中南大學
CENTRAL SOUTH UNIVERSITY

Probing light-quark dipole operators with nucleon energy correlators at the EIC

Yingsheng Huang

Central South University

In collaboration with:

Xuan-Bo Tong and **Hao-Lin Wang**

PRL 136 (2026), 131902

June 6th, 2026

第一届华中核理论中心区域研讨会

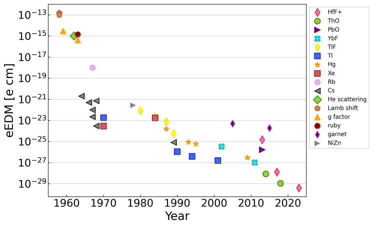
DIPOLE MOMENTS IN SEARCH FOR BSM

Dipole moments: stringent SM tests and sensitive BSM probes.

Electron electric dipole moment

d_e (Electron EDM)

- $|d_e| < 4.1 \times 10^{-30} \mathrm{e} \cdot \mathrm{cm}$ (JILA).
- Constraints on BSM at $\Lambda \sim 100$ TeV.



arXiv: 2505.22281

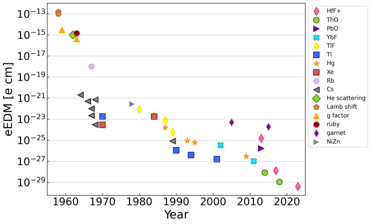
DIPOLE MOMENTS IN SEARCH FOR BSM

Dipole moments: stringent SM tests and sensitive BSM probes.

Electron electric dipole moment

d_e **(Electron EDM)**

- $|d_e| < 4.1 \times 10^{-30} \mathrm{e} \cdot \mathrm{cm}$ (JILA).
- Constraints on BSM at $\Lambda \sim 100$ TeV.

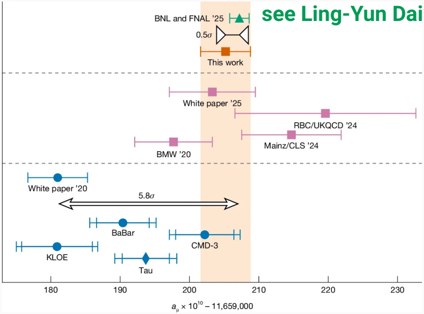


arXiv: 2505.22281

Lepton $g-2$ (magnetic dipole moments)

Muon $g-2$ (a_μ)

- FNAL '23 (190 ppb): $\sim 5\sigma$ tension.
- WP25: Tension eased (Lattice/CMD-3 HVP).



see Ling-Yun Dai's talk

Nature ***653****, 373 (2026)*

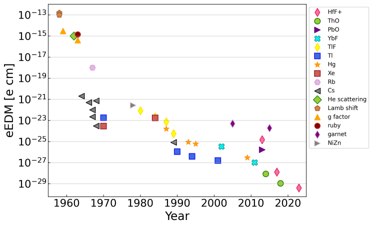
DIPOLE MOMENTS IN SEARCH FOR BSM

Dipole moments: stringent SM tests and sensitive BSM probes.

Electron electric dipole moment

d_e **(Electron EDM)**

- $|d_e| < 4.1 \times 10^{-30} \mathrm{e} \cdot \mathrm{cm}$ (JILA).
- Constraints on BSM at $\Lambda \sim 100$ TeV.



arXiv: 2505.22281

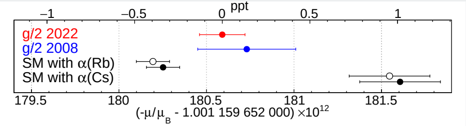
Lepton $g-2$ (magnetic dipole moments)

Muon $g-2\ (a_\mu)$

- FNAL ’23 (190 ppb): $\sim 5\sigma$ tension.
- WP25: Tension eased (Lattice/CMD-3 HVP).

Electron $g-2\ (a_e)$

- Gabrielse ’23 (0.13 ppt): Tension between α determined by Cs and Rb experiments.



*PRL **130**, 071801 (2023)*

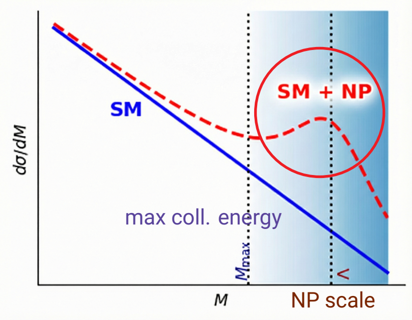
THE LIGHT QUARK SECTOR: CURRENT STATUS

Lepton dipoles have shown why dipoles
are powerful on probing/eliminating BSM
$\Rightarrow$ extend the program to **light-quark dipoles**

Low-energy constraints	Collider opportunity
• **Neutron EDM (nEDM):** stringent bounds on light-quark EDMs. • **Hadronic magnetic moments:** light-quark MDM bounds remain weak due to SM background.	• High-energy searches can probe *both* MDM and EDM. • Direct access to dipole effects in a complementary kinematic regime.

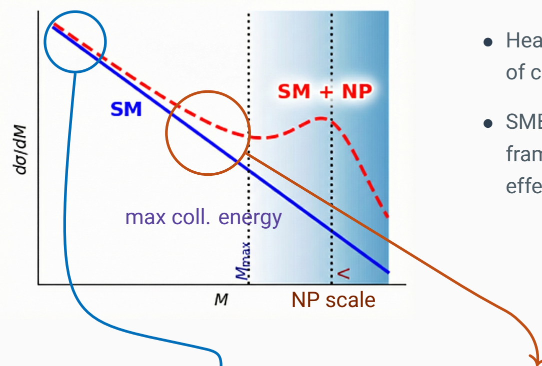
Goal: precision complementary to low-energy EDM searches

STANDARD MODEL EFFECTIVE FIELD THEORY (SMEFT)



- Heavy BSM beyond the energy reach of current collider experiments

STANDARD MODEL EFFECTIVE FIELD THEORY (SMEFT)



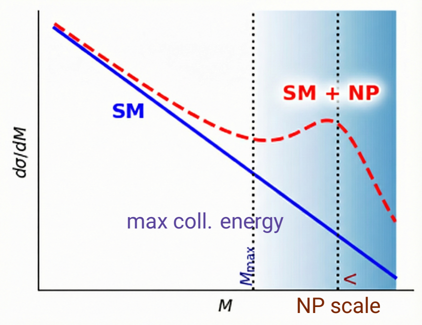
- Heavy BSM beyond the energy reach of current collider experiments
- SMEFT provides a systematic framework to parameterize these effects below the NP scale Λ

$$\mathcal{L}_{\text{SMEFT}} = \boxed{\mathcal{L}_{\text{SM}}} + \boxed{\frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)} + \frac{1}{\Lambda^4}\sum_i C_i^{(8)}\mathcal{O}_i^{(8)} + \dots}$$

BSM scale, EFT expansion parameter

Ignore baryon/lepton no. violating odd-dimensions.

