varepsilon_q - \mu)} + e^{-3\beta(\varepsilon_q - \mu)}\right] \\ + Tln\left[1 + 3\bar{\ell} e^{-\beta(\varepsilon_q + \mu)} + 3\ell e^{-2\beta(\varepsilon_q + \mu)} + e^{-3\beta(\varepsilon_q + \mu)}\right]\Big\}, \end{aligned} \tag{14}$$

$$V(\sigma, r_3, r_8) \equiv V_{\mathrm{PNJL}}(\sigma, r_3, r_8) + V_{\mathrm{csCF}}(r_3, r_8)\,.$$

the value of σ, r_3, r_8 is determined by the gap equation

$$0 = \frac{\partial V}{\partial \sigma} = \frac{\partial V}{\partial r_3} = \frac{\partial V}{\partial r_8}. \tag{15}$$



In this crossover region, one can define a pseudo-critical transition temperature $T_{\rm pc}(\mu)$ from the inflection of the corresponding order parameters

$$\left.\frac{\partial^2\sigma}{\partial T^2}\right|_{T^\sigma_{\rm pc}} = 0, \quad \left.\frac{\partial^2\ell}{\partial T^2}\right|_{T^\ell_{\rm pc}} = 0, \quad \text{or} \quad \left.\frac{\partial^2\bar\ell}{\partial T^2}\right|_{T^{\bar\ell}_{\rm pc}} = 0, \tag{16}$$

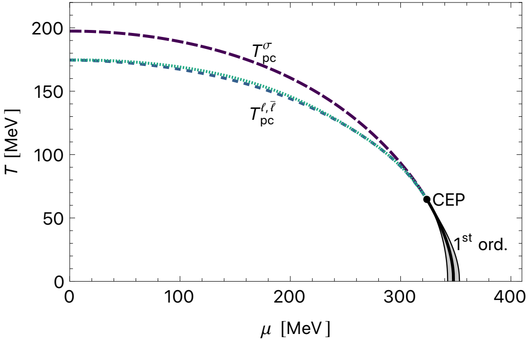


图: Phase diagram of the csCF-PNJL model in the (μ, T)-plane . The dashed lines mark the crossovers of the various order parameters σ, ℓ and $\bar\ell$ (lighter, dotted), the black dot labels the critical endpoint, and the solid lines mark the spinodal region and first-order transition

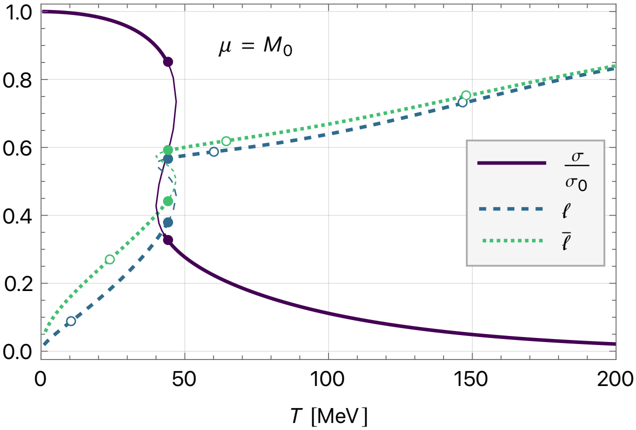


图: Temperature dependence of the order parameters at the chemical potential $\mu = M_0 \simeq 336$ MeV corresponding to the vacuum constituent quark mass, which crosses through the spinodal region. Thin lines represent metastable and unstable extrema. The filled points mark the first-order transition, and the open points mark additional inflection points.

T=0;

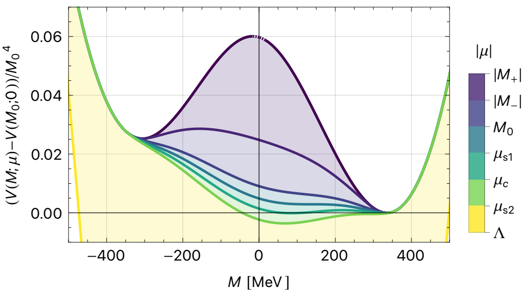


图: The zero-temperature potential $V(M;\mu) - V(M_0;0)$ normalized by M_0^4 for increasing values of $|\mu|$. The plain curves of decreasing darkness correspond to the potential at the special values of $|\mu| \in \{|M_+|, |M_-|, M_0, \mu_{s1}, \mu_c, \mu_{s2}, \Lambda\}$, see the main text for the definitions, and the dotted, light gray line between $(-M_+, M_+)$ shows the potential at $\mu = 0$ in the range where it deviates (slightly) from the one at M_+.

