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Novel Method for Centrality Determination at Low Collision Energy

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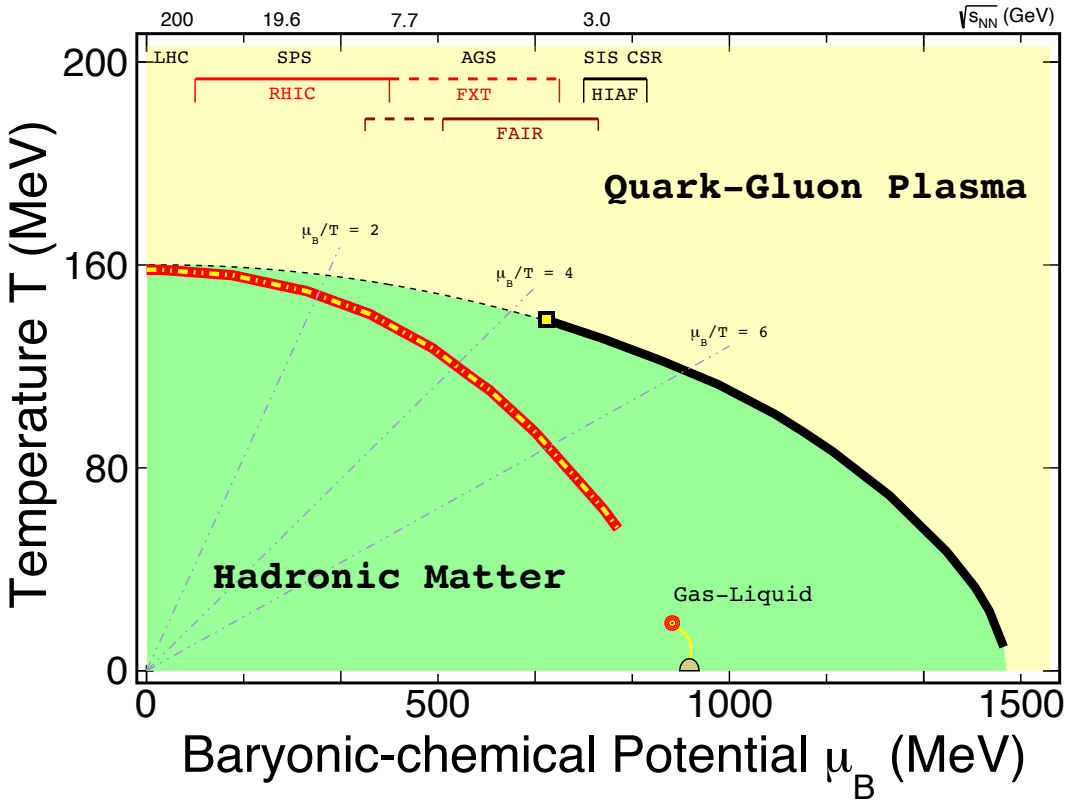
Central China Normal University

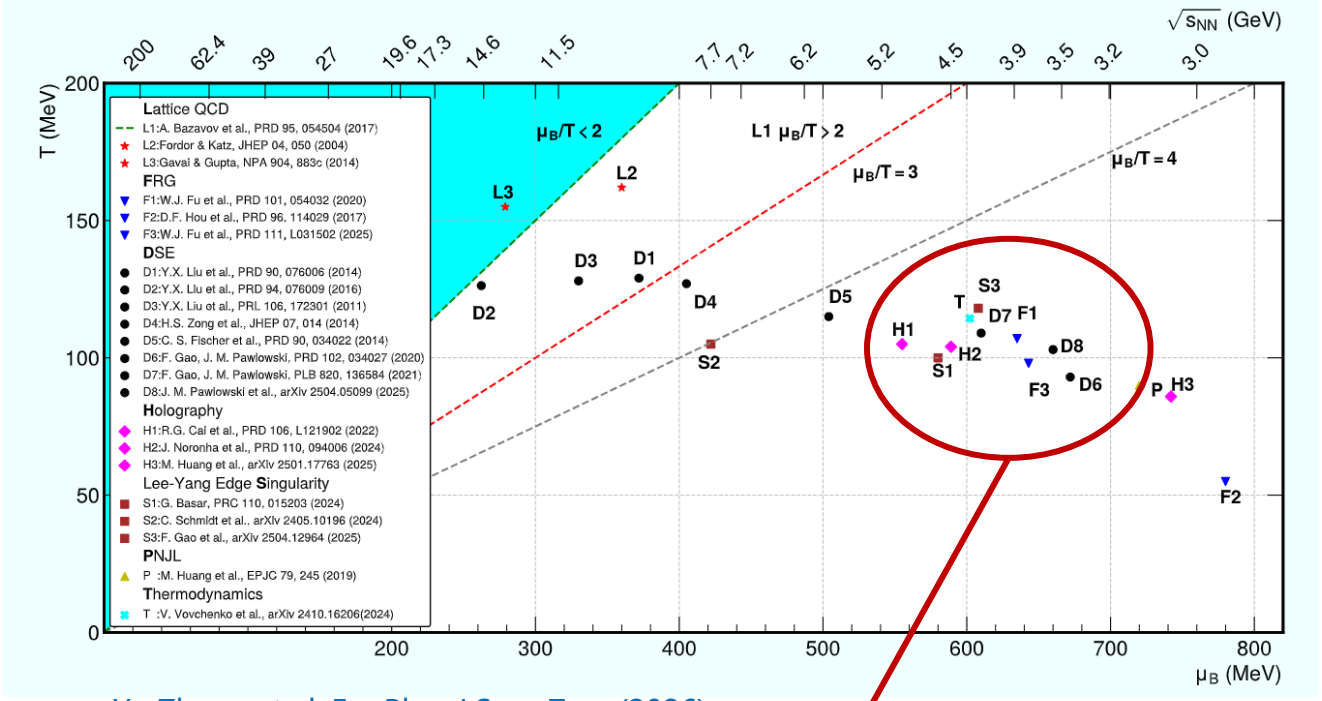
2026/11/11

Outline

- Motivation
- Introduction
- Method
- Closure Test
- Summary & Outlook

Motivation >> Model Prediction





Yu Zhang et al, Eur.Phys.J.Spec.Top. (2026).

Lattice QCD: A smooth crossover at small μ_B

Model predictions: First-order phase transition at high μ_B

CEP: The endpoint of the first-order phase transition boundary

Recent theoretical calculations converged to the RHIC-STAR fixed-target energy region!

