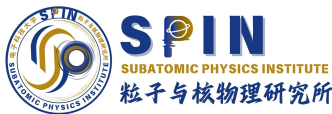


Glueball Dark Matter: From Thermal Resummation to Gravitational Waves and Direct Detection

Zhi-Wei Wang 王志伟

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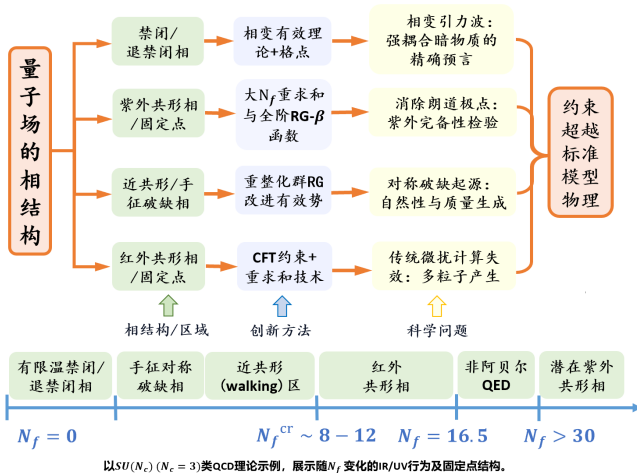
Oct.31th, 2026



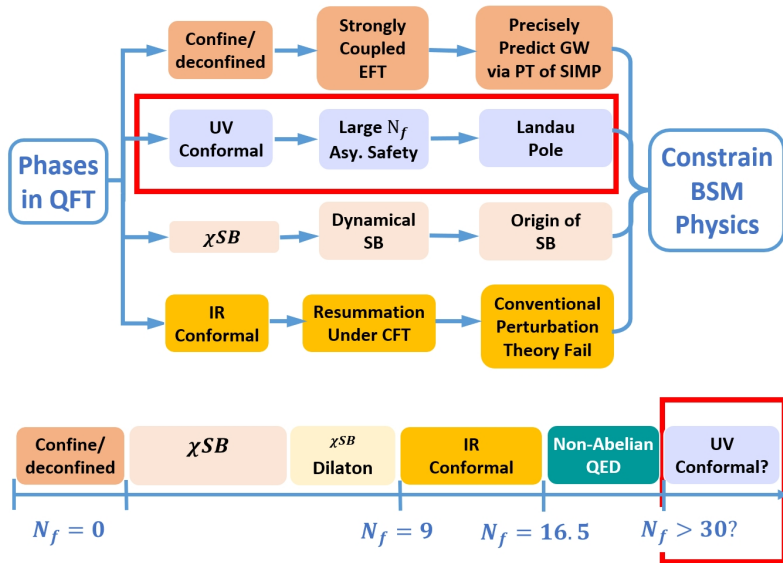
The 7th Joint Workshop on the Standard Model and Beyond
Oct.28th-Nov.1st, 2026, NanJing

A Landscape of Phases in QFT (QCD-like Theory) and its Relation to BSM Physics

“探索量子色动力学的不同相，是物理的前沿领域之一”
——李政道



UV Conformal Phase: Asymptotic Safety



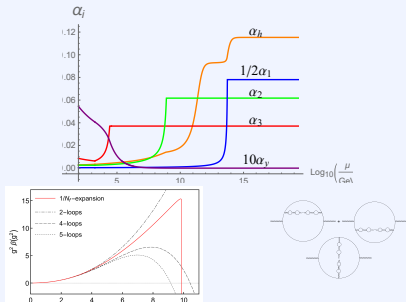
UV phase: Asymptotic Safety

Mann, Meffe, Sannino, Steele, Z-W W and Zhang, Phys. Rev. Lett. 119 (2017) 261802

Antipin, Dondi, Sannino, Thomsen, Z-W W, Phys. Rev. D 98 (2018) 016003

Fang, Z-W W, Zhou, JHEP 08 (2025) 042

Running, resummation and higher-order structure



Safe-SM running; resummed $g^2 \beta(g^2)$ versus finite-loop truncations; bubble-chain topologies in the large- N_f expansion.

Take-home

Asymptotic safety turns UV completion into predictive IR boundary conditions.

Safe SM and Safe GUTs

- **Safe SM:** vector-like fermions can render $\alpha_{1,2,3}$ and λ_H asymptotically safe; y_t is UV free.
- **Large- N_f RG:** bubble resummation extends to semi-simple gauge groups, Yukawa couplings and scalar quartics.
- **GUT embedding:** Pati-Salam and Trinification embed $U(1)_Y$ into non-Abelian UV-safe dynamics.
- **Predictivity:** UV fixed points act as boundary conditions for Higgs/top matching and BSM model building.

