

Power corrections in rare B -meson decays

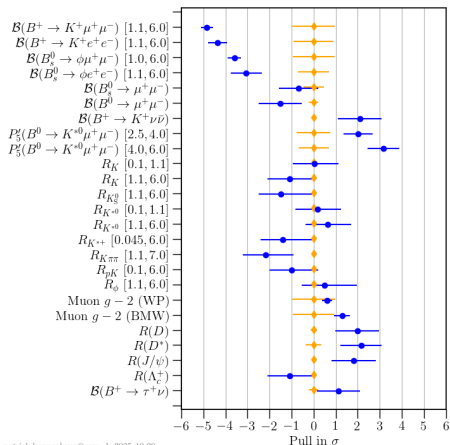
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Central China Normal University

第 6 届 LHCb 前沿物理研讨会 · 广州

2026.5.22-2026.5.25





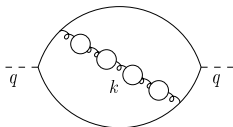
patrick.koppenburg@cern.ch 2025-10-20

Multi-scale problem:



Power correction:

- $\Lambda/m_b \sim \mathcal{O}(20\%)$
- IR renormalons [Beneke, 98']



LQCD: $am_b \gtrsim 1$ [FLAG, 24']

- Effective b quark action
- Quark mass extrapolation



Expansions in B decays

The perturbation expansion and power expansion

$$\alpha_s \sim \Lambda/m_b \sim 20\%$$

\mathcal{A}	LL (α_s)	NLL	NNLL
LP	LP@LL	LP@NLL	LP@NNLL
NLP	NLP@LL	NLP@NLL	
NNLP	NNLP@LL		

End-point singularity at NLP.

EOM vanishing operators at NNLP [Neubert, 93].

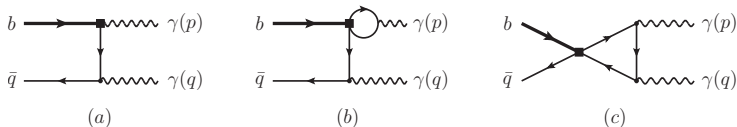


Rare $B \rightarrow \gamma\gamma$ decay: LP

The decay amplitudes [Descotes-Genon and Sachrajda, 03']

$$\bar{A}(\bar{B}_q \rightarrow \gamma\gamma) = -\frac{4 G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \epsilon_1^{*\alpha}(p) \epsilon_2^{*\beta}(q) \sum_{p=u,c} V_{pb} V_{pq}^* \sum_{i=1}^8 C_i T_{i,\alpha\beta}^{(p)}$$

The leading-order diagrams



The **leading power** factorization formula with **NLL resummation**

[Shen, Wang and YBW, 20']

$$\bar{A}_{\text{LP}}(\bar{B}_q \rightarrow \gamma\gamma) = i \frac{4 G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \epsilon_1^{*\alpha}(p) \epsilon_2^{*\beta}(q) \left[g_{\alpha\beta}^\perp - i \varepsilon_{\alpha\beta}^\perp \right] e_q f_{B_q} m_{B_q}^2 K^{-1}(m_b, \mu) \left[\sum_{p=u,c} V_{pb} V_{pq}^* \bar{m}_b(\nu) V_{7,\text{eff}}^{(p)}(m_b, \mu, \nu) \right] \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu) J(m_b, \omega, \mu)$$

Summary for the helicity amplitudes of $B_q \rightarrow \gamma\gamma$

- Final expressions for the factorized NLP corrections:

$$\begin{aligned}\sum_{i=1}^8 C_i F_{i,L}^{(p), \text{fac}, \text{NLP}} &= \mathcal{C}_7^{\text{eff}} \left[F_{7,L}^{\text{hc}, \text{NLP}} + F_{7,L}^{m_q, \text{NLP}} + F_{7,L}^{A2, \text{NLP}} + F_{7,L}^{\text{HT}, \text{NLP}} + F_{7,L}^{e_b, \text{NLP}} \right] \\ &\quad + \frac{f_{B_q}}{m_{B_q}} \left[\mathcal{F}_V^{(p), \text{WA}} - \mathcal{F}_A^{(p), \text{WA}} \right], \\ \sum_{i=1}^8 C_i F_{i,R}^{(p), \text{fac}, \text{NLP}} &= \frac{f_{B_q}}{m_{B_q}} \left[\mathcal{F}_V^{(p), \text{WA}} + \mathcal{F}_A^{(p), \text{WA}} \right].\end{aligned}$$

Large-recoil symmetry violation from the weak annihilation correction completely.