Motivation >> CEP Observables

N: Event-by-event multiplicity

$\delta N = N - \langle N \rangle$

- $C_1 = \langle N \rangle$
- $C_2 = \langle \delta N^2 \rangle$
- $C_1 = \langle \delta N^3 \rangle$
- $C_1 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2$

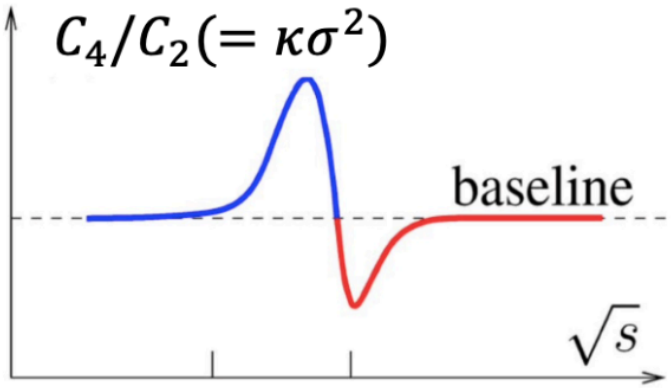
Cumulants

1) Related to correlation length ξ, $C_2 \sim \xi^2$ and $C_4 \sim \xi^7$
2) ξ diverges at CEP
3) Higher order: more sensitive

- $FC_1 = C_1$
- $FC_2 = -C_1 + C_2$
- $FC_3 = 2C_1 - 3C_2 + C_3$
- $FC_4 = -6C_1 + 11C_2 - 6C_3 + C_4$

Factorial cumulants

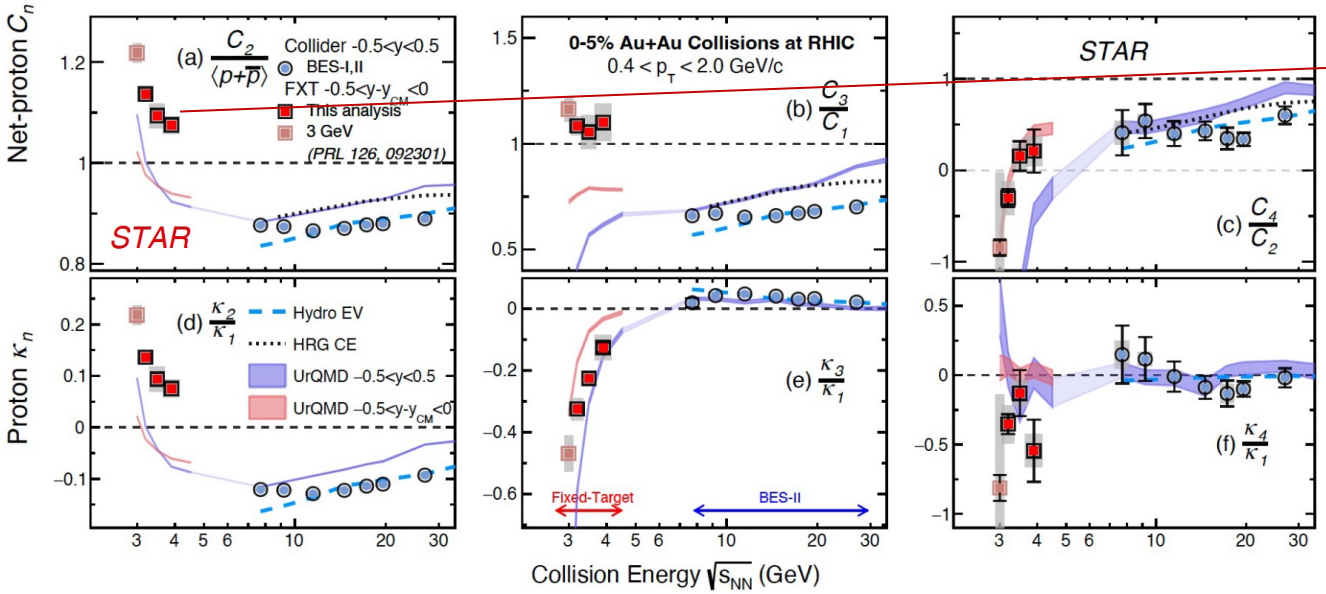
1) Can be expressed by linear combinations ordinary cumulant
2) Directly capture and are sensitive to genuine multi-particle correlations



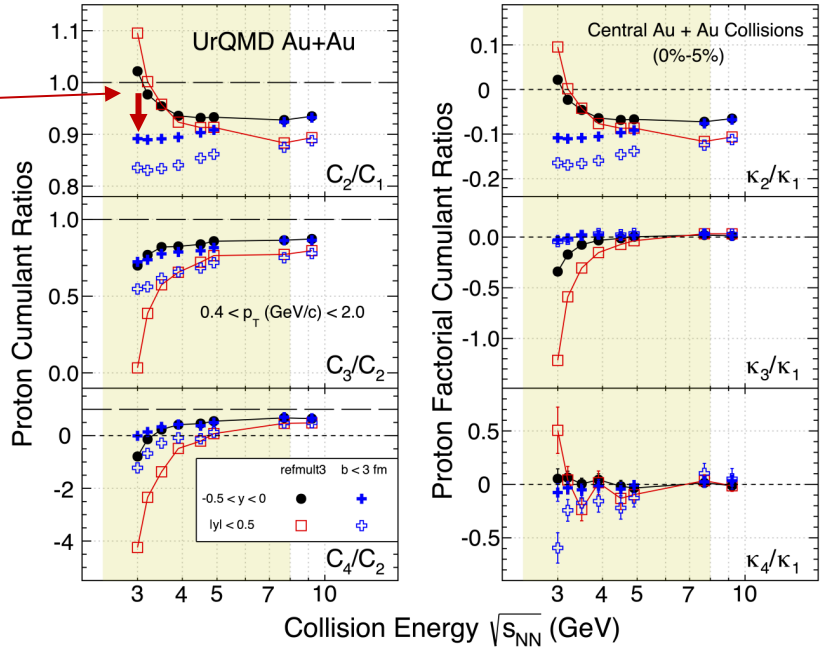
Non-monotonic energy dependence of C_4/C_2 of conserved quantity indicates the existence of critical region.[1]

[1] M. A. Stephanov, Phys.Rev.Lett, 107 (2011) 052301.

Motivation >> Previous Results



STAR: Z. Sweger, Quark Matter 2025

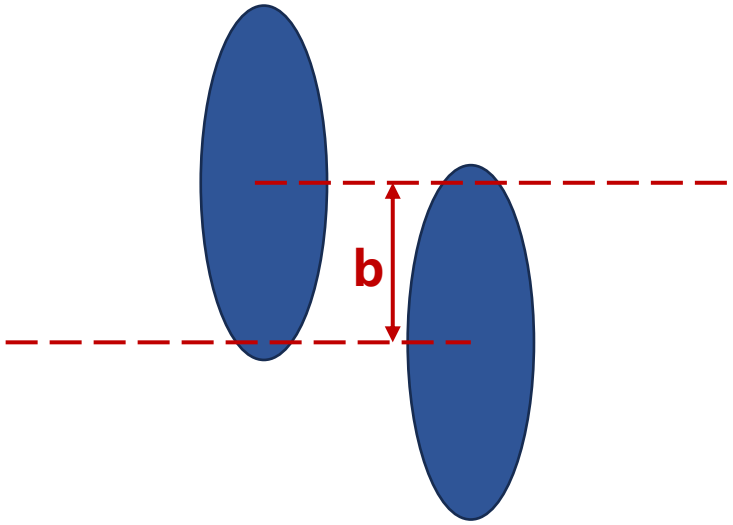


X. Zhang, Y. Zhang, X. Luo, N. Xu, Chinese Phys. C. 50 (2026) 011003

- C_2/C_1 in previous QM results shows a rapidly increasing trend as energy decreases.
- In UrQMD simulations, this increasing trend will disappear when initial collision geometry is applied.

