格点QCD五夸克态研究

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第6届LHCb前沿物理研讨会

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Outline

- ✦ Spectroscopy on lattice
- ✦ Hidden-charm pentaquarks

  H. Xing et al. arXiv:2210.08555

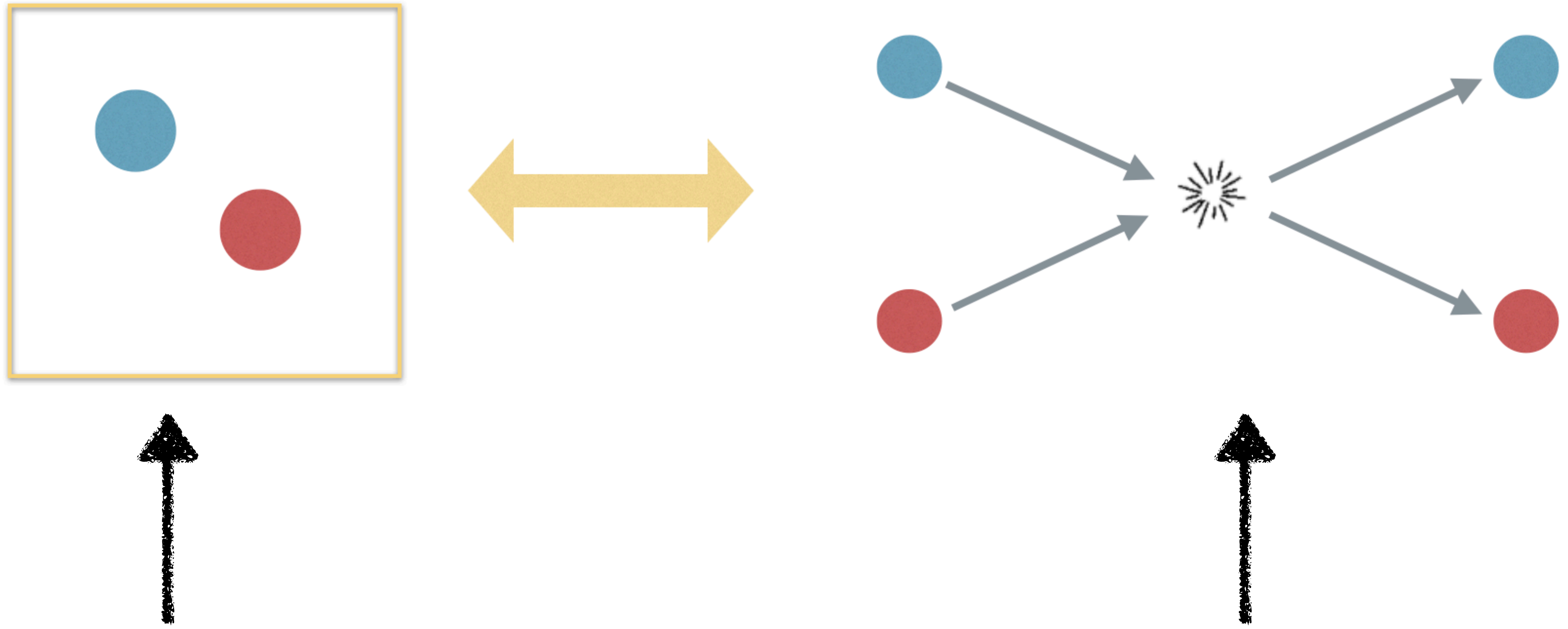
- ✦ Doubly-charmed pentaquarks

  J.-Y. Yi et al. JHEP03(2006)006

  J.-Y. Yi et al. work in progress

Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)



Finite volume spectrum

Infinite volume scattering parameters

Scattering on lattice

CLQCD

- build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = <0|\mathcal{O}_i\mathcal{O}_j^\dagger|0> = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

- Solve the generalized eigenvalue problem(GEVP):

$$C_{ij}v_j^n(t) = \lambda_n(t)C_{ij}^0 v_j^n(t)$$

- Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

- Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

Scattering on lattice

- General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

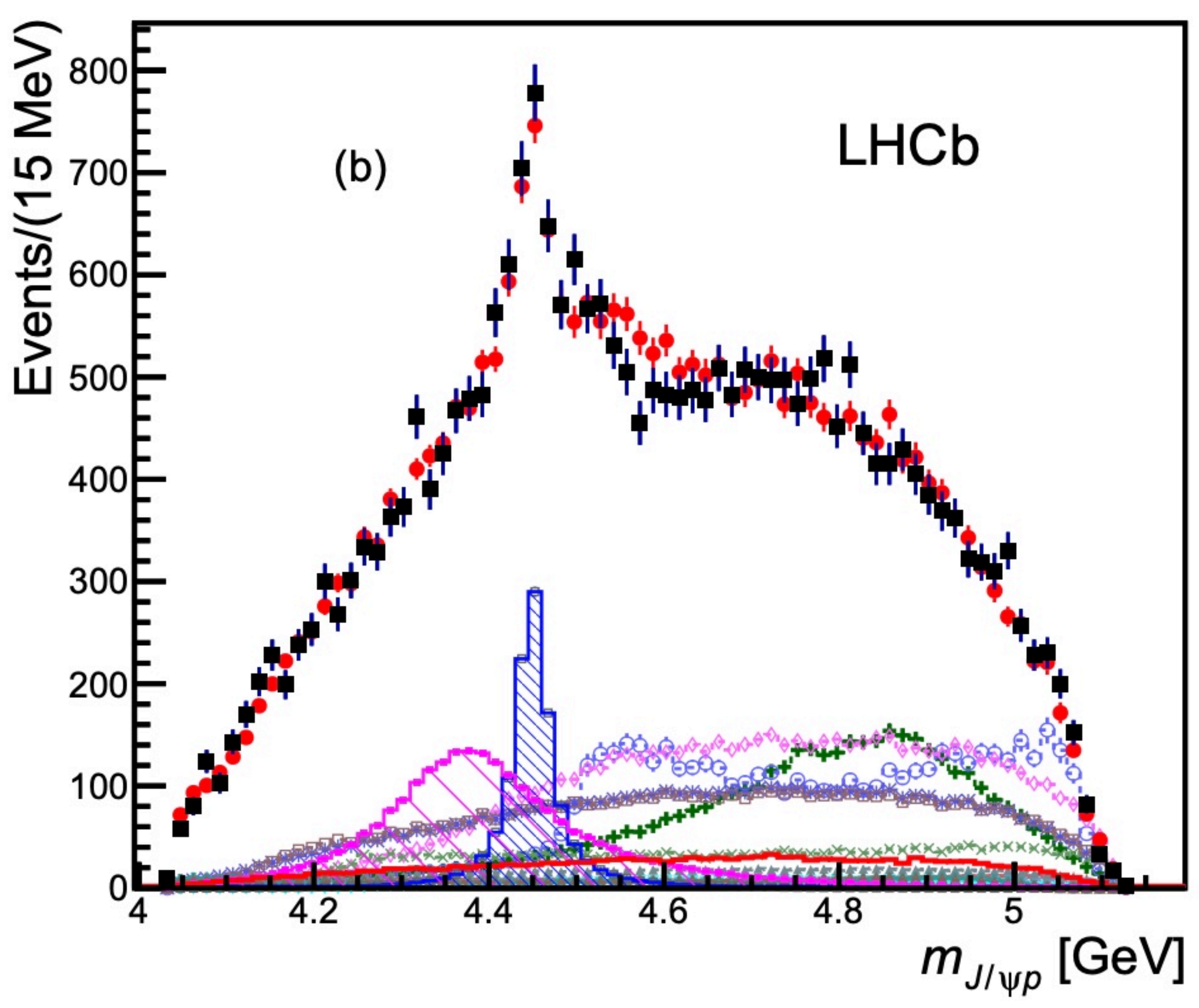
Diagonal matrix of phase-space factors $\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$