DIPOLE OPERATORS within SMEFT



SMEFT up to dimension-6:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \boxed{\frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)}} + \cdots$$

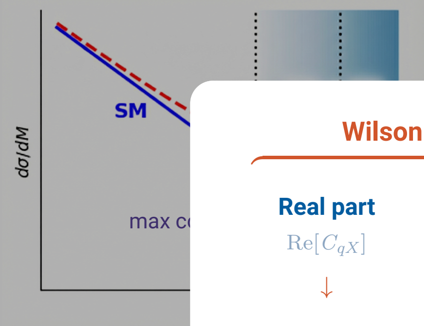
We focus on dimension-6 electroweak dipole operators ($+h.c.$):

$$\mathcal{O}_{uB} = (\bar{Q}\sigma^{\mu\nu}u)\tilde{H}B_{\mu\nu} \qquad \mathcal{O}_{uW} = (\bar{Q}\sigma^{\mu\nu}\tau^I u)\tilde{H}\,W^I_{\mu\nu}$$

$$\mathcal{O}_{dB} = (\bar{Q}\sigma^{\mu\nu}d)HB_{\mu\nu} \qquad \mathcal{O}_{dW} = (\bar{Q}\sigma^{\mu\nu}\tau^I d)HW^I_{\mu\nu}$$

Q: Left-handed doublet; u, d: Right-handed singlets, τ^I: SU(2) generators, $\tilde{H}_I = \epsilon_{IJ}H^*_J$.

DIPOLE OPERATORS within SMEFT



6:

$$\boxed{\sum C_i^{(6)}\mathcal{O}_i^{(6)}} + \cdots$$

We focus on di

$\mathcal{O}_{uB}$ $W^I_{\mu\nu}$

$\mathcal{O}_{dB} = (\bar{Q}\sigma^{\mu\nu} d)HB_{\mu\nu}$ $\mathcal{O}_{dW} = (\bar{Q}\sigma^{\mu\nu}\tau^I d)HW^I_{\mu\nu}$

Wilson coefficients

Real part	**Imaginary part**
$\mathrm{Re}[C_{qX}]$	$\mathrm{Im}[C_{qX}]$
↓	↓
MDM	**EDM**
(CP-even)	(CP-odd)
μ_q	d_q

Q: Left-handed doublet; u, d: Right-handed singlets, τ^I: SU(2) generators, $\tilde{H}_I = \epsilon_{IJ} H^*_J$.

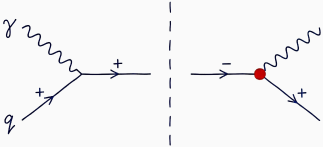
The Chirality Bottleneck at High Energy

However, precision collider searches face a fundamental suppression mechanism rooted in the operator structure.

Operator Structure:

- Chirality-flipping operators:

$$\mathcal{O}_{\text{dipole}} \sim \bar{\psi}_L \sigma^{\mu\nu} F_{\mu\nu} \psi_R$$



Chirality Mismatch:

- **SM:** γ^μ (odd no. of γ) $\implies$ Conserves chirality ($L \to L$).
- **Dipole:** $\sigma^{\mu\nu}$ (even no. of γ) $\implies$ Flips chirality ($L \to R$).

$\implies$ Interference term $\mathcal{M}_{\text{SM}} \times \mathcal{M}^*_{\text{dipole}} \to 0$ in massless limit

Interference $\propto$ Tr$[\not{k}' \gamma^\mu \not{k} \sigma^{\alpha\beta}] \xrightarrow{\text{assumed } m_q \to 0} 0$ (Trace of odd # of γs)

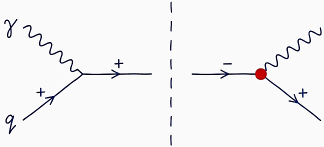
THE CHIRALITY BOTTLENECK AT HIGH ENERGY

However, precision collider searches face a fundamental suppression mechanism rooted in the operator structure.

Operator Structure:

- Chirality-flipping operators:

$$\mathcal{O}_{\text{dipole}} \sim \bar{\psi}_L \sigma^{\mu\nu} F_{\mu\nu} \psi_R$$



Chirality Mismatch:

- **SM:** γ^μ (odd no. of γ) $\Longrightarrow$ Conserves chirality ($L \to L$).
- **Dipole:** $\sigma^{\mu\nu}$ (even no. of γ) $\Longrightarrow$ Flips chirality ($L \to R$).

$\Longrightarrow$ Interference term $\mathcal{M}_{\text{SM}} \times \mathcal{M}^*_{\text{dipole}} \to 0$ in massless limit

Consequence: Quadratic Suppression

Observables are sensitive only to the square of the BSM amplitude:

$$\sigma \sim |\mathcal{M}_{\text{SM}}|^2 + \frac{1}{\Lambda^2}\left(\mathcal{M}_{\text{SM}} \times \mathcal{M}^*_{\text{dipole}} + \text{h.c.}\right) + \frac{1}{\Lambda^4}|\mathcal{M}_{\text{dipole}}|^2$$

i.e. Drell-Yan measurements **Boughezal et al., PRD 104, 095022 (2021)**

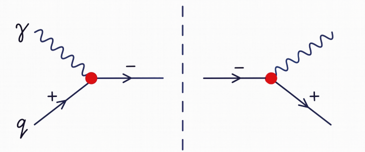
THE CHIRALITY BOTTLENECK AT HIGH ENERGY

However, precision collider searches face a fundamental suppression mechanism rooted in the operator structure.

Operator Structure:

- Chirality-flipping operators:

$$\mathcal{O}_{\text{dipole}} \sim \bar{\psi}_L \sigma^{\mu\nu} F_{\mu\nu} \psi_R$$



Chirality Mismatch:

- **SM:** γ^μ (odd no. of γ) $\Longrightarrow$ Conserves chirality ($L \to L$).
- **Dipole:** $\sigma^{\mu\nu}$ (even no. of γ) $\Longrightarrow$ Flips chirality ($L \to R$).

