

Weak and electric dipoles of tau at CEPC and FCCee

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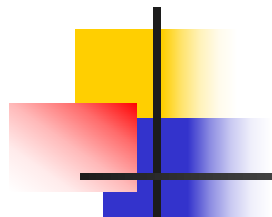
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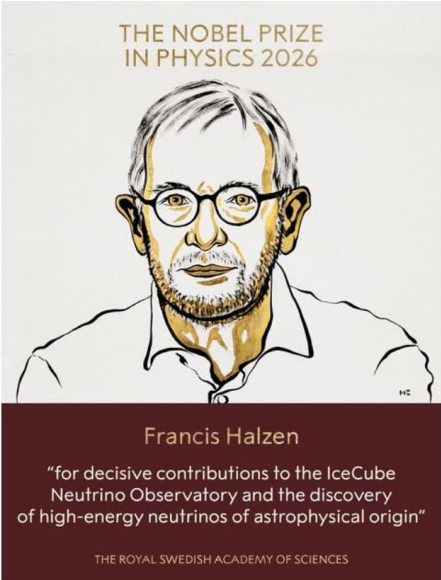
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- 1. The Electric and Weak Dipole Moments of a Fermion
- 2. Measurements of Hyperons and Tau EDM and WDM at Low Energies
- 3. Measurements of Tau EDM and WDM at/near Z pole



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1. The Electric and Weak Dipole Moments of a Fermion

Classically a EDM $\vec{D} = \int d^3x \vec{x} \rho(\vec{x})$ interacts with an electric field $\vec{E}$

The interaction energy is given by $H = \vec{D} \cdot \vec{E}$, allowed by P and T symmetries.

Under P, $\vec{D} \rightarrow -\vec{D}$ and $\vec{E} \rightarrow -\vec{E}$, H conserves both P and T.

A fundamental particle, $\vec{D}$ is equal to $d\vec{S}$, $H_{edm} = d\vec{S} \cdot \vec{E}$.

Since under P, $\vec{S} \rightarrow \vec{S}$ and under T, $\vec{S} \rightarrow -\vec{S}$

H_{edm} violates both P and T, CPT is conserved, CP is also violated!

Quantum field theory, $H_{edm} = -i\frac{1}{2}d\bar{\psi}\sigma^{\mu\nu}\psi\tilde{F}_{\mu\nu} = -i\frac{1}{2}d\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu}$

In non-relativistic limit H_{edm} reduce to $d\frac{\vec{\sigma}}{2} \cdot \vec{E} = d\vec{S} \cdot \vec{E}$.

One easily sees that H_{edm} violates P and T, violates CP, but conserve CPT.

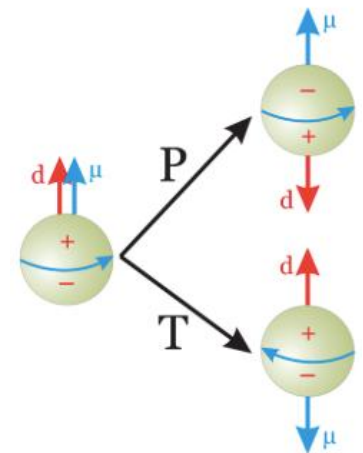
A non-zero fundamental particle EDM, violates P, T and CP!

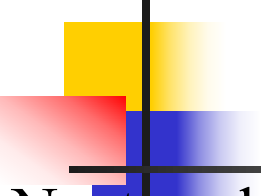
Magnetic Dipole conserves P and T

$$H_{mdm} = d_m \vec{S} \cdot \vec{B},$$

Under P: $\vec{B} \rightarrow \vec{B}$ and under T: $\vec{B} \rightarrow -\vec{B}$

Relativistic expression: $d_m \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu}$.





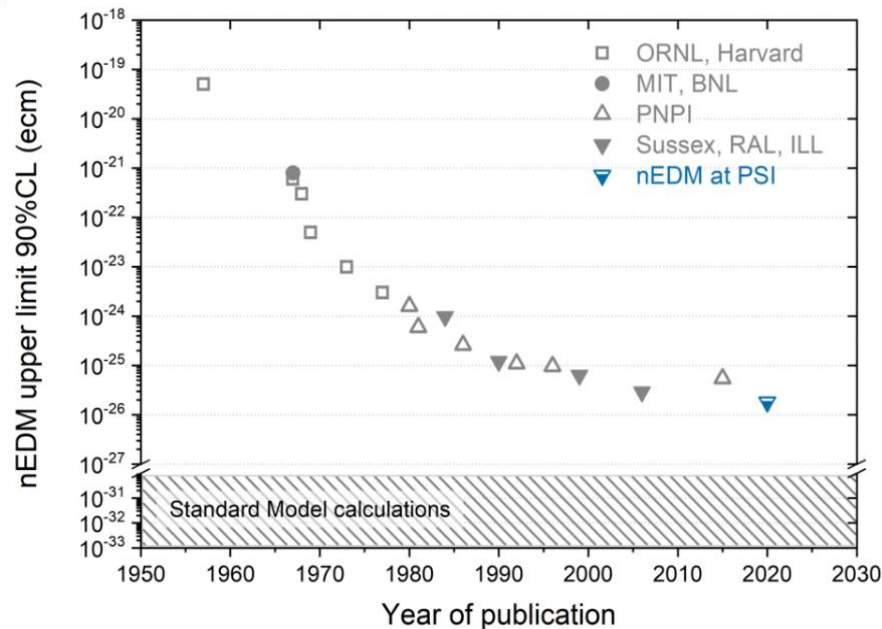
Neutron lives long enough, can watch Larmor precession, and switch electric field direction to measure. Proton is stable who EDM can be measured in proton beams... But hyperon decays, need different methods to measure it, through interactions

Electron EDM is usually not measured on free electrons, but instead on bound, unpaired valence electrons inside atoms and molecules. In these, one can observe the effect of $U = -\mathbf{d}_e \cdot \mathbf{E}$ as a slight shift of spectral lines.

Muon EDM measured in Storage ring and measured the precision by looking its decay.

But for fast decaying particles like hyperons and tauon other methods are needed, usually through interaction in decay or production processes.

History of Neutron EDM measurement



The first measurement started by Smith, Purcell, and Ramsey in 1951 (and published in 1957) obtaining a limit of

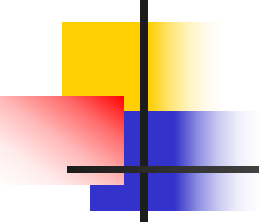
$$|d_n| < 5 \times 10^{-20} \text{ e} \cdot \text{cm}$$

1957, Landau realized edm of a fundamental particle violates CP

In order to extract the neutron EDM, one measures the Larmor precession of the neutron spin in the presence of parallel and antiparallel magnetic and electric fields.

$$h\nu = 2\mu_n B \pm 2d_n E \qquad d_n = \frac{h \Delta\nu}{4E}$$

Current Neutron $d_n < 1.8 \times 10^{-26} \text{ ecm}$



No measurement of a fundamental particle EDM, yet!

Current 90% C.L. limits on EDM:

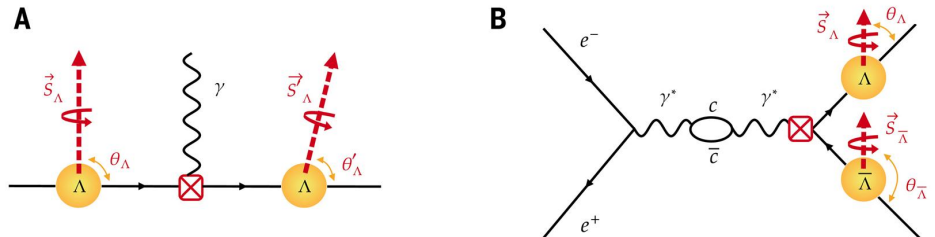
Proton $|d_p| < 2.1 \times 10^{-25}$ ecm,

Neutron $|d_n| < 1.8 \times 10^{-26}$ ecm,

Hyperon, like Lambda, measured long time ago with
Lambda $|d_\Lambda| < 1.5 \times 10^{-16}$ ecm,

Recently, BESIII improved the bound by more than two orders of magnitudes

$$|d_\Lambda| < 7.0 \times 10^{-19} \text{ e cm (95\% CL)}$$





A none zero value of EDM for a fundamental particle has not been measured!

Tauon EDM is very poor constrained compared with neutron and also its charge lepton partners, the electron and muon because very short lifetime! Have to be measured thorough interactions through scattering!

Electron $|d_e| < 1.1 \times 10^{-29}$ ecm, Muon $|d_\mu| < 1.8 \times 10^{-19}$ ecm

Tauon $\text{Re}(d_\tau) -2.2$ to 0.45×10^{-17} ecm, $-1.10 \times 10^{-3} < \text{Re } \tilde{d}_\tau^\gamma < 3.33 \times 10^{-3}$,
 $\text{Im}(d_\tau) -2.5$ to 0.08×10^{-17} ecm BELLE $-0.41 \times 10^{-3} < \text{Im } \tilde{d}_\tau^\gamma < 1.86 \times 10^{-3}$.

SM: $10^{-37} - 10^{-34}$ ecm

$$d_{\tau, \text{Belle}} = -e\tilde{d}_\tau^\gamma / (2m_\tau)$$

The WDM are also poorly constrained from ALEPH:

$$|\text{Re } \tilde{d}_\tau^W| < 0.91 \times 10^{-3}, \quad |\text{Im } \tilde{d}_\tau^W| < 2.01 \times 10^{-3}$$

Can still improve? Yes!

