

Freeze-in Millicharged Vector Dark Matter

With a Long-Range Dark Photon Mediator

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Outline

- The Model — SM+Dark Sector (暗扇区模型)

$$\text{SM} \otimes SU(2)_D + \Sigma_D$$

with **kinetic mixing**. Both sectors are **weakly** coupled to each other.

- **Sub-GeV Millicharged** Vector Dark Matter

$$W' = W^{p,m}$$

- Constraints (ΔN_{eff} , ellipticity of NGC720, $\Omega_{\text{DM}} h^2$, LEP precision data, direct detection, etc.)
- **Two**-Temperature Universe
 - Evolution of $\xi(T) \equiv T_h(T)/T$
 - Coupled Boltzmann equations of relics
 - Synchronized evolution
 - Freeze-in production of dark matter
- Numerical Results
- Summary & Outlook

The Model (1/2)

- SM is enlarged with a dark gauge sector $SU(2)_D$ broken by a real dark triplet Higgs $\vec{\Sigma} = (\Sigma^1, \Sigma^2, \Sigma^3)$,

$$\Sigma = \frac{1}{2}\Sigma^a\sigma^a = \frac{1}{2}\begin{pmatrix} \Sigma^0 + v_\Sigma & \sqrt{2}\Sigma^p \\ \sqrt{2}\Sigma^m & -\Sigma^0 - v_\Sigma \end{pmatrix}$$

- $\Sigma^0 = \Sigma^3$ mixes with SM Higgs $h_{\text{SM}} \xrightarrow{\langle\Phi\rangle, \langle\Sigma\rangle} h_{125}, h_D$ via an angle β ; $\Sigma^{p,m} = \frac{1}{\sqrt{2}}(\Sigma^1 \mp i\Sigma^2) \sim W_L^{p,m}$

- Dimension 5 operator induces Dimension 4 kinetic mixing term

$$\mathcal{O}_5 = \frac{c_5 g_D g'}{\Lambda} \text{Tr} \left[\Sigma W_D^{\mu\nu} \right] B_{\mu\nu} \xrightarrow{\langle\Sigma\rangle} -\frac{1}{2} \epsilon W_D^{3\mu\nu} B_{\mu\nu}$$

with $\epsilon = -\frac{c_5 g_D g' v_\Sigma}{\Lambda}$ referred as **Abelian-NonAbelian kinetic mixing**.

[Chen, Cline, and Frey 2009;
Zhang, Li, Cao, and Li 2010;
Ko, Nomura, and Okada 2021;
Zhou 2024;
Ardu, Rahat, Valori, and Vives 2024;
Tran and Yuan 2025; etc]

- The kinetic energy of the electroweak gauge sector in the extended model is

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}W_{D\mu\nu}^a W_D^{a\mu\nu} - \frac{1}{2}\epsilon W_D^{3\mu\nu} B_{\mu\nu}$$

The Model (2/2)

- Diagonalization of the kinetic terms

$$\begin{pmatrix} W_{D\mu}^3 \\ B_\mu \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{1-\epsilon^2}} & 0 \\ -\frac{\epsilon}{\sqrt{1-\epsilon^2}} & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \cdot \begin{pmatrix} W_{D\mu}^3 \\ B_\mu \end{pmatrix},$$

with α arbitrary

- Covariant derivative is modified as

$$D_\mu = \partial_\mu - igW_\mu^a T^a - ig_D \left(W_{D\mu}^1 T_D^1 + W_{D\mu}^2 T_D^2 \right) \\ - iW_{D\mu}^3 \left(\tilde{g}'' Y + g_D c_\alpha C_\epsilon T_D^3 \right) - iB_\mu \left(\tilde{g}' Y - g_D s_\alpha C_\epsilon T_D^3 \right)$$

where

$$\tilde{g}' = g' (c_\alpha + s_\alpha S_\epsilon)$$

$$\tilde{g}'' = g' (s_\alpha - c_\alpha S_\epsilon)$$

with

$$c_\alpha = \cos \alpha, s_\alpha = \sin \alpha, C_\epsilon = \frac{1}{\sqrt{1-\epsilon^2}}, S_\epsilon = \epsilon C_\epsilon.$$

Both $\tilde{g}'$ and $\tilde{g}''$ depend on α !!

Gauge Sector ($W^\pm$, $W^{(p,m)}$, photon γ , **massless** dark photon Z' , and massive Z boson)

- $\mathcal{L}_{\text{mass}}^{\text{gauge}} = \frac{g^2 v^2}{4} W_\mu^+ W^{\mu-} + g_D^2 v_\Sigma^2 W_\mu^p W^{\mu m} + \frac{v^2}{8} \left(-g W_\mu^3 + \tilde{g}' B_\mu + \tilde{g}'' W_{D\mu}^3 \right)^2$
- No Stueckelberg mass!
Feldman, Liu and Nath (2007);
Cheung & Yuan (2007), etc.

where $W_\mu^{p,m} = \frac{1}{\sqrt{2}} \left(W_{D\mu}^1 \mp W_{D\mu}^2 \right)$ is a **dark matter candidate** with mass $m_{W'} = 2g_D v_\Sigma$,

and $W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp W_\mu^2 \right)$ is the SM charged boson $W^\pm$ with mass $m_{W'} = gv/2$ with $v = 246$ GeV .

- In the basis of $V = (B, W^3, W_D^3)^T$, we have the mass mixing matrix for the neutral gauge bosons

$$M_1^2 = \frac{v^2}{4} \begin{pmatrix} \tilde{g}'^2 & -\tilde{g}'g & \tilde{g}'\tilde{g}'' \\ -\tilde{g}'g & g^2 & -g\tilde{g}'' \\ \tilde{g}'\tilde{g}'' & -g\tilde{g}'' & \tilde{g}''^2 \end{pmatrix} \Rightarrow \boxed{m_\gamma = 0, m_{Z'} = 0, m_Z = \frac{v}{2} \sqrt{g^2 + \bar{g}'^2}} \quad (\text{Independent of } \alpha!)$$

with $\bar{g}'^2 = \tilde{g}'^2 + \tilde{g}''^2 = \frac{g'^2}{1 - \epsilon^2}$

- M_1^2 can be diagonalized by $\mathcal{O}^T \cdot M_1^2 \cdot \mathcal{O} = M_{\text{Diag}}^2 = \text{Diag}(\mathbf{0}, m_Z^2, \mathbf{0})$ and $V = \mathcal{O} \cdot V_{\text{ph}}$ with $V_{\text{ph}} = (A, Z, Z')^T$.
- $\mathcal{O}$ involves just 2 mixing angles, θ_1 and θ_2 . θ_1 can be identified as the weak angle. θ_2 provides further mixing between W^3 and W_D^3

Millicharged Vector Dark Matter

— $\gamma W^p W^m$ coupling (“Making Dark Matter Out Of Light”)

- Diagonalization of the kinetic and mass mixing terms induces coupling between the dark matter $W^p W^m$ pair with the physical photon

$$\mathcal{L}_{\text{mil}} \sim -i g_D \left[\overbrace{\kappa_1 \left(W_{\mu\nu}^p W^{m\mu} - W_{\mu\nu}^m W^{p\mu} \right) A^\nu}^{\text{"minimal" coupling}} + \frac{1}{2} \kappa'_1 \underbrace{\left(W_\mu^p W_\nu^m - W_\mu^m W_\nu^p \right) F^{\mu\nu}}_{\text{"anomalous" magnetic moment}} \right]$$

- Remarks:

[Hagiwara, Peccei, Zeppenfeld, and Hikasa, NPB, 282, p253 – 307 (1987)]

$$(1) \kappa_1 = C_\epsilon \left(-s_\alpha \mathcal{O}_{11} + c_\alpha \mathcal{O}_{31} \right), \quad \kappa'_1 = \left(\epsilon c_\alpha - s_\alpha / C_\epsilon \right) \mathcal{O}_{11} + \left(\epsilon s_\alpha + c_\alpha / C_\epsilon \right) \mathcal{O}_{31}$$

Both depend on the arbitrary angle α explicitly!

(2) However, Pan, He, He, and Li, “Scrutinizing a massless dark photon: basis independence” ([arXiv.1807.11363](#)) demonstrated that such dependencies are **not** physical! **Holds in non-abelian models too!**

(3) SU(2) VDM: C. X. Yuan, Z. Zhang, C. Cai, Y. L. Tang and H. H. Zhang, [arXiv:2405.16165](#) [hep-ph]; J. H. Liang, Y. Liao, X. D. Ma and H. L. Wang, [arXiv:2501.13501](#) [hep-ph];.....

