From Functional QCD to heavy-ion collision observables

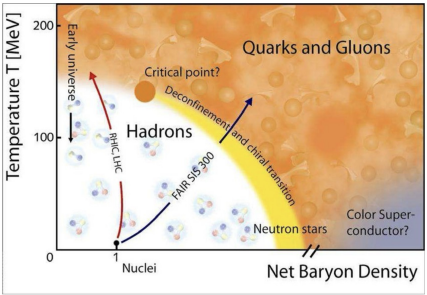
Fei Gao

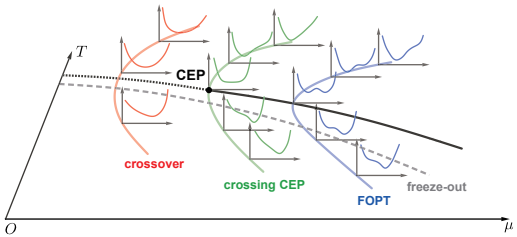
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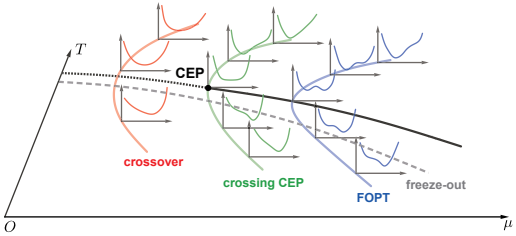
QCD in heavy ion collisions:

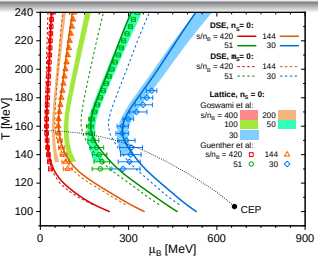




- The evolution for each collision is a trajectory in the $T - \mu_B$ plane (isentropic for ideal case): smaller collision energy, higher density.
- If the starting point is higher the QCD PT line, the evolution will cross crossover/CEP/FOPT region, and ends at so called chemical freeze out point.
- The typical measurements for phase transitions are the cumulants/fluctuations at freeze out line.

Ideal case is isentropic trajectory: for each collision energy, it corresponds to a specific s/n_B.





- This is the evolution of ideal hydrodynamics only with uniform space distribution.
- The real time dynamics gives a timing. When does the collision start/stop?
- The dynamical evolution gives the space-distribution of the matter, and different yield rate for different species.
- A fit between s/n_B and $\sqrt{s_{NN}}$: Since we know for which s/n_B the trajectory crosses the CEP, and we know the respective collision energy, and it is $\sqrt{s_{NN}} \sim 1 - 2$ GeV. (Criticality analysis is generally speaking a dead end)

Assuming the local equilibrium condition, one can actually build the effective potential from any local order parameter, for instance, the number density n_B. The effective potential can be written as:

$$\Phi[n_B] = \sum_k \frac{1}{k!}\alpha_k \left(n_B - \overline{n}\right)^k , \quad \alpha_k = \left.\frac{\partial^k \Phi}{\partial n_B^k}\right|_{n_B=\overline{n}} ,$$

For the uniform case, the coefficients of the effective potential can be expressed in terms of cumulants:

$$\omega_k = \alpha_k\, \overline{n}^{k-1}/T, \qquad \omega_2 = \frac{1}{R_{21}}, \qquad \omega_3 = -\frac{R_{32}}{R_{21}^2}, \qquad \omega_4 = \frac{3\,R_{32}^2 - R_{42}}{R_{21}^3}, \qquad \cdots ,$$

From the effective potential, one can determine the order of the phase transition via these coefficients, as it is determined by the number of the extrema of the potential. This is the microscopic mechanism for using cumulants to detect CEP.

