

# Ground-state and radially excited charmed mesons in B decays

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# Introduction

- The ground states  $D_{(s)}$  and  $D_{(s)}^*$  have been well determined in experiments, their productions and decays have been extensively studied.
- While the first radially excited states  $D_{(s)}(2S)$  and  $D_{(s)}^*(2S)$  have not been well confirmed.
- For example both  $D_0(2550)$  and  $D_J(2580)$  are considered as the candidates of  $D(2^1S_0)$  state.
- $D_{s0}(2590)$  observed by LHCb is suggested to a candidate of  $D_s(2^1S_0)$  state. Some theoretical works suggest that it may not be a pure state but rather has  $D^*K$  component.

# Introduction

- $D_1^*(2680)^0$ ,  $D^*(2650)^0$  and  $D_1^*(2600)^0$  observed by LHCb are considered as the candidates of neutral  $D(2^3S_1)$ . The  $D^*(2640)^\pm$  discovered by Delphi have masses consistent with the predictions of charged  $D(2^3S_1)$ , while it has not been confirmed in any other experiments.
- Except for the assignment of  $D_s(2^3S_1)$  to  $D_{s1}^*(2700)^\pm$ ,  $D_s(1^3D_1)$  and a mixture of  $D_s(2^3S_1)$  and  $D_s(1^3D_1)$  are also been proposed.
- We will assume these several particles discovered in experiments as the corresponding first excited D mesons in the calculations:

$$D_0(2550) \rightarrow D(2^1S_0), D_{s0}(2590)^\pm \rightarrow D_s(2^1S_0),$$

$$D_1^*(2600)^0, D^*(2640)^\pm \rightarrow D(2^3S_1), D_{s1}^*(2700)^\pm \rightarrow D_s(2^3S_1).$$

# Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions describing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at  $q^2 = 0$  is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transition form factors.

# The CLFQM general calculation procedure

# The CLFQM general calculation procedure

The Bauer-Stech-Wirble (BSW) form factors for  $B \rightarrow D$  and  $B \rightarrow D^*$  transitions are defined as follows,

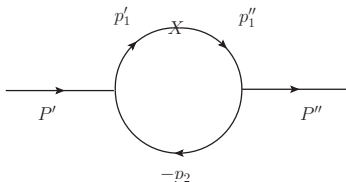
$$\begin{aligned}\langle D(P'') | V_\mu | B(P') \rangle &= \left( P_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right) F_1^{BD}(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0^{BD}(q^2), \\ \langle D^*(P'', \epsilon'') | V_\mu | B(P') \rangle &= -\frac{1}{m_{D^*} + m_B} \epsilon_{\mu\nu\alpha\beta} \epsilon''^{*\nu} P^\alpha q^\beta V^{BD^*}(q^2), \\ \langle D^*(P'', \epsilon'') | A_\mu | B(P') \rangle &= i \left\{ (m_{D^*} + m_B) \epsilon''^{*\mu} A_1^{BD^*}(q^2) - \frac{\epsilon''^{*\mu} \cdot P}{m_{D^*} + m_B} P_\mu A_2^{BD^*}(q^2) \right. \\ &\quad \left. - 2m_{D^*} \frac{\epsilon''^{*\mu} \cdot P}{q^2} q_\mu [A_3^{BD^*}(q^2) - A_0^{BD^*}(q^2)] \right\},\end{aligned}$$

where  $P = P' + P''$ ,  $q = P' - P''$ ,  $\epsilon''$  is the polarization vector. The four-momentum of the initial (final) meson is  $P' = p_1' + p_2$  ( $P'' = p_1'' + p_2$ ).

# The CLFQM general calculation procedure

The decay amplitude in the lowest order for the transitions  $B \rightarrow D$  and  $B \rightarrow D^*$

$$\begin{aligned} \mathcal{B}_\mu^{BD} &= -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h'_B h''_D}{N_1' N_1'' N_2} S_\mu^{BD}, \\ \mathcal{B}_\mu^{BD^*} &= -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p_1' \frac{h'_B (i h''_{D^*})}{N_1' N_1'' N_2} S_{\mu\nu}^{BD^*} \varepsilon^{\mu*\nu}, \end{aligned} \quad (1)$$



- $N_1^{(n)} = p_1^{(n)2} - m_1^{(n)2}$ ,  $N_2 = p_2^2 - m_2^2$  arise from the quark propagators.

# The CLFQM general calculation procedure

The traces  $S_{\mu}^{BD}$  and  $S_{\mu\nu}^{BD*}$  can be obtained directly using Lorentz contraction,

$$\begin{aligned} S_{\mu}^{BD} &= \text{Tr} [\gamma_5 (\not{p}'_1 + m''_1) \gamma_{\mu} (\not{p}'_1 + m'_1) \gamma_5 (-\not{p}_2 + m_2)], \\ S_{\mu\nu}^{BD*} &= (S_V^{BD*} - S_A^{BD*})_{\mu\nu} \\ &= \text{Tr} \left[ \left( \gamma_{\nu} - \frac{1}{W_V''} (p'_1 - p_2)_{\nu} \right) (\not{p}'_1 + m''_1) (\gamma_{\mu} - \gamma_{\mu} \gamma_5) (\not{p}'_1 + m'_1) \gamma_5 (-\not{p}_2 + m_2) \right], \end{aligned} \quad (2)$$

The covariant vertex function  $h''_M$  with  $M = D(1S, 2S), D^*(1S, 2S)$  is defined as

$$\begin{aligned} h''_M &= (M''^2 - M_0''^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2\tilde{M}_0''}} \varphi, \\ M_0''^2 &= (e_1'' + e_2) ^2 = \frac{p_{\perp}^{\prime 2} + m_1^{\prime\prime 2}}{x_1} + \frac{p_{\perp}^2 + m_2^2}{x_2}, \quad \tilde{M}_0'' = \sqrt{M_0''^2 - (m_1'' - m_2)^2}. \end{aligned} \quad (3)$$

# The CLFQM general calculation procedure

The phenomenological Gaussian-type wave function  $\varphi$  depicts the light-front momentum distribution amplitude for the S-wave mesons,

$$\varphi(1S) = 4 \left( \frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp_z}{dx_2}} \exp \left( -\frac{p_z^2 + p_{\perp}^2}{2\beta^2} \right),$$

$$\varphi(2S) = 4 \left( \frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp_z}{dx_2}} \exp \left( -\frac{p_z^2 + p_{\perp}^2}{2\beta^2} \right) \left( 3 - 2\frac{p_z^2 + p_{\perp}^2}{\beta^2} \right),$$

- The shape parameter  $\beta$  can be fixed by fitting the corresponding decay constant.

# Transition Form Factors

## Input parameters:

- The constituent quark masses(GeV):  $m_c = 1.4$ ,  $m_s = 0.37$ ,  $m_{u,d} = 0.25$ .
- The masses of the initial and the final mesons(GeV):  $m_D = 1.86966$ ,  
 $m_{D(2S)} = 2.549$ ,  $m_{D_s} = 1.96835$ ,  $m_{D_s(2S)} = 2.591$ ,  
 $m_{D_s^{*\pm}} = 2.1122$ ,  $m_{D^{*0}} = 2.0068$ ,  $m_{D^{*\pm}} = 2.0102$ ,  $m_B = 5.279$ ,  
 $m_{D_s^{*\pm}(2S)} = 2.732$ ,  $m_{D^{*0}(2S)} = 2.627$ ,  $m_{D^{*\pm}(2S)} = 2.637$ ,  $m_{B_s^0} = 5.4154$ .
- The CKM matrix element:  $|V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$ ,  $V_{cd} = 0.221 \pm 0.004$ ,  $V_{cs} = 0.975 \pm 0.006$ ,  $V_{ud} = 0.97373 \pm 0.00031$ .

# Transition Form Factors

## Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta_{D_s^{*\pm}(1S)} = 0.534_{-0.014}^{+0.014}, \beta_{D^{*0}(1S)} = 0.500_{-0.187}^{+0.140}, \beta_D = 0.466_{-0.021}^{+0.022},$$

$$\beta_{D_s^{*\pm}(2S)} = 0.473_{-0.041}^{+0.041}, \beta_{D^{*0}(2S)} = 0.456_{-0.003}^{+0.004}, \beta_{D_s} = 0.600_{-0.025}^{+0.026},$$

$$\beta_{D^{*\pm}(1S)} = 0.502_{-0.041}^{+0.041}, \beta_{D^{*\pm}(2S)} = 0.453_{-0.003}^{+0.004}, \beta_{\bar{B}_s^0} = 0.626_{-0.045}^{+0.045},$$

$$\beta_{D(2S)} = 0.297_{-0.041}^{+0.041}, \beta_{D_s(2S)} = 0.422_{-0.025}^{+0.026}, \beta_B = 0.555_{-0.060}^{+0.060}.$$

- mean life( $10^{-12}$ s):  $\tau_{B^0} = (1.519 \pm 0.004)$ ,  $\tau_{\bar{B}_s^0} = (1.520 \pm 0.005)$ ,  
 $\tau_{B^\pm} = (1.638 \pm 0.004)$ .