$\Longrightarrow$ Interference term $\mathcal{M}_{\text{SM}} \times \mathcal{M}^*_{\text{dipole}} \to 0$ in massless limit

Consequence: Quadratic Suppression

Observables are sensitive only to the square of the BSM amplitude:

$$\sigma \sim |\mathcal{M}_{\text{SM}}|^2 + \xcancel{\frac{1}{\Lambda^2}\left(\mathcal{M}_{\text{SM}} \times \mathcal{M}^*_{\text{dipole}} + \text{h.c.}\right)} + \frac{1}{\Lambda^4}|\mathcal{M}_{\text{dipole}}|^2$$

i.e. Drell-Yan measurements **Boughezal et al., PRD 104, 095022 (2021)**

THE CHIRALITY BOTTLENECK AT HIGH ENERGY

However, precision collider searches face a fundamental suppression mechanism rooted in the operator structure.

A few issues w/ vanishing linear order

- worse sensitivity
- depends on $|C|^2$, no sensitivity to the complex phase
  $\rightarrow$ cannot constrain $\mathrm{Re}[C]$ and $\mathrm{Im}[C]$ individually
- same order as linear dim-8 contributions ($1/\Lambda^4$)
  $\rightarrow$ cannot ignore dim-8 contribution

Observables are sensitive only to the square of the BSM amplitude:

$$\sigma \sim |\mathcal{M}_{\mathrm{SM}}|^2 + \xcancel{\frac{1}{\Lambda^2}\left(\mathcal{M}_{\mathrm{SM}} \times \mathcal{M}^*_{\mathrm{dipole}} + \text{h.c.}\right)} + \frac{1}{\Lambda^4}|\mathcal{M}_{\mathrm{dipole}}|^2$$

i.e. Drell-Yan measurements **Boughezal et al., PRD 104, 095022 (2021)**

INTERFERENCE RESTORATION

The linear Λ^{-2} interference can be restored by quark transverse spin.

- **Why?** Transverse spin is a superposition of helicity states: $|\uparrow\rangle \propto |-\rangle + i\,|+\rangle$.
- This allows the **helicity-conserving** SM amplitude and the **helicity-flipping** dipole amplitude to interfere!

Transverse spin S_T can be projected out by $u(p)\bar{u}(p) \propto \not{p}(1+\gamma^5 \not{S}_T)$

$\Longrightarrow$ restore even # of γs

INTERFERENCE RESTORATION

The linear Λ^{-2} interference can be restored by quark transverse spin at the **EIC**.

Existing Approaches:

Transverse spin asymmetry @EIC

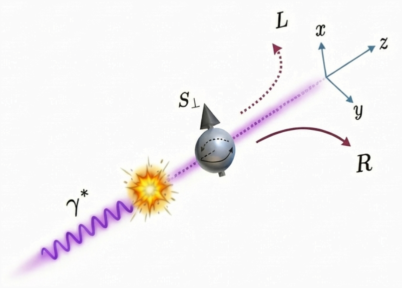
Boughezal et al. PRD 107, 075028 (2023)

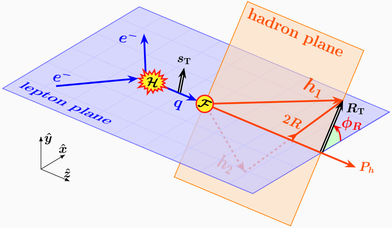
- quark transverse spin from a polarized nucleon beam
- *Limitation:* Requires polarized hadron beams, less events

Dihadron fragmentation @EIC

Wen et al., arXiv: 2408.07255

- transverse plane formed by dihadron, no need for polarized nucleon beam
- *Limitation:* requires hadron id. & tracking





INTERFERENCE RESTORATION

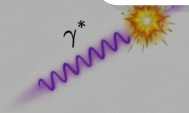
The linear Λ^{-2} interference can be restored by quark transverse spin at the EIC.

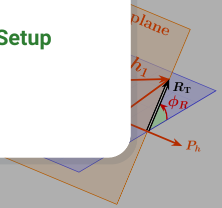
Existing

Transv *EIC*

Bough

- quark adron, no
  nucle am
- *Limita* & tracking
  beam





Our proposal

use **transversity nucleon energy correlators** **@EIC** to induce a transversely polarized struck quark from an unpolarized nucleon

---

Unpol. nucleon beam

Calorimeter

(No PID/tracking)

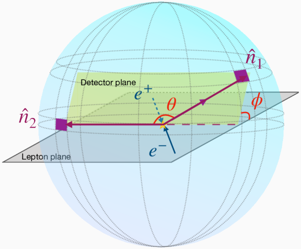
$\Longrightarrow$

Simpler Exp. Setup

More events

ENERGY CORRELATORS (see Moult & Zhu, arXiv:2506.09119 for a review)

Energy correlators measure the energy-weighted angular correlations of particles in the detector.



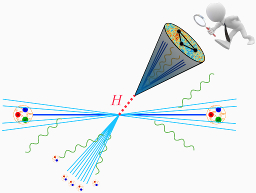
$$\langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) \rangle$$

Why use them? **see talks by Wen-Jing Xing & Weiyao Ke**

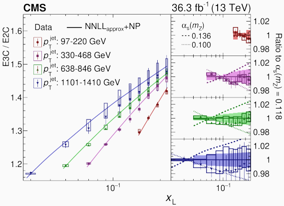
- Theoretically simple
- IRC safe → Insensitive to low-energy radiation
- No jet clustering ambiguities
- ...

Applications in HEP:

Jet Substructure

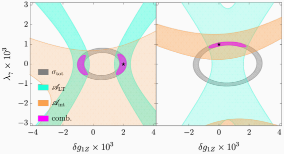


Measuring α_s



PRL 133, 071903 (2024)

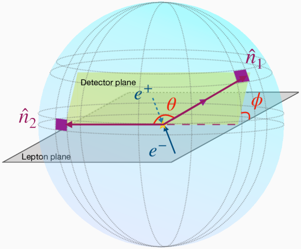
BSM w/ TEEC



PRD 106, 114010 (2022)

ENERGY CORRELATORS (see Moult & Zhu, arXiv:2506.09119 for a review)

Energy correlators measure the energy-weighted angular correlations of particles in the detector.



$$\langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\rangle$$

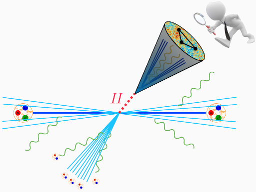
Why use them? see talks by Wen-Jing Xing & Weiyao Ke

- Theoretically simple
- IRC safe → Insensitive to low-energy radiation
- No jet clustering ambiguities
- ...