Millicharged Basis:

$$s_\alpha = \epsilon, \quad c_\alpha = \sqrt{1 - \epsilon^2}$$

$$\Rightarrow \tilde{g}' = \bar{g}' = \frac{g'}{\sqrt{1 - \epsilon^2}}, \quad \tilde{g}'' = 0$$

$$\Rightarrow \begin{aligned} s_1 &= \frac{\bar{g}'}{\sqrt{g^2 + \bar{g}'^2}} & c_1 &= \frac{g}{\sqrt{g^2 + \bar{g}'^2}} \\ s_2 &= 0 & c_2 &= 1 \end{aligned} \Rightarrow \mathcal{O} = \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{Dark photon} \\ & & & & & \text{doesn't couple to} \\ & & & & & \text{SM fermions!}$$

$$g\mathcal{O}_{21} \equiv gs_1 = e$$

$$\begin{aligned} \gamma W^p W^m : & \quad \kappa_1 = -\frac{\epsilon c_1}{\sqrt{1 - \epsilon^2}} & \kappa'_1 &= 0 & \Rightarrow W^{(p,m)} \text{ carries millicharged!} \\ Z W^p W^m : & \Rightarrow \kappa_2 = +\frac{\epsilon s_1}{\sqrt{1 - \epsilon^2}} & \kappa'_2 &= 0 & \text{All } \kappa_i, \kappa'_i \text{ are independent of } \alpha! \\ Z' W^p W^m : & \quad \kappa_3 = 1 & \kappa'_3 &= 1 \end{aligned}$$

Galaxy NGC720

- Ellipticity profile of the galaxy sets stringent bound on dark sector **self-interactions** (SIDM). Too strong interactions would **destroy** observed anisotropy.

- Following closely the evolution of the velocity anisotropy due to energy transfer, Agrawal, Cyr-Racine, Randall, and Scholtz (JCAP 1705(05), 022 (2017)) sets a limit for NGC720 of about

$$m_{\text{DM}} \left(\frac{0.01}{\alpha_D} \right)^{2/3} \gtrsim 300 \text{ GeV} , \quad \alpha_D = \frac{g_D^2}{4\pi}$$

- SIDM helps to solve WIMP **small scale problems** (core-cusp, too big to fail, missing satellites) [S. Tulin and H. B. Yu, Phys. Rept. 730, 1-57 (2018); $\dots$; Tran, Nguyen, and TCY, JCAP 05 (2024) 015; $\dots$]

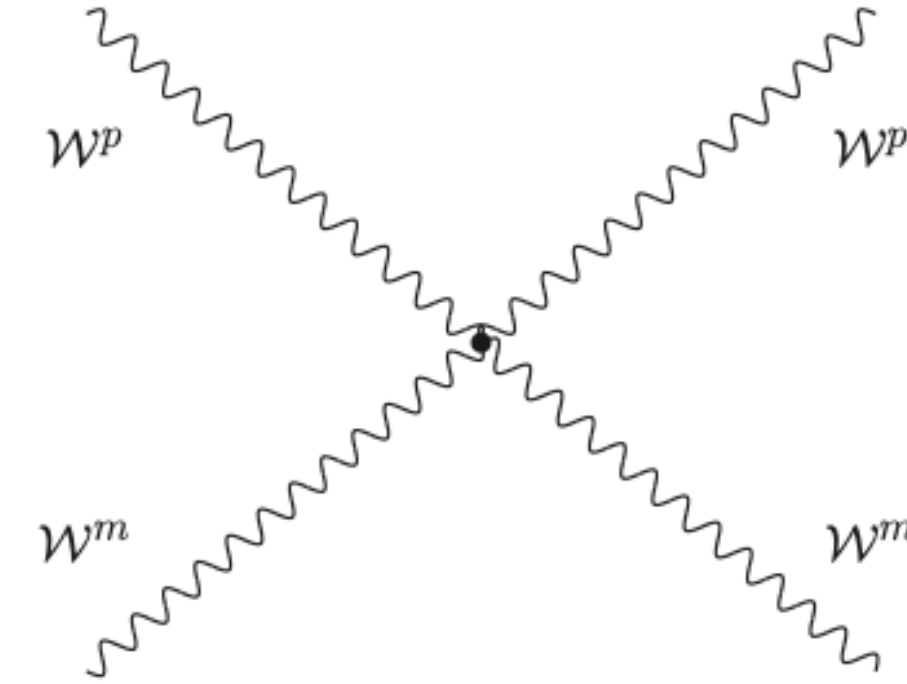


Figure 2. The seagull diagram for SIDM in the model.

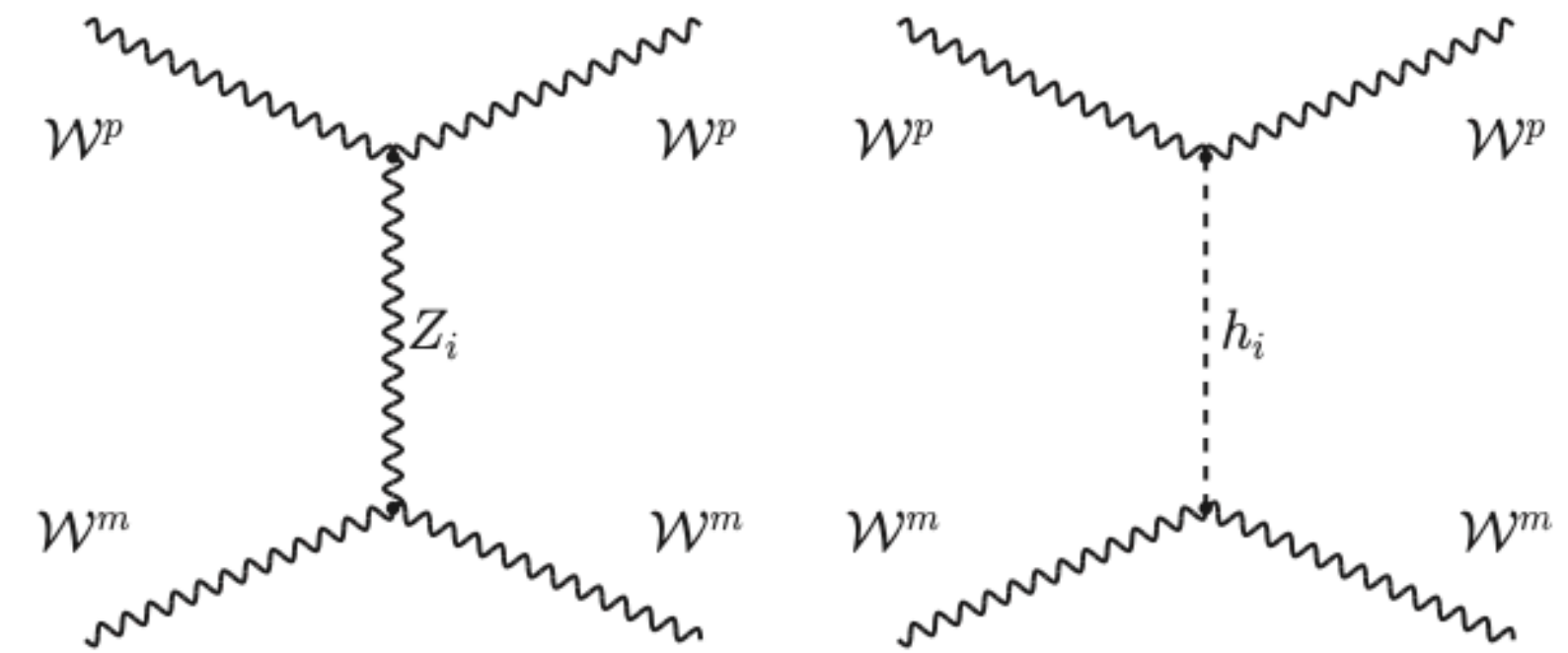
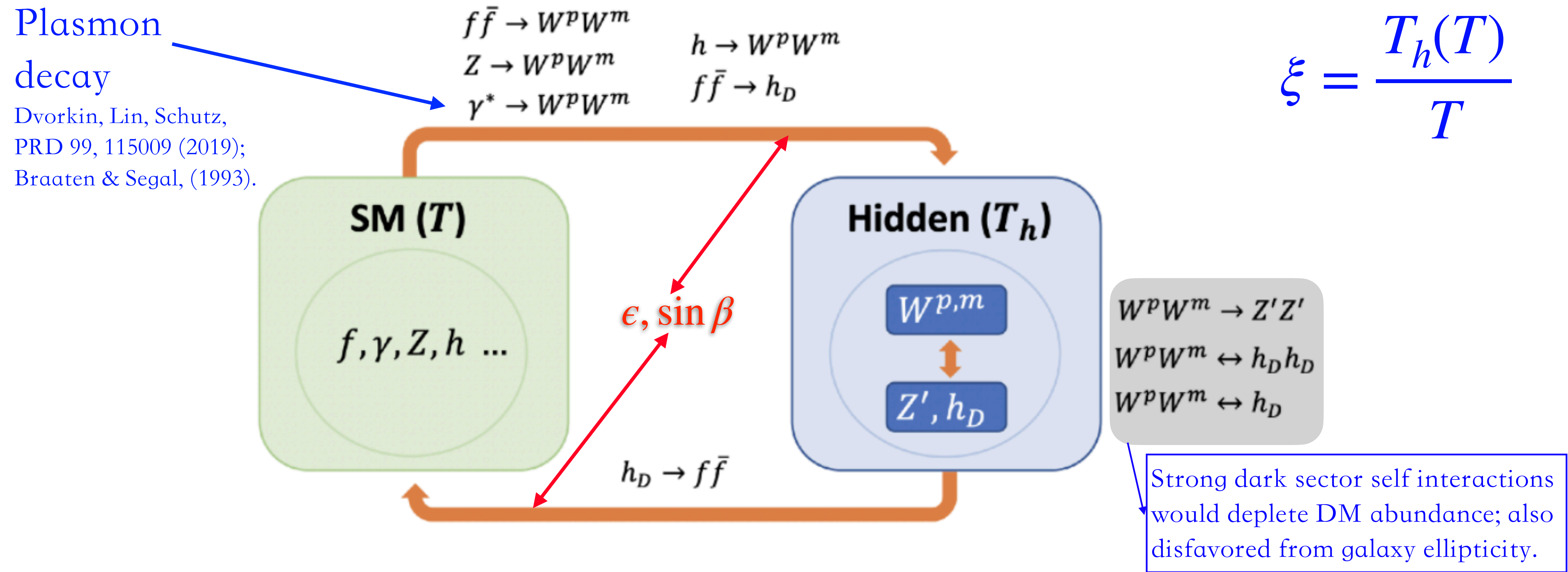


Figure 3. The t channel Feynman diagrams for SIDM in the model.

