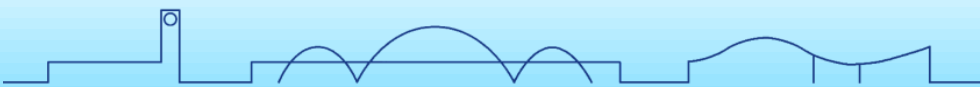


# The discussion of $D_s$ family

Jia-jun Wu (UCAS)

Collaborator: Ruhui Ni (UCAS), Xianhui Zhong(HUNNA)

**PRD 109 (2024) 11, 116006, 2605.17823**



# Outline

- Background: Bare state + Coupled channel
- Unquenched quark model for heavy-light mesons
- Family of  $D_s$
- Summary



# Background: Quark Model

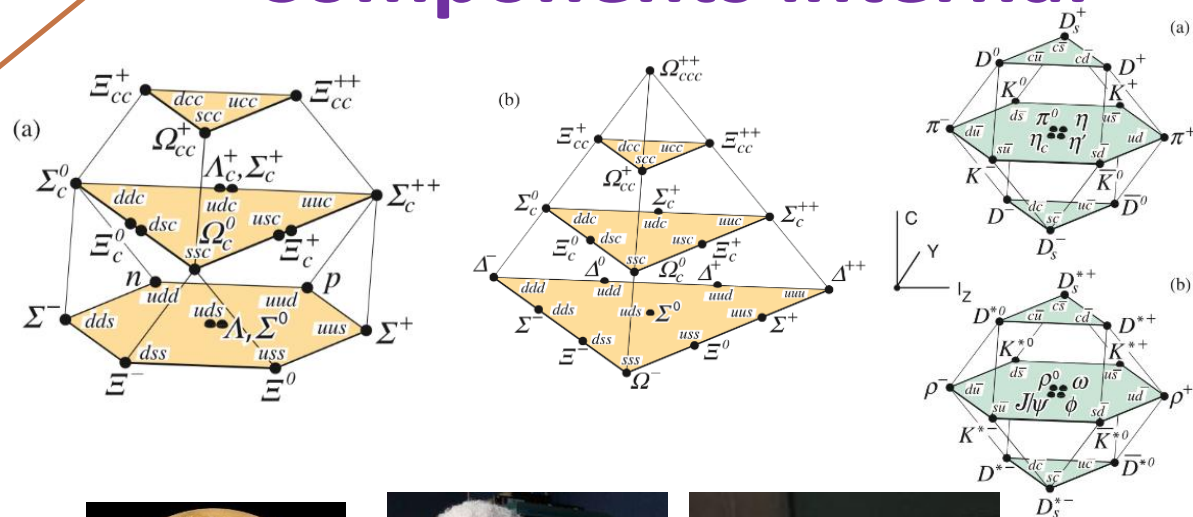
## Hadron spectroscopy--- Components internal

Hadron state:  
meson, baryon

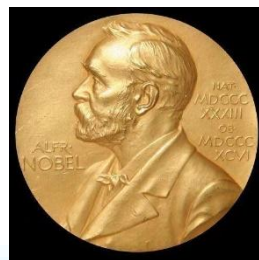
QCD, NP

Quark  
model

Quark/Gluon



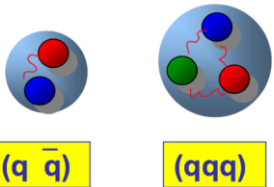
1969:  
Nobel prize



# Background: Beyond Quark Model

Hadron state:  
meson, baryon//Exotic

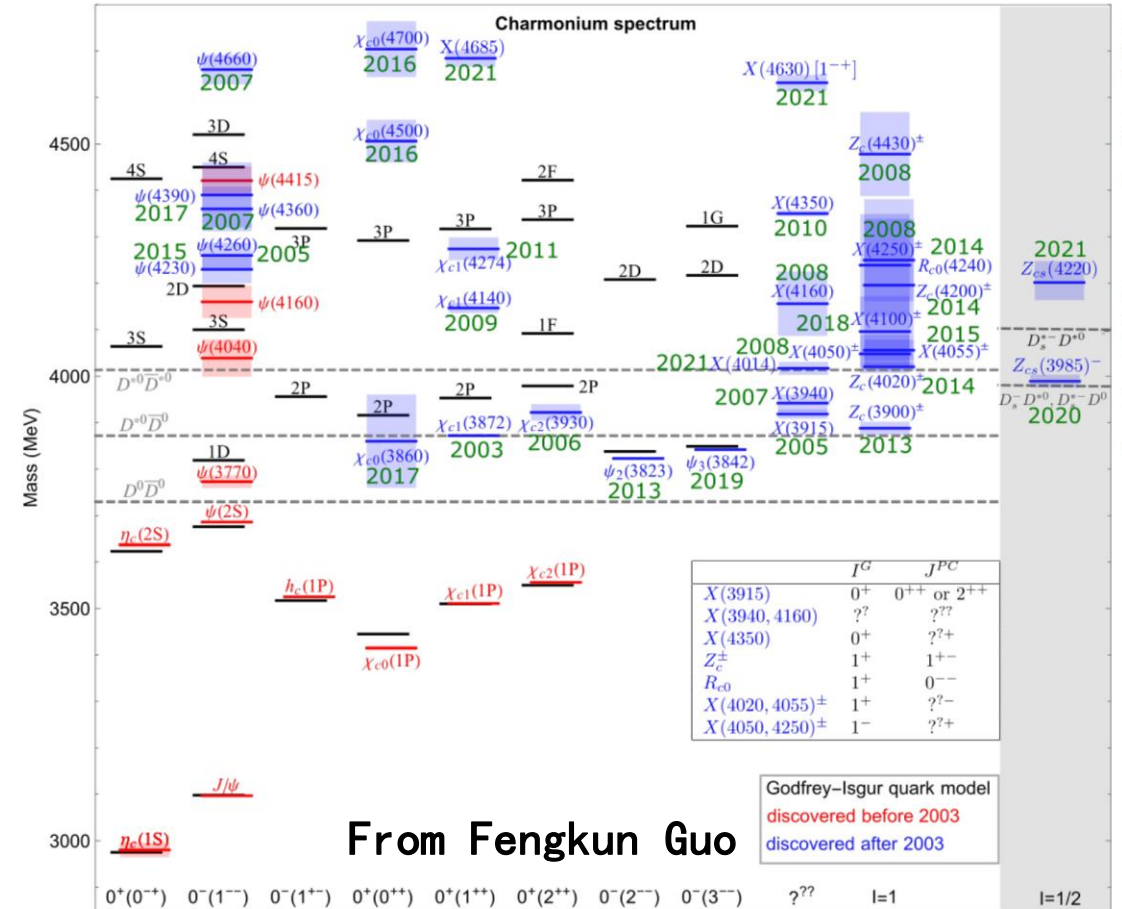
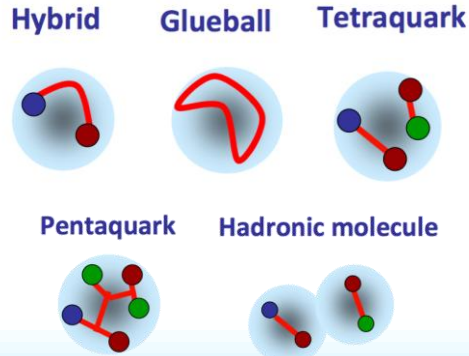
conventional hadron