Quantum field theory of EDM and WDM for a Tauon

The electric dipole moment: Photon interaction vertex with a tauon

$$\Gamma_{\tau}^{\gamma,\nu} = ie \left[Q_{\tau} \gamma^{\nu} + \frac{\tilde{d}_{\tau}^{\gamma}}{2m_{\tau}} \gamma^5 \sigma^{\nu\rho} q_{\rho} \right]$$

Similarly

The weak dipole moment: Z-boson interaction vertex with a tauon

$$\Gamma_{\tau}^{Z,\nu} = ie \left[v_{\tau} \gamma^{\nu} - a_{\tau} \gamma^{\nu} \gamma^5 + \frac{\tilde{d}_{\tau}^W}{2m_{\tau}} \gamma^5 \sigma^{\nu\rho} q_{\rho} \right]$$

In the Standard Model, tauon EDM and WDM are extremely small, four loop effects. A good place looking for new physics beyond SM.

Photon and Z offshell, q^2 not equal to zero, crossing tau pair thresholds, EDM and WDM can have imaginary parts

2. Measurements of Hyperons and Tau EDM and WDM at Low Energies

Hyperons EDM at Low Energies: psi and upsilon region

A new test of CP violation for Hyperon production (method applicable to tauon)

X-G He, J-P Ma, B. McKellar, PRD 47(1993) 1744; X-G He and J-P Ma, PLB839(2023)137834; Yong Du, X-G He, J-P Ma, X-Y Du, PRD

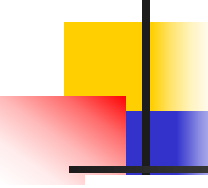
Testing of P and CP symmetries with $e^+e^- \rightarrow J/\psi \rightarrow \Lambda \bar{\Lambda}$

$$\mathcal{A}^\mu = \bar{u}(k_1) \left[\gamma^\mu F_V + \frac{i}{2m_\Lambda} \sigma^{\mu\nu} q_\nu H_\sigma + \gamma^\mu \gamma_5 F_A + \sigma^{\mu\nu} q_\nu \gamma_5 H_T \right] v(k_2),$$

$$G_1^B = F_V^B + H_\sigma^B, \quad G_2^B = G_1^B - \frac{(k_1 - k_2)^2}{4m_B^2} H_\sigma^B, \quad H_T = \frac{2e}{3m_{J/\psi}^2} g_V d_\Lambda$$

$$\Lambda \rightarrow p + \pi \text{ or } \bar{\Lambda} \rightarrow \bar{p} + \pi.$$

$\hat{l}_p$, $\hat{l}_{\bar{p}}$ and $\hat{k}$ momentum directions of p , $\bar{p}$ and Λ .



$e^+ e^- \rightarrow J/\psi$ Density matrix

$$\mathcal{T} = \epsilon^\mu \mathcal{A}_\mu, \quad R(\hat{\mathbf{p}}, \hat{\mathbf{k}}, \mathbf{s}_1, \mathbf{s}_2) = \mathcal{T} \mathcal{T}^\dagger = \rho^{ij} \mathcal{M}^{ij},$$

$$\mathcal{M}^{ij} = \mathcal{A}^i \mathcal{A}^{*j}, \quad \rho^{ij} = \epsilon^i \epsilon^{*j}. \quad \rho^{ij}(\hat{\mathbf{p}}) = \frac{1}{3} \delta^{ij} - i d_J \epsilon^{ijk} \hat{p}^k - \frac{c_J}{2} \left(\hat{p}^i \hat{p}^j - \frac{1}{3} \delta^{ij} \right),$$

d_J induced by Z exchange in SM $e^+ e^- \rightarrow Z \rightarrow J/\psi$

$$d_J = \frac{3 - 8 \sin^2 \theta_W}{32 \cos^2 \theta_W \sin^2 \theta_W} \frac{M_{J/\psi}^2}{M_Z^2} \approx 2.53 \times 10^{-4},$$

$$\hat{\mathbf{p}} = \frac{\mathbf{p}}{|\mathbf{p}|}, \quad \hat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|}, \quad \hat{\mathbf{n}} = \frac{\mathbf{p} \times \mathbf{k}}{|\mathbf{p} \times \mathbf{k}|}, \quad \omega = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}.$$

$$R(\hat{\mathbf{p}}, \hat{\mathbf{k}}, \mathbf{s}_1, \mathbf{s}_2) = a(\omega) + \mathbf{s}_1 \cdot \mathbf{B}_1(\hat{\mathbf{p}}, \hat{\mathbf{k}}) + \mathbf{s}_2 \cdot \mathbf{B}_2(\hat{\mathbf{p}}, \hat{\mathbf{k}}) + s_1^i s_2^j C^{ij}(\hat{\mathbf{p}}, \hat{\mathbf{k}}).$$

a , B_1 , B_2 and C^{ij} are functions of $F_{V,A}$, $H_{\sigma, \tau}$



$$\begin{aligned}
\mathbf{B}_1(\hat{\mathbf{p}}, \hat{\mathbf{k}}) &= \hat{\mathbf{p}} b_{1p}(\omega) + \hat{\mathbf{k}} b_{1k}(\omega) + \hat{\mathbf{n}} b_{1n}(\omega), \quad \mathbf{B}_2(\hat{\mathbf{p}}, \hat{\mathbf{k}}) = \hat{\mathbf{p}} b_{2p}(\omega) + \hat{\mathbf{k}} b_{2k}(\omega) + \hat{\mathbf{n}} b_{2n}(\omega), \\
C^{ij}(\hat{\mathbf{p}}, \hat{\mathbf{k}}) &= \delta^{ij} c_0(\omega) + \epsilon^{ijk} \left(\hat{\mathbf{p}}^k c_1(\omega) + \hat{\mathbf{k}}^k c_2(\omega) + \hat{\mathbf{n}}^k c_3(\omega) \right) + \left(\hat{\mathbf{p}}^i \hat{\mathbf{p}}^j - \frac{1}{3} \delta^{ij} \right) c_4(\omega) + \left(\hat{\mathbf{k}}^i \hat{\mathbf{k}}^j - \frac{1}{3} \delta^{ij} \right) c_5(\omega) \\
&\quad + \left(\hat{\mathbf{p}}^i \hat{\mathbf{k}}^j + \hat{\mathbf{k}}^i \hat{\mathbf{p}}^j - \frac{2}{3} \omega \delta^{ij} \right) c_6(\omega) + (\hat{\mathbf{p}}^i \hat{\mathbf{n}}^j + \hat{\mathbf{n}}^i \hat{\mathbf{p}}^j) c_7(\omega) + (\hat{\mathbf{k}}^i \hat{\mathbf{n}}^j + \hat{\mathbf{n}}^i \hat{\mathbf{k}}^j) c_8(\omega),
\end{aligned}$$

$$\begin{aligned}
a(\omega) &= E_c^2 \left[|G_1|^2 (1 + \omega^2) + |G_2|^2 y_m^2 (1 - \omega^2) \right], & c_8(\omega) &= 2E_c^2 \left| \hat{\mathbf{p}} \times \hat{\mathbf{k}} \right| \left\{ 2d_J y_m \text{Im}(G_1 G_2^*) - \beta \omega \text{Im}[F_A (G_1 - y_m G_2)^*] \right\}, \\
c_0(\omega) &= \frac{1}{3} a(\omega), & b_{1n}(\omega) &= b_{2n}(\omega) = 2 \left| \hat{\mathbf{p}} \times \hat{\mathbf{k}} \right| E_c^2 \omega y_m \text{Im}(G_1 G_2^*), \\
c_1(\omega) &= -4E_c^3 \beta \omega \text{Re}(H_T G_1^*), & b_{1p}(\omega) &= 2E_c^2 \left\{ 2y_m d_J \text{Re}(G_1 G_2^*) + \beta \omega [y_m \text{Re}(F_A G_2^*) + 2E_c \text{Im}(H_T G_1^*)] \right\}, \\
c_2(\omega) &= 4E_c^3 y_m \beta \left\{ \text{Re}(H_T G_2^*) + \omega^2 \text{Re}[H_T (G_1 - G_2)^*] \right\}, & b_{2p}(\omega) &= 2E_c^2 \left\{ 2y_m d_J \text{Re}(G_1 G_2^*) + \beta \omega [y_m \text{Re}(F_A G_2^*) - 2E_c \text{Im}(H_T G_1^*)] \right\}, \\
c_3(\omega) &= 0, & b_{1k}(\omega) &= 2E_c^2 \left\{ 2d_J \omega \left(|G_1|^2 - y_m \text{Re}(G_1 G_2^*) \right) + \beta \text{Re} \left[F_A \left((1 + \omega^2) G_1 - \omega^2 y_m G_2 \right)^* \right] \right. \\
c_4(\omega) &= 2E_c^2 |G_1|^2, & & \quad \left. - 2E_c \beta \text{Im} \left[H_T \left(\omega^2 G_1 + (1 - \omega^2) y_m G_2 \right)^* \right] \right\}, \\
c_5(\omega) &= 2E_c^2 \left[|G_1|^2 - y_m^2 |G_2|^2 + |G_1 - y_m G_2|^2 \omega^2 \right], & b_{2k}(\omega) &= 2E_c^2 \left\{ 2d_J \omega \left(|G_1|^2 - y_m \text{Re}(G_1 G_2^*) \right) + \beta \text{Re} \left[F_A \left((1 + \omega^2) G_1 - \omega^2 y_m G_2 \right)^* \right] \right. \\
c_6(\omega) &= -2E_c^2 \omega \left[|G_1|^2 - y_m \text{Re}(G_1 G_2^*) \right], & & \quad \left. + 2E_c \beta \text{Im} \left[H_T \left(\omega^2 G_1 + (1 - \omega^2) y_m G_2 \right)^* \right] \right\}, \\
c_7(\omega) &= 2E_c^2 \beta \left| \hat{\mathbf{p}} \times \hat{\mathbf{k}} \right| \text{Im}(F_A G_1^*),
\end{aligned}$$

On-shell production of J/ψ and observables

$$e^-(p_1) + e^+(p_2) \rightarrow J/\psi \rightarrow \Lambda(k_1, s_1) + \bar{\Lambda}(k_2, s_2),$$