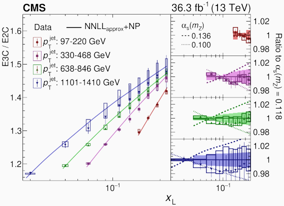
Applications in HEP:

one of few EC appls. to BSM

Jet Substructure

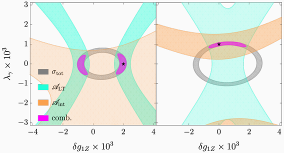


Measuring α_s



PRL 133, 071903 (2024)

BSM w/ TEEC

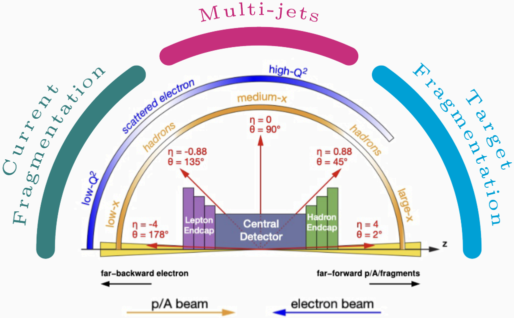


PRD 106, 114010 (2022)

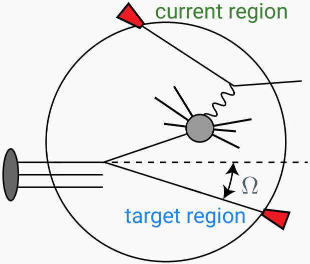
NUCLEON ENERGY CORRELATORS (NEC)

Liu & Zhu, PRL 130, 091901 (2023)

- Unlike Energy Correlators in the Current Region, **NECs** are defined in the **target fragmentation region (TFR)** and probe the **initial-state nucleon remnant**.



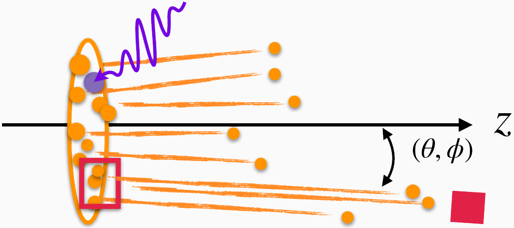
Kinematic regions @EIC. Figure from 2506.09119



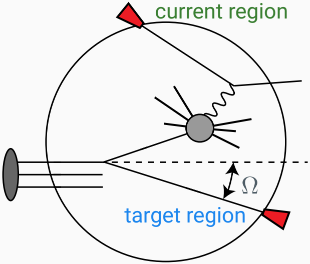
Geometry: target vs current region

NUCLEON ENERGY CORRELATORS (NEC) Liu & Zhu, PRL 130, 091901 (2023)

- Unlike Energy Correlators in the Current Region, **NECs** are defined in the **target fragmentation region (TFR)** and probe the **initial-state nucleon remnant**.



NEC: probability to find a quark with longitudinal momentum fraction x in the target, conditioned on energy flux at (θ, ϕ) in the TFR



Geometry: target vs current region

NEC: energy-weighted quark correlator

$$\mathcal{M}^{[\Gamma]}(x, \vec{\Omega}) = \int \frac{d\eta^-}{4\pi} e^{-ixP^+\eta^-} \times \langle P|\bar{\psi}(\eta^-)\Gamma \underbrace{\mathcal{E}(\vec{\Omega})}_{\text{energy flow op.}} \psi(0)|P\rangle$$

THE TRANSVERSITY NEC h_1^t YH, Tong & Wang, PRL 136, 131902

Leading-twist decomposition of the NEC correlator:

$$\mathcal{M}_{ij}(x,\vec{\Omega}) = \frac{1}{2}\left[(\gamma^-)_{ij}\mathcal{M}^{[\gamma^+]} + (\gamma_5\gamma^-)_{ij}\mathcal{M}^{[\gamma^+\gamma_5]} - i\frac{(\sigma_{\alpha_\perp}^{-}\gamma_5)_{ij}}{2}\mathcal{M}^{[i\sigma^{\alpha_\perp +}\gamma_5]}\right] + \cdots$$

- **Chiral-even** unpolarized NEC f_1

  isolated from $\Gamma = \gamma^+$:

  $$f_1^q(x,\theta^2) = \mathcal{M}^{[\gamma^+]}(x,\theta,\phi)$$

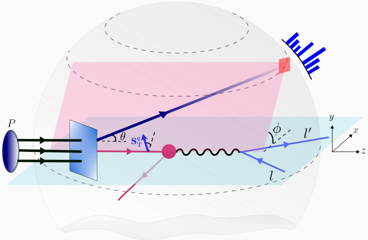
- **Chiral-odd** transversity NEC h_1^t

  isolated from $\Gamma = i\sigma^{\alpha +}\gamma_5$:

  $$\epsilon_\perp^{\alpha\rho}\hat{n}_{T,\rho}\, \boldsymbol{h_1^t(x,\theta^2)} \subset \mathcal{M}^{[i\sigma^{\alpha +}\gamma_5]}$$

$\boldsymbol{h_1^t(x,\theta^2)}$: a filter selecting the quark transverse spin in an **unpolarized nucleon**.

- ⇒ **Chiral-odd nature** enables interference with *dipole operators*.
  - **Linear order** (Λ^{-2}) contribution is restored.
- h_1^t also probes the remnant energy angular distribution.



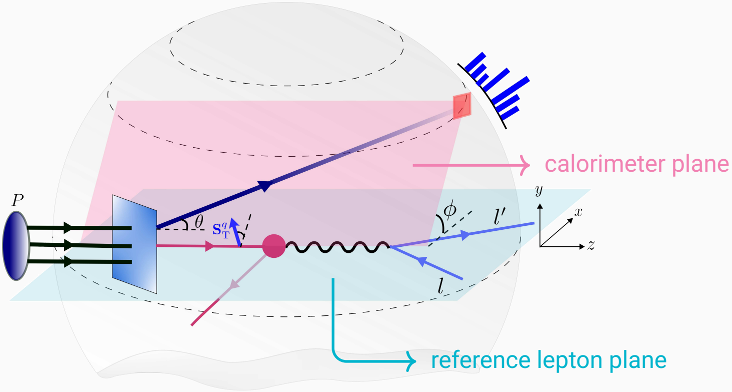
OBSERVABLE: ENERGY PATTERN XSEC.