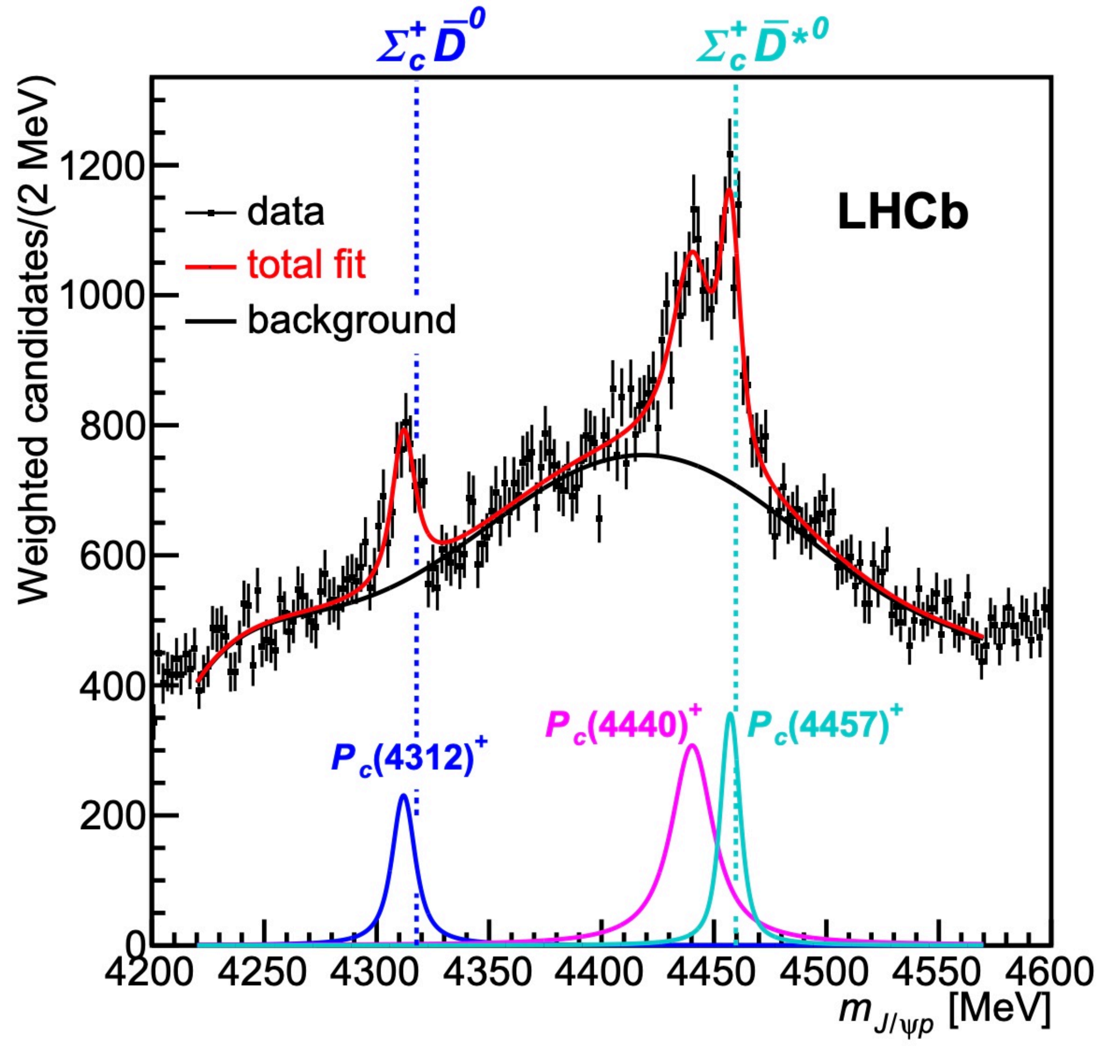
Infinite-volume scattering matrix

Finite volume information $M(E_{cm}, L)$

- Resonances/bound states are formally defined as poles in scattering amplitudes.

Hidden-charm Pentaquarks





$P_c(4380)$

$P_c(4450)$ → $P_c(4440)$, $P_c(4457)$

$P_c(4312)$

R. Aaij et al. (LHCb), Phys. Rev. Lett. 115, 072001 (2015)

R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 222001 (2019)

Theory interpretations:

- Molecule bound states
- Compact pentaquark states
- ……

$\Sigma_c^{(*)}\bar{D}^{(*)}$ molecules:

$\Sigma_c\bar{D}, J^P = \frac{1}{2}^-, P_c(4312)$

$\Sigma_c\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-), P_c(4440)/P_c(4457)$

$\Sigma_c^*\bar{D}, J^P = \frac{3}{2}^-, \quad \Sigma_c^*\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-),$

Hidden-charm Pentaquarks

CLQCD

$\Sigma_c \bar{D}$ and $\Sigma_c \bar{D}^*$ scattering$(J^P = \frac{1}{2}^-)$:

- Five operators:

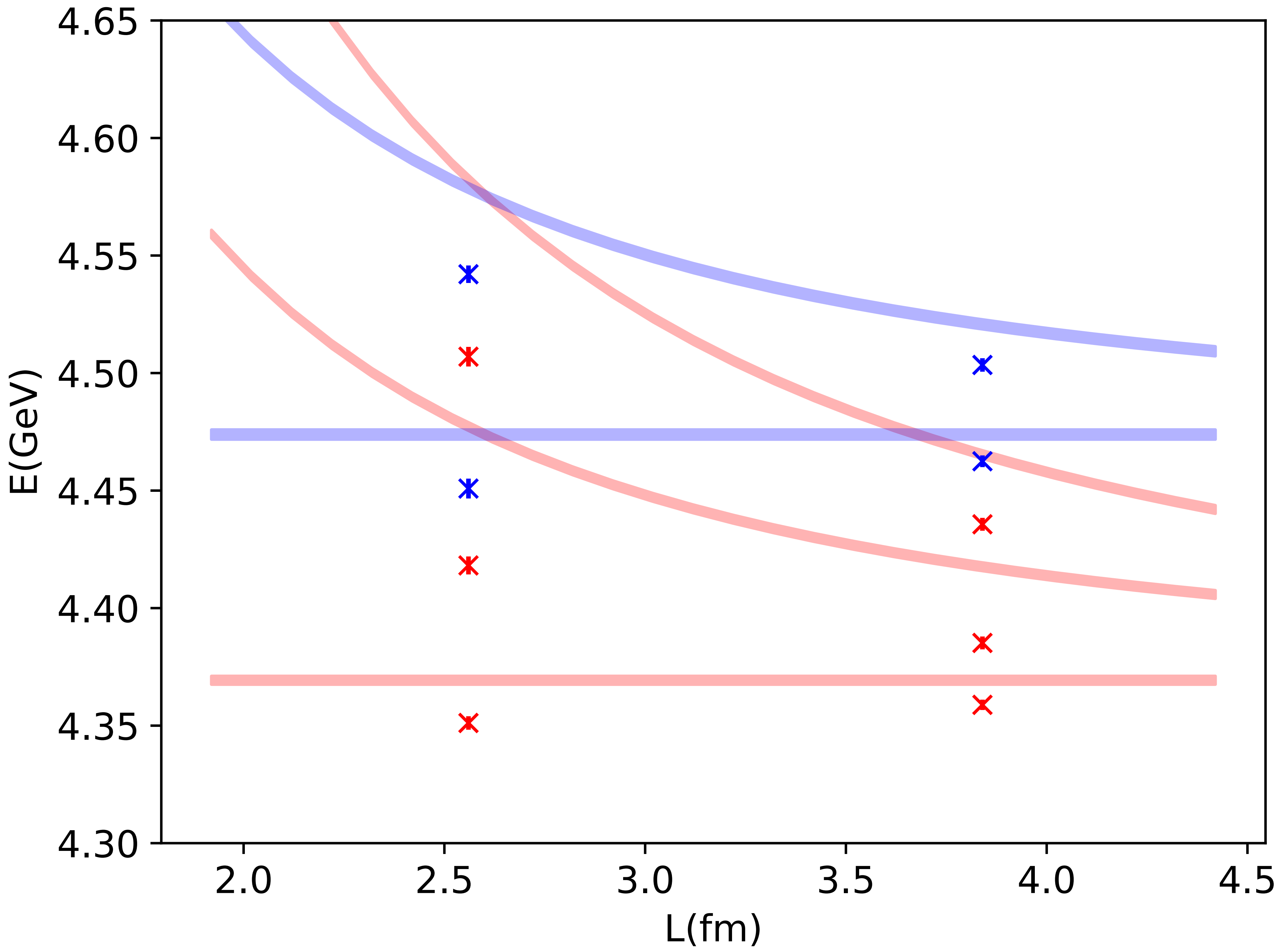
$\mathcal{O}_1 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = 0)$

$\mathcal{O}_2 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = 1)$

$\mathcal{O}_3 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = \sqrt{2})$

$\mathcal{O}_4 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (|\mathbf{p}| = 0)$

$\mathcal{O}_5 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (|\mathbf{p}| = 1)$



- The finite-volume energies lie below the free energies, indicating rather strong attractive interactions.

Hidden-charm Pentaquarks

CLQCD

Scattering amplitude:

$$T \sim \frac{1}{p\cot\delta - ip}$$

Bound state pole:

$$p = i|p_B|$$

Effective range expansion:

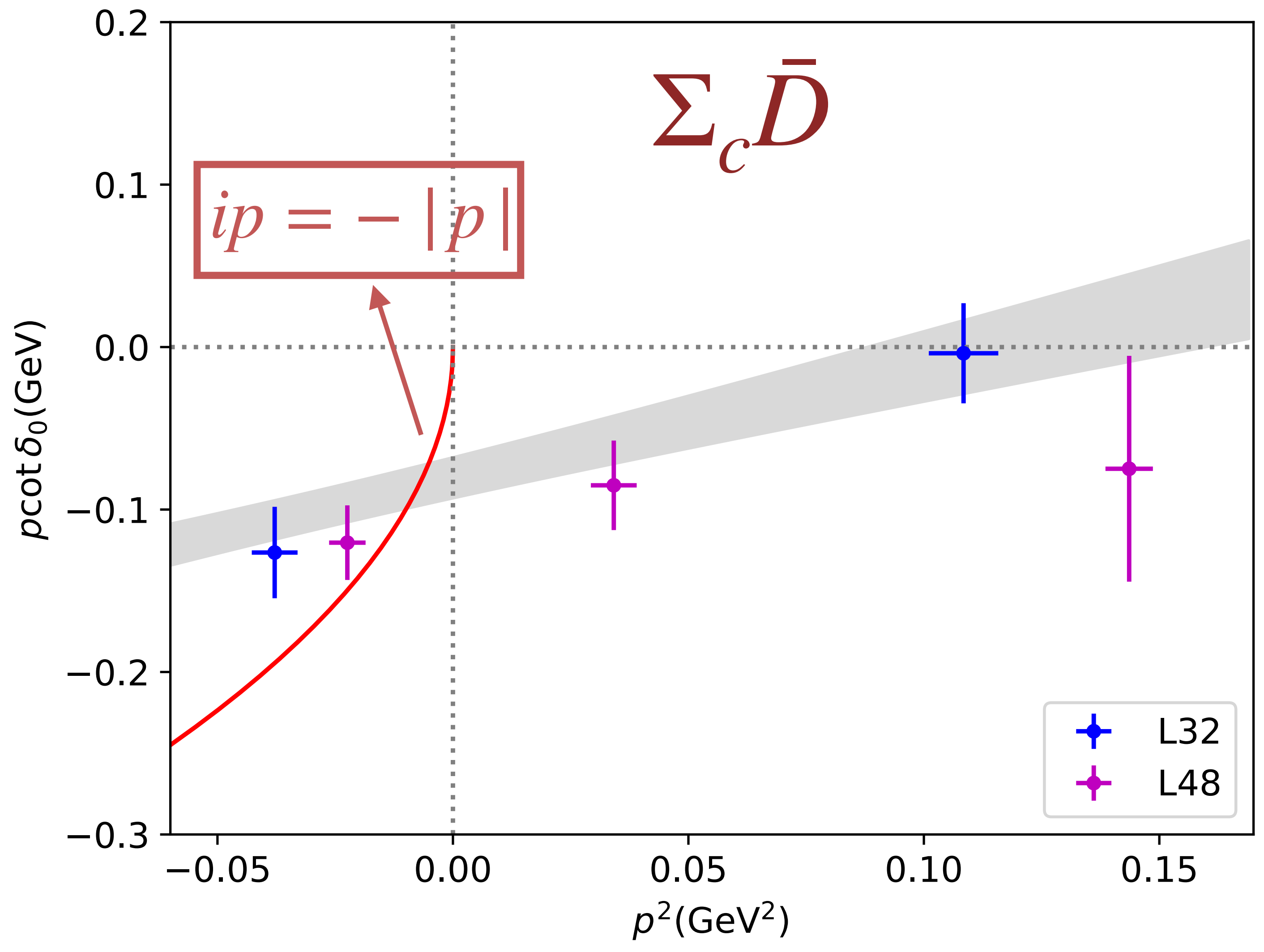
$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \cdots$$

