



PRECISION FRONTIER OF PQCD WITH NNLOJET

第6届LHCb前沿物理研讨会

Xuan Chen

Shandong University

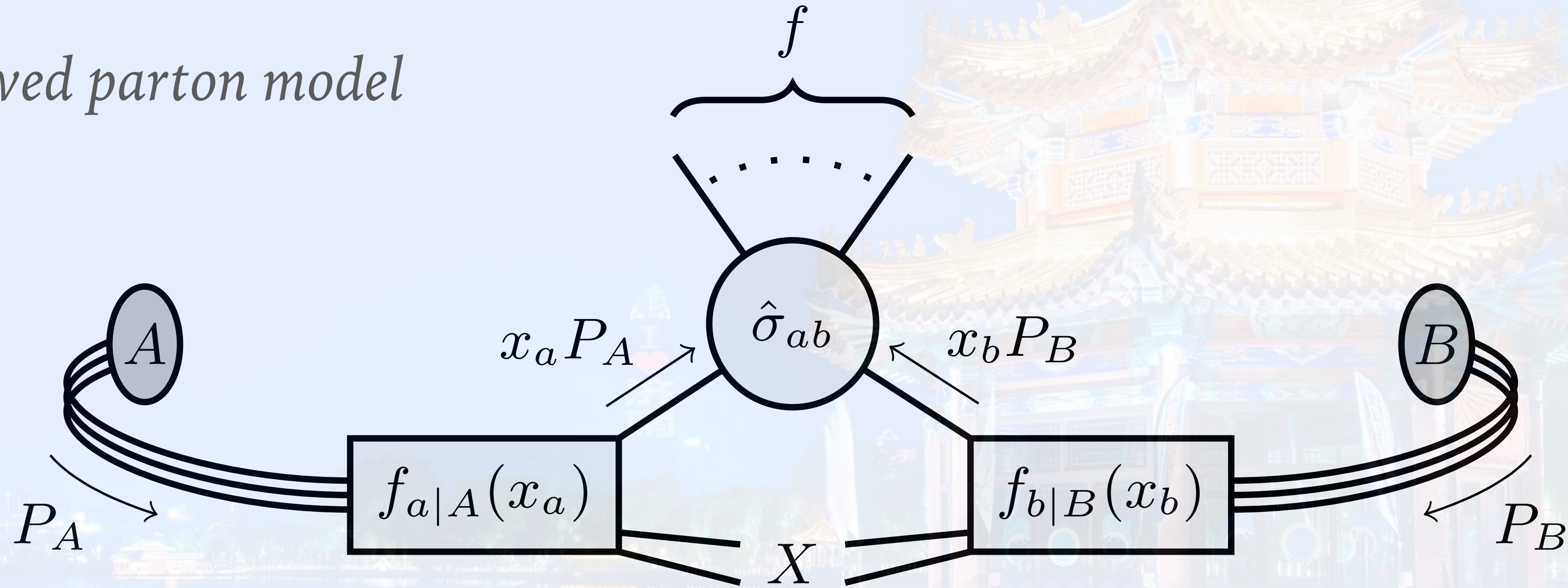
Guang Zhou, 24 May, 2026

NNLO
JET



Precision Predictions at Hadron Collider

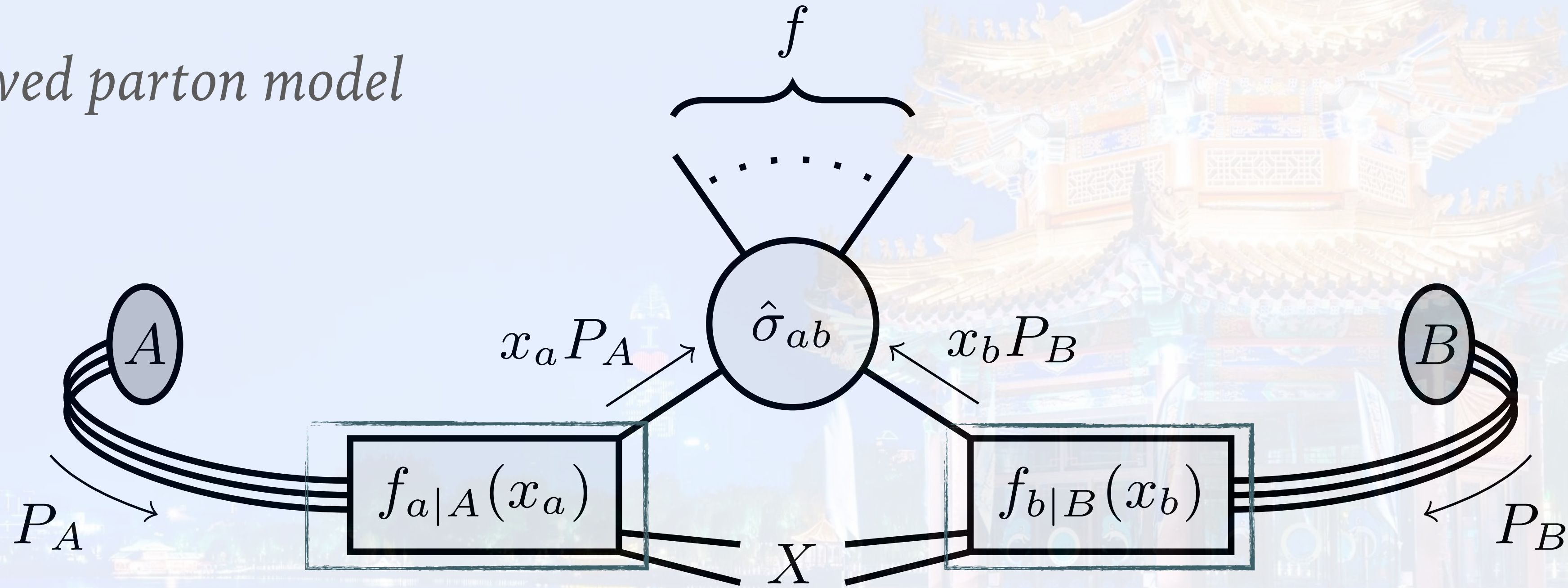
QCD improved parton model



$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

Precision Predictions at Hadron Collider

QCD improved parton model



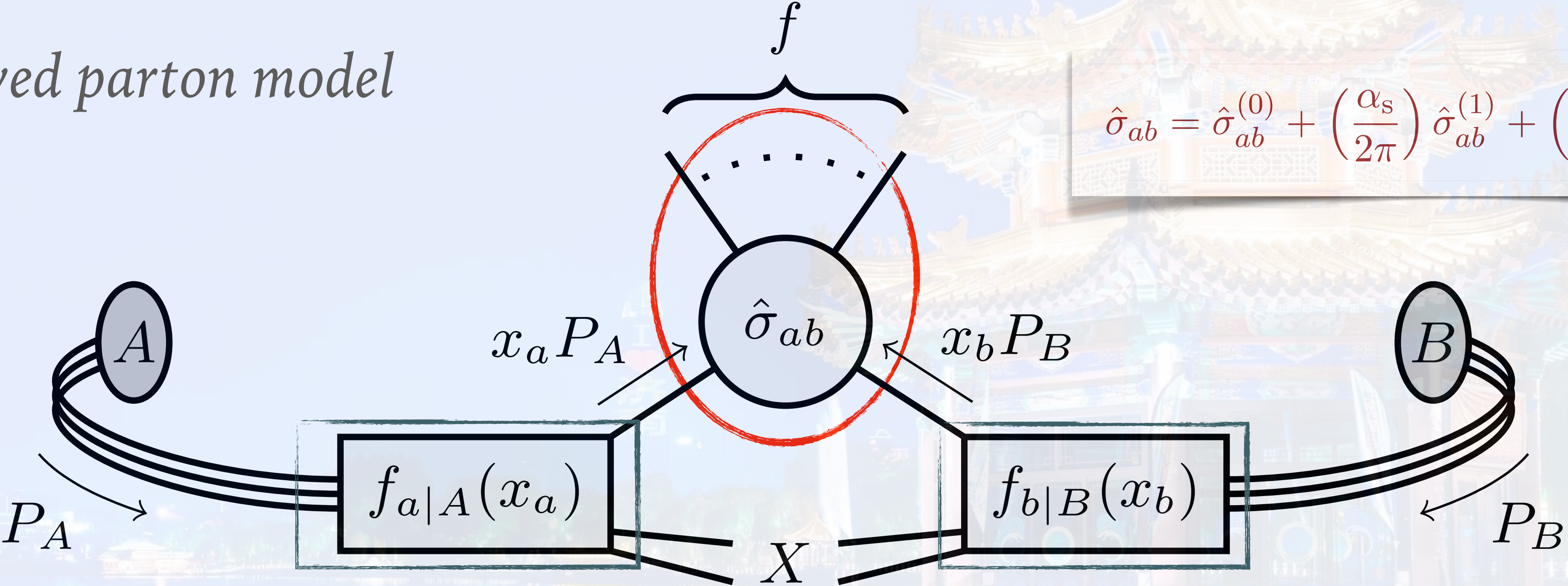
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Parton distribution functions
(Energy evolution from all exp.)

$\pm 3\text{-}5\%$ at LHC energy

Precision Predictions at Hadron Collider

QCD improved parton model



$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \dots$$

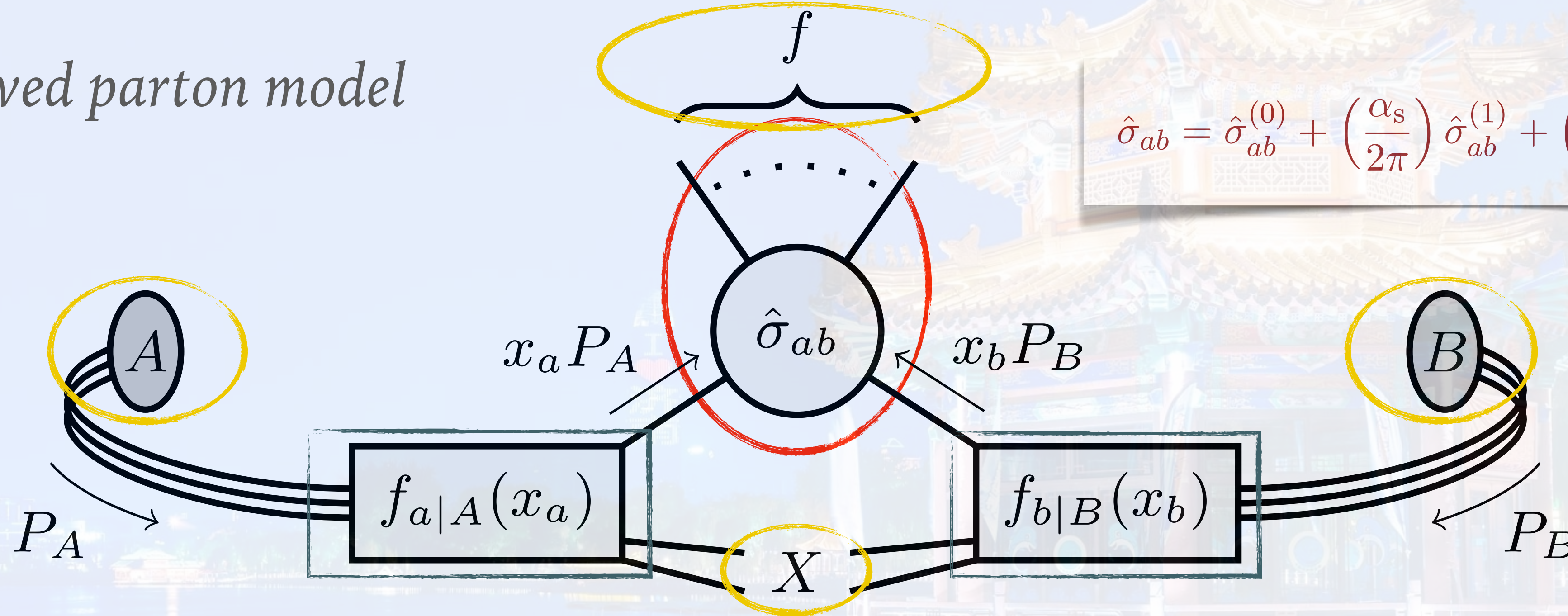
$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

Parton distribution functions
(Energy evolution from all exp.)
± 3-5 % at LHC energy

Hard scattering
(Perturbative quantum field theory)
± 1~3 % level!

Precision Predictions at Hadron Collider

QCD improved parton model



$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}_{ab}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}_{ab}^{(2)} + \dots$$

$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b f_{a|A}(x_a) f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

Parton distribution functions
(Energy evolution from all exp.)

$\pm 3\text{-}5\%$ at LHC energy

Hard scattering
(Perturbative quantum field theory)

$\pm 1\text{-}3\%$ level!

non-perturbative effects
(Fragmentation, hadronisation)

$\pm \Lambda_{\text{QCD}}/\sqrt{\hat{s}}$

State-of-the-Art QCD Calculations @ NNLO

Marcoli's slide @ Loop Summit 2

Antenna subtraction,
NNLOJET

$pp \rightarrow \Upsilon \Upsilon j$

$pp \rightarrow Htt$
soft approx.
massification (LC)

$pp \rightarrow Wtt$
soft approx.
massification (LC)

$pp \rightarrow Wbb$
massification (LC)

$pp \rightarrow Htt$
soft approx.
massification (LC)

$pp \rightarrow \Upsilon \Upsilon \Upsilon$
LC

qT slicing + PS, MiNNLOPS

$pp \rightarrow Zbb$
massification (LC)

qT slicing, MATRIX

<p>Precise predictions for $t\bar{t}H$ production at the LHC: inclusive cross section and differential distributions</p> <p>Simone Devoto^a, Massimiliano Grazzini^b, Stefan Kallweit^b, Javier Mazzitelli^c and Chiara Savoini^d</p> <p>^aDepartment of Physics and Astronomy, Ghent University</p> <p>PHYSICAL REVIEW LETTERS 131, 231901 (2023)</p>
<p>Precise Predictions for the Associated Production of a W Boson with a Top-Antitop Quark Pair at the LHC</p> <p>Luca Buonocore¹, Simone Devoto², Massimiliano Grazzini¹, Stefan Kallweit³, Javier Mazzitelli⁴, Luca Rottoli¹ and Chiara Savoini¹</p> <p>PHYSICAL REVIEW D 107, 074032 (2023)</p>
<p>Associated production of a W boson and massive bottom quarks at next-to-next-to-leading order in QCD</p> <p>Luca Buonocore¹, Simone Devoto², Stefan Kallweit³, Javier Mazzitelli⁴, Luca Rottoli¹ and Chiara Savoini¹</p> <p>PHYSICAL REVIEW LETTERS 130, 111902 (2023)</p>
<p>Higgs Boson Production in Association with a Top-Antitop Quark Pair in Next-to-Next-to-Leading Order QCD</p> <p>Stefano Catani^{1,2}, Simone Devoto^{2,1}, Massimiliano Grazzini^{3,1}, Stefan Kallweit^{4,1}, Javier Mazzitelli^{5,6,1} and Chiara Savoini^{5,1}</p> <p>PHYSICAL REVIEW LETTERS 127, 152001 (2021)</p>
<p>Triphoton production at hadron colliders in NNLO QCD</p> <p>Stefan Kallweit^{a,*}, Vasily Sotnikov^{b,*}, Marius Wiesemann^{b,*}</p> <p>PHYSICAL REVIEW LETTERS 127, 152001 (2021)</p>

$pp \rightarrow Hbb$
massification

Precision frontier of pQCD with NNLOJET

Sector improved residue subtraction,
STRIPPER

$pp \rightarrow \Upsilon jj$

$pp \rightarrow jjj$
LC

$pp \rightarrow Wbb$
LC, mb=0

$pp \rightarrow jjj$
LC

$pp \rightarrow \Upsilon \Upsilon j$
LC

$pp \rightarrow \Upsilon \Upsilon \Upsilon$
LC

$pp \rightarrow jjj$
LC, gluons

Isolated photon production in association with a jet pair through next-to-next-to-leading order in QCD
Simon Badger^a, Michał Czakon^b, Heribertus Bayu Hartanto^c, Ryan Moodie^a, Tiziano Peraro^d, Rene Poncelet^e and Simone Zoia^a

NNLO QCD corrections to event shapes at the LHC
Manuel Alvarez^a, Josu Cantero^d, Michał Czakon^a, Javier Llorente^c, Alexander Mitov^b and Rene Poncelet^b

Next-to-next-to-leading order QCD corrections to $Wb\bar{b}$ production at the LHC
Heribertus Bayu Hartanto^{1,2}, Rene Poncelet^{1,3}, Andrei Popescu^{1,3} and Simone Zoia^{2,3}

Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC
Michał Czakon^a, Alexander Mitov^b and Rene Poncelet^c

NNLO QCD corrections to diphoton production with an additional jet at the LHC
Herschel A. Chawdhry^a, Michał Czakon^b, Alexander Mitov^c and Rene Poncelet^d

NNLO QCD corrections to three-photon production at the LHC
Herschel A. Chawdhry^a, Michał Czakon^b, Alexander Mitov^c and Rene Poncelet^d

Precise Predictions for Event Shapes in Diphoton Production at the LHC
Federico Buccioni^{1,2}, Xuan Chen^{3,4}, Wei-Jie Feng^{3,4}, Thomas Gehrmann^{2,4}, Alexander Huss^{4,5} and Matteo Marcoli^{2,4}

Automation of antenna subtraction in colour space: gluonic processes
X. Chen^{a,b}, T. Gehrmann^a, E.W.N. Glover^a, A. Huss^a and M. Marcoli^a

Higgs boson production in association with massive bottom quarks at NNLO+PS
Christian Biello^a, Javier Mazzitelli^b, Aparna Sankar^{a,c}, Marius Wiesemann^a and Giulia Zanderighi^{a,c}

Next-to-next-to-leading order event generation for Z-boson production in association with a bottom-quark pair
Javier Mazzitelli¹, Vasily Sotnikov², Marius Wiesemann³

State-of-the-Art QCD Calculations @ N3LO

- Several phenomenologically relevant results despite the extreme complexity.
- Available techniques are applicable to limited cases with high quality EXP data.
- New approaches must be developed for more complicated scattering.

(10 M → X00 k CPU hours)

◇ qT slicing

⊙ τ slicing

* Projection-2-Born

† Antenna subtraction

Precision frontier of pQCD with NNLOJET

PHYSICAL REVIEW LETTERS 128, 052001 (2022)

Dilepton Rapidity Distribution in Drell-Yan Production to Third Order in QCD

Xuan Chen^{1,2,3,*}, Thomas Gehrmann^{1,†}, Nigel Glover^{4,‡}, Alexander Huss^{5,§}, Tong-Zhi Yang^{1,||} and Hua Xing Zhu^{6,¶}

PHYSICAL REVIEW LETTERS 128, 252001 (2022)

Third-Order Fiducial Predictions for Drell-Yan Production at the LHC

Xuan Chen^{1,2}, Thomas Gehrmann³, Nigel Glover⁴, Alexander Huss⁵, Pier Francesco Monni⁵, Emanuele Re^{6,7}, Luca Rottoli³ and Paolo Torrielli⁸

Physics Letters B 840 (2023) 137876

Transverse mass distribution and charge asymmetry in W boson production to third order in QCD

Xuan Chen^{a,b,c,*}, Thomas Gehrmann^c, Nigel Glover^d, Alexander Huss^e, Tong-Zhi Yang^c and Hua Xing Zhu^f

Third order QCD predictions for fiducial W-boson production

John Campbell^a and Tobias Neumann^b

Fully differential Higgs boson pair production at N³LO with top quark mass effects

Xuan Chen^a, Yuesheng Dai^a, Hai Tao Li^a, Shi-Yuan Li^a, Hua-Sheng Shao^b and Jian Wang^{a,c}

Next-to-next-to-next-to-leading order QCD corrections to photon-pair production

Michał Czakon^{1,‡}, Felix Eschment^{1,‡}, Terry Generet^{2,‡} and Rene Poncelet^{3,§}

pp → γγ

JHEP11

PHYSICAL REVIEW LETTERS 127, 072002 (2021)

pp → H

Fully Differential Higgs Boson Production to Third Order in QCD

X. Chen^{1,2,3}, T. Gehrmann¹, E. W. N. Glover⁴, A. Huss⁵, B. Mistlberger⁶ and
¹Department of Physics, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland
²Institute for Theoretical Physics, Karlsruhe Institute of Technology, 76131 Karlsruhe, Germany

PHYSICAL REVIEW LETTERS 134, 251905 (2025)

H → bb*

N³LO predictions for the decay of the Higgs boson to bottom quarks

PHYSICAL REVIEW LETTERS 134, 251905 (2025)

H → JJ*

Jet Rates in Higgs Boson Decay at Third Order in QCD

Elliot Fox¹, Aude Gehrmann-De Ridder^{2,3}, Thomas Gehrmann³, Nigel Glover¹, Matteo Marcoli¹ and Christian T. Preuss⁴

Phys. Lett. B 869 (2025) 139804

Letter

H → JJ*

Jet production at electron-positron colliders at next-to-next-to-next-to-leading order in QCD

Xuan Chen^a, Petr Jakubčík^{b,*}, Matteo Marcoli^c, Giovanni Stagnitto^d

^aSchool of Physics, Shandong University, Shandong, Jinan, 250100, China
^bPhysik-Institut, Universität Zürich, Winterthurerstrasse 190, Zürich, CH-8057, Switzerland
^cInstitute for Particle Physics Phenomenology, Department of Physics, Durham University, Durham, DH1 3LE, UK
^dUniversità degli Studi di Milano-Bicocca & INFN, Piazza della Scienza 3, Milano, 20126, Italy

ABSTRACT

We present the first application of antenna subtraction at next-to-next-to-next-to-leading order (N³LO) in QCD by computing fully differential predictions for two-jet production at electron-positron colliders. We illustrate the structure of the infrared counterterms and provide results for the N³LO correction to the two-jet production rate and to the leading-jet energy. Our work constitutes the first direct calculation of jet production at electron-positron colliders at N³LO and represents the first step in tackling arbitrary processes with jets at this perturbative order.

