

# Muonic Th-229 spectroscopy

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# Outline

- 1 Recap of previous results
- 2 Muonic atom spectroscopy
- 3 Simulation of muonic atom spectroscopy with cosmic muons
- 4 Demonstrative experimental setups with water target
- 5 The ultimate target: muonic Th-229 spectroscopy

# Recap of previous results

[Link to the last report](#)

## Motivation

- Muonic atoms provide **enhanced sensitivity to nuclear properties**;  $^{229}\text{Th}$  is especially interesting due to its 8 eV isomeric splitting.

## Method

- Integrated MuDirac (relativistic) into GEANT4, improving muonic cascade  $\gamma$  energies.
- Built minimal simulation setups for pure and doped  $^{229}\text{Th}$  targets.

## Results

- Successfully reproduced MuDirac X-ray spectra (e.g.,  $K_\alpha$ ,  $K_\beta$ ).
- For doped samples, **no generator-level background overlap** in signal regions; background can be simulated and subtracted.

## Primary limitations

- Phenomenological cascade model and GEANT4 limitations for low-energy muons, especially in the simulation of the slowing-down process.

## Muon capture by atoms and its simulation in Geant4

- A negative muon stopped in matter, if it does not decay, is captured into an atomic orbit, forming a **muonic atom**.
- In simplified models, the muon is assumed to be captured into a sufficiently high energy level (e.g.,  $n = 14$  conventionally), and then rapidly cascades to the ground state via **radiative transitions** and/or **Auger electron emission**.
- The cascade duration is  $\sim 37$  ns for muonic hydrogen atoms [PRA 75, 040501]. For heavier muonic atoms, the typical cascade time is less than a nanosecond [EPJ Plus 135, 10, 777]. This timescale is **comparable to the muon–matter interaction duration, but much shorter than the muon lifetime** ( $2.2 \mu\text{s}$ ).
- Current Geant4 releases include preliminary muonic capture models based on simplified non-relativistic theory. The **MuDirac project solves the Dirac equation to compute muonic atom energy levels and transition probabilities, and can be integrated into Geant4 to improve the accuracy of muonic cascade simulations** (see **this previous report** for details).

## Status of muon X-ray measurements

- Systematic muonic X-ray spectroscopy was first developed in the 1950s, starting with small-to-medium- $Z$  nuclei.
- The muonic X-ray energies (and the deduced root-mean-square radii of the nuclear charge distribution) of  $^{16}\text{O}$  were measured with high precision: 133.525(15) keV for  $K_\alpha$  and 158.408(15) keV for  $K_\beta$  [NPA 219 29]. The data were collected at the CERN PS using a low-energy ( $\sim 4$  MeV) muon beam, with X-rays measured by lithium-drifted germanium (Ge(Li)) detectors.
- A comprehensive compilation of nuclear ground-state charge radii (including results from muonic atoms) and a combined model-independent analysis is presented in [PRC 45 80].
- Modern muonic (medium-to-high- $Z$ ) X-ray spectroscopy is performed at PSI using high-intensity muon beams and germanium detector systems, with rare isotopes accessed via muon transfer techniques [PSI muX].

# Atmospheric cosmic rays and their simulation

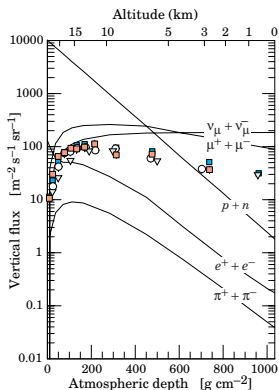


Figure: Vertical fluxes of cosmic rays in the atmosphere. From [PDG 2012].

- Muons are most abundant charged CR particles at sea level
- Produced at  $\sim 15$  km from decay of charged mesons ( $\pi^\pm, K^\pm$ )
- Mean energy at ground:  $\sim 4$  GeV
- Spectrum:
  - $\lesssim 1$  GeV: nearly “flat” (in the logarithmic scale)
  - $\gtrsim 10$  GeV: follows primary CR or drops steeper
- Vertical intensity:  $\sim 1 \text{ cm}^{-2} \text{ min}^{-1}$

## The CRY cosmic-ray shower library:

- Uses MCNPX-based data tables covering **multiple particle types and wide energy ranges**.
- Produces shower properties such as multiplicity, timing, and **arrival angles** over large areas.
- Includes geomagnetic cutoff and solar-cycle modulation (no East-West effect).