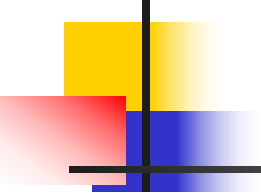
$$p_1^\mu = (E_c, \mathbf{p}), \quad p_2^\mu = (E_c, -\mathbf{p}), \quad k_1^\mu = (k^0, \mathbf{k}), \quad k_2^\mu = (k^0, -\mathbf{k}).$$

$$\Lambda \rightarrow p + \pi \quad \text{or} \quad \bar{\Lambda} \rightarrow \bar{p} + \pi$$

$$\frac{d\Gamma_\Lambda}{d\Omega_p}(\mathbf{s}_1, \hat{\mathbf{l}}_p) \propto 1 + \alpha \mathbf{s}_1 \cdot \hat{\mathbf{l}}_p, \quad \frac{d\Gamma_{\bar{\Lambda}}}{d\Omega_{\bar{p}}}(\mathbf{s}_2, \hat{\mathbf{l}}_{\bar{p}}) \propto 1 - \bar{\alpha} \mathbf{s}_2 \cdot \hat{\mathbf{l}}_{\bar{p}}$$

$\hat{\mathbf{l}}_p$ and $\hat{\mathbf{l}}_{\bar{p}}$ is the direction of the momentum of the proton or anti-proton in the rest frame of Λ or $\bar{\Lambda}$

$$\mathcal{A}(\mathcal{O}) = \frac{\mathcal{N}_{\text{event}}(\mathcal{O} > 0) - \mathcal{N}_{\text{event}}(\mathcal{O} < 0)}{\mathcal{N}_{\text{event}}(\mathcal{O} > 0) + \mathcal{N}_{\text{event}}(\mathcal{O} < 0)} = \frac{1}{\mathcal{N}} \int \frac{d\Omega_k d\Omega_p d\Omega_{\bar{p}}}{(4\pi)^3} \left(\theta(\mathcal{O}) - \theta(-\mathcal{O}) \right) \mathcal{W}(\Omega)$$



$$\langle \hat{\mathbf{l}}_b \cdot \hat{\mathbf{p}} \rangle = \frac{4\alpha_B}{9\mathcal{N}} E_c^2 d_J \left(4y_m \text{Re}(G_1 G_2^*) + |G_1|^2 \right),$$

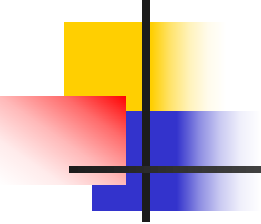
$$\langle \hat{\mathbf{l}}_{\bar{b}} \cdot \hat{\mathbf{p}} \rangle = -\frac{4\bar{\alpha}_B}{9\mathcal{N}} E_c^2 d_J \left(4y_m \text{Re}(G_1 G_2^*) + |G_1|^2 \right),$$

$$\langle \hat{\mathbf{l}}_b \cdot \hat{\mathbf{k}} \rangle = \frac{8\alpha_B \beta}{9\mathcal{N}} E_c^2 [\text{Re}(F_A G_1^*) - E_c \text{Im}(H_T G_1^* + y_m H_T G_2^*)],$$

$$\langle \hat{\mathbf{l}}_{\bar{b}} \cdot \hat{\mathbf{k}} \rangle = -\frac{8\bar{\alpha}_B \beta}{9\mathcal{N}} E_c^2 [\text{Re}(F_A G_1^*) + E_c \text{Im}(H_T G_1^* + y_m H_T G_2^*)],$$

$$\langle (\hat{\mathbf{l}}_b \times \hat{\mathbf{l}}_{\bar{b}}) \cdot \hat{\mathbf{k}} \rangle = -\frac{16\alpha_B \bar{\alpha}_B}{27\mathcal{N}} \beta y_m E_c^3 \text{Re}(H_T G_2^*),$$

$$\mathcal{N} = \frac{2}{3} E_c^2 \left(2|G_1|^2 + y_m^2 |G_2|^2 \right)$$



P violating observable

$$\mathcal{A}(\hat{\mathbf{l}}_b \cdot \hat{\mathbf{p}} - \hat{\mathbf{l}}_{\bar{b}} \cdot \hat{\mathbf{p}}) \equiv A_{\text{PV}}^{(1)} \simeq \frac{4\alpha_B}{3\mathcal{N}} E_c^2 d_J \left(4y_m \text{Re}(G_1 G_2^*) + |G_1|^2 \right),$$

$$\mathcal{A}(\hat{\mathbf{l}}_b \cdot \hat{\mathbf{k}} - \hat{\mathbf{l}}_{\bar{b}} \cdot \hat{\mathbf{k}}) \equiv A_{\text{PV}}^{(2)} \simeq \frac{8\alpha_B \beta}{3\mathcal{N}} E_c^2 \text{Re}(F_A G_1^*),$$

CP violating observable

$$\mathcal{A}(\hat{\mathbf{l}}_b \cdot \hat{\mathbf{k}} + \hat{\mathbf{l}}_{\bar{b}} \cdot \hat{\mathbf{k}}) \equiv A_{\text{CPV}}^{(1)} \simeq -\frac{4\alpha_B \beta}{3\mathcal{N}} E_c^3 \text{Im}(H_T G_1^* + y_m H_T G_2^*),$$

$$\mathcal{A}((\hat{\mathbf{l}}_b \times \hat{\mathbf{l}}_{\bar{b}}) \cdot \hat{\mathbf{k}}) \equiv A_{\text{CPV}}^{(2)} \simeq -\frac{8\alpha_B \bar{\alpha}}{9\mathcal{N}} \beta y_m E_c^3 \text{Re}(H_T G_2^*),$$

F_A in the SM

On-shell $e^+e^- \rightarrow J/\psi$ by a photon ($s = m_{J/\psi}^2$) and J/ψ decays into $\Lambda\bar{\Lambda}$

$$i \left(\frac{e^2 Q_e Q_q}{s} \bar{e} \gamma^\mu e \langle J/\Psi | \bar{c} \gamma_\mu c | 0 \rangle \right)$$



$$\times i \left(-\frac{g^2 g_V^c g_A^q}{4 \cos^2 \theta_W (s - m_Z^2)} \langle 0 | \bar{c} \gamma^\nu c | J/\Psi \rangle \langle \bar{\Lambda} \Lambda \bar{q} \gamma_\nu \gamma_5 q | 0 \rangle \right)$$

$$g_V^e = (1 - 4 \sin^2 \theta_W)/2, \quad g_A^u = -g_A^d = -g_A^s = 1/2$$

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | \Lambda \rangle = D w_q \bar{\Lambda} \gamma^\mu \gamma_5 \Lambda. \quad \text{Here } D \approx 0.80, \quad w_u = w_d = 1/6 \text{ and } w_s = 4/6$$

$$\langle 0 | \bar{c} \gamma_\mu c | J/\Psi \rangle = \epsilon_\mu^J g_V, \quad g_V = 1.25 \text{ GeV}^2$$

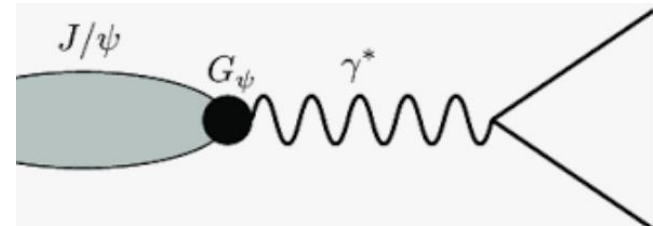
$$F_A^R = \frac{g_V^2}{s - m_{J/\Psi}^2 + i m_{J/\Psi} \Gamma_{J/\Psi} / 2} \frac{g^2 (1 - 8 \sin^2 \theta_W / 3)}{24 \cos^2 \theta_W (s - m_Z^2)} D$$

Dipole moment contribution to H_T

H_T is flavor conserving CP violating. It is extremely small in the SM.

Beyond SM, it may be large. Consider now Λ edm contribution.

$$L_{edm} = -i \frac{d_\Lambda}{2} \Lambda \sigma_{\mu\nu} \gamma_5 \Lambda F^{\mu\nu}$$



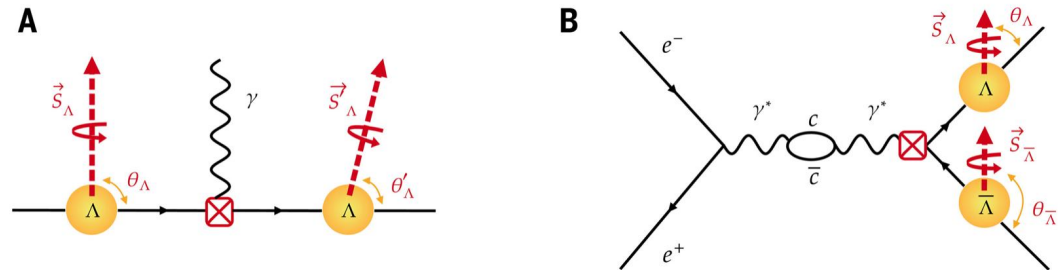
Exchange a photon

$$\frac{ed_\Lambda}{s} \bar{e} \gamma^\mu e \Lambda \sigma_{\mu\nu} \gamma_5 \Lambda q^\nu$$

Measurement of the Λ electric dipole moment with an entangled baryon-antibaryon system

BESIII COLLABORATION [Authors Info & Affiliations](#)