Net quark number

at low T :

$$V(\ell,\bar{\ell}) \simeq \partial_\ell \partial_{\bar{\ell}} V_{\text{gluon}} \times \ell\bar{\ell} - C\left(e^{\beta\mu} f_{\beta M_0} + e^{-2\beta\mu} f_{2\beta M_0}\right)\ell - C\left(e^{-\beta\mu} f_{\beta M_0} + e^{2\beta\mu} f_{2\beta M_0}\right)\bar{\ell}\,, \tag{17}$$

$$\Delta Q_q \equiv T\partial_\mu \ln \ell \quad \text{and} \quad \Delta Q_{\bar{q}} \equiv T\partial_\mu \ln \bar{\ell}\,, \tag{18}$$

which characterize the net quark number response of the medium when a quark or an antiquark is added, respectively. By adding or subtracting 1 to these quantities, one obtains the net quark numbers acquired by the system, including the quark or antiquark probe.

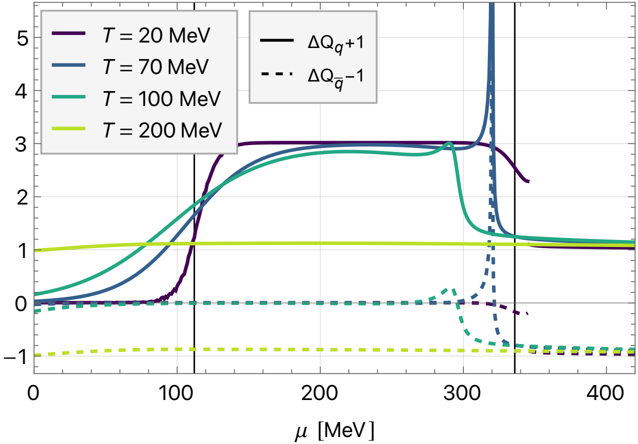


图: Net quark number gains as a function of the chemical potential μ at various temperatures T. Plain and dashed curves correspond respectively to the case of a quark (ΔQ_q) or antiquark $(\Delta Q_{\bar{q}})$ probe. The black vertical lines mark the points $\mu = M_0/3$ and $\mu = M_0$.

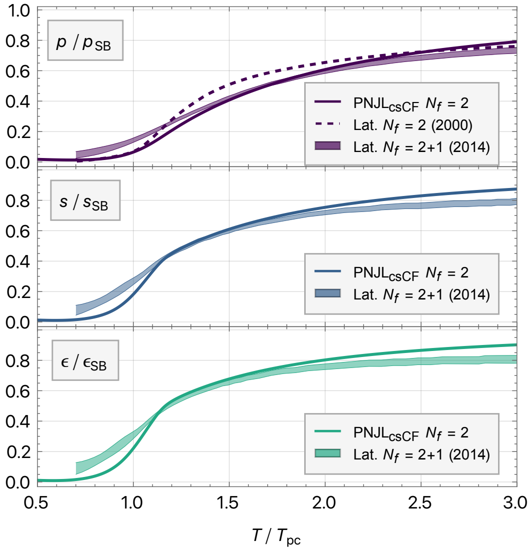


图: Pressure (top), entropy density (middle) and energy density (bottom) at $\mu = 0$ normalized to their SB limits as functions of $T/T_{\rm pc}$ compared to continuum extrapolated $N_f = 2+1$ lattice data (bands), and $N_f = 2$ pressure data on a $16^3 \times 4$ lattice (dashed).

From combinations of the basic functions p, s and ϵ, one can obtain other interesting functions that tell us about refined properties of the system.