Two-Temperature Universe — Freeze In VDM



Two-Temperature Universe — Freeze In VDM

Consistently Synchronized Evolution of Two Temperature Universe

- Temperature Evolution $\xi \equiv T_h/T$

$$\frac{d\xi}{dT} = \frac{1}{T} \left(-\xi + \frac{dT_h}{dT} \right) = \frac{1}{T} \left[-\xi + A \frac{d\rho_v}{dT} \left(\frac{d\rho_h}{dT_h} \right)^{-1} \right]$$

$$A = \frac{3H(\rho_h + p_h) - J_{vh}}{3H(\rho_v + p_v) + J_{vh}}$$

J_{vh} is the source term for the hidden sector accounts for energy exchange between SM and hidden sectors.

- $Y_i = \frac{n_i}{s}$ (comoving number density)

$$i \in \{W^{(p,m)}, Z', h_D\}$$

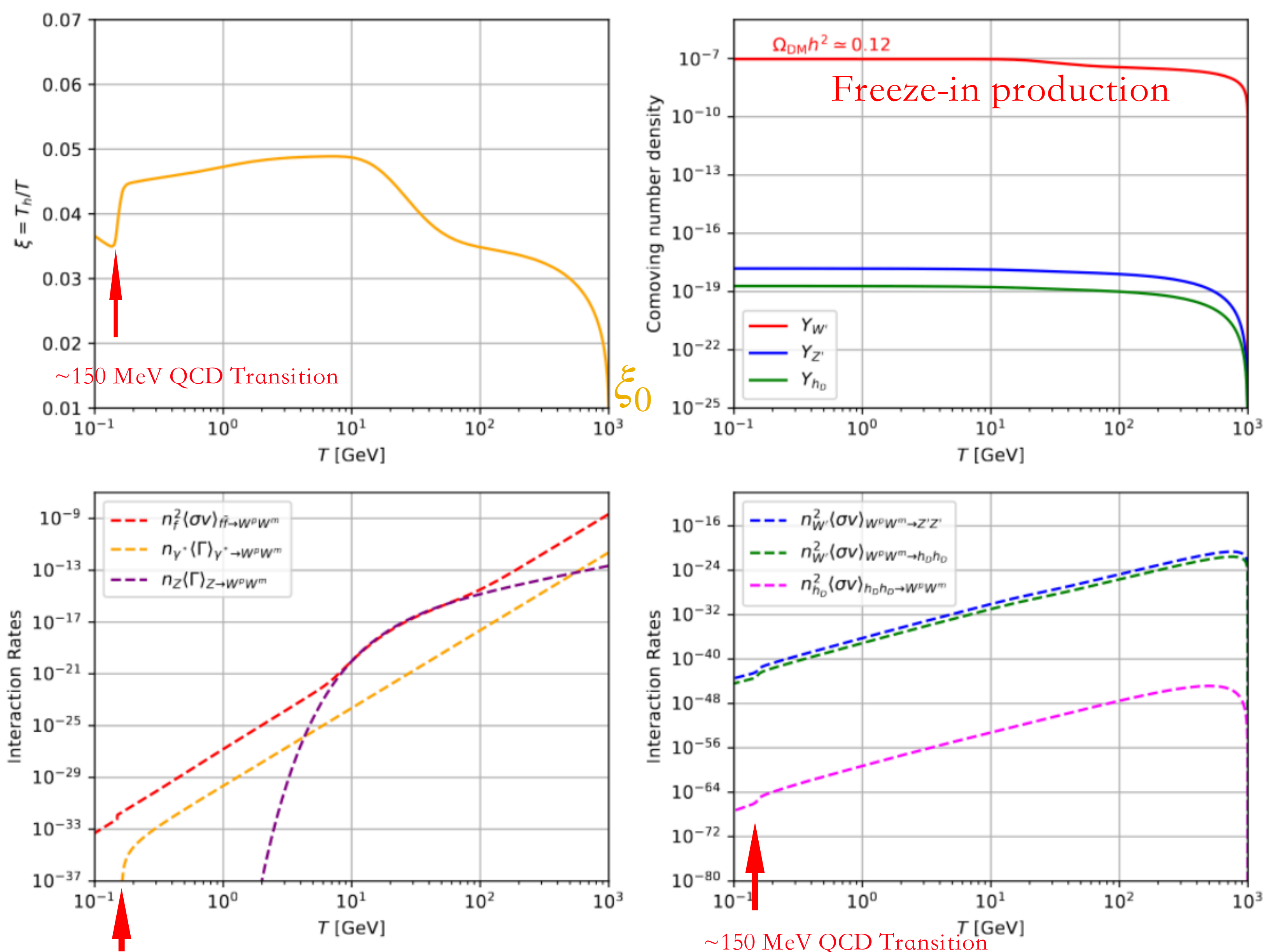
$$-s \frac{dY_i}{dT} [3H(\rho_v + p_v) + J_{vh}] \left(\frac{d\rho_v}{dT} \right)^{-1} = \mathcal{R}_i(T, T_h) .$$

Collision terms. Depends on **both** temperatures.

- Relic density is computed according to

$$\Omega_{\text{DM}} h^2 = \left(\frac{m_W Y_W^0 s_0}{\rho_c} \right) h^2$$

Thermal History BM1 $m_{W'} = 0.01 \text{ GeV}, m_{h_D} = m_{W'}/3, g_D = 10^{-7}, \epsilon = 3.8 \times 10^{-7}, \sin \beta = 0, T_{RH} = 1 \text{ TeV}, \xi_0 = 0.01$