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From 10^{10} J/psi collected
Improved by more than 2 orders of magnitudes
compared with previous measurement

$$\begin{aligned} \text{Re}(d_{\Lambda}) &= (-3.1 \pm 3.2 \pm 0.5) \times 10^{-19} \text{ e cm} & -8.6 \times 10^{-19} < \text{Re}(d_{\Lambda}) < 3.3 \times 10^{-19} \text{ e cm} \\ \text{Im}(d_{\Lambda}) &= (2.9 \pm 2.6 \pm 0.6) \times 10^{-19} \text{ e cm} & -2.5 \times 10^{-19} < \text{Im}(d_{\Lambda}) < 7.2 \times 10^{-19} \text{ e cm} \end{aligned}$$

$$|d_{\Lambda}| < 7.0 \times 10^{-19} \text{ e cm (95\% CL)}$$

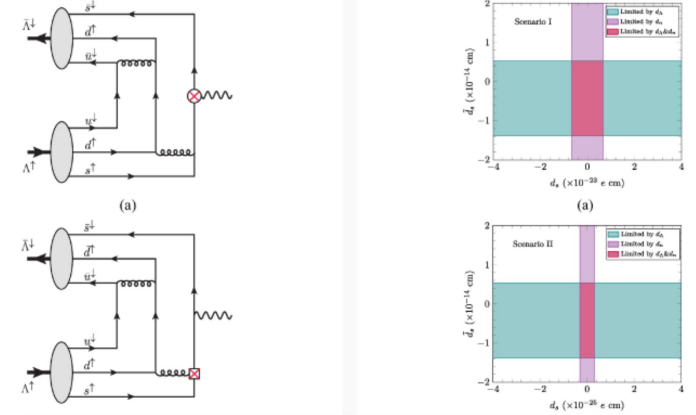
EDM of hyperon and quarks

Valence quark model

d_B	QM	Reduced Results	d_B	NR QCD & QM	Reduced Results
d_p^{qEDM}	$\frac{1}{3}(4d_u - d_d)$	—	d_p^{qCDM}	$-\frac{1}{3}(4Q_d f_d - Q_u f_u)$	—
d_n^{qEDM}	$\frac{1}{3}(4d_d - d_u)$	—	d_p^{qCDM}	$-\frac{1}{3}(4Q_u f_u - Q_d f_d)$	—
$d_{\Sigma^+}^{\text{qEDM}}$	$\frac{1}{3}(4d_u - d_s)$	$-\frac{1}{3}d_s$	d_p^{qCDM}	$-\frac{1}{3}(4Q_u f_u - Q_s f_s)$	$-\frac{1}{9}ef_s$
$d_{\Sigma^0}^{\text{qEDM}}$	$\frac{1}{3}(2d_u + 2d_d - d_s)$	$-\frac{1}{3}d_s$	d_p^{qCDM}	$-\frac{1}{3}(2Q_u f_u + 2Q_d f_d - Q_s f_s)$	$-\frac{1}{9}ef_s$
$d_{\Sigma^-}^{\text{qEDM}}$	$\frac{1}{3}(4d_d - d_s)$	$-\frac{1}{3}d_s$	d_p^{qCDM}	$-\frac{1}{3}(4Q_d f_d - Q_s f_s)$	$-\frac{1}{9}ef_s$
$d_{\Xi^0}^{\text{qEDM}}$	$\frac{1}{3}(4d_s - d_u)$	$\frac{4}{3}d_s$	d_p^{qCDM}	$-\frac{1}{3}(4Q_s f_s - Q_u f_u)$	$\frac{4}{9}ef_s$
$d_{\Xi^-}^{\text{qEDM}}$	$\frac{1}{3}(4d_s - d_d)$	$\frac{4}{3}d_s$	d_p^{qCDM}	$-\frac{1}{3}(4Q_s f_s - Q_d f_d)$	$\frac{4}{9}ef_s$
$d_{\Lambda^0}^{\text{qEDM}}$	d_s	d_s	d_p^{qCDM}	$-Q_s f_s$	$\frac{1}{3}ef_s$

Perturbative QCD calculations

KB Chen, XG He, JP Ma and XB Tong, PRL 136(2026)051902

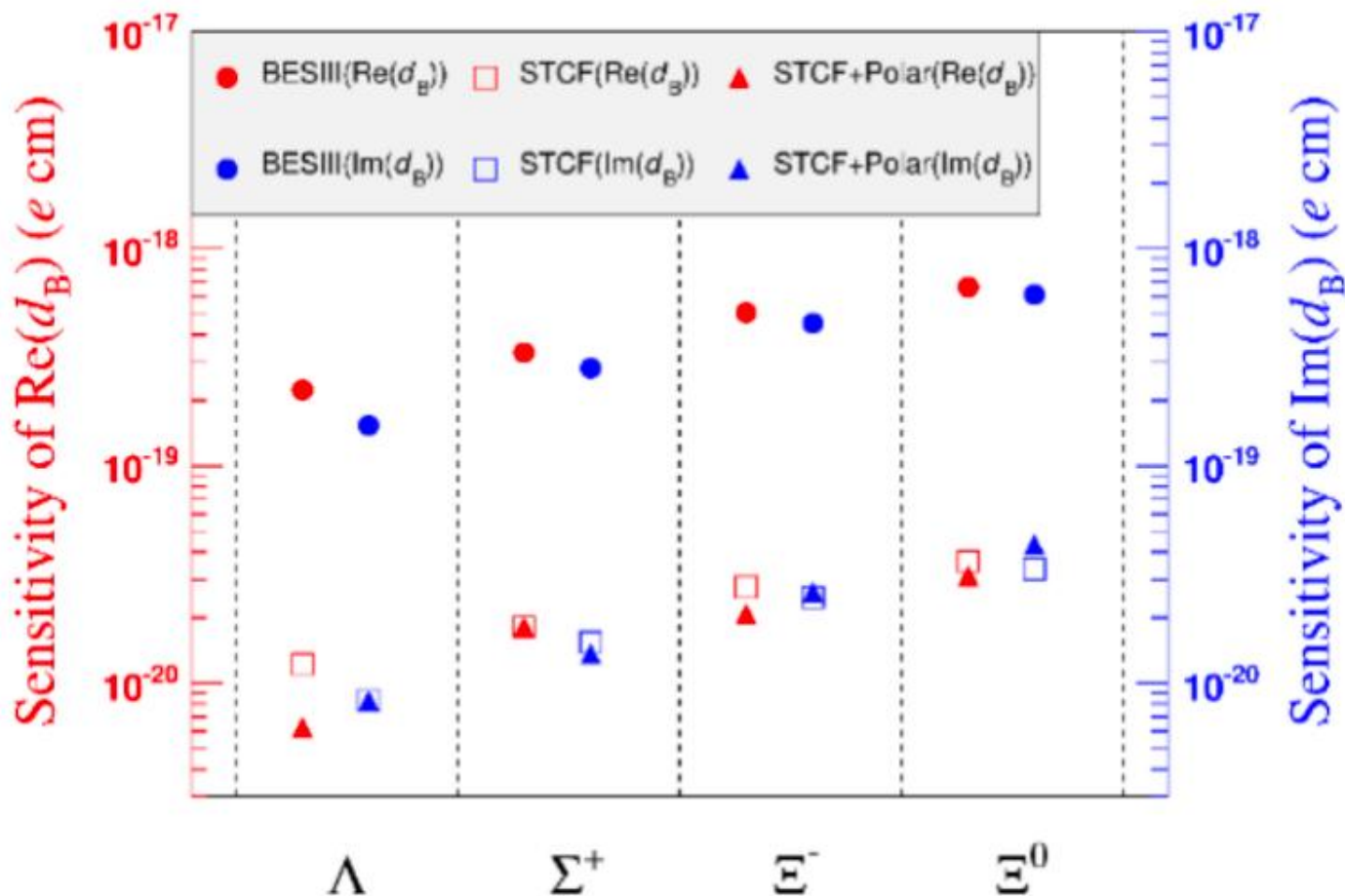


$$\phi_{00}(\mu_0) = 4.75 \times 10^{-3}, \quad \phi_{10}(\mu_0) = 0.563 \times 10^{-3}, \quad \phi_{11}(\mu_0) = 0.242 \times 10^{-3}, \quad \pi_{10}(\mu_0) = 0.237 \times 10^{-3},$$

again in GeV^2 . Using these lattice results at $\mu = Q = M_{J/\psi}$, we find

$$d_{\Lambda} = 5.29 \times 10^{-4}d_s + 4.61 \times 10^{-5}(d_u + d_d) + 6.21 \times 10^{-5}e\tilde{d}_s + 1.98 \times 10^{-5}e\tilde{d}_d - 2.14 \times 10^{-5}e\tilde{d}_u.$$

STCF sensitivities to Hyperon EDMs



Tau EDM measurement at $e^+e^- \rightarrow \tau^+ \tau^-$

XG He, CW Liu, LP Ma and ZY Zou JHEP04(2025)001

Use similar method replacing $J/\psi \rightarrow$ hyperon pairs by $\psi(2S) \rightarrow \tau^+ \tau^-$, or just $e^+e^- \rightarrow \gamma^* \rightarrow \tau^+ \tau^-$

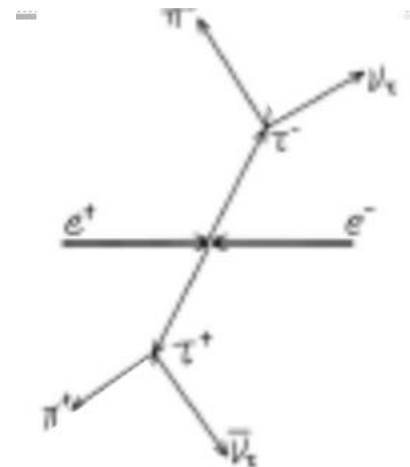
STCF 10 year running, expect 2×10^{10} $\psi(2S)$

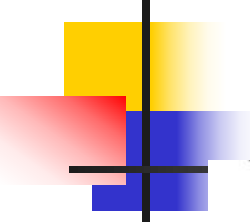
But because tauon decays always has a neutrino in the final state, harder to reconstruct moments, for example using $\tau(k) \rightarrow \pi(l) + \nu$.

Still if spacial resolution is better than $38.6 \mu\text{m}$ (the flying length of tauon from $\psi(2S)$ decay), there are substantial events to reconstruct tauon momenta,

STCF can reach a better resolution than current best bounds.

