CKM matrix elements from B decays

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CKM matrix

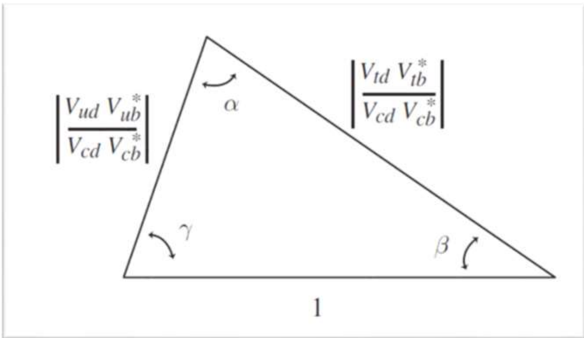
Charged current term

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} W_\mu^+ \, \bar{u}_L U_u^\dagger \gamma^\mu U_d d_L = \frac{g}{\sqrt{2}} W_\mu^+ \, \bar{u}_L \gamma^\mu V_{CKM} \, d_L$$

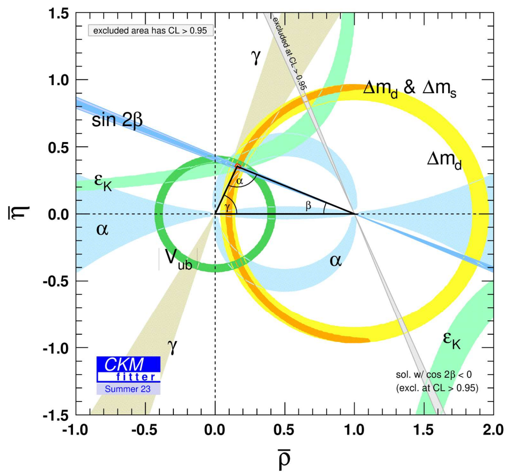
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Unitary Triangle