# Transition Form Factors

- All the calculations are carried out within the  $q^+ = 0$  reference frame, where the form factors can only be obtained at spacelike momentum transfers  $q^2 = -q_{\perp}^2 \leq 0$ ,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F(q^2) = \frac{F(0)}{(1 - q^2/m^2) [1 - a(q^2/m^2) + b(q^2/m^2)^2]}. \quad (4)$$

- $F(q^2)$  denotes different form factors, such as  $F_1(q^2)$ ,  $F_0(q^2)$ ,  $V(q^2)$ ,  $A_0(q^2)$ ,  $A_1(q^2)$ ,  $A_2(q^2)$ .

# Transition Form Factors

- The form factors of the transitions  $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$  in the CLFQM. The uncertainties are from the decay constants of  $B_{(s)}$  and final state mesons.

|                     | $F(0)$                           | $F(q_{max}^2)$                   | $a$                               | $b$                               |
|---------------------|----------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| $F_1^{BD}$          | $0.66^{+0.00+0.01}_{-0.01-0.01}$ | $0.81^{+0.01+0.01}_{-0.00-0.01}$ | $0.80^{+0.01+0.04}_{-0.02-0.04}$  | $0.86^{+0.01+0.02}_{-0.01-0.02}$  |
| $F_0^{BD}$          | $0.66^{+0.00+0.01}_{-0.01-0.01}$ | $0.70^{+0.02+0.01}_{-0.03-0.01}$ | $0.46^{+0.14+0.00}_{-0.12-0.01}$  | $0.77^{+0.01+0.05}_{-0.01-0.05}$  |
| $F_1^{BD(2S)}$      | $0.26^{+0.01+0.01}_{-0.02-0.02}$ | $0.34^{+0.02+0.01}_{-0.03-0.03}$ | $0.99^{+0.04+0.16}_{-0.10-0.18}$  | $0.66^{+0.07+0.17}_{-0.12-0.15}$  |
| $F_0^{BD(2S)}$      | $0.26^{+0.01+0.02}_{-0.01-0.02}$ | $0.32^{+0.03+0.02}_{-0.00-0.04}$ | $0.65^{+0.03+0.05}_{-0.01-0.04}$  | $-0.24^{+0.02+0.01}_{-0.03-0.03}$ |
| $F_1^{B_s D_s}$     | $0.67^{+0.00+0.01}_{-0.00-0.01}$ | $0.81^{+0.00+0.00}_{-0.01-0.01}$ | $0.82^{+0.00+0.02}_{-0.01-0.02}$  | $0.96^{+0.01+0.03}_{-0.02-0.03}$  |
| $F_0^{B_s D_s}$     | $0.67^{+0.00+0.01}_{-0.00-0.01}$ | $0.71^{+0.01+0.01}_{-0.02-0.01}$ | $0.48^{+0.00+0.08}_{-0.01-0.08}$  | $0.85^{+0.02+0.06}_{-0.02-0.06}$  |
| $F_1^{B_s D_s(2S)}$ | $0.26^{+0.02+0.02}_{-0.02-0.02}$ | $0.29^{+0.02+0.01}_{-0.02-0.02}$ | $0.59^{+0.12+0.05}_{-0.16-0.01}$  | $0.35^{+0.11+0.04}_{-0.13-0.02}$  |
| $F_0^{B_s D_s(2S)}$ | $0.26^{+0.01+0.01}_{-0.02-0.02}$ | $0.25^{+0.02+0.01}_{-0.02-0.02}$ | $-0.09^{+0.22+0.22}_{-0.14-0.26}$ | $-0.07^{+0.13+0.18}_{-0.21-0.08}$ |

# Transition Form Factors

- The form factors of the transitions  $B \rightarrow D^*(1S, 2S)$  in the CLFQM. The uncertainties are from the decay constants of  $B$  and final state mesons.

| F                | $F(0)$                            | $F(q_{max}^2)$                   | $a$                               | $b$                               |
|------------------|-----------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| $V^{BD^*}$       | $0.77^{+0.00+0.01}_{-0.00-0.01}$  | $0.94^{+0.00+0.01}_{-0.00-0.01}$ | $0.78^{+0.00+0.03}_{-0.01-0.03}$  | $0.82^{+0.04+0.14}_{-0.08-0.16}$  |
| $A_0^{BD^*}$     | $0.75^{+0.00+0.06}_{-0.01-0.02}$  | $0.79^{+0.00+0.05}_{-0.02-0.02}$ | $0.17^{+0.01+0.01}_{-0.01-0.00}$  | $0.12^{+0.07+0.02}_{-0.06-0.03}$  |
| $A_1^{BD^*}$     | $0.67^{+0.00+0.01}_{-0.01-0.11}$  | $0.76^{+0.00+0.01}_{-0.01-0.01}$ | $0.38^{+0.01+0.01}_{-0.01-0.01}$  | $0.19^{+0.05+0.09}_{-0.05-0.09}$  |
| $A_2^{BD^*}$     | $0.58^{+0.00+0.00}_{-0.01-0.02}$  | $0.69^{+0.01+0.01}_{-0.01-0.02}$ | $0.68^{+0.02+0.00}_{-0.02-0.00}$  | $0.66^{+0.25+0.10}_{-0.12-0.16}$  |
| $V^{BD^*(2S)}$   | $0.19^{+0.05+0.01}_{-0.06-0.01}$  | $0.19^{+0.01+0.03}_{-0.04-0.01}$ | $0.11^{+0.05+0.03}_{-0.06-0.05}$  | $0.34^{+0.00+0.03}_{-0.04-0.06}$  |
| $A_0^{BD^*(2S)}$ | $0.27^{+0.04+0.01}_{-0.05-0.00}$  | $0.28^{+0.00+0.01}_{-0.02-0.00}$ | $0.24^{+0.05+0.00}_{-0.06-0.00}$  | $-0.07^{+0.15+0.03}_{-0.29-0.06}$ |
| $A_1^{BD^*(2S)}$ | $0.16^{+0.04+0.01}_{-0.05-0.00}$  | $0.15^{+0.05+0.02}_{-0.04-0.02}$ | $-0.23^{+0.04+0.00}_{-0.04-0.00}$ | $0.17^{+0.03+0.01}_{-0.01-0.01}$  |
| $A_2^{BD^*(2S)}$ | $-0.05^{+0.04+0.01}_{-0.04-0.00}$ | $0.01^{+0.00+0.00}_{-0.00-0.00}$ | $-1.10^{+0.02+0.01}_{-0.01-0.01}$ | $-0.63^{+0.02+0.01}_{-0.01-0.00}$ |

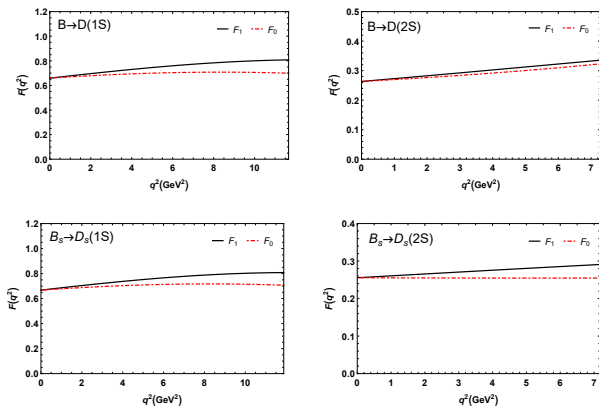
# Transition Form Factors

- The form factors of the transitions  $B_s \rightarrow D_s^*(1S, 2S)$  in the CLFQM. The uncertainties are from the decay constants of  $B_s$  and final state mesons.