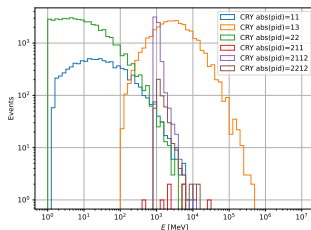


Figure: CRY energy spectrum generated from 100k events.

## MuDirac data for elements in pure water

We can first demonstrate the hardware and software designs by measuring muon capture and cascade processes in pure water.

Isotope	Abundance	$K_\alpha$ [keV]	$K_\beta$ [keV]	$L_\alpha$ [keV]
$^1\text{H}$	0.999855	2	2	0
$^2\text{H}$	0.000115	2	2	0
$^{16}\text{O}$	0.99757	134	158	25
$^{17}\text{O}$	0.00038	134	158	25
$^{18}\text{O}$	0.00205	134	158	25

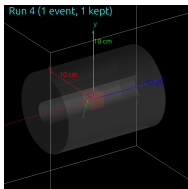
Remark: We round the characteristic X- or  $\gamma$ -ray energies to keV according to the expected detector resolution. Note that the energies of  $K_\alpha$  and  $L_\alpha$  sum to  $K_\beta$ , which can be observed collectively as  $K_\beta$  on the detector.

# Geant4 setup with muon beam, water target, and HPGe detector

Repository: <https://github.com/PKMunon/mudet/tree/muwater>

## Geometry

- Target: H<sub>2</sub>O (l), defined by NIST
- Dimensions: 2 × 2 × 2 cm<sup>3</sup>



## Muon source

- Particle:  $\mu^-$
- Kinetic energy: 4 MeV
- 2 cm of H<sub>2</sub>O (l), stops the muon
- Effective energy range: 0–4 MeV

## HPGe detector

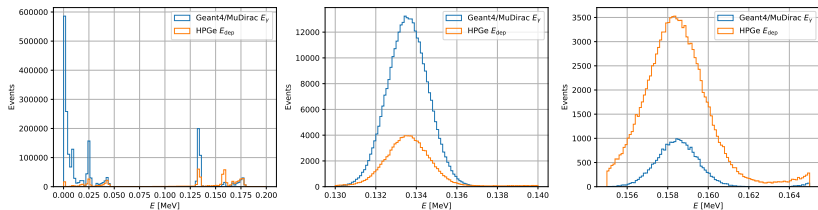
- Tubular, centered on the target
- Minimum inner radius: 1.5 cm
- Adjustable outer radius: e.g., 4.5 cm
- Adjustable length: e.g., 15 cm

## Muonic atom physics (based on Geant4 v11.3.2)

- MuDiracEmCaptureCascade (overriding G4EmCaptureCascade)
- MuDiracMuonMinusAtomicCapture (overriding G4MuonMinusAtomicCapture)
- MuDiracMuonMinusAtomicCaptureConstructor
  - constructs generic muonic atom
  - registers MuDiracMuonMinusAtomicCapture for  $\mu^-$
- MuDiracEmCaptureCascade customized using MuDirac spectra

# HPGe energy deposit for muon beam on water target

[Optimized] Length: 15 cm, thickness: 2 cm (1m events, see backup for comparisons)



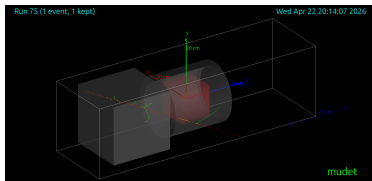
- The measured  $K_\beta$  peak is higher than the MC truth due to the **additional contribution from  $K_\alpha$  and  $L_\alpha$** .  $K_\alpha$  and  $K_\beta$  are the two most significant peaks.
- A thickness of 2 cm balances the efficiency with the cost and is used in the following studies.
- A length of 15 cm is sufficient to capture the characteristic X- or  $\gamma$ -rays, and increasing the length does not significantly improve the efficiency.