$V_{CKM}^\dagger V_{CKM}$=1

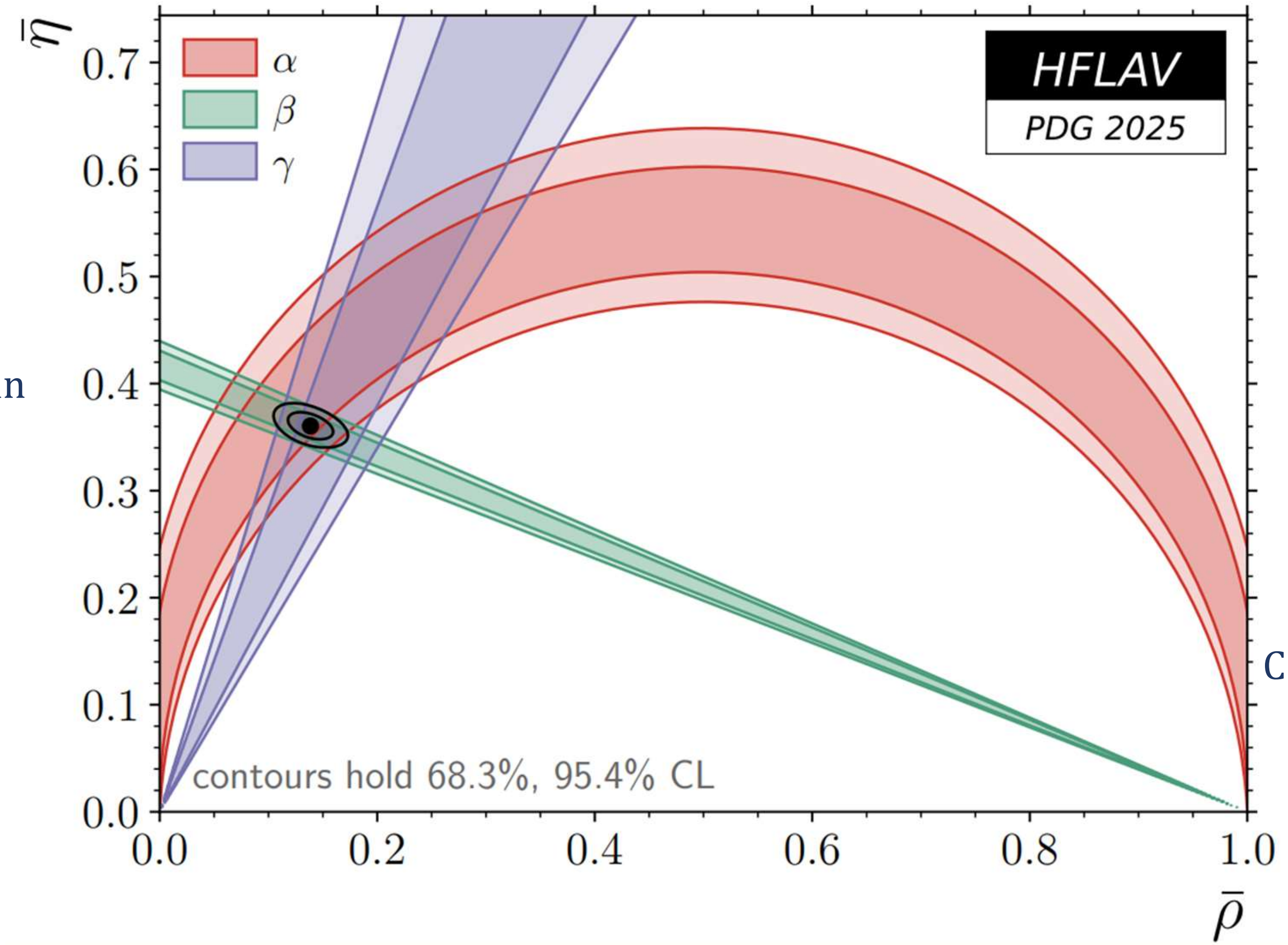


$$V_{ud}\, V_{ub}^* + V_{cd}\, V_{cb}^* + V_{td}\, V_{tb}^* = 0$$



$B \to DK$: GLW, ADS,BPGGSZ

Mixing induced CPV S_f in $B \to J/\psi K_s$



CPV in $B \to \pi\pi, \pi\rho, \rho\rho$

Puzzles in the determination of CKM matrix elements

$|V_{cb}|$

From inclusive decays

$$10^3\ |V_{cb}|^{\rm incl} = 42.0 \pm 0.5$$

From exclusive decays

$$10^3\ |V_{cb}|^{\rm excl} = 39.5 \pm 0.5$$

$$10^3\ |V_{cb}|^{\rm aver} = 40.7 \pm 1.3$$

$|V_{ub}|$

From inclusive decays

$$10^3\ |V_{ub}|^{\rm incl} = 4.06 \pm 0.11 \pm 0.13 \pm 0.18$$

From exclusive decays

$$10^3\ |V_{ub}|^{\rm excl} = 3.75 \pm 0.06 \pm 0.19$$

$$10^3\ |V_{ub}|^{B_s,\Lambda_b} = 3.43 \pm 0.32$$

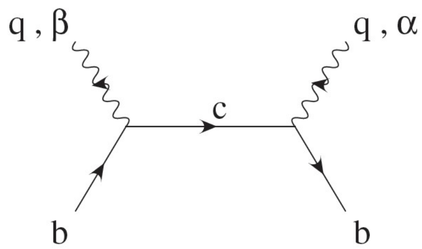
$$10^3\ |V_{ub}|^{aver} = 3.89 \pm 0.16$$

Inclusive decays $B \to X_c \ell \nu$

$$\frac{d\Gamma}{dq^2 dE_\ell dE_\nu} = 2G_F^2 |V_{cb}|^2 W^{\alpha\beta} L_{\alpha\beta}$$

$$W^{\alpha\beta} = \frac{1}{\pi} \mathrm{Im}\, T^{\alpha\beta} \text{(left cut)}$$

$$T^{\alpha\beta} = -i \int d^4x e^{-i\ \cdot x} T[J_{L\alpha}^\dagger(x) J_{L\beta}(0)]$$