| F                     | $F(0)$                            | $F(q_{max}^2)$                   | $a$                               | $b$                               |
|-----------------------|-----------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| $V^{B_s D_s^*}$       | $0.78_{-0.01-0.01}^{+0.01+0.01}$  | $0.87_{-0.00-0.01}^{+0.00+0.00}$ | $0.86_{-0.01-0.04}^{+0.01+0.04}$  | $1.11_{-0.01-0.02}^{+0.01+0.02}$  |
| $A_0^{B_s D_s^*}$     | $0.74_{-0.01-0.01}^{+0.01+0.01}$  | $0.69_{-0.01-0.00}^{+0.01+0.01}$ | $0.23_{-0.01-0.01}^{+0.01+0.01}$  | $0.21_{-0.00-0.01}^{+0.00+0.01}$  |
| $A_1^{B_s D_s^*}$     | $0.66_{-0.02-0.02}^{+0.01+0.01}$  | $0.95_{-0.00-0.01}^{+0.00+0.00}$ | $0.81_{-0.01-0.02}^{+0.01+0.02}$  | $0.93_{-0.01-0.01}^{+0.01+0.01}$  |
| $A_2^{B_s D_s^*}$     | $0.57_{-0.01-0.00}^{+0.00+0.00}$  | $0.68_{-0.01-0.00}^{+0.01+0.00}$ | $0.80_{-0.01-0.03}^{+0.00+0.03}$  | $0.96_{-0.01-0.02}^{+0.01+0.03}$  |
| $V^{B_s D_s^*(2S)}$   | $0.26_{-0.03-0.04}^{+0.03+0.04}$  | $0.28_{-0.09-0.10}^{+0.00+0.02}$ | $0.25_{-0.01-0.04}^{+0.02+0.04}$  | $0.30_{-0.01-0.00}^{+0.03+0.04}$  |
| $A_0^{B_s D_s^*(2S)}$ | $0.31_{-0.03-0.02}^{+0.02+0.01}$  | $0.33_{-0.01-0.07}^{+0.00+0.08}$ | $0.21_{-0.05-0.00}^{+0.01+0.03}$  | $-0.09_{-0.02-0.14}^{+0.04+0.10}$ |
| $A_1^{B_s D_s^*(2S)}$ | $0.21_{-0.03-0.03}^{+0.02+0.03}$  | $0.20_{-0.07-0.06}^{+0.00+0.01}$ | $-0.16_{-0.17-0.02}^{+0.20+0.04}$ | $0.12_{-0.01-0.07}^{+0.00+0.12}$  |
| $A_2^{B_s D_s^*(2S)}$ | $-0.01_{-0.02-0.06}^{+0.02+0.06}$ | $0.01_{-0.01-0.01}^{+0.00+0.03}$ | $-4.03_{-0.47-1.38}^{+0.51+0.75}$ | $4.31_{-0.00-2.44}^{+0.00+2.13}$  |

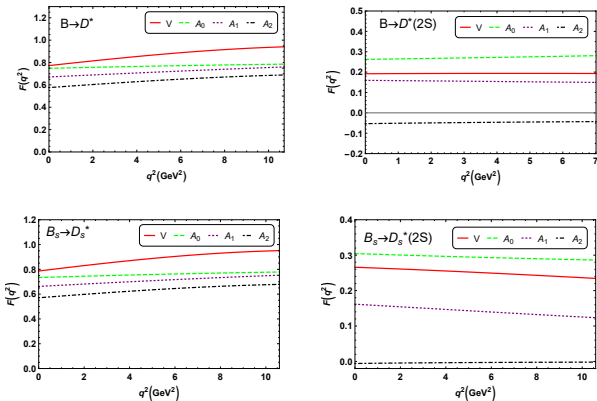
# Transition Form Factors

图: The  $q^2$ -dependence of the  $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$  transition form factors.



# Transition Form Factors

图: The  $q^2$ -dependence of the  $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)$  transition form factors.



# Semi-leptonic Decays

$$B \rightarrow D^{(*)}(1S, 2S)l\nu_l$$

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

The differential decay widths of the semileptonic  $B_{(s)}$  decays can be obtained by the combinations of the helicity amplitudes via the form factors

$$\begin{aligned}\frac{d\Gamma(B_{(s)} \rightarrow D_{(s)}\ell\nu)}{dq^2} &= (1 - \hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)} G_F^2 |V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \left\{ (\hat{m}_\ell^2 + 2) \lambda(q^2) F_1^2(q^2) \right. \\ &\quad \left. + 3\hat{m}_\ell^2 (m_{B_{(s)}}^2 - m_{D_{(s)}}^2)^2 F_0^2(q^2) \right\}, \\ \frac{d\Gamma_L(B_{(s)} \rightarrow D_{(s)}^*\ell\nu)}{dq^2} &= (1 - \hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)} G_F^2 |V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \left\{ 3\hat{m}_\ell^2 \lambda(q^2) A_0^2(q^2) + (\hat{m}_\ell^2 + 2) \right. \\ &\quad \times \left[ \frac{1}{2m_{D_{(s)}^*}} \left[ (m_{B_{(s)}}^2 - m_{D_{(s)}^*}^2 - q^2)(m_{B_{(s)}} + m_{D_{(s)}^*}) A_1(q^2) \right. \right. \\ &\quad \left. \left. - \frac{\lambda(q^2)}{m_{B_{(s)}} + m_{D_{(s)}^*}} A_2(q^2) \right] \right]^2 \left. \right\},\end{aligned}$$

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

$$\frac{d\Gamma^\pm(B_{(s)} \rightarrow D_{(s)}^*\ell\nu)}{dq^2} = (1 - \hat{m}_\ell^2)^2 \frac{\sqrt{\lambda(q^2)} G_F^2 |V_{cb}|^2}{384 m_{B_{(s)}}^3 \pi^3} \left\{ (m_\ell^2 + 2q^2) \lambda(q^2) \right. \\ \left. \times \left| \frac{V(q^2)}{m_{B_{(s)}} + m_{D_{(s)}^*}} \mp \frac{(m_{B_{(s)}} + m_{D_{(s)}^*}) A_1(q^2)}{\sqrt{\lambda(q^2)}} \right|^2 \right\},$$

where  $\lambda(q^2) = (m_{B_{(s)}}^2 + m_{D_{(s)}^*}^2 - q^2)^2 - 4m_{B_{(s)}}^2 m_{D_{(s)}^*}^2$ ,  $\hat{m}_\ell^2 = m_\ell^2/q^2$  and  $m_\ell$  is the mass of the lepton  $\ell$  with  $\ell = e, \mu, \tau$ . The combined transverse and total differential decay widths are defined as

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}, \quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \quad (5)$$

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

For the  $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)\ell\nu_\ell$  decays, defining the polarization fraction is important owing to the existence of different polarizations

$$f_L = \frac{\Gamma_L}{\Gamma_L + \Gamma_+ + \Gamma_-}. \quad (6)$$

The analytical expression of the forward-backward asymmetry is defined as

$$A_{FB} = \frac{\int_0^1 \frac{d\Gamma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\Gamma}{d\cos\theta} d\cos\theta}{\int_{-1}^1 \frac{d\Gamma}{d\cos\theta} d\cos\theta} = \frac{\int b_\theta(q^2) dq^2}{\Gamma_{B \rightarrow D^{(*)}\ell\nu_\ell}}. \quad (7)$$

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays  $B \rightarrow D(1S)\ell^+\nu_\ell$  (%).

| Modes            | $B(B^0 \rightarrow D^-(1S)e^+\nu_e)$    | $B(B^0 \rightarrow D^-(1S)\mu^+\nu_\mu)$ | $B(B^0 \rightarrow D^-(1S)\tau^+\nu_\tau)$ |
|------------------|---|--|--|
| This work(CLFQM) | $1.96^{+0.06}_{-0.06} (\sim 1.5\sigma)$ | $1.96^{+0.06}_{-0.06} (\sim 1.5\sigma)$  | $0.51^{+0.02}_{-0.02} (\sim 2.2\sigma)$    |
| PQCD             | 2.19                                    | 2.19                                     | 0.82                                       |
| PQCD+Lattice     | 1.95                                    | 1.95                                     | 0.62                                       |
| CQM              | 2.74                                    | 2.74                                     | 0.73                                       |
| HQET             | –                                       | –  | 0.64                                       |
| PQCD             | 2.03                                    | 2.03                                     | 0.87                                       |
| CPQCD            | –                                       | 1.65                                     | 0.554                                      |
| PDG              | $2.10 \pm 0.07$                         | $2.10 \pm 0.07$                          | $0.98 \pm 0.21$                            |
| Modes            | $B(B^+ \rightarrow D^0(1S)e^+\nu_e)$    | $B(B^+ \rightarrow D^0(1S)\mu^+\nu_\mu)$ | $B(B^+ \rightarrow D^0(1S)\tau^+\nu_\tau)$ |
| This work(CLFQM) | $2.11^{+0.06}_{-0.06} (\sim 1.6\sigma)$ | $2.11^{+0.06}_{-0.06} (\sim 1.6\sigma)$  | $0.55^{+0.02}_{-0.02} (\sim 0.9\sigma)$    |
| RQM              | 2.53                                    | 2.53                                     | 0.68                                       |
| PQCD             | 2.29                                    | 2.29                                     | 0.86                                       |
| PQCD+Lattice     | 2.10                                    | 2.10                                     | 0.69                                       |
| LQCD             | –                                       | –  | 0.65                                       |
| HQET             | –                                       | –  | 0.66                                       |
| PQCD             | 2.19                                    | 2.19                                     | 0.95                                       |
| PDG              | $2.26 \pm 0.07$                         | $2.26 \pm 0.07$                          | $0.77 \pm 0.25$                            |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays  $B_s \rightarrow D_s(1S)\ell^+\nu_\ell(\%)$ .