Bell II can also use $\Upsilon(4S)$ to do similar things





$$\tau(k) \rightarrow \pi(l) + \nu, \rho(l) + \nu$$

For the the imaginary part, we have

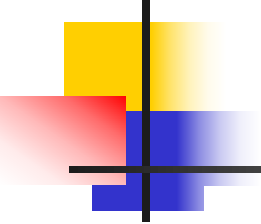
$$\text{Im}(d_\tau) = \frac{-e(3s + 6m_\tau^2)}{4s\sqrt{s - 4m_\tau^2}} \left(\frac{\langle \hat{l}_{h-} \cdot \hat{k} \rangle}{\alpha_h} + \frac{\langle \hat{l}_{h'+} \cdot \hat{k} \rangle}{\bar{\alpha}_{h'}} \right). \quad (6)$$

There are two different methods to extract the real part of the EDM from the distributions:

$$\text{Re}(d_\tau)^a = e \frac{9}{4} \frac{s + 2m_\tau^2}{\alpha_h \alpha_{h'} m_\tau \sqrt{s^2 - 4sm_\tau^2}} \langle (\hat{l}_- \times \hat{l}_+) \cdot \hat{k} \rangle, \quad (7)$$

and

$$\text{Re}(d_\tau)^b = -e \frac{45}{4} \frac{(s + 2m_\tau^2) \langle (\hat{p} \cdot \hat{k})(\hat{l}_- \times \hat{l}_+) \cdot \hat{p} \rangle}{\alpha_h \alpha_{h'} m_\tau (\sqrt{s} - 2m_\tau) \sqrt{s - 4m_\tau^2}}. \quad (8)$$



$$\hat{l}_{\mp} \cdot \hat{k} = \pm \frac{1}{\sqrt{E_{\mp}^2 - m_h^2}} \left(\frac{4E_{\mp} m_{\tau}^2 / \sqrt{s} - m_{\tau}^2 - m_h^2}{2m_{\tau} \sqrt{1 - 4m_{\tau}^2/s}} \right)$$

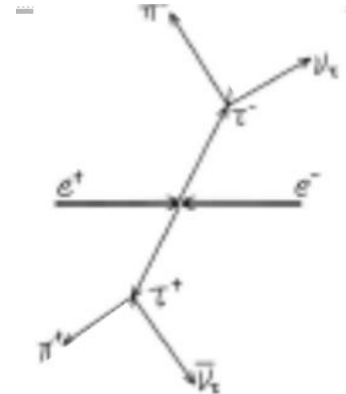
$$\hat{k} = u\hat{l}_{+} + v\hat{l}_{-} + \text{sgn} \left((\hat{l}_{-} \times \hat{l}_{+}) \cdot \hat{k} \right) w\hat{l}_{-} \times \hat{l}_{+}$$

We note that it suffices for measurements to determine $\text{sgn}((\hat{l}_{-} \times \hat{l}_{+}) \cdot \hat{k})$ for reconstructing $\hat{k}$. The proportion of $\hat{k}$ being measured is then given by

$$P_{\tau} = 1 - \left(\frac{1}{l_0} \int_0^D \exp \left(-\frac{l}{l_0} \right) dl \right)^2, \quad (13)$$

where $l_0 = v\gamma/\Gamma$ with Γ the total decay width of τ^{-} , $\gamma = \sqrt{s}/(2m_{\tau})$, and $v = \sqrt{\gamma^2 - 1}/\gamma$. l_0 is the τ mean pass length. The integral represents the probability of τ^{-} decaying before its flight distance reaches D in the lab frame, and the square is because it suffices to probe the momenta of either τ^{-} or τ^{+} .

$\text{Im}(d\tau)$ easy to measure.
 $\text{Re}(d\tau)$ needs to reconstruct the momentum direction k .



$$\delta_{\text{Im}} = \frac{3s + 6m_\tau^2}{4s\sqrt{s - 4m_\tau^2}} \sqrt{\frac{4}{3 \sum_h^{\pi, \rho} \left(\alpha_h^2 N_{\text{Im}}^h + \bar{\alpha}_h^2 \bar{N}_{\text{Im}}^h \right)}}.$$

The event number is given by

$$\begin{aligned} N_{\text{Im}}^h &= \epsilon L \sigma \mathcal{B}(\tau^- \rightarrow h^- \nu_\tau), \\ \bar{N}_{\text{Im}}^h &= \epsilon L \sigma \mathcal{B}(\tau^+ \rightarrow h^+ \nu_{\bar{\tau}}), \end{aligned}$$

The standard deviations of $\text{Re}(d_\tau)^{a,b}$ in order are

$$\delta_{\text{Re}}(D)^a = \frac{3e}{4} \frac{s + 2m_\tau^2}{m_\tau \sqrt{s^2 - 4sm_\tau^2}} \sqrt{\frac{2}{N_{\text{Re}}^{\text{eff}}}},$$

and

$$\delta_{\text{Re}}(D)^b = \frac{3e}{4} \frac{\sqrt{s^2 + 3sm_\tau^2 + 2m_\tau^4}}{m_\tau (\sqrt{s} - 2m_\tau) \sqrt{s - 4m_\tau^2}} \sqrt{\frac{20}{N_{\text{Re}}^{\text{eff}}}},$$

where the effective number of events is given by

$$N_{\text{Re}}^{\text{eff}} = P_\tau \epsilon L \left(\alpha_\pi^2 \mathcal{B}(\tau^- \rightarrow \pi^- \nu_\tau) + \alpha_\rho^2 \mathcal{B}(\tau^- \rightarrow \rho^- \nu_\tau) \right)^2$$

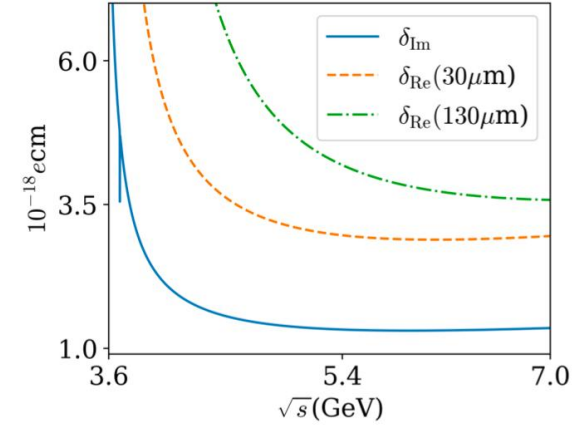


Fig. 1. The expected precision of d_τ with $L\epsilon = 0.63 \text{ ab}^{-1}$.

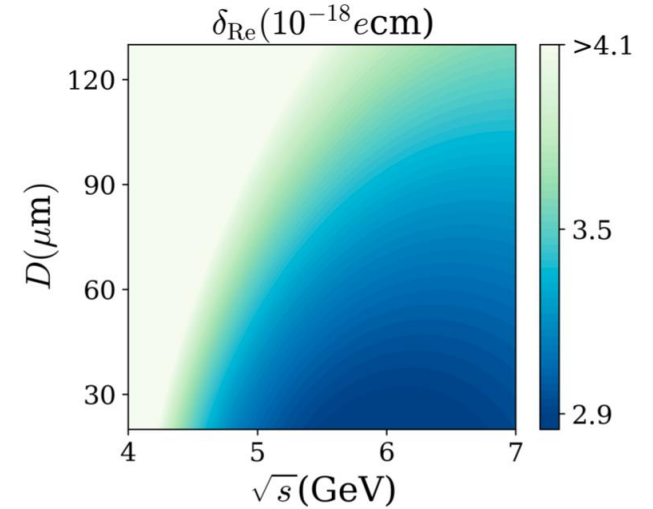


Fig. 2. The precision of $\text{Re}(d_\tau)$ may be achieved with $L\epsilon = 0.63 \text{ ab}^{-1}$. The color indicates the values of δ_{Re} , as shown in the color bar on the right.

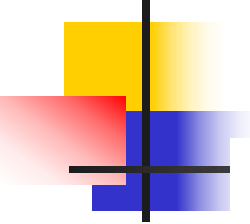


Table I. The precision of d_τ that may be achieved with $L\epsilon = 0.63 \text{ ab}^{-1}$ is given in units of $10^{-18} e \text{ cm}$. The absolute value is defined as $\delta_{|d_\tau|}^2(D) = \delta_{\text{Re}}(D)^2 + \delta_{\text{Im}}(D)^2$, where D is in units of μm . The case $D = 0$ corresponds to situations where the τ -lepton momentum can be reconstructed with 100% accuracy which is shown only as a reference number.

$\sqrt{s}$	$m_{\psi(2S)}$	4.2 GeV	4.9 GeV	5.6 GeV	6.3 GeV	7 GeV
δ_{Im}	3.5	1.8	1.4	1.3	1.3	1.4
$\delta_{\text{Re}}(180)$	234	14.7	6.6	4.9	4.3	4.1
$\delta_{\text{Re}}(130)$	82	9.4	5.0	4.0	3.7	3.6
$\delta_{\text{Re}}(80)$	29	6.2	3.9	3.3	3.2	3.2
$\delta_{\text{Re}}(30)$	11	4.4	3.2	2.9	2.9	3.0
$\delta_{\text{Re}}(0)$	7.7	4.0	3.0	2.8	2.8	2.9
$\delta_{ d_\tau }(80)$	30	6.5	4.1	3.6	3.5	3.5