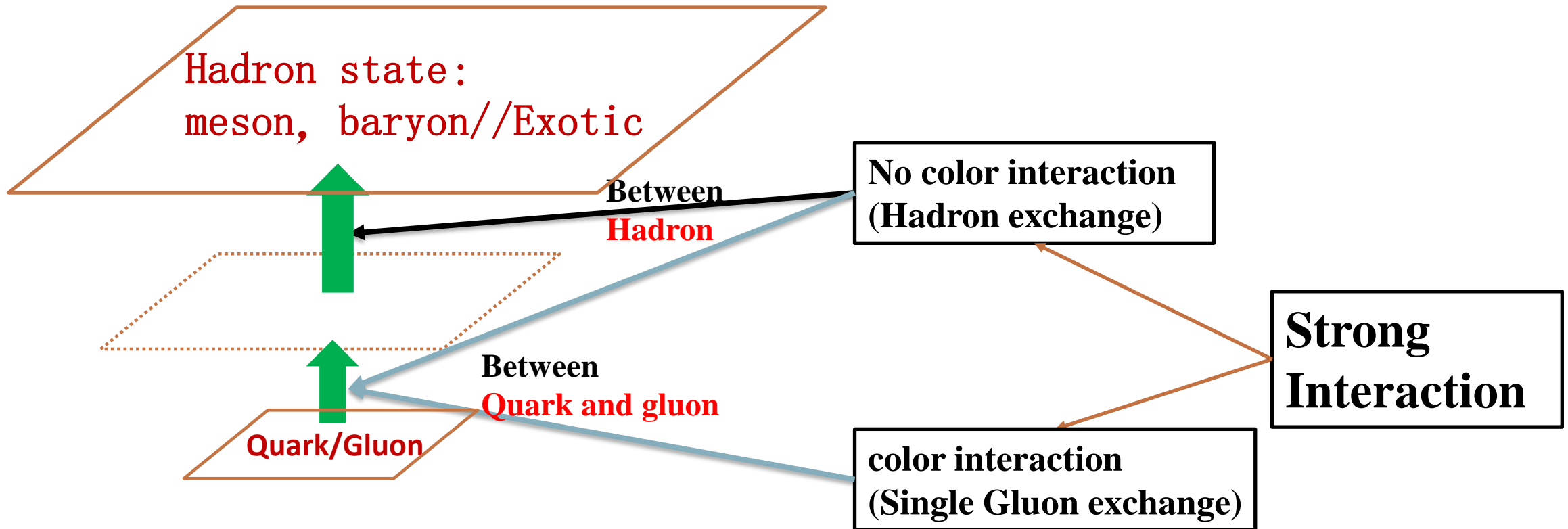
Quark model?