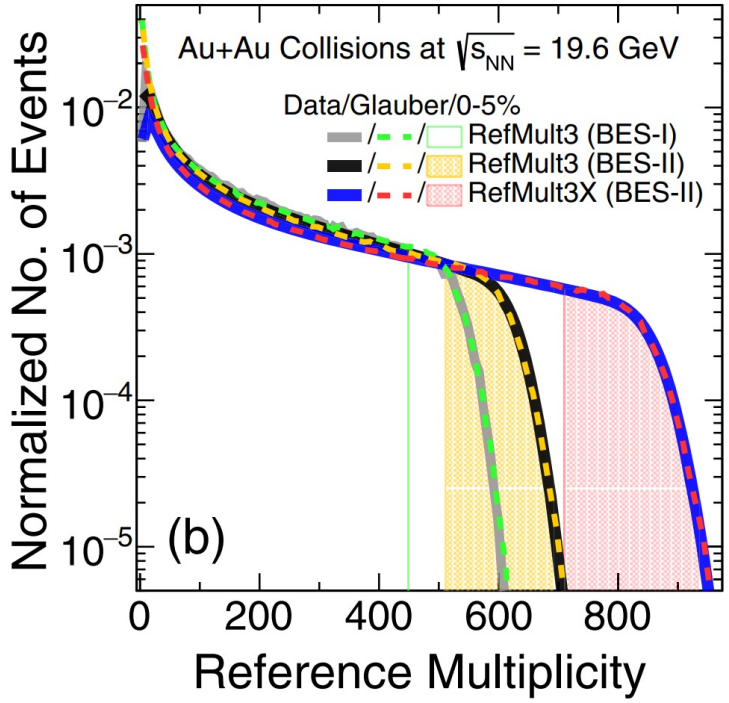
Introduction >> Centrality Definition

Theoretically(Initial state):



b: impact parameter to describe the collision geometry centrality

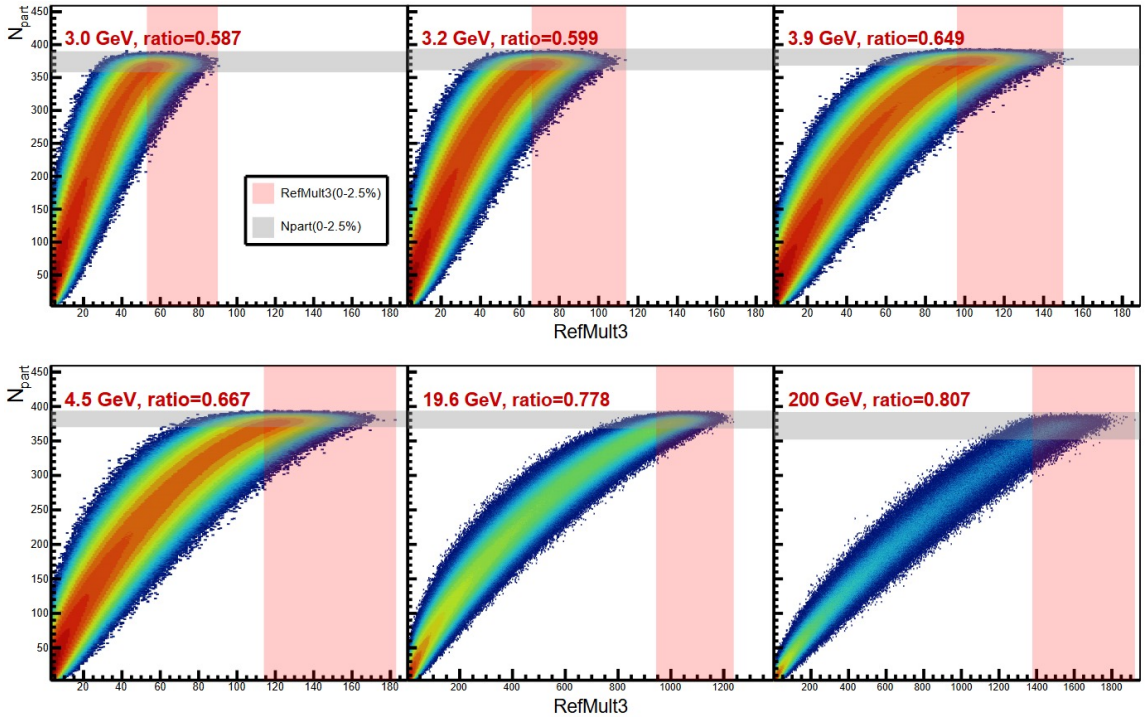
Experimentally(Final state):

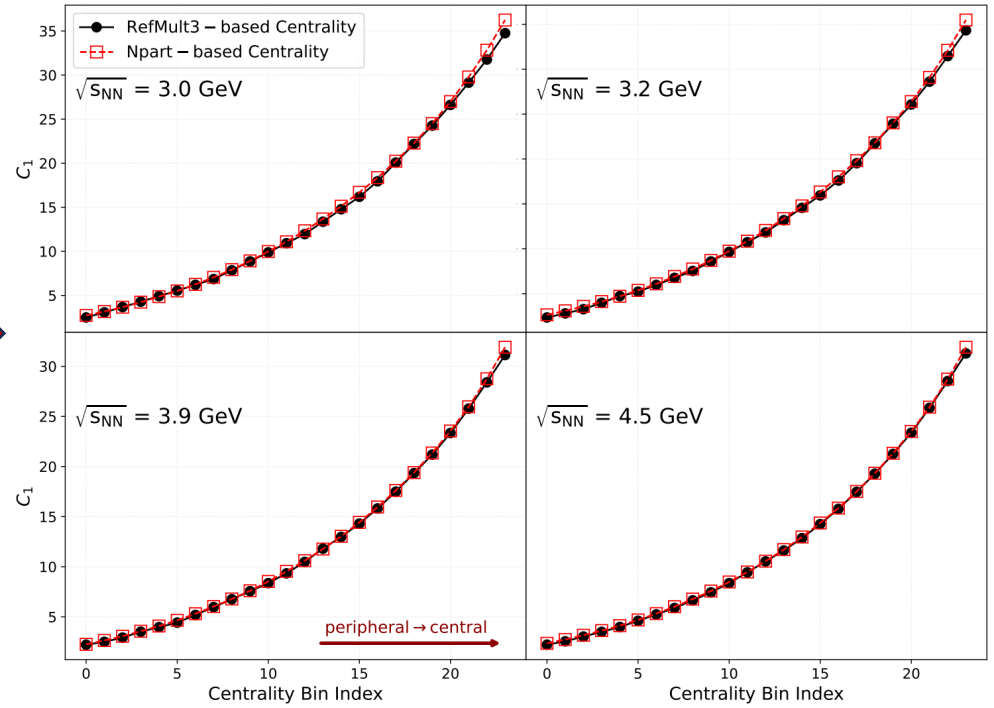


STAR Collaboration. Phys.Rev.Lett. 135 (2025) 142301

Introduction >> Initial Volume Fluctuation

What happens when the charged particle multiplicity becomes less?





UrQMD simulation in Au+Au collision

$\text{ratio} = (\text{Events}_{\text{gray \& red}})/(\text{Events}_{\text{gray}})$

- RefMult3-based centrality: disrupted event ordering

- Using C_1 as a proxy for centrality and incorporating event weight fractions, the event ordering can be restored.