Y. Lu, FG, X. Luo, L. Chang, Y. Liu, SCPMA 68, 251012 (2025), arXiv:2211.03401

The quark propagator sees the order parameter of chiral phase transition and "deconfinement" (gluon) phase transition.

$$G_q^{-1}(p) = i(\omega_p + i\mu_q + gA_0)\gamma_4 Z_q^E(p) + i\vec{\gamma}\cdot\vec{p}\, Z_q^M(p) + Z_q^E(p)M_q(p).$$

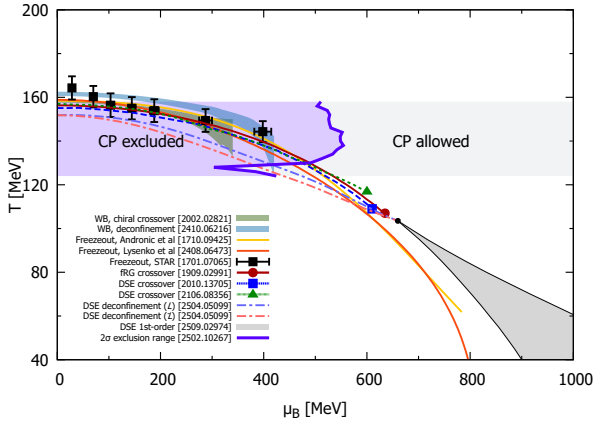
And then we obtain the number density directly by the trace of the propagator:

$$n_q^f(T,\mu_B) \simeq -N_c Z_2^f\, T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{tr}_D \left[\gamma_4 S^f(p)\right]$$

$$P(T,\boldsymbol{\mu}) = P_{Latt.}(T,\mathbf{0}) + \sum_q \int_0^{\mu_q} n_q(T,\mu)\, d\mu$$

Then we have all the thermodynamic quantities.

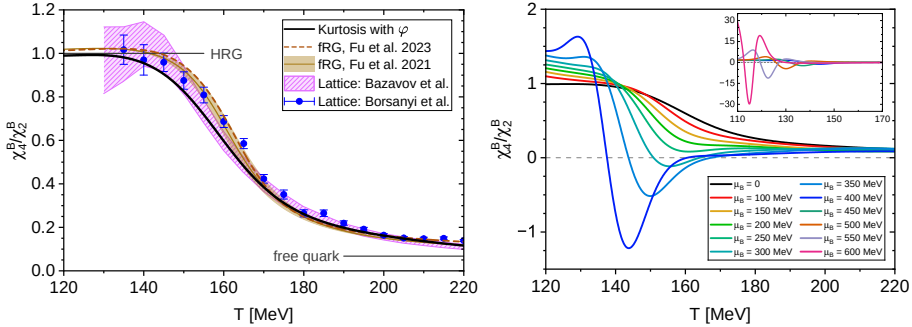
Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



- CEP: (T, μ_B)
  $\sim (100 - 110, 600 - 700)$ MeV
- First order phase transition at high baryon density with Coexistence region slightly above liquid gas transition.
- At zero temperature: Ideal phase transition at $\mu_B \approx 1050$ MeV

Look into the fluctuations/cumulants

Kurtosis at the whole $T - \mu_B$ plane:



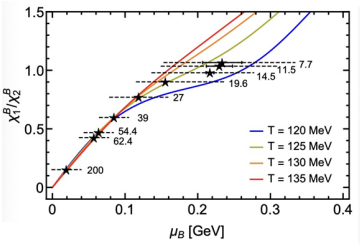
- Currently obtained at $\mu_S = 0$ but has been improved to the strangeness neutral case $n_S = 0$.
- Equilibrium calculation without time-space distribution (which also gives the canonical ensemble correction), net proton/baryon difference and other non equilibrium effects.
- With this caveat in mind, we can still map our results to experimental data.

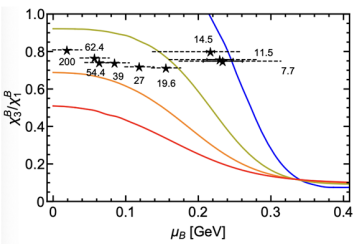
To compare with experiment, one needs to know the freeze out line.