OPE & Heavy quark expansion

$$\Gamma = \frac{\mathrm{G_F^2 m_b^5}}{192\pi^3} |\eta_{EW}|^2 |\mathrm{V}_{cb}|^2 [Z_0(r) + Z_2(r) + Y_2(r) + Z_3(r) + Y_3(r) + \cdots]$$

$$Z_0(r) = z_0^{(0)}(r) + \frac{\alpha_s}{\pi} z_0^{(1)}(r) + \frac{\alpha_s^2}{\pi^2} z_0^{(2)}(r) + \cdots$$

$$Z_2(r) = \frac{\mu_\pi^2}{m_b^2}\left[z_2^{(0)}(r) + \frac{\alpha_s}{\pi} z_2^{(1)}(r) + \cdots\right]$$

$$Y_2(r) = \frac{\mu_G^2}{m_b^2}\left[y_2^{(0)}(r) + \frac{\alpha_s}{\pi} y_2^{(1)}(r) + \cdots\right]$$

$$Z_2(r) = \frac{\rho_D^3}{m_b^3}\left[z_3^{(0)}(r) + \frac{\alpha_s}{\pi} z_3^{(1)}(r) + \cdots\right]$$

$$Y_3(r) = \frac{\rho_{LS}^3}{m_b^3}\left[y_3^{(0)}(r) + \frac{\alpha_s}{\pi} y_3^{(1)}(r) + \cdots\right]$$

$$\bar{\Lambda} = \mathrm{M_B} - \mathrm{m_b}$$

$$\mu_\pi^2 = -\langle B|\bar{b}(iD_\perp)^2 b|B\rangle$$

$$\mu_G^2 = -\langle B|\bar{b}(iD_\perp^\mu)(iD_\perp^\nu)\sigma_{\mu\nu} b|B\rangle$$

$$\rho_D^3 = -\langle B|\bar{b}(iD_\perp^\mu)(iv\cdot D)(iD_{\perp\mu}) b|B\rangle$$

$$\rho_{LS}^3 = -\langle B|\bar{b}(iD_\perp^\mu)(iv\cdot D)(iD_\perp^\nu)\sigma_{\mu\nu} b|B\rangle$$

Theoretical uncertainty:

- Power correction
- Nonperturbation parameter
- Quark mass scheme…
- Quark hadron duality violation?

$B \to D^{(*)}\ell\nu$

$$\frac{d\Gamma(B \to D^{(*)}\ell\nu)}{dw} = [\text{calculable terms}]|V_{cb}|\begin{cases}(w^2-1)^{\frac{1}{2}}\mathcal{F}_*^2(w) & \text{for } B \to D^* \\ (w^2-1)^{\frac{3}{2}}\mathcal{F}^2(w) & \text{for } B \to D\end{cases} \qquad w = v \cdot v'$$

HQET constraints

$$\mathcal{F}_{(*)}^2(1) = 1 \text{ in heavy quark limit}$$

Including QCD and power corrections

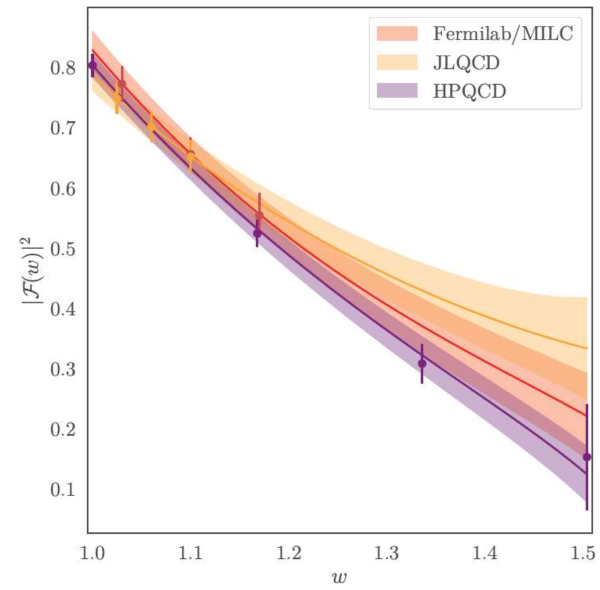
$$\mathcal{F}_*(1) = 1 + c_A(\alpha_s) + \frac{0}{m_{c,b}}(\text{Luke}) + O(\Lambda_{QCD}^2/m_{c,b}^2)$$

$$\mathcal{F}(1) = 1 + c_V(\alpha_s) + O(\Lambda_{QCD}^2/m_{c,b}^2)$$

Lattice simulation Fermilab, MILC, 2105.14019; JLQCD, 2306.05657;HPQCD, 2105.11433