ARTICLE INFO

Editor: Feng Bo

e⁺e⁻ → JJ†

liders. Its fundamental ingredients are antenna functions [46–48], which encode the singular behavior of real-emission matrix elements analytically integrated over the phase space of the unresolved order to cancel the explicit IR singularities of antenna functions. We present the first application of antenna subtraction to the production of physical cross sections which was computed

NNLOJET: Parton Level Event Generator



A parton-level event generator
for jet cross sections at NNLO QCD accuracy

About NNLOJET is a parton-level event generator for jet cross sections using the antenna subtraction method. It can be used to compute a large number of jet cross sections and related observables in e^+e^- , ep and pp collisions at next-to-next-to-leading order in QCD. NNLOJET contains routines for Monte Carlo phase-space integration, event handling and analysis.

Citation If you are using NNLOJET for a scientific paper, please cite:

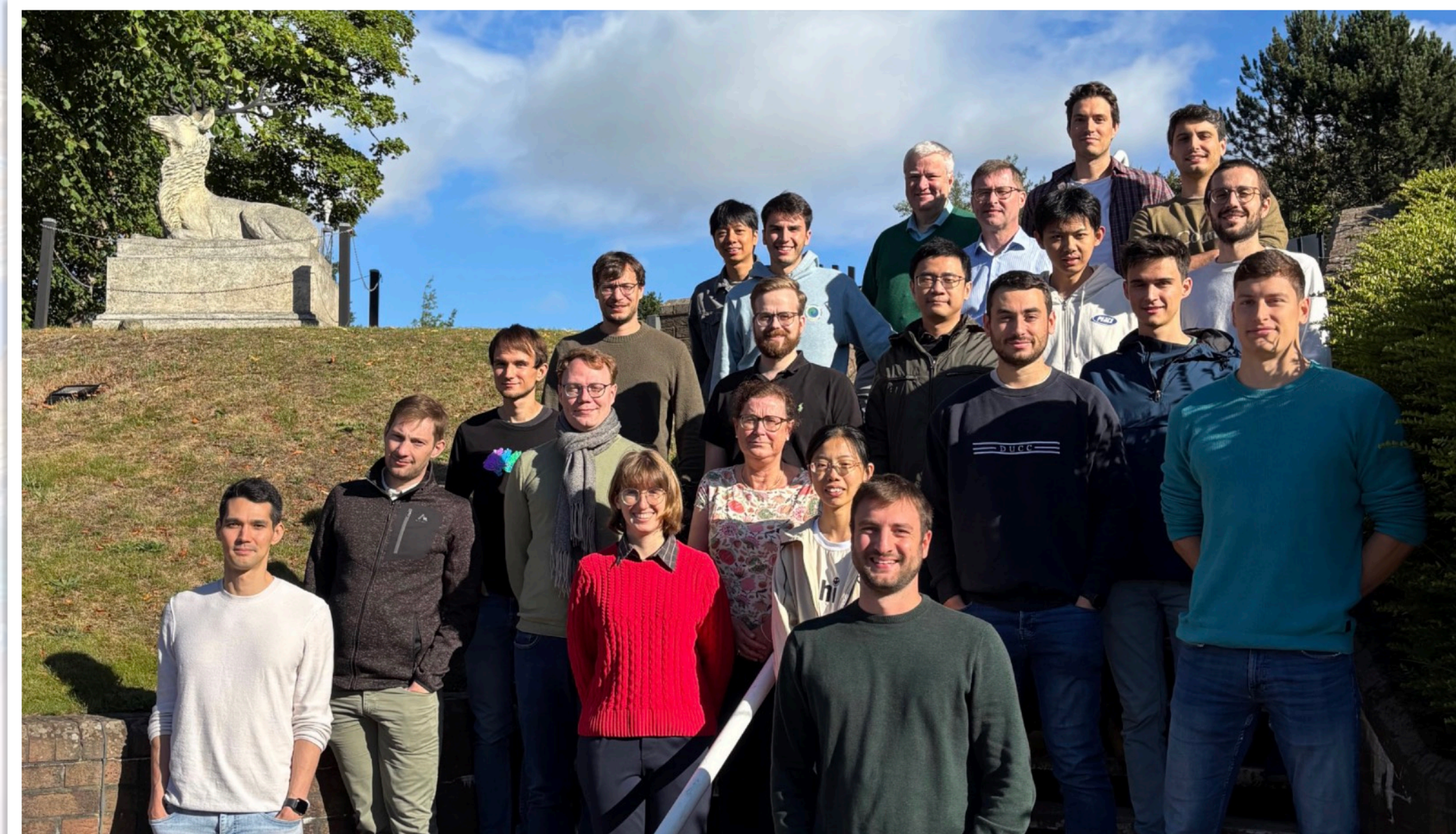
A. Huss et al. (NNLOJET Collaboration)
NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy
[arXiv:2503.22804](https://arxiv.org/abs/2503.22804) [INSPIRE]

Please also cite the relevant references for each process (as included in the .bib file which is automatically written when running NNLOJET through the automatic workflow)

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Contact Please send comments, questions and suggestions to nnlojet-support@cern.ch

<https://nnlojet.hepforge.org/index.html>



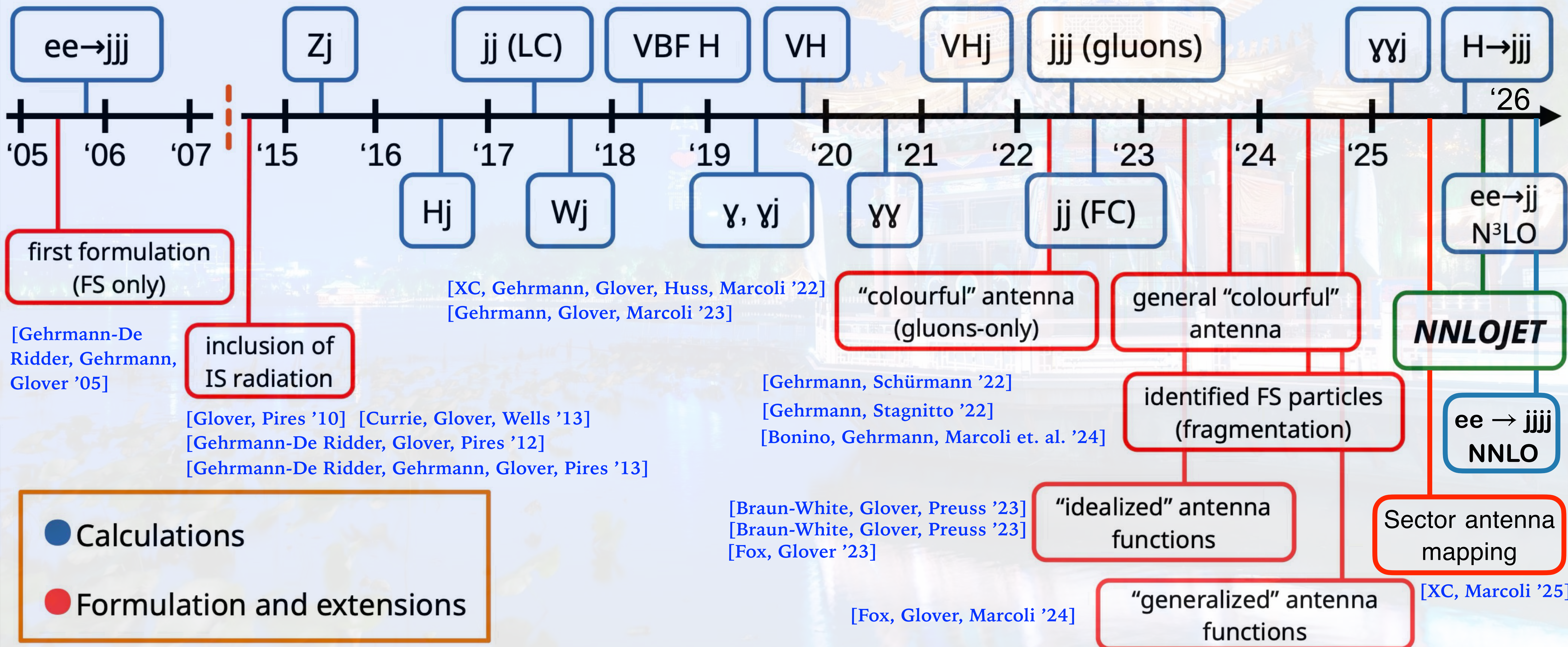
A.Huss, L.Bonino, O.Braun-White, S.Caletti, X.C., J.Cruz-Martinez, J.Currie, Y.S., Dai, W.Feng, G.Fontana, E.Fox, R.Gauld, A.Gehrmann-De Ridder, T. Gehrmann, E.W.N.Glover, M.Höfer, P.Jakubcik, M.Jaquier, M.Löchner, F.Lorkowski, I.Majer, M.Marcoli, F. Merlotti, P.Meinzinger, J.Mo, T. Morgan, J.Niehues, J.Pires, C.Preuss, A.Rodriguez Gracia, K.Schönwald, R.Schürmann, V.Sotnikov, G.Stagnitto, H.S. Sun, D.Walker, J.Whitehead, T.Z.Yang, H.Zhang,

- NNLO parton level event generator
 - Based on antenna subtraction
- Provides infrastructure
 - Process management
 - Phase space, histogram routines
 - Validation and testing
- Parallel computing (MPI) support
- Typical runtimes: 60 k ~ 250 k core-hours

Development of Antenna Subtraction Method

Based on Marcoli's slide @ Loop Summit 2

Successfully applied at NNLO to a variety of processes within the **NNLOJET** Monte Carlo framework



Application of **NNLOJET** at the LHC

$$pp \rightarrow JJ @ \text{NNLO}$$

$$pp \rightarrow \gamma\gamma + \text{jet} @ \text{NNLO}$$

$$pp \rightarrow Z + \text{c-jet} @ \text{NNLO}$$

Coupling Constants of the SM

➤ Great precision for α or G_F

➤ Electromagnetic interaction

$$\alpha \approx 1/137 \quad \Delta\alpha/\alpha = 0.15 \times 10^{-9}$$

➤ Weak interaction

$$G_F \approx 1.17 \times 10^{-5} \text{ GeV}^2 \quad \Delta G_F/G_F = 0.51 \times 10^{-6}$$

➤ Least known SM parameter

$$\alpha_s \approx 0.118 \quad \Delta\alpha_s/\alpha_s = 0.76 \times 10^{-2}$$

➤ Enters all LHC processes

➤ Significant source of Higgs XS uncertainty

➤ Scale dependence predicted

➤ Running of α_s tested only up to ~ 200 GeV pre LHC

➤ Need highest-energy probe to further advance into asymptotic regime

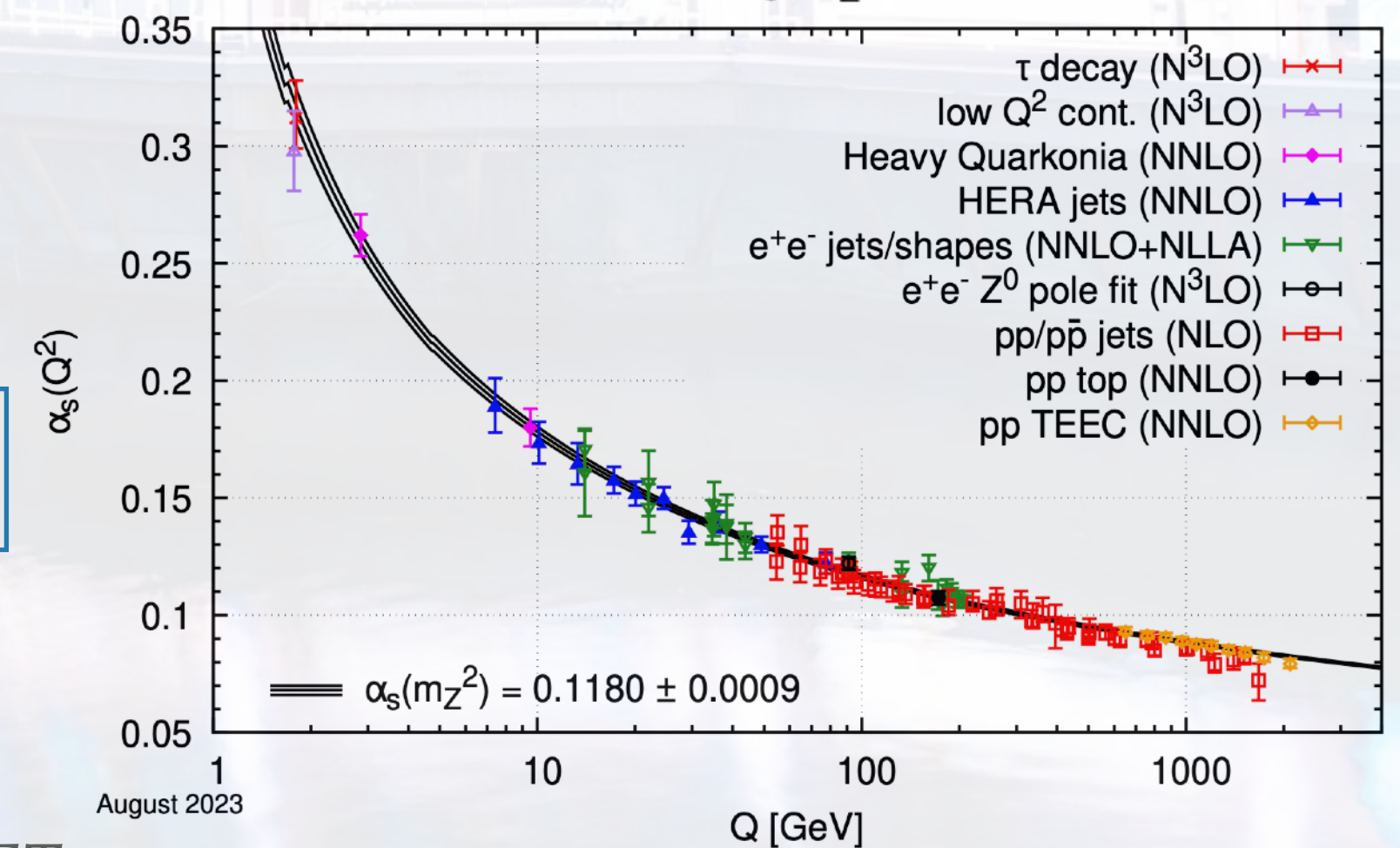
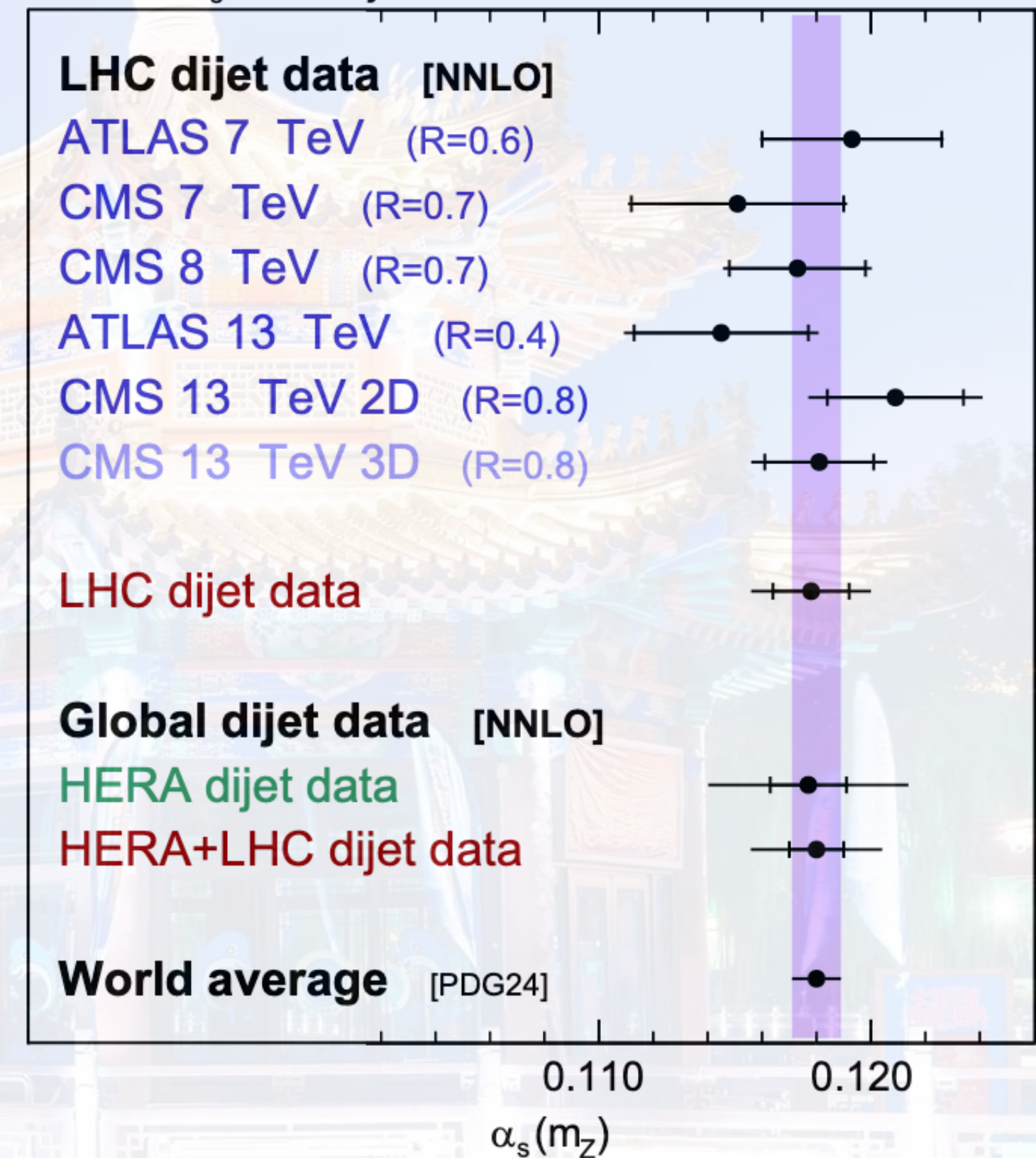
➤ Latest progress

➤ NNLO accuracy for XS prediction including jets

➤ Huge LHC datasets at 13 TeV (and 13.6 TeV)

Run 3 to finish by June 2026 ...
Up to now: $\sim 318 \text{ fb}^{-1}$ at 13.6 TeV