- The freeze out condition can be determined by matching the theoretical calculation of fluctuations with the experimental data as firstly proposed in Lattice simulations.

  A. Bazavov et al, PRL 109, 192302 (2012); S.Borsanyi et al, PRL 111, 062005 (2013); S. Borsanyi, et al, PRL 113, 052301 (2014)

- This matching gives an exact 2 to 2 mapping after scanning the T-μ_B plane: (Jing Chen, FG, Yuxin Liu, *arXiv:1510.07543(2015); Yi Lu, et al, PRD 105, 034012 (2022)*)



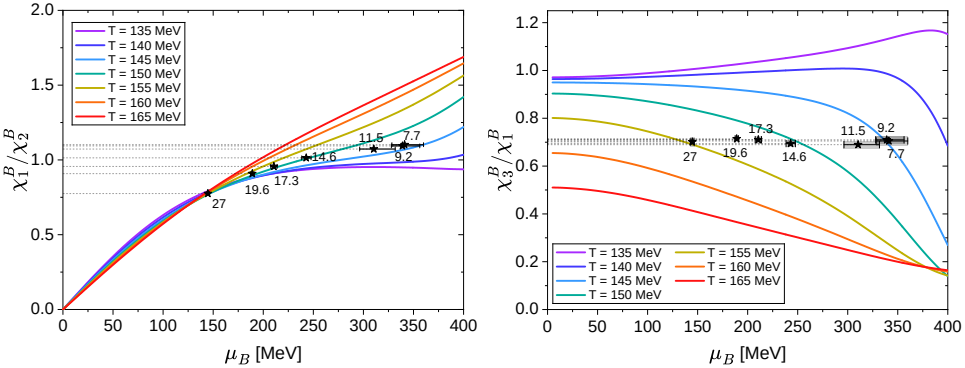


$$\frac{\chi_2^B}{\chi_1^B}(T_f, \mu_{B,f}) = \frac{C_2}{C_1}\left(\sqrt{s_{\rm NN}}\right)$$

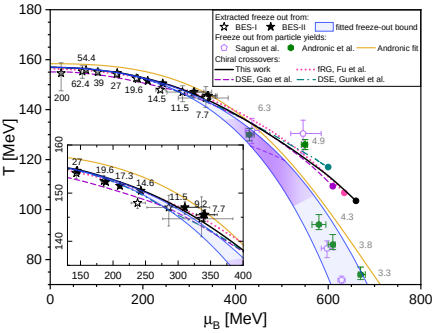
$$\frac{\chi_3^B}{\chi_1^B}(T_f, \mu_{B,f}) = \frac{C_3}{C_1}\left(\sqrt{s_{\rm NN}}\right)$$

A self consistent determination of freeze out line and the prediction of kurtosis and also higher order fluctuations at freeze out line.

C_1/C_2 mainly determines the chemical potential, and C_3/C_1 determines the temperature:



The direct extraction and the fitting procedure of freeze out line



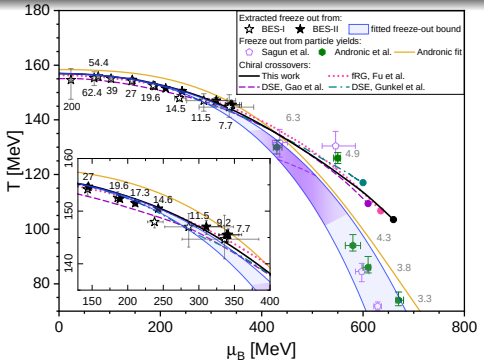
$$T_f = T_0 \left[1 - \kappa_2^f \left(\frac{\mu_{B,f}}{T_0} \right)^2 - \kappa_4^f \left(\frac{\mu_{B,f}}{T_0} \right)^4 + \cdots \right]$$