Quark/Gluon

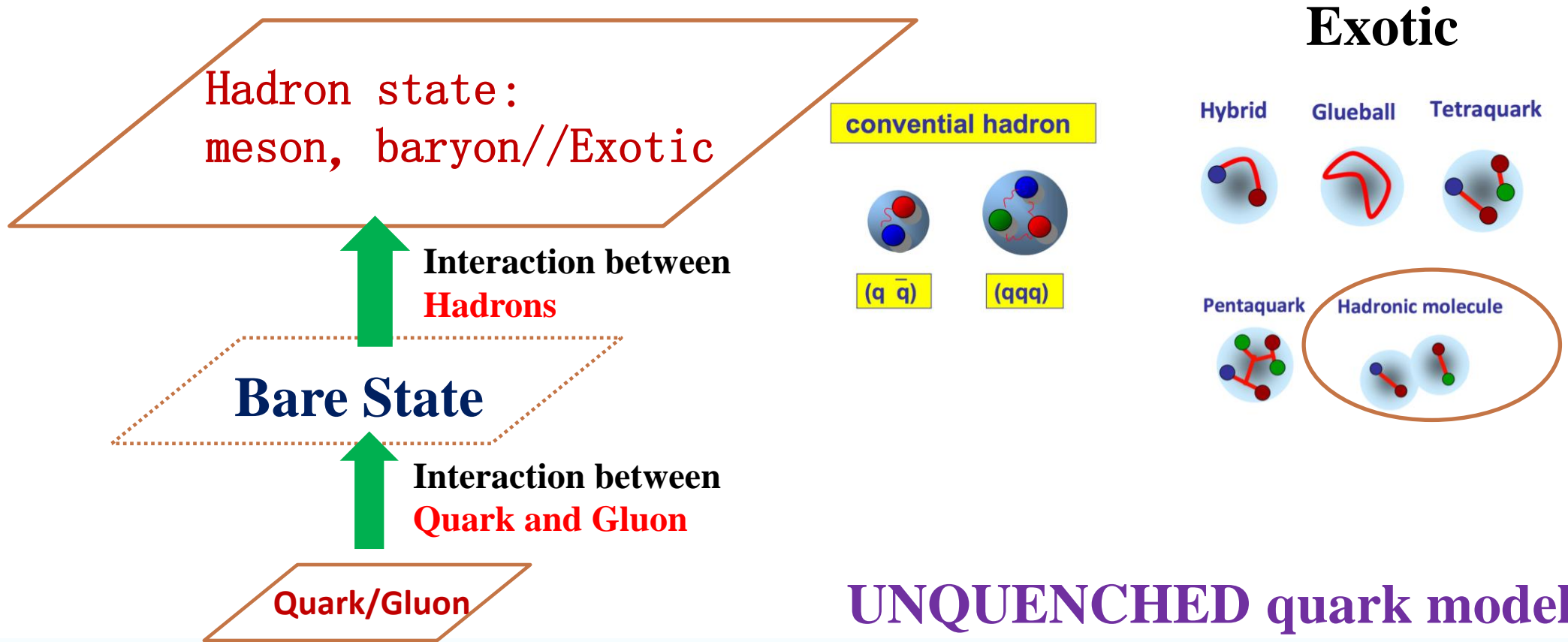
Exotic



# Background: Strong Interaction



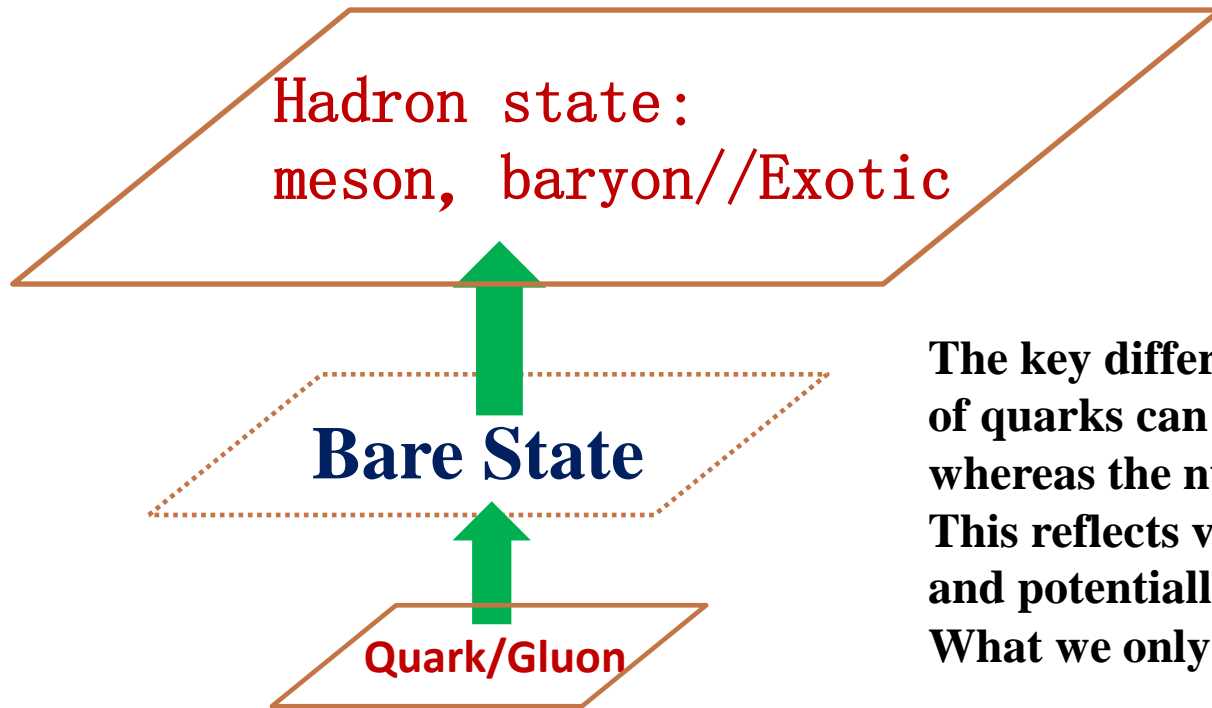
# Background: Bare state + Coupled channel



**UNQUENCHED quark model**



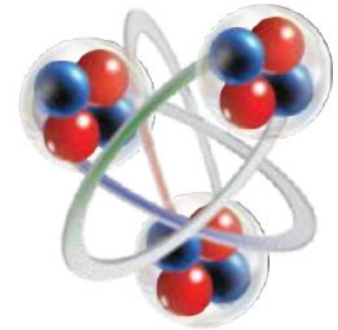
# Background: Strong-Spectrum-Reaction



Nuclear Physics:

$12C \leq 6p \ \& \ 6n$

$\leq 3 \alpha$  Cluster



The key difference from Hadronic and Atomic system: the number of quarks can **fluctuate** (only quark quantum numbers conserved), whereas the number of nucleons is fixed.

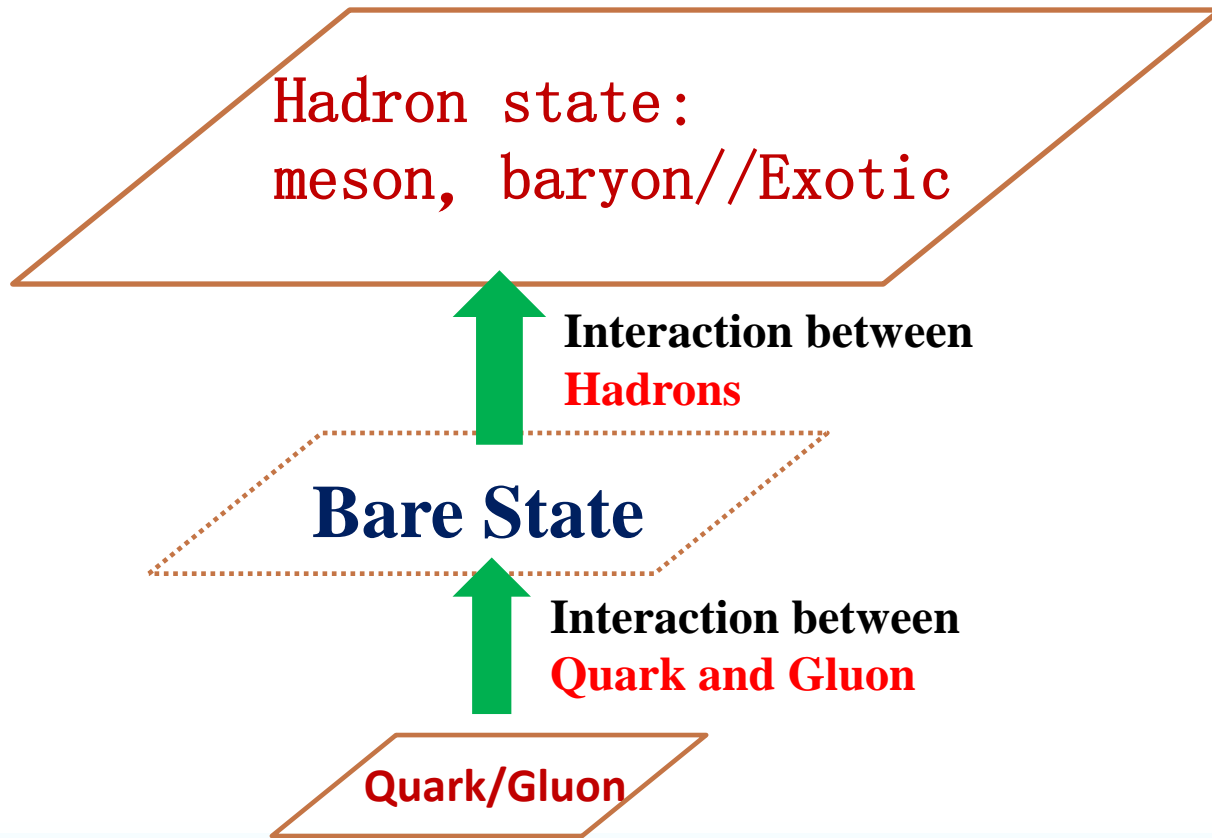
This reflects vacuum fluctuations involving **quark-antiquark pairs** and potentially **glueballs** as well.

What we only can define is **the lowest number of constitute quark**.