Extrapolation: BGL parameterization

$$\mathcal{F}(z) = \frac{1}{P_F(z)\phi_F(z)}\sum_{n=0}^{\infty} a_n z^n \qquad \sum_{n=0}^{\infty} a_n^2 \leq 1$$



Light-cone sum rules: valid at large recoil region

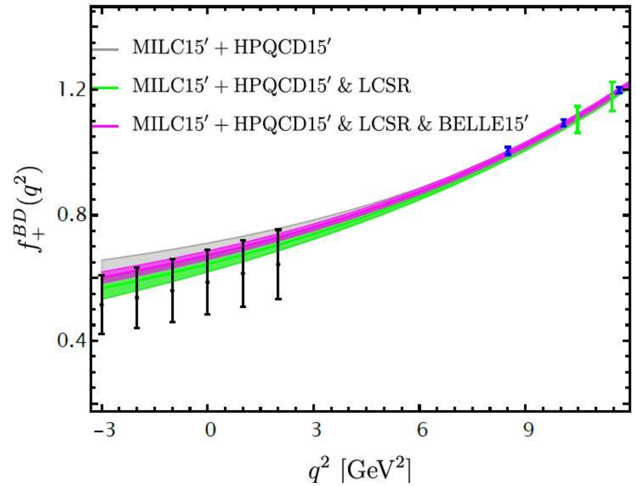
The correlation function

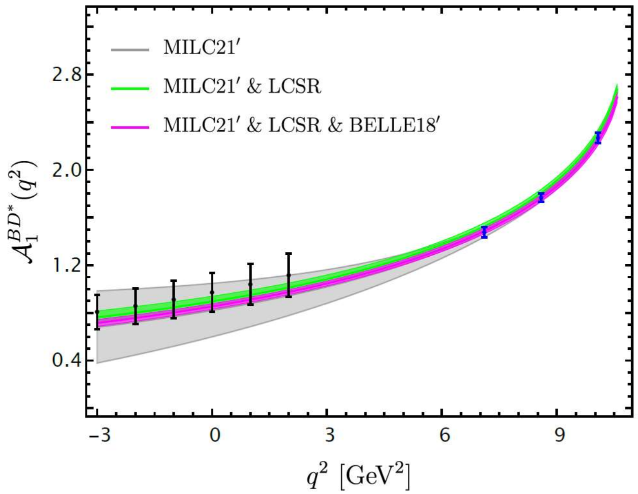
$\int d^4x e^{iP\cdot x}\langle \bar{M}(P)|T\{J_{\text{weak}}(0), J_M(x)\}|0\rangle$

$\int d^4x e^{iP\cdot x}\langle 0|T\{J_{\text{weak}}(0), J_B(x)\}|\bar{B}(P)\rangle$

Quark level
OPE ⇔ Quark hadron
duality ⇔ Hardon level
Parameterization

- B $\to D$ form factors: leading power, NLO corrections [Wang, Wei, YLS,2016]
- B $\to D$ form factors: LP-NLO+NLP [Gao, Huber, Ji, Wang, Wang, Wei, 2022]
- B $\to D(D^*)$ form factors: SCET+NLO+NLP [Cui, huang, Wang, Zhao, 2023]