YH, Tong & Wang, PRL 136, 131902

Observable: **Energy Pattern xsec.** $\Sigma(\theta, \phi)$

$$\Sigma(\theta, \phi) = \sum_{i \in X} \int \mathrm{d}\sigma^{l+p \to l'+X} \frac{E_i}{E_N} \delta(\theta^2 - \theta_i^2) \delta(\phi - \phi_i)$$

Σ: angular distribution of the total energy deposited in the calorimeters



calorimeter plane

reference lepton plane

FACTORIZE THE ENERGY PATTERN

YH, Tong & Wang, PRL 136, 131902

Factorization of the **energy pattern** cross section (similar to coll. factorization)

- General NEC:

Liu & Zhu, PRL 130, 091901 (2023)
Chen, Ma & Tong, JHEP 08, 227 (2024)

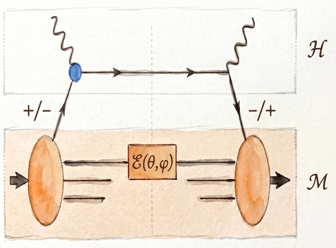
$$\frac{\mathrm{d}\Sigma(\theta,\phi)}{\mathrm{d}x_B\mathrm{d}Q^2} \propto L_{\mu\nu}\mathcal{H}^{\mu\nu}_{ji}\otimes\mathcal{M}_{ij}$$

- Transversity NEC ($\hat{n}_T = (\cos\phi, \sin\phi)$):

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}x_B\mathrm{d}Q^2} \propto L_{\mu\nu}\,\mathrm{Tr}\left[\mathcal{H}^{\mu\nu}\sigma^{-\alpha}\gamma_5\right] \times \epsilon_\perp^{\alpha\beta}\hat{n}_{T\beta}\; h_1^t(x_B,\theta^2)$$

ϕ-dependence from transverse spin direction

θ-dependence



Azimuthal Modulation of the energy pattern

$$\Sigma(\theta,\phi) = \Sigma_{UU}(\theta) + \Sigma_{UU}^{\sin\phi}(\theta)\,\sin\phi + \Sigma_{UU}^{\cos\phi}(\theta)\,\cos\phi$$

SM, unpolarized NEC

Re($c_{qZ/\gamma}$), h_1^t

Im($c_{qZ/\gamma}$), h_1^t

FACTORIZE THE ENERGY PATTERN YH, Tong & Wang, PRL 136, 131902

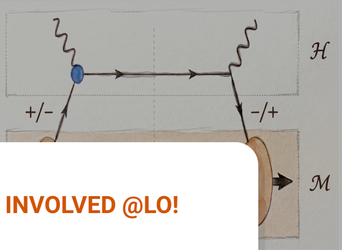
Factorization of the **energy pattern** cross section (similar to coll. factorization)

- General NEC:

Liu & Zhu, PRL 130, 091901 (2023)
Chen, Ma & Tong, JHEP 08, 227 (2024)

$$\frac{d\Sigma(\theta,\phi)}{dx_B dQ^2} \propto L_{\mu\nu}\mathcal{H}^{\mu\nu}_{ji} \otimes \mathcal{M}_{ij}$$

- Transversity NEC ($\hat{n}_T = (\cos\phi, \sin\phi)$):



NO OTHER SMEFT OPERATORS INVOLVED @LO!

ϕ-dependence from transverse spin direction

θ-dependence

Azimuthal Modulation of the energy pattern

$$\Sigma(\theta,\phi) = \Sigma_{UU}(\theta) + \Sigma_{UU}^{\sin\phi}(\theta)\sin\phi + \Sigma_{UU}^{\cos\phi}(\theta)\cos\phi$$

SM, unpolarized NEC

Re($c_{qZ/\gamma}$), h_1^t

Im($c_{qZ/\gamma}$), h_1^t

NON-PERTURBATIVE INPUT: resort to TMD PDFs

General Correspondence: θ-weighted moments of **NECs** $\leftrightarrow$ $k_\perp$-weighted moments of **transverse momentum dependent PDFs (TMDs)**.

Liu et al. 2403.08874

$$E_N \int d\theta^2 \, |\sin\theta| \, \overbrace{f_1(x, \theta^2)}^{\text{unpol. NEC}} = \int \frac{d^2 k_\perp}{2\pi} \, |\mathbf{k}_\perp| \, \overbrace{f_1(x, k_\perp^2)}^{\text{unpol. TMD}}$$

$$E_N \int d\theta^2 \, |\sin\theta| \, \underbrace{h_1^t(x, \theta^2)}_{\text{transversity NEC}} = \int \frac{d^2 k_\perp}{2\pi} \, \frac{k_\perp^2}{M} \, \underbrace{h_1^\perp(x, k_\perp^2)}_{\text{Boer-Mulders TMD}}$$

YH, Tong & Wang, PRL 136, 131902

We weight $\Sigma(\theta, \phi)$ with $|\sin\theta|$ (to utilize the correspondence) and only look at

$$\Sigma(\phi) = \int \mathrm{d}\theta^2 |\sin\theta| \Sigma(\theta, \phi).$$

TMD input for numerical analysis:

- We use the fit from **Barone et al., PRD 81, 114026 (2010)**.

AZIMUTHAL ASYMMETRIES

YH, Tong & Wang, PRL 136, 131902

Transverse spin superposition states

⇓

$\Sigma(\phi)$ contains chirality-conserving contribution

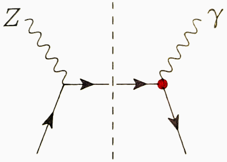
⇓

Extract chirality-flipping BSM effects w/ azimuthal asymmetries

Unpolarized beams (A_{UU}):

$$A_{UU}^{u} = \frac{\pi}{2} \frac{\Sigma(u>0) - \Sigma(u<0)}{\Sigma(u>0) + \Sigma(u<0)}$$

$$u \in \begin{cases} \sin\phi & \rightsquigarrow \quad \mathrm{Re}\, C \\ \cos\phi & \rightsquigarrow \quad \mathrm{Im}\, C \end{cases}$$



Leading contribution: γZ interference $\implies$ Q^2/m_Z^2 suppression.