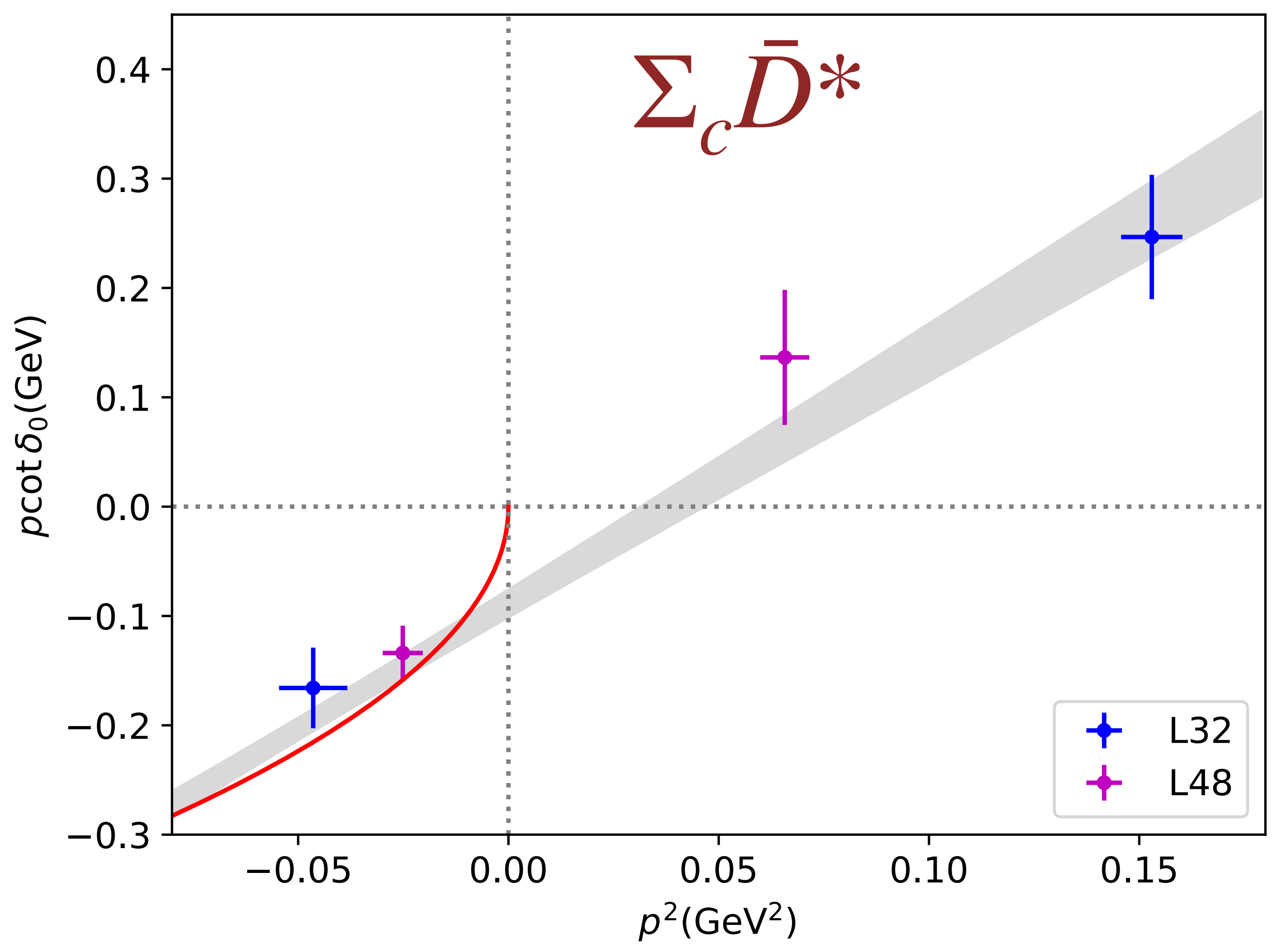
$\Sigma_c\bar{D}$: $P_c(4312)$?
$a_0 = -2.0(3)(5)\mathrm{fm}$
$E_B = 6(2)(2)\mathrm{MeV}$

Lüscher formula:

$$p\cot\delta(p) = \frac{2Z_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

$\Sigma_c\bar{D}^*$: $P_c(4440)/P_c(4457)$?
$a_0 = -2.3(5)(1)\mathrm{fm}$
$E_B = 7(3)(1)\mathrm{MeV}$