| Modes            | $B(B_s^0 \rightarrow D_s^-(1S)e^+\nu_e)$ | $B(B_s^0 \rightarrow D_s^-(1S)\mu^+\nu_\mu)$ | $B(B_s^0 \rightarrow D_s^-(1S)\tau^+\nu_\tau)$ |
|------------------|--|--|--|
| This work(CLFQM) | $2.05^{+0.12}_{-0.13}$                   | $2.04^{+0.11}_{-0.11}(\sim 1.1\sigma)$       | $0.53^{+0.04}_{-0.07}$                         |
| NCQM             | 2.32                                     | –  | 0.67   |
| LCSR             | 1.817                                    | 1.817  | 0.606  |
| QCDSR            | 2.03                                     | 2.03   | –  |
| LCSR             | 1.0                                      | 1.0  | 0.33   |
| pQCD             | 2.13                                     | 2.13   | 0.84   |
| RQM              | 2.1                                      | 2.1  | 0.62   |
| CQM              | 2.73 – 3.00                              | 2.73 – 3.00                                  | –  |
| pQCD             | 1.97                                     | 1.97   | 0.72   |
| pQCD+Lattice     | 1.84                                     | 1.84   | 0.63   |
| LQCD             | 2.013 – 2.469                            | 2.013 – 2.469                                | 0.619 – 0.724                                  |
| BS               | 1.4 – 1.7                                | 1.4 – 1.7                                    | 0.47 – 0.55                                    |
| LQCD             | 2.31                                     | 2.31   | 0.69   |
| CQM              | 2.89                                     | 2.88   | 0.78   |
| PDG              | –  | $2.29 \pm 0.21$                              | –  |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays  $B \rightarrow D^*(1S)\ell^+\nu_\ell$  (%).

| Modes        | $B^0 \rightarrow D^{*-}(1S)e^+\nu_e$    | $B^0 \rightarrow D^{*-}(1S)\mu^+\nu_\mu$ | $B^0 \rightarrow D^{*-}(1S)\tau^+\nu_\tau$ |
|--------------|---|--|--|
| This work    | $4.97^{+0.36}_{-0.16} (\sim 0.3\sigma)$ | $4.95^{+0.36}_{-0.16} (\sim 0.3\sigma)$  | $1.13^{+0.07}_{-0.04} (\sim 3.1\sigma)$    |
| RQM          | 6.28                                    | 6.28                                     | 1.45                                       |
| PQCD         | 5.32                                    | 5.32                                     | 1.53                                       |
| PQCD+Lattice | 4.63                                    | 4.63                                     | 1.25                                       |
| PQCD         | 4.52                                    | 4.52                                     | 1.36                                       |
| HQET         | –                                       | –  | 1.29                                       |
| CQM          | 6.64                                    | 6.64                                     | 1.57                                       |
| CPQCD        | –                                       | 4.33                                     | 1.175                                      |
| PDG          | $4.87 \pm 0.09$                         | $4.87 \pm 0.09$                          | $1.48 \pm 0.09$                            |
| Modes        | $B^+ \rightarrow D^{*0}(1S)e^+\nu_e$    | $B^+ \rightarrow D^{*0}(1S)\mu^+\nu_\mu$ | $B^+ \rightarrow D^{*0}(1S)\tau^+\nu_\tau$ |
| This work    | $5.37^{+0.39}_{-0.17} (\sim 0.3\sigma)$ | $5.35^{+0.38}_{-0.17} (\sim 0.3\sigma)$  | $1.22^{+0.08}_{-0.04} (\sim 3.1\sigma)$    |
| RQM          | 6.81                                    | 6.77                                     | 1.52                                       |
| PQCD         | 5.53                                    | 5.53                                     | 1.60                                       |
| PQCD+Lattice | 4.89                                    | 4.89                                     | 1.34                                       |
| PQCD         | 4.87                                    | 4.87                                     | 1.47                                       |
| HQET         | –                                       | –  | 1.34                                       |
| PDG          | $5.26 \pm 0.10$                         | $5.26 \pm 0.10$                          | $1.88 \pm 0.20$                            |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays  $B_s \rightarrow D_s^*(1S)\ell^+\nu_\ell$  (%)

| Modes            | $\mathcal{B}(B_s^0 \rightarrow D_s^{*-}(1S)e^+\nu_e)$ | $\mathcal{B}(B_s^0 \rightarrow D_s^{*-}(1S)\mu^+\nu_\mu)$ | $\mathcal{B}(B_s^0 \rightarrow D_s^{*-}(1S)\tau^+\nu_\tau)$ |
|------------------|---|---|---|
| This work(CLFQM) | $5.02_{-0.54}^{+0.48}$                                | $5.00_{-0.53}^{+0.48} (\sim 0.3\sigma)$                   | $1.13_{-0.10}^{+0.09}$                                      |
| NCQM             | 6.26  | –   | 1.53  |
| PQCD             | 4.76  | 4.76  | 1.44  |
| RQM              | 5.3   | 5.3   | 1.3   |
| RQM              | 2.54  | 2.54  | 0.70  |
| CQM              | 7.49 – 7.66   | 7.49 – 7.66   | –   |
| pQCD             | 5.04  | 5.04  | 1.45  |
| pQCD+Lattice     | 4.42  | 4.42  | 1.20  |
| BS               | 5.1 – 5.8   | 5.1 – 5.8   | 1.2 – 1.3   |
| LQCD             | 5.25  | 5.25  | 1.31  |
| CQM              | 1.89 – 6.61   | 1.89 – 6.61   | –   |
| CQM              | 6.42  | 6.39  | 1.53  |
| PDG              | –   | $5.2 \pm 0.5$   | –   |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- The branching ratios of the decays  $B \rightarrow D^{(*)}(2S)\ell\nu_\ell(10^{-3})$ .

| Modes   | This work                                  | BS     | Modes   | This work                                  | BS     |
|---|--|--------|---|--|--------|
| $B(B^0 \rightarrow D^-(2S)e^+\nu_e)$          | $1.44_{-0.00-0.19-0.19}^{+0.00+0.18+0.15}$ | 0.139  | $B(B^+ \rightarrow D^0(2S)e^+\nu_e)$          | $1.55_{-0.00-0.21-0.20}^{+0.00+0.19+0.17}$ | 0.15   |
| $B(B^0 \rightarrow D^-(2S)\mu^+\nu_\mu)$      | $1.43_{-0.00-0.19-0.19}^{+0.00+0.18+0.15}$ | 0.138  | $B(B^+ \rightarrow D^0(2S)\mu^+\nu_\mu)$      | $1.54_{-0.00-0.21-0.20}^{+0.00+0.19+0.17}$ | 0.149  |
| $B(B^0 \rightarrow D^-(2S)\tau^+\nu_\tau)$    | $0.18_{-0.00-0.03-0.03}^{+0.00+0.03+0.03}$ | 0.0135 | $B(B^+ \rightarrow D^0(2S)\tau^+\nu_\tau)$    | $0.19_{-0.00-0.04-0.04}^{+0.00+0.03+0.03}$ | 0.0149 |
| $B(B^0 \rightarrow D^{*-}(2S)e^+\nu_e)$       | $2.67_{-0.00-0.24-0.14}^{+0.00+0.20+0.11}$ | 0.1942 | $B(B^+ \rightarrow D^{*0}(2S)e^+\nu_e)$       | $2.93_{-0.00-0.26-0.16}^{+0.00+0.21+0.13}$ | 0.2123 |
| $B(B^0 \rightarrow D^{*-}(2S)\mu^+\nu_\mu)$   | $2.65_{-0.00-0.24-0.14}^{+0.00+0.20+0.11}$ | 0.1932 | $B(B^+ \rightarrow D^{*0}(2S)\mu^+\nu_\mu)$   | $2.91_{-0.00-0.26-0.16}^{+0.00+0.21+0.13}$ | 0.2113 |
| $B(B^0 \rightarrow D^{*-}(2S)\tau^+\nu_\tau)$ | $0.19_{-0.00-0.02-0.11}^{+0.00+0.02+0.10}$ | 0.0137 | $B(B^+ \rightarrow D^{*0}(2S)\tau^+\nu_\tau)$ | $0.21_{-0.00-0.03-0.11}^{+0.00+0.02+0.11}$ | 0.0155 |

- The branching ratios of the decays  $B_s \rightarrow D_s^{(*)}(2S)\ell\nu_\ell(10^{-3})$ .