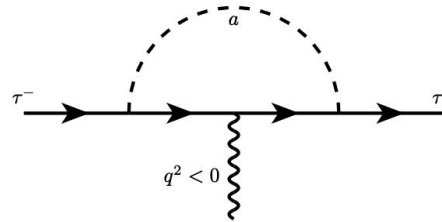
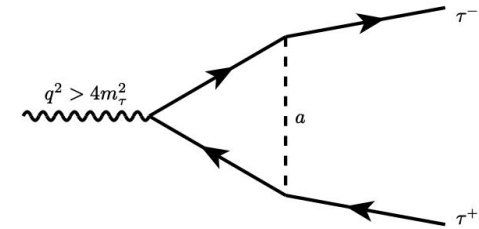
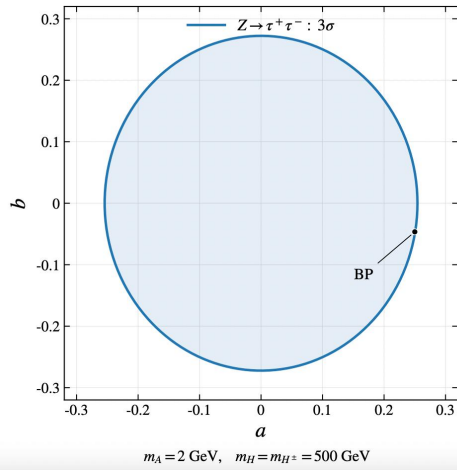
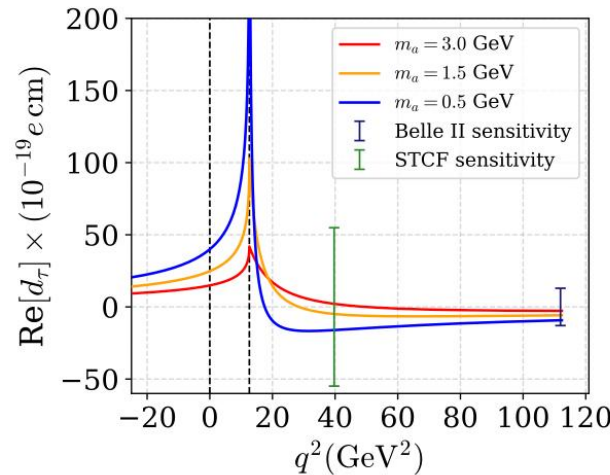
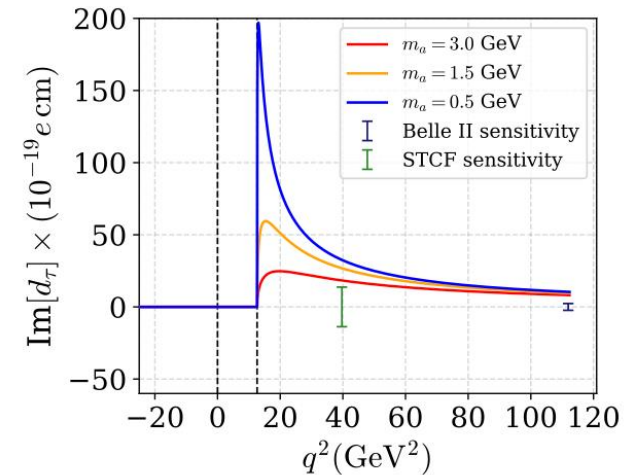
Can reach a sensitivity one order better than current BELLE bounds
 Using similar method, BELLE can improve their sensitivities

Theoretically how large tau EDM can be, reaching current bounds?

ZL Huang, XY Du, XG He, CW Liu, ZY Zou, CPL 43(2026)030201
 XY Du, XG He, ZL Huang, ZY Zou, arXiv: 2606.0178

In beyond SM, like two Higgs doublet model, the answer is **YES!**

$$\mathcal{L}_A = A\bar{\tau}(a - ib\gamma_5)\tau$$



A two Higgs doublet model realization

$$\Phi'_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} (v + h + iG^0) \end{pmatrix}, \quad \Phi'_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (H + iA) \end{pmatrix}$$

$$\begin{aligned} V(\Phi'_1, \Phi'_2) = & M_{11}^2 \Phi_1'^{\dagger} \Phi'_1 + M_{22}^2 \Phi_2'^{\dagger} \Phi'_2 - \left(M_{12}^2 \Phi_1'^{\dagger} \Phi'_2 + \text{c.c.} \right) \\ & + \frac{1}{2} \Lambda_1 \left(\Phi_1'^{\dagger} \Phi'_1 \right)^2 + \frac{1}{2} \Lambda_2 \left(\Phi_2'^{\dagger} \Phi'_2 \right)^2 + \Lambda_3 \left(\Phi_1'^{\dagger} \Phi'_1 \right) \left(\Phi_2'^{\dagger} \Phi'_2 \right) + \Lambda_4 \left(\Phi_1'^{\dagger} \Phi'_2 \right) \left(\Phi_2'^{\dagger} \Phi'_1 \right) \\ & + \left(\frac{1}{2} \Lambda_5 \left(\Phi_1'^{\dagger} \Phi'_2 \right)^2 + \left(\Lambda_6 \Phi_1'^{\dagger} \Phi'_1 + \Lambda_7 \Phi_2'^{\dagger} \Phi'_2 \right) \left(\Phi_1'^{\dagger} \Phi'_2 \right) + \text{c.c.} \right). \end{aligned}$$

$$-\mathcal{L}_Y = \bar{l} \hat{M}_l l + \bar{l} \left(\frac{\text{Re}(Y_2)}{\sqrt{2}} + i \frac{\text{Im}(Y_2)}{\sqrt{2}} \gamma_5 \right) l H + \bar{l} \left(-\frac{\text{Im}(Y_2)}{\sqrt{2}} + i \frac{\text{Re}(Y_2)}{\sqrt{2}} \gamma_5 \right) l A$$

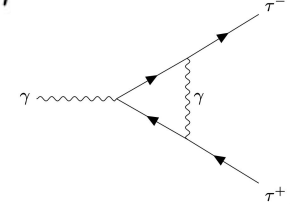
$$a = -\frac{\text{Im}(Y_2)_{33}}{\sqrt{2}}, \quad b = -\frac{\text{Re}(Y_2)_{33}}{\sqrt{2}}$$

The model satisfies known constraints!

$a_\tau = (g-2)/2$ of tauon

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau} + F_3(q^2)\sigma^{\mu\nu}q_\nu\gamma_5 + F_4(q^2)(q^2\gamma^\mu - a^\mu \not{d})\gamma_5$$

$$a_\tau(q^2) = F_2(q^2)$$



$g-2$ of tauon is a fundamental property, CP conserving, and can be generated at one loop QED level

$$a_\tau^{\text{QED}_{1\text{-loop}}}(s) = \frac{\alpha_{\text{em}} m_\tau^2}{\pi s \beta_\tau} \ln \left(\frac{\beta_\tau - 1}{1 + \beta_\tau} \right),$$

$$\text{Re}[a_\tau^{\text{QED}_{1\text{-loop}}}(s)] = \frac{\alpha_{\text{em}} m_\tau^2}{\pi s \beta_\tau} \ln \left(\frac{1 - \beta_\tau}{1 + \beta_\tau} \right),$$

$$\text{Im}[a_\tau^{\text{QED}_{1\text{-loop}}}(s)] = \frac{\alpha_{\text{em}} m_\tau^2}{s \beta_\tau} \Theta(s - 4m_\tau^2).$$

Theoretical one loop value: 1.1614×10^{-3}

Experimental constraint: $-0.057 < a_\tau < 0.024$, C.L. = 95%.

Not a satisfactory situation! Can we improve this? **Yes**

New method to measure a_τ in $e^+ e^- \rightarrow \gamma^*/\psi(2S) \rightarrow \tau^+ \tau^-$

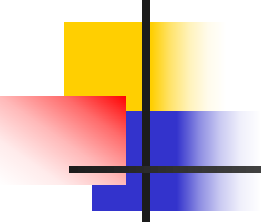
$$\mathcal{O}_5 = (\hat{\mathbf{p}} \cdot \hat{\mathbf{k}})^2, \quad \mathcal{O}_6 = \frac{(\hat{\mathbf{p}} \cdot \hat{\mathbf{k}}) [\hat{\mathbf{l}}_\mp \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}})]}{|\hat{\mathbf{p}} \times \hat{\mathbf{k}}|}.$$

$$\begin{aligned} \text{Re}(a_\tau) &= \frac{2(s + 2m_\tau^2)(s + m_\tau^2)}{s(s - 4m_\tau^2)} - \frac{5(s + 2m_\tau^2)^2}{s(s - 4m_\tau^2)} \left\langle (\hat{\mathbf{p}} \cdot \hat{\mathbf{k}})^2 \right\rangle, \quad \Delta \text{Re}(a_\tau) = \frac{(s + 2m_\tau^2)}{s(s - 4m_\tau^2)} \sqrt{\frac{32m_\tau^4 + 64m_\tau^2 s + 17s^2}{14N_{\text{eff}}^{(1)}}}, \\ \text{Im}(a_\tau) &= -\frac{64m_\tau(2m_\tau^2 + s)}{\pi\alpha_{h^\pm}\sqrt{s}(s - 4m_\tau^2)} \left\langle \frac{(\hat{\mathbf{p}} \cdot \hat{\mathbf{k}}) [\hat{\mathbf{l}}_\mp \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}})]}{|\hat{\mathbf{p}} \times \hat{\mathbf{k}}|} \right\rangle. \quad \Delta \text{Im}(a_\tau) = \frac{64m_\tau}{\pi(s - 4m_\tau^2)\sqrt{s}} \sqrt{\frac{4m_\tau^4 + 6m_\tau^2 s + 2s^2}{30N_{\text{eff}}^{(2)}}}. \end{aligned}$$

$\sqrt{q^2}$ [GeV]	$ \text{Re}(a_\tau^{QED}) $	$\Delta \text{Re}(a_\tau)$	$ \text{Im}(a_\tau^{QED}) $	$\Delta \text{Im}(a_\tau)$
$m_{\psi(2S)}(3.686\text{GeV})$	1.11×10^{-3}	2.17×10^{-3}	6.39×10^{-3}	4.90×10^{-3}
$m_{\Upsilon(4S)}(10.58\text{GeV})$	2.44×10^{-4}	6.29×10^{-5}	2.18×10^{-4}	5.02×10^{-5}

STCF 2×10^{10} , BELLE 4.5×10^{10} tau pairs

Can also probe q^2 dependence!