α_s from Dijet Cross Sections in NNLO



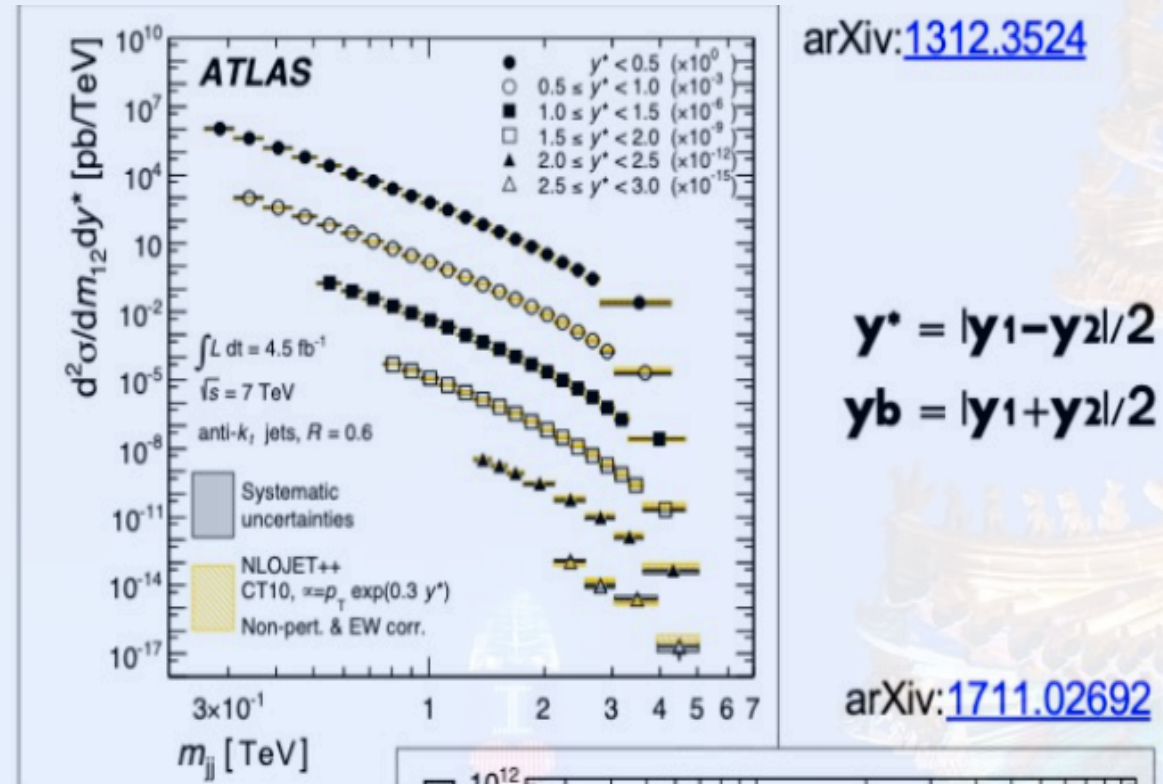
Ahmadova, Britzger, XC, Gaßler et. al.,
Phys. Rev. Lett. 135 (2025) 031903

PDG 2023 update

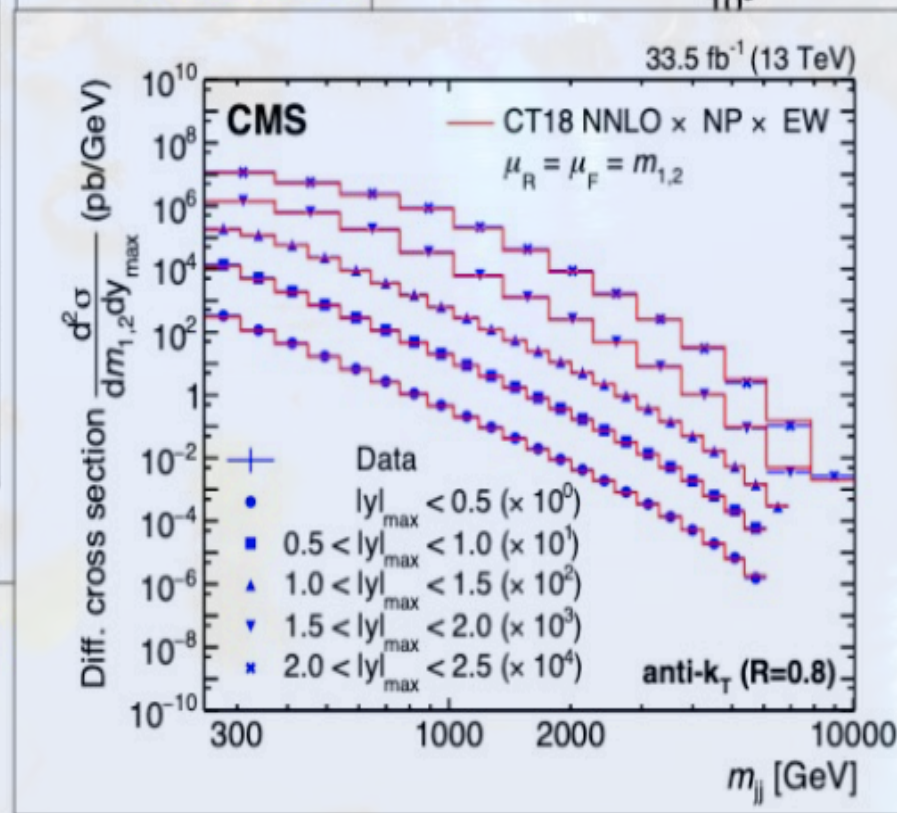
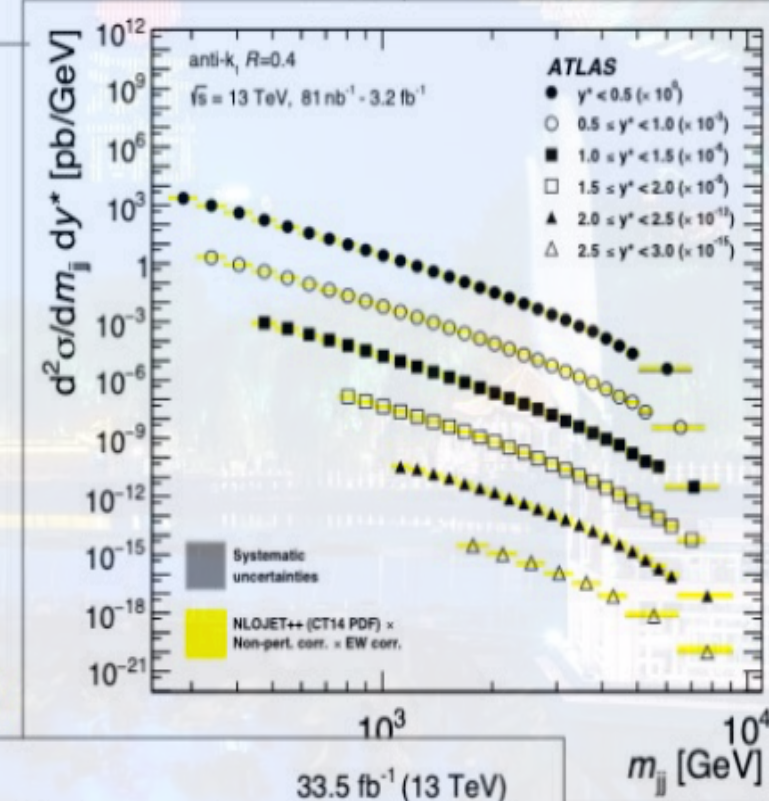
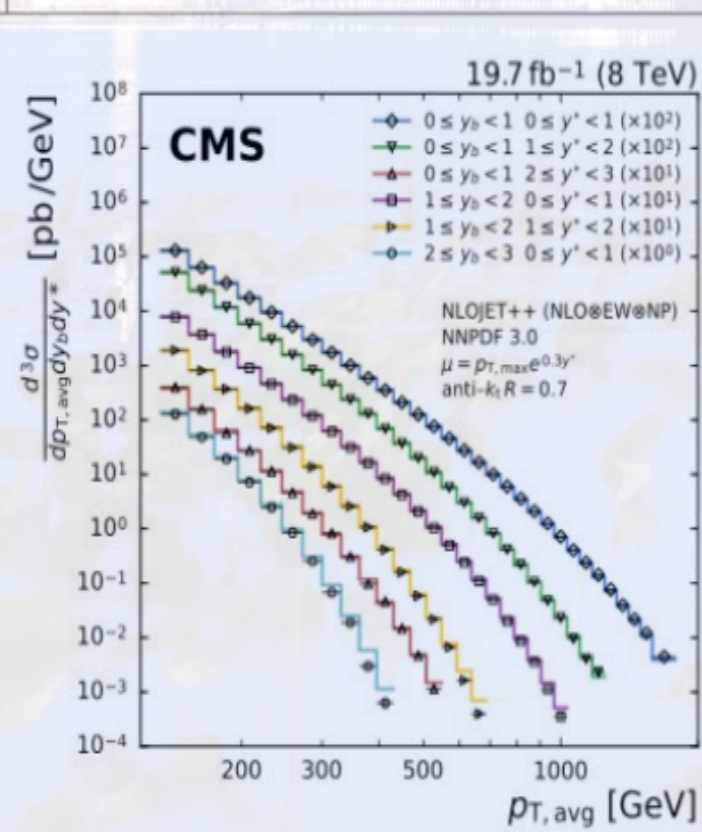
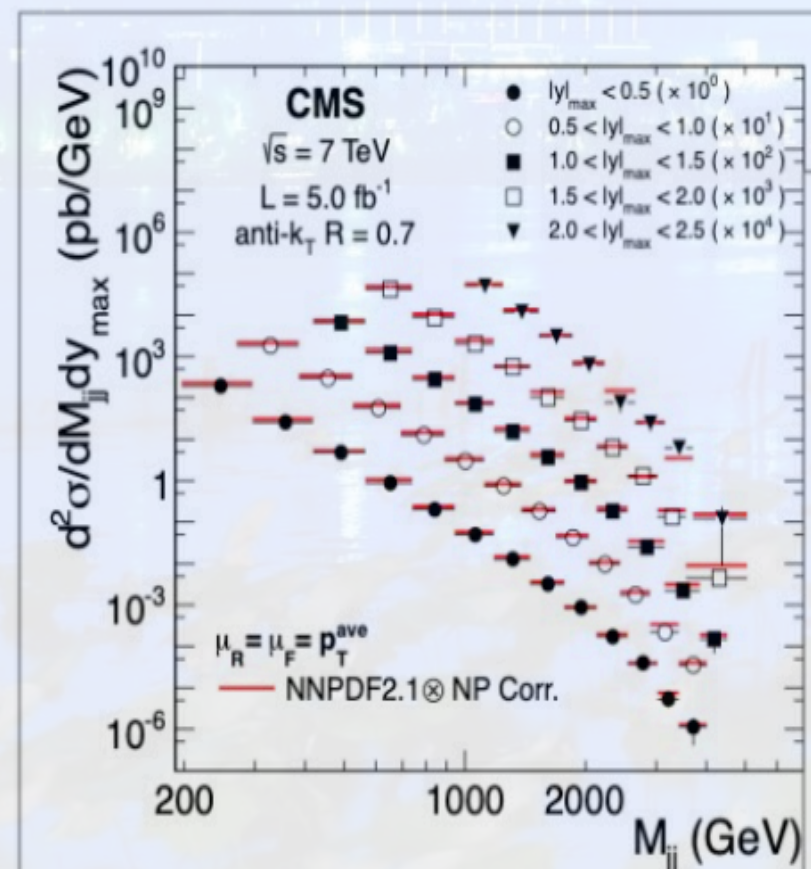
Precise Determination of the α_s

- Abundant production of jets, typically well described by pQCD calculations

Data	\sqrt{s} [TeV]	$d\sigma$	R	\mathcal{L}
ATLAS	7	$\frac{d^2\sigma}{dm_{jj}dy^*}$	0.6	$4.5 \text{ fb}^{-1} \pm 1.8 \%$
CMS	7	$\frac{d^2\sigma}{dm_{jj}dy_{\text{max}}}$	0.7	$5.0 \text{ fb}^{-1} \pm 2.2 \%$
CMS	8	$\frac{d^3\sigma}{d(p_T)_{1,2}dy^*dy_b}$	0.7	$19.7 \text{ fb}^{-1} \pm 2.6 \%$
ATLAS	13	$\frac{d^2\sigma}{dm_{jj}dy^*}$	0.4	$3.2 \text{ fb}^{-1} \pm 2.1 \%$
CMS	13	$\frac{d^2\sigma}{dm_{jj}dy_{\text{max}}}$	0.8	$33.5 \text{ fb}^{-1} \pm 1.2 \%$
CMS	13	$\frac{d^3\sigma}{dm_{jj}dy^*dy_b}$	0.8	$29.6 \text{ fb}^{-1} \pm 1.2 \%$

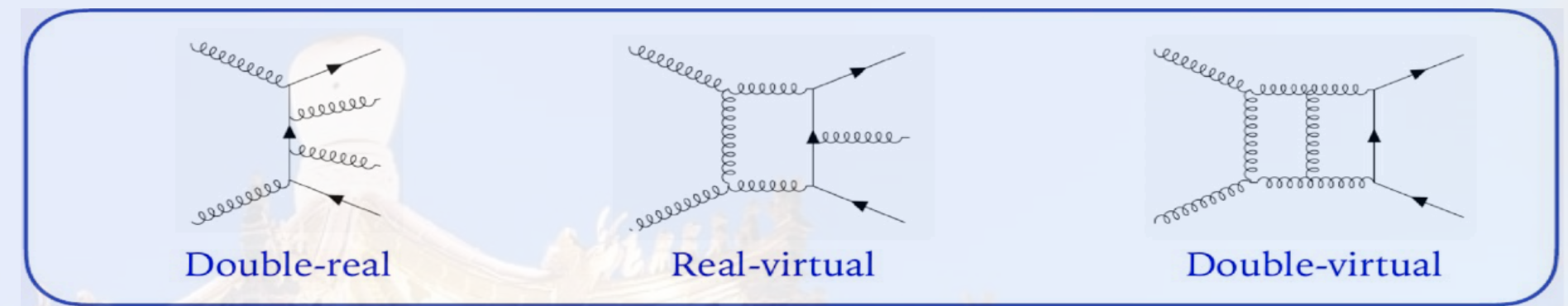


used for additional fit variant
 (2D and 3D cannot be used simultaneously due to unknown correlations)



selection: y_{max} or $y^* = |y_1 - y_2|/2 < 2$; $y_b = |y_1 + y_2|/2 < 1$:

367 LHC cross sections used in α_s determination



- NNLO di-jet cross sections obtained with NNLOJET
NNLOJET Collaboration 2503.22804
- Include all sub-leading colour contributions for the first time in α_s determination with LHC data
XC, Gehrmann, Glover, Huss, Mo JHEP 09 (2022) 025 and JHEP 10 (2022) 040
- Fit algorithm for α_s requires re-calculation of cross sections for varying $\alpha_s(m_Z)$, PDFs, DGLAP evolution
- NNLOJET interfaced to APPLfast library for fast re-computation grids
Britzger, Currie et. al. EPJC 79 (2019) 10, 845 and EPJC 82 (2022) 10, 930
- New APPLfast grids are publicly available in multiple formats on Ploughshare
Ahmadova, Britzger, XC, Gaßler et. al., Phys. Rev. Lett. 135 (2025) 3

Ploughshare
 for all your interpolation grid needs
 Ploughshare allows users from the HEP community to share fast interpolation grids in a standardised way. PDF fitters and those from the experimental collaborations are able to upload their validated grids and access the grids of others quickly and with minimal fuss.

https://ploughshare.web.cern.ch/

Precise Determination of the α_s

PDF grids: APPLfast and PineAPPL

Data	\sqrt{s} [TeV]	$d\sigma$	R	\mathcal{L}
ATLAS [10]	7	$\frac{d^2\sigma}{dm_{jj}dy^*}$	0.6	$4.5 \text{ fb}^{-1} \pm 1.8\%$
CMS [12]	7	$\frac{d^2\sigma}{dm_{jj}dy_{\max}}$	0.7	$5.0 \text{ fb}^{-1} \pm 2.2\%$
CMS [13]	8	$\frac{d^3\sigma}{d\langle p_T \rangle_{1,2} dy^* dy_b}$	0.7	$19.7 \text{ fb}^{-1} \pm 2.6\%$
ATLAS [11]	13	$\frac{d^2\sigma}{dm_{jj}dy^*}$	0.4	$3.2 \text{ fb}^{-1} \pm 2.1\%$
CMS [14]	13	$\frac{d^2\sigma}{dm_{jj}dy_{\max}}$	0.8	$33.5 \text{ fb}^{-1} \pm 1.2\%$
CMS [14]	13	$\frac{d^3\sigma}{dm_{jj}dy^* dy_b}$	0.8	$29.6 \text{ fb}^{-1} \pm 1.2\%$

367 LHC data

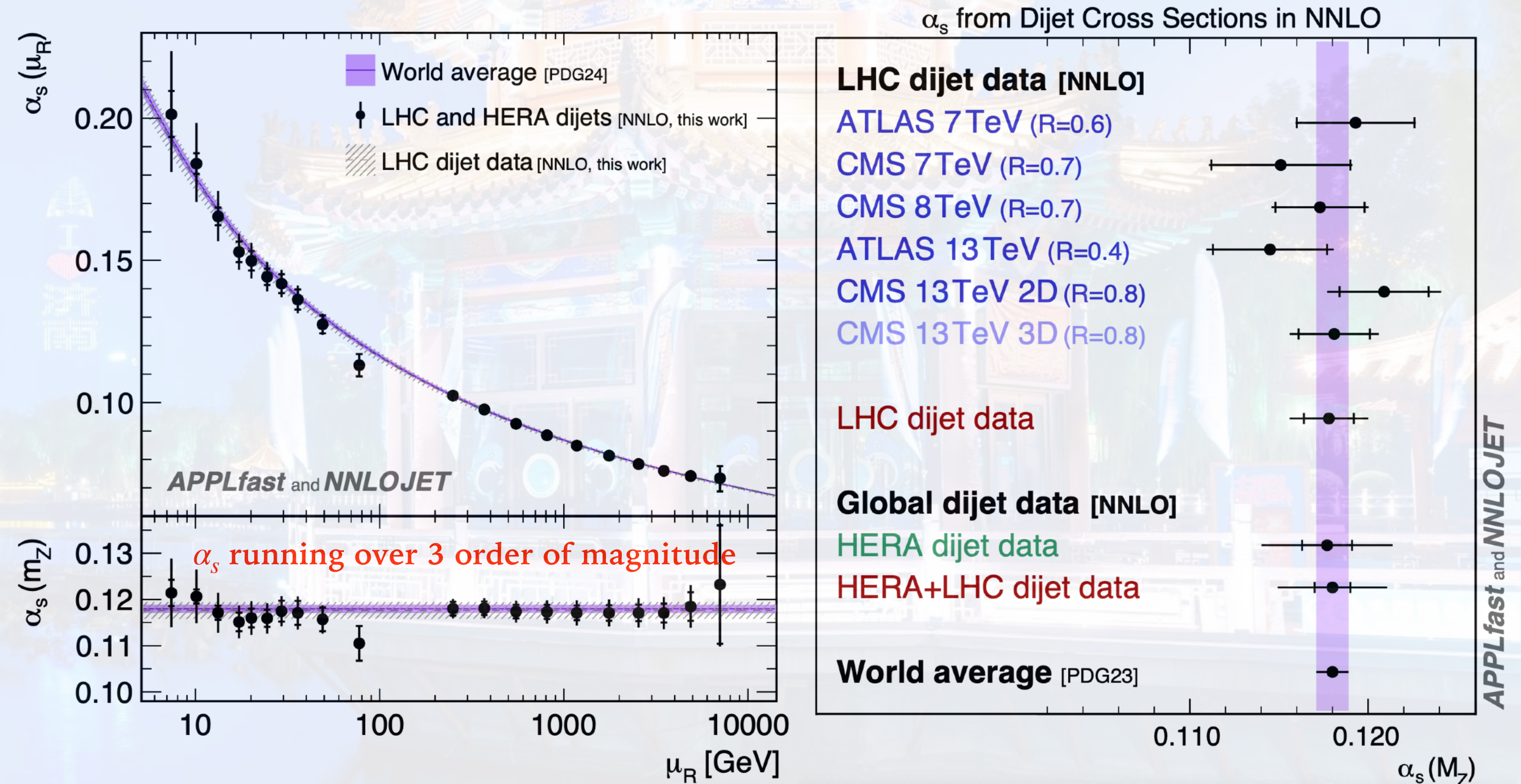


119 HERA data

Data set	χ^2/n_{dof}	$\alpha_s(m_Z)$
ATLAS 7 TeV	74.7/ 77	0.1193 (33) (4) (6)
ATLAS 13 TeV	87.7/106	0.1145 (32) (4) (16)
CMS 7 TeV	50.7/ 45	0.1151 (39) (1) (9)
CMS 8 TeV	37.0/ 56	0.1173 (25) (0) (11)
CMS 13 TeV (2D)	71.6/ 78	0.1209 (25) (2) (20)
CMS 13 TeV (3D)	137.7/112	0.1181 (20) (1) (15)
LHC dijets (CMS13-2D)	335.3/366	0.1178 (14) (0) (17)
LHC dijets (CMS13-3D)	397.9/400	0.1172 (14) (0) (14)
HERA	92.8/118	0.1177 (14) (1) (34)
LHC+HERA (CMS13-2D)	428.4/485	0.1180 (10) (0) (29)
LHC+HERA (CMS13-3D)	491.0/519	0.1177 (10) (0) (27)

$$\chi^2 = \sum_{i,j} \log \frac{S_i}{\sigma_i} (V_{\text{exp}} + V_{\text{NP}} + V_{\text{NNLOstat}} + V_{\text{PDF}})^{-1}_{ij} \log \frac{S_j}{\sigma_j}$$