# Geant4 setup with cosmic muons, water target, and HPGe detector

Repository: <https://github.com/PKMuon/mudet/tree/muwater>

## Geometry

- Target:  $\text{H}_2\text{O}$  (l), defined by NIST
- Dimensions:  $10 \times 10 \times 10 \text{ cm}^3$



## Muon source

- Particle: all sea-level cosmic rays
- Kinetic energy: up to  $\sim 1 \text{ TeV}$
- Decelerated by a 20 cm thick Pb block
- New secondary muons may be produced

## HPGe detector

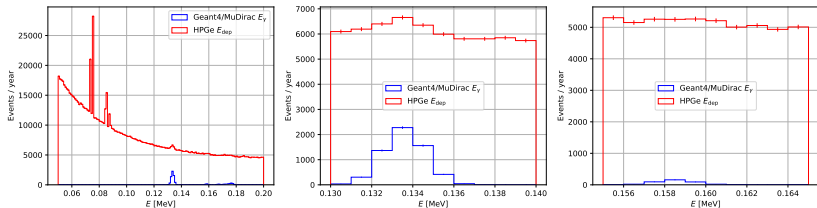
- Tubular, centered on the target
- Compact inner radius: 7.5 cm
- Optimized outer radius: e.g., 9.5 cm
- Preliminary target length: e.g., 20 cm

## Muonic atom physics (based on Geant4 v11.3.2)

- MuDiracEmCaptureCascade (overriding G4EmCaptureCascade)
- MuDiracMuonMinusAtomicCapture (overriding G4MuonMinusAtomicCapture)
- MuDiracMuonMinusAtomicCaptureConstructor
  - constructs generic muonic atom
  - registers MuDiracMuonMinusAtomicCapture for  $\mu^-$
- MuDiracEmCaptureCascade customized using MuDirac spectra

# HPGe energy deposit for cosmic muons on water target

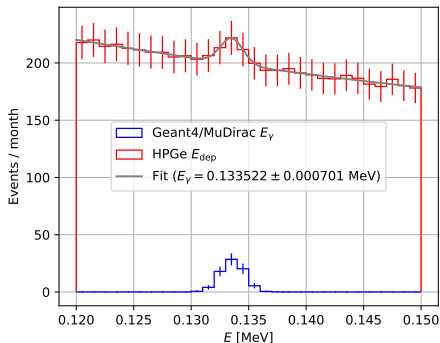
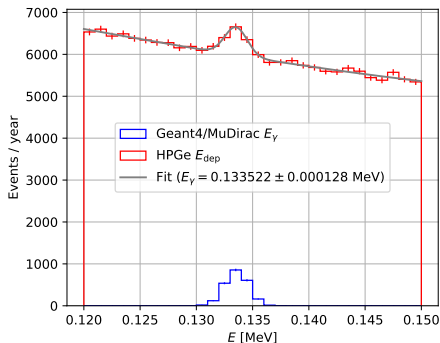
[Preliminary] Length: 20 cm, thickness: 2 cm (351560897 / 189741600 muon events)



- Expect  $19 \times 19 \div 60 \times 86400 \times 365 = 189\,741\,600$  muon events annually
- 351 560 897 muon events generated by simulating  $10^9$  cosmic-ray shower events with CRY, propagated to ground level in a  $19 \times 19 \text{ cm}^2$  detecting area
- Pure Poisson uncertainties are assigned to each bin, neglecting Monte Carlo statistical uncertainties due to sufficiently large sample size

# Prospected O-16 K-alpha energy fit

[Preliminary] Length: 20 cm, thickness: 2 cm (351560897 / 189741600 muon events)



- Uniformly binned fit over the range  $[0.12, 0.15]$  MeV with a bin width of 1 keV
- Assumes a **cubic background** and a **Gaussian signal shape**; no smearing applied
- **Consistent with MuDirac**: K1–L2 at 133506 eV and K1–L3 at 133545 eV
- **One-month data** already appears promising (needs validation with more toy datasets)

## A brief summary of the ultimate target of muonic Th-229 spectroscopy

- We have succeeded in generating muonic Th-229 cascade gamma energies with MuDirac and validated its integration in Geant4 (see [this previous report](#) for details)
- We are exploring ways to account for the low density of Th-229 (a high-intensity muon beam can help a lot; may learn from PSI)
- We are studying various Th-229(m) theoretical models and searching for possible sensitive physics observables; we are trying to generate gamma rays produced by muonic Th-229m