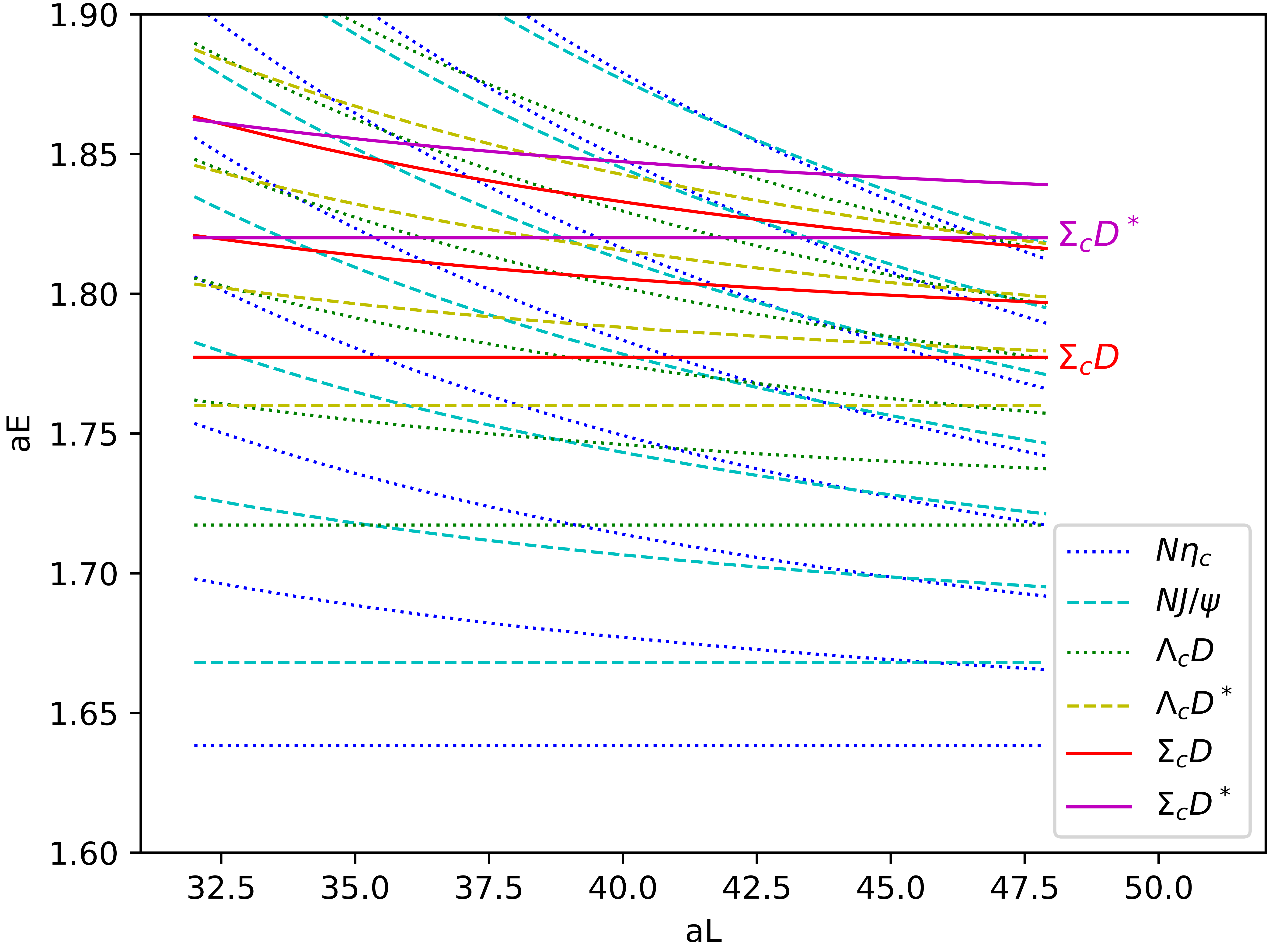


Hidden-charm Pentaquarks

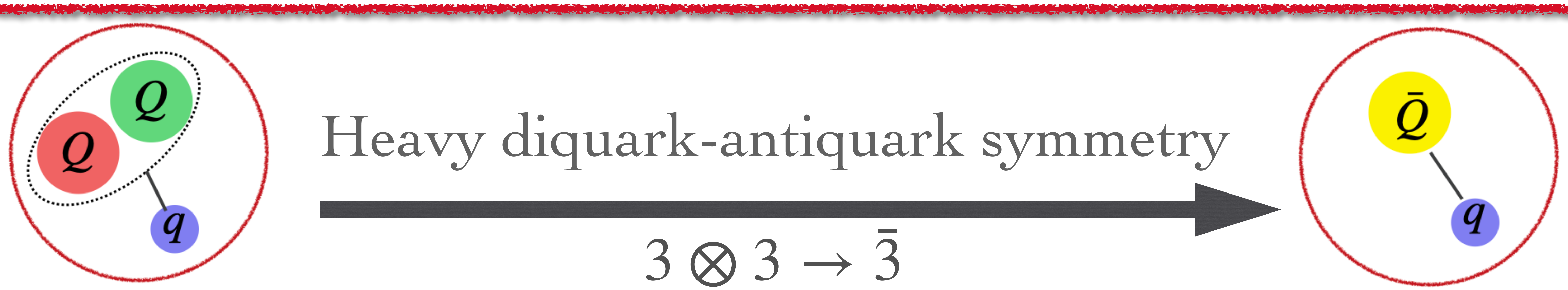
Coupled channels: $\eta_c N, J/\psi N, \Lambda_c \bar{D}, \Lambda_c \bar{D}^*, \Sigma_c \bar{D}, \Sigma_c \bar{D}^*$

Non-interacting energy levels:

$$E_{free} = \sqrt{m_1^2 + p_1^2} + \sqrt{m_1^2 + p_2^2}$$



Doubly charmed Pentaquarks



$I = 1/2$	$\Omega_{cc}\bar{K}$	$D_s K$
$I = 1$	$\Xi_{cc}K$	$D\bar{K}$
$I = 0$	$\Xi_{cc}K$	$D\bar{K}$
$I = 3/2$	$\Xi_{cc}\pi$	$D\pi$
$I = 0$	$\Xi_{cc}\bar{K},\ \Omega_{cc}\eta$	$DK,\ D_s\eta$
$I = 1$	$\Xi_{cc}\bar{K},\ \Omega_{cc}\pi,$	$DK,\ D_s\pi$
$I = 1/2$	$\Xi_{cc}\pi,\ \Xi_{cc}\eta,\ \Omega_{cc}K$	$D\pi,\ D\eta,\ D_s\bar{K}$

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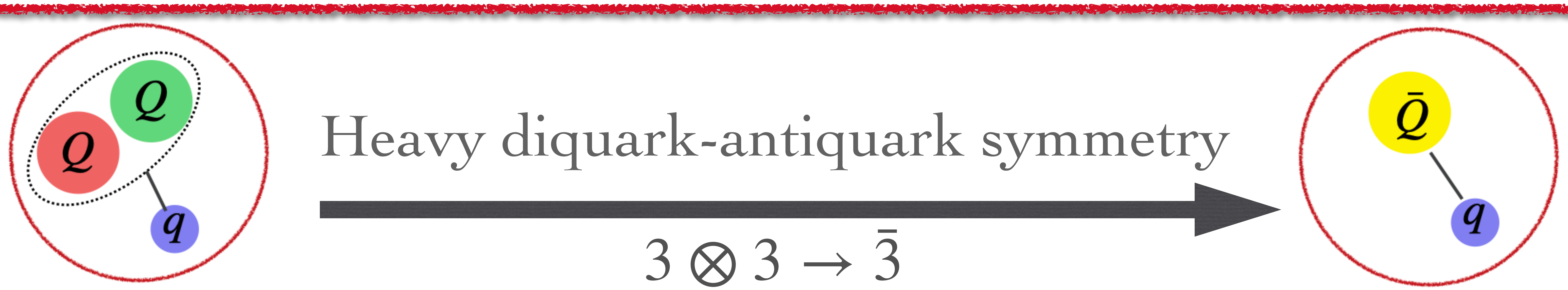