ζ_i LHC or HERA jet data
 σ_i NNLO theory
 V covariance matrices



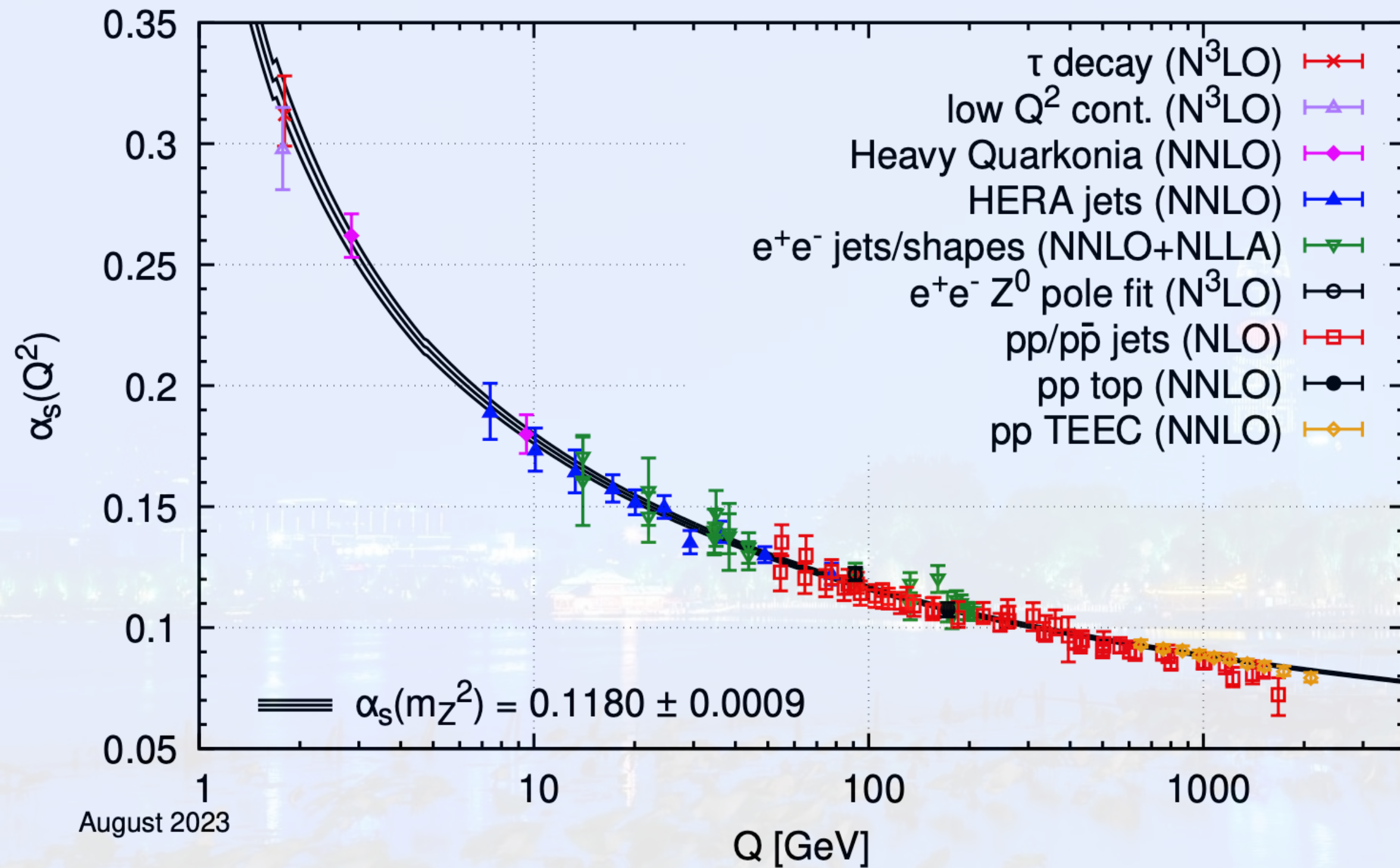
- All LHC and HERA di-jet double- and triple-differential measurements.
- APPLfast PDF grids from NNLOJET NNLO di-jet predictions.
- Fitting of the strong coupling constant: $\alpha_s(m_Z) = 0.1178 \pm 0.0022$.

Ahmadova, Britzger, XC, Gaßler et. al., Phys.Rev.Lett. 135 (2025) 031903

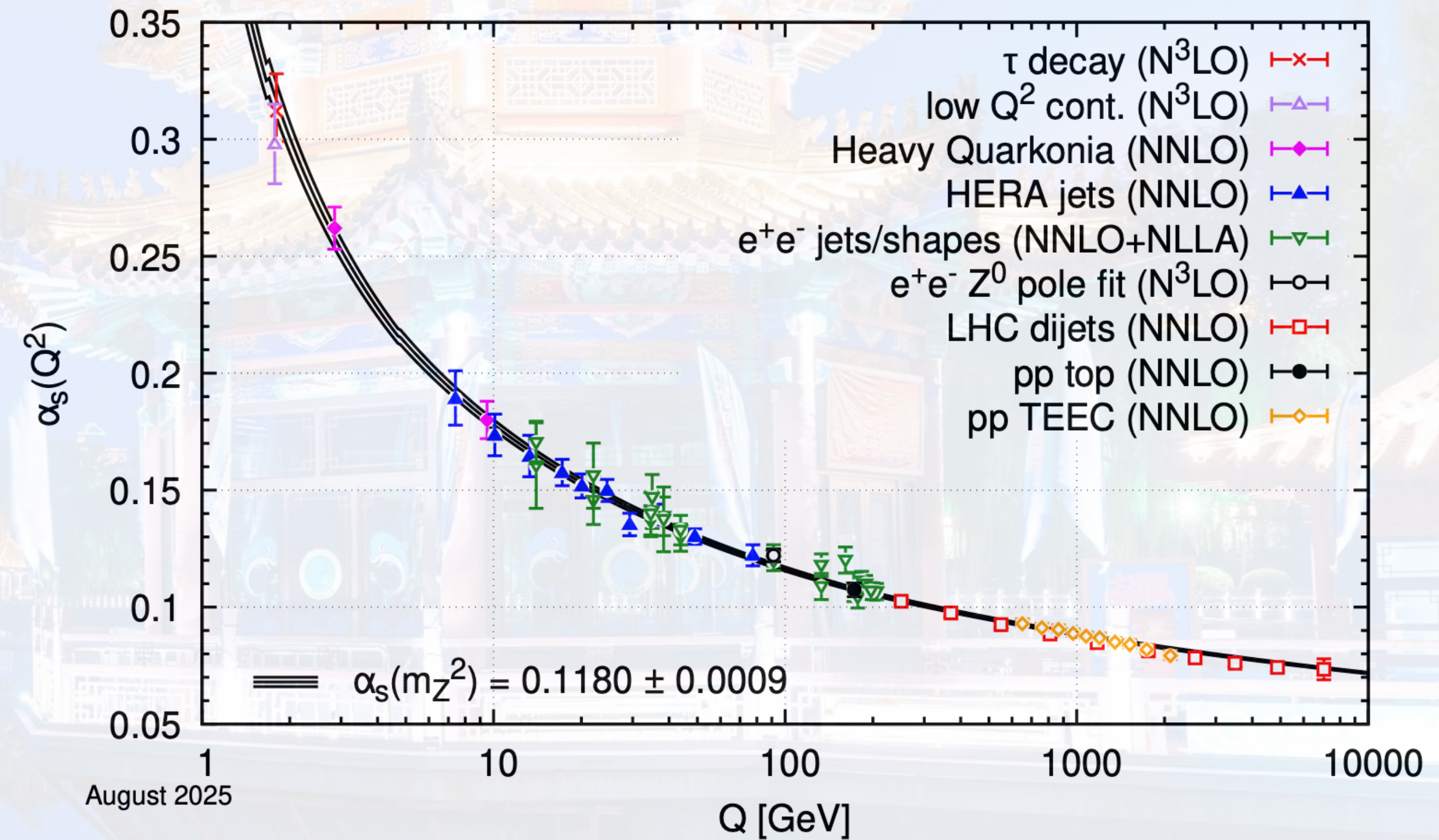
Precise Determination of the α_s

Based on Klaus's slide @ alphas-2025 Workshop

PDG 2023 α_s running



PDG 2025 α_s running



- Strong coupling α_s determined from di-jet data for the first time based on complete NNLO calculations.
- Consistent with the expectation from the RGE running at all scales from 2 GeV to 7 TeV.
- Di-jet 2-D and 3-D re-computation grids also provides NNLO accuracy benchmark for PDFs fittings.
- Improved α_s and PDFs accuracy will benefit precision predictions of the Higgs boson.

Application of **NNLOJET** at the LHC

$pp \rightarrow JJ @ \text{NNLO}$

$pp \rightarrow \gamma\gamma + \text{jet} @ \text{NNLO}$

$pp \rightarrow Z + \text{c-jet} @ \text{NNLO}$

$\gamma\gamma$ Production at NNLO QCD

$pp \rightarrow \gamma\gamma$ @ NNLO QCD (α_s^2)

► Achieved by multiple groups with slicing and local subtraction

► **1st** calculation in 2011 by 2γ NNLO (erratum in 2016):

► $\sqrt{s} = 14$ TeV, MSTW2008 PDF with match fixed order.

► **Smooth γ isolation**: $E_T^{\text{iso}} = p_T^\gamma/2$, $\Delta R \leq 0.4$, $n = 1$

► $M_{\gamma\gamma} \in [20, 250]$ GeV, $|y^\gamma| \in [0, 2.5]$, $\alpha = 1/137$

► $p_T^{\gamma_1} \geq 40$ GeV, $p_T^{\gamma_2} \geq 25$ GeV, $q_T^{\text{cut}} \geq 4 \times 10^{-2}$ GeV

► **2nd** calculation in 2016 by MCFM:

► Find disagreement with 2γ NNLO \rightarrow erratum

► $q_T^{\text{cut}} \geq 1 \times 10^{-2}$ or $\tau^{\text{cut}} \geq 4 \times 10^{-4}$ GeV extrapolated to 0

► **3rd** calculation in 2017 by MATRIX: [MATRIX: M. Grazzini, S. Kallweit, M. Wiesemann, EPJC. 78 \(2018\) 537](#)

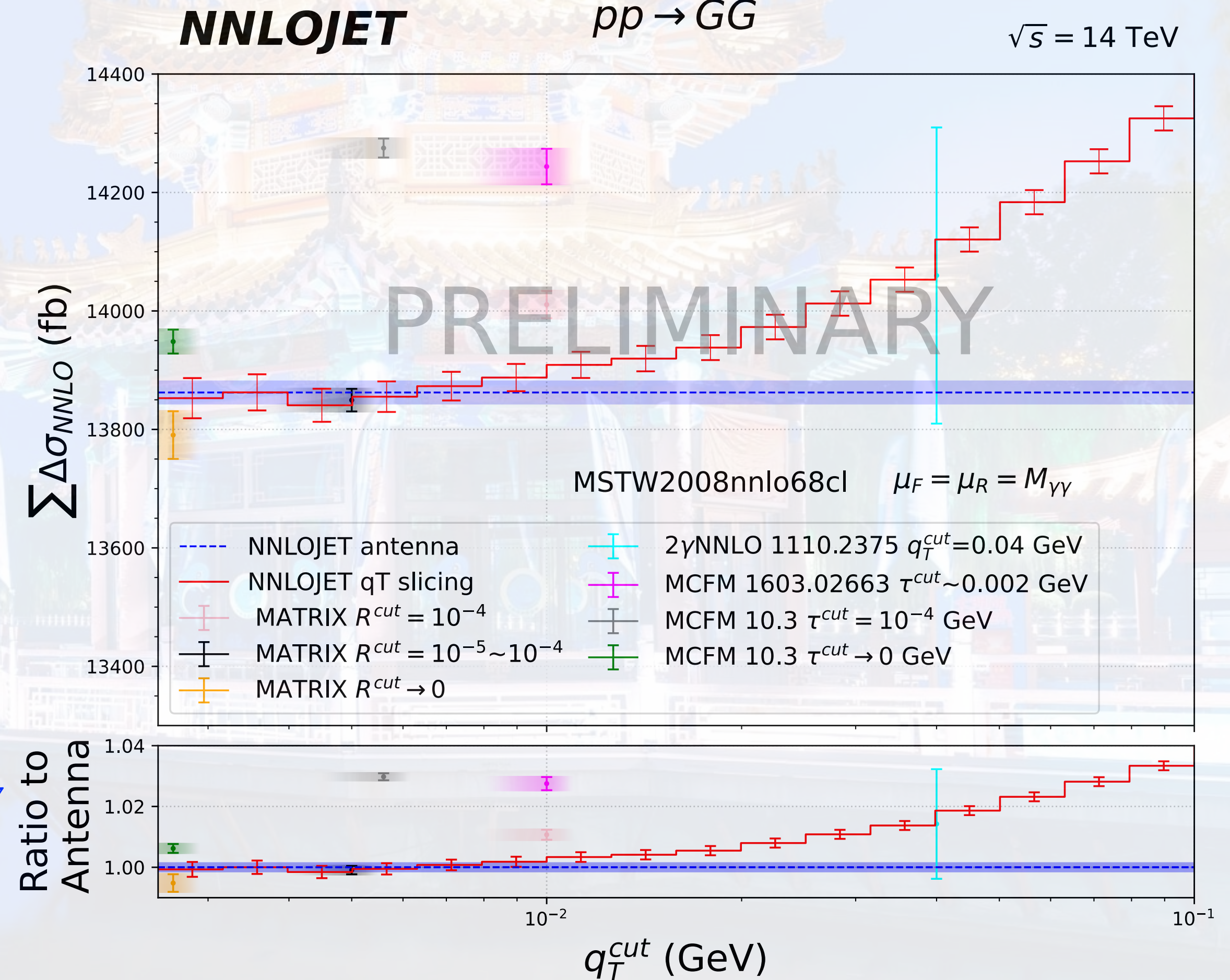
► $r^{\text{cut}} \geq 5 \times 10^{-4}$ ($r^{\text{cut}} = q_T^{\text{cut}}/M_{\gamma\gamma}$) and extrapolation

► **4th** calculation in 2020 by NNLOJET:

► First **local subtraction** implementation (no slicing dependence of r^{cut} , q_T^{cut} , τ^{cut})

► **Including** 2-loop colour singlet and gluon induced contribution

[NNLOJET: T. Gehrmann, N. Glover, A. Huss, J. Whitehead, JHEP. 01 \(2021\) 108](#)



Translate all slicing parameters to q_T^{cut} , all published results are questionable!

Newer version of public tools can make correct predictions

$\gamma\gamma$ Production up to N3LO QCD

- Apply q_T -slicing at N3LO with **SCET factorisation** and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_p \sigma_{LO}^{\gamma\gamma} H_{q\bar{q}} \left[\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} I_{pk}(z_1, b_T^2, \mu) f_{k/h_1}(x_1/z_1, \mu) \right. \\ \left. \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} I_{pj}(z_2, b_T^2, \mu) f_{j/h_2}(x_2/z_2, \mu) S(b_\perp, \mu) \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- All factorised functions are known @ N3LO:

1) 3-loop hard function $H_{q\bar{q}}^{(3)}$:

Baikov, Chetyrkin et. al., Phys.Rev.Lett. 102 (2009) 212002, Gehrmann, Glover et. al., JHEP 06 (2010) 094

2) TMD soft function $S(b_\perp, \mu)$ at α_s^3 : *Li, Zhu, Phys.Rev.Lett. 118 (2017) 022004*

3) Matching kernel of TMD beam function $I_{pk}(z, b_\perp, \mu)$ at α_s^3 :

Luo, Yang, Zhu, Zhu, Phys.Rev.Lett 124 (2020) 092001, Ebert, Mistlberger, Vita, JHEP 09 (2020) 146

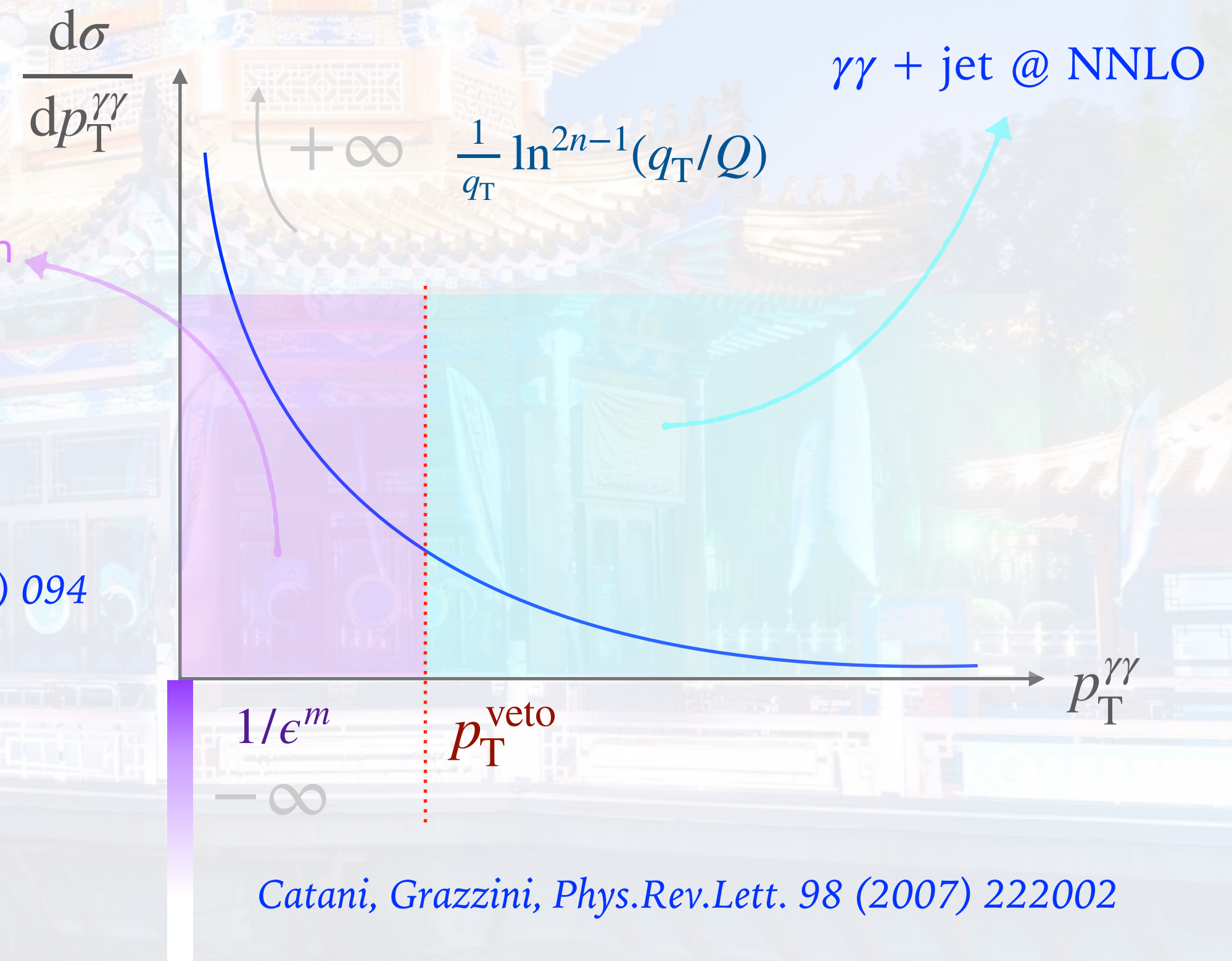
- Apply p_T^{veto} to factorise N3LO contribution into two parts:

$$d\sigma_{N^3LO}^{\gamma\gamma, a} = [\mathcal{H}^{\gamma\gamma} \otimes d\sigma^{\gamma\gamma}]_{N^3LO} \Big|_{\delta(p_T^{\gamma\gamma})} + [d\sigma_{NNLO}^{\gamma\gamma+jet} - d\sigma_{N^3LO}^{\gamma\gamma CT}]_{p_T^{\gamma\gamma} > p_T^{\text{veto}}} + \mathcal{O}((p_T^{\text{veto}}/Q)^2)$$

Competing interests: p_T^{veto} as small as possible \leftrightarrow p_T^{veto} as large as possible

\hookrightarrow suppress power corrections

\hookrightarrow numerical stability & efficiency



Catani, Grazzini, Phys.Rev.Lett. 98 (2007) 222002

$\gamma\gamma$ Production up to N3LO QCD

$pp \rightarrow \gamma\gamma$ @ NNLO QCD (α_s^2)

► Achieved by multiple groups with slicing and local subtraction

► **1st** calculation in 2011 by 2γ NNLO (erratum in 2016):

► $\sqrt{s} = 14$ TeV, MSTW2008 PDF with match fixed order.