Introduction >> Initial Volume Fluctuation

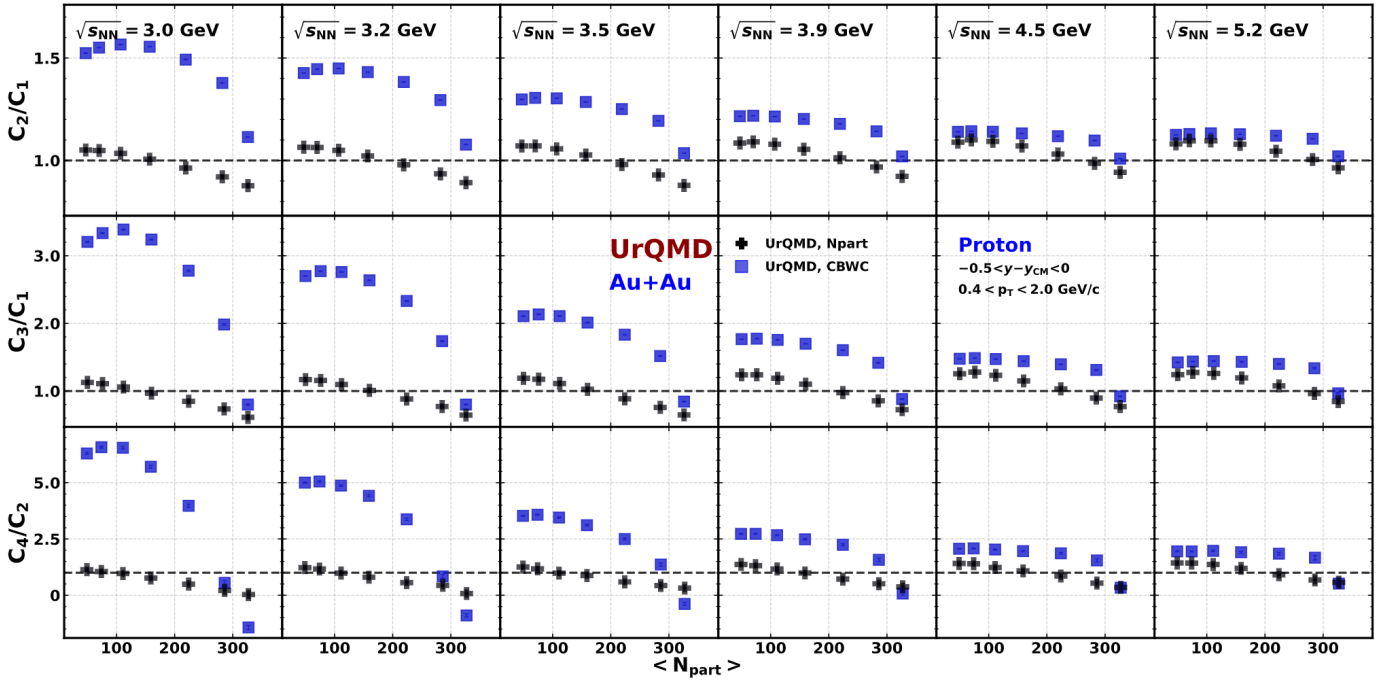
Centrality Bin Width Correction (CBWC)

$$C_n = \sum_i^N w_i C_{n,i}\,,\ N \text{ is total multiplicity bins}$$

$$w_i = \frac{n_i}{\sum_i^N n_i}\,,\ n_i \text{ is events in the ith bin}$$

$C_{n,i}$: nth order cumulant in the ith bin

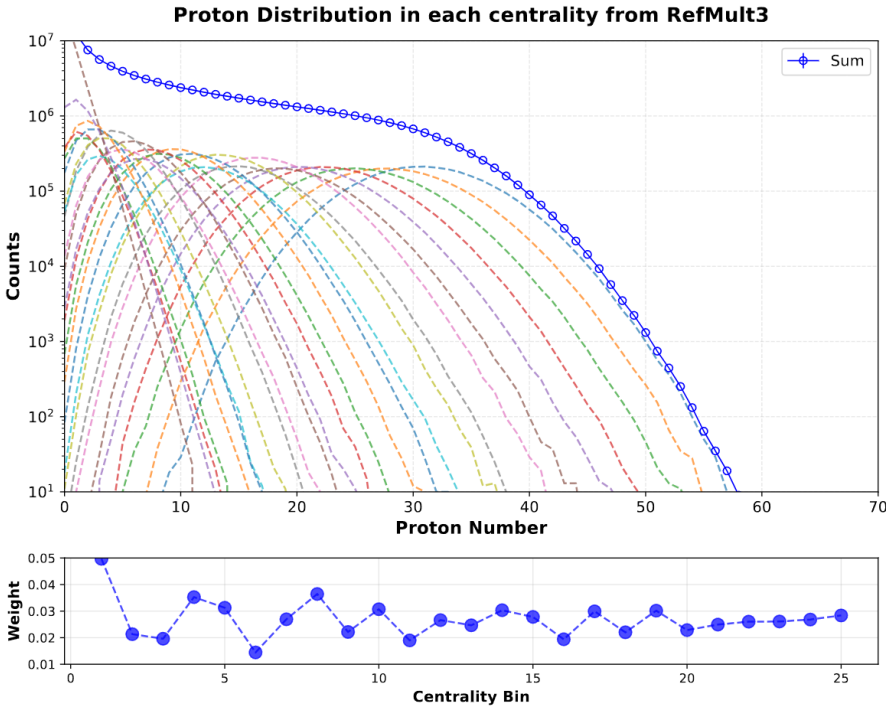
X. Luo, J. Xu, B. Mohanty, N. Xu. J. Phys. G. 40 (2013) 105104

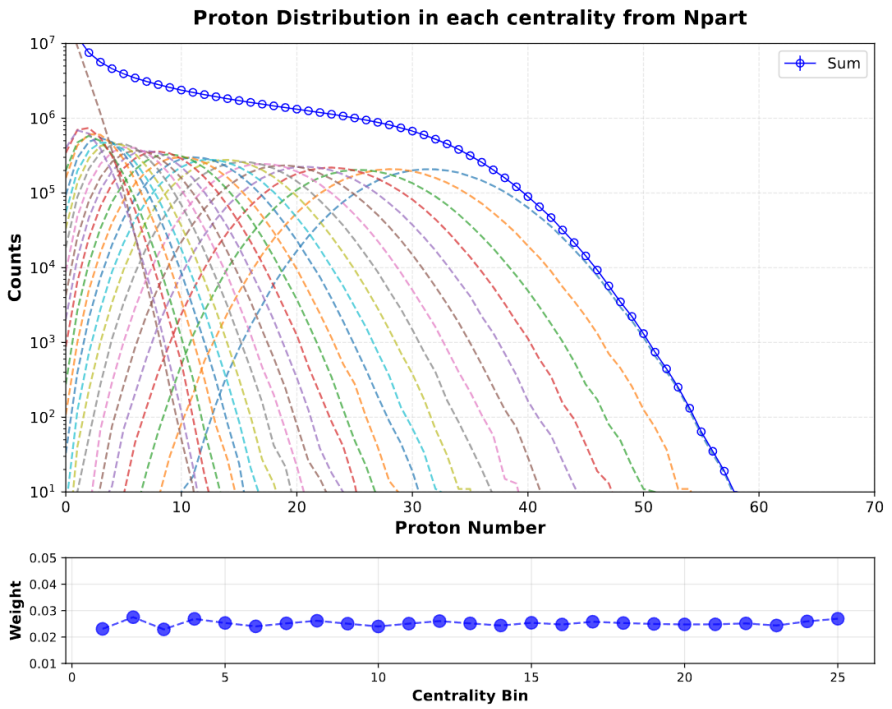


The effectiveness of CBWC method decreases at lower energies.