## UNQUENCHED quark model



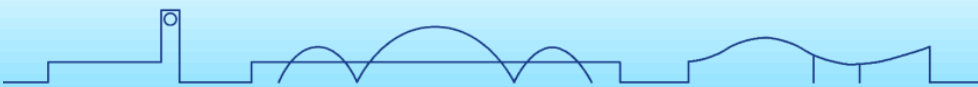
# Background: Bare state + Coupled channel



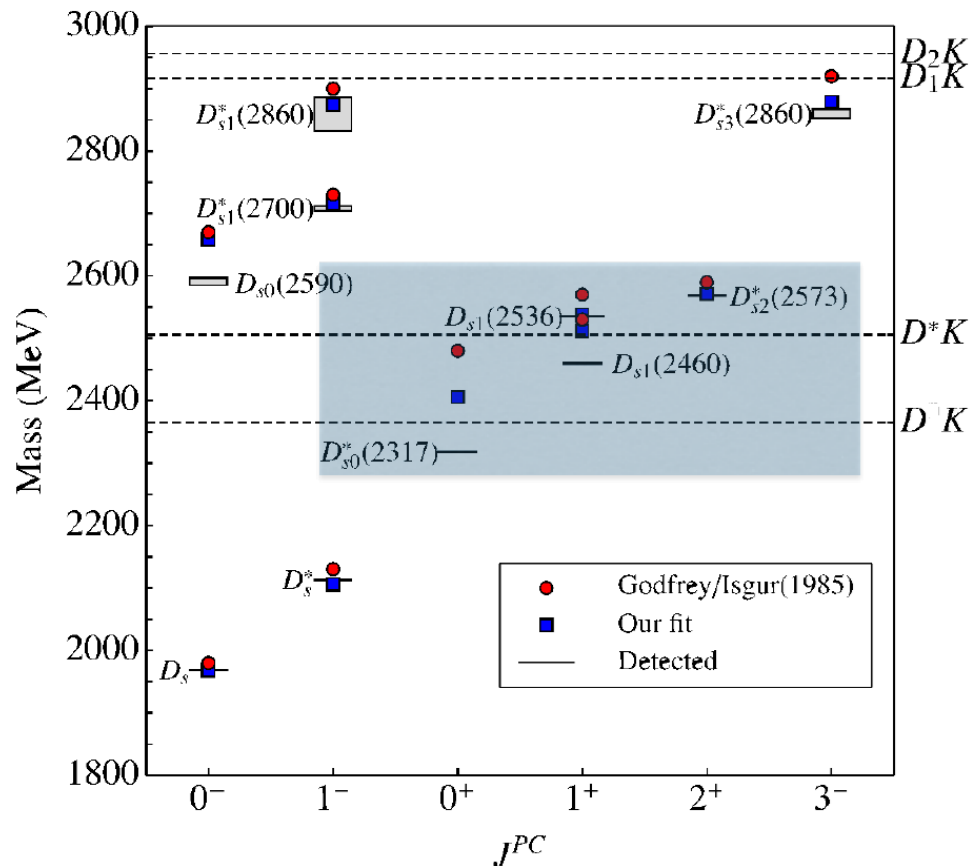
Bare state(Single) + multi-hadron

The number of hadrons is  
**not conserved!**

**Coupled channel**



# Non-Perturbative Hamiltonian Framework



**V contain various parameters**

HEFT  $\rightarrow$  NPHF

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_{\alpha})\rangle \left[ \sqrt{m_{\alpha 1}^2 + k_{\alpha}^2} + \sqrt{m_{\alpha 2}^2 + k_{\alpha}^2} \right] \langle \alpha(k_{\alpha})|$$

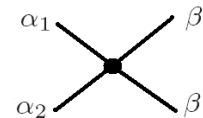
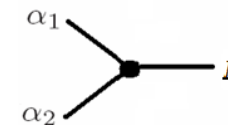
$|B_i\rangle$  bare state, bare mass  $m_i$

$|\alpha(k_{\alpha})\rangle$  non-interaction channels

$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[ |\alpha(k_{\alpha})\rangle g_{i,\alpha}^+ \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_{\alpha})| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_{\alpha})\rangle v_{\alpha,\beta} \langle \beta(k_{\beta})|$$



Yang, Wang, Wu, Oka, Zhu, PRL128(2022),112001



中国科学院大学  
University of Chinese Academy of Sciences

# Unquenched quark model for heavy-light mesons

$$\begin{pmatrix} \mathcal{H}_0 & \mathcal{H}_I \\ \mathcal{H}_I & \mathcal{H}_c \end{pmatrix} \begin{pmatrix} c_A(M)|A\rangle \\ \sum_{BC} \int c_{BC}(\mathbf{q}, M) d^3\mathbf{q} |BC, \mathbf{q}\rangle \end{pmatrix} = M \begin{pmatrix} c_A(M)|A\rangle \\ \sum_{BC} \int c_{BC}(\mathbf{q}, M) d^3\mathbf{q} |BC, \mathbf{q}\rangle \end{pmatrix}$$

$\mathcal{H}_0 \gg$  **OGE Potential**  $\mathcal{H}_0 = \sqrt{\mathbf{p}_1^2 + m_q^2} + \sqrt{\mathbf{p}_2^2 + m_{\bar{q}}^2} + V_0(r) + V_{sd}(r)$

$$V_0(r) = -\frac{4\alpha_s(r)}{3r} + br + C_0 \quad V_{sd}(r) = \frac{32\pi\alpha_s(r)\sigma^3 e^{-\sigma^2 r^2}}{9\sqrt{\pi^3} \tilde{m}_1 m_2} \mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{4\alpha_s(r)}{3\tilde{m}_1 m_2} \frac{1}{r^3} S_{12} + H_{LS},$$

$$H_{LS} = H_{\text{sym}} + H_{\text{anti}},$$

$$H_{\text{sym}} = \frac{\mathbf{S}_+ \cdot \mathbf{L}}{2} \left[ \left( \frac{1}{2\tilde{m}_1^2} + \frac{1}{2m_2^2} \right) G(r) + \frac{8\alpha_s(r)}{3\tilde{m}_1 m_2 r^3} \right],$$

$$H_{\text{anti}} = \frac{\mathbf{S}_- \cdot \mathbf{L}}{2} \left( \frac{1}{2\tilde{m}_1^2} - \frac{1}{2m_2^2} \right) G(r),$$