► **Smooth γ isolation**: $E_T^{\text{iso}} = p_T^\gamma/2$, $\Delta R \leq 0.4$, $n = 1$

► $M_{\gamma\gamma} \in [20, 250]$ GeV, $|y^\gamma| \in [0, 2.5]$, $\alpha = 1/137$

► $p_T^{\gamma_1} \geq 40$ GeV, $p_T^{\gamma_2} \geq 25$ GeV, $q_T^{\text{cut}} \geq 4 \times 10^{-2}$ GeV

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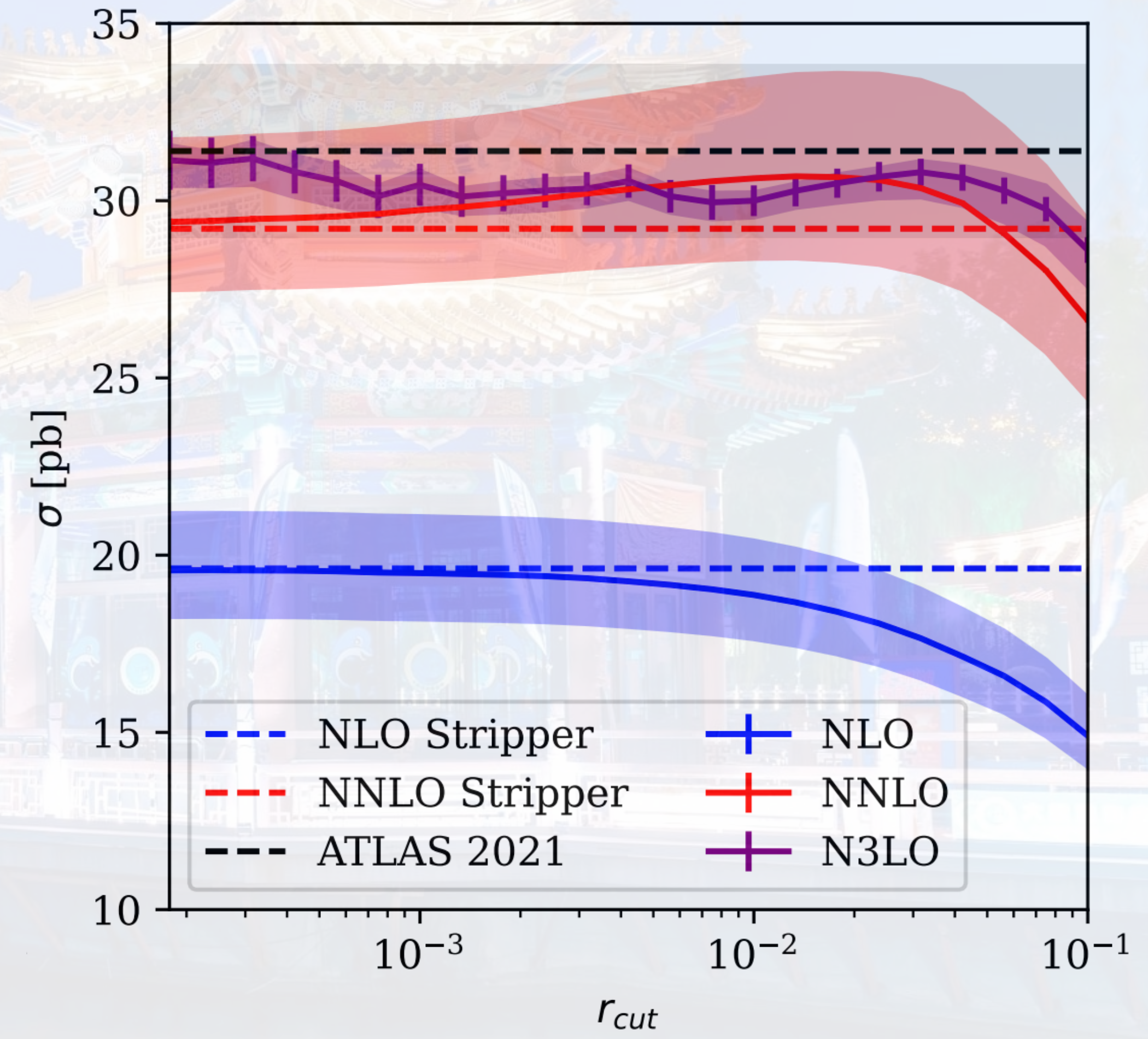
► $r^{\text{cut}} \geq 5 \times 10^{-4}$ ($r^{\text{cut}} = q_T^{\text{cut}}/M_{\gamma\gamma}$) and extrapolation

$pp \rightarrow \gamma\gamma$ @ N3LO QCD (α_s^3)

► The first di-photon production with N3LO QCD correction

► Also with **smooth γ isolation** and **q_T -slicing** at $r^{\text{cut}} \geq 2 \times 10^{-4}$

► Excellent numerical stability control with **10M CPU hours**



M. Czakon, F. Eschment, T. Generet, R. Poncelet, 2604.12613

$$d\sigma_{N^3LO}^{\gamma\gamma,a} = [\mathcal{H}^{\gamma\gamma} \otimes d\sigma^{\gamma\gamma}]_{N^3LO} \Big|_{\delta(p_T^{\gamma\gamma})} + [d\sigma_{NNLO}^{\gamma\gamma+jet} - d\sigma_{N^3LO}^{\gamma\gamma CT}]_{p_T^{\gamma\gamma} > p_T^{\text{veto}}} + \mathcal{O}((p_T^{\text{veto}}/Q)^1) + \mathcal{O}((p_T^{\text{veto}}/Q)^2)$$

Smooth isolation q_T factorisation
power correction power correction

γ + Jet Production at NNLO QCD

Photon isolation: Smooth cone vs. Experiment cone

► Smooth cone isolation [[hep-ph/9801442](https://arxiv.org/abs/hep-ph/9801442)]:

► Cone-dependent threshold energy :

For $0 < r_d < R$:

$$E_T^{had} < E_T^{max}(r_d) = \epsilon_d E_T^\gamma \left(\frac{1 - \cos(r_d)}{1 - \cos(R)} \right)^n$$

► IR safe and fragmentation free: $E_T^{max}(r_d) = \begin{cases} \epsilon_d E_T^\gamma & (r_d \rightarrow R) \\ 0 & (r_d \rightarrow 0) \end{cases}$

► **Adjust** ϵ_d, n to approximate mismatched region.

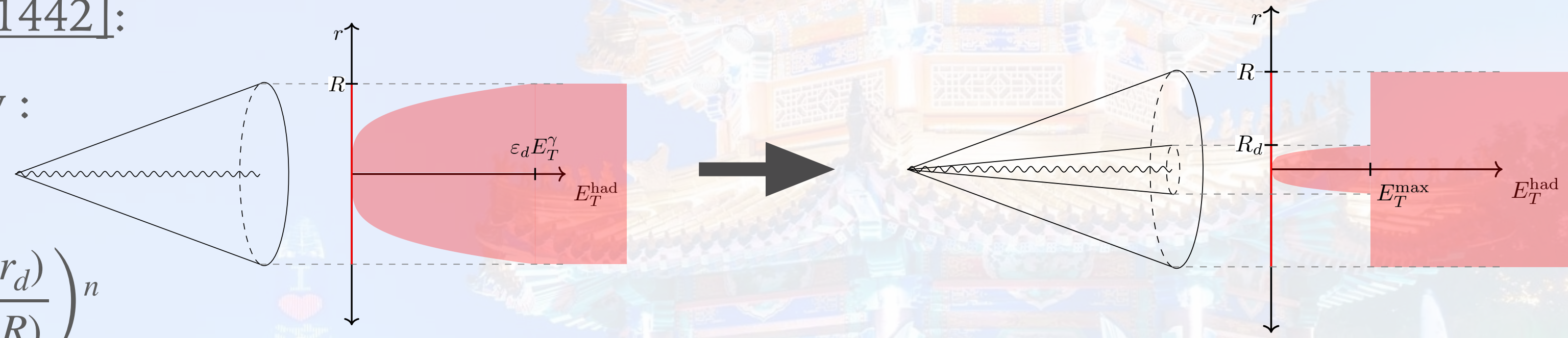
► Fixed cone isolation used by all measurement:

► Apply threshold energy to allow soft partons:

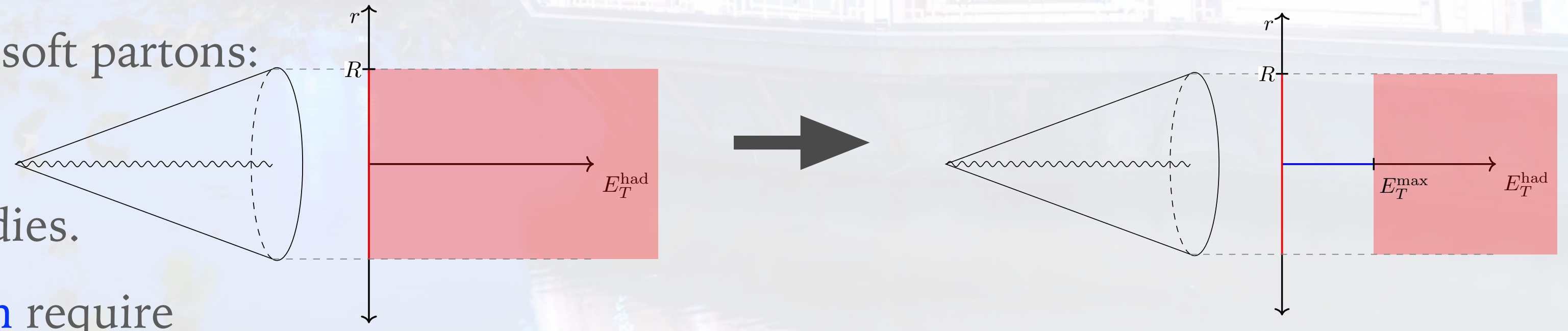
$$E_T^{had} < E_T^{max} = \epsilon E_T^\gamma + E_T^{thres}$$

► Current prescription in EXP studies.

► IR divergence of **collinear photon** require **photon fragmentation** \rightarrow theoretically complicated



Reduced mismatch
between EXP and TH



$\gamma + \text{Jet}$ Production at NNLO QCD

Photon isolation: Smooth cone vs. Experiment cone

► Smooth cone isolation [hep-ph/9801442]:

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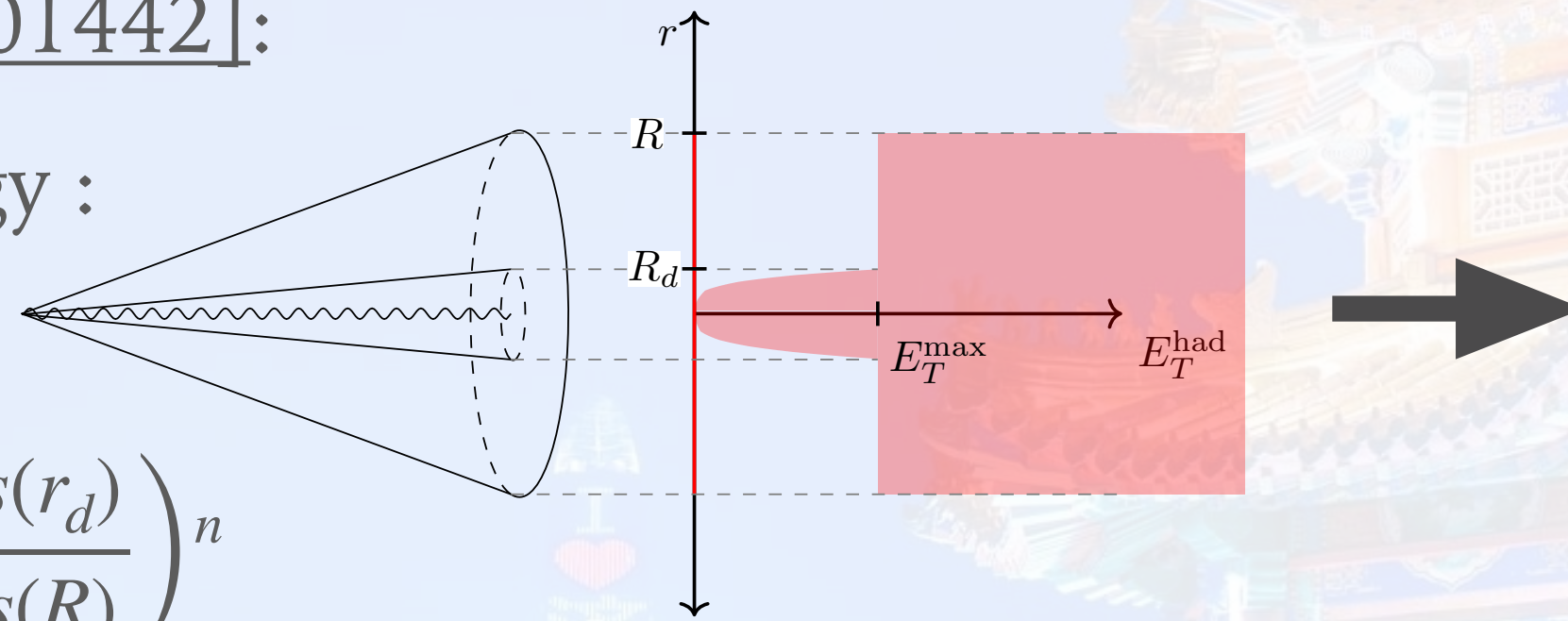
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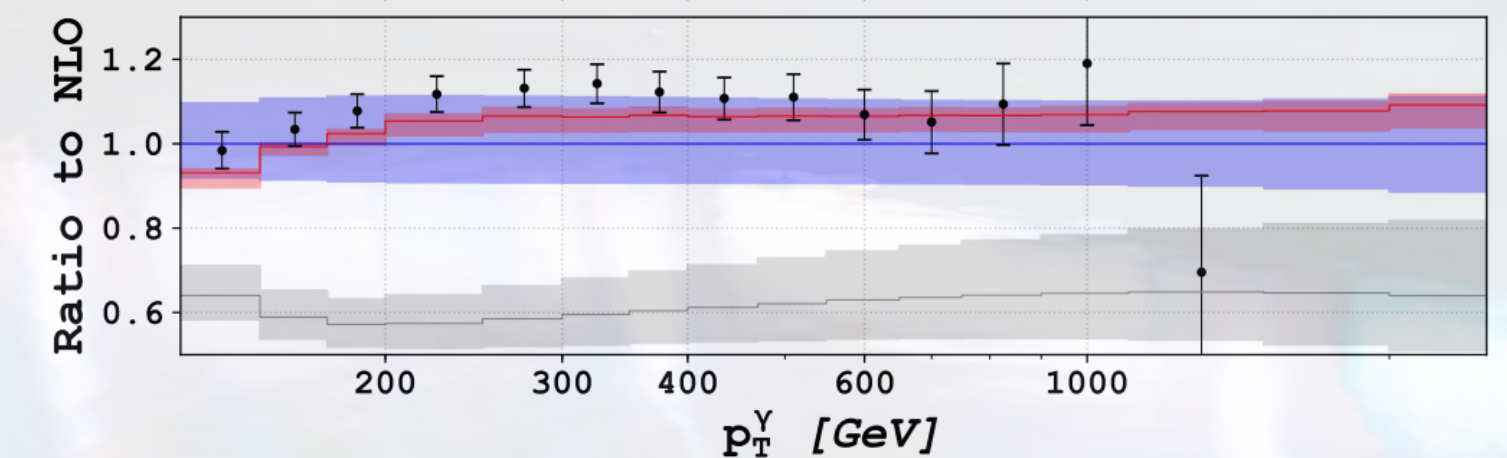
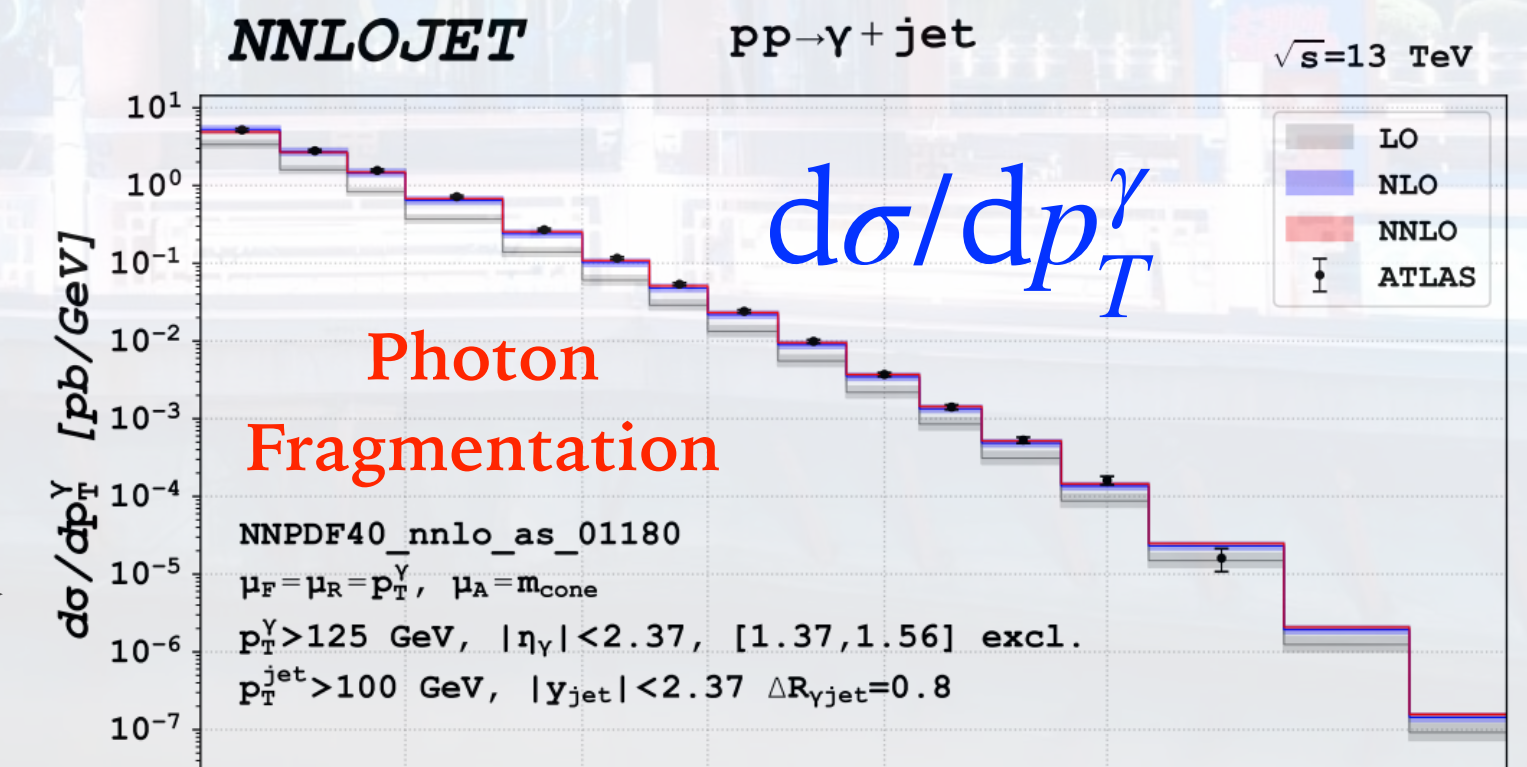
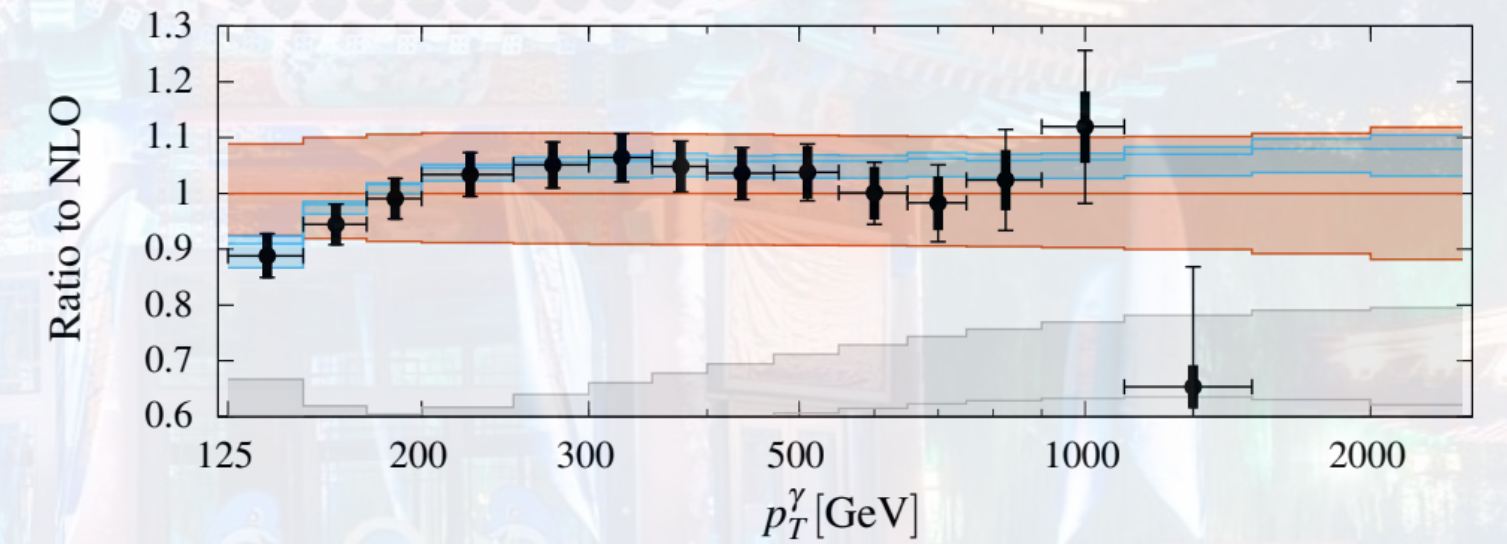
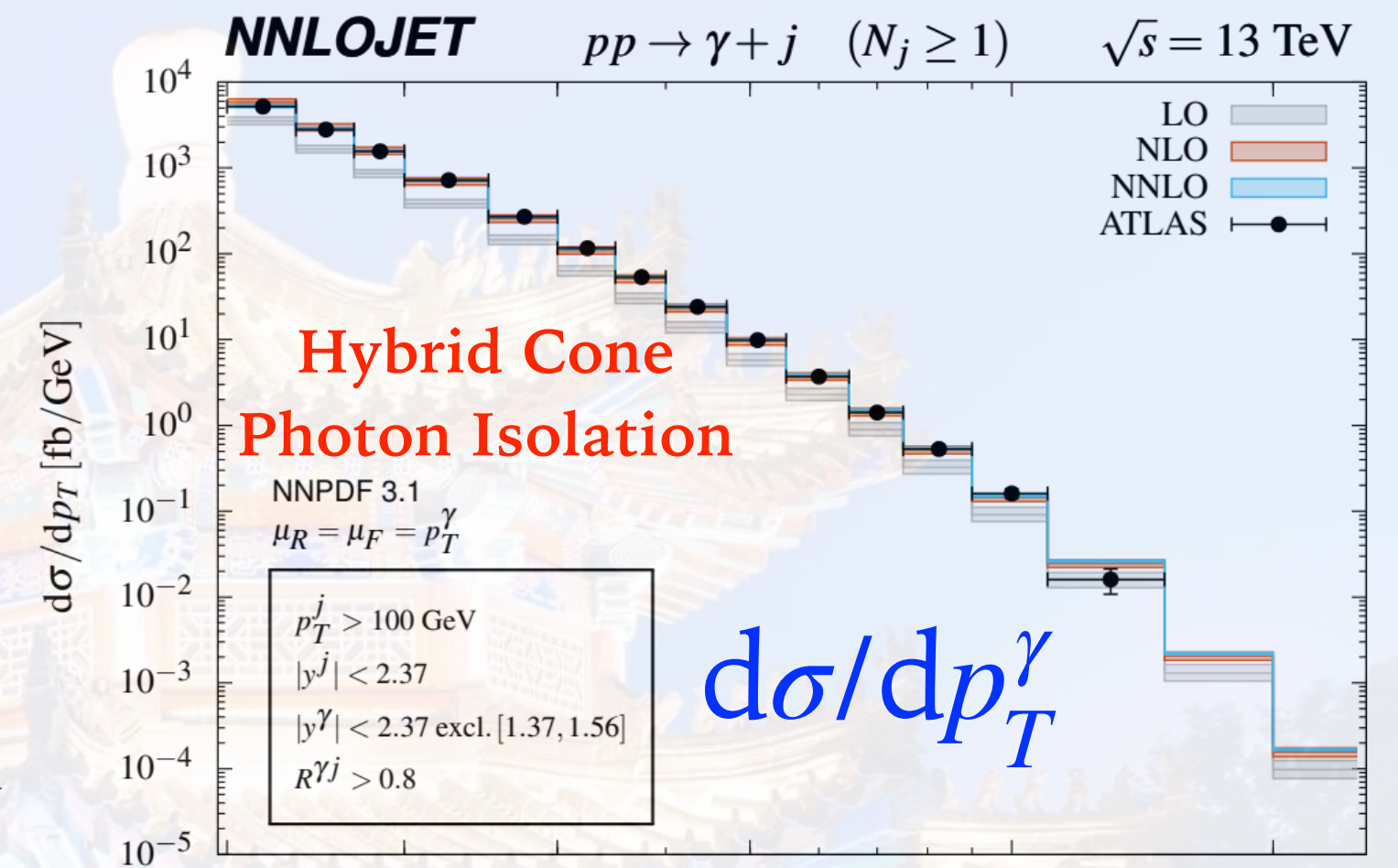
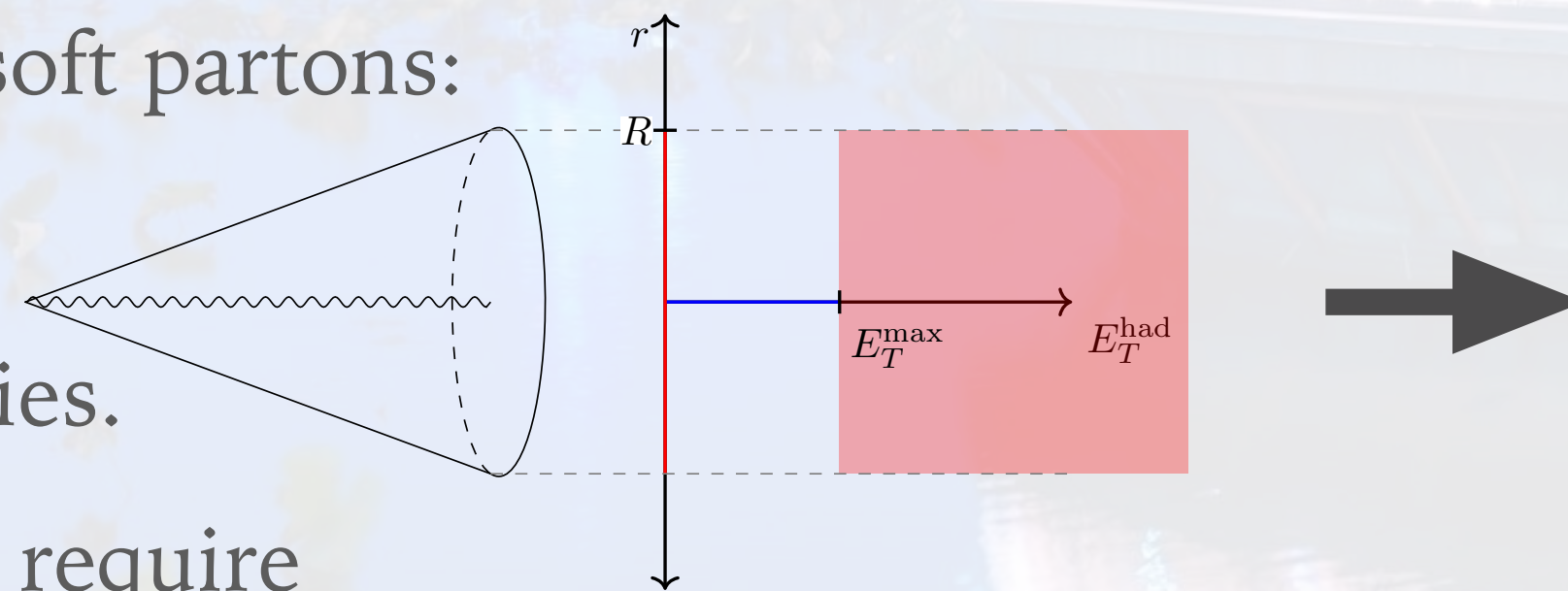
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► Current prescription in EXP studies.