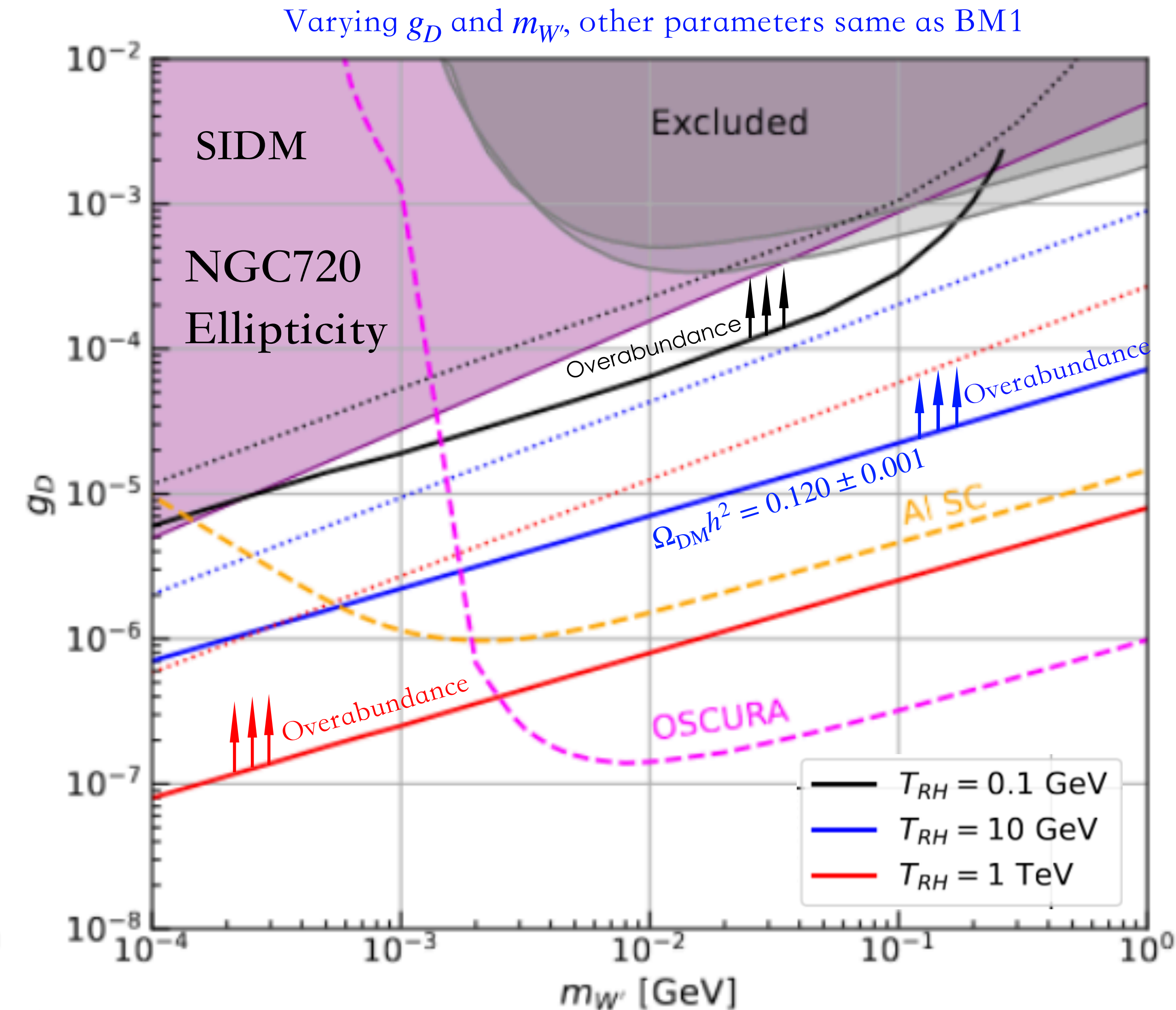


- Dominated by milli-charged annihilation process $f\bar{f} \xrightarrow{\gamma} W^p W^m$
- Z and plasmon decays subdominated

- Freeze-in production of W' is dominant relic component via millicharged annihilation of fermion-antifermion pair
- Relic of dark photon and dark Higgs are subdominant as suppressed by tiny g_D

Direct Detection of Milli-charged sub-GeV VDM via searches of electron recoils

- Based on electron recoils from the process $W' + e^- \rightarrow W' + e^-$ for sub-GeV DM. Dominated by photon exchange, while Z and scalar mediated contributions being sub-leading.
- Regions with smaller value of T_{RH} are excluded regions from current DMDD data from SENS, DAMIC, EDELWEISS, SuperCDMS, CDEX, DarkSide50, XENONnT, and PandaX-II
- Purple regions excluded by galaxy ellipticity (NGC720)
- Relic density is sensitive to T_{RH} (Dark, Blue, Red lines)
- Dotted lines: Boundary where dark sector self-interaction rates are comparable to Hubble rate. Thermal vs. non-thermal !
- SIDM and DMDD can set lower limit on T_{RH}
- Projected sensitivities from OSCURA and ALSC (Aluminum based superconducting detectors)



(1) B. A. Cervantes-Vergara et al. (Oscura), JINST 18, P08016 (2023), arXiv:2304.04401

(2) Y. Hochberg et al. PRL 127, 151802 (2021), arXiv:2101.08263

Summary & Outlook

- A simple yet realistic VDM model of **two-temperature universe** based on $\text{SM} \otimes \textcolor{red}{SU(2)}_D + \textcolor{red}{\Sigma}_D$ is studied
- Coupled Boltzmann equations with two temperatures are solved numerically to obtain the correct relic density of **freeze-in** dark matter which is the **millicharged vectorial** particle/antiparticle pair. Dominated process is $\bar{f}f \xrightarrow{\gamma} W^p W^m$. **Making VDM out of light! Plasmon** decays into VDN is studied in detail and showed to be subdominant.
- All constraints from **cosmology** (relic density, ΔN_{eff} from BBN & CMB), **astrophysics** (galactic ellipticity of observed halos), **collider physics** (LEP EW precision data, LHC Higgs signal strength) and **direct detection experiments** are satisfied.
- Direct detection of the vectorial dark matter at the **sub-GeV** mass scale (via electron recoils) at future OSCURA and ALSC detectors are discussed.
- Interesting topics to study
 - (1) **Making freeze-in millicharged DM with other spins out of light**
 - (2) **Strongly first order phase transition in the early universe and possible GW signals in this two-temperature setup**

Thank You!

Back Up Slides

UV Completion

- Introduce a vector-like Dirac lepton $\Psi(\mathbf{1}, \mathbf{2}, y)$

$$\mathcal{L}_\Psi = \bar{\Psi}(i \not{D} - M_\Psi)\Psi - (y_\Psi \bar{\Psi} \Sigma \Psi + H.c.) \text{ with } \not{D} = \gamma^\mu (\partial_\mu - ig_D W_{D\mu}^a T_D^a - ig' y B_\mu) \Psi.$$

- Integrating out the heavy lepton Ψ running inside a triangular loop with external lines of W_D, B and Σ gives rise to the dimension 5 operator $\mathcal{O}_5 = \frac{c_5 g_D g'}{\Lambda} \text{Tr} \left[\Sigma W_D^{\mu\nu} \right] B_{\mu\nu}$ with a coefficient $\frac{c_5}{\Lambda} \simeq \frac{y \text{Re}[y_\Psi]}{12\pi^2 M_\Psi}$.
- With $\langle \Sigma \rangle \neq 0$, $\mathcal{O}_5$ induces $\mathcal{L}_{\text{mix}} = -\frac{\epsilon}{2} W_D^{\mu\nu} B_{\mu\nu}$ with $\epsilon = -\frac{c_5}{\Lambda} g_D g' v_\Sigma$.
- Take $y = -1, y_\Psi \sim 5, M_\Psi \sim \mathcal{O}(1)\text{TeV}$, one obtains $\epsilon \sim 10^{-7}$ for $m_{W'} \sim 0.01\text{GeV}$.
- **Caveat:** If $v_\Sigma \gg \Lambda/c_5$, fine-tuning is unavoidable in the parameter space to keep the physical mass of the heavy lepton not driving far away from its bare mass M_Ψ .

Higgs Sector ($\Phi, \vec{\Sigma}$)

- Scalar Potential

$$V(\Phi, \Sigma) = -\mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \Delta V(\Phi, \Lambda)$$

$$\Delta V(\Phi, \Sigma) = -\frac{1}{2}m_\Sigma^2 \vec{\Sigma} \cdot \vec{\Sigma} + \frac{1}{4}\lambda_\Sigma (\vec{\Sigma} \cdot \vec{\Sigma})^2 + \frac{1}{2}\lambda_{\Phi\Sigma} |\Phi|^2 \vec{\Sigma} \cdot \vec{\Sigma}$$

- Perturbative unitarity, bounded from below

$$0 < \lambda_\Phi, \lambda_\Sigma \leq \frac{4\pi}{3}$$

$$-2\sqrt{\lambda_\Phi \lambda_\Sigma} < \lambda_{\Phi\Sigma} \leq 8\pi$$

- Scalar Mass Matrix

$$M_S^2 = \begin{pmatrix} 2\lambda_\Phi v^2 - \frac{1}{2}\lambda_{\Phi\Sigma} v_\Sigma^2 & -\lambda_{\Phi\Sigma} v v_\Sigma \\ -\lambda_{\Phi\Sigma} v v_\Sigma & 2\lambda_\Sigma v_\Sigma^2 - \frac{1}{2}\lambda_{\Phi\Sigma} v^2 \end{pmatrix}$$

$$M_{1,2}^2 = \frac{1}{4} \left(v^2 (4\lambda_\phi - \lambda_{\phi\Sigma}) + v_\Sigma^2 (4\lambda_\Sigma - \lambda_{\phi\Sigma}) \right)$$

$$\pm \frac{1}{4} \sqrt{v^4 (4\lambda_\phi + \lambda_{\phi\Sigma})^2 - 2v^2 v_\Sigma^2 \left(4\lambda_{\phi\Sigma} (\lambda_\Sigma + \lambda_\phi) + 16\lambda_\Sigma \lambda_\phi - 7\lambda_{\phi\Sigma}^2 \right) + v_\Sigma^4 (4\lambda_\Sigma + \lambda_{\phi\Sigma})^2}$$

- Higgs Mass Mixing

$$\begin{pmatrix} h \\ h_D \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi \\ \Sigma^0 \end{pmatrix}$$

h is identified as SM 125 GeV Higgs boson;

h_D is an extra Higgs from hidden triplet

- Higgs signal strength requires

$$|\sin \beta| \lesssim 0.2 \quad [\text{ATLAS 2022}]$$

Benchmark Points

	g_D	ϵ		$\sin \beta$	$m_{Z'}$ (GeV)	$m_{W'}$ (GeV)	m_{h_D} (GeV)	$T_0 = T_{\text{RH}}$ (TeV)	$\xi_0 = (T_{h0}/T_0)_{\text{RH}}$
BM1	10^{-7}	3.8×10^{-7}		0	0	0.01	0.003	1	0.01
BM2	10^{-7}	1.2×10^{-6}		10^{-9}	0	0.1	0.4	1	0.01

Table II. Values of model parameters of benchmark points **BM1** and **BM2**.

- W' is sub-GeV DM.
- BM1: $h_D \nrightarrow W'W'$ (kinematic), $f\bar{f}$ ($\sin \beta = 0$); BM2: $h_D \rightarrow W'W', f\bar{f}, \dots$.
Loop processes $h_D \rightarrow \gamma\gamma, \gamma Z', Z'Z'$ are ignored.