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$I = 1$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$,	DK, $D_s\pi$
$I = 1/2$	$\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$	$D\pi$, $D\eta$, $D_s\bar{K}$

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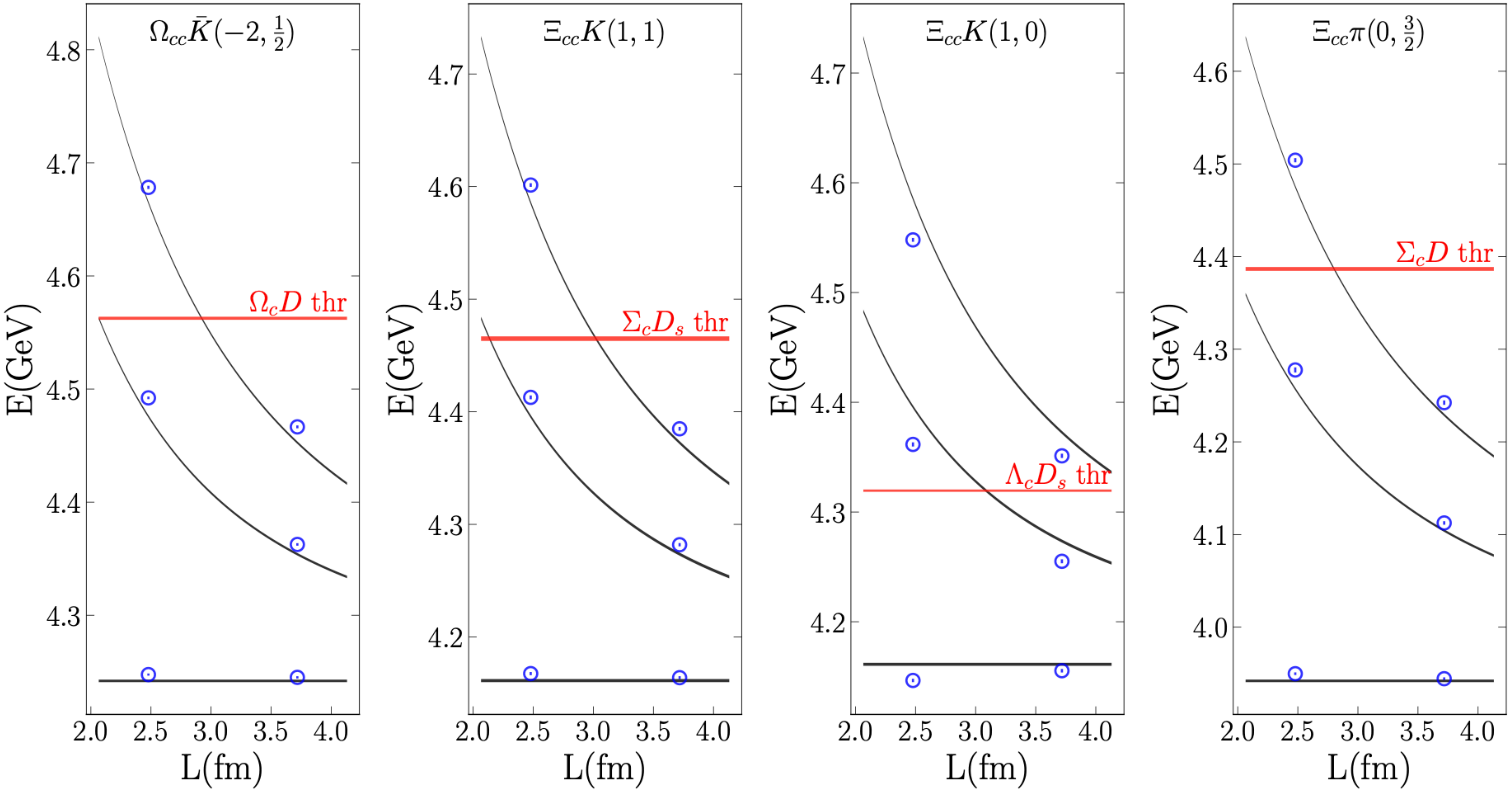

Doubly charmed Pentaquarks

- Scattering lengths from lattice QCD and BChPT

(S, I)	Processes	$M_\pi \sim 300$ MeV	$M_\pi \sim 210$ MeV	EOMS	HB
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	$-0.161(20)$	$-0.136(12)$	$-0.09^{+0.12}_{-0.13}$	-0.20(1)
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	$-0.177(23)$	$-0.212(14)$	-0.60 ± 0.13	$-0.25(1)$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	$0.63(10)$	$0.694(90)$	1.03 ± 0.19	$0.92(2)$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	$-0.140(15)$	$-0.143(24)$	-0.16 ± 0.02	$-0.10(2)$