► IR divergence of **collinear photon** require **photon fragmentation** → theoretically complicated



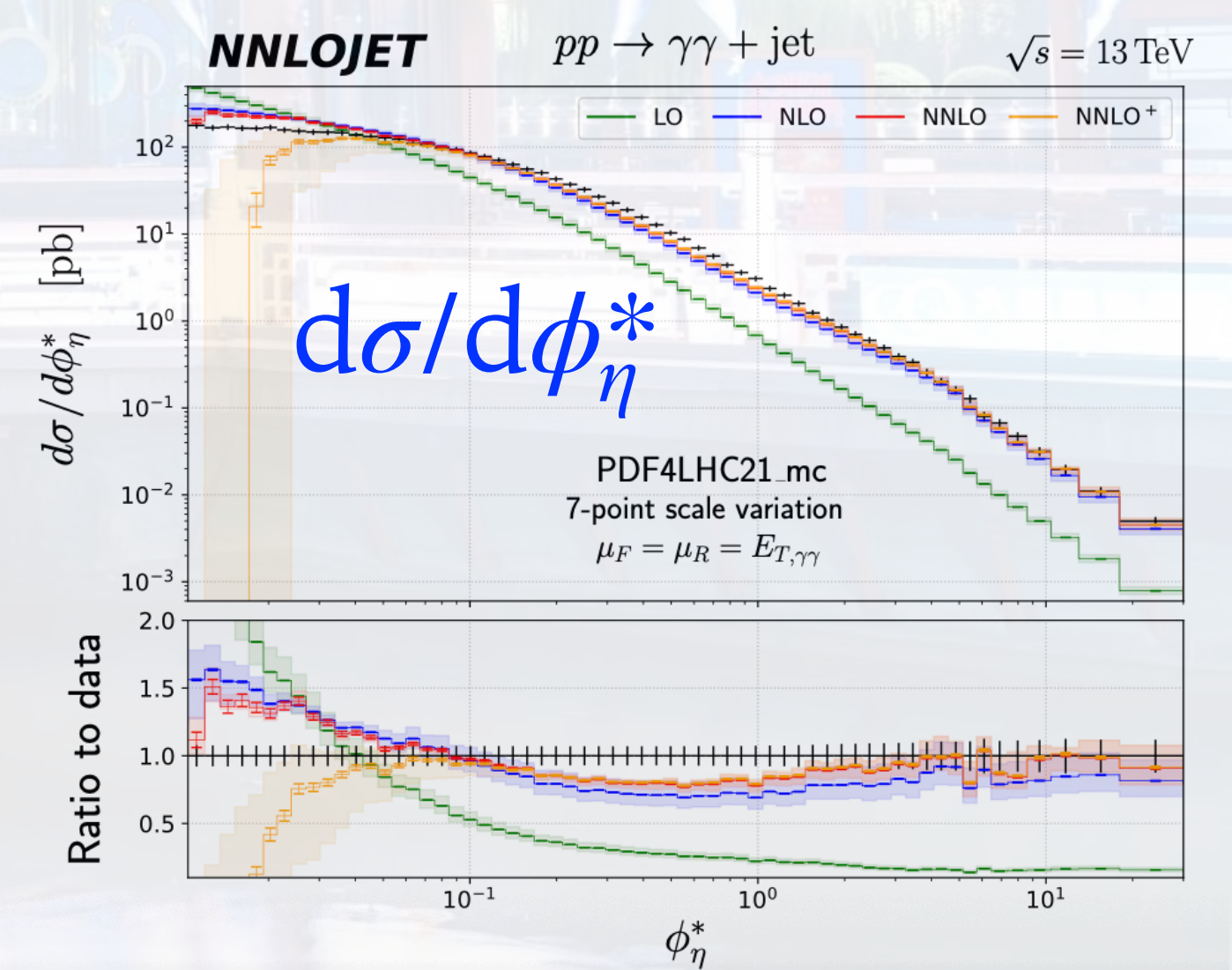
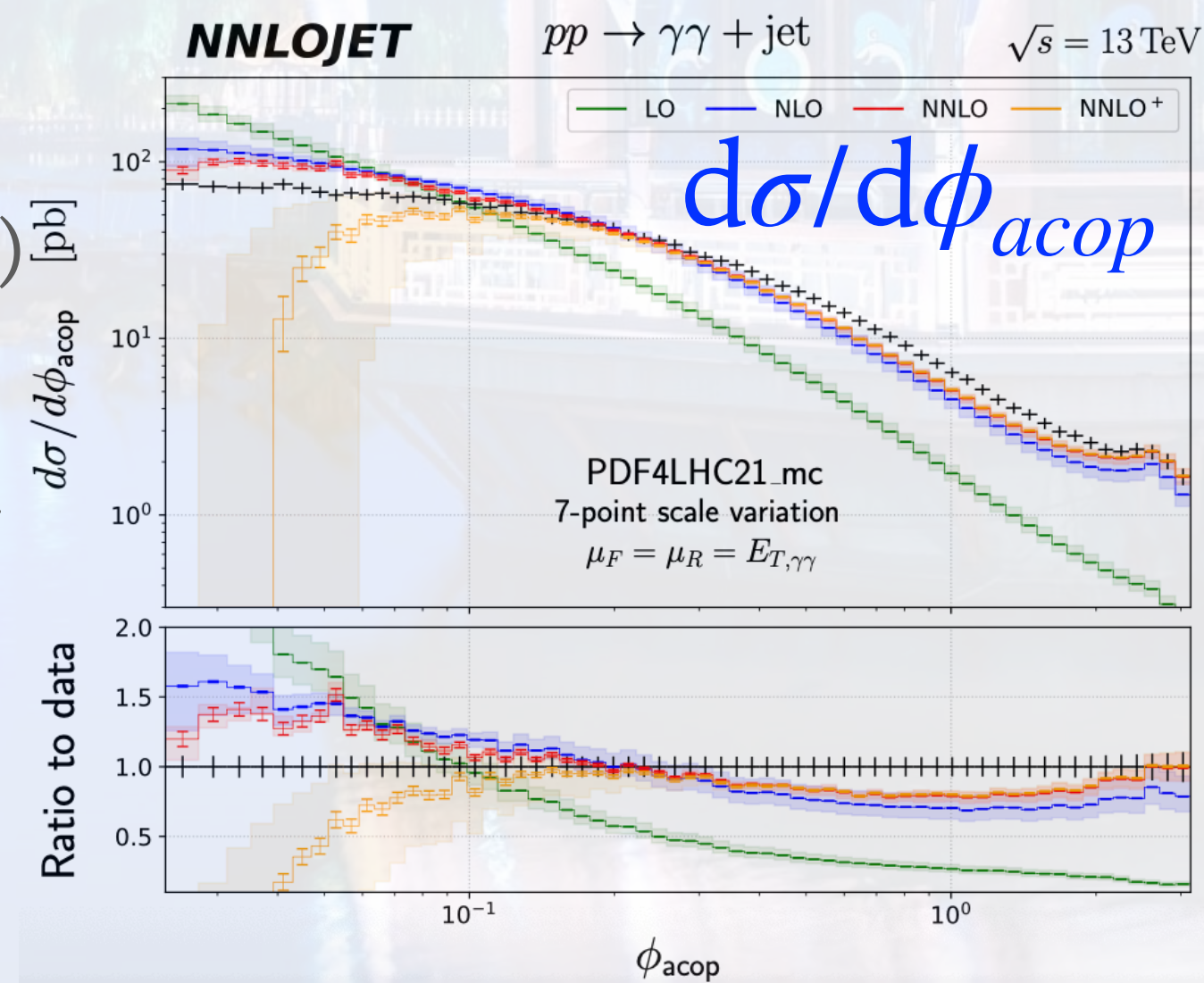
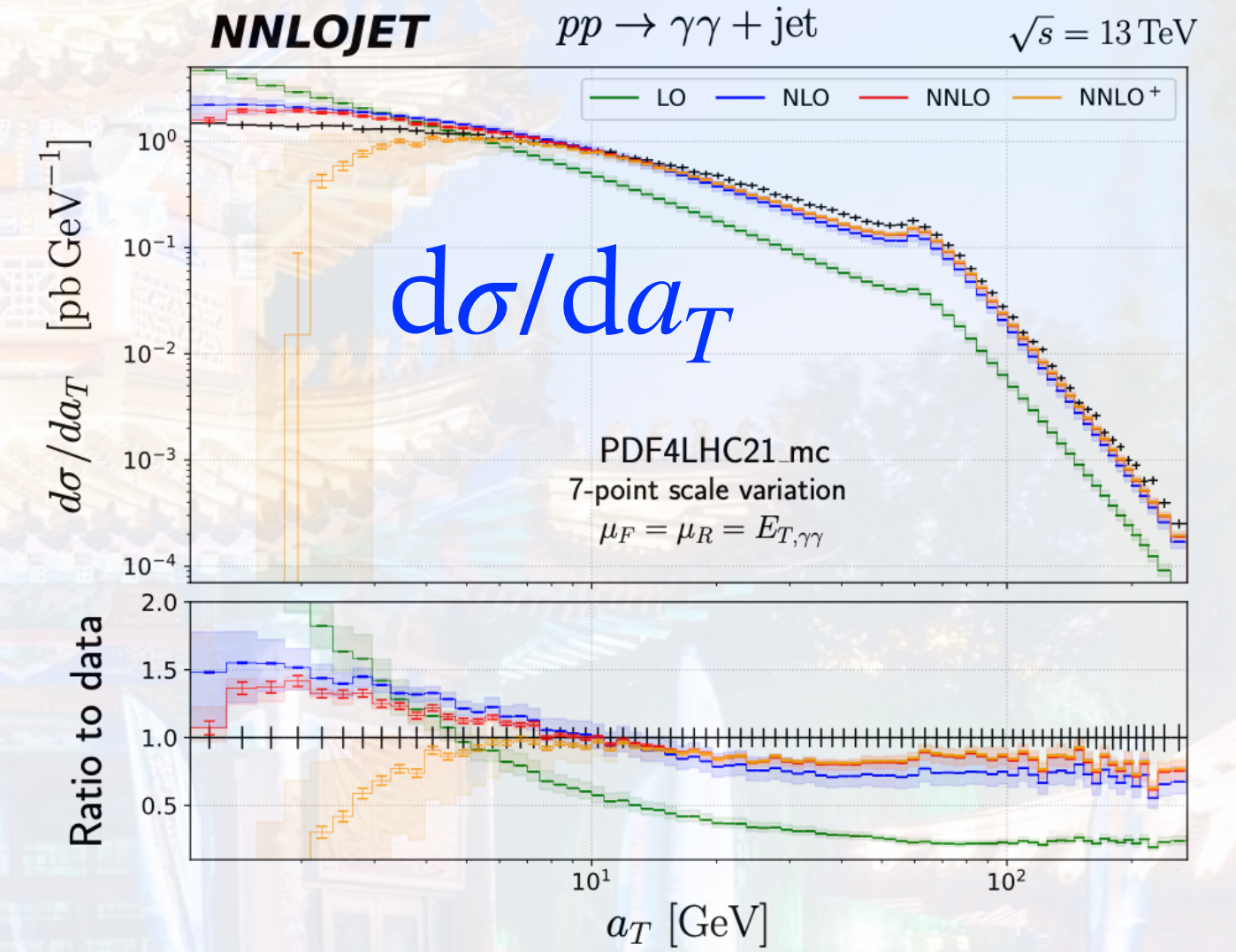
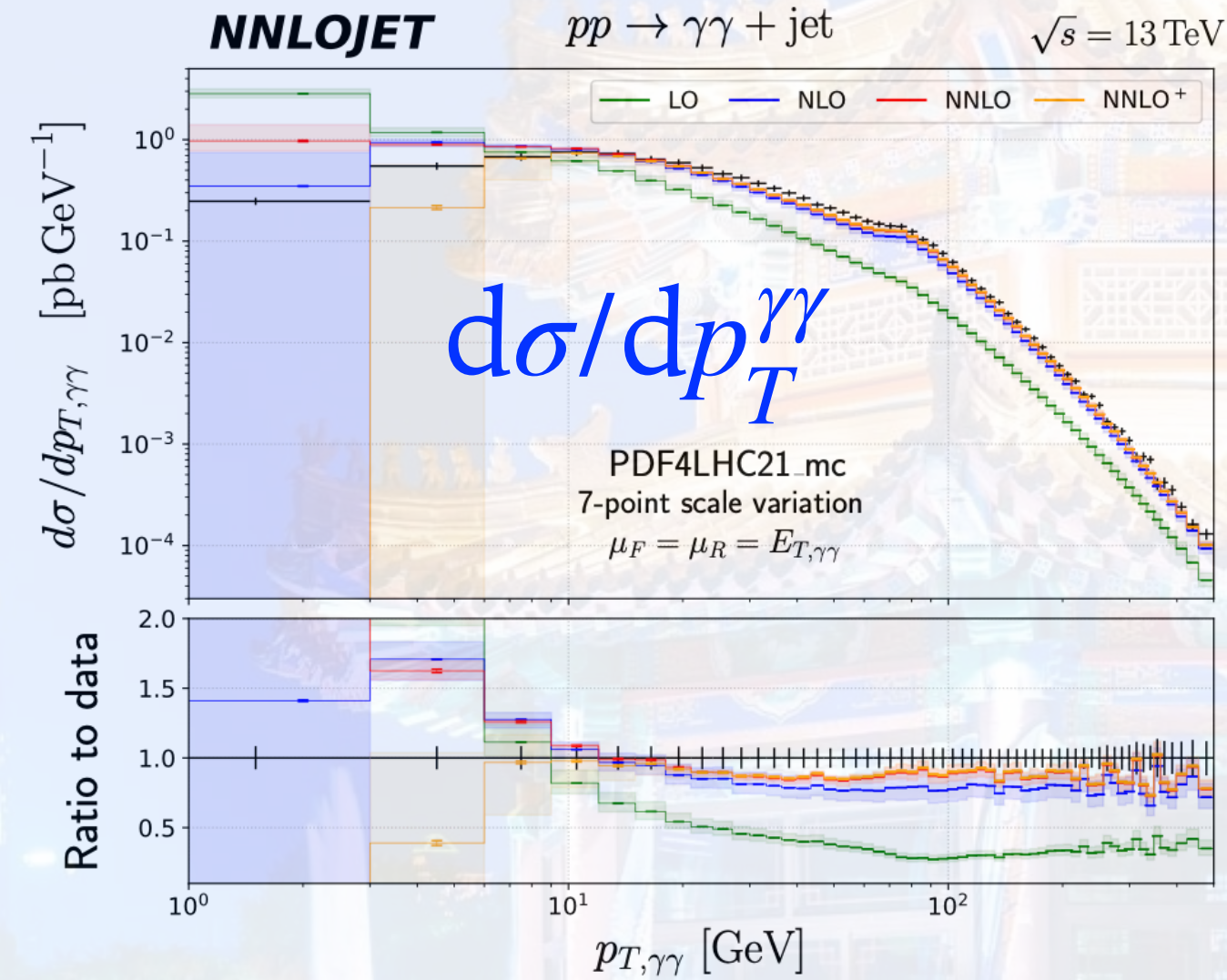
Comparison with ATLAS data
Phys.Lett.B 780 (2018) 578