Remarks:

- Kinetic mixing ϵ (gauge portal), Higgs mixing $\sin \beta$ (or quartic coupling $\lambda_{\Phi\Sigma}$) (Higgs portal) but **no** Stueckelberg mass portal ($m_{Z'} = 0$).
- Reheating temperature T_{RH} , whence freeze-in DM is expected to be first produced, is **not** exactly known.
Theoretically $T_{\text{BBN}} (\sim 4 \text{ MeV}) < T_{\text{RH}} < \Lambda_{\text{GUT/Inflation}} (\sim 10^{16}\text{GeV})$

Cosmology constraints on ΔN_{eff}

- Massless dark photon contributes to effective number of relativistic species.
Sensitively depends on the evolution of $\xi(T)$.
- Current BBN and CMB analyses demand

$$\Delta N_{\text{eff}} = N_{\text{eff}}^{\text{exp}} - N_{\text{eff}}^{\text{SM}} < 0.18 \text{ at } 95 \% \text{ CL}$$

$$\Delta N_{\text{eff}}(T, \xi) \Big|_{\text{BBN}} = \frac{4}{7} \left(\frac{11}{4} \right)^{4/3} g_{\text{eff}}^h(\xi(T)T) \xi(T)^4 \Big|_{\text{BBN}}, \quad \xi(T) \equiv \frac{T_h(T)}{T}$$

Other constraints:

- Lifetime of light dark Higgs τ_{h_D}

In order to avoid disrupting light-element abundance, dark Higgs must decay before BBN. This requires,

$$\tau_{h_D} \lesssim 0.1 \text{ sec for } 2m_\mu \lesssim m_{h_D} \lesssim m_h/2$$

Typically, we find τ_{h_D} is $\sim 10^{-9}$ sec in the model.

- Limits from direct detection experiments (SENSI, DAMIC, EDELWEISS, SuperCDMS, CDEX, Darkside-50, XENONnT, PandaX—II, ...) for sub-GeV DM

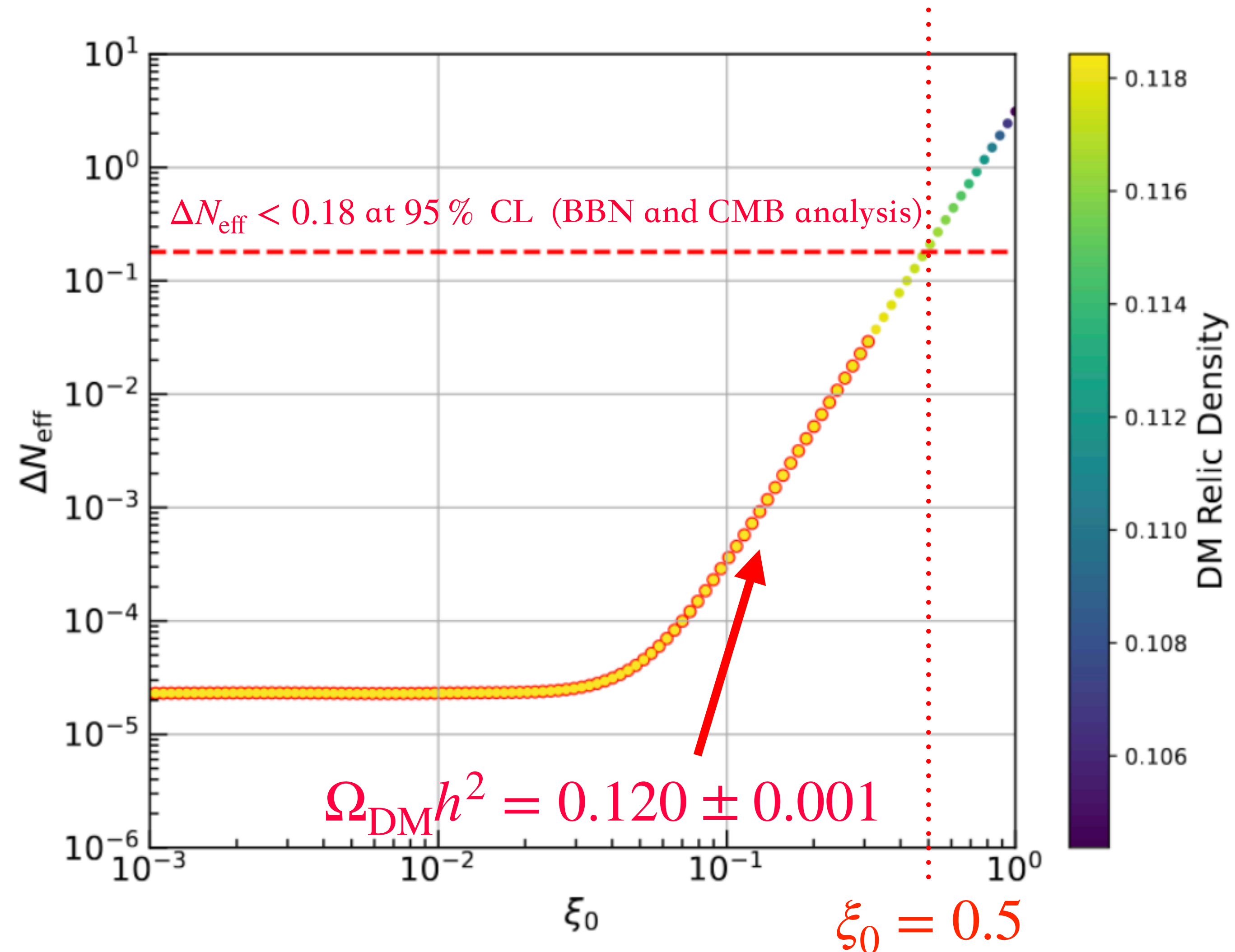
In the model, we use the reference elastic cross section $W'e^- \rightarrow W'e^-$ (electron recoil) which is **dominated** by the photon exchange

$$\bar{\sigma}_e = \frac{4g_D^2 \kappa_1^2 \alpha_{\text{EM}} \mu_{W'e}^2}{(\alpha_{\text{EM}} m_e)^4} \quad \text{with } \mu_{W'e} = \frac{m'_W m_e}{m'_W + m_e}$$

