

Instantons on Black holes

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Based on work with Elden Loomes: [2604.02841](#) + in progress

7th Joint Workshop on the Standard Model & Beyond

NNU, 29 October 2026

Classical black holes

- Classical black holes are quite boring objects – (electro) vacuum solutions of classical GR
- They are also rather ‘bald’ – very few classical ‘hair’: mass, spin, electric/magnetic charge
- All of them associated with the gauge constraints (the respective Gauss’ laws):
 - Mass/energy – Hamiltonian constraint in GR
 - Electric charge – Electric Gauss’ law of EM

Black holes with quantum fields

- In semi-classical GR and no backreaction limit, i.e.,

$$G \rightarrow 0, \quad M \rightarrow \infty, \quad \text{with } r_S = 2GM \text{ --- finite}$$

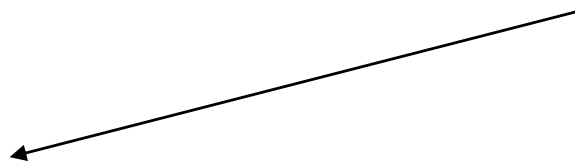
gives nonsensical result: pure quantum state of matter can be turned into a mixed thermal state (Hawking radiation) in sharp contradiction to the rules of quantum mechanics

- The emergence of quantum 'hair': typically topological solutions of quantum fields that cannot be disturbed by local perturbations (e.g., skyrmion hair).
- Quantum hair can be 'detected' by doing 'quantum measurements, e.g., interference pattern, entanglement...
- Question: As the classical black holes are the vacuum solutions, does the quantum vacuum manifests in this semiclassical limit?

Topology and vacuum tunnelling in curved spacetimes: General considerations

- Gauge theory based on gauge group G on spacetime manifold

$$\mathcal{M} = \mathcal{T} \times \Sigma_3$$



- Globally defined time:
 - Real time correlators $\Leftrightarrow$ Imaginary time (Euclidean) correlators
- Compact imaginary time, $\mathcal{T} \simeq S^1$:
 - Euclidean correlators $\Leftrightarrow$ Correlators at finite temperature

Topology and vacuum tunnelling in curved spacetimes: General considerations

- Euclidean description in quasi-classical approximation requires positive action

$$\mathcal{A} \propto e^{-S_E}, \quad S_E > 0$$

- Positive energy/action theorem: Spacetime manifolds must be asymptotically (locally) flat:

$$R_{\mu\nu\rho\sigma} \xrightarrow{|\vec{x}| \rightarrow \infty} 0$$

Schon and Yau, '79
Witten, '81

- Hence, the gauge connection must be asymptotically flat (pure vacuum configuration):

$$F_{\mu\nu} \xrightarrow{|\vec{x}| \rightarrow \infty} 0$$

- Spatial section (topologically) compact

$$\tilde{\Sigma}_3 \simeq \Sigma_3 \cup \{\infty\}$$

Topology and vacuum tunnelling in curved spacetimes: General considerations

- Vacuum gauge field configurations (in temporal gauge)

$$A \xrightarrow{|\vec{x}| \rightarrow \infty} g^{-1} dg, \quad g \in G$$

$$g : \tilde{\Sigma}_3 \rightarrow G$$

- Vacuum configurations are distinguished by homotopy classes of such maps of the set

$$[\tilde{\Sigma}_3, G]$$

- In general, such maps depend on the topology of both $\tilde{\Sigma}_3$ and G

Topology and vacuum tunnelling in curved spacetimes: General considerations

- Conveniently the homotopy set is given by the n th cohomology group of $\tilde{\Sigma}_3$

$$\begin{aligned} [\tilde{\Sigma}_3, G] &= \sum_n H^n(\tilde{\Sigma}_3, \pi_n(G)) \\ &= H^1(\tilde{\Sigma}_3, \pi_1(G)) \oplus H^3(\tilde{\Sigma}_3, \pi_3(G)) \end{aligned}$$

Isham, '83

[Note: $\pi_2(G) = 0$]

Topology and vacuum tunnelling in curved spacetimes: General considerations

- Familiar case of flat space R^4 and $G=SU(N)$:

$$\tilde{\Sigma}_3 = R^3 \cup \{\infty\} = S^3, \quad \pi_3(SU(N)) = \mathbb{Z}$$

- Hence, $H^3(S^3, \pi_3(G)) = \mathbb{Z}$, and the vacuum configurations are labelled by integer topological numbers $n \in \mathbb{Z}$.