$\mathcal{H}_I \gg$  **Chiral quark model**

$$\mathcal{L}_{Pqq} = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu^j \gamma_5^j \psi_j \vec{\tau} \cdot \partial^\mu \vec{\phi}_m$$

$$\mathcal{H}_I^{NR} = g \sum \left[ - \left( 1 + \frac{\omega_m}{4\mu_a} \right) \sigma_j \cdot \mathbf{q} + \frac{\omega_m}{2\mu_a} \sigma_j \cdot \mathbf{p}_j \right] \hat{I}_j \phi_m$$

the leading nonrelativistic operator

$$\mathcal{H}_I^{RC} = \frac{g}{32\mu_q^2} \sum_j \left[ -m_{\mathbb{P}}^2 (\sigma_j \cdot \mathbf{q}) - 2\sigma_j \cdot (\mathbf{q} - 2\mathbf{p}_j) \times (\mathbf{q} \times \mathbf{p}_j) \right] \hat{I}_j \phi_m$$

the relativistic correction terms kept up to order 1/m<sup>2</sup>

$\mathcal{H}_c \gg \sqrt{m_B^2 + \mathbf{q}^2} + \sqrt{m_C^2 + \mathbf{q}^2}$

Ni, Wu, Zhong, PRD109, 116006(2024)

Ni, Li, Zhong, PRD105 056006 (2022),

Zhong, Zhao, PRD78 014029 (2008),



# Unquenched quark model for heavy-light mesons

M.R.Pennington and D.J.Wilson, *Decay channels and charmonium mass-shifts*, PRD 76 077502 (2007)

$$M = M_A + g(M)$$

$$\text{Re } g(M) \equiv \Delta M(M)$$

$$= \sum_{BC} \mathcal{P} \int_0^\infty \frac{|\mathcal{M}_{A \rightarrow BC}(\mathbf{q})|^2}{M - E_{BC}} q^2 dq,$$

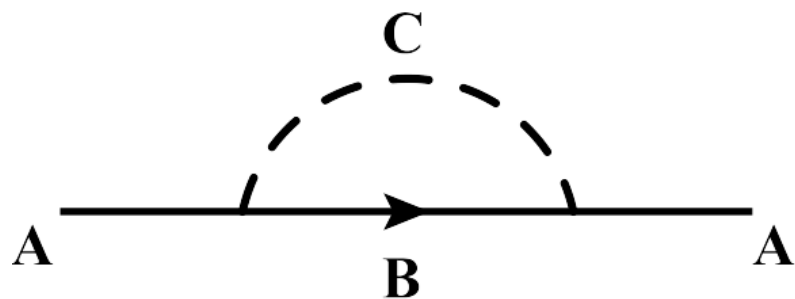
$$\text{Im } g(M) = - \sum_{BC} \pi \frac{|\mathbf{q}| E_B E_C}{E_{BC}} |\mathcal{M}_{A \rightarrow BC}(\mathbf{q})|^2 \Big|_{M_i = E_{BC}}$$

$$\Delta \Pi_n(s, s_0) \equiv \Pi_n(s) - \Pi_n(s_0)$$

$$= \frac{(s - s_0)}{\pi} \int_{s_n}^\infty ds' \frac{\text{Im} \Pi_n(s')}{(s' - s)(s' - s_0)} \quad (3)$$

Then

$$\sum_{n=1} \Delta \Pi_n(s, s_0) = \mathcal{M}^2(s) - \mathcal{M}_{\text{charmonium}}^2 \equiv \Delta \mathcal{M}^2(s). \quad (4)$$



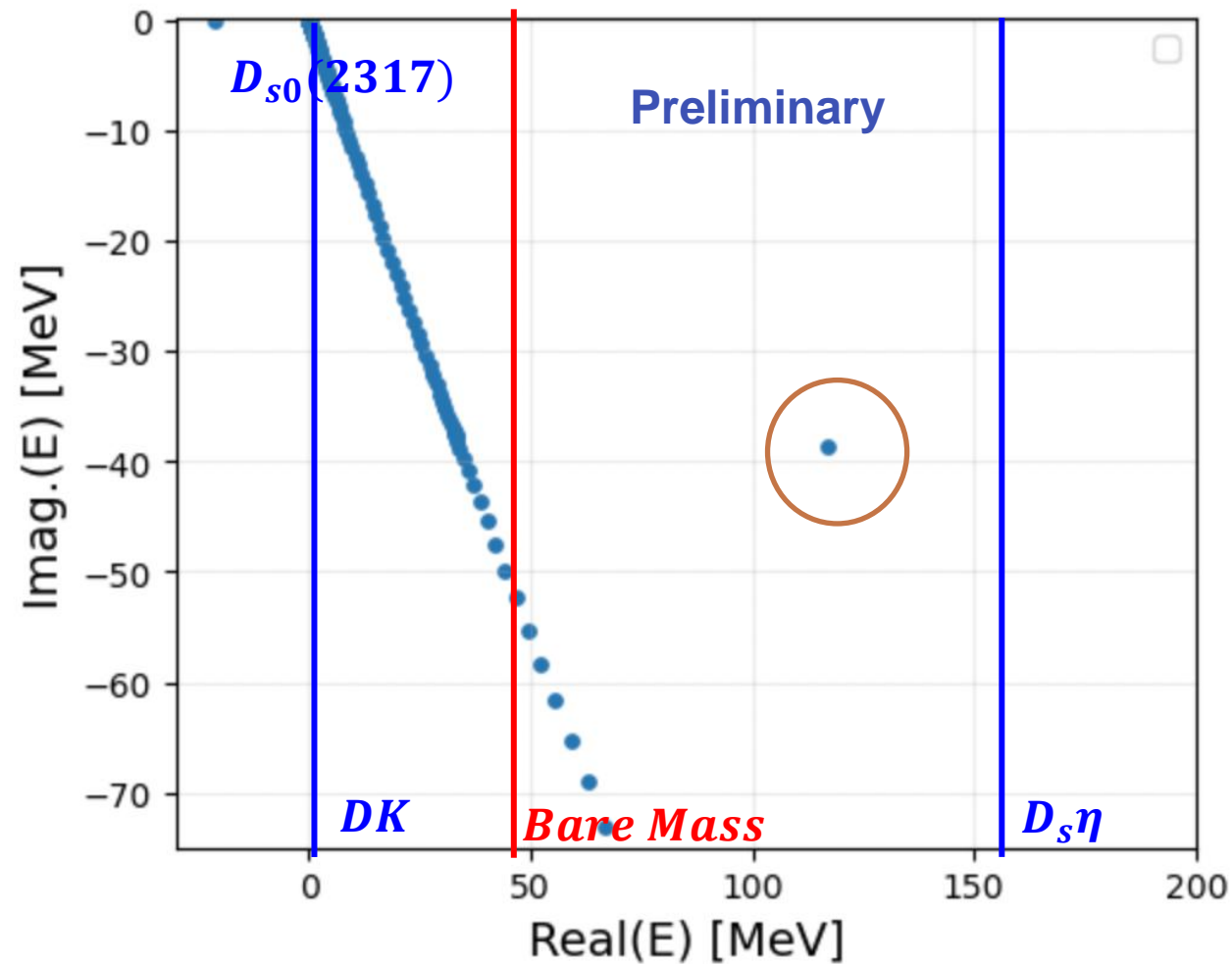
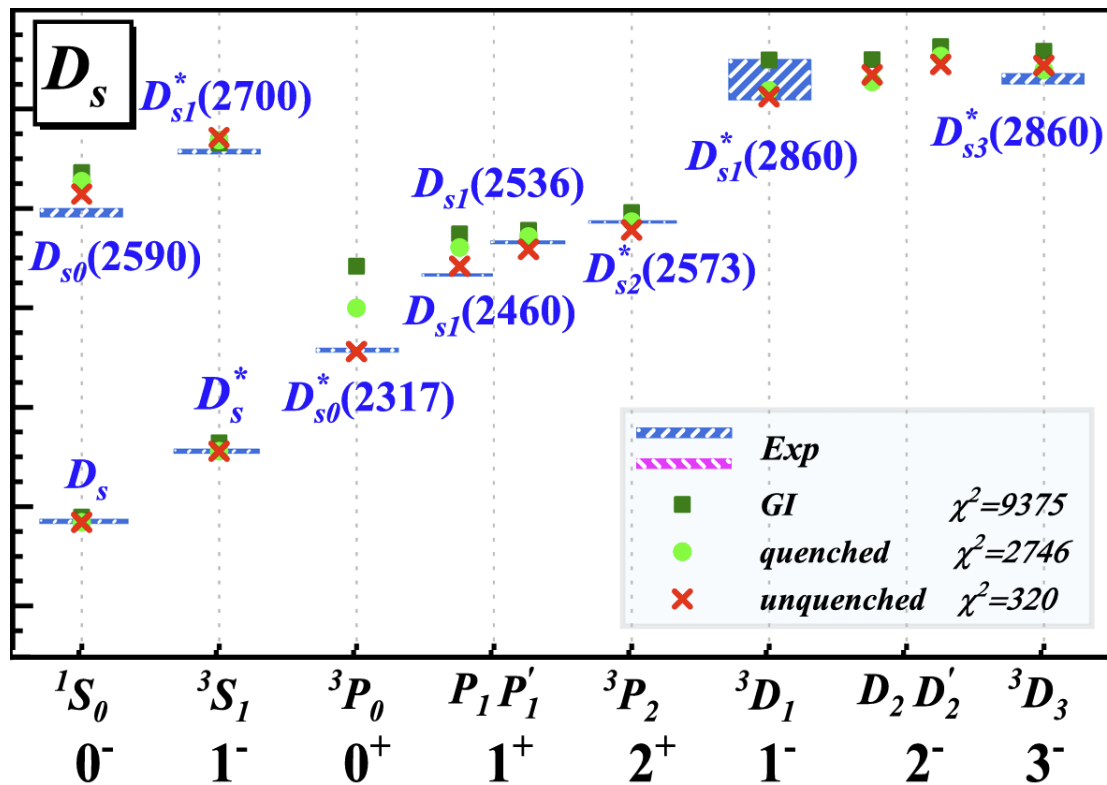
$$\langle BC, \mathbf{q} | \mathcal{H}_I | A \rangle \rightarrow \langle BC, \mathbf{q} | \mathcal{H}_I e^{-q^2/(2\Lambda^2)} | A \rangle$$