Large- N_f mechanism

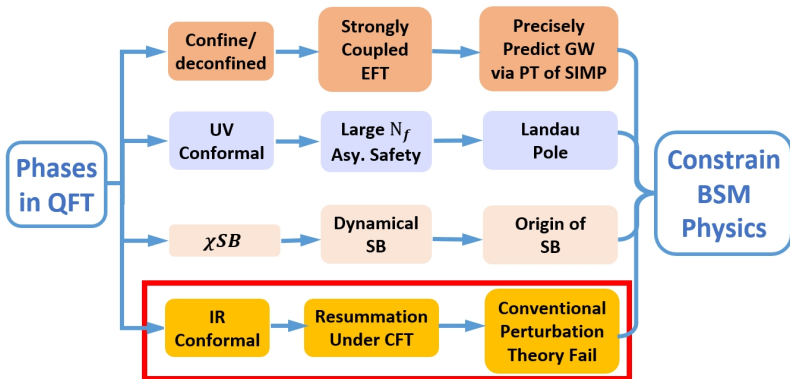
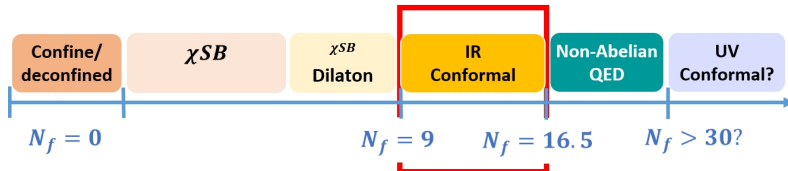
$$K_i \equiv \frac{\alpha_i}{\pi} N_{F_i} T_i, \quad T_i \equiv T(R_i),$$

$$D_{\text{dress}} = D_0 \sum_{n \geq 0} (\Pi_f D_0)^n, \quad (\Pi_f D_0)^n \sim K_i^n = O(N_f^0),$$

$$\beta_{K_i} = \frac{2}{3} K_i^2 \left[1 + \frac{1}{N_{F_i}} H_i(K_i) + \dots \right], \quad H_i \rightarrow -\infty \Rightarrow K_i^* < K_c.$$

Infinite bubble chains are same order in $1/N_f$; the fixed point is a resummed effect.

QCD Conformal Window

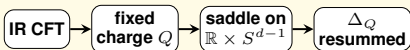


IR phase: large-charge resummation

Antipin, Bersini, Sannino, Z-W W and Zhang, Phys. Rev. D 102 (2020) 045011

Antipin, Bersini, Panopoulos, Sannino, Z-W W, JHEP 02 (2024) 168

**Fixed-charge sector Q ,
large Q double scaling with $A = \lambda Q$**



$$G_Q(\tau) \equiv \langle O_Q^\dagger(\tau) O_Q(0) \rangle_{\text{cyl}} \sim e^{-\Delta_Q \tau / R},$$

$$\Delta_Q = Q F_{-1}(A) + F_0(A) + O(Q^{-1}), \quad A = \lambda_* Q.$$

The fixed point is the anchor: once Δ_Q is obtained at $g_i = g_i^*$, one can trade g_i^* for $g_i(\mu)$, giving the coupling-dependent scaling/anomalous dimension in a general QFT.

$$\sum_{\ell} \lambda^\ell P_\ell(Q) \implies \sum_{k \geq -1} Q^{-k} F_k(\lambda Q)$$

Large-parameter analogy

- large N_f : fixed $g^2 N_f \Rightarrow$ resummed gauge β functions.
- large Q : fixed $\lambda Q \Rightarrow$ resummed CFT dimensions Δ_Q .

Charged scalar sectors and the SM Higgs

- $O(N)$ and $U(N) \times U(M)$: LO/NLO in charge, all orders in λQ , with charge-configuration recipe.
- SM Higgs: charged-sector dimensions for $H^{I_1} \dots H^{I_Q}$; $Q = 1$ matches known three-loop terms.
- Long goal: toward the “Holy Grail” function for multi-Higgs production.

$$\Delta_Q = Q + \left\{ \frac{1}{3} \lambda Q^2 + \left[N_f^u + N_f^d + \mathcal{Y}_l - \frac{3}{4} g^2 - \frac{\lambda}{3} \right] Q \right\} - \left\{ \frac{2}{9} \lambda^2 Q^3 - \left[2N_f^u + 2N_f^d + 2\mathcal{Y}_l \right] \right. \\ \left. - \frac{2}{3} \lambda (N_f^u + N_f^d + \mathcal{Y}_l) - \frac{1}{3} \lambda g^2 + \frac{g^4}{16} + \frac{\lambda^2}{9} \right\} Q^2 + C_{22} Q + \left\{ \frac{8}{27} \lambda^3 Q^4 + \left[\frac{1}{16} g^6 (9\zeta(3) - 1) \right. \right. \\ \left. \left. - \frac{1}{6} g^4 \lambda (1 + 3\zeta(3)) + \frac{1}{3} g^2 \lambda^2 (3 - 2\zeta(3)) + \frac{4}{27} \lambda^3 (9\zeta(3) - 8) + \frac{4}{27} (3N_f^u (\lambda^2 \mathcal{Y}_u - 3\lambda \mathcal{Y}_{uu}) \right. \right. \\ \left. \left. + 9\zeta(3) (\lambda \mathcal{Y}_{uv} - 2\mathcal{Y}_{uvu})) + 3N_f^d (\lambda^2 \mathcal{Y}_d - 3\lambda \mathcal{Y}_{dd}) + 9\zeta(3) (\lambda \mathcal{Y}_{dv} - 2\mathcal{Y}_{dvd}) \right] \right\} Q^3 + C_{23} Q^2 + C_{33} Q \left\} + O(\kappa_I^2 Q^2). \quad (7.1)$$

where red, blue, and orange colors highlight the terms stemming from the small $\kappa_I Q$ expansion of Δ_{-1} , Δ_0^{fm} , and Δ_0^{bos} .

