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Hadronic production of fully charm tetraquarks with soft gluon resummation

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第6届LHCb前沿物理研讨会 2026.05.22-25@广州

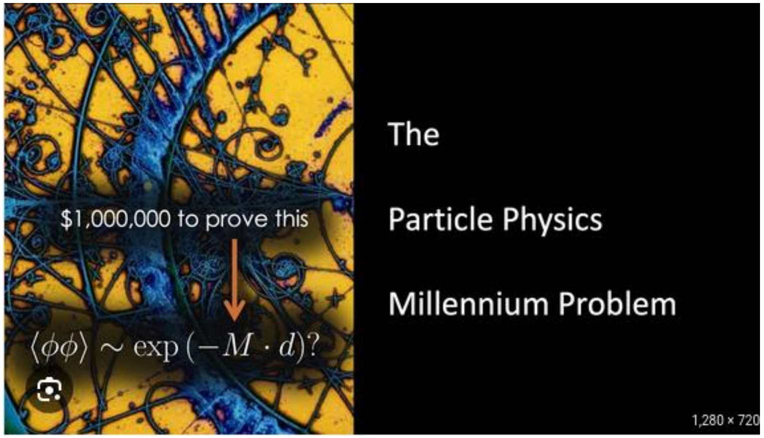
Color confinement

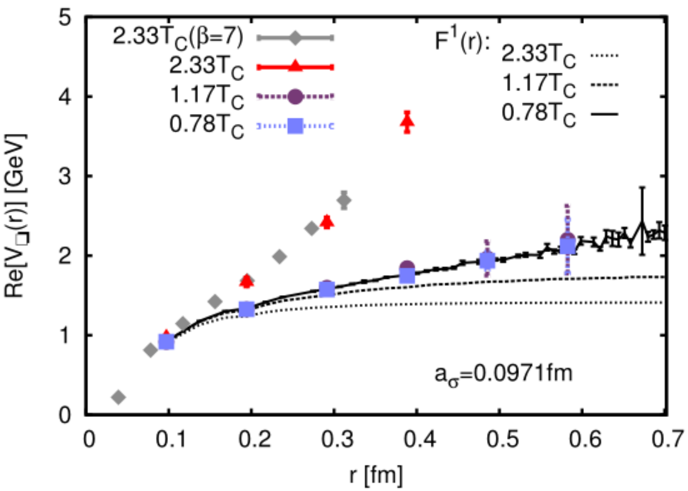
- **One of Seven Millennium Prize Problems**

- **Distinct from other fundamental interactions; Its understanding is not clear**

1)Quark confining potential is not derived analytically

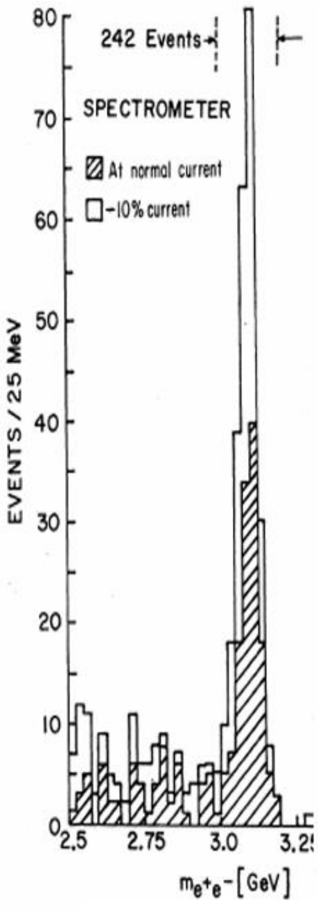
2)Heavy quark system provides a vivid picture to the confining mechanism; linear confining potential is support from Lattice QCD

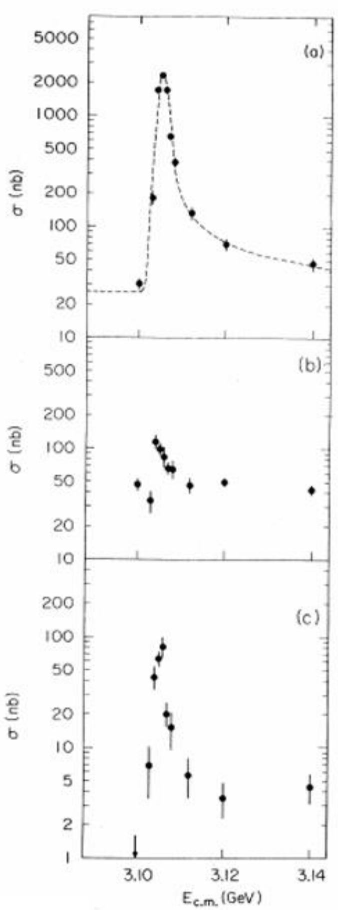


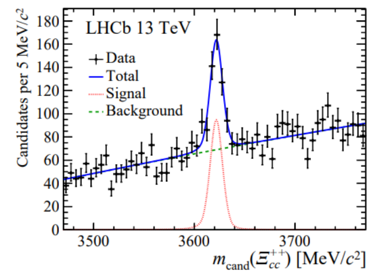


arXiv, 1108.1579

Charm family spectrum







gure 3: Invariant mass distribution of $\Lambda_c^+ K^- \pi^+ \pi^+$ candidates with fit projections overlaid.

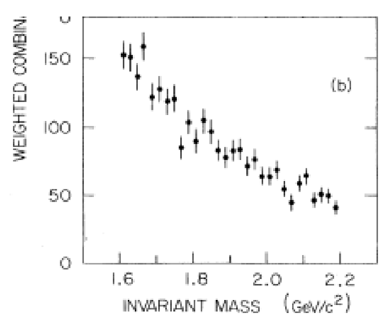
$\Omega_{ccc} \quad (CCC)$???

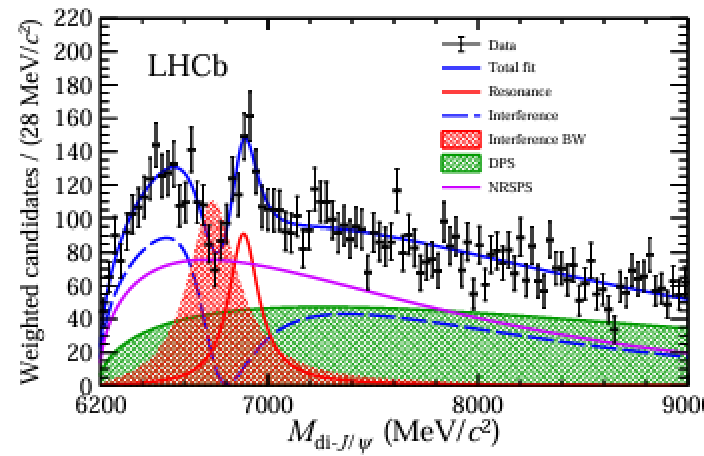
$T_{3c} \quad (CC\bar{C}\bar{q})$???

$\Xi_{cc} \quad (CCq)$

2017

中国科学十大进展





$J/\psi \quad (C\bar{C})$  **(1976)**

$D \quad (C\bar{q})$

$T_{4c} \quad (CC\bar{C}\bar{C})$

1974 by Ting and Richter

1976 at SLAC

LHCb, 2006.16957

Discovery of X(6900)

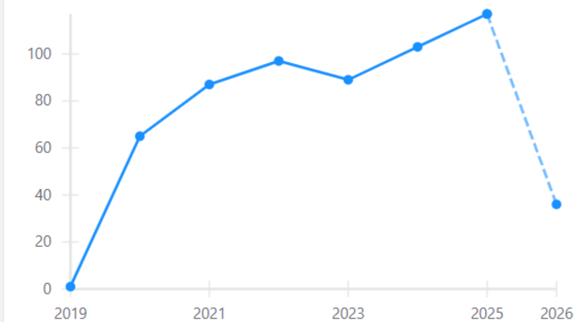
Observation of structure in the J/ψ -pair mass spectrum

LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) Show All(998)

Jun 30, 2020

11 pages
Published in: *Sci.Bull.* 65 (2020) 23, 1983-1993
Published: Dec 15, 2020
e-Print: 2006.16957 [hep-ex]
DOI: 10.1016/j.scib.2020.08.032 (publication)
Report number: CERN-EP-2020-115, LHCb-PAPER-2020-011
Experiments: CERN-LHC-LHCb
View in: CERN Document Server, HAL Science Ouverte, ADS Abstract Service

Citations per year



Abstract: (arXiv)
Using proton-proton collision data at centre-of-mass energies of $\sqrt{s} = 7, 8$ and $13\,\mathrm{TeV}$ recorded by the LHCb experiment at the Large Hadron Collider, corresponding to an integrated luminosity of $9\,\mathrm{fb}^{-1}$, the invariant mass spectrum of J/ψ pairs is studied. A narrow structure around $6.9\,\mathrm{GeV}/c^2$ matching the lineshape of a resonance and a broad structure just above twice the J/ψ mass are observed. The deviation of the data from nonresonant J/ψ-pair production is above five standard deviations in the mass region between 6.2 and $7.4\,\mathrm{GeV}/c^2$, covering predicted masses of states composed of four charm quarks. The mass and natural width of the narrow $X(6900)$ structure are measured assuming a Breit--Wigner lineshape.