| Modes  | This work                                  | BS     | RQM  | Modes   | This work                                  | BS     | RQM  |
|--|--|--------|------|---|--|--------|------|
| $B(B_s^0 \rightarrow D_s^-(2S)e^+\nu_e)$       | $1.35_{-0.00-0.23-0.22}^{+0.00+0.25+0.14}$ | 0.314  | 2.7  | $B(B_s^0 \rightarrow D_s^{*-}(2S)e^+\nu_e)$       | $2.25_{-0.01-0.70-0.80}^{+0.01+1.09+0.64}$ | 0.5873 | 3.8  |
| $B(B_s^0 \rightarrow D_s^-(2S)\mu^+\nu_\mu)$   | $1.34_{-0.00-0.23-0.22}^{+0.00+0.25+0.14}$ | 0.312  | –    | $B(B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu)$   | $2.23_{-0.01-0.70-0.80}^{+0.01+1.09+0.64}$ | 0.5842 | –    |
| $B(B_s^0 \rightarrow D_s^-(2S)\tau^+\nu_\tau)$ | $0.14_{-0.00-0.03-0.03}^{+0.00+0.03+0.02}$ | 0.0244 | 0.11 | $B(B_s^0 \rightarrow D_s^{*-}(2S)\tau^+\nu_\tau)$ | $0.15_{-0.00-0.07-0.06}^{+0.00+0.05+0.06}$ | 0.0405 | 0.15 |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- For the decays  $B_{u,d} \rightarrow D\ell'\nu_{\ell'}$  and  $B_s \rightarrow D_s\ell'\nu_{\ell'}$  with  $\ell' = e, \mu$ , their branching ratios deviate from the PDG data by about  $1.5\sigma$  and  $1.1\sigma$ . For the decays  $B_{u,d} \rightarrow D^*\ell'\nu_{\ell'}$  and  $B_s \rightarrow D_s^*\ell'\nu_{\ell'}$ , the deviations are about  $0.3\sigma$ .
- The branching ratios of all the considered decays with  $\tau\nu_\tau$  involved exhibit systematically smaller than the experimental values. The deviations between our predictions and the data for the decays  $B^0 \rightarrow D^{(*)-}\tau^+\nu_\tau$ ,  $B^+ \rightarrow D^{*0}\tau^+\nu_\tau$  exceed  $2\sigma$  or even  $3\sigma$ .
- The branching ratios of the decays  $B \rightarrow D^{(*)}(2S)\ell\nu_\ell$  lie in the range  $10^{-4} \sim 10^{-3}$ , which are about one order larger than those predicted by BS equation. For the decays  $B_s \rightarrow D_s^{(*)}(2S)\ell\nu_\ell$ , our predictions are  $3 \sim 5$  times of those given by BS equation, while are consistent with RQM calculations.

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)l\nu_l$

- Forward-backward asymmetries  $A_{FB}$  for the decays  $B \rightarrow D^{(*)}(1S)l\nu_l$ .

|              |  |  |  |
|--------------|--|--|--|
| Channels     | $B^0 \rightarrow D^-(1S)e^+\nu_e$                        | $B^0 \rightarrow D^-(1S)\mu^+\nu_\mu$        | $B^0 \rightarrow D^-(1S)\tau^+\nu_\tau$        |
| $A_{FB}$     | $4.4^{+0.0+0.0+0.1}_{-0.0-0.0-0.1} \times 10^{-7}$       | $0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$   | $0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$     |
| CQM          | $-11.7 \times 10^{-7}$                                   | -  | -0.36  |
| PQCD         | -  | -  | 0.35   |
| PQCD+Lattice | -  | -  | 0.36   |
| Channels     | $B^+ \rightarrow D^0(1S)e^+\nu_e$                        | $B^+ \rightarrow D^0(1S)\mu^+\nu_\mu$        | $B^+ \rightarrow D^0(1S)\tau^+\nu_\tau$        |
| $A_{FB}$     | $4.4^{+0.00+0.00+0.01}_{-0.00-0.00-0.00} \times 10^{-7}$ | $0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$   | $0.36^{+0.00+0.04+0.01}_{-0.00-0.05-0.01}$     |
| RQM          | $-9.8 \times 10^{-7}$                                    | -0.013                                       | -0.37  |
| Channels     | $B_s^0 \rightarrow D_s^-(1S)e^+\nu_e$                    | $B_s^0 \rightarrow D_s^-(1S)\mu^+\nu_\mu$    | $B_s^0 \rightarrow D_s^-(1S)\tau^+\nu_\tau$    |
| $A_{FB}$     | $4.4^{+0.0+0.0+0.1}_{-0.0-0.0-0.1} \times 10^{-7}$       | $0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$   | $0.36^{+0.00+0.04+0.02}_{-0.00-0.02-0.01}$     |
| RQM          | $-9.7 \times 10^{-7}$                                    | -0.013                                       | -0.36  |
| PQCD         | -  | -  | 0.36   |
| PQCD+Lattice | -  | -  | 0.36   |
| Channels     | $B^0 \rightarrow D^{*-}(1S)e^+\nu_e$                     | $B^0 \rightarrow D^{*-}(1S)\mu^+\nu_\mu$     | $B^0 \rightarrow D^{*-}(1S)\tau^+\nu_\tau$     |
| $A_{FB}$     | $-0.20^{+0.00+0.00+0.00}_{-0.00-0.01-0.01}$              | $-0.20^{+0.00+0.00+0.00}_{-0.00-0.01-0.01}$  | $-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.04}$    |
| CQM          | 0.19   | -  | 0.027  |
| PQCD         | -  | -  | -0.085   |
| PQCD+Lattice | -  | -  | -0.054   |
| HQET         | -  | -  | -0.084   |
| Channels     | $B^+ \rightarrow D^{*0}(1S)e^+\nu_e$                     | $B^+ \rightarrow D^{*0}(1S)\mu^+\nu_\mu$     | $B^+ \rightarrow D^{*0}(1S)\tau^+\nu_\tau$     |
| $A_{FB}$     | $-0.20^{+0.00+0.01+0.00}_{-0.00-0.00-0.01}$              | $-0.20^{+0.00+0.01+0.00}_{-0.00-0.00-0.01}$  | $-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.04}$    |
| RQM          | -0.22  | -0.23  | -0.32  |
| Channels     | $B_s^0 \rightarrow D_s^{*-}(1S)e^+\nu_e$                 | $B_s^0 \rightarrow D_s^{*-}(1S)\mu^+\nu_\mu$ | $B_s^0 \rightarrow D_s^{*-}(1S)\tau^+\nu_\tau$ |
| $A_{FB}$     | $-0.20^{+0.00+0.01+0.02}_{-0.00-0.00-0.01}$              | $-0.20^{+0.00+0.01+0.02}_{-0.00-0.00-0.01}$  | $-0.14^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$    |
| RQM          | -0.26  | -0.27  | -0.32  |
| PQCD         | -  | -  | -0.083   |
| PQCD+Lattice | -  | -  | -0.050   |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)l\nu_l$

- Forward-backward asymmetries  $A_{FB}$  for the decays  $B \rightarrow D^{(*)}(2S)l\nu_l$ .

|                    |  |  |  |
|--------------------|--|--|--|
| Channel            | $B^0 \rightarrow D^-(2S)e^+\nu_e$                  | $B^0 \rightarrow D^-(2S)\mu^+\nu_\mu$        | $B^0 \rightarrow D^-(2S)\tau^+\nu_\tau$        |
| $\mathcal{A}_{FB}$ | $6.6^{+0.0+0.8+0.8}_{-0.0-0.9-0.7} \times 10^{-7}$ | $0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$   | $0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$     |
| Channels           | $B^+ \rightarrow D^0(2S)e^+\nu_e$                  | $B^+ \rightarrow D^0(2S)\mu^+\nu_\mu$        | $B^+ \rightarrow D^0(2S)\tau^+\nu_\tau$        |
| $\mathcal{A}_{FB}$ | $6.6^{+0.0+0.8+0.7}_{-0.0-0.9-0.8} \times 10^{-7}$ | $0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$   | $0.35^{+0.00+0.05+0.05}_{-0.00-0.06-0.06}$     |
| Channels           | $B_s^0 \rightarrow D_s^-(2S)e^+\nu_e$              | $B_s^0 \rightarrow D_s^-(2S)\mu^+\nu_\mu$    | $B_s^0 \rightarrow D_s^-(2S)\tau^+\nu_\tau$    |
| $\mathcal{A}_{FB}$ | $6.6^{+0.0+1.1+0.6}_{-0.0-1.0-1.0} \times 10^{-7}$ | $0.02^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$   | $0.37^{+0.00+0.07+0.06}_{-0.00-0.08-0.08}$     |
| Channels           | $B^0 \rightarrow D^{*-}(2S)e^+\nu_e$               | $B^0 \rightarrow D^{*-}(2S)\mu^+\nu_\mu$     | $B^0 \rightarrow D^{*-}(2S)\tau^+\nu_\tau$     |
| $\mathcal{A}_{FB}$ | $-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$        | $-0.11^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$  | $-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$    |
| Channels           | $B^+ \rightarrow D^{*0}(2S)e^+\nu_e$               | $B^+ \rightarrow D^{*0}(2S)\mu^+\nu_\mu$     | $B^+ \rightarrow D^{*0}(2S)\tau^+\nu_\tau$     |
| $\mathcal{A}_{FB}$ | $-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$        | $-0.12^{+0.00+0.01+0.06}_{-0.00-0.02-0.05}$  | $-0.05^{+0.00+0.00+0.03}_{-0.00-0.02-0.03}$    |
| Channels           | $B_s^0 \rightarrow D_s^{*-}(2S)e^+\nu_e$           | $B_s^0 \rightarrow D_s^{*-}(2S)\mu^+\nu_\mu$ | $B_s^0 \rightarrow D_s^{*-}(2S)\tau^+\nu_\tau$ |
| $\mathcal{A}_{FB}$ | $-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$        | $-0.13^{+0.00+0.04+0.06}_{-0.00-0.06-0.05}$  | $-0.07^{+0.00+0.02+0.04}_{-0.00-0.06-0.03}$    |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- The values of  $\mathcal{A}_{FB}(B_{(s)} \rightarrow D_{(s)}\ell\nu_\ell)$  exhibit a clear hierarchical relationship, While the difference among  $\mathcal{A}_{FB}(B_{(s)} \rightarrow D_{(s)}^*\ell\nu_\ell)$  is not significant.
- $\mathcal{A}_{FB}(B_{(s)} \rightarrow D_{(s)}\ell\nu_\ell)$  are positive, while  $\mathcal{A}_{FB}(B_{(s)} \rightarrow D_{(s)}^*\ell\nu_\ell)$  are negative.
- Our predictions are consistent with those given by the PQCD, PQCD+Lattice and HQET approaches, while have the opposite sign compared to the calculations in CQM and RQM except for those for the decays  $B_{(s)} \rightarrow D_{(s)}^*\ell\nu_\ell$ .
- The forward-backward asymmetries of the decays  $B_{(s)} \rightarrow D_{(s)}^{(*)}(2S)\ell\nu_\ell$  are similar with those of the corresponding decays  $B_{(s)} \rightarrow D_{(s)}^{(*)}(1S)\ell\nu_\ell$ .