- Final expressions for the two helicity amplitudes:

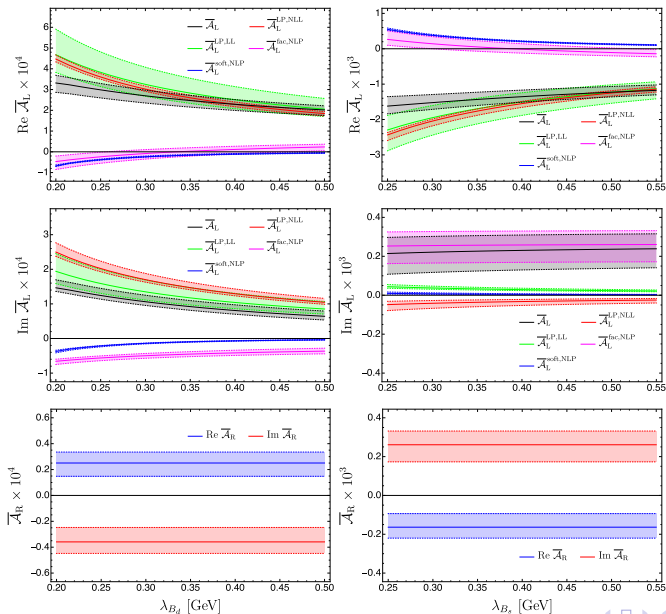
$$\begin{aligned}\tilde{\mathcal{A}}_L &= \sum_{p=u,c} V_{pb} V_{pq}^* \sum_{i=1}^8 C_i \left[F_{i,L}^{(p), \text{LP}} + F_{i,L}^{(p), \text{fac}, \text{NLP}} + F_{i,L}^{(p), \text{soft}, \text{NLP}} \right], \\ \tilde{\mathcal{A}}_R &= \sum_{p=u,c} V_{pb} V_{pq}^* \sum_{i=1}^8 C_i \left[F_{i,R}^{(p), \text{LP}} + F_{i,R}^{(p), \text{fac}, \text{NLP}} + F_{i,R}^{(p), \text{soft}, \text{NLP}} \right].\end{aligned}$$

- The fundamental nonperturbative functions: **HQET B -meson LCDAs**.

- Key hadronic inputs for exclusive B -meson decay phenomenologies, for instance $B \rightarrow \pi \ell \nu$, $B \rightarrow D^{(*)} \ell \nu$, $B \rightarrow K^* \gamma$, $B \rightarrow K^{(*)} \ell \ell$, $B \rightarrow \pi \pi$ [Lü, Shen, Wang, YMW, 2023].



$B_{(d,s)} \rightarrow \gamma\gamma$ decay



- **NLL:** reduce the μ dependence
- **Large error** from B -LCDA
- Factorizable NLP: **30%**
- NLP: **strong phase**



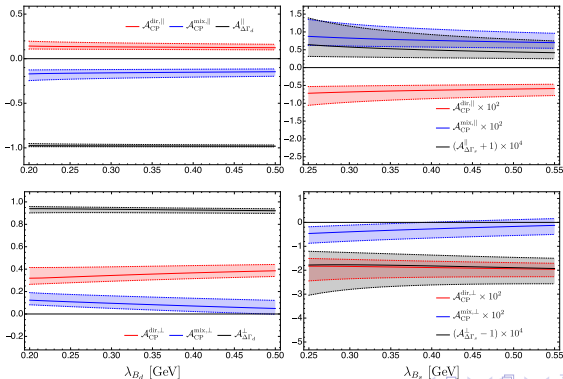
$B_{(d,s)} \rightarrow \gamma\gamma$ decay

Branch ratios

$$\mathcal{BR}(\bar{B}_d \rightarrow \gamma\gamma) = 1.44_{-0.80}^{+1.35} \times 10^{-8}, \quad \mathcal{BR}(\bar{B}_s \rightarrow \gamma\gamma) = 3.17_{-1.74}^{+1.96} \times 10^{-7}$$