Note: All figures and tables, along with any supplementary material and additional information, are available at https://cern.ch/lhcbproject/Publications/p/LHCb-PAPER-2020-011.html (LHCb public pages)

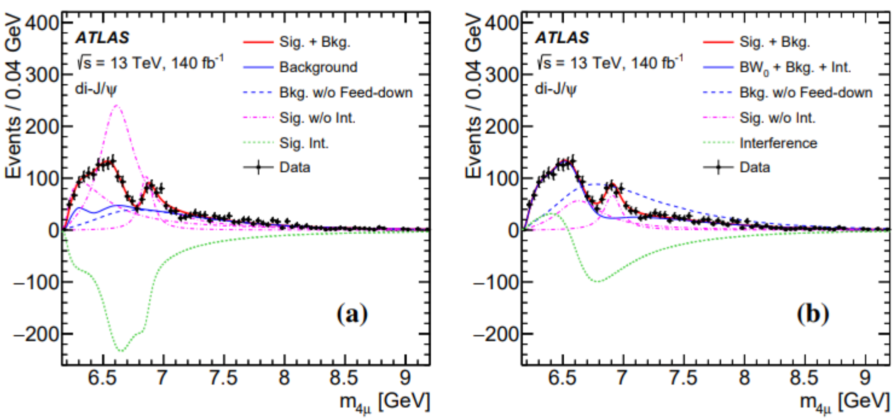
QCD | Exotics | Tetraquark | Spectroscopy | Quarkonium | Particle and resonance production | p p: scattering | p p: colliding beams | J/psi(3100): pair production | resonance: production | Show all (25)

The first discovery of X(6900): a fully charm tetraquark candidate

LHCb,2006.16957 (Science Bulletin)

Current spectrum from LHCb/ATLAS/CMS

Exp.	Fit	M_{BW_1}	Γ_{BW_1}	$M_{\mathrm{X}(6900)}$	$\Gamma_{\mathrm{X}(6900)}$	M_{BW_3}	Γ_{BW_3}
LHCb	No interf.	—	—	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	—	—
CMS	No interf.	$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 184$	$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$
LHCb	Interf.	6741 ± 6	288 ± 16	$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	—	—
CMS	Interf.	6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$	6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}	7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}
ATLAS	Fit-A	$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$	$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	$7220 \pm 30^{+10}_{-40}$	$90 \pm 60^{+60}_{-50}$
ATLAS	Fit-B	$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$	$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	—	—



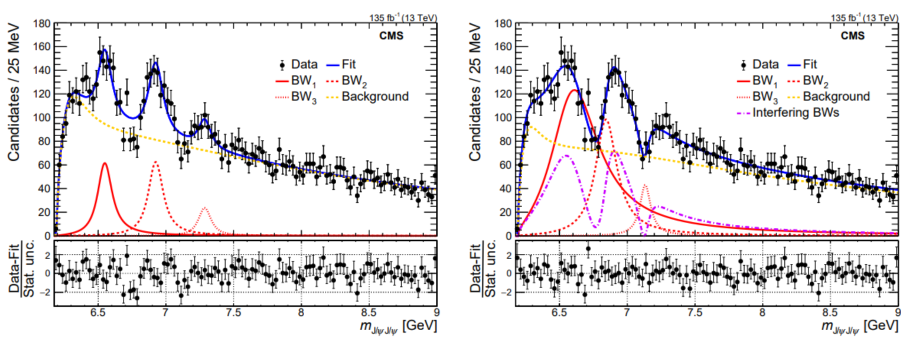
LHCb, 2006.16957; 9fb-1

rapidity range $2.0 < y^{J/\psi} < 4.5$

ATLAS, 2304.08962; 140fb-1 data;

P_t(mu1,2,3,4)>4,4,3,3GeV; |eta(mu1,2,3,4)|<2.5;

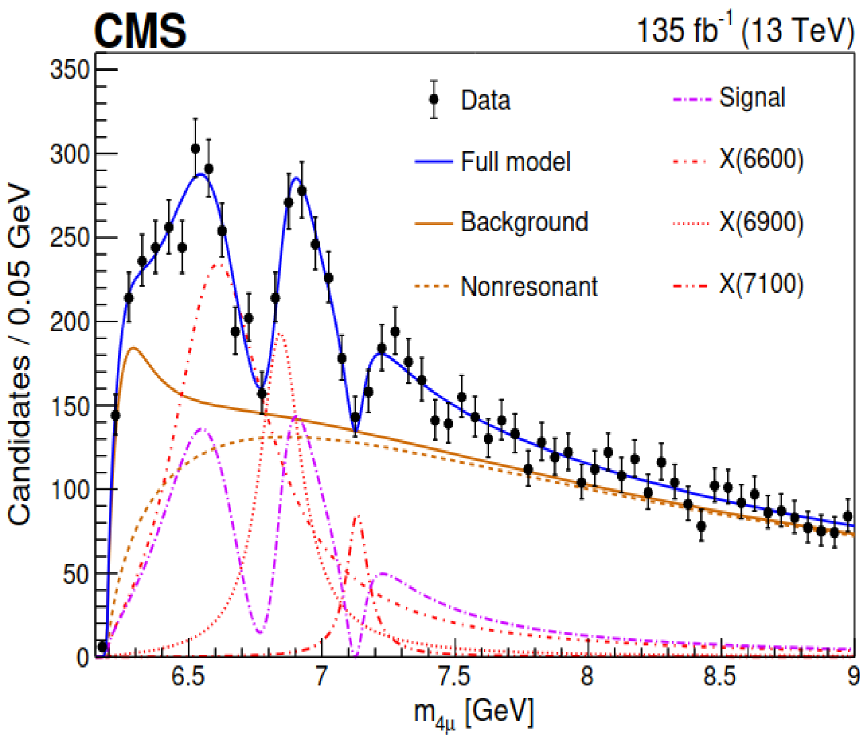
2.94(3.56)GeV<M(dimuon)<3.25(3.80)GeV

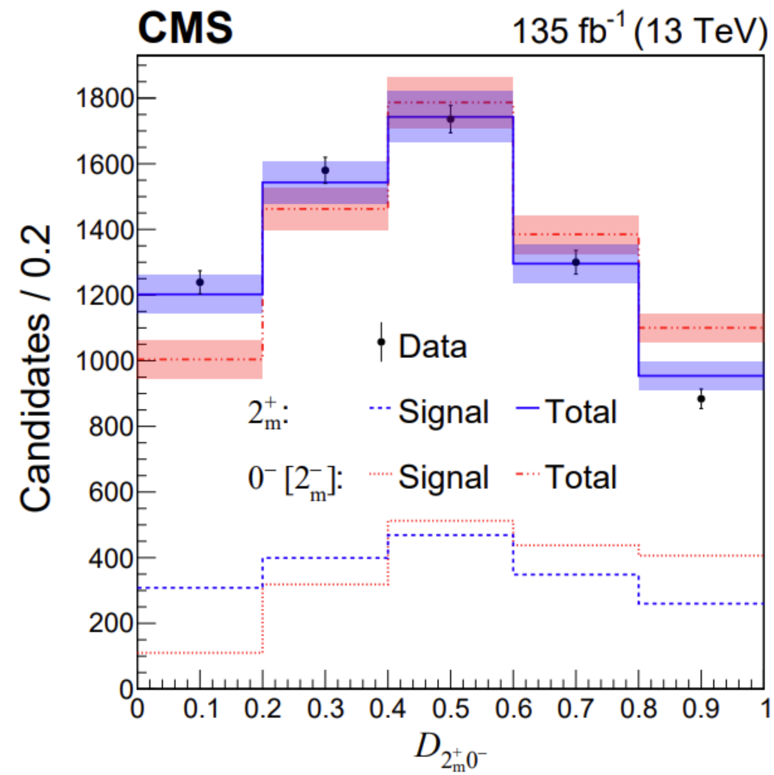


CMS, 2306.07164; 135fb-1 data;

P_t(muon)>2GeV;|eta(muon)|<2.4;

P_t(di muon)>3.5GeV;

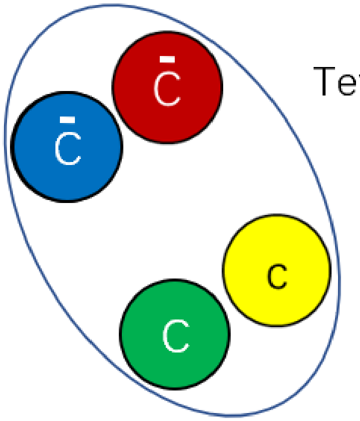




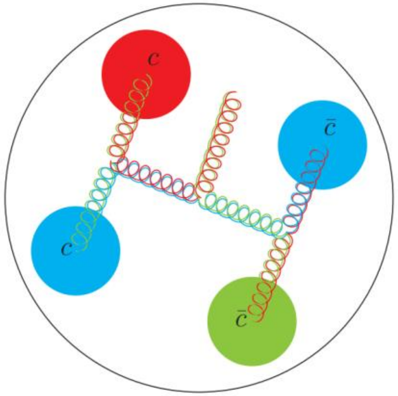
The spin-parity for three states is determined as 2^{++}

CMS,2506.07944 (Nature)

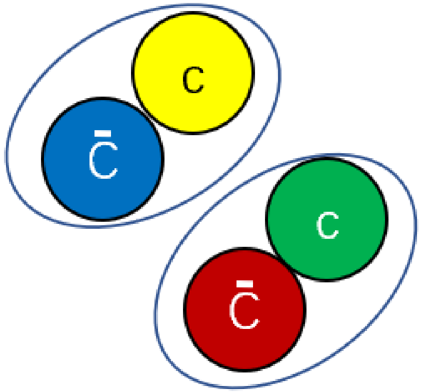
How to explain these exotic states?



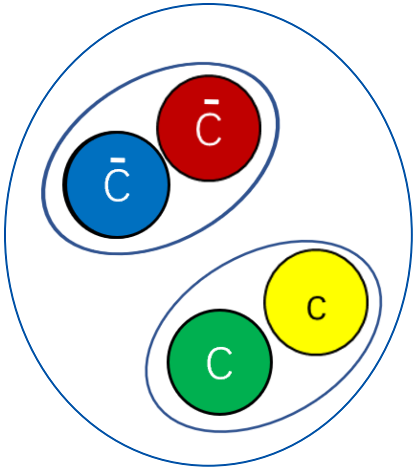
Tetraquark



Gluonic Tetracharm Hybrid

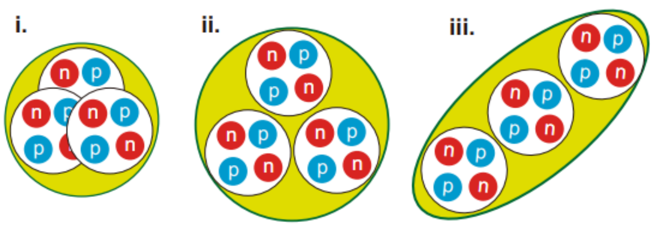


Charmonia Molecule



Diquark-antidiquark

c. Cluster model for C-12 nucleus



Similar to alpha cluster;
Different in diquark-antidiquark (color-confining) tetraquark