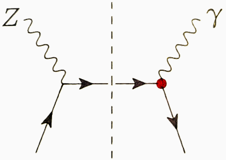
AZIMUTHAL ASYMMETRIES

YH, Tong & Wang, PRL 136, 131902

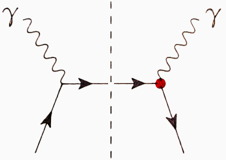
Unpolarized beams (A_{UU}):

$$A^{u}_{UU} = \frac{\pi}{2}\frac{\Sigma(u>0)-\Sigma(u<0)}{\Sigma(u>0)+\Sigma(u<0)}$$

$$u \in \begin{cases} \sin\phi & \rightsquigarrow \ \mathrm{Re}\, C \\ \cos\phi & \rightsquigarrow \ \mathrm{Im}\, C \end{cases}$$



Leading contribution: γZ interference $\Longrightarrow$ Q^2/m_Z^2 suppression.



Long. polarized electron beam (A_{LU}): Analogous definition for longitudinally polarized electron beam.

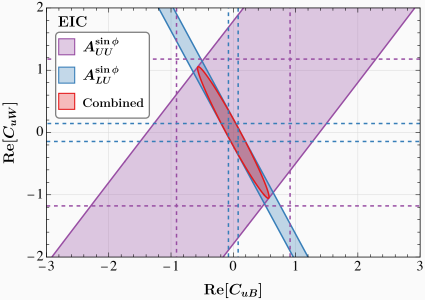
- Parity-violation induced by polarized electron $\Longrightarrow$ **nonzero $\gamma\gamma$ channel**
- Enhances sensitivity to dipole couplings (avoids Q^2/m_Z^2 suppression).

CONSTRAINTS W/ EIC

Correlations & Flat Directions:

- $\mathbf{A}_{\mathbf{UU}}^{\sin\phi}$: Strong correlation between C_{uW} and C_{uB}, leaving a "flat direction" unconstrained.
- $\mathbf{A}_{\mathbf{LU}}^{\sin\phi}$: Polarized electron beam $\Longrightarrow$ nonzero $\gamma\gamma$ interference $\Longrightarrow$ seemingly tighter constraints pointing at another direction.
- Single-operator constraints reach $\mathcal{O}(10^{-2})$ at $\Lambda = 1$ TeV.

$$\sqrt{s} = 105 \text{ GeV}, \mathcal{L} = 100 \text{ fb}^{-1}, P_e = 70\%$$
$$\delta A \simeq \frac{\pi}{2\sqrt{\mathcal{E}_{\text{total}}\mathcal{L}}} \text{ (stat. only)}$$

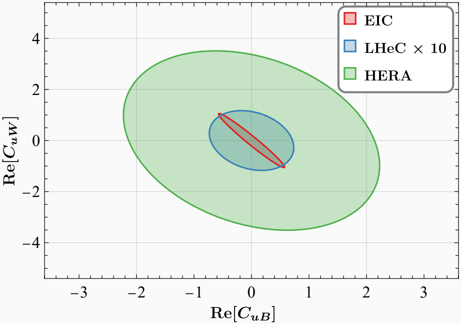


Synergy:

- **The combination (Red)** breaks the degeneracy, improving parameter resolution by orders of magnitude.

COMPARISON BTW. ELECTRON-ION COLLIDERS

	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [fb^{-1}]	P_e
HERA	**318**	**0.4**	**40%**
EIC	**105**	**100**	**70%**
LHeC	**1300**	**50**	**80%**



Collider Sensitivity:

- **HERA:** High Q^2 coverage but limited by luminosity and lack of polarization.
- **EIC:** Polarization breaks γ/Z degeneracy; high $\mathcal{L}$ drives precision.
- **LHeC:** Potential to improve bounds by another order of magnitude.

SUMMARY

- Chiral-odd transversity NEC in the TFR probes light-quark dipole operators in DIS, enabling $\mathcal{O}(\Lambda^{-2})$ (linear) BSM interference with SM using unpolarized nucleons.
- Characteristic $\sin\phi$ & $\cos\phi$ azimuthal asymmetries in energy flow access the real (MDM) and imaginary (EDM) parts of the dipole coupling.
- Advantages:
  - no need for polarized nucleon beams;
  - inclusive calorimetric measurement;
  - no mixing with other SMEFT operators.
- Though not traditionally a strong BSM probe, the **EIC** can be competitive in our proposal.
- Opens a new avenue for Nucleon Energy Correlators (and energy correlators in general) in BSM searches.

Backup

Backup: Light Quark Dipole Operators in SMEFT

We focus on dimension-6 electroweak dipole operators ($+h.c.$):

$$\mathcal{O}_{uB} = (\bar{\mathcal{Q}}\sigma^{\mu\nu} u)\tilde{H} B_{\mu\nu} \qquad \mathcal{O}_{uW} = (\bar{\mathcal{Q}}\sigma^{\mu\nu}\tau^I u)\tilde{H} W^I_{\mu\nu}$$

$$\mathcal{O}_{dB} = (\bar{\mathcal{Q}}\sigma^{\mu\nu} d) H B_{\mu\nu} \qquad \mathcal{O}_{dW} = (\bar{\mathcal{Q}}\sigma^{\mu\nu}\tau^I d) H W^I_{\mu\nu}$$

$\mathcal{Q}$: Left-handed doublet; u, d: Right-handed singlets, τ^I: SU(2) generators.

Effective couplings after EWSB:

$$\mathcal{L}_{\text{eff}} \supset \sum_{q=u,d} \bar{q}_L \, \sigma^{\mu\nu} q_R \left(c_{q\gamma} F_{\mu\nu} + c_{qZ} Z_{\mu\nu} \right) + h.c.$$

The complex coefficients ($c_{q\gamma}, c_{qZ}$) are rotations of SMEFT (C_{qB}, C_{qW}):

$$c_{q\gamma} = (v/\sqrt{2}\Lambda^2)\,(c_W C_{qB} \pm s_W C_{qW})\,,$$

$$c_{qZ} = (v/\sqrt{2}\Lambda^2)\,(-s_W C_{qB} \pm c_W C_{qW})\,,$$

v: Higgs vev., c_W, s_W: cosine and sine of Weinberg angle θ_W.