Relic Density vs. ΔN_{eff} and vs. Reheating Temperature T_{RH}

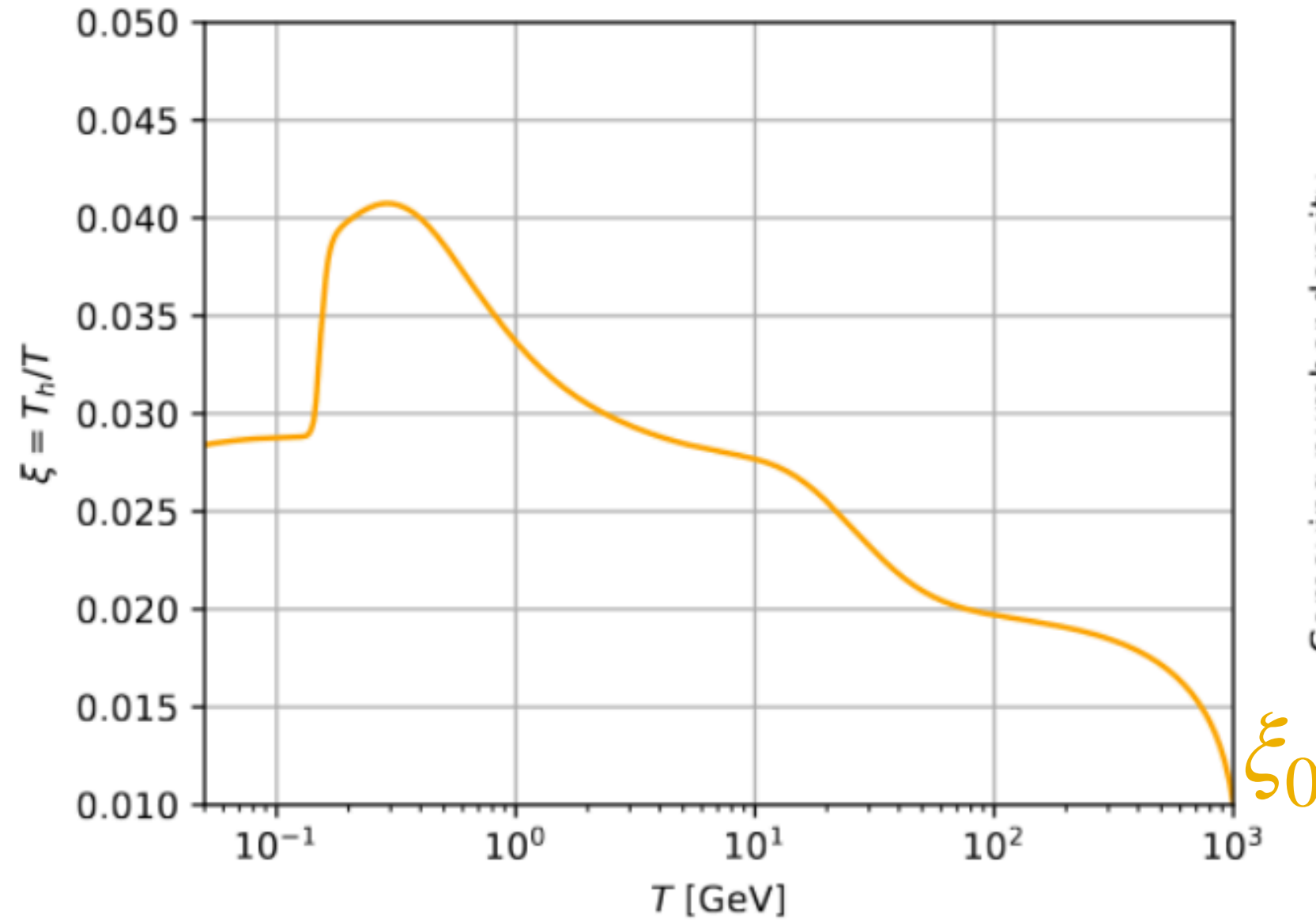
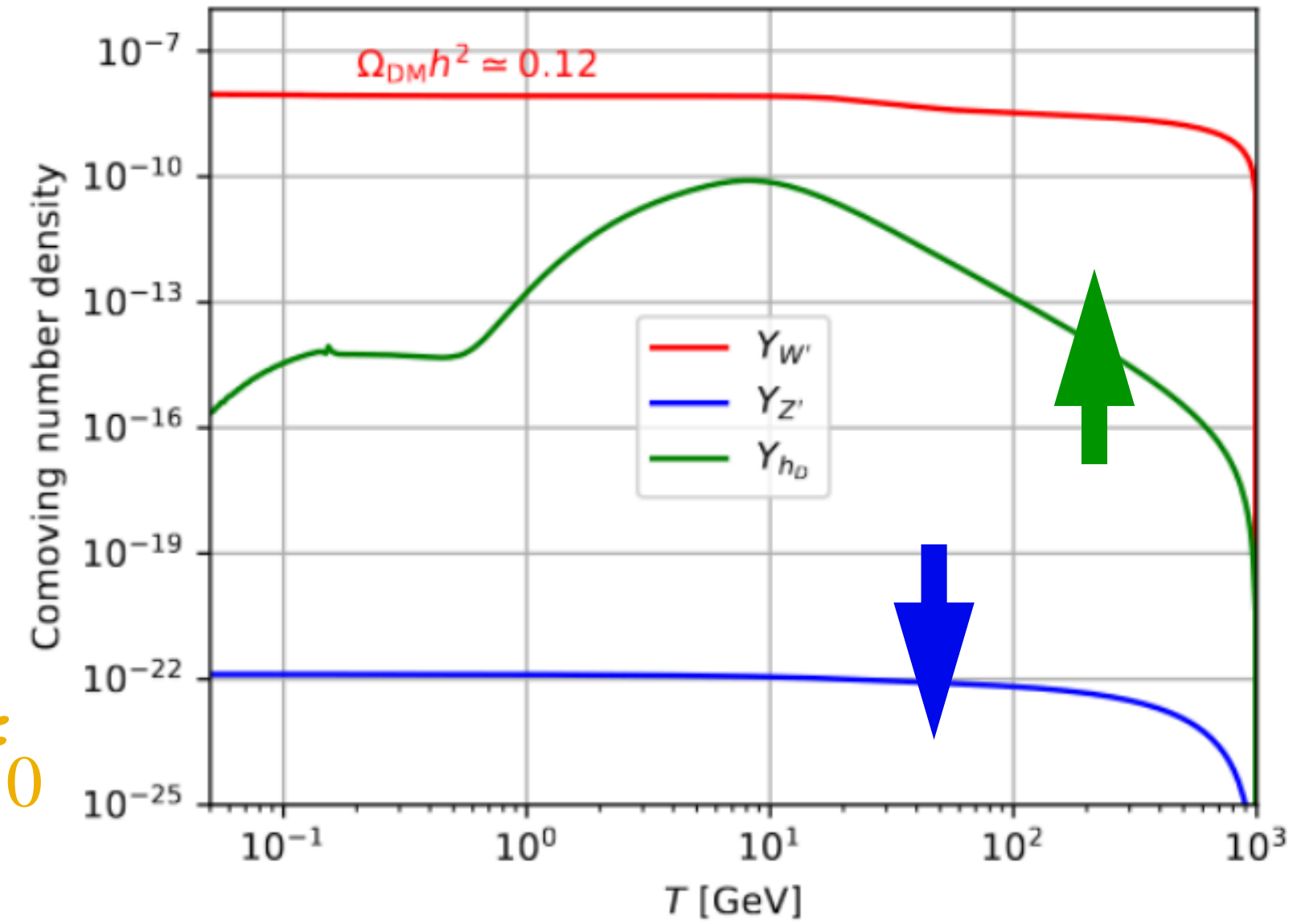
$$\Delta N_{\text{eff}}(T, \xi) \Big|_{\text{BBN}} = \frac{4}{7} \left(\frac{11}{4} \right)^{4/3} g_{\text{eff}}^h(T_h) \xi(T)^4 \Big|_{\text{BBN}} \quad \text{where } \xi(T) = T_h(T)/T \quad \xi_0 = (T_{h0}/T_0)_{\text{RH}}, T_0 = T_{\text{RH}} = 1 \text{ TeV}$$

- Massless dark photon affects effective number of relativistic species N_{eff}
- Varying ξ_0 , other parameters same as BM1
- Reheating temperature is NOT exactly known. We set $T_0 = T_{\text{RH}} = 1 \text{ TeV}$ in this plot
- Larger ξ_0 enhances ΔN_{eff} while suppressed $\Omega_{\text{DM}} h^2$
- $\xi_0 \gtrsim 0.5$ is excluded by BBN and CMB
- Cosmology tightly constrains hidden sector initial conditions while leaving ample parameter space consistent with freeze-in mechanism