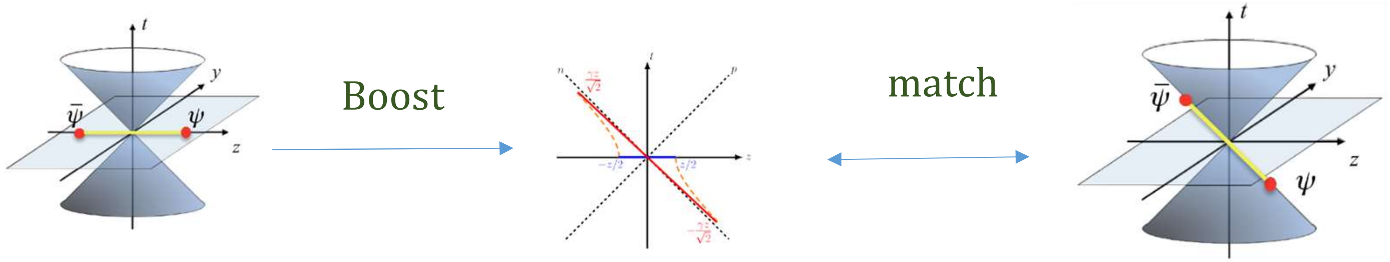


Light-cone sum rules: valid at large recoil region

- Main uncertainty of LCSR: LCDA

$$\phi_B(\omega) = \frac{1}{iF_B m_B}\int \frac{dt}{2\pi} e^{-i\omega t}\langle 0|\bar{q}(tn)\gamma\cdot n\gamma_5[tn,0]b_v(0)|\bar{B}(v)\rangle$$

- LCDA from Lattice   The Large momentum effective theory [Ji, 2013]



Quasi-PDA

$$\varphi^+(\xi,\mu) = \int_0^1 du\, H(\xi,u,\mu/P^z)\,\phi^+(u,\mu) + O\left(\frac{m_H^2}{(P^z)^2},\frac{\Lambda^2}{(uP^z,\bar{u}P^z)^2}\right)$$

D meson LCDAs from Lattice: LPC collaboration, 2604.25802

- $B \to D$ form factors from D meson LCDAs   Qu etc., in preparation

Inclusive decays $B \to X_u \ell \nu$

two kinematical regions
- (local) OPE region — Similar to $B \to X_c \ell \nu$, large background
- Shape function region — QCD factorization B. Lange, Nuebert, Paz, hep-ph/0504071

$$T = \sum H(u_1, \dots u_i) J(u_1, \dots u_i; \omega_1, \dots \omega_i) \otimes S(\omega_1, \dots \omega_i)$$

Alternative methods Gambino etc.,0707.2493; Aderson etc., hep-ph/05093601

- Shape functions — Leading Shape functions

$$\langle \bar{B} | \bar{h}_v Y(x_-)_a \; Y^{\dagger} h_v(0)_{b\beta} | \bar{B} \rangle = \frac{\delta_{ba}}{N_c} \frac{1}{2} \left(\frac{1 + v \cdot \gamma}{2} \right)_{\beta\alpha} \tilde{S}(x_+)$$

More Shape functions are defined by suppressed operators

- Modeling shape functions and perspective form Lattice simulation: Ji Xu's talk at HFCPV2025
- Weak annihilation contributions

Babar+Belle+CLEO

$$|V_{ub}| = (4.06 \pm 0.12_{\rm exp} \pm 0.13_{\rm theo} \pm 0.19_{\Delta \rm model}) \times 10^{-3} \quad ({\rm inclusive})$$

Exclusive decays: $B \to \pi\ell\nu$ & $\mathrm{B_s} \to K\ell\nu$

Partial decay width(massless lepton)

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|p_\pi|^3|f_+(q^2)|^2$$

Partial decay width

$$\langle\pi(p)|V^\mu|B(P)\rangle = f_+(q^2)\left(P^\mu + p^\mu - \frac{M_B^2 - m_\pi^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - m_\pi^2}{q^2}q^\mu$$

Lattice simulation

- FNAL/MLIC, 1507.07839; 1901.02561
- HPQCD: 1406.2279; 1501.05373; 2303.11280
- JLQCD: 2203.04938

Extrapolation: BCL parameterization

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}}\sum_{k=0}^{N-1} b_k^+\left[z^k - \frac{(-1)^{k-N}k}{N}z^N\right]$$