Previous theoretical studies

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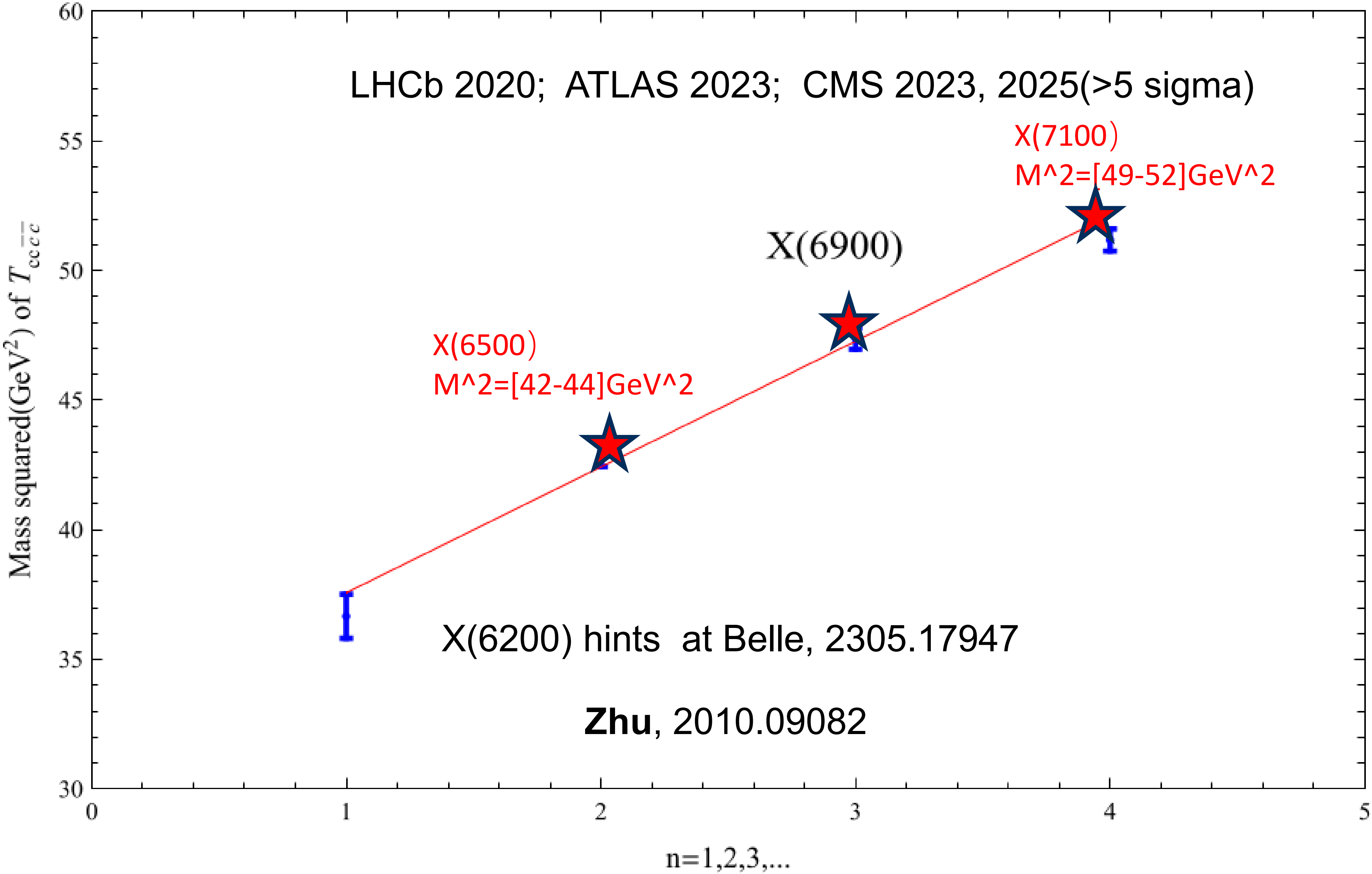
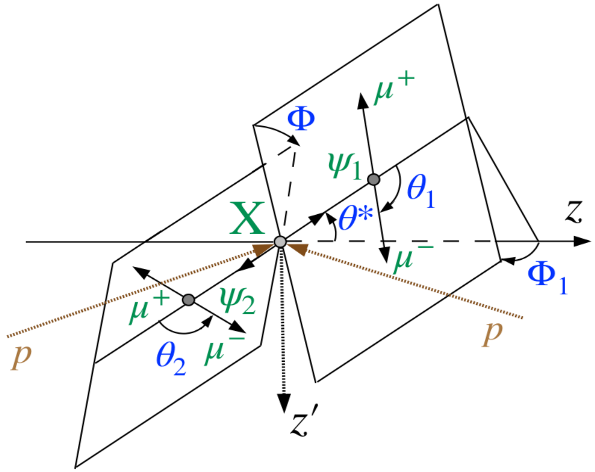
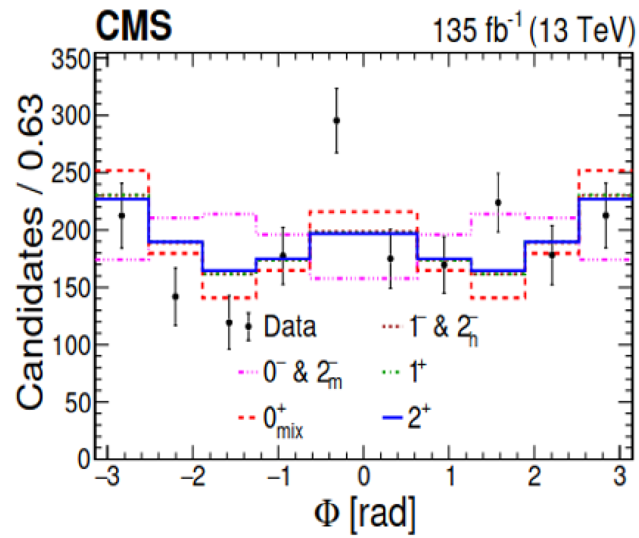
F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y.Zhang, arXiv:2009.08450; F. Feng et al, 2304.11142

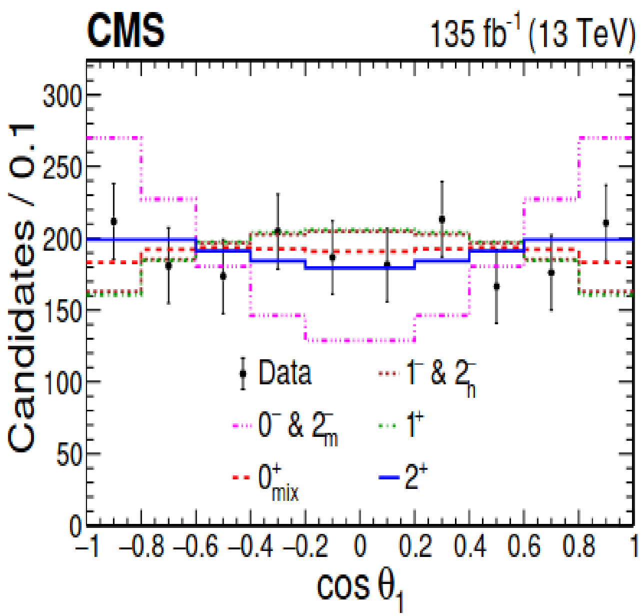
Current perspectives and challenges

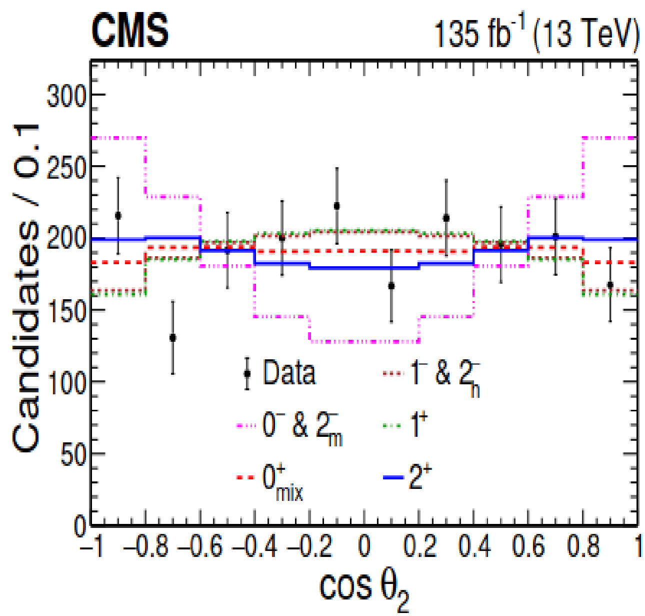


Fully charm tetraquark family: Linear Regge trajectories?



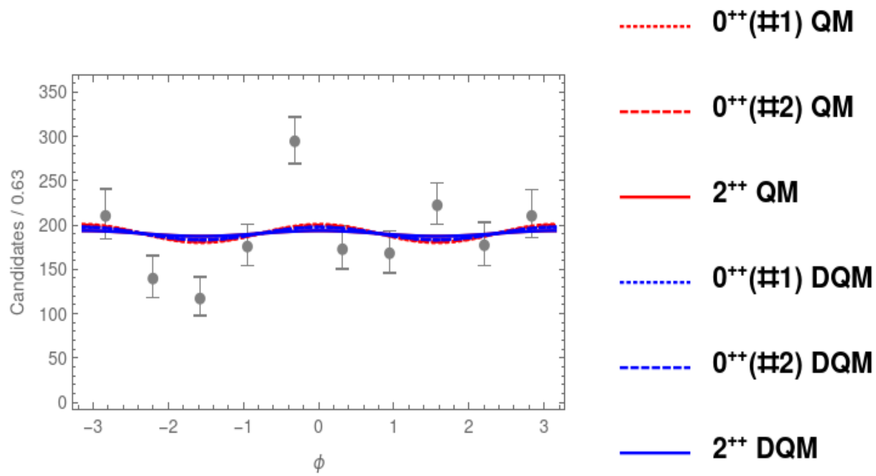


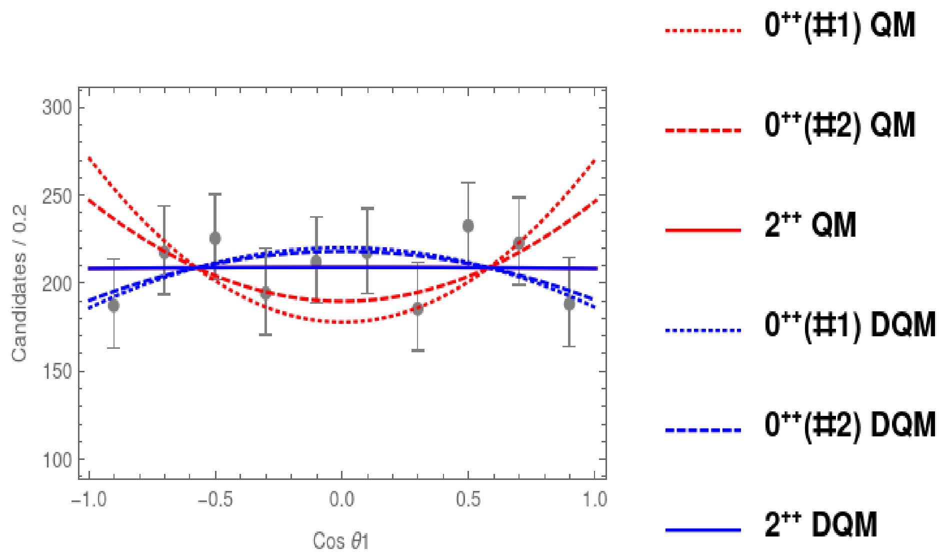




CMS data supports spin-parity 2^++;