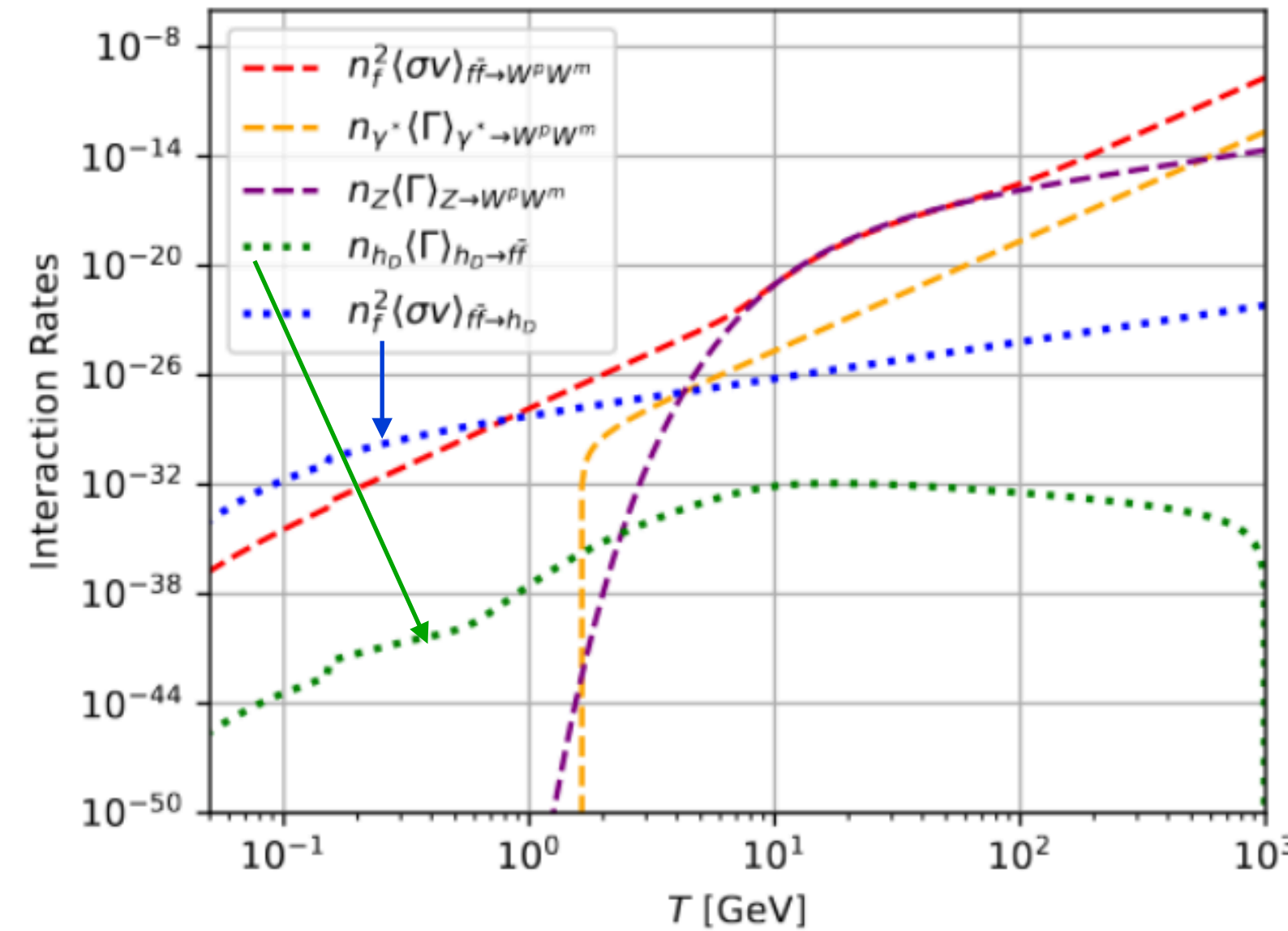
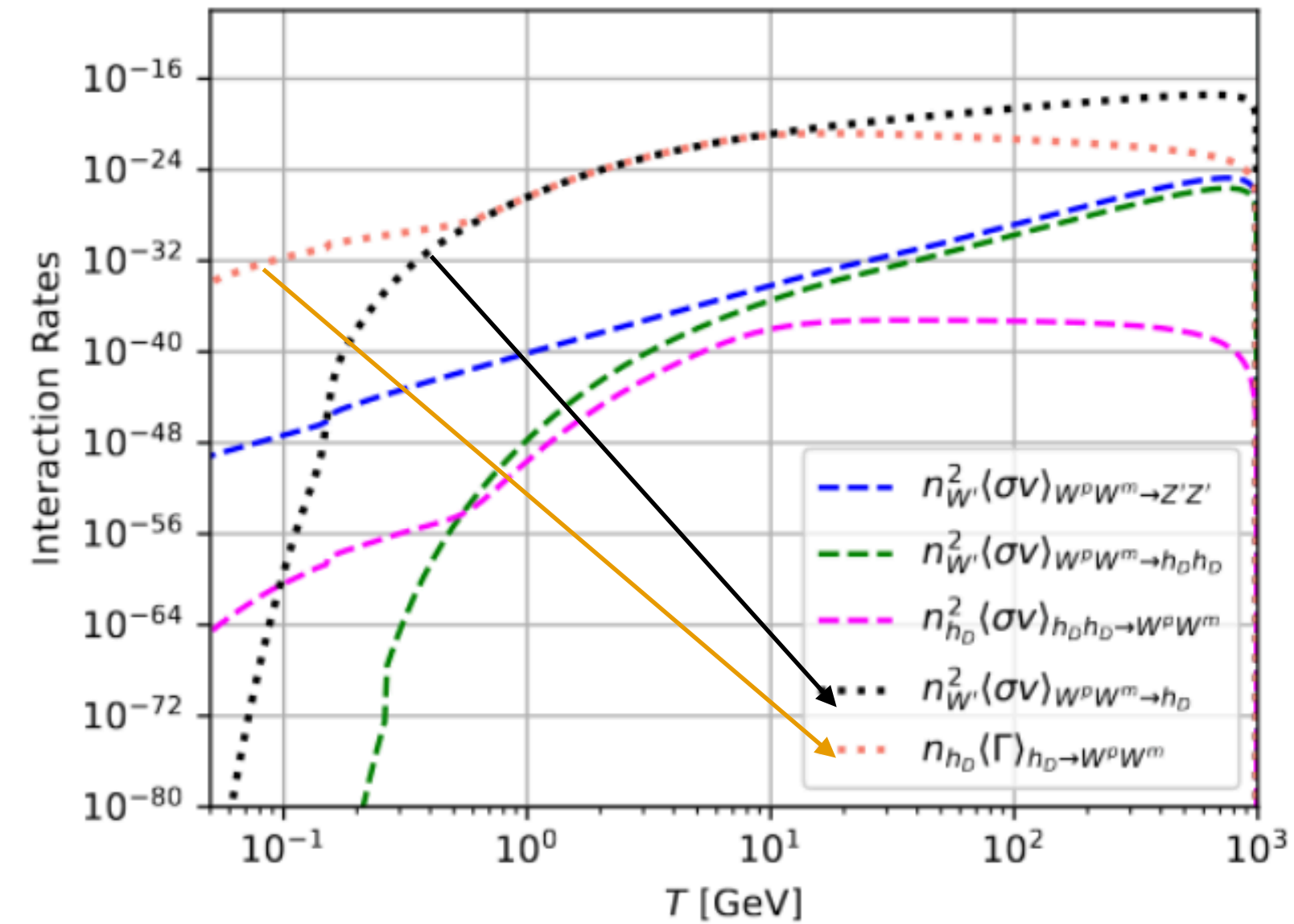
Thermal History BM2 $m_{W'} = 0.1 \text{ GeV}, m_{h_D} = 0.4 \text{ GeV}, g_D = 10^{-7}, \epsilon = 1.2 \times 10^{-6}, \sin \beta = 10^{-9}, T_{RH} = 1 \text{ TeV}, \xi_0 = 0.01$

- Richer structure in $\xi(T)$ evolution as compared with BM1



- Relic h_D is significantly enhanced as compared with BM1. Yet W' is still the dominant freeze-in relic component among the three dark species

- New contributions $h_D \leftrightarrow f\bar{f}$ are needed to consider since $\sin \beta \neq 0$



- Other new but much smaller contributions are $W^p W^m \leftrightarrow h_D$ which is kinematically allowed in BM2

Coupled Boltzmann Equations (1/3)

Two Temperature Universe $\xi \equiv T_h/T$

- Temperature Evolution

$$\frac{d\xi}{dT} = \frac{1}{T} \left(-\xi + \frac{dT_h}{dT} \right) = \frac{1}{T} \left[-\xi + A \frac{d\rho_v}{dT} \left(\frac{d\rho_h}{dT_h} \right)^{-1} \right]$$

$$A = \frac{3H(\rho_h + p_h) - J_{vh}}{3H(\rho_v + p_v) + J_{vh}}$$

J_{vh} is the source term for the hidden sector accounts for energy exchange between SM and hidden sectors.

- $Y_i = \frac{n_i}{s}$ (comoving number density)

$$i \in \{W^{(p,m)}, \gamma', h_D\}$$

$$-s \frac{dY_i}{dT} [3H(\rho_v + p_v) + J_{vh}] \left(\frac{d\rho_v}{dT} \right)^{-1} = \mathcal{R}_i(T, T_h) .$$

Collision terms. Depends on both temperatures.

- Relic density

$$\Omega_{\text{DM}} h^2 = \left(\frac{m_W Y_W^0 s_0}{\rho_c} \right) h^2$$

Coupled Boltzmann Equations (2/3)

Interaction Rates $\mathcal{R}_{W',Z',h_D}(T, T_h)$ (number of interactions per unit volume per unit time that produce W', γ', h_D)

$$\begin{aligned} \mathcal{R}_{W'}(T, T_h) = & \sum_{f \in \text{SM}} \left[n_f^2 \langle \sigma v \rangle_{f\bar{f} \rightarrow W^p W^m}(T) \right] + n_Z \langle \Gamma \rangle_{Z \rightarrow W^p W^m}(T) + n_h \langle \Gamma \rangle_{h \rightarrow W^p W^m}(T) \\ & + n_{\gamma^*} \langle \Gamma \rangle_{\gamma^* \rightarrow W^p W^m}(T) + n_{h_D} \langle \Gamma \rangle_{h_D \rightarrow W^p W^m}(T_h) \\ & + n_{h_D}^2 \langle \sigma v \rangle_{h_D h_D \rightarrow W^p W^m}(T_h) - n_{W'}^2 \left[\langle \sigma v \rangle_{W^p W^m \rightarrow h_D}(T_h) \right. \\ & \left. + \langle \sigma v \rangle_{W^p W^m \rightarrow Z' Z'}(T_h) + \langle \sigma v \rangle_{W^p W^m \rightarrow h_D h_D}(T_h) \right], \end{aligned}$$

Plasmon decay

$$\mathcal{R}_{Z'}(T, T_h) = n_{W'}^2 \langle \sigma v \rangle_{W^p W^m \rightarrow Z' Z'}(T_h),$$

$$\begin{aligned} \mathcal{R}_{h_D}(T, T_h) = & \sum_{f \in \text{SM}} \left[n_f^2 \langle \sigma v \rangle_{f\bar{f} \rightarrow h_D}(T) \right] + n_{W'}^2 \left[\langle \sigma v \rangle_{W^p W^m \rightarrow h_D h_D}(T_h) \right. \\ & \left. + \langle \sigma v \rangle_{W^p W^m \rightarrow h_D}(T_h) \right] - n_{h_D}^2 \langle \sigma v \rangle_{h_D h_D \rightarrow W^p W^m}(T_h) \\ & - n_{h_D} \left[\langle \Gamma \rangle_{h_D \rightarrow W^p W^m}(T_h) + \sum_{f \in \text{SM}} \langle \Gamma \rangle_{h_D \rightarrow f\bar{f}}(T_h) \right]. \end{aligned}$$

Depletion of
DM within the
hidden sector

Depletion of
 h_D within/to the
hidden/SM sector

Coupled Boltzmann Equations (3/3)