Doubly charmed Pentaquarks

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Doubly charmed Pentaquarks

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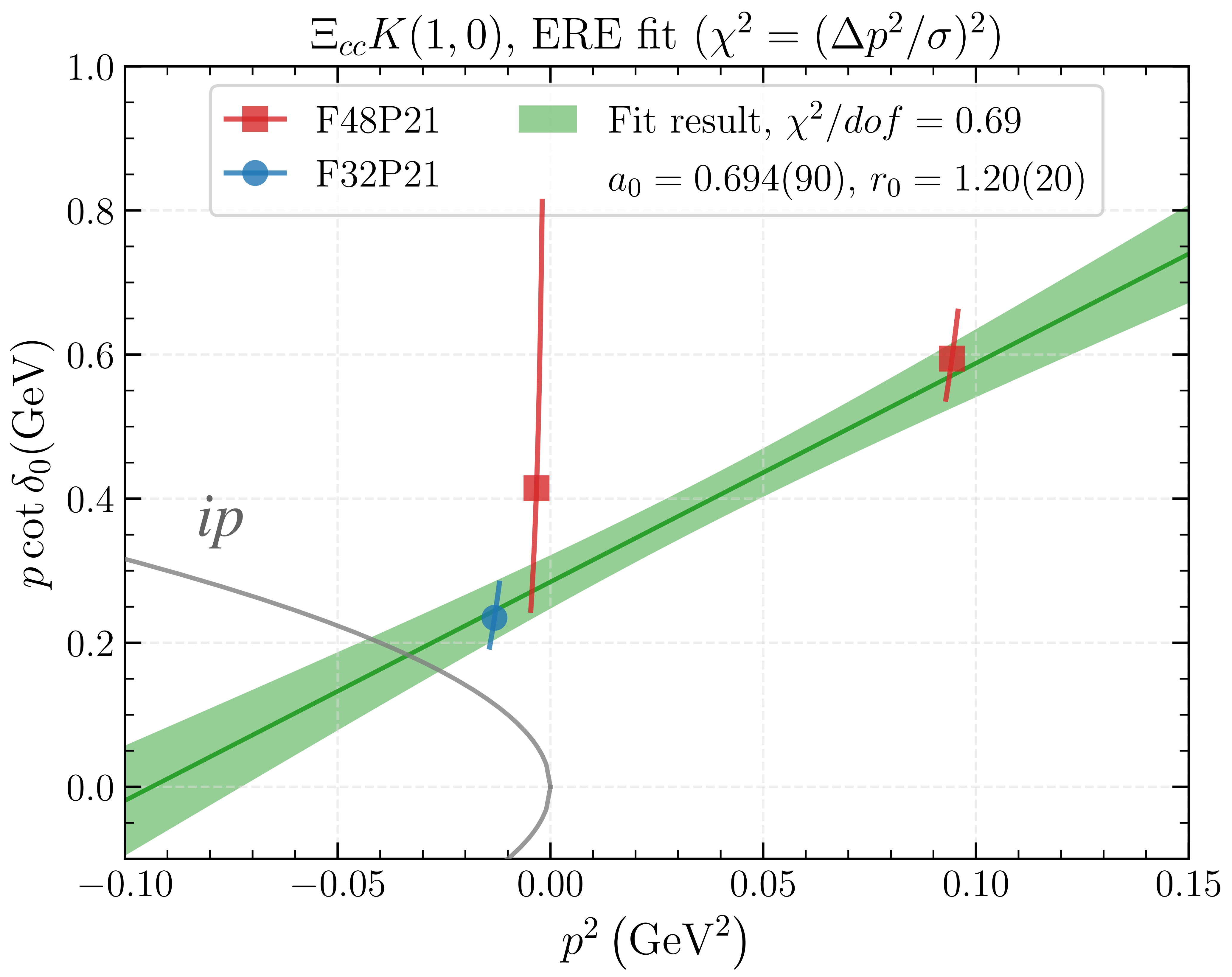
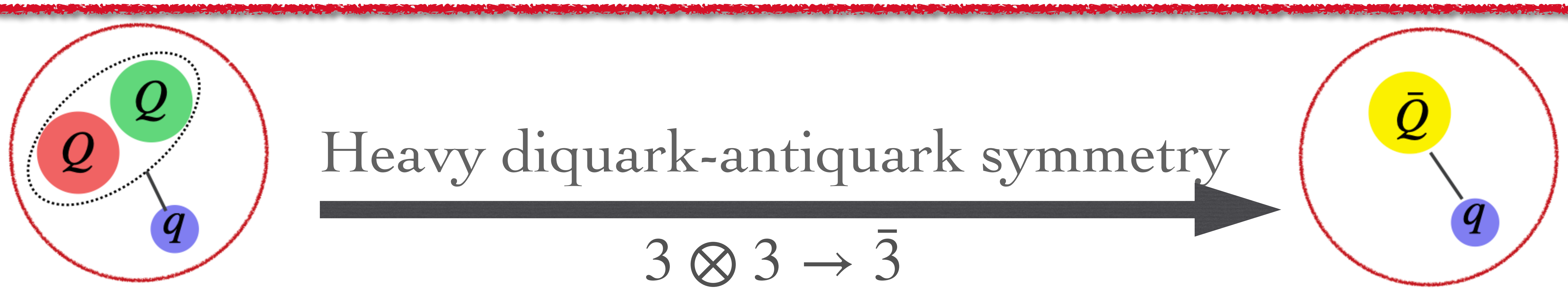
Luscher's formula:

$$p\cot\delta(p) = \frac{2Z_{00}(1; (\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$



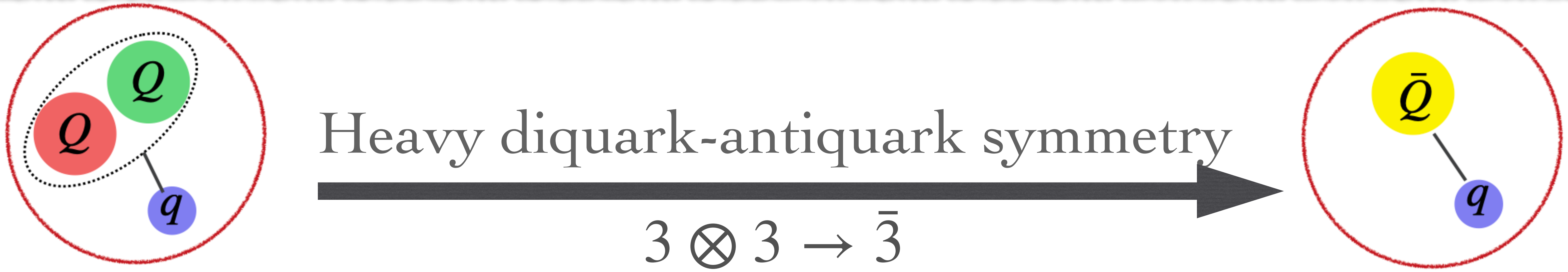
$m_\pi = 210\,\mathrm{MeV}$

Doubly charmed Pentaquarks



$I = 1/2$	$\Omega_{cc}\bar{K}$	**Repulsive**	$D_s K$
$I = 1$	$\Xi_{cc}K$	**Repulsive**	$D\bar{K}$
$I = 0$	$\Xi_{cc}K$	**Attractive, virtual pole**	$D\bar{K}$
$I = 3/2$	$\Xi_{cc}\pi$	**Repulsive**	$D\pi$
$I = 0$	$\Xi_{cc}\bar{K}, \Omega_{cc}\eta$		$DK, D_s\eta$
$I = 1$	$\Xi_{cc}\bar{K}, \Omega_{cc}\pi,$		$DK, D_s\pi$
$I = 1/2$	$\Xi_{cc}\pi, \Xi_{cc}\eta, \Omega_{cc}K$		$D\pi, D\eta, D_s\bar{K}$