However, derivation from the data and MC

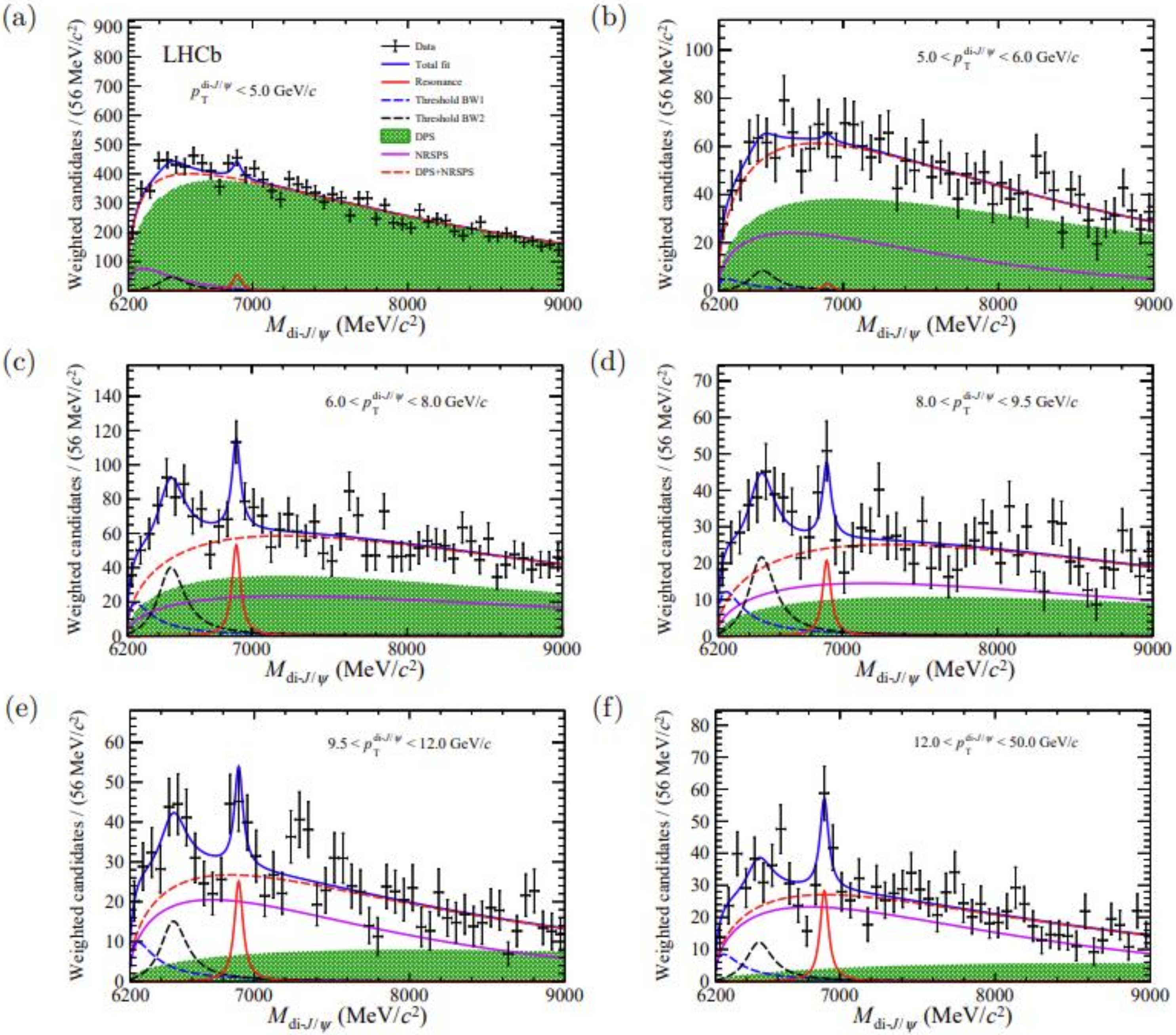




$$\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\Phi} = \frac{9P_H}{256\pi^2 M_T^2}\Big\{h_{00}^{00}\sin^2\theta_1\sin^2\theta_2 + \frac{1}{2}h_{11}^{11}(1+\cos^2\theta_1)(1+\cos^2\theta_2) + \frac{1}{8}h_{11}^{11}\sin^2\theta_1\sin^2\theta_2\cos 2\Phi + \frac{1}{4}h_{00}^{11}\sin 2\theta_1\sin 2\theta_2\cos\Phi\Big\}. \quad (8)$$

the derivation appears from the data and (diquark)quark model

Chen-Liu-Zhao-Zhong-**Zhu**-Zou:2412.13455



Significance	
$p_T^{di-J/\psi}$-threshold	$p_T^{di-J/\psi}$-binned
3.4 σ	6.0 σ
6.4 σ	6.9 σ
6.0 σ	6.5 σ
5.1 σ	5.4 σ

the signal significance is different for different transverse momentum cuts

Outline

- Transverse momentum resummation formulae for fully charm tetraquarks production at hadron-hadron colliders
- Extraction of LDMEs and comparison with experimental data
- Summary and Outlook

(1)Transverse momentum resummation formulae for fully charm tetraquarks production at hadron-hadron colliders

Hadronic production cross-section

QCD factorization:

$$\sigma\left(p(K_1)+p(K_2)\to T_{4c}(P)+X\right)=\sum_{ij}\int_0^1 dx_1\int_0^1 dx_2 f_{i/p}(x_1,\mu_F)f_{j/p}(x_2,\mu_F)$$
$$\times\,\hat{\sigma}_{i+j\to T_{4c}+X}(\mu_R,\mu_F,x_1,x_2,\hat{s}=x_1x_2S),$$

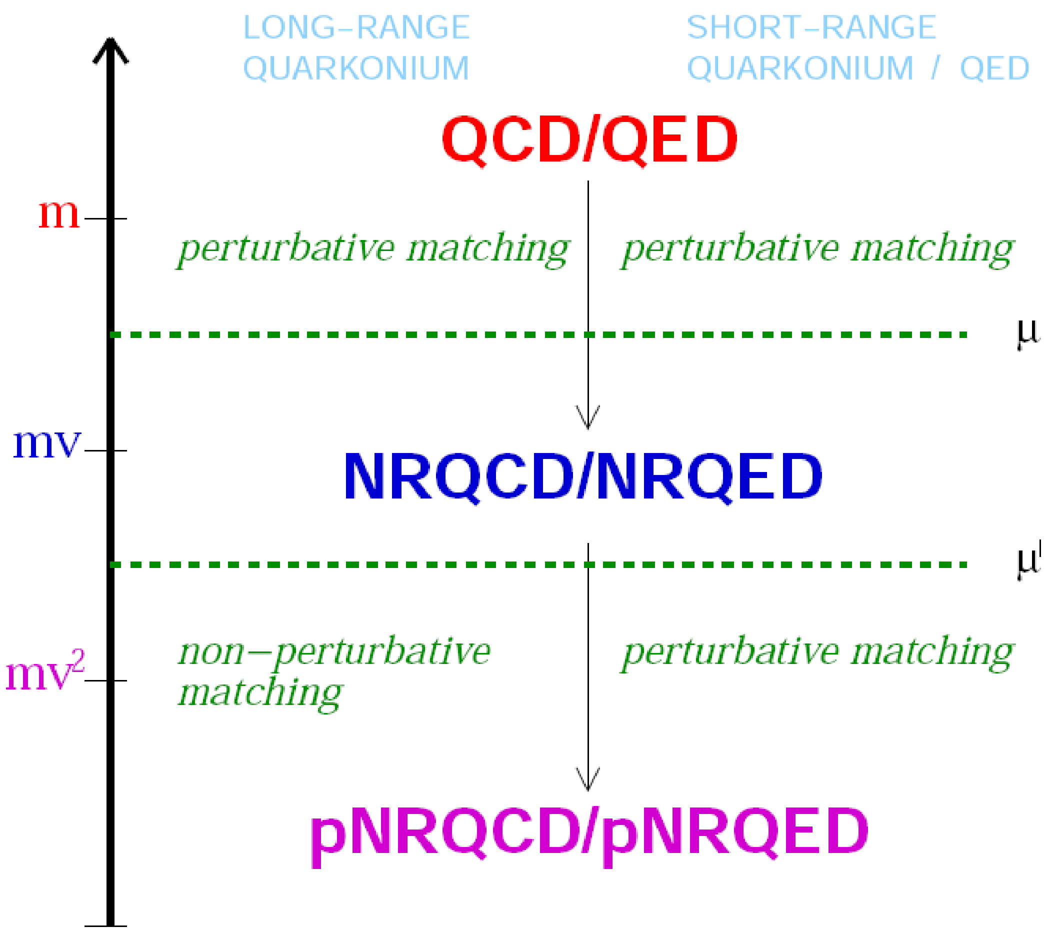
Partonic processes:

LO+NLO virtual: $g+g\to T_{4c}$ and $q+\bar{q}\to T_{4c}$.

NLO real: $g+g\to T_{4c}+g,\ q+\bar{q}\to T_{4c}+g$

$q+g\to T_{4c}+q,\ \bar{q}+g\to T_{4c}+\bar{q}$

NRQCD/pNRQCD factorization



$$\alpha_s(mv) \sim v$$

$v^2 \approx 0.1$ for the Υ

Bodwin-Braaten-Lapage 1995

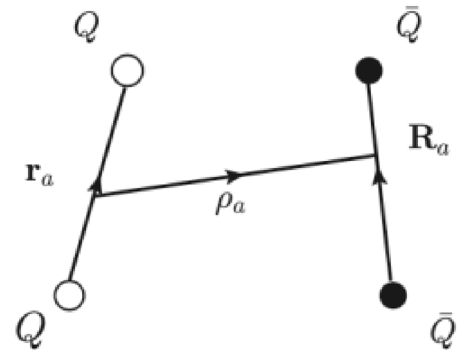
Pineda-Soto-Brambilla-Vairo 2000

Quark configurations from symmetry

Color-antisymmetry-symmetry basis:

$$\bar{\mathbf{3}} \otimes \mathbf{3} \to \mathbf{1} + \cdots \text{ and } \mathbf{6} \otimes \bar{\mathbf{6}} \to \mathbf{1} + \cdots.$$

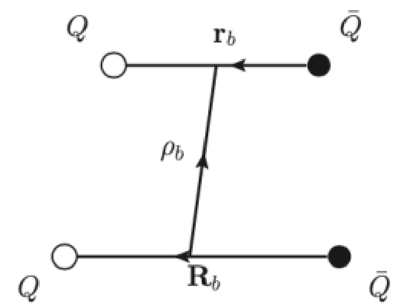
$$|\bar{\mathbf{3}}, \mathbf{3}\rangle = |(Q_1 Q_2)_{\bar{\mathbf{3}}}; (\bar{Q}_3 \bar{Q}_4)_{\mathbf{3}}\rangle,$$
$$|\mathbf{6}, \bar{\mathbf{6}}\rangle = |(Q_1 Q_2)_{\mathbf{6}}; (\bar{Q}_3 \bar{Q}_4)_{\bar{\mathbf{6}}}\rangle,$$



Color-singlet-octet basis:

$$\mathbf{1} \otimes \mathbf{1} \to \mathbf{1} + \cdots \text{ and } \mathbf{8} \otimes \mathbf{8} \to \mathbf{1} + \cdots.$$

$$|\mathbf{1}, \mathbf{1}\rangle = |(Q_1 \bar{Q}_3)_{\mathbf{1}}; (Q_2 \bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{8}, \mathbf{8}\rangle = |(Q_1 \bar{Q}_3)_{\mathbf{8}}; (Q_2 \bar{Q}_4)_{\mathbf{8}}\rangle,$$



1)They are equivalent according to Fierz Transformation when deal with local observables.