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)l\nu_l$

- The longitudinal polarization fractions  $f_L$  for the decays  $B_{(s)} \rightarrow D_{(s)}^*(nS)l\nu_l$  in Region 1, Region 2 and total physical region.

| Observables   | Region 1 | Region 2 | Total                                      | Observables   | Region 1 | Region 2 | Total                                      |
|---|----------|----------|--|---|----------|----------|--|
| $f_L(B^0 \rightarrow D^{*-}(1S)\ell^+\nu_\ell)$     | 0.35     | 0.20     | $0.55^{+0.00+0.00+0.03}_{-0.00-0.04-0.02}$ | $f_L(B^0 \rightarrow D^{*-}(2S)\ell^+\nu_\ell)$     | 0.50     | 0.18     | $0.68^{+0.00+0.06+0.03}_{-0.00-0.05-0.03}$ |
| $f_L(B^0 \rightarrow D^{*-}(1S)\tau^+\nu_\tau)$     | 0.20     | 0.25     | $0.45^{+0.00+0.00+0.01}_{-0.00-0.03-0.02}$ | $f_L(B^0 \rightarrow D^{*-}(2S)\tau^+\nu_\tau)$     | 0.22     | 0.29     | $0.51^{+0.00+0.03+0.01}_{-0.00-0.06-0.02}$ |
| PQCD  | —        | —        | 0.42                                       | —   | —        | —        | —  |
| PQCD+Lattice  | —        | —        | 0.43                                       | —   | —        | —        | —  |
| SM  | —        | —        | 0.457                                      | —   | —        | —        | —  |
| SM  | —        | —        | 0.441                                      | —   | —        | —        | —  |
| Belle   | —        | —        | $0.60 \pm 0.08 \pm 0.04$                   | —   | —        | —        | —  |
| LHCb  | —        | —        | $0.41 \pm 0.06 \pm 0.03$                   | —   | —        | —        | —  |
| Observables   | Region 1 | Region 2 | Total                                      | Observables   | Region 1 | Region 2 | Total                                      |
| $f_L(B^+ \rightarrow D^{*0}(1S)\ell^+\nu_\ell)$     | 0.35     | 0.20     | $0.55^{+0.00+0.00+0.03}_{-0.00-0.03-0.02}$ | $f_L(B^+ \rightarrow D^{*0}(2S)\ell^+\nu_\ell)$     | 0.49     | 0.18     | $0.68^{+0.00+0.05+0.03}_{-0.00-0.06-0.03}$ |
| RQM   | —        | —        | 0.55                                       | —   | —        | —        | —  |
| $f_L(B^+ \rightarrow D^{*0}(1S)\tau^+\nu_\tau)$     | 0.21     | 0.25     | $0.46^{+0.00+0.00+0.01}_{-0.00-0.03-0.06}$ | $f_L(B^+ \rightarrow D^{*0}(2S)\tau^+\nu_\tau)$     | 0.22     | 0.29     | $0.51^{+0.00+0.44+0.03}_{-0.00-0.06-0.03}$ |
| RQM   | —        | —        | 0.47                                       | —   | —        | —        | —  |
| Observables   | Region 1 | Region 2 | Total                                      | Observables   | Region 1 | Region 2 | Total                                      |
| $f_L(B_s^0 \rightarrow D_s^{*-}(1S)\ell^+\nu_\ell)$ | 0.34     | 0.20     | $0.54^{+0.00+0.04+0.05}_{-0.00-0.03-0.07}$ | $f_L(B_s^0 \rightarrow D_s^{*-}(2S)\ell^+\nu_\ell)$ | 0.51     | 0.17     | $0.68^{+0.00+0.31+0.15}_{-0.00-0.20-0.22}$ |
| RQM   | —        | —        | 0.49                                       | —   | —        | —        | —  |
| $f_L(B_s^0 \rightarrow D_s^{*-}(1S)\tau^+\nu_\tau)$ | 0.19     | 0.25     | $0.45^{+0.00+0.00+0.05}_{-0.00-0.02-0.03}$ | $f_L(B_s^0 \rightarrow D_s^{*-}(2S)\tau^+\nu_\tau)$ | 0.23     | 0.28     | $0.51^{+0.01+0.12+0.11}_{-0.01-0.18-0.11}$ |
| PQCD  | —        | —        | 0.42                                       | —   | —        | —        | —  |
| PQCD+Lattice  | —        | —        | 0.43                                       | —   | —        | —        | —  |
| RQM   | —        | —        | 0.42                                       | —   | —        | —        | —  |

# Semi-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)\ell\nu_\ell$

- Region 1  $m_\ell^2 < q^2 < \frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2}$   
Region 2  $\frac{(m_B - m_{D^*(nS)})^2 + m_\ell^2}{2} < q^2 < (m_B - m_{D^*(nS)})^2$  with  $n = 1, 2$ .
- For the decays  $B_{(s)} \rightarrow D_{(s)}^*(nS)\ell'\nu_{\ell'}$ , the longitudinal polarization fractions from Region 1 are larger than those from Region 2, while it is contrary to the decays  $B_{(s)} \rightarrow D_{(s)}^*(nS)\tau\nu_\tau$ .
- Relations  $f_L(B_{(s)} \rightarrow D_{(s)}^*(nS)\ell'\nu_{\ell'}) > f_L(B_{(s)} \rightarrow D_{(s)}^*(nS)\tau\nu_\tau)$ ,  $f_L(B_{(s)} \rightarrow D_{(s)}^*(2S)\ell\nu_\ell) > f_L(B_{(s)} \rightarrow D_{(s)}^*(1S)\ell\nu_\ell)$ .
- Our predictions for the considered decays are consistent with other theoretical calculations.  $f_L(B^0 \rightarrow D^{*-}(1S)\tau\nu_\tau)$  is in good agreement with the experimental data given by LHCb, but smaller than Belle measurement.