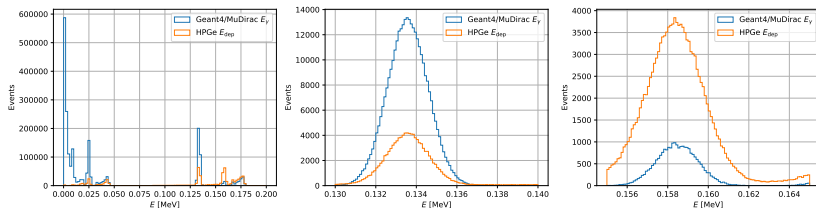
Isotope	Abundance	$K_\alpha$ [keV]	$K_\beta$ [keV]
$^{229}\text{Th}$	trace	6148/6369	9309/9365
$^{229m}\text{Th}$	trace	ongoing	ongoing
$^{230}\text{Th}$	$\sim 0.0002$	6137/6357	9297/9352
$^{232}\text{Th}$	$\sim 1$	6120/6340	9277/9333

Note:  $K_\alpha$  and  $K_\beta$  exhibit double peaks from different magnetic quantum numbers in the  $L$  or  $M$  shells.

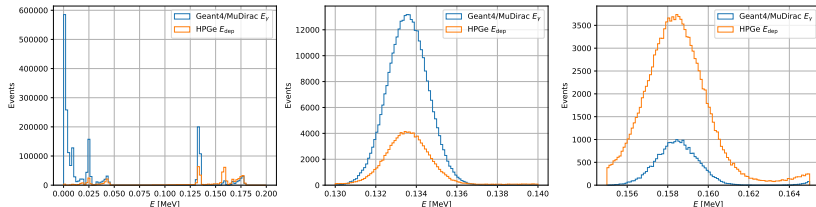
Thanks for your attention!

# HPGe energy deposit for muon beam on water target

Length: 15 cm, thickness: 6 cm, 1m events



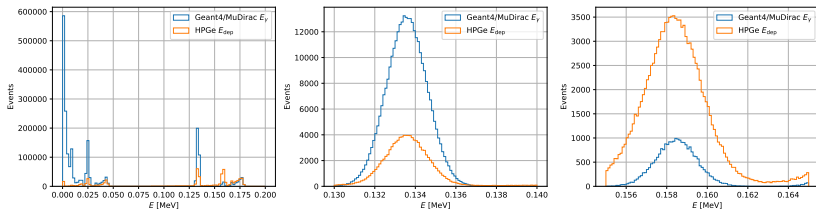
Length: 15 cm, thickness: 4 cm, 1m events



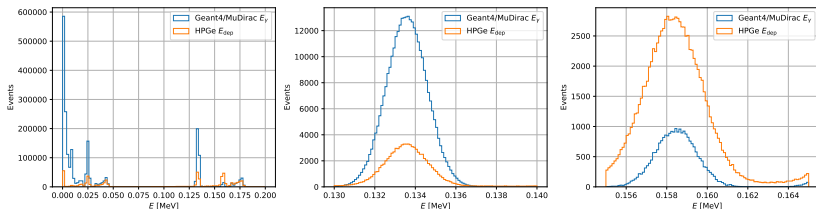
Remark: The measured  $K_\beta$  peak is higher than the MC truth due to the **additional contribution from  $K_\alpha$  and  $L_\alpha$** .  $K_\alpha$  and  $K_\beta$  are the two most significant peaks.

# HPGe energy deposit for muon beam on water target

Length: 15 cm, thickness: 2 cm, 1m events



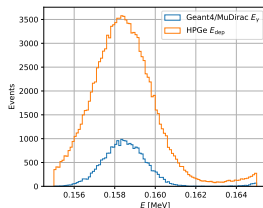
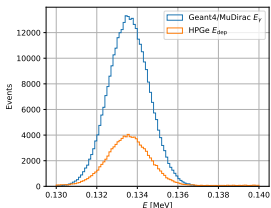
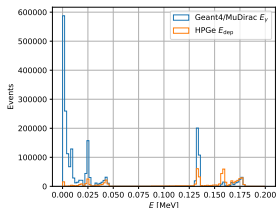
Length: 15 cm, thickness: 1 cm, 1m events



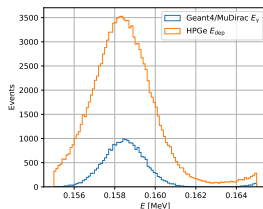
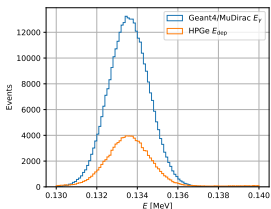
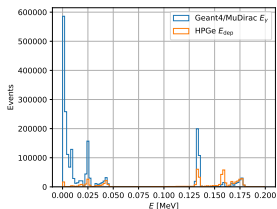
Remark: A thickness of 2 cm balances the efficiency with the cost and is used in the following studies.