Key Question: How to choose coupled channel ?

One only need consider the **OZI-allowed** two-body hadronic channels with mass thresholds **below or just above** the bare |A⟩ states.

# Family of $D_s$

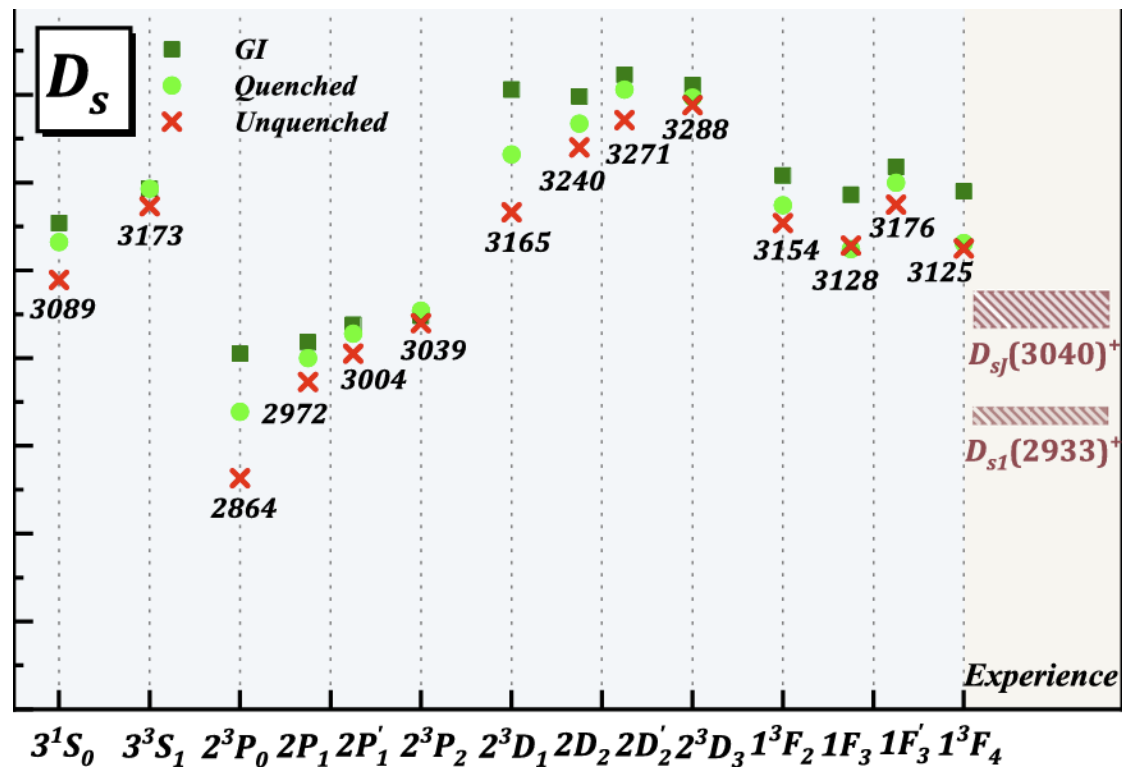
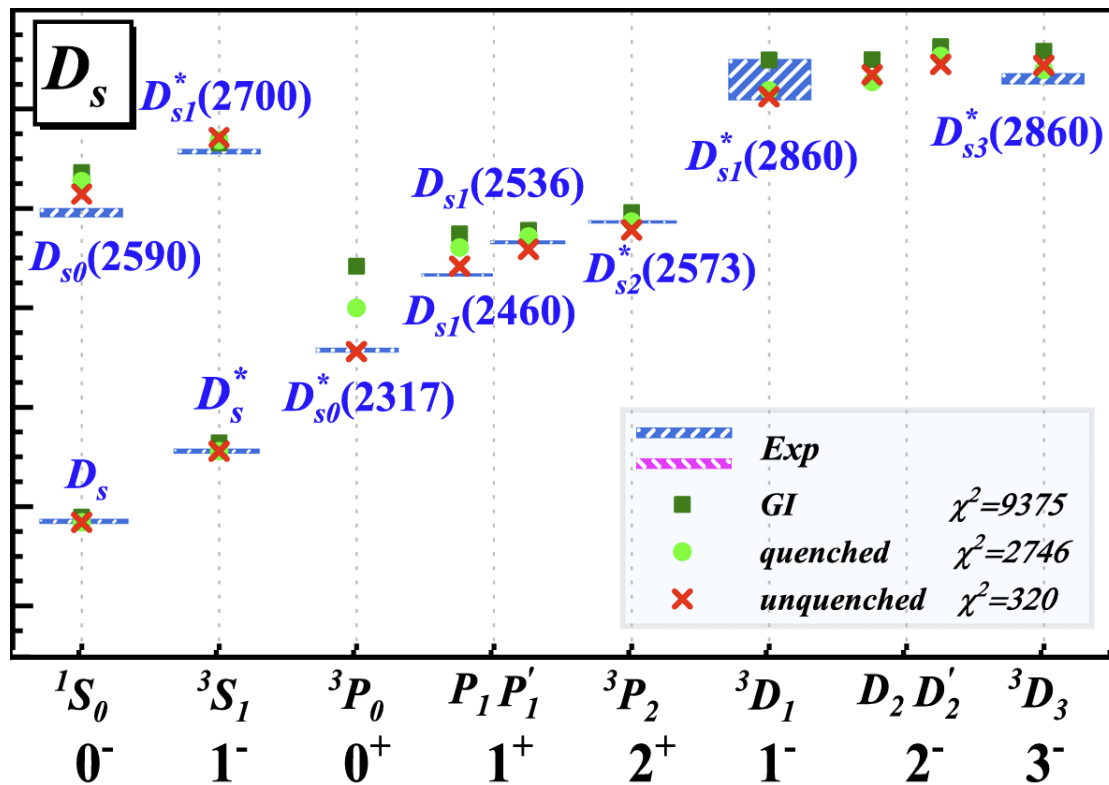
NPHF