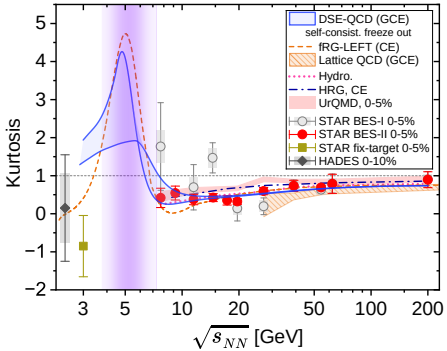
- The direct extraction from data gives:
  $T_0 = T_c(0) = 157$ MeV;
  $\kappa_2^f \leq \kappa_2 = 0.0153$.
- Assumption: freeze out after phase transition:
  $\kappa_2^f \geq \kappa_2 = 0.0153$.

$T_0 = T_c(0) = 157$ MeV; $\kappa_2^f = \kappa_2 = 0.0153$, with only κ_4^f that is determined by one point from the estimate of statistic model at 6.3 *GeV from (A. Andronic, et al, Nature 561, 321 (2018).)*

- Up to 7.7 GeV, the freeze out line and the phase transition line coincide, and the deviation caused possibly by non equilibrium effects appears from 10 GeV.
- The only deviation is the point from statistical model used for low energy, hence the CE/GCE, proton/baryon effects are incorporated.

The freeze out line and the kurtosis





- The determination of the freeze out line in fact absorbs the effects of CE/GCE, proton/baryon, and some other non equilibrium effect by mapping $\sqrt{s_{NN}}$ to the effective T' and μ_B'.
- The peak of kurtosis is located at 4-5 GeV, which can be observed in experiments, as the signals of new dynamics (CEP or new phases)
- The kurtosis would still be affected by these effects, for example, CE/GCE correction could reduce the peak height by $1/3$.

How to incorporate these effects?

- Proton/Baryon difference: binomial filtering of the (anti)baryon fluctuations.

$$\delta C_{n,m}^{B^{\pm},p^{\pm},\mathrm{gce}}(x_i;\Delta p_{\mathrm{acc}}) = \delta_{m,0}\delta C_{n}^{B^{\pm},\mathrm{gce}}(x_i;\Delta p_{\mathrm{acc}})$$

$$+\sum_{l=1}^{m}\delta C_{n+l}^{B^{\pm},\mathrm{gce}}(x_i;\Delta p_{\mathrm{acc}})B_{m,l}\left(\gamma'_{t_p},\cdots,\gamma_{t_p}^{(m-l+1)}\right),$$

- CE/GCE correction: subensemble acceptance method.

$$C_{n,m}^{B,p,\mathrm{ce}}(\Delta p_{\mathrm{acc}}) = \mathcal{S}[C_{n,m}^{B,p,\mathrm{gce}}(\Delta p_{\mathrm{acc}}), C_{n,m}^{B,p,\mathrm{gce}}(\overline{\Delta p_{\mathrm{acc}}})]$$

The apples to apples comparison between theory and experiment:

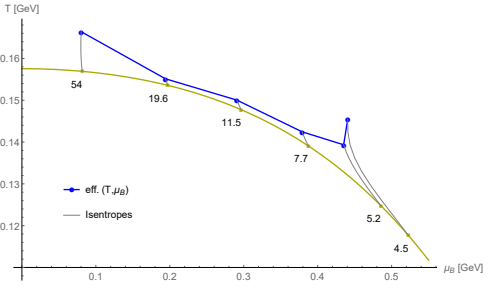
- Obtaining the baryon number fluctuations $\chi_{1\ldots4}$ calculated in GCE with functional QCD method.
- Adopting the above correction method to gives the corrections for each fluctuation at each collision energy.
- Converting $\chi_{1\ldots4}$ into proton number fluctuations in CE, which can be then directly compared to the experimental data.