# Non-leptonic Decays

$$B \rightarrow D^{(*)}(1S, 2S)M$$

# Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

Combining with the form factors, we can obtain the partial widths for our considered non-leptonic decays, which are written as

$$\Gamma(B \rightarrow DP) = \frac{G_F^2 (m_B^2 - m_D^2)^2 |\vec{p}_c|}{16\pi m_B^2} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_P^2 F_0^2(m_P^2), \quad (8)$$

$$\Gamma(B \rightarrow D^*P) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_P^2 A_0^2(m_P^2), \quad (9)$$

$$\Gamma(B \rightarrow DV) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_V^2 F_+^2(m_V^2), \quad (10)$$

$$\Gamma(B \rightarrow D^*V) = \frac{G_F^2 |\vec{p}_c|}{16\pi m_B^2} |V_{uq}^* V_{cb}|^2 \left( |H_0|^2 + |H_+|^2 + |H_-|^2 \right), \quad (11)$$

# Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

Here  $\vec{p}_c$  is the momentum of either of the two final state meson in the  $B$  rest frame and  $H_{0,\pm}$  are the helicity amplitudes,

$$H_0 = \frac{if_V a_1}{2m_{D^*}} \left[ (m_B^2 - m_{D^*}^2 - m_V^2) (m_B + m_{D^*}) A_1^{BD^*} (m_V^2) - \frac{4m_B^2 p_c^2}{m_B + m_{D^*}} A_2^{BD^*} (m_V^2) \right], \quad (12)$$

$$H_{\pm} = if_V m_V a_1 \left[ - (m_B + m_{D^*}) A_1^{BD^*} (m_V^2) \mp \frac{2m_B p_c}{m_B + m_{D^*}} V^{BD^*} (m_V^2) \right]. \quad (13)$$

The polarization fractions are defined as  $f_{L,\parallel,\perp} = \frac{H_{0,\parallel,\perp}}{H_0 + H_{\parallel} + H_{\perp}}$ , where  $H_{\parallel}$  and  $H_{\perp}$  are parallel and perpendicular amplitudes, respectively, and can be obtained through

$$H_{\parallel,\perp} = \frac{(H_- \pm H_+)}{\sqrt{2}}.$$

# Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- The branching ratios of the decays  $B \rightarrow D(1S, 2S)M$  with  $M = \pi, K, \rho, K^*$ .

| Modes                                 | This work                                   | Bethe-Salpeter | QCDF  | PQCD  | Exp.  | Unit        |
|---------------------------------------|---|----------------|-------|-------|-------|-------------|
| $\bar{B}^0 \rightarrow D^+(1S)\pi^-$  | $4.37^{+0.01+0.01+0.01}_{-0.01-0.09-0.01}$  | 3.24           | 3.93  | 2.69  | 2.52  | $(10^{-3})$ |
| $\bar{B}^0 \rightarrow D^+(1S)K^-$    | $0.34^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$  | 0.245          | 0.30  | 0.243 | 0.186 |             |
| $\bar{B}^0 \rightarrow D^+(1S)\rho^-$ | $10.09^{+0.03+0.10+0.25}_{-0.03-0.20-0.28}$ | 7.91           | 10.42 | 6.96  | 7.6   |             |
| $\bar{B}^0 \rightarrow D^+(1S)K^{*-}$ | $0.56^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$  | 0.431          | 0.53  | 0.407 | 0.45  |             |
| $\bar{B}^0 \rightarrow D^+(2S)\pi^-$  | $4.76^{+0.01+0.48+0.33}_{-0.01-0.62-0.57}$  | 0.458          | —     | —     | —     | $(10^{-4})$ |
| $\bar{B}^0 \rightarrow D^+(2S)K^-$    | $0.37^{+0.00+0.04+0.04}_{-0.00-0.05-0.03}$  | 0.034          | —     | —     | —     |             |
| $\bar{B}^0 \rightarrow D^+(2S)\rho^-$ | $10.54^{+0.03+1.06+0.73}_{-0.03-1.37-1.26}$ | 1.03           | —     | —     | —     |             |
| $\bar{B}^0 \rightarrow D^+(2S)K^{*-}$ | $0.58^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$  | 0.0557         | —     | —     | —     |             |
| $B^- \rightarrow D^0(1S)\pi^-$        | $4.71^{+0.01+0.02+0.12}_{-0.01-0.09-0.13}$  | 3.49           | —     | 5.11  | 4.68  | $(10^{-3})$ |
| $B^- \rightarrow D^0(1S)K^-$          | $0.37^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$  | 0.264          | —     | 0.400 | 0.363 |             |
| $B^- \rightarrow D^0(1S)\rho^-$       | $10.88^{+0.03+0.00+0.27}_{-0.03-0.20-0.31}$ | 8.40           | —     | 11.3  | 13.4  |             |
| $B^- \rightarrow D^0(1S)K^{*-}$       | $0.60^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$  | 0.466          | —     | 0.696 | 0.53  |             |
| $B^- \rightarrow D^0(2S)\pi^-$        | $5.14^{+0.01+0.52+0.36}_{-0.01-0.67-0.61}$  | 0.488          | —     | —     | —     | $(10^{-4})$ |
| $B^- \rightarrow D^0(2S)K^-$          | $0.40^{+0.00+0.04+0.03}_{-0.00-0.05-0.05}$  | 0.0362         | —     | —     | —     |             |
| $B^- \rightarrow D^0(2S)\rho^-$       | $11.36^{+0.03+1.14+0.79}_{-0.03-1.48-1.35}$ | 1.09           | —     | —     | —     |             |
| $B^- \rightarrow D^0(2S)K^{*-}$       | $0.62^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$  | 0.0595         | —     | —     | —     |             |

# Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- The branching ratios of the decays  $\bar{B}_s^0 \rightarrow D_s^+(1S, 2S)(\pi, \rho, K^{(*)})^-$ .

| Modes                                     | This work                                   | BS    | RQM  | NCQM | QCDSRs | Instant-BS | PQCD  | QCDF | Exp.              | Unit        |
|---|---|-------|------|------|--------|------------|-------|------|-------------------|-------------|
| $\bar{B}_s^0 \rightarrow D_s^+(1S)\pi^-$  | $4.57^{+0.02+0.04+0.07}_{-0.02-0.09-0.09}$  | 2.92  | 3.5  | 5.3  | 5      | 2.7        | 1.7   | 4.39 | $2.98 \pm 0.14$   | $(10^{-3})$ |
| $\bar{B}_s^0 \rightarrow D_s^+(1S)K^-$    | $0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$  | 0.221 | 0.28 | 0.4  | 0.4    | 0.21       | 0.13  | 0.33 | $0.225 \pm 0.012$ |             |
| $\bar{B}_s^0 \rightarrow D_s^+(1S)\rho^-$ | $10.55^{+0.04+0.09+0.16}_{-0.04-0.23-0.21}$ | 7.04  | 9.4  | 12.6 | 13     | 6.4        | 4.7   | —    | $6.8 \pm 1.4$     |             |
| $\bar{B}_s^0 \rightarrow D_s^+(1S)K^{*-}$ | $0.59^{+0.00+0.01+0.01}_{-0.00-0.01-0.01}$  | 0.392 | 0.47 | 0.8  | 0.6    | 0.38       | 0.281 | —    | —                 |             |
| $\bar{B}_s^0 \rightarrow D_s^+(2S)\pi^-$  | $4.70^{+0.02+0.61+0.50}_{-0.02-0.68-0.59}$  | 1.13  | 7    | —    | —      | —          | —     | —    | —                 | $(10^{-4})$ |
| $\bar{B}_s^0 \rightarrow D_s^+(2S)K^-$    | $0.37^{+0.00+0.05+0.04}_{-0.00-0.05-0.05}$  | 0.084 | 0.5  | —    | —      | —          | —     | —    | —                 |             |
| $\bar{B}_s^0 \rightarrow D_s^+(2S)\rho^-$ | $10.43^{+0.04+1.36+1.11}_{-0.04-1.52-1.31}$ | 2.49  | 17   | —    | —      | —          | —     | —    | —                 |             |
| $\bar{B}_s^0 \rightarrow D_s^+(2S)K^{*-}$ | $0.57^{+0.00+0.07+0.06}_{-0.00-0.08-0.07}$  | 0.134 | 0.8  | —    | —      | —          | —     | —    | —                 |             |

- The branching ratios  $(10^{-3})$  of the decays  $\bar{B}_s^0 \rightarrow D_s^{*+}(1S, 2S)(\pi, \rho, K^{(*)})^-$ .

| Modes  | This work                                   | RQM  | QCDSR | 3P QCDSR | PQCD  | BS      | RIQ   | Exp.  |
|--|---|------|-------|----------|-------|---------|-------|-------|
| $\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\pi^-$  | $4.04^{+0.02+0.03+0.14}_{-0.02-0.09-0.15}$  | 2.7  | 2     | 2.11     | 1.89  | 3.37    | —     | 1.9   |
| $\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^-$    | $0.31^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$  | 0.21 | 0.2   | 0.159    | 0.164 | 0.249   | —     | 0.132 |
| $\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\rho^-$ | $11.22^{+0.05+0.08+0.36}_{-0.05-0.25-0.38}$ | 8.7  | 13    | —        | 5.23  | 7.26    | 11.73 | 9.5   |
| $\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^{*-}$ | $0.66^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$  | 0.48 | 0.56  | 0.163    | 0.322 | 0.688   | 0.69  | —     |
| $\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\pi^-$  | $0.61^{+0.00+0.10+0.06}_{-0.00-0.11-0.09}$  | 0.8  | —     | —        | —     | 0.108   | —     | —     |
| $\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^-$    | $0.05^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$  | 0.05 | —     | —        | —     | 0.00777 | —     | —     |
| $\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\rho^-$ | $1.51^{+0.00+0.27+0.19}_{-0.00-0.29-0.25}$  | 2.2  | —     | —        | —     | 0.0475  | 1.05  | —     |
| $\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^{*-}$ | $0.09^{+0.00+0.02+0.01}_{-0.00-0.02-0.01}$  | 0.08 | —     | —        | —     | 0.00332 | 0.06  | —     |

# Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- Polarization fractions ( $10^{-2}$ ) of the decays  $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\rho, K^*)$ .

| Channels            | $\bar{B}^0 \rightarrow D^{*+}(1S)\rho^-$     | $\bar{B}^0 \rightarrow D^{*+}(1S)K^{*-}$     | $\bar{B}^0 \rightarrow D^{*+}(2S)\rho^-$     | $\bar{B}^0 \rightarrow D^{*+}(2S)K^{*-}$     |
|---------------------|--|--|--|--|
| $f_L$ [%]           | 90.59  | 88.11  | 93.52  | 91.56  |
| PDG                 | $88.5 \pm 2.0$                               | $92^{+38}_{-32}$                             | -  | -  |
| $f_{\parallel}$ [%] | 8.27   | 10.47  | 6.42   | 8.36   |
| Channel             | $B^- \rightarrow D^{*0}(1S)\rho^-$           | $B^- \rightarrow D^{*0}(1S)K^{*-}$           | $B^- \rightarrow D^{*0}(2S)\rho^-$           | $B^- \rightarrow D^{*0}(2S)K^{*-}$           |
| $f_L$ [%]           | 90.60  | 88.12  | 93.25  | 91.21  |
| PDG                 | $89.2 \pm 2.4$                               | $86.0 \pm 6.7$                               | -  | -  |
| $f_{\parallel}$ [%] | 8.26   | 10.46  | 6.68   | 8.70   |
| Channels            | $\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\rho^-$ | $\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^{*-}$ | $\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\rho^-$ | $\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^{*-}$ |
| $f_L$ [%]           | 90.68  | 88.22  | 92.68  | 90.52  |
| BS+FA               | 87.40  | 84.10  | -  | -  |
| BS+PQCD             | 85.40  | 85.70  | -  | -  |
| PQCD                | 87   | 83   | -  | -  |
| Bell                | $105^{+6}_{-11}$                             | -  | -  | -  |
| $f_{\parallel}$ [%] | 8.22   | 10.41  | 7.32   | 9.48   |
| BS+FA               | 10.40  | 13.30  | -  | -  |
| BS+PQCD             | 11.30  | 10.40  | -  | -  |

# Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- Although there exist significant difference between the form factors of the transitions  $B_{(s)} \rightarrow D_{(s)}^*(1S)$  and  $B_{(s)} \rightarrow D_{(s)}^*(2S)$ , the similar polarization behaviors can be observed in these decays  $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\rho, K^*)$ .
- The longitudinal polarization is dominant, reaching approximately 90%, while the transverse parallel and perpendicular polarization fractions are only a few percent or roughly 10%.

- The form factors of the transitions  $B_{(s)} \rightarrow D_{(s)}(2S), D_{(s)}^*(2S)$  are much smaller than those of the corresponding transitions  $B_{(s)} \rightarrow D_{(s)}(1S), D_{(s)}^*(1S)$  because of the different structures of the wave functions between the ground and radially excited charmed mesons.
- The branching ratios of the decays  $B_{(s)} \rightarrow D_{(s)}\ell'\nu_{\ell'}$  and  $B_{(s)} \rightarrow D_{(s)}^*\ell'\nu_{\ell'}$  are consistent well with the experimental data, while those of the decays  $B_{(s)} \rightarrow D_{(s)}\tau\nu_{\tau}$  and  $B_{(s)} \rightarrow D_{(s)}^*\tau\nu_{\tau}$  are systematically smaller than the data.
- The branching ratios of the decays  $B_{(s)} \rightarrow D_{(s)}(2S)\ell\nu_{\ell}$  and  $B_{(s)} \rightarrow D_{(s)}^*(2S)\ell\nu_{\ell}$  lie in the range  $10^{-4} \sim 10^{-3}$ , which can be observed by current LHCb and Belle II experiments.
- There exist similar forward-backward asymmetry (polarization) characteristics between the decays  $B \rightarrow D^{(*)}(1S)\ell\nu_{\ell}$  and  $B \rightarrow D^{(*)}(2S)\ell\nu_{\ell}$ .

# Summary

- Except for the branching ratios of the neutral decays  $B_{(s)}^0 \rightarrow D_{(s)}^{(*)\pm}(1S)(\pi, \rho, K^{(*)})^\mp$ , which have some excess compared to the experimental data, overall our predictions for the decays  $B_{(s)} \rightarrow D_{(s)}^{(*)}(1S)(\pi, \rho, K^{(*)})$  are consistent with the experimental measurements.
- Most the branching ratios of the decays  $B_{(s)} \rightarrow D_{(s)}^{(*)}(2S)(\pi, \rho, K^{(*)})$  lie in the range of  $10^{-5}$  to  $10^{-4}$ , which are likely to be detected by the present LHCb and Belle II experiments. Our predictions for these decays are larger than the results given by the BS equation, but agree well with the RQM and RIQ calculations.
- Although there exist obvious difference in the branching ratios between the decays  $B_{(s)} \rightarrow D_{(s)}^*(1S)(\rho, K^*)$  and  $B_{(s)} \rightarrow D_{(s)}^*(2S)(\rho, K^*)$ , the similar polarization behaviours can be observed in these decays  $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\rho, K^*)$ .

Thank you for your attention!

- $f_D = 0.2058 \pm 0.0089 \text{ GeV}$ ,  $f_{D_s} = 0.2499 \pm 0.0005 \text{ GeV}$ ,  $f_{D^*} = 0.245^{+0.023}_{-0.022} \text{ GeV}$ ,  
 $f_{D_s^*} = 0.272^{+0.039}_{-0.038} \text{ GeV}$ .

The effective Hamiltonian for the decays  $B_{(s)} \rightarrow D_{(s)}^{(*)}(1S, 2S)M$  with  $M = \pi, K, \rho, K^*$  can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} \{ C_1 Q_1 + C_2 Q_2 \}, \quad (14)$$

The local tree four-quark operators  $Q_{1,2}$  are defined by :

$$Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha], \quad (15)$$

$$Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta], \quad (16)$$

where  $\alpha$  and  $\beta$  are color indices.

- As one of the pillars of the SM, the ratio  $\mathcal{R}(D^{(*)})$  is a powerful test of the LFU,

$$\mathcal{R}(D^{(*)}) = \frac{Br(B \rightarrow D^{(*)} \tau \nu)}{Br(B \rightarrow D^{(*)} \ell' \nu_{\ell'})}. \quad (17)$$

|              | $\mathcal{R}(D^-(1S))$             | $\mathcal{R}(D^0(1S))$      | $\mathcal{R}(D^{*-}(1S))$          | $\mathcal{R}(D^{*0}(1S))$   |
|--------------|------------------------------------|-----------------------------|------------------------------------|-----------------------------|
| This work    | $0.260 \pm 0.014 (\sim 3.1\sigma)$ | $0.261 \pm 0.013$           | $0.228 \pm 0.026 (\sim 2.1\sigma)$ | $0.228 \pm 0.026$           |
| RQM          | -                                  | 0.269                       | 0.231                              | 0.224                       |
| PQCD+Lattice | 0.318                              | 0.329                       | 0.270                              | 0.274                       |
| PQCD         | 0.429                              | 0.434                       | 0.301                              | 0.302                       |
| Belle-II     | 0.418                              | -                           | 0.306                              | -                           |
| LHCb         | 0.249                              | -                           | 0.402                              | -                           |
| HFLAF        | $0.347 \pm 0.025$                  | -                           | $0.288 \pm 0.012$                  | -                           |
|              | $\mathcal{R}(D^-(2S))$             | $\mathcal{R}(D^0(2S))$      | $\mathcal{R}(D^{*-}(2S))$          | $\mathcal{R}(D^{*0}(2S))$   |
| This work    | $0.125 \pm 0.037$                  | $0.123 \pm 0.039$           | $0.072 \pm 0.0431$                 | $0.072 \pm 0.043$           |
| BS           | 0.097                              | 0.099                       | 0.071                              | 0.073                       |
|              | $\mathcal{R}(D_s^-(1S))$           | $\mathcal{R}(D_s^{*-}(1S))$ | $\mathcal{R}(D_s^-(2S))$           | $\mathcal{R}(D_s^{*-}(2S))$ |
| This work    | $0.259 \pm 0.033$                  | $0.226 \pm 0.031$           | $0.104 \pm 0.038$                  | $0.067 \pm 0.052$           |
| RQM          | 0.276                              | 0.276                       | -                                  | -                           |
| RQM          | 0.295                              | 0.245                       | -                                  | -                           |
| pQCD+Lattice | 0.342                              | 0.271                       | -                                  | -                           |
| BS           | -                                  | -                           | 0.078                              | 0.069                       |