Only one  $D_{s0}(2317)$  one state with  $J^P = 0^+$  below 2700 MeV !

Fitted

# Family of $D_s$

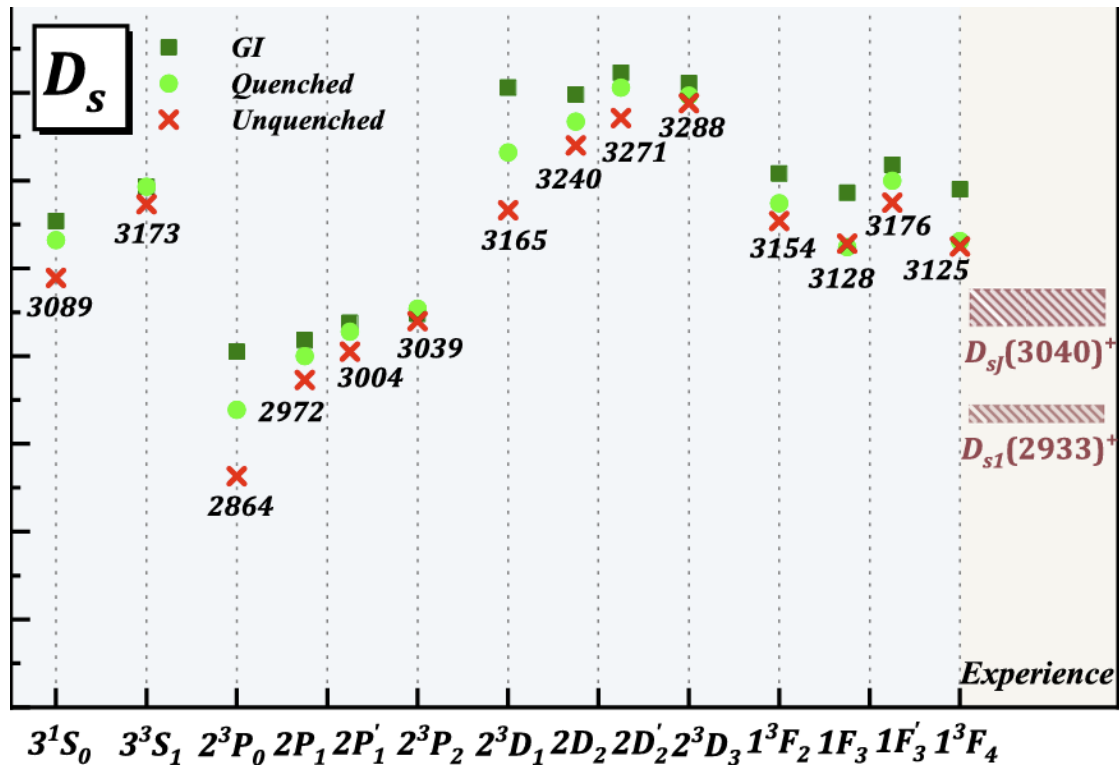


There are so many predicted states !

Fitted

Predicted

# Family of $D_s$



There are so many predicted states !

**Predicted**

$1^+$   $2933_{-5}^{+6}$  MeV,  $72_{-12}^{+18}$  MeV

LHCb arXiv:2604.21257

$2P_1$  2972 MeV 92 MeV

Channel	$D_s(2P_1)$ as $D_{s1}(2933)^+$
$D^*K$	53.7
$D_0^*(2300)K$	0.1
$D_1(2430)K$	1.5
$D_1(2420)K$	$4.4 \times 10^{-2}$
$D_2^*(2460)K$	–
$D_s^*\eta$	8.7
$D_{s0}^*(2317)\eta$	$1.5 \times 10^{-2}$
$D_{s1}(2460)\eta$	–
$DK^*$	15.8
$D^*K^*$	$9.4 \times 10^{-2}$
$D_s\phi$	–
Total	79.8

$J^P = ?$   $3044 \pm 8_{-5}^{+30}$  MeV,  $239 \pm 35_{-42}^{+46}$  MeV

BaBar, PRD80, 092003 (2009)

$2P_1'$  3004 MeV 116 MeV

$3^1S_0$  3089 MeV 160 MeV

$D_s(2^3P_2)(3039)$  :  $D_2^*(2460)K$  [~ 26%],  $DK$  [~ 20%],

$D_s(2^3P_0)(2864)$  :  $DK$  [~ 88%],  $D_s\eta$  [~ 12%].

$D_s(2^3D_1)(3165)$  :  $D_1(2420)K$  [~ 85%],

$D_s(2D_2)(3240)$  :  $D_2^*(2460)K$  [~ 76%].

$D_s(2D_2')(3271)$  :  $D_2^*(2460)K$  [~ 17%].