CP violation

$$A_{\text{CP}}^{\chi}(t) = \frac{\bar{\Gamma}^{\chi}(\bar{B} \rightarrow \gamma\gamma) - \Gamma^{\chi}(B \rightarrow \gamma\gamma)}{\bar{\Gamma}^{\chi}(\bar{B} \rightarrow \gamma\gamma) + \Gamma^{\chi}(B \rightarrow \gamma\gamma)} = -\frac{\mathcal{A}_{\text{CP}}^{\text{dir}, \chi} \cos(\Delta m_q t) + \mathcal{A}_{\text{CP}}^{\text{mix}, \chi} \sin(\Delta m_q t)}{\cosh(\Delta\Gamma_q t/2) + \mathcal{A}_{\Delta\Gamma}^{\chi} \sinh(\Delta\Gamma_q t/2)}$$

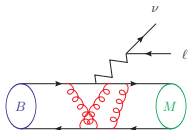


HQET LCDA of heavy meson

The leading-twist heavy-meson **HQET LCDA** [Grozin and Neubert, 96']

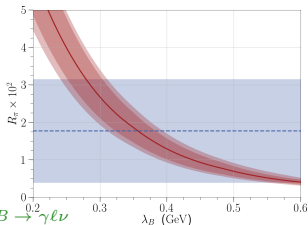
$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i \tilde{f}_H(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$

HQET LCDA: appears in exclusive B decays



- $B \rightarrow \pi K \dots$
- $B \rightarrow D^{(*)} \ell \nu \dots$
- $B \rightarrow \gamma \ell \nu \dots$

First inverse momentum $\lambda_B \sim \mathcal{O}(\Lambda_{\text{QCD}})$: **NOT** well determined



[Belle, 18']: $B \rightarrow \gamma \ell \nu$



Study of HQET LCDA

HQET LCDA: non-perturbative physics

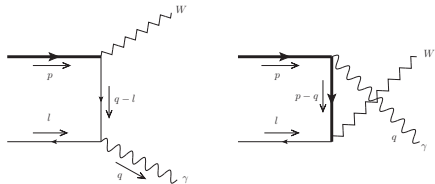
- **Inverse moments**: [Wang and Shen, 18'], [Beneke, Braun, Ji, and **YBW**, 18'], [Belle, 18'], [Galda, Neubert and Wang, 22'], [Han, et al., 24']
- EOM [Kawamura, Kodaira, Qiao and Tanaka, 02'], [Lü, Shen, Wang and **YBW**, 18']
- RG evolution properties: [Braun, Ji and Manashov, 19'], [Galda and Neubert, 20'], [Cui, Shen, Wang and **YBW**, 23']
- generalized LCDA [Beneke, Böer, Toelstede, and Vos, 22'], [Qin, Shen, Wang and Wang, 22'], [Huang, Ji, Shen, Wang and Wang, 23']

Determination of λ_B

- Extract from the experiments: clean process $B \rightarrow \gamma l \nu$, $W \rightarrow B \gamma$
- Calculate with LQCD [LPC collaboration, 22']



$$B \rightarrow \gamma \ell \nu$$

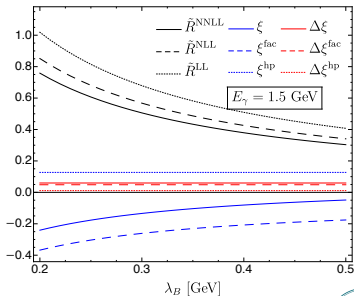


[Wang and Shen, 18'], [Beneke, Braun, Ji, and YBW, 18'], [Galda, Neubert and Wang, 22'], [Cui, Shen, Wang, and YBW, 23']

Decay form factor

$$F_{V/A}(E_\gamma) = R(E_\gamma) + [\xi(E_\gamma) \pm \Delta\xi(E_\gamma)]$$

- Factorizable contributions:
higher-twist B -meson LCDA,
QCD \rightarrow HQET: b quark field
- Non-factorizable contributions:
hadronic structure of the photon



[Cui, Shen, Wang, and YBW, 23']



$$B \rightarrow \gamma \ell \nu$$

$$\langle H_v | \bar{h}_v(0) \not{n}_+ \gamma^5 q_s(tn_+) | 0 \rangle = -i \tilde{f}_B(\mu) n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v} \varphi_+(\omega)$$

Exclusive B decays: B -meson **HQET** decay constant \tilde{f}_B :

$$\tilde{f}_B(\mu) = K^{-1}(\mu) f_B$$

QCD decay constant f_B calculated with LQCD [FLAG].