# HPGe energy deposit for muon beam on water target

Length: 20 cm, thickness: 2 cm, 1m events



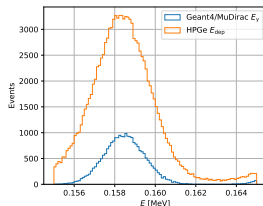
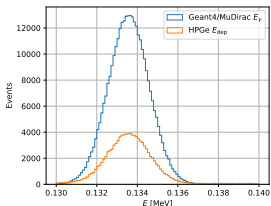
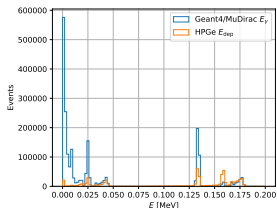
Length: 15 cm, thickness: 2 cm, 1m events



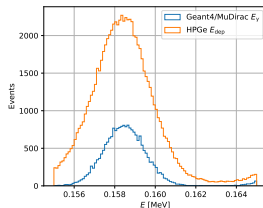
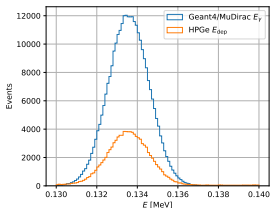
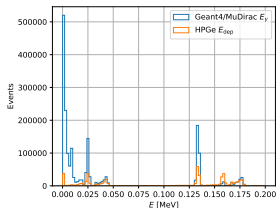
Remark: A length of 15 cm is sufficient to capture the characteristic X- or  $\gamma$ -rays, and increasing the length does not significantly improve the efficiency.

# HPGe energy deposit for muon beam on water target

Length: 10 cm, thickness: 2 cm, 1m events



Length: 5 cm, thickness: 2 cm, 1m events



Remark: A length of 15 cm is sufficient to capture the characteristic X- or  $\gamma$ -rays, and decreasing the length reduces the efficiency.

## MuDirac benchmark results

Line	Mudirac	Computed	Experimental
$^8\text{O}, \text{N}_6 - \text{O}_8$	4.0224	–	4.0243
$^8\text{O}, \text{N}_7 - \text{O}_8$	4.0216	–	4.0227
$^8\text{O}, \text{N}_7 - \text{O}_9$	4.0218	–	4.0237
$^{12}\text{Mg}, \text{L}_2 - \text{M}_4$	56.389	56.392	56.392
$^{12}\text{Mg}, \text{L}_3 - \text{M}_5$	56.214	56.216	56.216
$^{26}\text{Fe}, \text{K}_1 - \text{L}_3$	1256.89	–	1257.9
$^{26}\text{Fe}, \text{K}_1 - \text{M}_3$	1522.1	–	1523.5
$^{26}\text{Fe}, \text{K}_1 - \text{N}_3$	1614.94	–	1616.7
$^{50}\text{Sn}, \text{M}_4 - \text{N}_6$	349.964	349.980	349.975
$^{50}\text{Sn}, \text{M}_5 - \text{N}_7$	345.252	345.256	345.254
$^{82}\text{Pb}, \text{N}_6 - \text{O}_8$	437.757	437.749	437.749
$^{82}\text{Pb}, \text{N}_7 - \text{O}_9$	431.359	431.336	431.328
$^{82}\text{Pb}, \text{K}_1 - \text{L}_2$	5779.1	–	5777.9
$^{82}\text{Pb}, \text{K}_1 - \text{L}_2$	5779.1	–	5776.8
$^{82}\text{Pb}, \text{K}_1 - \text{L}_3$	5963.81	–	5962.7
$^{82}\text{Pb}, \text{K}_1 - \text{L}_3$	5963.81	–	5961.9

[X-Ray Spectrom. 50, 180–196]