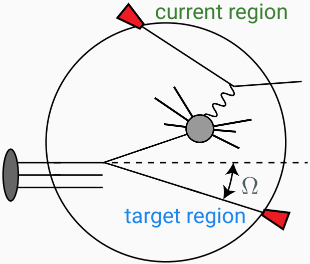
BACKUP: Nucleon Energy Correlators (NEC)

- Unlike Energy Correlators in the Current Region, NECs are defined in the **target fragmentation region (TFR)** and probe the **initial-state nucleon remnant**.
- **Definition:** Probability of finding a quark with momentum fraction x inside the target, given energy flux at (θ, ϕ) in the TFR.

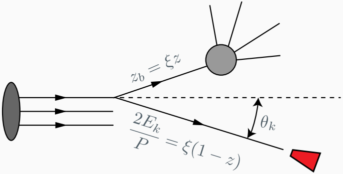
$$\mathcal{M}_{ij}(x,\theta,\phi) = \int \frac{d\eta^-}{2\pi} e^{-ixP^+\eta^-} \langle PS|\bar{\psi}_j(\eta^-) \\ \mathcal{L}_n^\dagger(\eta^-)\mathcal{E}(\theta,\phi)\mathcal{L}_n(0)\psi_i(0)|PS\rangle \, ,$$

where the *Energy Flow Operator* $\mathcal{E}$ follows

$$\mathcal{E}(\theta,\phi)|X\rangle = \sum_{i\in X} \frac{E_i}{E_N}\delta(\theta_i^2-\theta^2)\delta(\phi_i-\phi)|X\rangle$$



Geometry: target vs current region



Energy flux from target remnants measured in forward calorimeters

BACKUP: NEC DECOMPOSITION

$$
\begin{aligned}
\mathcal{M}_{ij}(x,\theta,\phi,S) = \frac{1}{2}\Bigg[& \overbrace{(\gamma^-)_{ij}\mathcal{M}^{[\gamma^+]}}^{\text{unpol. quark}} + \overbrace{(\gamma_5\gamma^-)_{ij}\mathcal{M}^{[\gamma^+\gamma_5]}}^{\text{long. quark}} \\
& \underbrace{- i\frac{g^{\perp}_{\alpha\beta}}{2}(\sigma^{\beta-}\gamma_5)_{ij}\mathcal{M}^{[i\sigma^{\alpha+}\gamma_5]}}_{\text{trans. quark}}\Bigg] .
\end{aligned}
$$

$$
\begin{aligned}
\overbrace{\mathcal{M}^{[i\sigma^{\alpha+}\gamma_5]}}^{\text{trans. quark}}(x,\theta,\phi,S) = & \overbrace{S_T^{\alpha}h_1(x,\theta)}^{\text{trans. proton}} + \overbrace{\frac{S_L\mathbf{n}_t^{\alpha}}{|\mathbf{n}_t|}h_{1L}^{t}(x,\theta)}^{\text{long. proton}} \\
& - \underbrace{\left(\frac{g_T^{\alpha\rho}}{2} + \frac{\mathbf{n}_t^{\alpha}\mathbf{n}_t^{\rho}}{|\mathbf{n}_t|^2}\right)S_{T\rho}h_{1T}^{t}(x,\theta)}_{\text{trans. proton}} + \underbrace{\frac{\epsilon_{\perp}^{\alpha\rho}\mathbf{n}_{t,\rho}}{|\mathbf{n}_t|}h_1^{t}(x,\theta)}_{\text{unpol. proton}} .
\end{aligned}
$$

BACKUP: TRANSVERSITY FROM UNPOL. NUCLEONS

How does an unpolarized nucleon provide polarized quarks?

- **Decomposition:** The quark correlator $\mathcal{M}_{ij}$ contains a chiral-odd term:

$$\mathcal{M}^{[\Gamma^{\alpha}_{\perp}]} = \frac{1}{2}\mathrm{Tr}\left[(i\sigma^{\alpha_{\perp}+}\gamma_5)\mathcal{M}\right]$$

- **Spin Projector:** This operator projects onto transverse spin states along direction $\alpha_{\perp}$:

$$i\sigma^{\alpha_{\perp}+}\gamma_5 = \gamma^{+}(\underbrace{\mathcal{Q}^{\alpha}_{+}}_{\text{Spin}\uparrow} - \underbrace{\mathcal{Q}^{\alpha}_{-}}_{\text{Spin}\downarrow})$$

  where $\mathcal{Q}^{\alpha}_{\pm} = \frac{1}{2}(1 \mp \gamma^5\gamma^{\alpha}_{\perp})$ selects quarks with spin $\parallel$ or anti-$\parallel$ to the transverse vector $\alpha_{\perp}$.

BACKUP: TRANSVERSITY FROM UNPOL. NUCLEONS

How does an unpolarized nucleon provide polarized quarks?

- **Decomposition:** The quark correlator $\mathcal{M}_{ij}$ contains a chiral-odd term:

$$\mathcal{M}^{[\Gamma^{\alpha}_{\perp}]} = \frac{1}{2}\text{Tr}\left[(i\sigma^{\alpha_{\perp}+}\gamma_5)\mathcal{M}\right]$$

- **Symmetry Argument:** In an unpolarized nucleon, the *only* transverse reference vector is the energy flow direction $\hat{n}_T = (\cos\phi, \sin\phi)$.
- **Form Factor:** Rotational invariance dictates the structure:

$$\mathcal{M}^{[\Gamma^{\alpha}_{\perp}]} \propto \epsilon_{\perp}^{\alpha\beta}\,\hat{n}_{T,\beta}\,h_1^t(x,\theta^2)$$

- **Mechanism:** Selecting an energy flow direction ϕ breaks rotational symmetry and ”filters” the quark bath, preferentially selecting quarks with transverse spin correlated with ϕ.