- Another familiar case: flat space R^4 and $G=U(1)$:

$$\tilde{\Sigma}_3 = R^3 \cup \{\infty\} = S^3, \quad \pi_3(U(1)) = 0$$

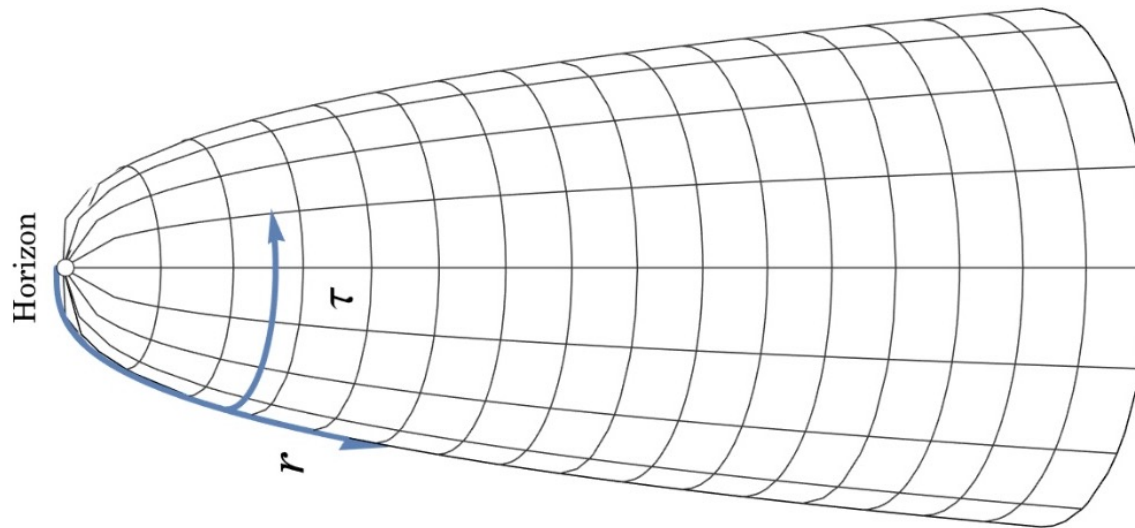
- Hence, all cohomology groups are trivial as well. The vacuum is topologically trivial

Euclidean Schwarzschild black hole

- Euclidean description in quasi-classical approximation requires positive action

$$ds^2 = \left(1 - \frac{r_h}{r}\right) d\tau^2 + \left(1 - \frac{r_h}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad r_h = 2GM$$

- Imaginary time is periodic, $\mathcal{T} = S^1$: $\tau \sim \tau + 4\pi r_h$, $T_H = 1/4\pi r_h$



Electromagnetic instantons (Euclidean dyons)

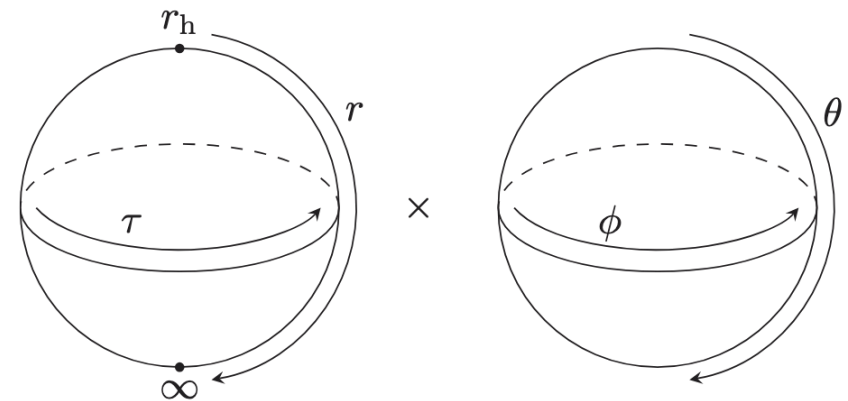
Charap and M. J. Duff, '77

- Closed (co-closed), but not exact (co-exact) gauge strength 2-forms

$$F = \frac{n}{2r^2} d\tau \wedge dr + \frac{m}{2} \sin \theta d\theta \wedge d\phi .$$

$$n = \frac{1}{2\pi} \int_{S^2_{\theta,\phi}} *F = \frac{1}{2\pi} \int_{S^2_{\tau,r}} F$$

$$n = \frac{1}{2\pi} \int_{S^2_{\tau,r}} *F = \frac{1}{2\pi} \int_{S^2_{\theta,\phi}} F$$



Electromagnetic instantons (Euclidean dyons)

- Actions: $S_{\text{EM}} = \frac{2\pi^2}{e^2}(n^2 + m^2)$

- Euclidean energy-momentum tensor:

$$T_{\mu\nu} = (m^2 - n^2)t_{\mu\nu}$$

$$t_{\tau\tau} = \left(1 - \frac{r_h}{r}\right)^2, \quad t_{rr} = \frac{1}{2r^4} \left(1 - \frac{r_h}{r}\right), \quad t_{\phi\phi} = \sin^2 \theta, \quad t_{\theta\theta} = -\frac{\sin^2 \theta}{2r^2}.$$