Rui-zhe Zhao, Shi Yin, Lipei Du, Xiaofeng Luo, Wei-jie Fu, arXiv:2607.26912.

A step forward to understand the collision via considering more observables

- The mean p_T as effective temperature of collisions: $< p_T > \sim 3T$, this can also be obtained within the functional method via calculation $p_T \sim 3T$,
- Considering one specific collision energy, we know the freeze out T and μ_B from the cumulants as described above, and then

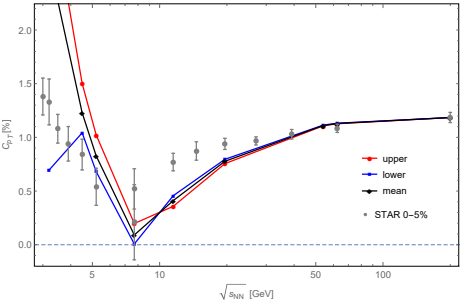
With the isentropic trajectory, one can trace back to the point which characterizes the collision, i.e. the effective T and μ_B.



- For collision energy above 54 GeV, the effective temperature is high, around 200 MeV.
- Between 54 GeV and 7.7 GeV, the effective T and μ_B are closely above the PT line.
- For below, the equilibrium interpretation for p_T certainly fails, as the collision is supposedly to start even from the hadronic phase.
- The turning point is around 5 GeV.

Preliminary results for C_{pT}

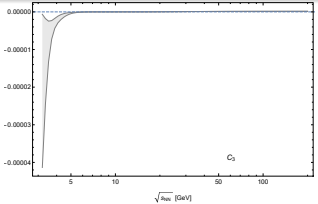
One can also calculate the higher order correlations of transverse momentum p_T (Temperature fluctuations as pointed out in Chen, Fu, Yin, Zhang, PRD 114 (2026) 1, 014064).
Here we show the results of C_{p_T} on freeze out line:

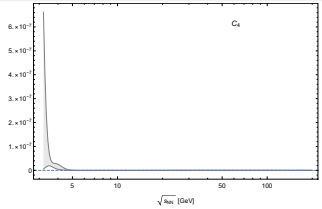


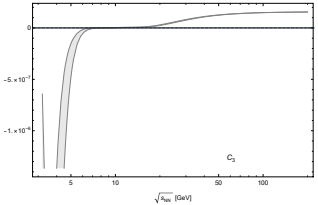
Y. Lu, F. Gao, J. M. Pawlowski, C. J. Zhang, in preparation.

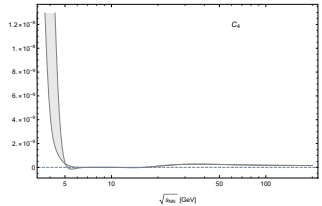
- The minimum point is not related to the CEP, instead, it is a trivial zero in the correlation.
- If working with the effective temperature obtained above, one sees a maximum around 5 GeV which is in contradiction to the trend of experimental data.
- The consistency of the results at freeze out means that below 5 GeV, it is mainly the hadronic dynamics in charge.

Higher order correlations:









- The general magnitude is small due to the factor of volume in the correlations.
- The third order correlation changes sign at around 7 GeV, from positive to negative as energy decreases.
- The four order correlation increases quickly for low energy.

The conclusions and discussions:

To understand QCD phase transitions by combining fQCD computations with BESII measurements

- The chiral PT is with a CEP at $\mu_B \approx 600 - 700$ MeV.
- Freeze out line is consistent with phase transition line up to 7.7 GeV.
- The computed kurtosis is consistent with the current measurements in BESII, and we find a peak is located at around 5 GeV.
- CE/GCE, proton/baryon corrections should be incorporated to really compare theoretical results to experimental data, which can be done now.
- A new measurement, correlations of p_T, share similar behavior to the cumulants, which might be a complementary messenger of QCD phase transition.
- A detailed measurement of p_T may help to further understand the heavy ion collision in terms of QCD phase transition.

Thank you!