2)Heavy diquark is very different to light diquark(isospin symmetry): good diquark is 1^+ for heavy sector but 0^+ for light sector

Amplitudes calculation in NRQCD

$$\mathcal{A} = \sum_n \langle T_{4c} + X | c\bar{c}c\bar{c}[n] \rangle \langle c\bar{c}c\bar{c}[n] | \overline{\psi}_i \psi_j \overline{\psi}_k \psi_l | 0 \rangle \mathcal{T}_{ijkl}$$

$$\equiv \sum_n \langle T_{4c} + X | c\bar{c}c\bar{c}[n] \rangle \mathrm{Tr}\left[\mathcal{T} \mathcal{P}^{(n)} \right].$$

$$|\mathcal{A}|^2 = \sum_X | \sum_n \langle T_{4c} + X | c\bar{c}c\bar{c}[n] \rangle |^2 | \mathrm{Tr}\left[\mathcal{T} \mathcal{P}^{(n)} \right] |^2$$

$$= \sum_{m,m'} \langle 0 | O^{4c}_{m,n'} | 0 \rangle \left| \mathcal{M}^m \mathcal{M}^{m'\,\dagger} \right|,$$

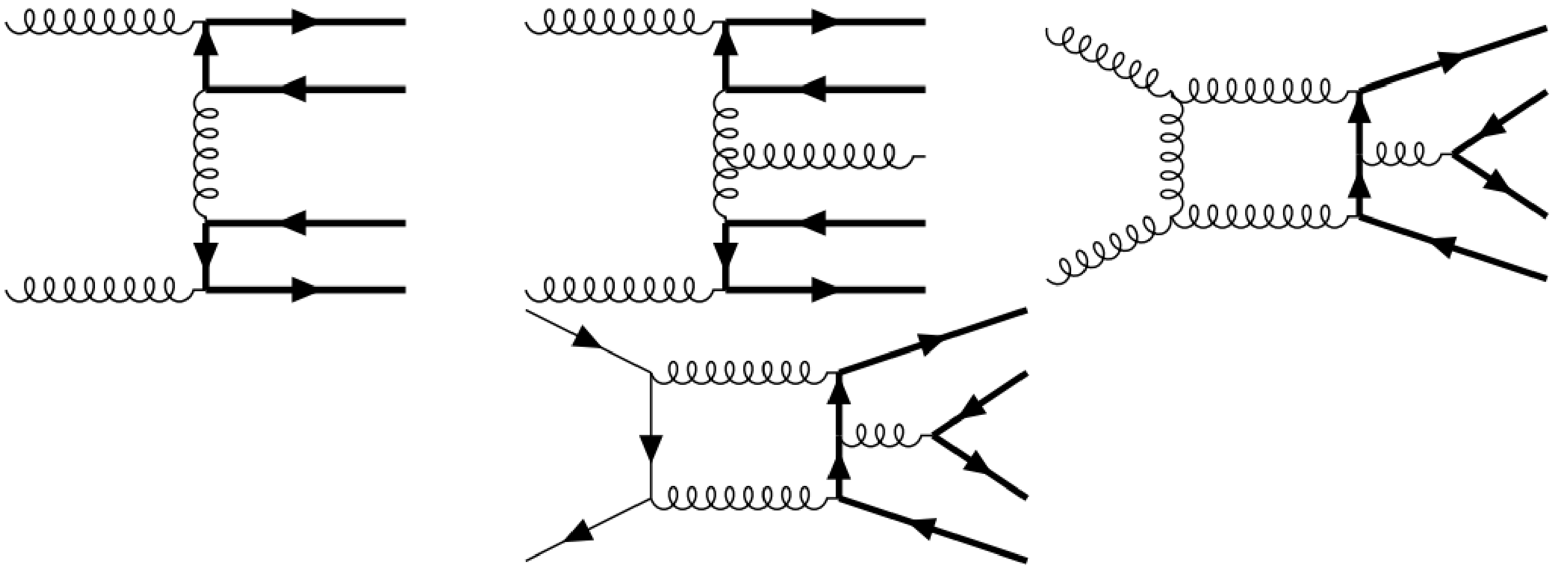
$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}} \otimes \mathbf{3}} = -\frac{1}{\sqrt{3}} \left[\psi_a^T \left(i\sigma^2 \right) \sigma^i \psi_b \right] \left[\chi_c^\dagger \sigma^i \left(i\sigma^2 \right) \chi_d^* \right] \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}} \otimes \mathbf{3}},$$

$$\mathcal{O}^{(2;ij)}_{\bar{\mathbf{3}} \otimes \mathbf{3}} = \left[\psi_a^T \left(i\sigma^2 \right) \sigma^m \psi_b \right] \left[\chi_c^\dagger \sigma^n \left(i\sigma^2 \right) \chi_d^* \right] \Gamma^{ij;mn} \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}} \otimes \mathbf{3}},$$

$$\mathcal{O}^{(0)}_{\mathbf{6} \otimes \bar{\mathbf{6}}} = \left[\psi_a^T \left(i\sigma^2 \right) \psi_b \right] \left[\chi_c^\dagger \left(i\sigma^2 \right) \chi_d^* \right] \mathcal{C}^{ab;cd}_{\mathbf{6} \otimes \bar{\mathbf{6}}},$$

Feng, Huang, Jia, Sang, Xiong and Zhang, arXiv:2009.08450;
Zhu, 2010.09082

Complete NLO calculation



LO:64(gg)，4(qqbar)

NLO Virtual:2008(gg)，170(qqbar)

NLO Real: 618(gg)， 98(qqbar, qg)

Exact IR cancellation and no additional renormalization at NLO

$$K^{\mathrm{LH3}}_{\mathrm{gg,virtual}} = \frac{3}{\epsilon^2} - \frac{1}{\epsilon}\left(3\log\left(\frac{\mu}{4m_c}\right)^2 - \frac{n_l}{3} + \frac{11}{2}\right) - \frac{3}{2}\log^2\left(\frac{\mu}{4m_c}\right)^2$$
$$+\left(\frac{11}{2} - \frac{2n_h + n_l}{3}\right)\log\left(\frac{\mu}{m_c}\right)^2 + \frac{4719}{256}\left(\mathrm{Li}_2\left(2\sqrt{2}-2\right) + \mathrm{Li}_2\left(-2\sqrt{2}-2\right)\right)$$
$$+\cdots$$

There are soft divergences when $k_g = 0$ or $z = 1$, and collinear divergences when $y_\theta = \cos\theta_{k_n k_g} = \pm 1$ with $k_n = k_1, k_2, k_T$.

$$\hat{\sigma}_{\mathrm{soft}}\ (i + j \to T_{4c} + k) = -C\frac{1}{2\epsilon_{\mathrm{IR}}}\delta(1-z)\frac{4^{-\epsilon}\Gamma(1-\epsilon)\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{B}_{ij}\left(z=1, y_\theta\right)$$
$$= \Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^\epsilon \frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left(\frac{1}{\epsilon_{\mathrm{IR}}^2} - \frac{\pi^2}{3}\right)C_{ij}$$

$$\hat{\sigma}_{\text{hard col. } y_\theta=\pm1}\left(i + j \to T_{4c} + k\right)$$
$$= -C\frac{4^{-\epsilon}}{2\epsilon_{\mathrm{IR}}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z, y_\theta = \pm 1\right)$$
$$= -C\frac{4^{-\epsilon}}{2\epsilon_{\mathrm{IR}}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z=1, y_\theta\right)\frac{b_{ij}^{\mathrm{collinear}}}{z^3}$$
$$= \Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^\epsilon\frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left[-2C_{ij}b_{ij}^{\mathrm{collinear}}\left(\left(\frac{1}{1-z}\right)_+\frac{1}{\epsilon_{\mathrm{IR}}} - 2\left(\frac{\log(1-z)}{1-z}\right)_+\right)\right]$$

$$\hat{\sigma}^{\mathrm{AP-CT}} = \frac{1}{\epsilon}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_R^2}{\mu_F^2}\right)^\epsilon\Gamma(1+\epsilon)\hat{\sigma}^{(0)}zP_{ij}(z),$$

The renormalization constant for four heavy quarks in a color singlet is 1 at NLO

Soft and collinear gluon radiation produces singularities

$$
\begin{aligned}
\frac{d\sigma}{dydP_\perp^2}\Big|^{NLO,R}_{Singular} &= \left(\frac{d\sigma}{dydP_\perp^2}\Big|^{NLO,R}\right)_{P_\perp\to 0} \\
&= \int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2} f_{i'}(x_1,\mu_F) f_{j'}(x_2,\mu_F)\frac{M^2}{S}\hat{\sigma}^{(0)}_{ij}\frac{\alpha_s(\mu_F)}{2\pi} \\
&\times\frac{1}{P_\perp^2}\Big[P_{jj'}(\xi_2)\delta_{ii'}\delta(1-\xi_1)+P_{i'i}(\xi_1)\delta_{j'j}\delta(1-\xi_2) \\
&-2C_{i'j'}\delta_{i'i}\delta_{j'j}\ln\left(\frac{P_\perp^2}{M^2}\right)\delta(1-\xi_1)\delta(1-\xi_2)\Big] .
\end{aligned}
$$

At low transverse momentum P_\perp, the soft and collinear gluon radiations generate the divergence, which should be resumed for a reliable prediction.