Doubly charmed Pentaquarks



$I=1/2$	$\Omega_{cc}\bar{K}$	**Repulsive**	D_sK	
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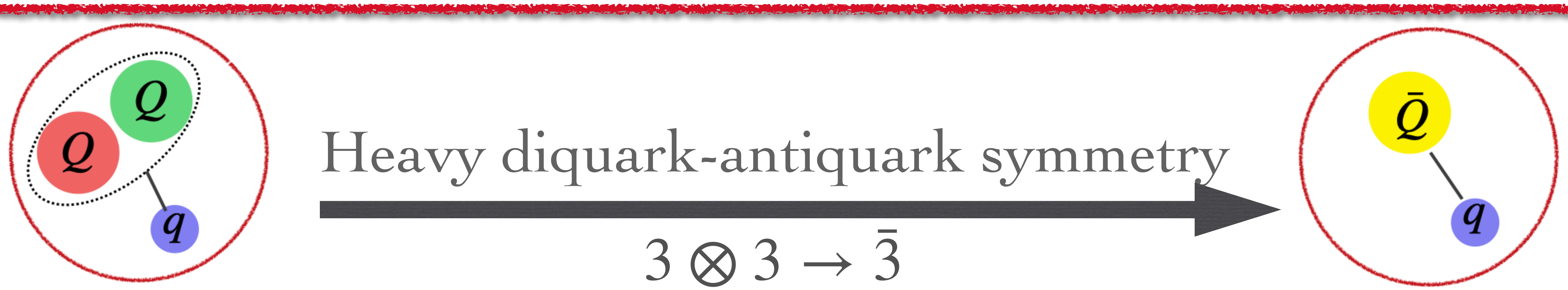
$I=0$	$\Xi_{cc}K$	**Attractive, virtual pole**	$D\bar{K}$	
$I=3/2$	$\Xi_{cc}\pi$	**Repulsive**	$D\pi$	
$I=0$	$\Xi_{cc}\bar{K},\ \Omega_{cc}\eta$	??	$DK,\ D_s\eta$	$D_{s0}^*(2317)$
$I=1$	$\Xi_{cc}\bar{K},\ \Omega_{cc}\pi,$		$DK,\ D_s\pi$	
$I=1/2$	$\Xi_{cc}\pi,\ \Xi_{cc}\eta,\ \Omega_{cc}K$	??	$D\pi,\ D\eta,\ D_s\bar{K}$	$D_0^*(2300)$

Doubly charmed Pentaquarks



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$I = 0$	$\Xi_{cc}K$	Attractive, virtual pole	$D\bar{K}$	
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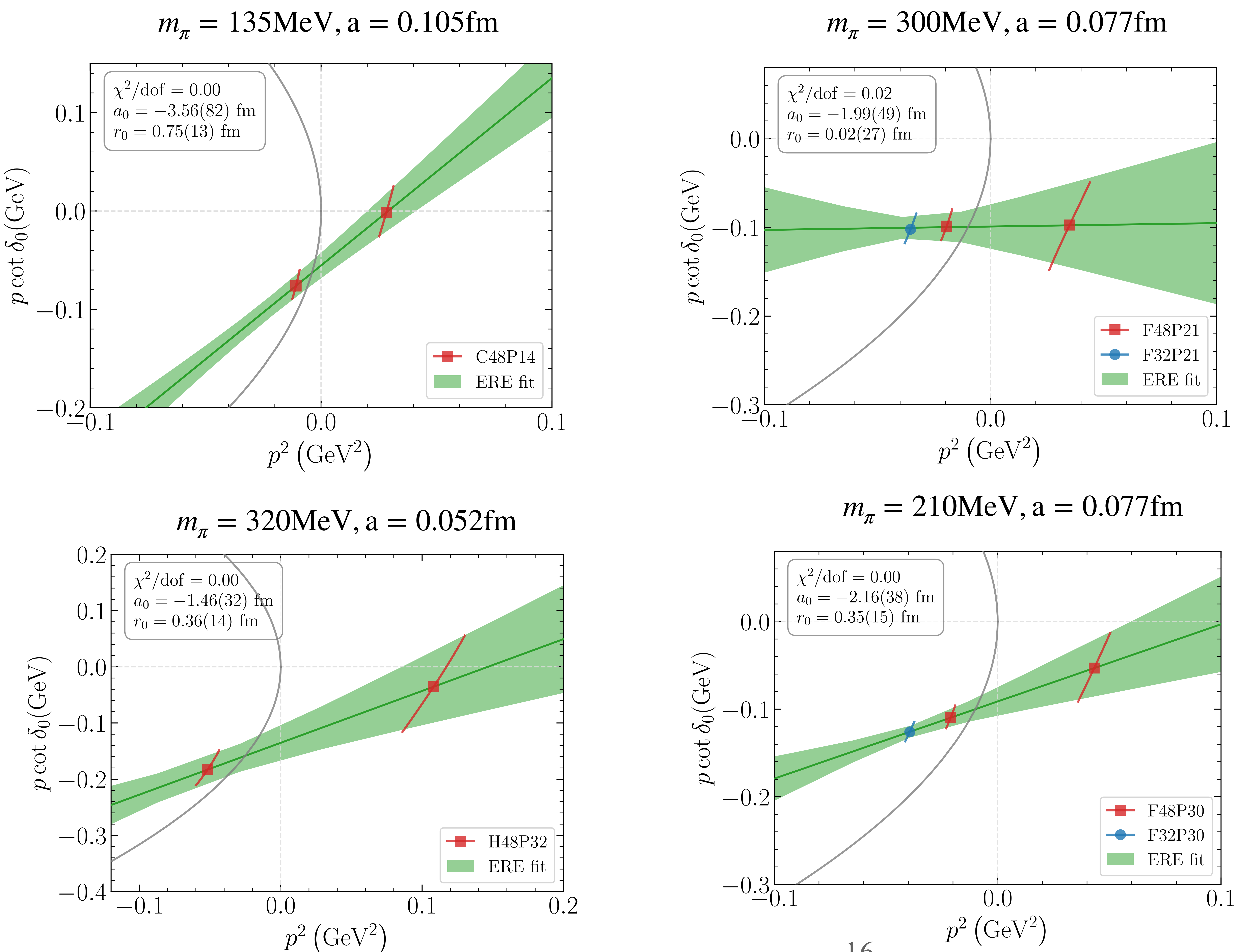
$I = 0$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\eta$??		DK, $D_s\eta$	$D_{s0}^*(2317)$
$I = 1$	$\Xi_{cc}\bar{K}$, $\Omega_{cc}\pi$,		DK, $D_s\pi$	
$I = 1/2$	$\Xi_{cc}\pi$, $\Xi_{cc}\eta$, $\Omega_{cc}K$??		$D\pi$, $D\eta$, $D_s\bar{K}$	$D_0^*(2300)$

Doubly charmed Pentaquarks



$I = 0\ \ \Xi_{cc}\bar{K}$

A bound state pole is found at various pion masses and lattice spacings.

Summary

- $J^P = \frac{1}{2}^-$ $\Sigma_c\bar{D}$ and $\Sigma_c\bar{D}^*$ scattering are studied, bound states are found in both channels; coupled channels need to be considered in the future.

- The interactions between the doubly charmed baryon (Ξ_{cc}, Ω_{cc}) and the light psuedoscalar mesons $(\pi, K, \bar{K})$ resemble the interactions beteween (D, D_s) and $(\pi, K, \bar{K})$; a bound state is found in the $I = 0$ $\Xi_{cc}\bar{K}$ channel.

Thanks!