The source term J_{vh} (account for all possible processes that exchange energy between the SM and the hidden sector)

$$J_{vh} \simeq \sum_f \left\{ n_f^2 \left[2J_{f\bar{f} \rightarrow W^p W^m}(T) + J_{f\bar{f} \rightarrow h_D}(T) \right] \right. \\ \left. + 2n_Z J_{Z \rightarrow W^p W^m}(T) + 2n_h J_{h \rightarrow W^p W^m}(T) \right. \\ \left. + 2n_{\gamma^*} J_{\gamma^* \rightarrow W^p W^m}(T) - n_{h_D} J_{h_D \rightarrow f\bar{f}}(T_h) \right\} .$$

Plasmon decay

$$n_a n_b J_{ab \rightarrow cd} = \frac{g_a g_b T}{128 \pi^4} \int_{s_0}^{\infty} ds \sigma_{cd \rightarrow ab}(s) s(s - s_0) K_2(\sqrt{s}/T) ,$$

$$n_f^2 J_{f\bar{f} \rightarrow h_D} = \frac{T}{32 \pi^4} \int_{s_0}^{\infty} ds \sigma_{f\bar{f} \rightarrow h_D}(s) s(s - s_0) K_2(\sqrt{s}/T) ,$$

$$n_a J_{a \rightarrow bc} = n_a m_a \Gamma_{a \rightarrow bc} ,$$

$$n_{\gamma^*} J_{\gamma^* \rightarrow W^p W^m} = n_{\gamma_\ell^*} m_\ell \langle \Gamma \rangle_{\gamma_\ell^* \rightarrow W^p W^m} + n_{\gamma_t^*} m_t \langle \Gamma \rangle_{\gamma_t^* \rightarrow W^p W^m} ,$$

$$V = \mathcal{O} \cdot E (1/2)$$

- $V = (B, W_L^3, W_D^3)^T$, $E = (A, Z, Z')^T$, $Z' \equiv A'$
- Orthogonal transformation $\mathcal{O}$

$$\mathcal{O} = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_2 & -\sin \theta_2 \\ 0 & \sin \theta_2 & \cos \theta_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \cos \theta_2 & \sin \theta_1 \sin \theta_2 \\ \sin \theta_1 & \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 \\ 0 & \sin \theta_2 & \cos \theta_2 \end{pmatrix} ,$$

$$s_1 \equiv \sin \theta_1 = \frac{\tilde{g}'}{g_M} , \quad c_1 \equiv \cos \theta_1 = \frac{g}{g_M} , \quad g_M = \sqrt{g^2 + \tilde{g}'^2} , \quad \bar{g}_M = \sqrt{g^2 + \bar{g}'^2}$$

$$s_2 \equiv \sin \theta_2 = -\frac{\tilde{g}''}{\bar{g}_M} , \quad c_2 \equiv \cos \theta_2 = \frac{g_M}{\bar{g}_M} . \quad \bar{g}'^2 = \tilde{g}'^2 + \tilde{g}''^2 = \frac{g'^2}{1 - \epsilon^2}$$

$\bar{g}'$, $\bar{g}_M$ no α dependence!

- Neutral current interaction

$$\mathcal{L}_{\text{NC}} = \sum_i \sum_f \bar{f} \gamma_\mu \left(v_i^f - \gamma_5 a_i^f \right) f E_i^\mu , \quad v_i^f = (g \mathcal{O}_{2i} - \tilde{g}' \mathcal{O}_{1i} - \tilde{g}'' \mathcal{O}_{3i}) \frac{T_f^3}{2} + (\tilde{g}' \mathcal{O}_{1i} + \tilde{g}'' \mathcal{O}_{3i}) Q_f ,$$

$$a_i^f = (g \mathcal{O}_{2i} - \tilde{g}' \mathcal{O}_{1i} - \tilde{g}'' \mathcal{O}_{3i}) \frac{T_f^3}{2} ,$$

$$V = \mathcal{O} \cdot E (2/2)$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}'^2} .$$

$$\begin{aligned} \Rightarrow g_{\text{SM}} &= g , g'_{\text{SM}} = \tilde{g}' \\ \Rightarrow \cos \theta_1 &= \frac{g_{\text{SM}}}{\sqrt{g_{\text{SM}}^2 + g_{\text{SM}}'^2}} \end{aligned}$$

- One can take $\alpha_{\text{EM}} = e^2/4\pi$, G_F , m_W at the Z-pole as inputs; and θ_1 as weak mixing angle.
- Z boson mass $m_Z^2 = \frac{1}{4}v^2 [g_{\text{SM}}^2 + g_{\text{SM}}'^2/(1 - \epsilon^2)]$. Precision LEP data for Z mass implies $\epsilon < 6.4 \times 10^{-3}$!
 ϵ measures the degree of violation of custodial symmetry.

- γ , Z and Z' ve

$$v_A^f \equiv v_1^f = \tilde{g}' c_1 Q_f = g s_1 Q_f ,$$

$$a_A^f \equiv a_1^f = 0 .$$

$$v_Z^f \equiv v_2^f = \frac{g}{c_1 c_2} \left[\frac{T_f^3}{2} - (s_1^2 c_2^2 + s_2^2) Q_f \right] = \bar{g}_M \left(\frac{T_f^3}{2} - \frac{\bar{g}'^2}{\bar{g}_M^2} Q_f \right) ,$$

$$a_Z^f \equiv a_2^f = \left(\frac{g}{c_1 c_2} \right) \frac{T_f^3}{2} = \bar{g}_M \frac{T_f^3}{2} ,$$

$$v_{Z'}^f \equiv v_3^f = \tilde{g}'' c_1^2 c_2 Q_f = \frac{\tilde{g}'' g^2}{g_M \bar{g}_M} Q_f ,$$

$$a_{Z'}^f \equiv a_3^f = 0 .$$

While v_Z^f and a_Z^f do not have α -dependence, v_A^f and $v_{Z'}^f$ do through $\tilde{g}'$ and $\tilde{g}''$!! However this α -dependence cancel in physical processes!

Millicharged basis: $g s_1 = e$
 $\tilde{g}'' = 0$

Gauge Bosons Self-Interactions (3 points)

$$\begin{aligned} \mathcal{L}_{3g} = & ig\mathcal{O}_{2i} \left[(W_{\mu\nu}^+ W^{-\mu} - W_{\mu\nu}^- W^{+\mu}) E_i^\nu + \frac{1}{2} (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) E_i^{\mu\nu} \right] \\ & + ig_D \left[\kappa_i (W_{\mu\nu}^p W^{m\mu} - W_{\mu\nu}^m W^{p\mu}) E_i^\nu + \frac{1}{2} \kappa'_i (W_\mu^p W_\nu^m - W_\mu^m W_\nu^p) E_i^{\mu\nu} \right], \end{aligned}$$

where

$$\begin{aligned} \kappa_i &= \frac{1}{\sqrt{1-\epsilon^2}} (-s_\alpha \mathcal{O}_{1i} + c_\alpha \mathcal{O}_{3i}) \ , \\ \kappa'_i &= \left(\epsilon c_\alpha - \sqrt{1-\epsilon^2} s_\alpha \right) \mathcal{O}_{1i} + \left(\epsilon s_\alpha + \sqrt{1-\epsilon^2} c_\alpha \right) \mathcal{O}_{3i} \ . \end{aligned}$$

Millicharged basis: All κ_i, κ'_i are independent of α !

$\gamma W^p W^m :$	$\kappa_1 = -\frac{\epsilon c_1}{\sqrt{1-\epsilon^2}}$	$\kappa'_1 = 0$	$\implies W^{(p,m)}$ carries millicharged!
$Z W^p W^m :$	$\implies \kappa_2 = +\frac{\epsilon s_1}{\sqrt{1-\epsilon^2}}$	$\kappa'_2 = 0$	
$Z' W^p W^m :$	$\kappa_3 = 1$	$\kappa'_3 = 1$	

Caveat:
 $\kappa'_i \neq \kappa_i$ is due to Abelian—non-Abelian kinetic mixing term where the non-Abelian field strength $W_D^{3\mu\nu}$ gives rise to additional contributions to κ'_i ! This might spoil the renormalizability of the model. But we will take the EFT view in this model.

Gauge Bosons Self-Interactions (4 points)

$$\begin{aligned}
 \mathcal{L}_{4g} = & -\frac{1}{2}g^2 \left[(W_\mu^+ W^{-\mu})^2 - W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu} \right. \\
 & \left. + 2 \mathcal{O}_{2i} \mathcal{O}_{2j} (W_\mu^+ W^{-\mu} E_{i\nu} E_j^\nu - E_{i\mu} W^{+\mu} E_{j\nu} W^{-\nu}) \right] \\
 & -\frac{1}{2}g_D^2 \left[(W_\mu^p W^{m\mu})^2 - W_\mu^p W^{p\mu} W_\nu^m W^{m\nu} \right. \\
 & \left. + 2 \kappa_i \kappa_j (W_\mu^p W^{m\mu} E_{i\nu} E_j^\nu - E_{i\mu} W^{p\mu} E_{j\nu} W^{m\nu}) \right] .
 \end{aligned}$$

- Note that the ZZW^+W^- and ZZW^pW^m couplings are independent of α !