Heavy-to-light form factors at large recoil

Factorization of heavy-to-light form factors at leading power

$$f_+(E) = C_{f_+}^{(A0)}(E)\,\xi_P(E) + \int d\tau\, C_{f_+}^{(B1)}(E,\tau)\,\Xi_P(\tau,E)$$

$$\frac{m_B}{2E}\, f_0(E) = C_{f_0}^{(A0)}(E)\,\xi_P(E) + \int d\tau\, C_{f_0}^{(B1)}(E,\tau)\,\Xi_P(\tau,E)$$

$$\frac{m_B}{m_B+m_P}\, f_T(E) = C_{f_T}^{(A0)}(E)\,\xi_P(E) + \int d\tau\, C_{f_T}^{(B1)}(E,\tau)\,\Xi_P(\tau,E)$$

Soft form factor: large recoil symmetry

Symmetry breaking part

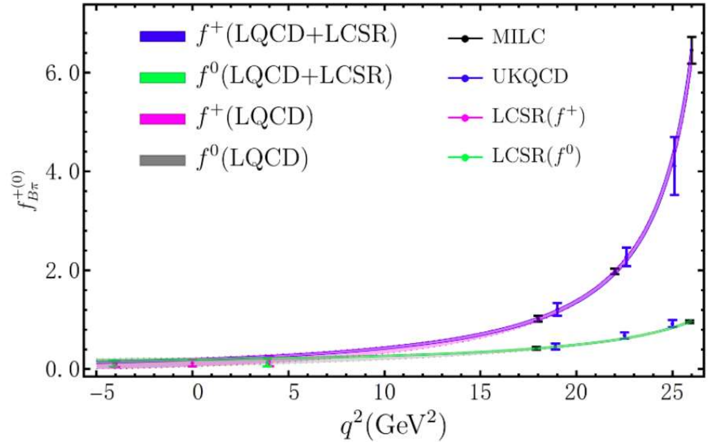
Light-cone sum rules for $B \to \pi$, K form factors

- B-meson LCSR: Wang, YLS, 1506.00667; YLS, Wei, 1607.08727; Lü, YLS, Wang, Wei, 1810.00819; Cui, Huang, YLS, Wang, Wang, 2212.11624
- π, K -meson LCSR: Duplancic etc., 0801.1769; Duplancic, 0805.4170; Khodjamirian etc., 1103.2655; Leljak etc., 2102.07233

Heavy-to-light form factors at large recoil

B-meson light-cone sum rules for $B_{(s)} \to \pi, \mathrm{K}$

Cui, Huang, YLS, Wang, Wang, 2212.11624

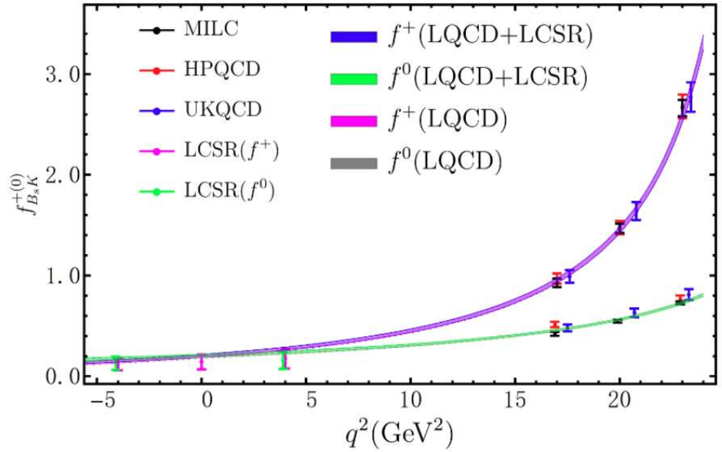


- Leading power contribution to the correlation functions: NLO+NLL
- Next-to-leading power corrections:

  higher twist;

  heavy quark expansion of the hard-collinear quark propagator;

  the subleading matrix element of the effective heavy-to-light current…
- Combined analysis of Lattice and LCSR



$$|V_{ub}|_{B\to\pi\ell\bar{\nu}_\ell} = (3.76 \pm 0.13) \times 10^{-3}\,, \qquad \text{(BCL fit with } N = 3)$$