- Only self-dual, $n=m$, and anti-selfdual, $n=-m$, dyons are on-shell solutions. Note however, that off-shell dyons do also contribute quantum mechanically

Electromagnetic instantons (Euclidean dyons)

- Physical interpretation: ground state around black hole is represented by a statistical ensemble of virtual electromagnetic dyons, which contribute to the thermal correlators (gravitational part is factored out)

$$\langle \mathcal{O} \rangle \simeq \frac{1}{Z} \sum_{n,m} \mathcal{O} e^{-S_{\text{EM}}}, \quad Z \simeq \sum_{n,m} e^{-S_{\text{EM}}}$$

$$\langle T_{\mu\nu} \rangle \simeq \frac{1}{Z} \sum_{n,m} T_{\mu\nu} e^{-S_{\text{EM}}} = \frac{t_{\mu\nu}}{Z} \sum_{n,m} (m^2 - n^2) \exp \left[-\frac{2\pi^2}{e^2} (n^2 + m^2) \right] = 0,$$

$$\langle n \rangle \simeq \frac{1}{Z} \sum_{n,m} n e^{-S_{\text{EM}}} = \frac{1}{Z} \sum_{n,m} n \exp \left[-\frac{2\pi^2}{e^2} (n^2 + m^2) \right] = 0$$

- This is consistent with the fact that we are working with Schwarzschild black hole in the semiclassical GR

Electromagnetic instantons (Euclidean dyons)

- Topological Pontryagin charge:

$$P \equiv \frac{1}{8\pi^2} \int_{\mathcal{M}} F \wedge F = \frac{1}{4\pi^2} \int_{\mathcal{M}} F_e \wedge F_m = \left(\frac{1}{2\pi} \int_{S^2_{(\tau,r)}} F_e \right) \left(\frac{1}{2\pi} \int_{S^2_{(\theta,\phi)}} F_m \right) = nm$$

- Bogomolny–Prasad–Sommerfeld bound

$$S_{\text{EM}} \geq \frac{4\pi^2}{e^2} |P|$$

is saturated by (anti-)selfdual dyons, $n=|m|$.

- The dyonic configurations support electromagnetic θ -term:

$$S_{\theta_{\text{EM}}} = -\frac{i\theta_{\text{EM}}}{8\pi^2} \int_{\mathcal{M}} F \wedge F$$

Electromagnetic instantons (Euclidean dyons)

- The electromagnetic θ_{EM} has potentially observable implications.
- Ensemble averaged Pontryagin charge:

$$\begin{aligned}\langle P \rangle &\simeq \frac{1}{Z} \sum_{n,m} nm e^{-S_{\text{EM}}} \simeq 2e^{-4\pi^2/e^2 + i\theta_{\text{EM}}} - 2e^{-4\pi^2/e^2 - i\theta_{\text{EM}}} \\ &\simeq 4i e^{-4\pi^2/e^2} \sin \theta_{\text{EM}}\end{aligned}$$

- This Pontryagin charge is responsible for photon helicity flux, which is described by the current,

$$J_{\text{h}} = * (A \wedge F), \quad d * J_{\text{h}} = F \wedge F$$

Electromagnetic instantons (Euclidean dyons)

- For a specific (n, m) dyon:

$$J_h^r = \frac{1}{\sqrt{g}} A_\tau F_{\theta\phi} = -\frac{1}{r^2} \frac{nm}{2r_h} \left(1 - \frac{r_h}{r}\right)$$

- Averaged helicity flux (circularly polarised photons):

$$\Phi_h = \int_{r \rightarrow \infty} \langle J_h^r \rangle r^2 d\Omega_2 = -32\pi^2 i T_H e^{-4\pi^2/e^2} \sin \theta_{\text{EM}}$$

- This, *in thesi*, observable feature in Hawking radiation arises from the non-trivial topological structure of Maxwell theory in the background of a non-rotating, neutral black hole.

Conclusion

- Non-trivial topology of curved manifolds can give rise to topological gauge field configurations that are absent in flat spacetime
- This may rearrange the vacuum structure of the theory and give rise to new non-perturbative observable effects.
- We have shown the simplest gauge theory of quantum electrodynamics admits virtual dyon solutions in the background of Euclidean Schwarzschild black hole
- The statistical ensemble virtual dyons support CP-violating electromagnetic θ -term
- The physical consequence of this is the feature of the black hole to emit polarised flux of thermal photons
- Other interesting effects after inclusion of fermions that exhibit topological zero modes