$\gamma\gamma + \text{Jet Production at NNLO QCD}$

$pp \rightarrow \gamma\gamma + \text{recoil} @ \text{NNLO QCD}$

- ▶ Di-photon production at the LHC with $p_T^{\gamma\gamma} > 1 \text{ GeV}$
 - ▶ **Hybrid photon isolation** to reduce mismatch
 - ▶ Large pQCD and photon fragmentation corrections
 - ▶ **NNLO** = $\mathcal{O}(\alpha_s^3)$ terms:
 - ▶ NNLO in $q\bar{q}$ and qg channels:
 - Tree and 1-loop ME: OpenLoops
 - Two-loop, five-point **full colour contributions**
 - ▶ **LO** loop-induced process for gg channel
 - ▶ **NNLO⁺** = NNLO + NLO (loop-induced new result)
- ▶ Comparison with ATLAS 13 TeV data:
 - ▶ Better agreement but still with systematic deviation
 - ▶ Loop-induced corrections are crucial for the reduction of systematic uncertainties.
- ▶ Future plan (to be included in public release) :
 - ▶ Impact of various photon-isolation algorithm
 - ▶ Extend to $p_T^{\gamma\gamma} \in [0,1] \text{ GeV}$ to complete N3LO



F. Buccioni, XC, W.J. Feng, T. Gehrmann et. al., Phys. Rev. Lett. 134 (2025) 17

Application of **NNLOJET** at the LHC

$$pp \rightarrow JJ @ \text{NNLO}$$

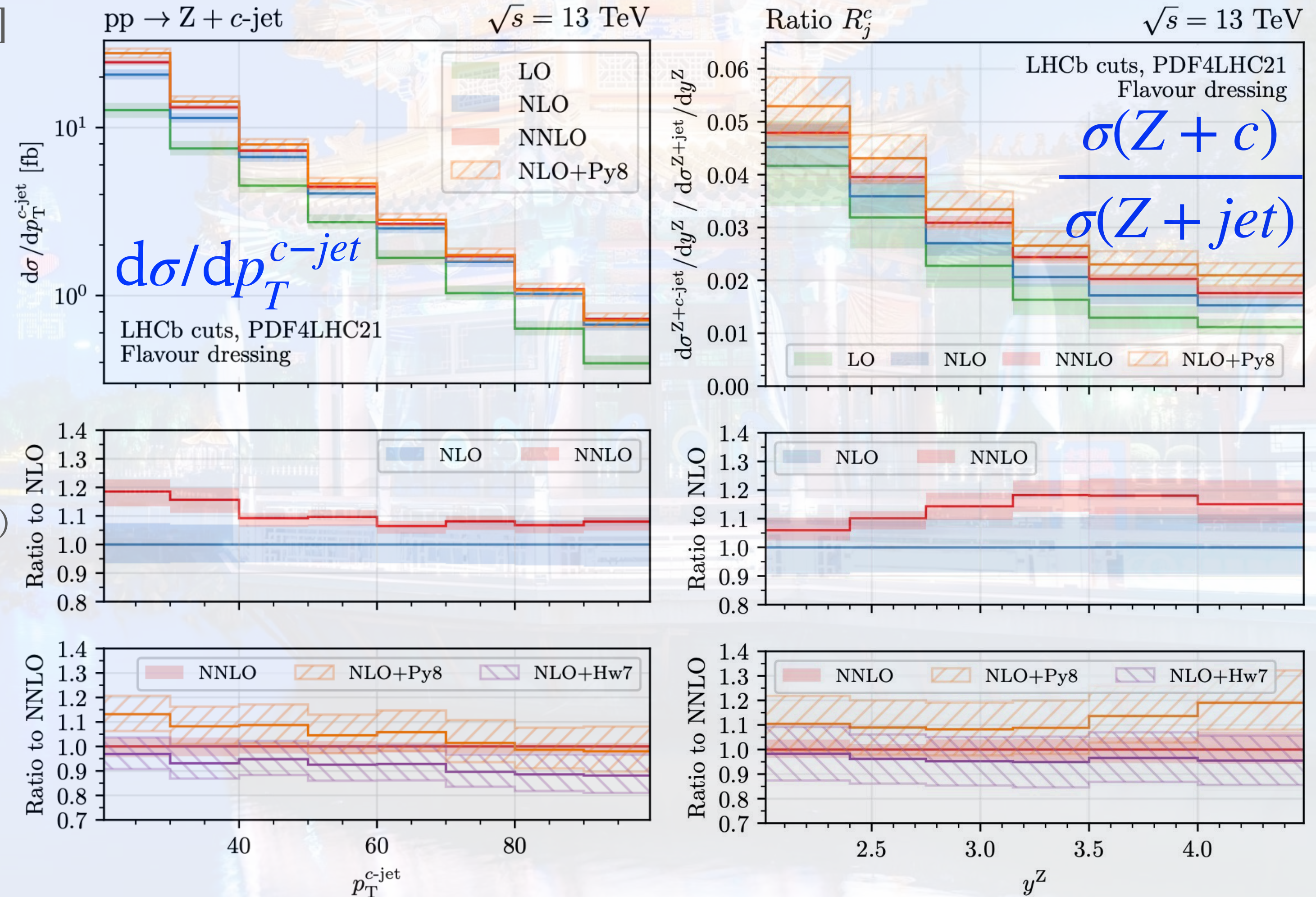
$$pp \rightarrow \gamma\gamma + \text{jet} @ \text{NNLO}$$

$$pp \rightarrow Z + c\text{-jet} @ \text{NNLO}$$

Z + Charm Jet Production at NNLO QCD

- ▶ LHCb measured ratio of $\sigma_{Z+c}/\sigma_{Z+jet}$
 - ▶ In forward Z-boson **rapidity region** $y(Z) \in [2, 4.5]$
 - ▶ At **13 TeV** with integrated luminosity of $6 fb^{-1}$
 - ▶ With **anti-kT jet algorithm (IR unsafe)**
 - LHCb collaboration, Phys.Rev.Lett. 128 (2022) 8*
- ▶ Flavour tagged jet require **IR safe** jet algorithm:
 - ▶ **SoftDrop Flavour** [2205.01109] (EPJC)
 - ▶ **Flavoured anti-kT** [2205.11879] (JHEP)
 - ▶ **Flavour dressing** [2208.11138] (PRL)
 - ▶ **Interleaved Flavour Neutralisation** [2306.07314] (PRD)
 - ▶ **Winner-Take-All flavour** [2205.01117] (JHEP)
 - ▶ All available now in **FastJet** package [2506.13449]
- ▶ Z + charm jet production @ NNLO QCD
 - ▶ NNLOJET: Apply LHCb fiducial cuts
 - ▶ Apply **flavour dressing** IR safe jet algorithm
 - ▶ Improved theory uncertainty (-75%) at NNLO compare to NLOPS at measured fiducial regions


NNLOJET Event Generator



R. Gauld, A. Gehrmann-De Ridder, N. Glover, et al., EPJC. 83 (2023) 4

CONCLUSIONS AND FUTURE PROSPECTS

- Precision phenomenology requires improvements in multiple frontier (PDFs, α_s , quark mass etc.)
- **Event generators** serve as Swiss knives to push the prediction power to new level of accuracy.
- First few applications of **fully-differential N3LO QCD calculation** are available with nontrivial corrections.
- The shortest panel of the bucket is shifting to **mismatch between TH and EXP definition** of observables.

- Precision is not the ultimate goal → identify anomaly then understand
- The most famous **failed experiment**: Michelson–Morley in **1887**, foundation of special relativity. → 1907 Nobel Prize to **Albert A. Michelson** . 
- “... it seems probable that **most of the grand underlying principles have been firmly established** and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice. ... An eminent physicist remarked that **the future truths of physical science are to be looked for in the sixth place of decimals.**” — **Albert A. Michelson, 1894**
University of Chicago



Thank You for Your Attention



BACK UP SLIDES

State-of-the-Art QCD Calculations @ NNLO

- NNLO QCD predictions for $2 \rightarrow 2$ processes (NNLO revolution since 2015)
 - Accomplished during past 10 years on case-by-case basis
 - As parton-level event generators (fully differential final state information)
 - Current frontier at NNLO $2 \rightarrow 3$

- Typical size of corrections and uncertainty
 - NLO corrections: 10~100%, uncertainty: 10~30%
 - NNLO corrections: 2~15%, uncertainty: 3~8%
 - expect N3LO to yield uncertainty at level of 1%

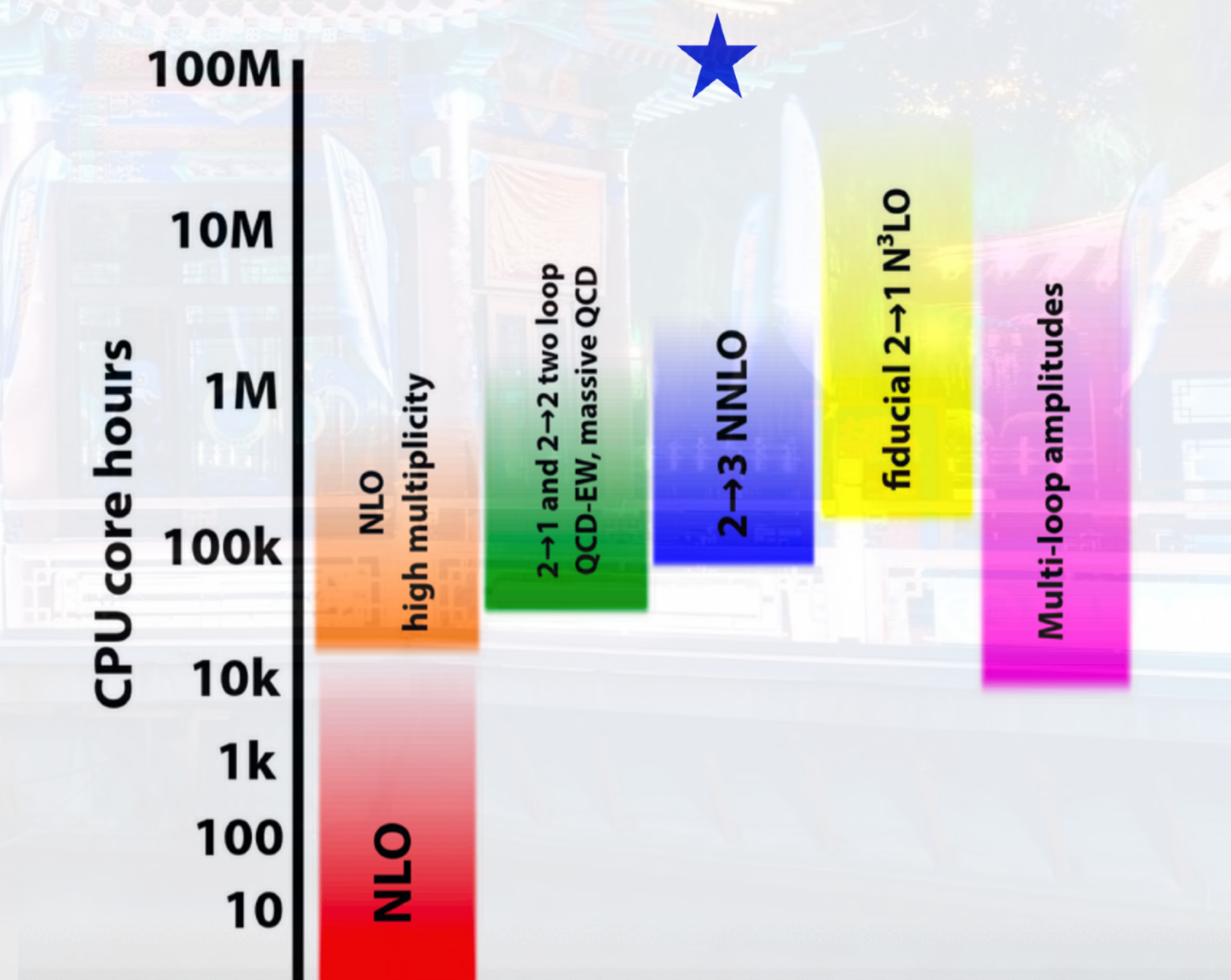
➤ So, is NNLO solved?

- In principle **yes**: STRIPPER, given the relevant amplitudes and enough computational resources, the NNLO calculation is streamlined.

➤ **But:**

- Prohibitive computational cost (loop AMP, IR subtraction)
- Missing cross-validation (many years between 1st and 2nd)
- Still a long way to automated NNLO event generation

pp \rightarrow jjj event shapes with STRIPPER



Snowmass White Paper, [Comput. Softw. Big Sci. 6 \(2022\)](#)

$\gamma\gamma$ + Recoil Production at NNLO QCD

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► **Missing** 2-loop colour singlet and gluon induced contribution

$\mu_F = \mu_R = M_{\gamma\gamma}$	Fiducial cross-section σ [fb]			
	Code	LO	NLO	NNLO
2γ NNLO [202]	5712 ± 2	26402 ± 25	40269 ± 250	40269 ± 250
MCFM [203]	5710 ± 1	26444 ± 12	40453 ± 30	40453 ± 30
MATRIX	5714 ± 1	26475 ± 3	40477 ± 10	40184 ± 160
NNLOJET	5712 ± 1	26474 ± 7	40328 ± 22	40328 ± 22

MATRIX: M. Grazzini, S. Kallweit, M. Wiesemann, EPJC. 78 (2018) 537

NNLOJET: T. Gehrmann, N. Glover, A. Huss, J. Whitehead, JHEP. 01 (2021) 108

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NNLOJET: T. Gehrmann, N. Glover, A. Huss, J. Whitehead, JHEP. 01 (2021) 108

σ [fb]	LO	NLO	NNLO
$\mu_F = \mu_R = m_{\gamma\gamma}/2$	5045 ± 1	26581 ± 23	45588 ± 97
$\mu_F = \mu_R = m_{\gamma\gamma}$	5712 ± 2	26402 ± 25	43315 ± 54
$\mu_F = \mu_R = 2m_{\gamma\gamma}$	6319 ± 2	26045 ± 24	41794 ± 77

S. Catani, L. Cieri, D. de Florian, M. Grazzini, Phys. Rev. Lett. 108 (2012) 072001

σ [fb]	LO	NLO	NNLO
$\mu_F = \mu_R = m_{\gamma\gamma}/2$	5043 ± 1	26578 ± 13	42685 ± 35
$\mu_F = \mu_R = m_{\gamma\gamma}$	5710 ± 1	26444 ± 12	40453 ± 30
$\mu_F = \mu_R = 2m_{\gamma\gamma}$	6315 ± 2	26110 ± 13	38842 ± 27

J. M. Campbell, R. K. Ellis, C. Williams, JHEP. 07 (2016) 148

σ (fb)	LO	NLO	NNLO
$\mu_F = \mu_R = M_{\gamma\gamma}/2$	5045 ± 1	26581 ± 23	42238 ± 330
$\mu_F = \mu_R = M_{\gamma\gamma}$	5712 ± 2	26402 ± 25	40269 ± 250
$\mu_F = \mu_R = 2M_{\gamma\gamma}$	6319 ± 2	26045 ± 24	38901 ± 310

S. Catani, L. Cieri, D. de Florian, M. Grazzini, Phys. Rev. Lett. 117 (2016) erratum

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3(4)LL}$

FO	α_s^n	$H(m_V, \mu)$	$I_{ilj}^{(n)}(x, b)$	$\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu} / \bar{\mu} (A(\alpha_s(\bar{\mu})) \ln \frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu})))$						
$\frac{d\hat{\sigma}_{NLO}^V}{dq_T}$	NLO	✓	✓	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1				
$\frac{d\hat{\sigma}_{NNLO}^V}{dq_T}$	N2LO	✓	✓	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1			
$\frac{d\hat{\sigma}_{N^3LO}^V}{dq_T}$	N3LO	✓	✓	$\ln^4(b^2 m_V^2)$	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1		
$\frac{d\hat{\sigma}_{N^4LO}^V}{dq_T}$	N4LO	✓	✗	$\ln^5(b^2 m_V^2)$	$\ln^4(b^2 m_V^2)$	$\ln^3(b^2 m_V^2)$	$\ln^2(b^2 m_V^2)$	$\ln(b^2 m_V^2)$	1	
...
$\frac{d\hat{\sigma}_{N^kLO}^V}{dq_T}$	NKLO			$\ln^{k+1}(b^2 m_V^2)$	$\ln^k(b^2 m_V^2)$	$\ln^{k-1}(b^2 m_V^2)$	$\ln^{k-2}(b^2 m_V^2)$	$\ln^{k-3}(b^2 m_V^2)$
...
Resum				LL	NLL	NNLL	N3LL	N4LL	...	$N^{k+1}LL$
A				A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	...	A_{k+2}
B					B1 ✓	B2 ✓	B3 ✓	B4 ✓	...	B_{k+1}