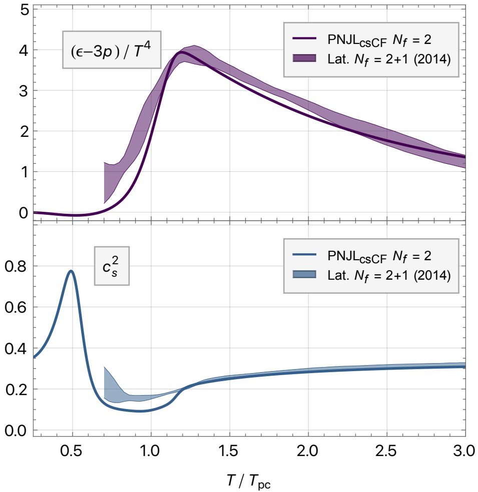


图: Temperature normalized trace anomaly $(\epsilon - 3p)/T^4$ (top), and speed of sound squared (bottom) at $\mu = 0$ as functions of $T/T_{\rm pc}$ compared to continuum extrapolated $N_f = 2 + 1$

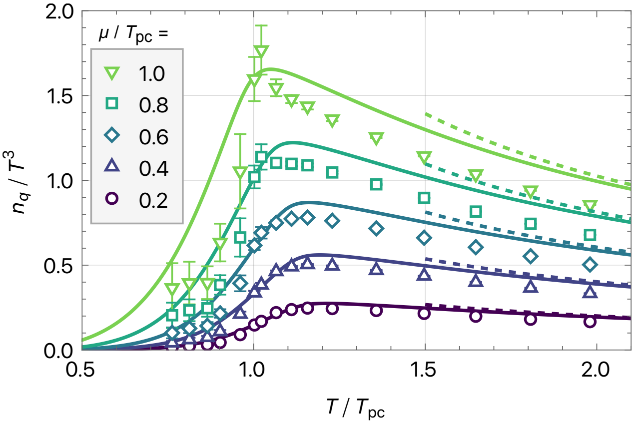
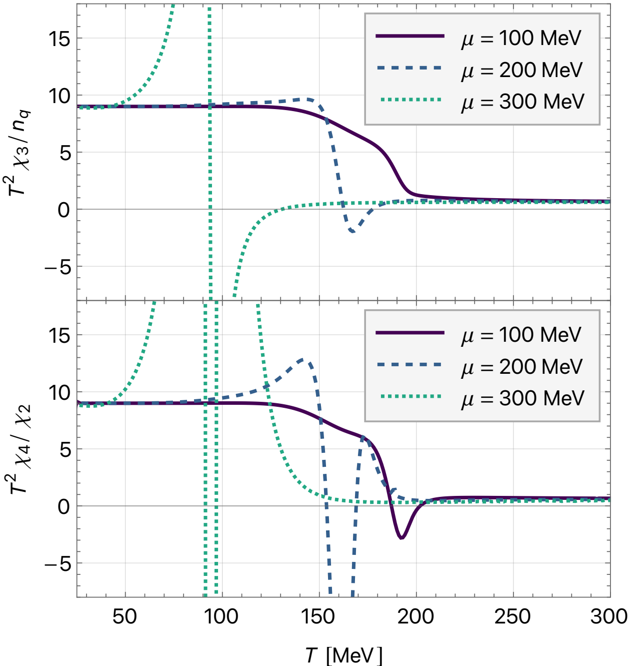


图: Quark number density n_q/T^3 as a function of T/T_{pc} for five μ's up to T_{pc} (solid lines), compared to $N_f = 2$ lattice dat (symbols). The dashed lines show the SB limit, which is approached faster in the PNJL model.



$\mu = 100$ MeV
$\mu = 200$ MeV
$\mu = 300$ MeV
$T^2\,\chi_3/\,n_q$
$T^2\,\chi_4/\,\chi_2$
15
10
5
0
−5
50
100
150
200
250
300
T [MeV]

Conclusion

表: csCF-PNJL Model Core Parameters

Item	m	G	Λ	m_g
Value	5 MeV	10.99 GeV^{-2}	631 MeV	390 MeV
	quark mass	4-quark interaction	Momentum cutoff	Gluon mass