Tauon WDM: $\Gamma_{\tau}^{Z,\nu} = ie \left[v_{\tau} \gamma^{\nu} - a_{\tau} \gamma^{\nu} \gamma^5 + \frac{\tilde{d}_{\tau}^W}{2m_{\tau}} \gamma^5 \sigma^{\nu\rho} q_{\rho} \right]$

At low energies not a good place to look for tauon WDM

Change $e^+e^- \rightarrow \gamma^* (\Psi(2S)) \rightarrow \tau^+ \tau^-$ to

$e^+e^- \rightarrow Z^* \rightarrow \tau^+ \tau^-$

there is a $(m_{\Psi(2S)}^2 / m_Z^2)$ suppression!

3. Measurements of Tau EDM and WDM at/near Z pole

XY Du, ZY Zou, XG He, KP Xie, arXiv:2609.18892

For e^+e^- machine at the Z pole, a lot of Z boson will be produced which has large branching ratio of 3.37% to $\tau^+ \tau^-$.

FCCee and CEPC will run at the Z pole and collect of order 6.0×10^{12} and 4.1×10^{12} Z bosons. These provide excellent opportunities to study WDM and also EDM

$$\Gamma_{\tau}^{Z,\nu} = ie \left[v_{\tau} \gamma^{\nu} - a_{\tau} \gamma^{\nu} \gamma^5 + \frac{\tilde{d}_{\tau}^W}{2m_{\tau}} \gamma^5 \sigma^{\nu\rho} q_{\rho} \right]$$

$$v_{\ell} = -0.04431, \quad a_{\ell} = -0.60039,$$

WDM weakly constrained:

$$\text{WDM bounds (ALEPH data)} \quad |\text{Re } \tilde{d}_{\tau}^W| < 0.91 \times 10^{-3}, \quad |\text{Im } \tilde{d}_{\tau}^W| < 2.01 \times 10^{-3},$$

$$-1.10 \times 10^{-3} < \text{Re } \tilde{d}_{\tau}^{\gamma} < 3.33 \times 10^{-3},$$

EDM bounds (BELLE II)

$$-0.41 \times 10^{-3} < \text{Im } \tilde{d}_{\tau}^{\gamma} < 1.86 \times 10^{-3}.$$

New methods to measure WDM

$$e^-(p_1)e^+(p_2) \rightarrow \gamma^*, Z \rightarrow \tau^-(k_1)\tau^+(k_2) \quad \tau^\mp \rightarrow \pi^\mp \nu$$

$$\hat{z} = \hat{k}_-, \quad \hat{y} = \frac{\hat{p}_1 \times \hat{k}_-}{|\hat{p}_1 \times \hat{k}_-|}, \quad \hat{x} = \hat{y} \times \hat{z},$$

$$\mathcal{O}_1 = (\hat{\ell}^- + \hat{\ell}^+) \cdot \hat{z} = \cos \theta_- + \cos \theta_+,$$

$$\mathcal{O}_2 = (\hat{\ell}^- \times \hat{\ell}^+) \cdot \hat{z} = \sin \theta_- \sin \theta_+ \sin(\phi_+ - \phi_-),$$

$$\mathcal{O}_3 = (\hat{\ell}^- + \hat{\ell}^+) \cdot \hat{x} = \sin \theta_- \cos \phi_- + \sin \theta_+ \cos \phi_+,$$

$$\mathcal{O}_4 = \text{sgn}(\cos \theta) (\hat{\ell}^- + \hat{\ell}^+) \cdot \hat{y} = \text{sgn}(\cos \theta) \mathcal{O}_5,$$

$$\mathcal{O}_5 = (\hat{\ell}^- + \hat{\ell}^+) \cdot \hat{y} = \sin \theta_- \sin \phi_- + \sin \theta_+ \sin \phi_+.$$

$$\langle \mathcal{O}_1 \rangle = -\frac{4}{3} \frac{\beta v_\tau}{\Delta_0} \text{Im} [\tilde{d}_\tau^W + i\epsilon_v \tilde{d}_\tau^\gamma], \quad (11)$$

$$\langle \mathcal{O}_2 \rangle = -\frac{4}{9} \frac{\beta v_\tau}{\Delta_0} \text{Re} [\tilde{d}_\tau^W + i\epsilon_v \tilde{d}_\tau^\gamma], \quad (12)$$

$$\langle \mathcal{O}_3 \rangle = -\frac{\pi}{2} \frac{m_Z}{2m_\tau} \frac{a_\tau A_e \beta^2}{\Delta_0} \text{Im} [\tilde{d}_\tau^W + i\epsilon_a \tilde{d}_\tau^\gamma], \quad (13)$$

$$\langle \mathcal{O}_4 \rangle = -\frac{2}{3} \frac{m_Z}{2m_\tau} \frac{a_\tau \beta^2}{\Delta_0} \text{Re} [\tilde{d}_\tau^W + i\epsilon_v \tilde{d}_\tau^\gamma], \quad (14)$$

$$\langle \mathcal{O}_5 \rangle = -\frac{\pi}{2} \frac{m_Z}{2m_\tau} \frac{v_\tau A_e \beta}{\Delta_0} (\text{Re} [\tilde{d}_\tau^W + i\epsilon_a \tilde{d}_\tau^\gamma] + \xi \text{Im} \tilde{d}_\tau^W).$$

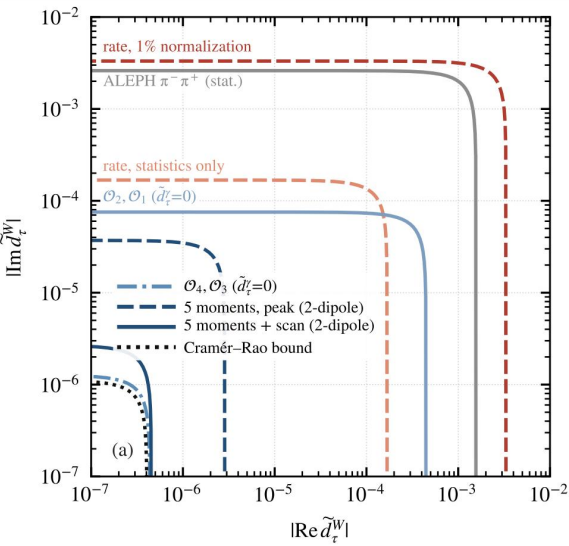
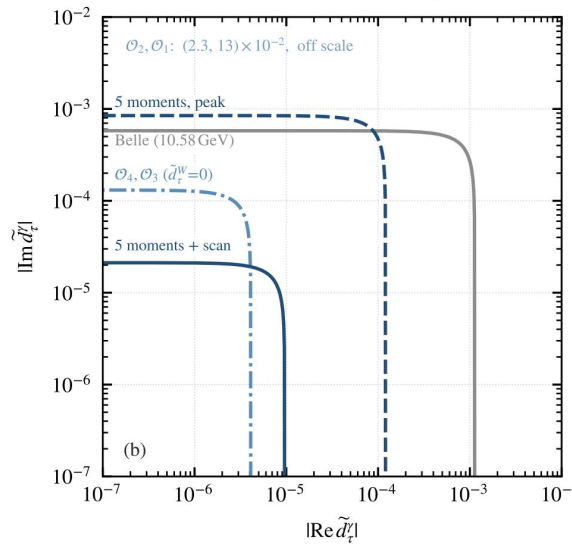
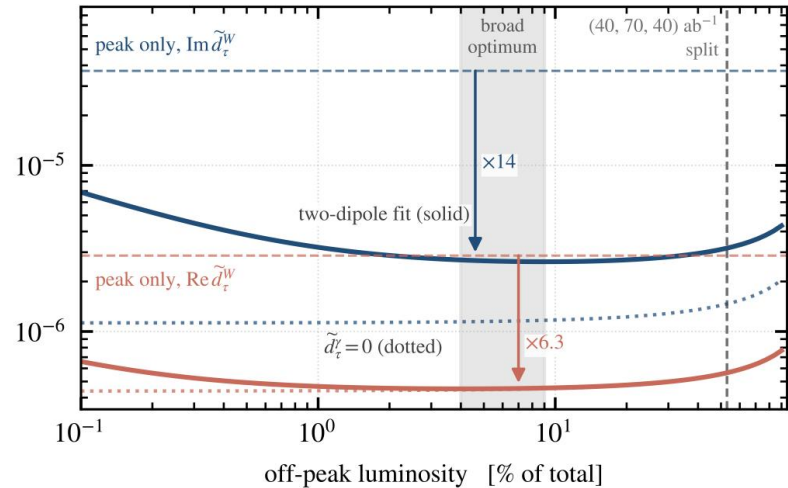
$\mathcal{O}_{1,2}$ similar to the ones used for low energy EDM measurement, but chirally suppressed, $\mathcal{O}_{3,4,5}$ no chiral suppression, need non-zero a_τ



$$N_Z = 6.0 (4.1) \times 10^{12} \text{ at FCC-ee (CEPC)}$$

Can increase the sensitivity
by as high as 3 orders of
manitudes!