Transverse momentum resummation in various processes

$$\frac{d\sigma}{dM^2 dy dp_t^2} = \frac{8}{27}\frac{\alpha^2\alpha_s}{M^2}\frac{1}{p_T^2}\int_{x_a^{min}}^1 dx_a P^{DIS}\frac{1}{x_a - x_1}\left(1 + \frac{\tau^2}{(x_a x_b)^2} - \frac{x_T^2}{2x_a x_b}\right)$$

- with $$x_a^{min} = \frac{x_a x_2 - \tau}{x_a - x_1}$$

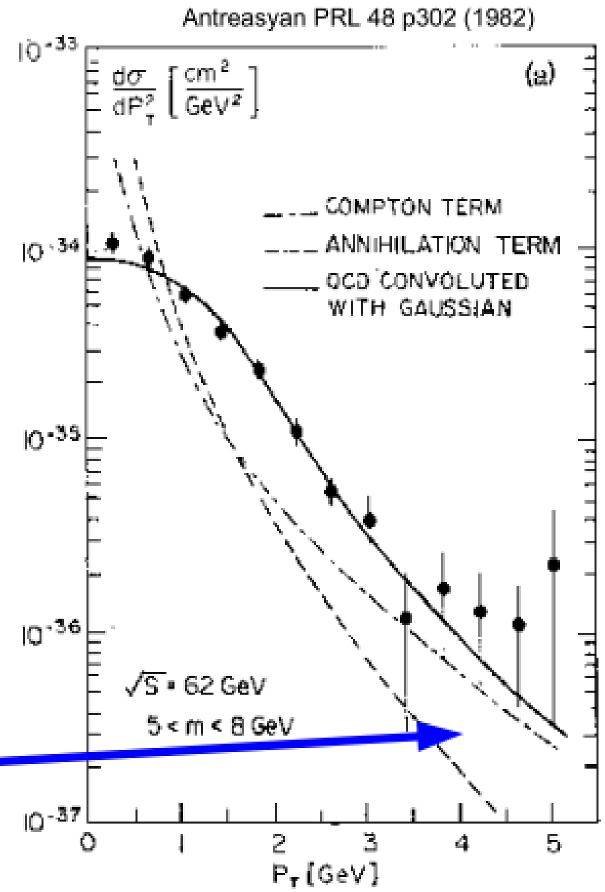
$$p_t^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$$

$$x_t = \frac{2p_t}{\sqrt{s}}$$

$$P^{DIS} = \sum e_q^2 \left(q_i(x_a, Q^2)\bar{q}_i(x_b, Q^2) + \bar{q}_i(x_a, Q^2)q_i(x_b, Q^2)\right)$$

- large pt

R. Field, Appl. of pQCD, p195 ff



Drell-Yan, vector boson production, DIS processes, Higgs production,…
Collins, Soper 81; Collins, Soper, Sterman 85
CP Yuan 1992; Ji-Ma-Yuan 2004; CS Li, HX Zhu, DY Shao, P Sun, Gao, Wang, Cao, Liu, Li, Yan, et al, 2010s,…….

Fourier transformation to impact-parameter b space

$$W(b)|^{NLO,R}_{Singular} = \int d^2P_\perp e^{-i\vec{P_\perp}\cdot\vec{b}} \frac{d\sigma}{dyd^2P_\perp}|^{NLO,R}_{Singular}$$

$$= \int_{x_A}^1 \frac{d\xi_1}{\xi_1} \int_{x_B}^1 \frac{d\xi_2}{\xi_2} f_{i'}(\frac{x_A}{\xi_1}, \mu_F) f_{j'}(\frac{x_B}{\xi_2}, \mu_F) \frac{M^2}{S} \hat{\sigma}^{(0)}_{ij} \frac{\alpha_s(\mu_F)}{2\pi} \Gamma(1+\epsilon)(4\pi)^\epsilon \pi$$

$$\times \Big[P_{jj'}(\xi_2)\delta_{i'i}\delta(1-\xi_1)\left(-\frac{1}{\epsilon} + \ln\left(\frac{4e^{-2\gamma_E}}{b^2\mu_R^2}\right)\right) + (\xi_1 \leftrightarrow \xi_2, i \leftrightarrow j, i' \leftrightarrow j')$$

$$+2C_{i'j'}\delta_{i'i}\delta_{j'j}\delta(1-\xi_1)\delta(1-\xi_2)\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\left(\frac{\mu_R^2}{M^2}\right) + \frac{1}{2}\ln^2\left(\frac{\mu_R^2}{M^2}\right)\right.$$

$$\left. - \frac{\pi^2}{6} - \frac{1}{2}\ln^2\left(\frac{M^2b^2e^{2\gamma_E}}{4}\right)\right)\Big]. \qquad ($$

$$W(b)|^{NLO,V} = \int d^2P_\perp e^{-i\vec{P_\perp}\cdot\vec{b}} \frac{d\sigma}{dydP_\perp^2}|^{NLO,V}$$

$$= \int_{x_A}^1 \frac{d\xi_1}{\xi_1} \int_{x_B}^1 \frac{d\xi_2}{\xi_2} f_i(\frac{x_A}{\xi_1}, \mu_F) f_j(\frac{x_B}{\xi_2}, \mu_F) \frac{M^2}{S} \hat{\sigma}^{(0)}_{ij} \frac{\alpha_s(\mu_F)}{\pi} \Gamma(1+\epsilon)(4\pi)^\epsilon \pi$$

$$\times K^{V,(J)}_{ij,g_n} \delta(1-\xi_1)\delta(1-\xi_2).$$

Collins-Soper evolution equation

$$W(b) = W(b)|^{NLO,R}_{Singular} + W(b)|^{NLO,V} + W(b)|^{PDFs}_{CT}$$
$$= \int_{x_A}^{1} \frac{d\xi_1}{\xi_1} \int_{x_B}^{1} \frac{d\xi_2}{\xi_2} f_{i'}(\frac{x_A}{\xi_1}, \mu_F) f_{j'}(\frac{x_B}{\xi_2}, \mu_F) \frac{M^2}{S} \hat{\sigma}^{(0)}_{ij} \frac{\alpha_s(\mu_F)}{2\pi} \Gamma(1+\epsilon)(4\pi)^{\epsilon}\pi$$
$$\times \left[P_{jj'}(\xi_2) \ln\left(\frac{4e^{-2\gamma_E}}{b^2\mu_F^2}\right) \delta_{i'i}\delta(1-\xi_1) + P_{ii'}(\xi_1) \ln\left(\frac{4e^{-2\gamma_E}}{b^2\mu_F^2}\right) \delta_{j'j}\delta(1-\xi_2) \right.$$
$$\left. + 2C_{ij}\delta_{i'i}\delta_{j'j}\delta(1-\xi_1)\delta(1-\xi_2) \left(-\frac{\pi^2}{6} - \frac{1}{2}\ln^2\left(\frac{M^2 b^2 e^{2\gamma_E}}{4}\right) + Fin^{V,(J)}_{ij,g_n} \right) \right],$$

$$\frac{\partial}{\partial \ln M^2} W(b, M) = \left[K(b\mu, \alpha_s(\mu)) + G\left(\frac{M}{\mu}, \alpha_s(\mu)\right) \right] W(b, M)$$

$$\mu \frac{d}{d\mu} K(b\mu, \alpha_s(\mu)) = -\gamma_K(\alpha_s(\mu)),$$

$$\mu \frac{d}{d\mu} G(b\mu, \alpha_s(\mu)) = +\gamma_K(\alpha_s(\mu)).$$

Separation of scales around 1/b and M respectively

Collins, Soper 81
Collins, Soper, Sterman 85

Sukakov form factor

$$\frac{\partial W(b,M)}{\partial \ln M^2} = \left[-\frac{1}{2}\int_{C_1^2/b^2}^{C_2^2M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\gamma_K(\alpha_s(\bar{\mu})) + K(C_1,\alpha_s(C_1/b)) + G(1/C_2,\alpha_s(C_2M))\right]W(b,M).$$

$$\ln\frac{W(M,b)}{W(Q_0,b)} = \int_{Q_0^2}^{Q^2} d\ln q^2\left[\int_{C_1^2/b^2}^{C_2^2q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}A(\alpha_s(\bar{\mu})) + B(\alpha_s(C_2q))\right]$$

$$W(b,M) = e^{-S(b,M,C_1,C_2)}W\left(b,\frac{C_1}{C_2b}\right),$$

where the Sudakov exponent is

$$S(b,M,C_1,C_2) = \int_{C_1^2/b^2}^{C_2^2M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln\left(\frac{C_2^2M^2}{\bar{\mu}^2}\right) + B(\alpha_s(\bar{\mu}))\right]$$

Nonperturbative Sukakov form factor

$$W(b,M)=e^{-S(b,M,C_1,C_2)}W\left(b,\frac{C_1}{C_2 b}\right),$$

where the Sudakov exponent is

$$S(b,M,C_1,C_2)=\int_{C_1^2/b^2}^{C_2^2M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln\left(\frac{C_2^2M^2}{\bar{\mu}^2}\right)+B(\alpha_s(\bar{\mu}))\right]$$

Solve the nonperturbative problem when $b \gg 1/\Lambda_{\rm QCD}$

$$W(b,M)=W(b_*,M)W^{NP}(b,M)=W(b_*,M)e^{-S_{NP}(b,M)},$$

$$b_*=\frac{b}{\sqrt{1+(b/b_{\rm max})^2}}.$$

$$S_{NP}^{SIYY-g}(b,M)=g_1b^2+g_2b^2\ln(M/(2Q_0))+g_3b^2\ln(100x_1x_2).$$

Transverse momentum resummation formula for T4c

$$\frac{d\sigma(p+p\to T_{4c}+X)}{dydP_\perp^2}$$
$$=\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P_\perp}\cdot\vec{b}}W(b,M)+\frac{d\sigma}{dydP_\perp^2}|_{Regular}$$
$$=\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P_\perp}\cdot\vec{b}}\sum_{ij}\int_{x_A}^{1}\frac{d\xi_1}{\xi_1}\int_{x_B}^{1}\frac{d\xi_2}{\xi_2}f_i(\frac{x_A}{\xi_1},\mu_F)f_j(\frac{x_B}{\xi_2},\mu_F)\frac{M^2}{S}\hat{\sigma}^{(0)}_{i'j'}\pi$$
$$\times W^{NP}(b,M)e^{-\int_{C_1^2/b_*^2}^{C_2^2M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A_{ij}(\alpha_s(\bar{\mu}))\ln\left(\frac{C_2^2M^2}{\bar{\mu}^2}\right)+B_{ij}(\alpha_s(\bar{\mu}))\right]}C_{i'i}\left(\xi_1,b_*,\frac{C_1}{C_2},\mu\right)$$
$$\times C_{j'j}\left(\xi_2,b_*,\frac{C_1}{C_2},\mu\right)+Y(P_\perp,M,x_A,x_B),$$
and
$$Y(P_\perp,M,x_A,x_B)=\frac{d\sigma}{dydP_\perp^2}|_{Regular}$$
$$=\sum_{ij}\int_{x_A}^{1}\frac{d\xi_1}{\xi_1}\int_{x_B}^{1}\frac{d\xi_2}{\xi_2}f_i(\frac{x_A}{\xi_1},\mu_F)f_j(\frac{x_B}{\xi_2},\mu_F)R_{ij}(P_\perp,M,x_A,x_B).$$

(2)Extraction of LDMEs and comparison with experimental data

Extraction of LDMEs

Table 5: Differential cross-sections $\mathrm{d}\sigma/\mathrm{d}p_{\mathrm{T}}^{\text{di-}J/\psi}$ of di-J/ψ production. The first uncertainties are statistical, and the second systematic.