Baryonic decays $\Lambda_b \to \Lambda_c \ell\nu, \Lambda_b \to p\ell\nu$

Form factors

$$\langle F(p',s')|\bar{q}\gamma_\mu b|\Lambda_b(p,s)\rangle = u_F(p',s')\left[f_0\frac{(M-m)q_\mu}{q^2} + f_+\frac{M+m}{s_+}\left(p_\mu + p'_\mu - \frac{q_\mu}{q^2}\right) + f_\perp\left(\gamma_\mu - \frac{2m}{s_+}p^\mu - \frac{2M}{s_+}p'_\mu\right)\right]u_{\Lambda_b}(p,s)$$

$$\langle F(p',s')|\bar{q}\gamma_\mu\gamma_5 b|\Lambda_b(p,s)\rangle = -u_F(p',s')\left[g_0\frac{(M+m)q_\mu}{q^2} + g_+\frac{M-m}{s_-}\left(p_\mu + p'_\mu - \frac{(M^2-m^2)q_\mu}{q^2}\right) + g_\perp\left(\gamma_\mu + \frac{2m}{s_-}p^\mu - \frac{2M}{s_-}p'_\mu\right)\right]u_{\Lambda_b}(p,s)$$

Heavy quark limit for $\Lambda_b \to \Lambda_c$

Large recoil limit for $\Lambda_b \to p$

$$f_0 = f_+ = f_\perp = g_0 = g_+ = g_\perp = \xi_F$$

Lattice simulation

$$\frac{Br(\Lambda_b \to p\mu\bar{\nu})_{q^2>15G\quad 2}}{Br(\Lambda_b \to \Lambda_c\mu\bar{\nu})_{q^2>7GeV^2}} = (1.471 \pm 0.095 \pm 0.109)\left|\frac{V_{ub}}{V_{cb}}\right|^2$$

W. Detmold etc. 1503.01421

LHCb measurement

$$\frac{Br(\Lambda_b \to p\mu\bar{\nu})_{q^2>15GeV^2}}{Br(\Lambda_b \to \Lambda_c\mu\bar{\nu})_{q^2>7GeV^2}} = (0.92 \pm 0.04 \pm 0.07) \times 10^{-2} \Rightarrow \left|\frac{V_{ub}}{V_{cb}}\right| = 0.079 \pm 0.004 \pm 0.004$$

LHCb, 1709.01920

LCSR

- Large theoretical uncertainty form baryonic LCDAS
- Unknown power suppressed contributions

Huang etc., 2205.06095; Miao etc., 2206.12189

- Other processes

$B \to (\rho, \omega)\ell\nu$

$$|V_{ub}| = (3.05\,^{+1.34}_{-1.30}|_{\rm th}\,^{+0.19}_{-0.20}|_{\rm exp}) \times 10^{-3}, \qquad [\text{from } B \to \rho\ell\bar{\nu}_\ell]$$

$$|V_{ub}| = (2.54\,^{+1.09}_{-1.05}|_{\rm th}\,^{+0.18}_{-0.19}|_{\rm exp.}) \times 10^{-3}. \qquad [\text{from } B \to \omega\ell\bar{\nu}_\ell]$$

SCET sum rules: Gao etc., 1907. 11092

$B \to \mu\nu$

QED corrections are about few percent, theoretically clean

Cornella etc., 2601.14361

- Improve the precision of LCSR predictions

  More precise LCDAs of B meson and light mesons;

  more systematic study on power suppressions;

  Reduction of the uncertainty from quark hadron dualities： e.g. reverse problem approach

Summary

- There still exist discrepancy between inclusive and exclusive determination of V_{cb} and V_{ub}

- LCSR can reduce the uncertainty of the form factors from Lattice at large recoil region.

- Improving the precision of LCDAs and other nonperturbabtion parameters is required to deduce the uncertainties