Method >> Weights Fluctuations

How to solve initial volume fluctuation effect ?





$$\mathbf{weight_i = A_i}$$

$\mathbf{A_i}$: the amplitude of the i-th sub-distribution

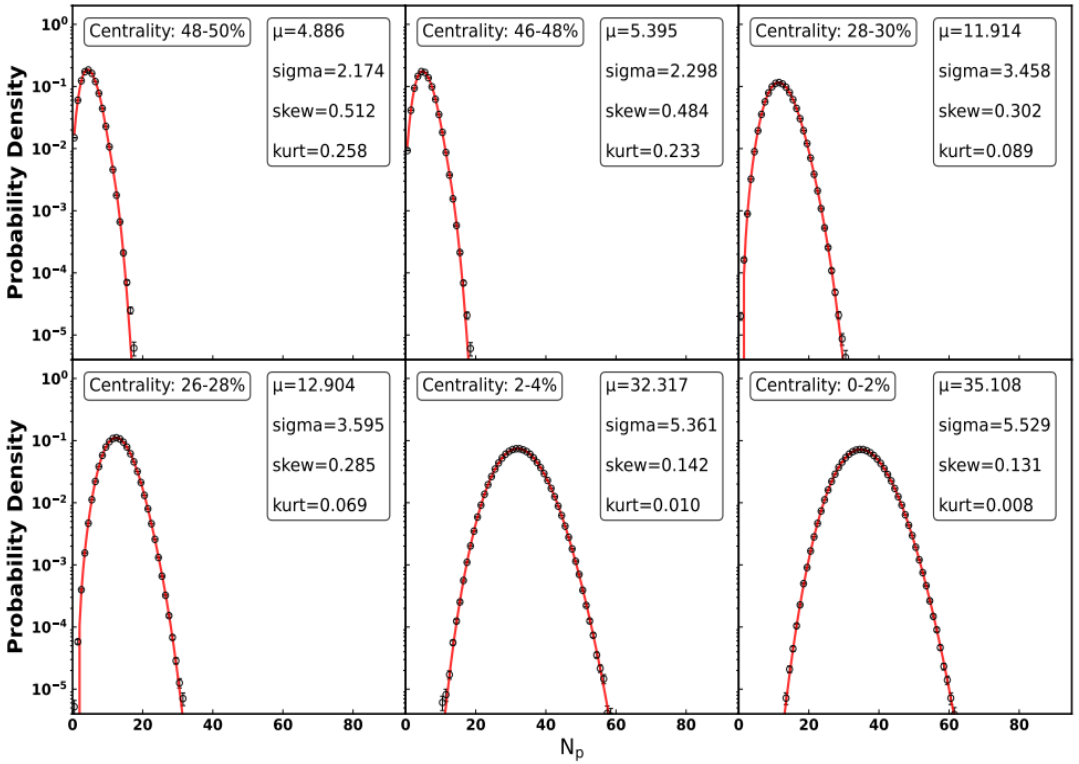
The distribution obtained from N_{part}-based centrality is flatter compared to the distribution obtained from RefMult3.

We can start from the probability distribution itself.

Method >> Mathematical Tool

How to solve initial volume fluctuation effect ?

Edgeworth Expansion: reconstruct the distribution based on cumulant information



Proton distribution in each centrality from UrQMD

Edgeworth expansion:

$$\sigma p(\sigma x) = Z(x)\left\{1+\sum_{s=1}^{\infty}\left[\sigma^s \sum_{\{k_m\}} He_{s+2r}(x) \prod_{m=1}^{s} \frac{1}{k_m!}\left(\frac{S_m+2}{m+2}\right)^{k_m}\right]\right\}$$

where Chebyshev-Hermite polynomials H_{e_n} is defined as:

$$H_{e_n}(x) = (-1)^n exp(x^2/2)\frac{d^n}{dx^n}exp(-x^2/2)$$

Edgeworth expansion can describe near-Gaussian distributions well !

Method >> Parameterization

How to solve initial volume fluctuation effect ?

Probability distribution reconstruction at each centrality bin relies on cumulant information.
For 20 centrality bins using cumulants up to the 4th order, it will require 4×20 parameters.

Polynomial assumption(reduce parameters):

C_2 reflects two-particle correlation$(\ C_2 \sim \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2\ )$

C_3 reflects three-particle correlation$(C_3 \sim \beta_0 + \beta_1 C_1 + \beta_2 C_1^2 + \beta_3 C_1^3)$

C_4 reflects four-particle correlation$(C_4 \sim \gamma_0 + \gamma_1 C_1 + \gamma_2 C_1^2 + \gamma_3 C_1^3 + \gamma_4 C_1^4)$.

We assume that these correlation coefficients are independent on centrality.

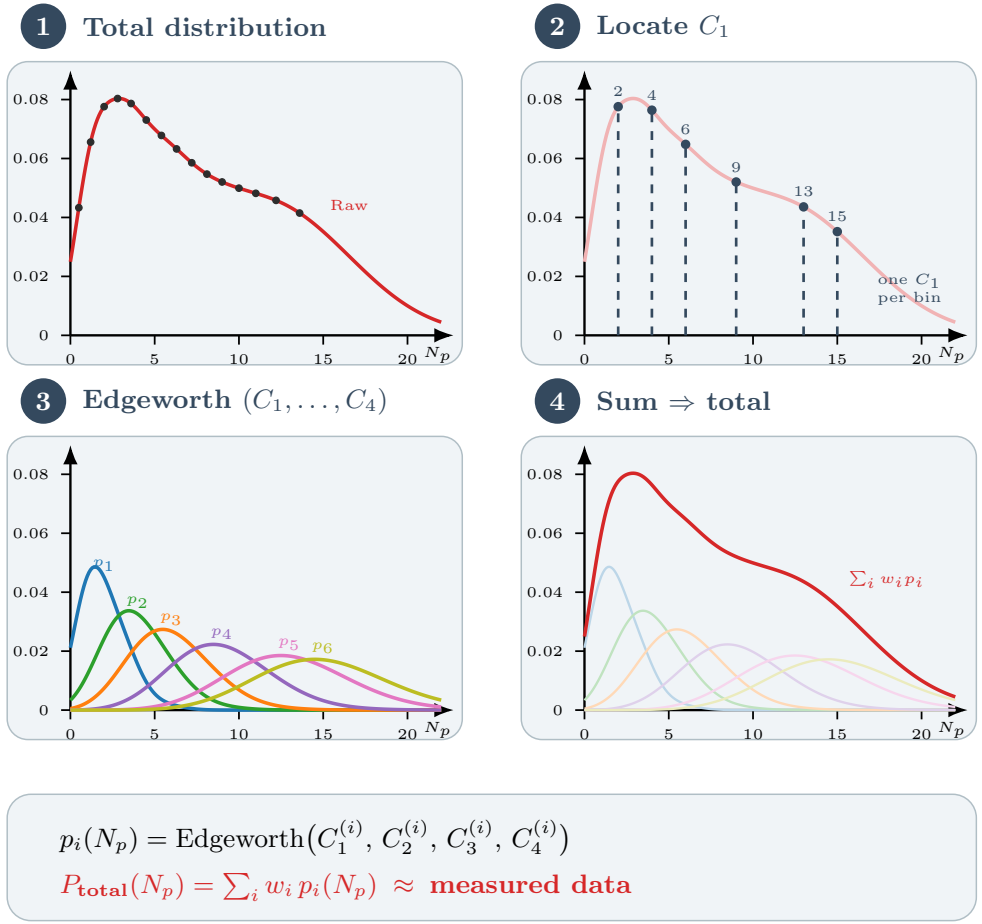
This naturally leads to using C1 as a direct proxy for centrality.