$D_s(1^3F_2)(3154)$  :  $D_1(2420)K$  [~ 57%],

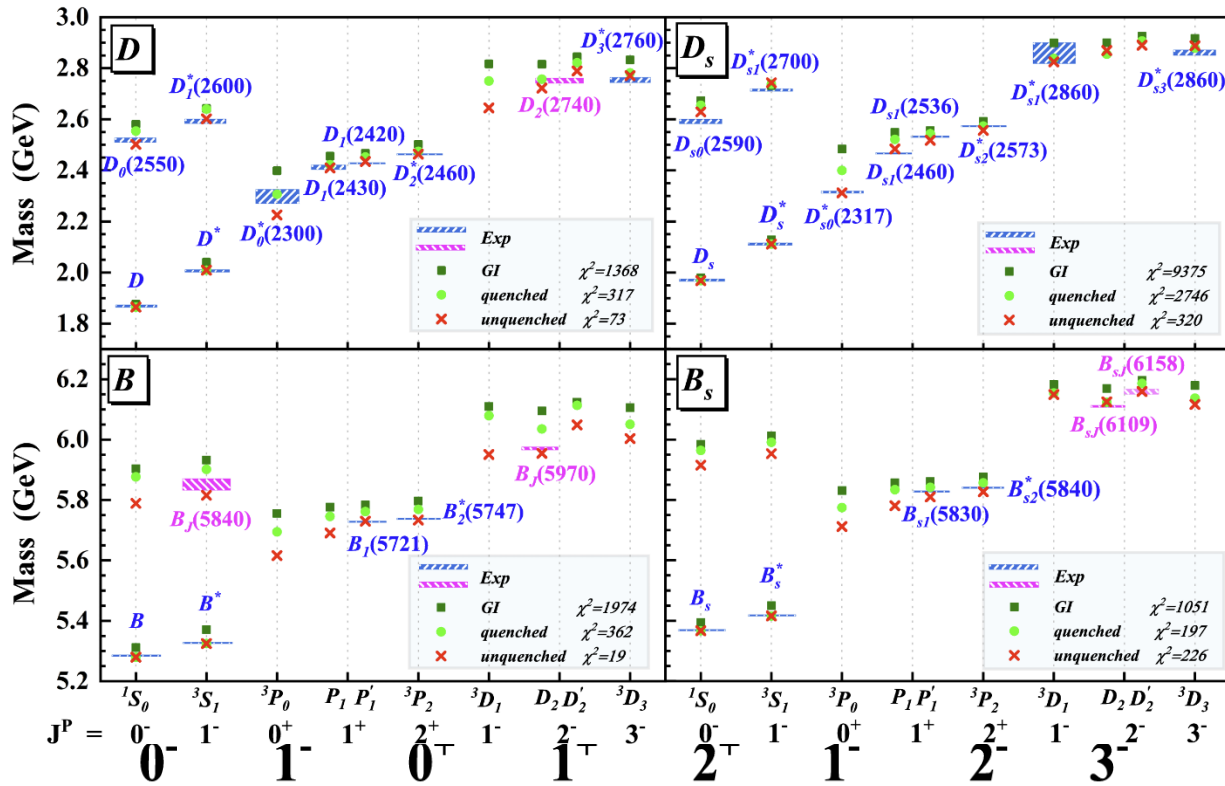
$D_s(1F_3)(3128)$  :  $D_2^*(2460)K$  [~ 82%],

$D_s(1F_3')(3176)$  :  $D^*K$  [~ 41%],  $D_1(2420)K$  [~ 14%],

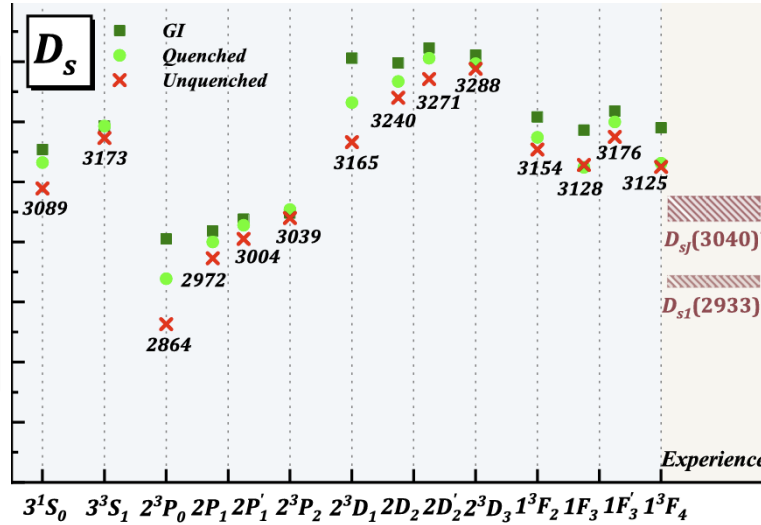
$D_s(1^3F_4)(3125)$  :  $D^*K$  [~ 31%],  $D_1(2430)K$  [~ 26%].



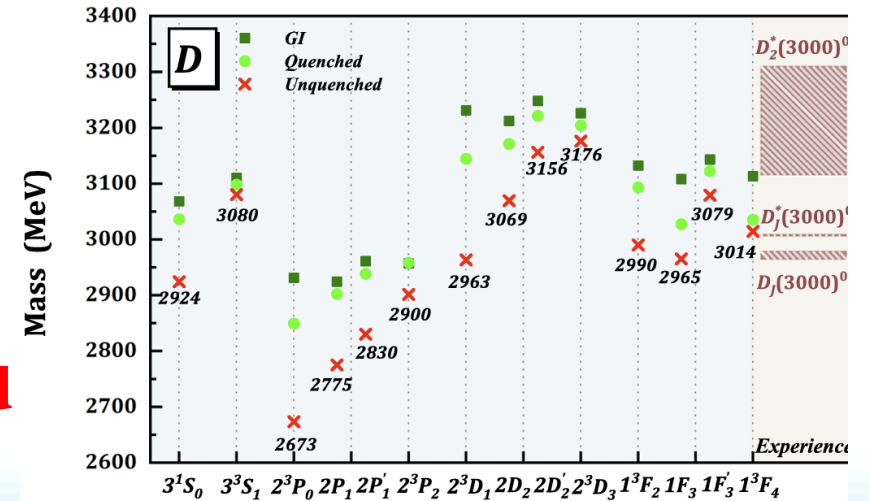
# Other $D, B, B_s$ states



**Fitted**



**Predicted**



# Summary

- Introduce an unquenched quark model.
- $D_{s0}(2317)$  should need  $DK$  channel, while higher state exist or not should be checked to understand how large interaction of  $DK \rightarrow DK$
- We predict the  $D_s(2933)$  as a  $2P_1$  state, the predicted mass and width are all similar, and  $D_s(3044)$  looks like a  $3^1S_0$  state.
- We also predicted various high angular momentum  $D_s$  states.

THANKS !