The relation receive

power corrections

[Becher, Neubert and Petrov, 00']

$$\bar{q} \gamma^\alpha Q \cong \sum_{i=1,2} C_i(\mu) J_i + \frac{1}{2m_Q} \sum_{j=1}^{10} B_j(\mu) Q_j$$

$$f_B = K(\mu) \tilde{f}_B(\mu) + \text{NLP}$$

Two loop renormalization of heavy light currents at order $1/m(Q)$ in the heavy quark expansion

Thomas Becher (Cornell U., LNS), Matthias Neubert (Cornell U., LNS), Alexey A. Petrov (Cornell U., LNS)

Dec, 2000

16 pages

Published in: *Nucl.Phys.B* 611 (2001) 367-382

e-Print: [hep-ph/0012183](https://arxiv.org/abs/hep-ph/0012183) [hep-ph]

DOI: [10.1016/S0550-3213\(01\)00341-8](https://doi.org/10.1016/S0550-3213(01)00341-8)

Report number: CLNS-00-1713

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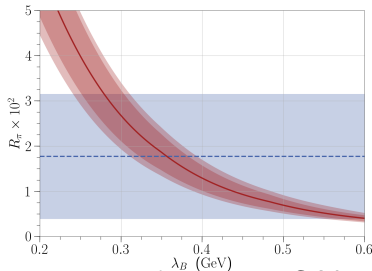
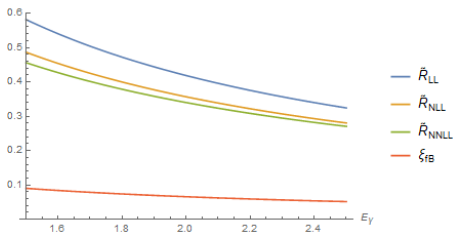
$$B \rightarrow \gamma \ell \nu$$

Power correction in f_B

⇒ 20% increase to $F_{V/A}$

⇒ 40% increase to decay width

[Liu, Su and YBW, in progress]



[Belle, 18']: $\lambda_B > 0.24$ GeV

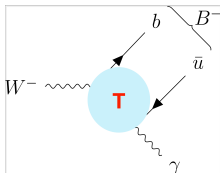
Also affect the exclusive extraction of $|V_{ub}|$ and $|V_{cb}|$.



$$W \rightarrow B\gamma$$

Rare decay $W^- \rightarrow B^- \gamma$: B -meson QCD LCDA

[Grossman, König and Neubert, 15']



Match from QCD LCDA to HQET LCDA [Ishaq, Jia, Xiong and Yang, 19'],

[Zhao, 19'], [Beneke, Finauri, Vos and YBW, 23'], [Deng, Wang, YBW and Zeng, 24']

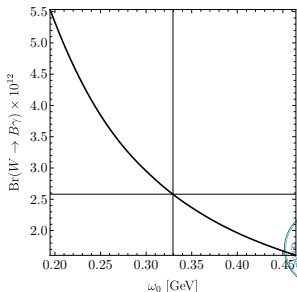
$$\phi_{\text{QCD}}(u) = \mathcal{J}(u, \omega) \otimes \varphi_{\text{HQET}}(\omega)$$

Our result: with NLL resummation [Beneke,

Finauri, Vos and YBW, 23']

$$\text{Br} = (2.58 \pm 0.21_{\text{in}} \begin{matrix} +0.05 \\ -0.08 \end{matrix} \begin{matrix} +2.95 \\ -0.98 \end{matrix} \mu_h \lambda_B) \cdot 10^{-12}$$

30% increase



Summary

* Rare $B \rightarrow \gamma\gamma$ decay

- **NLL resummation**: stabilize the factorization-scale dependence
- **Power correction**: factorization contribution and soft-overlap contribution

* Determination of λ_B

- **Power correction** in $B \rightarrow \gamma\ell\nu$: 40% increase of decay width
- $W \rightarrow B\gamma$ decay

Thank you!