Importantly, C1 shows little dependence on centrality resolution.

Method

How to suppress initial volume fluctuation(IVF) effect ?

1. **Feature:** Weights show small fluctuations across different sub-centrality bins.
2. **Tool:** Distribution can be reconstructed from cumulant information.
3. **Parameterization:** Parameterize the variation of cumulants as polynomial forms.
4. **Centrality proxy :** C_1 is used as a proxy for centrality.



$$p_i(N_p) = \text{Edgeworth}\left(C_1^{(i)}, C_2^{(i)}, C_3^{(i)}, C_4^{(i)}\right)$$

$$P_{\text{total}}(N_p) = \sum_i w_i\, p_i(N_p) \approx \text{measured data}$$

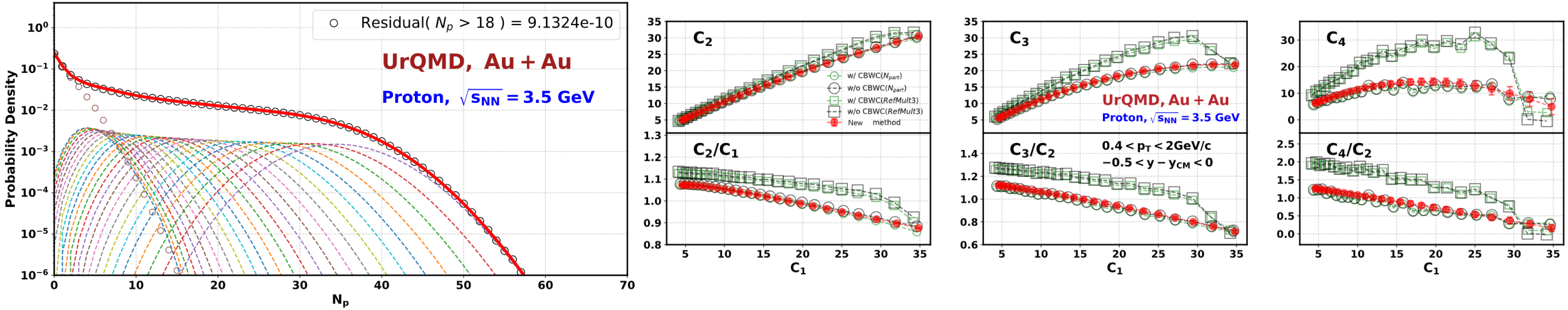
More details of this approach can be found here:

[1] Z. Wang. Talk at CPOD 2026, for the STAR Collaboration, 2026.

[2] Zhaohui Wang, Xiaofeng Luo. Phys.Lett.B.871(2025)139984.

Closure Test >> UrQMD

UrQMD Test in Au+Au Collision @ $\sqrt{s_{NN}} = 3.5$ GeV:

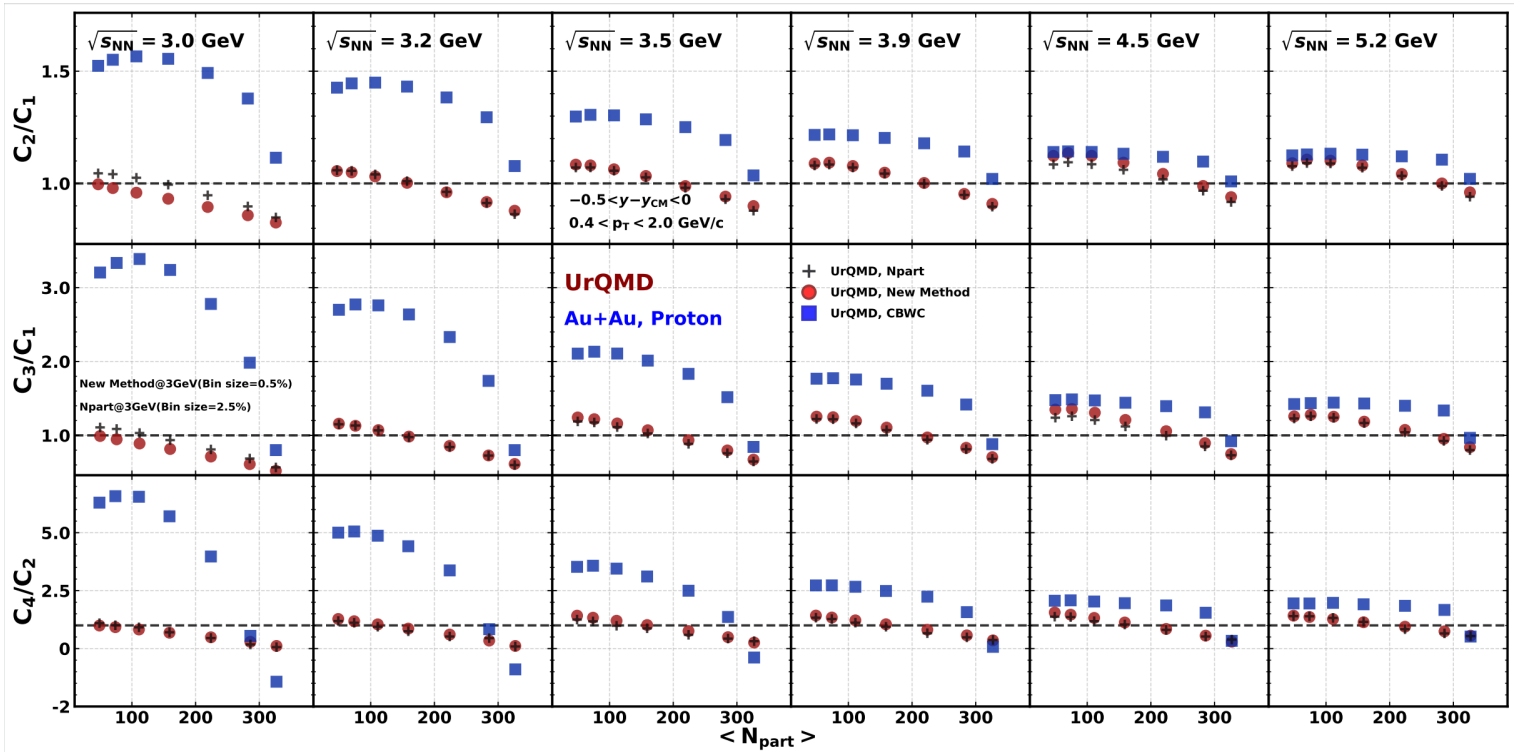


The results of fitting the total distribution by reconstructing the sub-distributions with the Edgeworth expansion are close to the results based on N_{part} information and follow the trend.