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

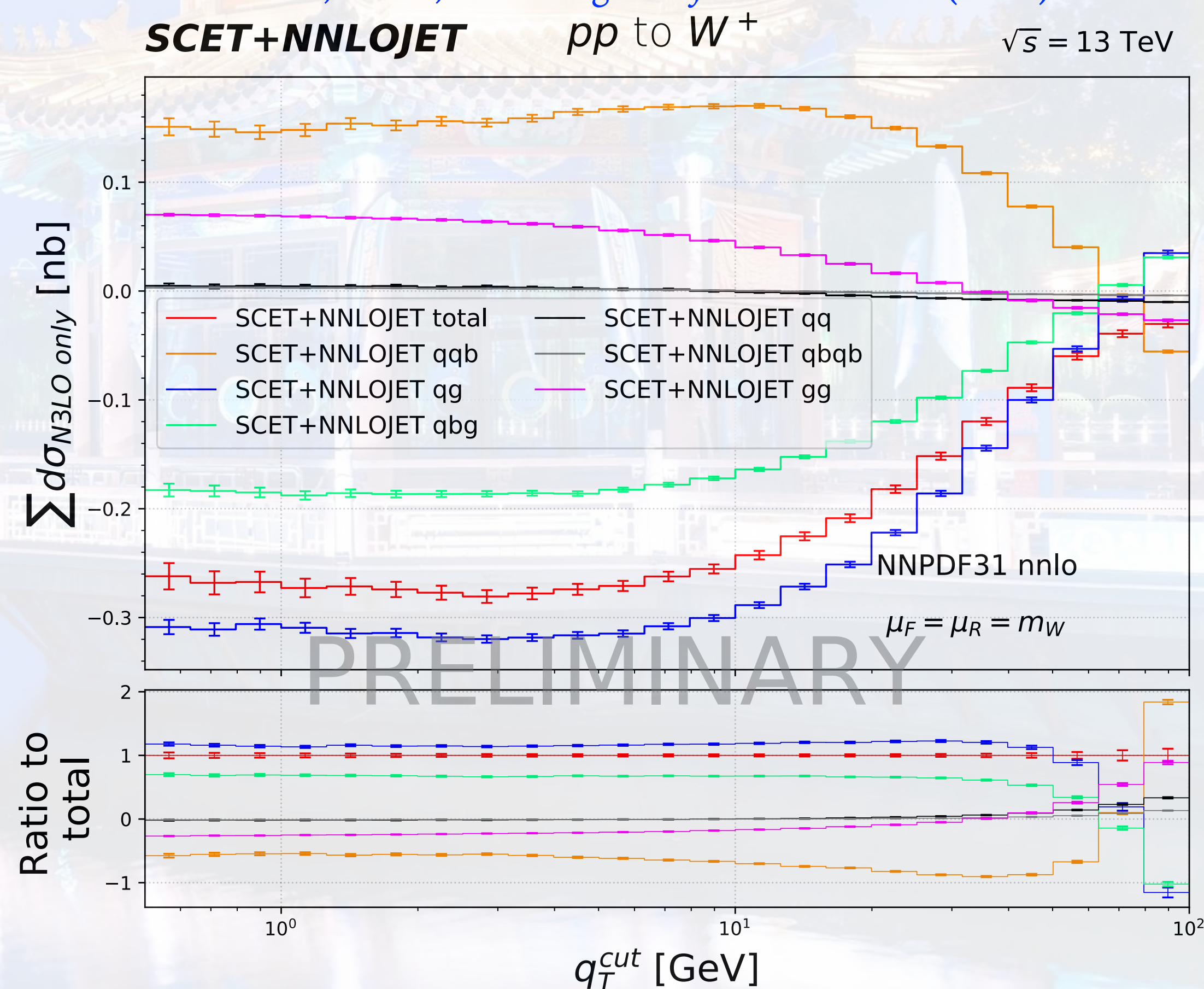
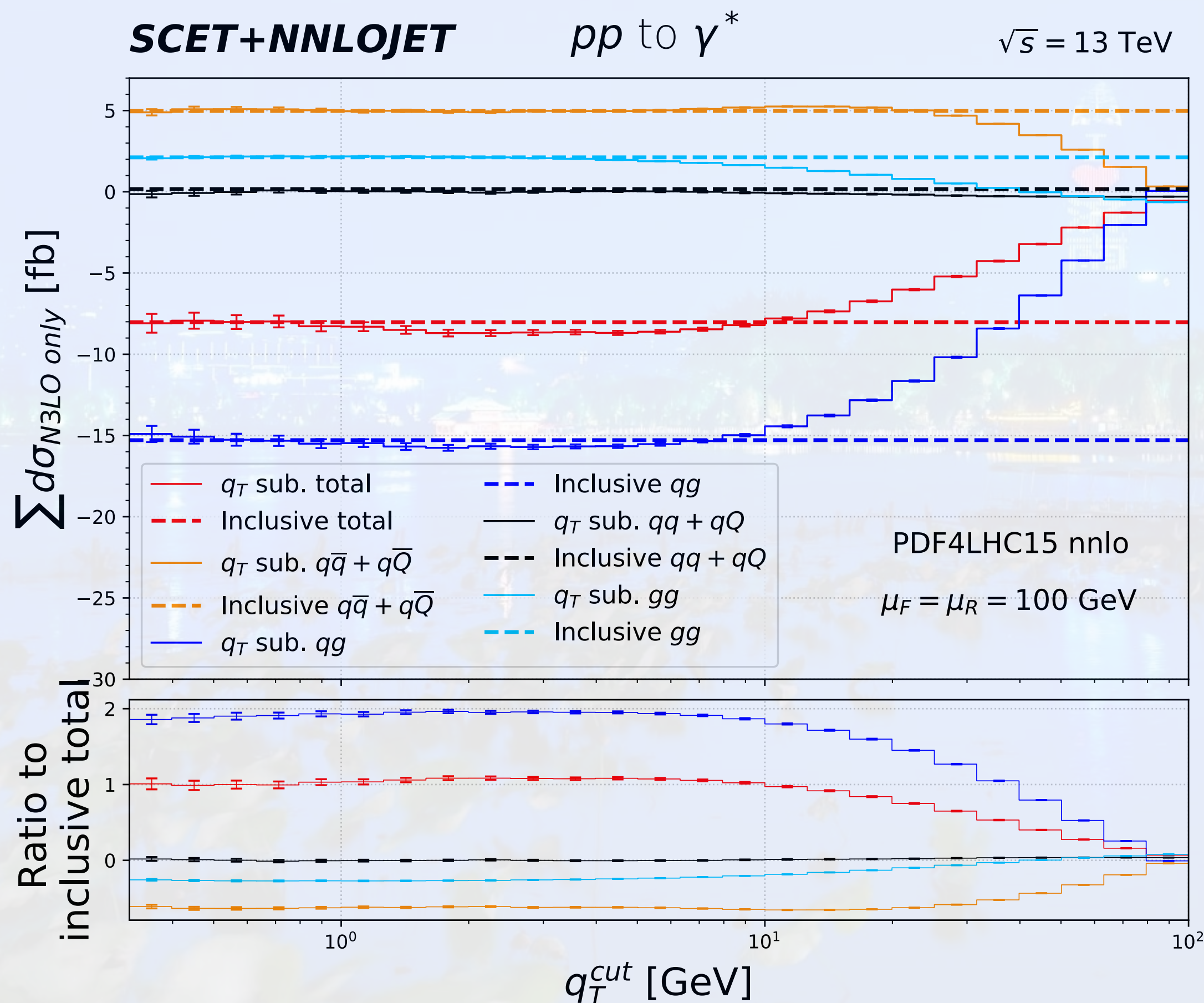
► qT slicing at N3LO for neutral and charged current production (NNLOJET)

$$\sum d\sigma_{N^3LO}^V \equiv \sum_{dp_{T,V}} d\sigma_{NNLO}^{V+jet} / dp_{T,V} |_{p_{T,V} > q_T^{cut}} + \sum_{dp_{T,V}} d\sigma_{N^3LO}^{V SCET} / dp_{T,V} |_{p_{T,V} \in [0, q_T^{cut}]}$$

NC and CC Validated against inclusive XS within $\pm 5\%$ uncertainty

$$\Delta\sigma_{N^3LO}^{\gamma^*} = -7.98 \pm 0.36 \text{ fb vs. } -8.03 \text{ fb}$$

Duhr, Dulat, Mistlberger *Phys.Rev.Lett.* 125 (2020)



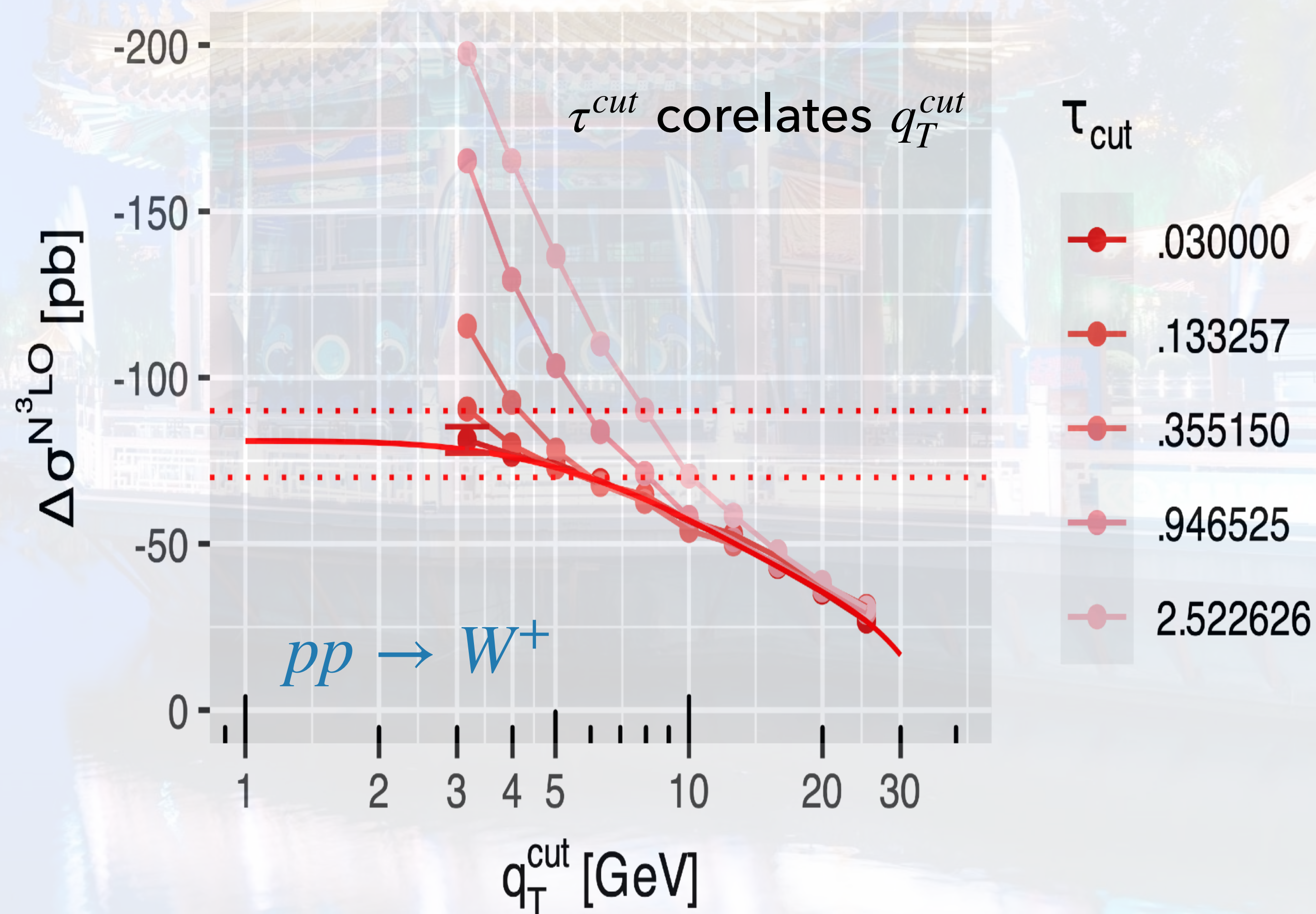
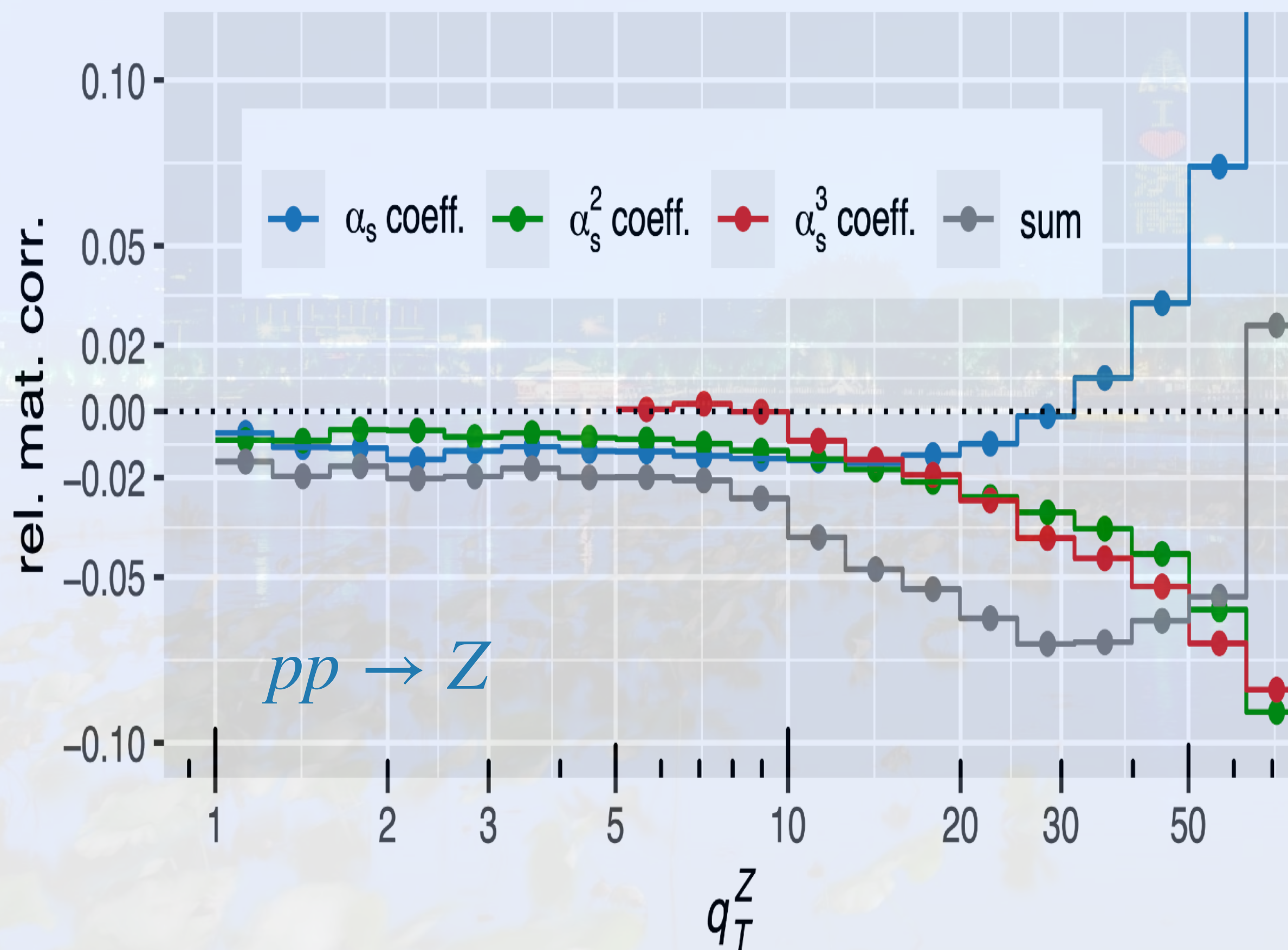
PRELIMINARY

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

► qT slicing at N3LO for neutral and charged current production (MCFM)

$$\sum d\sigma_{N^3LO}^V \equiv \sum_{dp_{T,V}} d\sigma_{NNLO}^{V+jet} / dp_{T,V} |_{p_{T,V} > q_T^{cut}} + \sum_{dp_{T,V}} d\sigma_{N^3LO}^{V SCET} / dp_{T,V} |_{p_{T,V} \in [0, q_T^{cut}]}$$

NC MCFM: $-22.6 \text{ pb} \pm 1.4 \text{ pb (num.)} \pm 1 \text{ pb (slicing)}$
 NC NNLOJET: $-18.7 \text{ pb} \pm 1.1 \text{ pb (num.)} \pm 0.9 \text{ pb (slicing)}$
 CC agree to inclusive XS within $\pm 60\%$ uncertainty of $\Delta(\alpha_s^3)$

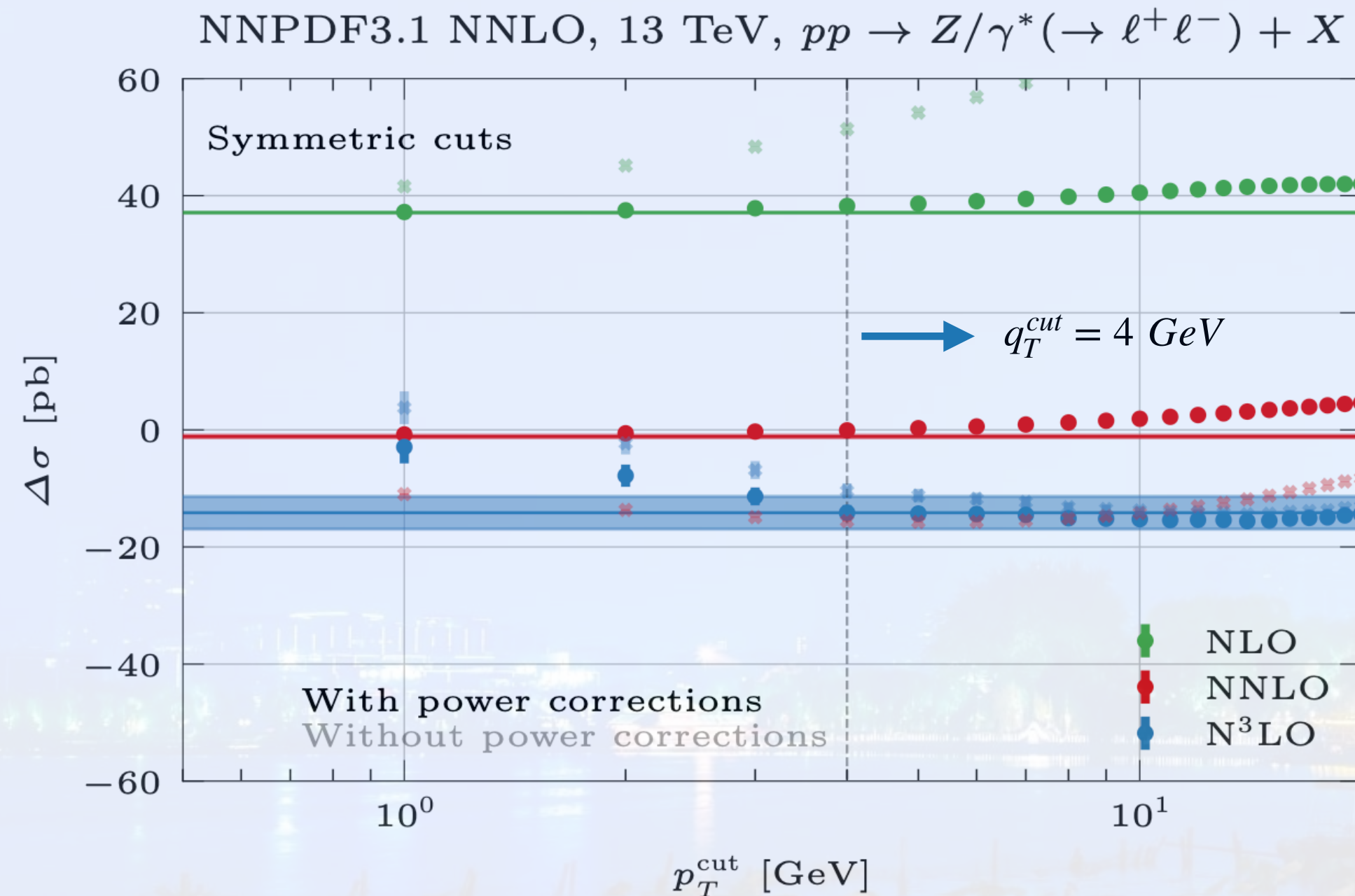


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Precision Predictions at Hadron Collider

$2 \rightarrow 1$ @ N3LO (+ N3LL) QCD



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

DYTURBO result with fiducial power correction

Order	N ³ LO
q_T subtr. ($q_T^{cut} = 4 \text{ GeV}$)	$747.1 \pm 0.7 \text{ pb}$
recoil q_T subtr.	$745.7 \pm 0.7 \text{ pb}$

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- Solid horizontal lines: NLO, NNLO at 1 GeV, N3LO at 4 GeV with MC error.
- N3LO shows no plateau in 1905.05171
- Pale dots are **values used by DYTURBO** in 2103.04974 and 2303.12781 (taken from 1905.05171).
- Fiducial power corrections are not included.
- Leads to 30% difference of N3LO coefficients at $q_T^{cut} = 4 \text{ GeV}$.
- Solid dots are corrected values with fiducial power correction.
- Central value shifts **2 pb** starting from NLO (the dominant error).
- **$\pm 2.1 \text{ pb}$** uncertainty from MC and q_T^{cut} (estimated from [3,5] GeV region).
- Not consistent with DYTURBO update result of **$\pm 0.7 \text{ pb}$** uncertainty.

DYTURBO result without fiducial power correction cited in ATLAS α_s fitting

Order	NLO	NNLO	N ³ LO
$\sigma(pp \rightarrow Z/\gamma^* \rightarrow l^+l^-)$ [pb]	766.3 ± 1	757.4 ± 2	746.1 ± 2.5
Order	NLL+NLO	NNLL+NNLO	N ³ LL+N ³ LO
$\sigma(pp \rightarrow Z/\gamma^* \rightarrow l^+l^-)$ [pb]	773.7 ± 1	759.8 ± 2	749.6 ± 2.5

S. Camarda, L. Cieri, G. Ferrera Phys. Rev. D 104, L111503 (2021)