$p_{\mathrm{T}}^{\text{di-}J/\psi}$ [GeV/c]	$\mathrm{d}\sigma/\mathrm{d}p_{\mathrm{T}}^{\text{di-}J/\psi}$ [nb/(GeV/c)]
$0-1$	$1.408 \pm 0.089 \pm 0.083$
$1-2$	$2.523 \pm 0.126 \pm 0.150$
$2-3$	$2.858 \pm 0.158 \pm 0.164$
$3-4$	$2.542 \pm 0.110 \pm 0.146$
$4-5$	$2.017 \pm 0.081 \pm 0.120$
$5-6$	$1.527 \pm 0.073 \pm 0.091$
$6-7$	$1.085 \pm 0.048 \pm 0.065$
$7-8$	$0.738 \pm 0.038 \pm 0.044$
$8-10$	$0.424 \pm 0.018 \pm 0.027$
$10-12$	$0.200 \pm 0.011 \pm 0.013$
$12-14$	$0.093 \pm 0.007 \pm 0.005$
$14-24$	$0.017 \pm 0.001 \pm 0.001$

LHCb, 2311.14085

$$\mathcal{R} \equiv \sigma(T_{4c})/\sigma(2J/\psi)\mathcal{B}(T_{4c} \to 2J/\psi)$$

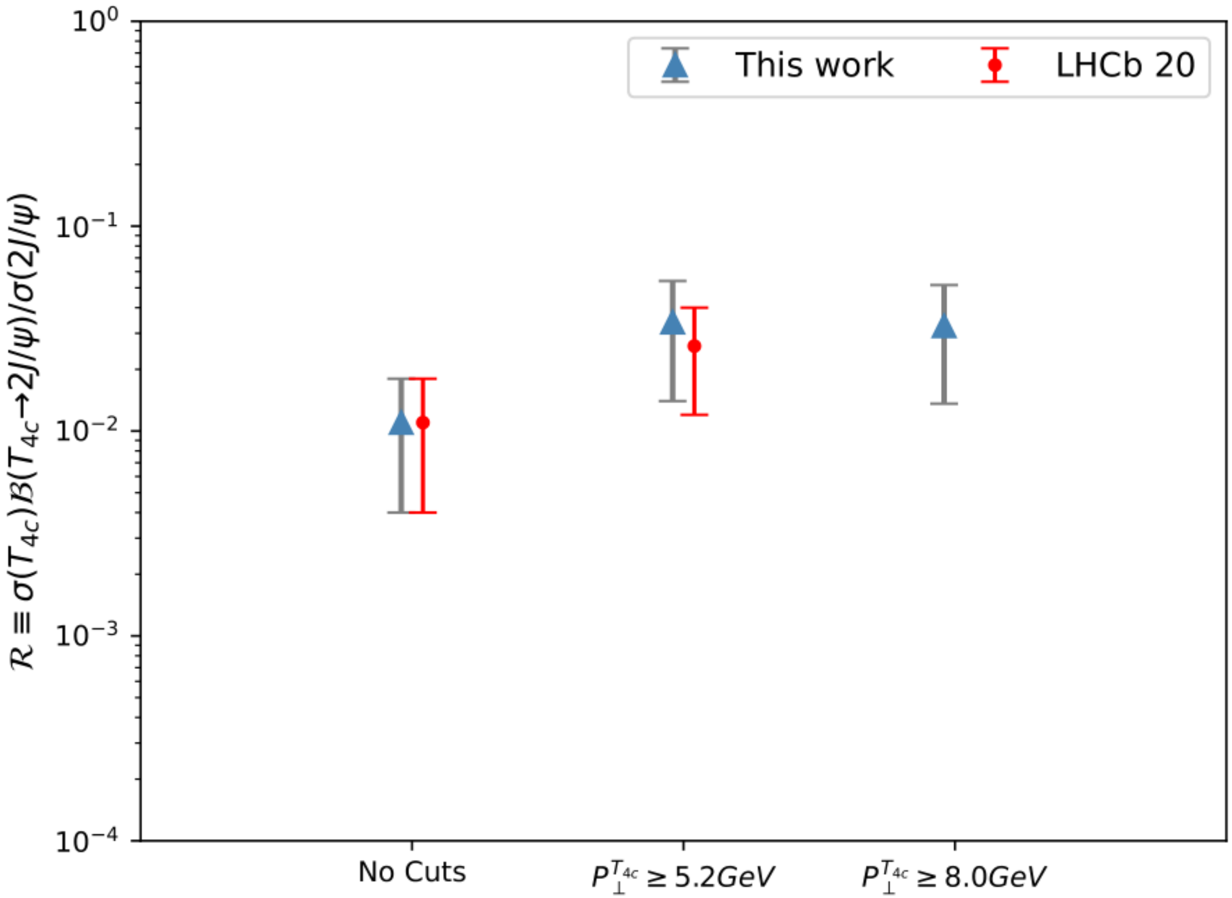
LHCb, 2006.16957

$$\mathcal{R}^{\text{LHCb}} = [1.1 \pm 0.4(\text{stat}) \pm 0.3(\text{syst})]\%.$$

Using the 2J/\psi cross section and R-value data from LHCb, we extracted the LDMEs based on the NLO+NLL QCD formula

$$\begin{aligned}\langle \mathcal{O}_{3\bar{3}}^{T4c} \rangle \left({}^5S_2, 2^{++}\right) \mathcal{B}(T_{4c}) &= \langle 0 | O^2_{\bar{\mathbf{3}}\otimes\mathbf{3},\bar{\mathbf{3}}\otimes\mathbf{3}} | 0 \rangle \mathcal{B}(T_{4c}) \\ &= (2.22 \pm 0.80^{+1.29+0.62}_{-0.57-0.00}) \times 10^{-4} GeV^9,\end{aligned}$$

A test of the predictive power of our formula



$P_\perp^{2J/\psi} > 5.2GeV$ case

$$\mathcal{R}^{Theo.} = [3.2 \pm 2.0]\%,$$

$$\mathcal{R}^{LHCb} =$$

$$[2.6 \pm 0.6(stat) \pm 0.8(syst)]\%$$

Well agreement with LHCb measurement

Another test but currently lack of decay information

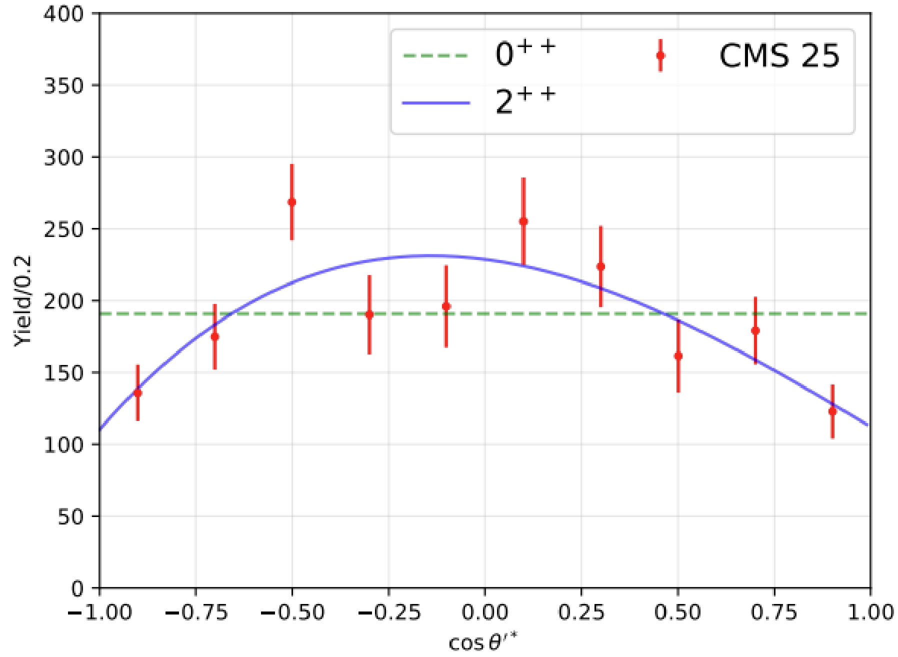
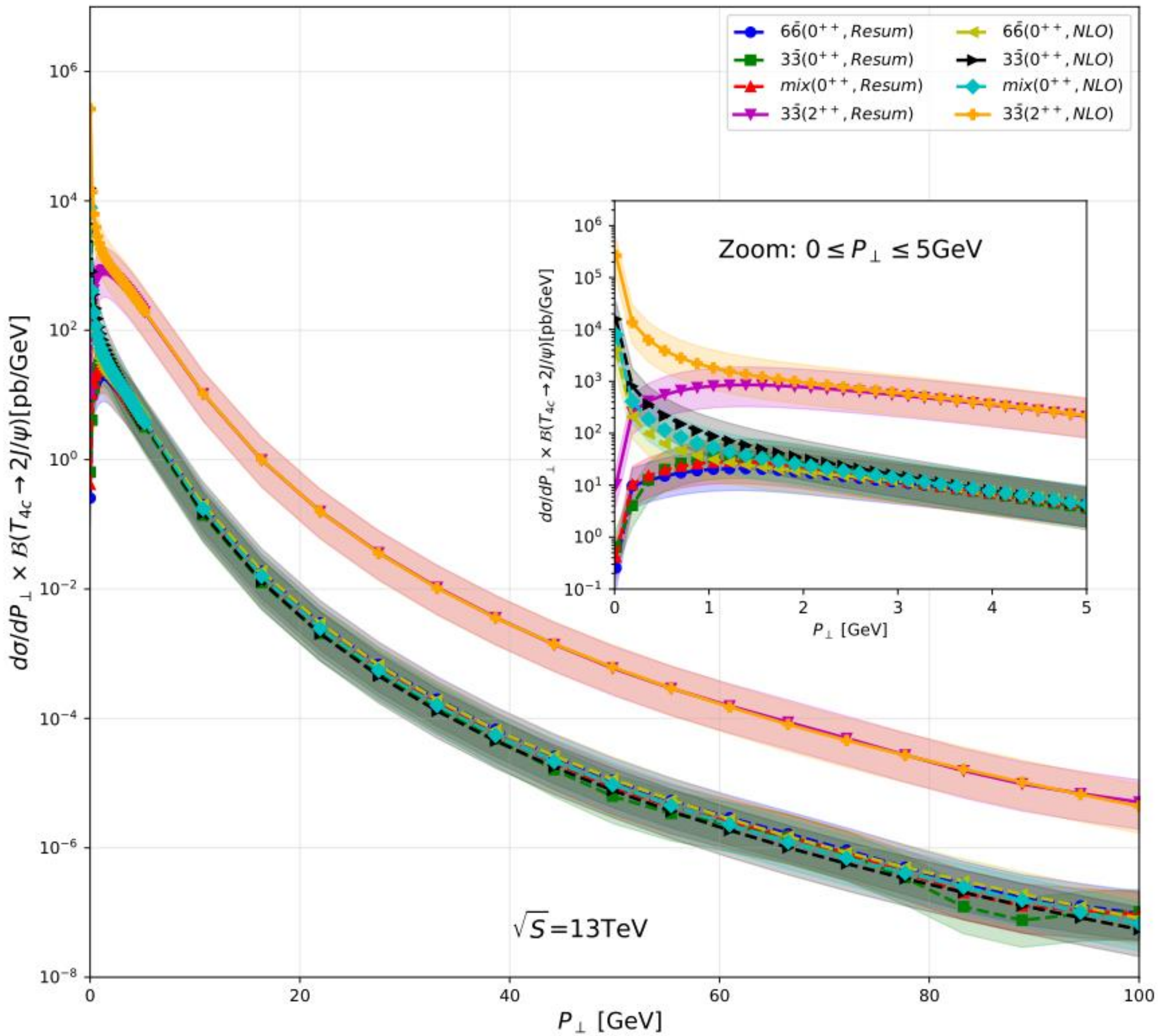