Single centrality interval: 2%

Number of centrality bins used: 25

Closure Test >> UrQMD



- A new baseline has been established.
- The results from the new method are consistent with those based on N_{part} information.
- As the energy increases, the results gradually converge with the CBWC results.

Closure Test >> Detector Effect

How to use for experiment? (The main issue comes from detector efficiency)

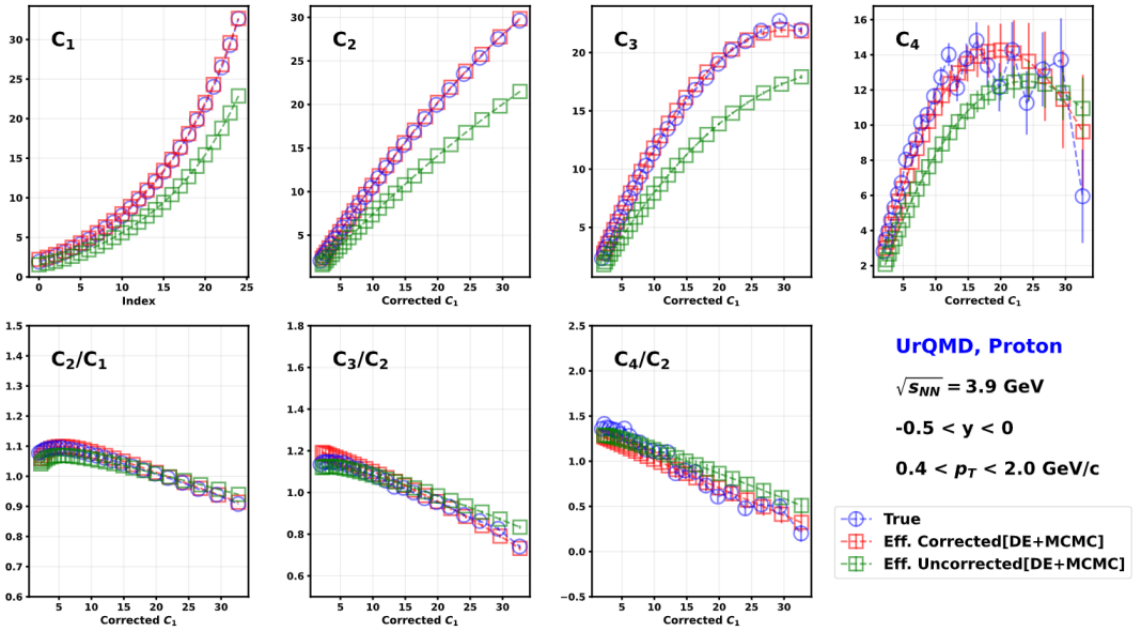
Apply an efficiency to particles in UrQMD to simulate detector effects

Input raw total probability distribution

Obtain the efficiency uncorrected cumulants

Using the average efficiency correction formula

Obtain the efficiency corrected cumulants



$$\lambda_1 = \frac{1}{\epsilon},\ \lambda_2 = \frac{1}{\epsilon^2},\ \lambda_3 = \frac{1}{\epsilon^3},\ \lambda_4 = \frac{1}{\epsilon^4},$$

$$C_{1,corr} = C_{1,uncorr} * \lambda_1$$

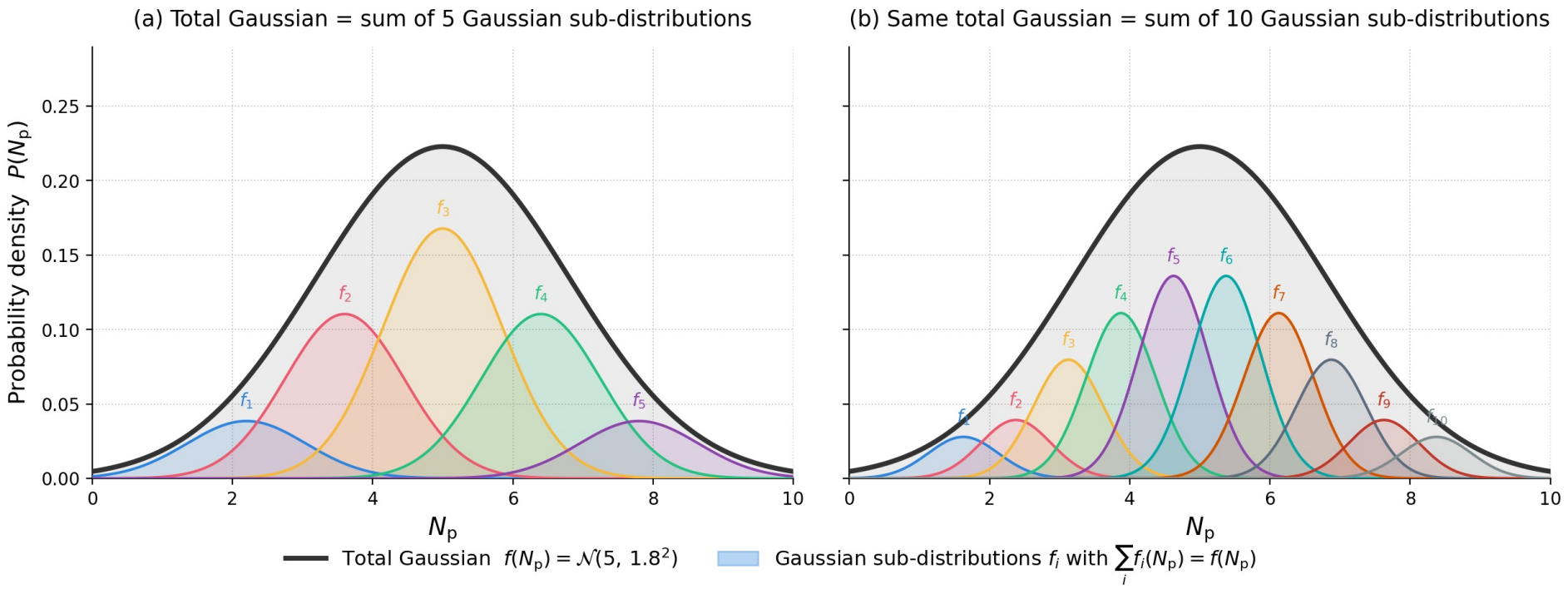
$$C_{2,corr} = C_{2,uncorr} * \lambda_2 + (\lambda_1 - \lambda_2) * C_{1,uncorr}$$

$$C_{3,corr} = C_{3,uncorr} * \lambda_3 + (\lambda_2 * 3 - \lambda_3 * 3) * C_{2,uncorr} + (\lambda_1 + \lambda_3 * 2 - \lambda_2 * 3) * C_{1,uncorr}$$

$$C_{4,corr} = C_{4,uncorr} * \lambda_4 + (\lambda_3 * 6 - \lambda_4 * 6) * C_{3,uncorr} + (\lambda_2 * 7 - \lambda_3 * 18 + \lambda_4 * 11) * C_{2,uncorr} + (\lambda_1 - \lambda_2 * 7 + 12 * \lambda_3 - 6 * \lambda_4) * C_{1,uncorr}$$