Backup: Factorizatize the Energy Pattern

Factorization of the **energy pattern** cross section

- General NEC:

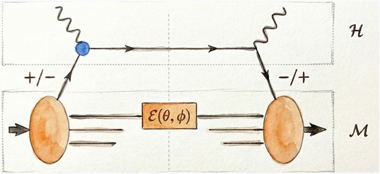
$$\frac{\mathrm{d}\Sigma(\theta,\phi)}{\mathrm{d}x_B\mathrm{d}Q^2} \propto \int_{x_B}^{1}\frac{\mathrm{d}x}{x}L_{\mu\nu}(l,l')\mathcal{H}^{\mu\nu}_{ji}\left(\frac{x_B}{x}\right)\delta(1-\frac{x_B}{x})\mathcal{M}_{ij}(x,\theta,\phi)$$

- Transversity NEC:

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}x_B\mathrm{d}Q^2} \propto L_{\mu\nu}\,\mathrm{Tr}\left[\mathcal{H}^{\mu\nu}i\sigma^{-\alpha}\gamma_5\right]\epsilon_{\perp}^{\alpha\beta}\hat{n}_{T\beta}h_1^t(x_B,\theta^2)$$

Coupling of h_1^t with dipole-SM interference

$$\mathrm{d}\Sigma \propto \left[\underbrace{\mathcal{C}_1}_{\mathrm{Re}(c_{qZ/\gamma})}\sin\phi + \underbrace{\mathcal{C}_2}_{\mathrm{Im}(c_{qZ/\gamma})}\cos\phi\right]\underbrace{h_1^t}_{\text{Transversity NEC}}(x_B,\theta^2)$$



BACKUP: ANALYTICAL RESULTS

SM Background:

$$\begin{aligned}\frac{\mathrm{d}\Sigma_{UU}}{\mathrm{d}x_B\mathrm{d}Q^2} = \frac{2\pi\alpha_{\text{em}}^2}{Q^4}\sum_q f_1^q(x_B,\theta^2)\Big\{ & Q_q^2(y^2-2y+2) + \frac{2Q^2}{Q^2+m_Z^2}\frac{Q_q}{(c_W s_W)^2}\Big[g_A^e g_A^q(y-2)y \\ & - g_V^e g_V^q(y^2-2y+2)\Big] \quad + \frac{1}{(c_W s_W)^4}\left(\frac{Q^2}{Q^2+m_Z^2}\right)^2\Big[(y^2-2y+2)[(g_A^e)^2 \\ & + (g_V^e)^2][(g_A^q)^2+(g_V^q)^2] - 4y(y-2)g_A^e g_A^q g_V^e g_V^q\Big]\Big\}\end{aligned}$$

SMEFT Signal (Interference):

$$\begin{aligned}\frac{\mathrm{d}\Sigma_{UU}^{\sin\phi}}{\mathrm{d}x_B\mathrm{d}Q^2} = \frac{4\pi\alpha_{\text{em}}^2}{e c_W s_W}\frac{y\sqrt{1-y}}{Q(Q^2+m_Z^2)}\sum_q h_1^{t,q}(x_B,\theta^2)\Big\{ & \Big[\frac{2-y}{y}g_A^q g_V^e + g_V^q g_A^e\Big]\frac{\text{Re}[c_{q\gamma}]}{c_W s_W} - Q_q g_A^e\text{Re}[c_{qZ}] \\ & + \frac{1}{(c_W s_W)^2}\frac{Q^2}{Q^2+m_Z^2}\Big[\frac{2-y}{y}[(g_A^e)^2+(g_V^e)^2]g_A^q + 2g_A^e g_V^e g_V^q\Big]\text{Re}[c_{qZ}]\Big\}\end{aligned}$$

BACKUP: EIC SIMULATION PARAMETERS

- **CM Energy:** $\sqrt{s} = 105$ GeV (optimal for EIC).
- **Luminosity:** $\mathcal{L} = 100\,\text{fb}^{-1}$.
- **Phase Space & Binning:**
  - $Q \in [10, 60]$ GeV, $x \in [0.01, 0.5]$.
  - Inelasticity cut: $0.1 \leq y \leq 0.9$.
  - $k_\perp$ limits (for the TMD moments): $k_\perp^{\mathrm{min}} \sim E_{\mathrm{min}}\theta_{\mathrm{min}}, k_\perp^{\mathrm{max}} \sim Q\theta_{\mathrm{max}}$ Practically: $k_\perp \in [0, 5]\ \mathrm{GeV}$.
- **Sensitivity Analysis:**
  - Dominated by statistical uncertainty: $\delta A \simeq \frac{\pi}{2\sqrt{\mathcal{E}_{\text{total}}}}$.
  - $\mathcal{E}_{\text{total}} = (\Sigma(u > 0) + \Sigma(u < 0)) \times \mathcal{L}$.
  - Determine limits via $\chi^2 = \sum_i \left[\frac{A_{\mathrm{th},i} - A_{\mathrm{exp},i}}{\delta A_i}\right]^2$.

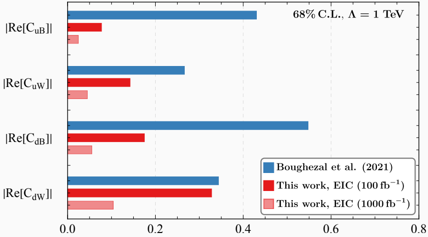
Observables:

1. $A_{UU}^{\sin\phi}$: Unpolarized e^- & unpolarized p.
2. $A_{LU}^{\sin\phi}$: Long. polarized e^- $(P_e = 70\%)$ & unpolarized p.

BACKUP: COMPARISON WITH OTHER PROBES

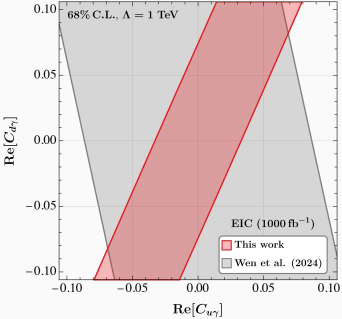
vs. LHC Drell-Yan ($pp \to Z/\gamma^* \to \ell\ell$)

- DY: Dipole effects at $\mathcal{O}(\Lambda^{-4})$ (squared) $\implies$ weaker bounds despite higher energy.
- Contamination from other dim-6/8 operators.
- Cannot resolve complex phase.
- **NEC Advantage:** Leading $\mathcal{O}(\Lambda^{-2})$ contrib., clean separation of Re/Im.



vs. EIC Dihadron ($ep \to e(h_1 h_2)X$)

- Relies on interference in dihadron fragmentation.
- Requires particle ID and tracking.
- **NEC Advantage:** Simpler inclusive calorimetric measurement; $\sim 10\times$ better sensitivity (for same lumi).



BACKUP: NON-PERTURBATIVE UNCERTAINTIES

Input Dependence:

- Input: Boer-Mulders TMD $h_1^{\perp}$.
- Comparison of 3 fits: Barone et al. (default), Christova et al., Zhang et al.
- Result: Constraints are robust against fit choice.
- Reason: Observable sensitive to $k_{\perp}$-weighted moment, integrating out local shape differences.