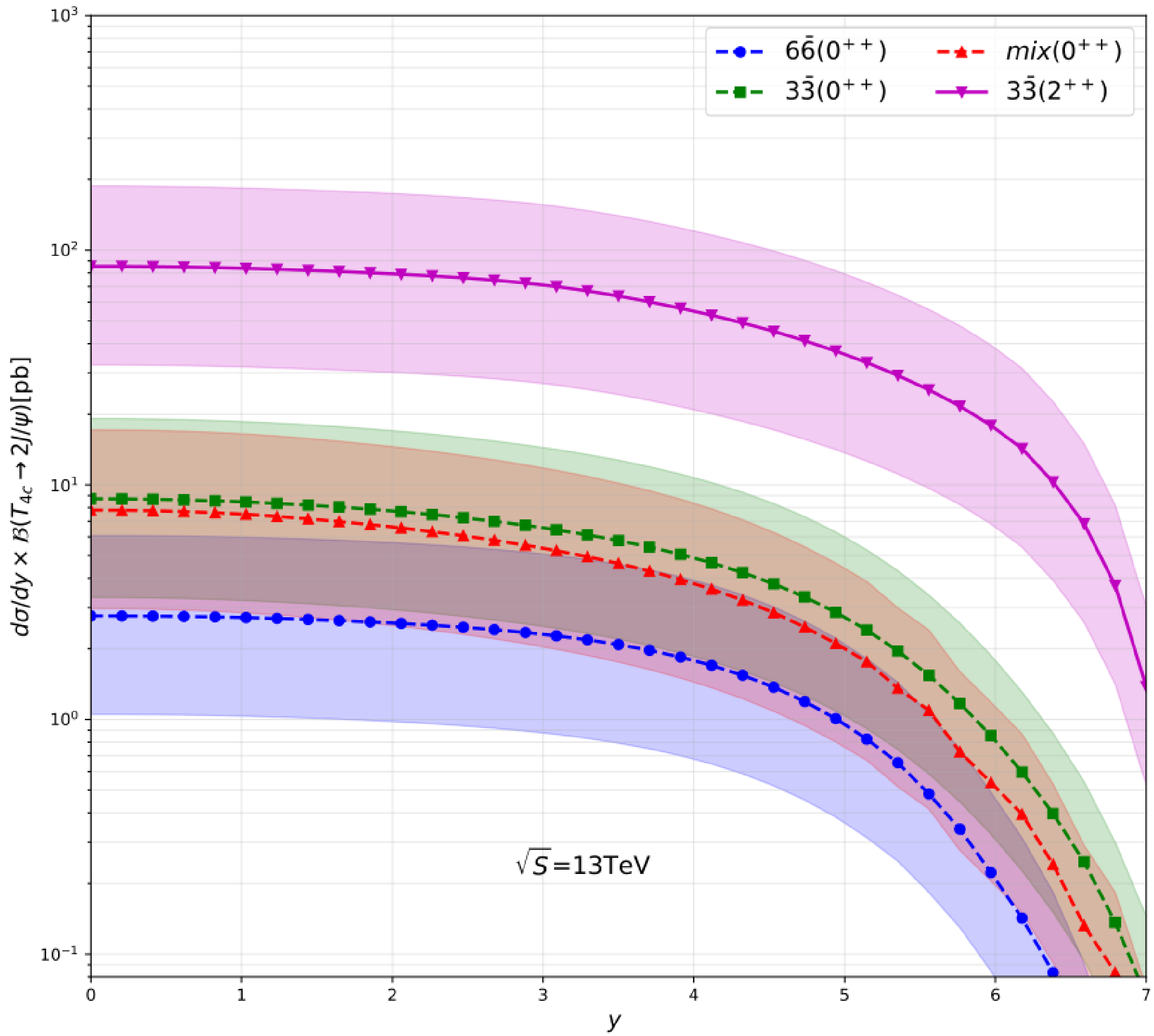
FIG. 6. Distribution of the polar angle θ'^* between the J/ψ and the fully charm tetraquark momenta. The CMS 25 data are from the very recent measurement by the CMS experiment [7].

$$dN/d\cos\theta'^* = \Sigma_{i=1}^{4} L_i \cos[(i-1)\theta'^*],$$

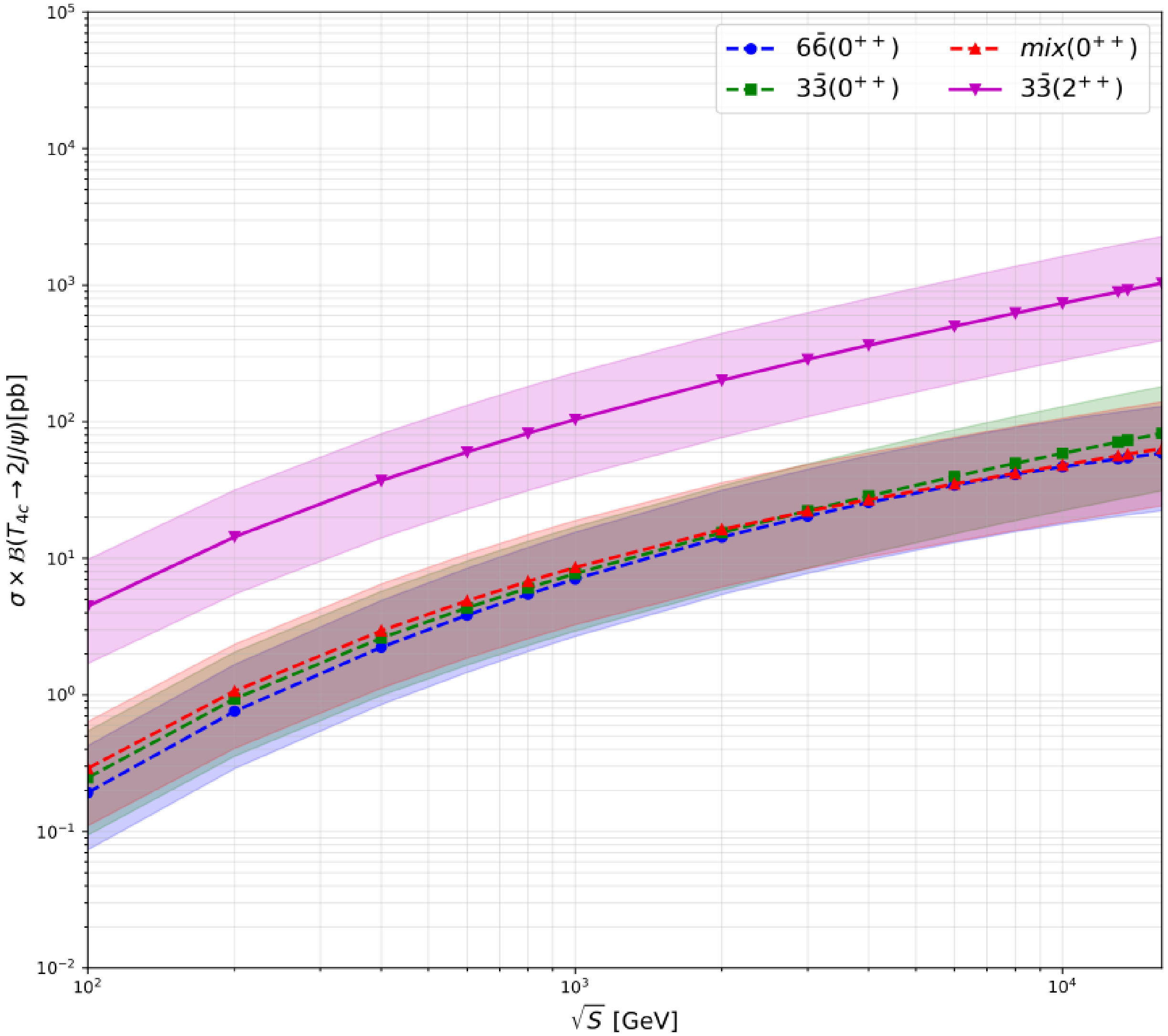
Transverse momentum distribution results at NLO+NLL



Rapidity distribution results



Total cross section dependence on center-of-mass energy



Summary and outlook

- ✓ QCD resummation formulae for T4c production at LHC are established.
- ✓ The LDMEs are extracted from the LHCb data, then the transverse momentum, rapidity distributions are predicted.

Outlook: 1) production differential cross section and decay angular distribution; 2J/psi entanglement
2)EM and hadronic transitions between T4c
3)T4c near threshold? T4b, T_{2b2c}?

Thank you a lot!