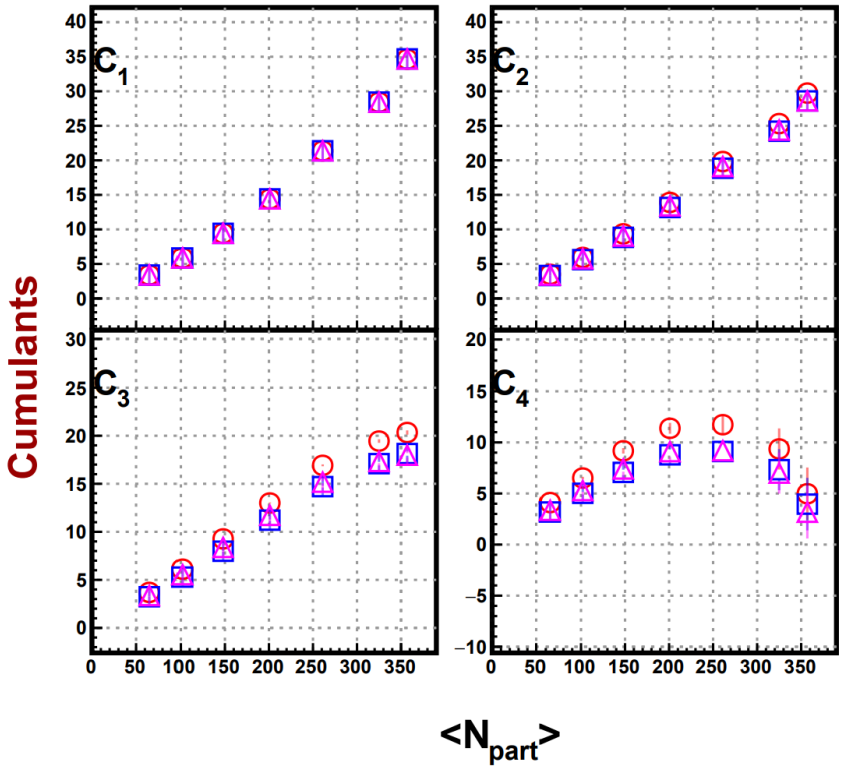
Effectively address the detector effect problem.

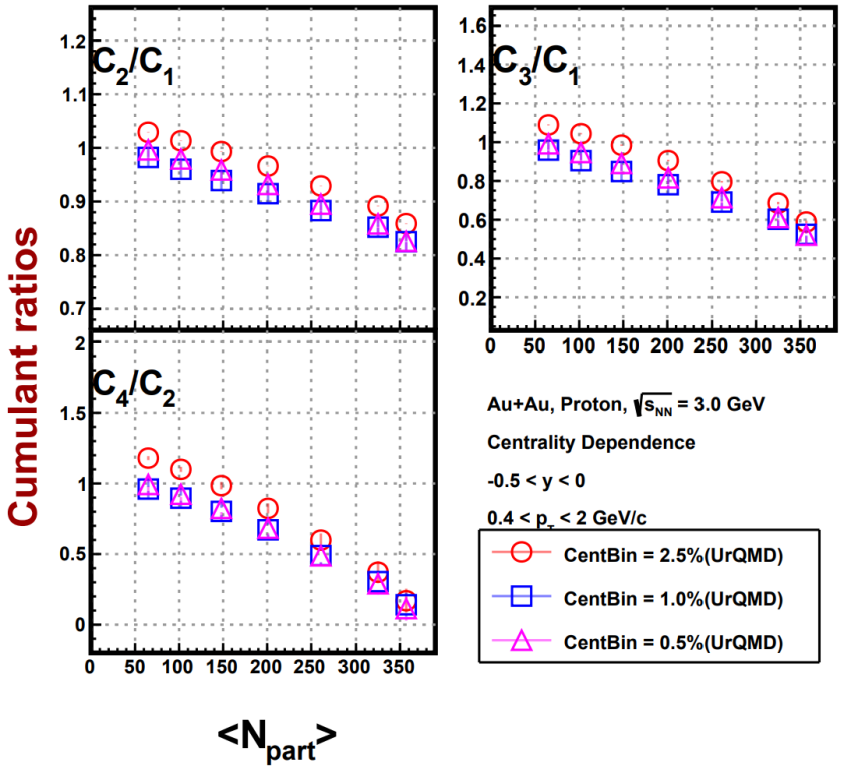
Closure Test >> Bin Size Effect



For a given total distribution, when it is split into more sub-components, each component will have a smaller volume.

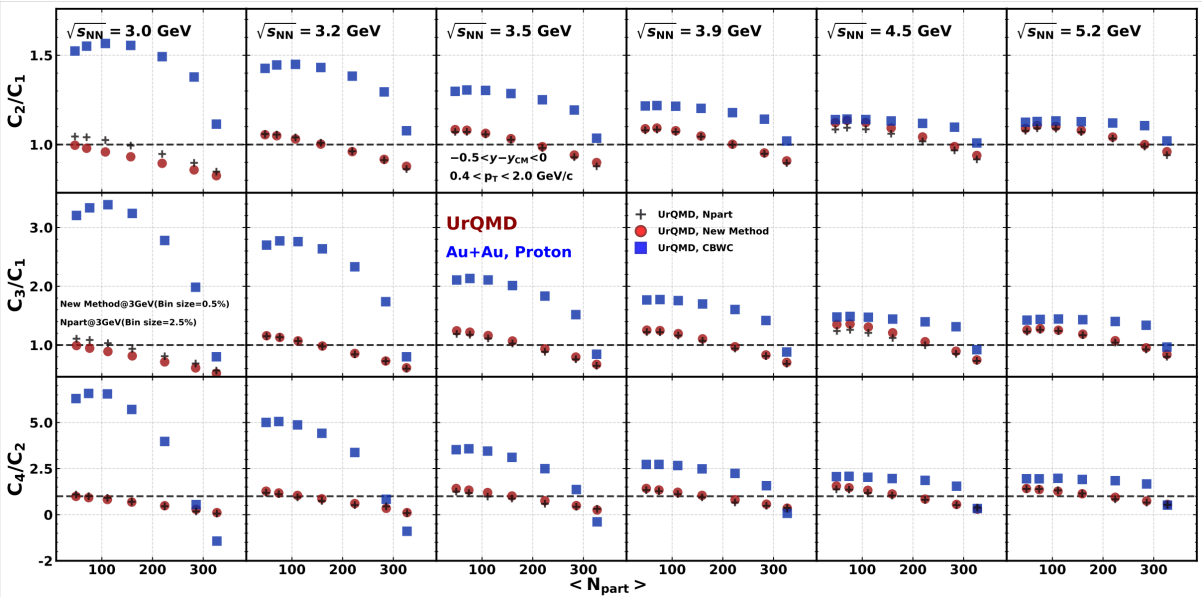
Closure Test >> Bin Size Effect





1. C_1 is not affected by volume.
2. C_2/C_1 and C_3/C_1 show a certain decrease as the volume decreases.
3. For most central C_4/C_2, the result is only slightly affected by volume.

Summary & Outlook



Summary

1. A novel method[1] has been proposed to suppress the IVF effect.
2. The closure tests verified the effectiveness of the method as well as its feasibility for application in experiments.

Outlook

- Finalize the analysis for experimental data and the new collected high statistics FXT energies in 2026: 4.2, 4.5, and 5.2 GeV.

Thanks for your attention!

[1] Zhaohui Wang, Xiaofeng Luo. Phys.Lett.B. 871(2025)139984