Sample	Estimator	Re $\tilde{d}_\tau^W$	Im $\tilde{d}_\tau^W$	Re $\tilde{d}_\tau^\gamma$	Im $\tilde{d}_\tau^\gamma$
FCC-ee	$\mathcal{O}_2, \mathcal{O}_1$	4.5×10^{-4}	7.5×10^{-5}	2.3×10^{-2}	1.3×10^{-1}
	$\mathcal{O}_4, \mathcal{O}_3$	4.4×10^{-7}	1.3×10^{-6}	4.1×10^{-6}	1.3×10^{-4}
	Cramér–Rao	4.1×10^{-7}	1.1×10^{-6}	—	—
	5 moments, peak	2.9×10^{-6}	3.7×10^{-5}	1.2×10^{-4}	8.5×10^{-4}
	5 moments + scan	4.5×10^{-7}	2.7×10^{-6}	9.6×10^{-6}	2.1×10^{-5}
CEPC	$\mathcal{O}_2, \mathcal{O}_1$	5.4×10^{-4}	9.2×10^{-5}	2.7×10^{-2}	1.6×10^{-1}
	$\mathcal{O}_4, \mathcal{O}_3$	5.2×10^{-7}	1.5×10^{-6}	5.0×10^{-6}	1.6×10^{-4}
	Cramér–Rao	4.9×10^{-7}	1.3×10^{-6}	—	—
	5 moments, peak	3.5×10^{-6}	4.5×10^{-5}	1.5×10^{-4}	1.0×10^{-3}
	5 moments + scan	5.5×10^{-7}	3.2×10^{-6}	1.2×10^{-5}	2.6×10^{-5}
ALEPH	published stat.	1.6×10^{-3}	2.6×10^{-3}	—	—
	Cramér–Rao	9.7×10^{-4}	1.6×10^{-3}	—	—
Belle	published limit	—	—	1.1×10^{-3}	5.8×10^{-4}



Measurements of Tau Magnetic dipole and Magnetic Weakdipole at Z pole (the CEPC/FCCee)

$$\Gamma_{\tau}^{Z,\nu} = ie \left[v_{\tau} \gamma^{\nu} - a_{\tau} \gamma^{\nu} \gamma^5 + \frac{ia_{\tau}^W}{2m_{\tau}} \sigma^{\nu\rho} q_{\rho} + \frac{\tilde{d}_{\tau}^W}{2m_{\tau}} \gamma^5 \sigma^{\nu\rho} q_{\rho} \right]$$

$$\Gamma_{\tau}^{\gamma,\nu} = ie \left[Q_{\tau} (\gamma^{\nu} + \frac{ia_{\tau}^{\gamma}}{2m_{\tau}} \sigma^{\nu\rho} q_{\rho}) + \frac{\tilde{d}_{\tau}^{\gamma}}{2m_{\tau}} \gamma^5 \sigma^{\nu\rho} q_{\rho} \right],$$

Current bounds:

$$-0.057 < a_{\tau} < 0.024, \quad \text{C.L.} = 95\%.$$

$$|\text{Re } a_{\tau}^W| < 1.14 \times 10^{-3}, \quad |\text{Im } a_{\tau}^W| < 2.65 \times 10^{-3}.$$

SM predictions:

$$a_{\tau}^{\gamma}(0) = 1177.171(39) \times 10^{-6}$$

$$a_{\tau}^W(m_Z^2) = -(2.10 + 0.61 i) \times 10^{-6}$$

$$a_{\tau}^{\text{QED}_{1\text{-loop}}}(s) = \frac{\alpha_{\text{em}} m_{\tau}^2}{\pi s \beta_{\tau}} \ln \left(\frac{\beta_{\tau} - 1}{1 + \beta_{\tau}} \right),$$

$$\text{Re}[a_{\tau}^{\text{QED}_{1\text{-loop}}}(s)] = \frac{\alpha_{\text{em}} m_{\tau}^2}{\pi s \beta_{\tau}} \ln \left(\frac{1 - \beta_{\tau}}{1 + \beta_{\tau}} \right),$$

$$\text{Im}[a_{\tau}^{\text{QED}_{1\text{-loop}}}(s)] = \frac{\alpha_{\text{em}} m_{\tau}^2}{s \beta_{\tau}} \Theta(s - 4m_{\tau}^2).$$

Measuring various moments

$$\mathcal{E}_1 = (\hat{\ell}^- - \hat{\ell}^+) \cdot \hat{z},$$

$$\mathcal{E}_2 = \hat{\ell}_x^- \hat{\ell}_y^+ + \hat{\ell}_y^- \hat{\ell}_x^+,$$

$$\mathcal{E}_3 = (\hat{\ell}^- - \hat{\ell}^+) \cdot \hat{y},$$

$$\mathcal{E}_4 = \text{sgn}(\cos \theta) (\hat{\ell}^- - \hat{\ell}^+) \cdot \hat{x} = \text{sgn}(\cos \theta) \mathcal{E}_5,$$

$$\mathcal{E}_5 = (\hat{\ell}^- - \hat{\ell}^+) \cdot \hat{x},$$

$$\mathcal{E}_6 = \text{sgn}(\cos \theta) (\hat{\ell}^- - \hat{\ell}^+) \cdot \hat{y} = \text{sgn}(\cos \theta) \mathcal{E}_3.$$

Can improve the bound at the

CEPC and FCCee!



$$\langle \mathcal{E}_1 \rangle \simeq -\frac{8a_\tau \beta}{3\Delta_0} \left[v_\tau + \left(1 - \frac{6v_\tau^2}{\Delta_0} \right) \text{Re} (a_\tau^W + iQ_\tau \epsilon_v a_\tau^\gamma) \right], \quad (13)$$

$$\langle \mathcal{E}_2 \rangle \simeq \frac{4a_\tau \beta}{9\Delta_0} \left[Q_\tau \epsilon_v + \text{Im} (a_\tau^W + iQ_\tau \epsilon_v a_\tau^\gamma) \right], \quad (14)$$

$$\langle \mathcal{E}_3 \rangle \simeq -\frac{\pi m_Z a_\tau A_e \beta}{4m_\tau \Delta_0} \left[\frac{4m_\tau^2}{m_Z^2} Q_\tau \epsilon_a + \text{Im} (a_\tau^W + iQ_\tau \epsilon_a a_\tau^\gamma) \right], \quad (15)$$

$$\langle \mathcal{E}_4 \rangle \simeq \frac{m_Z a_\tau \beta}{3m_\tau \Delta_0} \left[\frac{4m_\tau^2}{m_Z^2} v_\tau + \text{Re} (a_\tau^W + iQ_\tau \epsilon_v a_\tau^\gamma) \right], \quad (16)$$

$$\langle \mathcal{E}_5 \rangle \simeq \frac{\pi m_Z v_\tau A_e}{4m_\tau \Delta_0} \left[\frac{4m_\tau^2}{m_Z^2} v_\tau + (2 - \beta^2) \{ \text{Re} (a_\tau^W + iQ_\tau \epsilon_a a_\tau^\gamma) + \xi \text{Im} a_\tau^W \} \right], \quad (17)$$

$$\langle \mathcal{E}_6 \rangle \simeq -\frac{m_Z \beta^2}{3m_\tau \Delta_0} \left[v_\tau \text{Im} (a_\tau^W + iQ_\tau \epsilon_v a_\tau^\gamma) - Q_\tau \epsilon_v \text{Re} a_\tau^W + f_\gamma \text{Im} a_\tau^\gamma \right], \quad (18)$$

$$v_\ell = -0.04431, \quad a_\ell = -0.60039.$$

where $D^\gamma/D^Z = i\Gamma_Z/m_Z$ gives

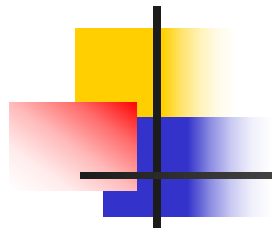
$$\epsilon_v = \frac{\Gamma_Z}{m_Z} \frac{Q_e v_e}{v_e^2 + a_e^2} = 3.35 \times 10^{-3},$$

$$\epsilon_a = \frac{\Gamma_Z}{m_Z} \frac{Q_e}{2v_e} = 0.309, \quad \xi = \frac{\Gamma_Z}{m_Z} \frac{Q_e Q_\tau}{2v_e v_\tau} = 7.0,$$

$$f_\gamma = \left(\frac{\Gamma_Z}{m_Z} \right)^2 \frac{(Q_e Q_\tau)^2}{v_e^2 + a_e^2} = 2.07 \times 10^{-3}.$$

$N_Z = 6.0(4.1) \times 10^{12}$ at FCC-ee (CEPC)

Sample	Estimator	$\text{Re} a_\tau^W$	$\text{Im} a_\tau^W$	$\text{Re} a_\tau^\gamma$	$\text{Im} a_\tau^\gamma$
FCC-ee	$\mathcal{E}_1, \mathcal{E}_2$ (1 dipole)	2.8×10^{-6}	3.3×10^{-5}	3.7×10^{-3}	8.5×10^{-4}
	$\mathcal{E}_3, \mathcal{E}_4$ (1 dipole)	4.4×10^{-7}	1.3×10^{-6}	4.1×10^{-6}	1.3×10^{-4}
	Cramér–Rao (1 dipole)	4.0×10^{-7}	9.9×10^{-7}	—	—
	6 moments, peak (2 dipoles)	4.9×10^{-7}	3.2×10^{-6}	1.1×10^{-5}	7.2×10^{-5}
	6 moments + scan (2 dipoles)	4.4×10^{-7}	2.4×10^{-6}	8.8×10^{-6}	2.0×10^{-5}
CEPC	$\mathcal{E}_1, \mathcal{E}_2$ (1 dipole)	3.4×10^{-6}	4.0×10^{-5}	4.5×10^{-3}	1.0×10^{-3}
	$\mathcal{E}_3, \mathcal{E}_4$ (1 dipole)	5.3×10^{-7}	1.5×10^{-6}	5.0×10^{-6}	1.6×10^{-4}
	Cramér–Rao (1 dipole)	4.9×10^{-7}	1.2×10^{-6}	—	—
	6 moments, peak (2 dipoles)	6.0×10^{-7}	3.8×10^{-6}	1.3×10^{-5}	8.7×10^{-5}
	6 moments + scan (2 dipoles)	5.4×10^{-7}	2.9×10^{-6}	1.1×10^{-5}	2.4×10^{-5}
ALEPH	95% CL/1.96	5.8×10^{-4}	1.4×10^{-3}	—	—
CMS	$a_\tau^\gamma(0)$, 1σ	—	—	3.2×10^{-3}	—
SM	one loop	-2.10×10^{-6}	-0.61×10^{-6}	-6.95×10^{-6}	2.77×10^{-6}



There are great potentials at the STCF, FCCee and CEPC for studying the Hyperons and tau EDM and WDM, improving sensitivities by orders of magnitudes!