IR phase: Banks-Zaks or Coleman-Weinberg

Hansen, Janowski, Langæble, Mann, Sannino, Steele, Z-W W, Phys. Rev. D 97 (2018) 065014.

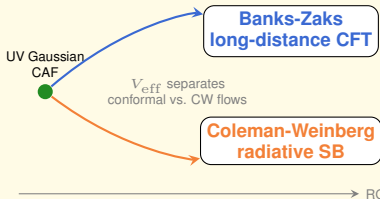
CAF $SU(N_c)$ model

$$\mathcal{L} = -\frac{1}{2} \text{Tr} F_{\mu\nu}^2 + \overline{Q} \not{D} Q + \text{Tr}(D_\mu S^\dagger D^\mu S) - v(\text{Tr} S^\dagger S)^2 - u \text{Tr}(S^\dagger S)^2.$$

$SU(N_c) + N_f$ quarks + N_s scalar quarks

UV: $\alpha, \lambda_1, \lambda_2 \rightarrow 0$, $\lambda_i/\alpha = \text{fixed flow}$.

One UV boundary, two IR fates



UV free $\rightarrow$ IR structure

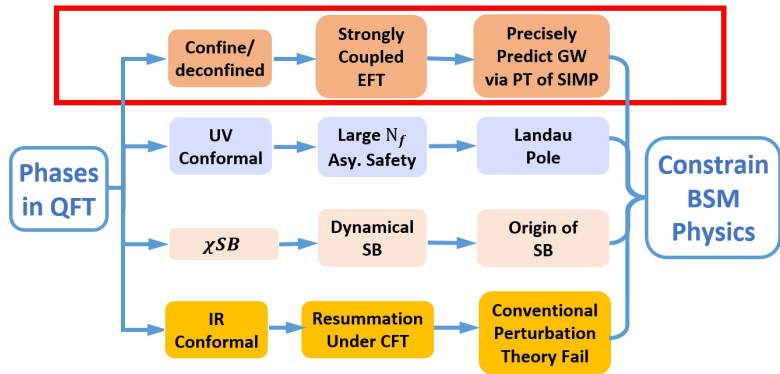
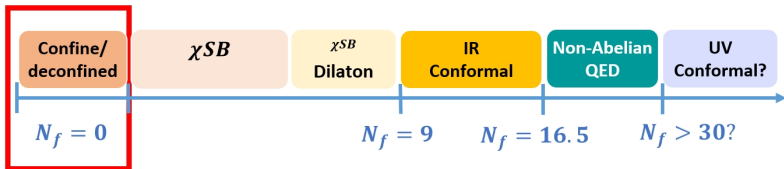
- Perturbative phase diagram of completely asymptotically free gauge theories with fermions and scalar quarks.
- CAF fixes the UV Gaussian boundary and allowed fixed-flow trajectories for $\alpha, \lambda_1, \lambda_2$.
- Gauge Banks-Zaks fixed point plus scalar fixed points give long-distance conformality.
- Quantum-corrected Coleman-Weinberg/Gildener-Weinberg potential separates conformal flows from radiative symmetry breaking.

$$\beta_\alpha = -B\alpha^2 + C\alpha^3 + \dots, \quad \alpha_*^{\text{BZ}} = B/C.$$

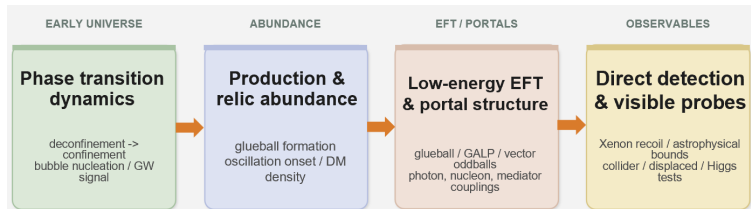
UV free $\rightarrow$ IR BZ CFT

or IR CW breaking

Confine/Deconfined Phase



Outline: From Phase Transition Dynamics to Detection

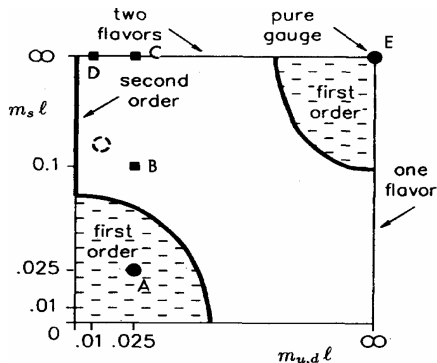


Dark confinement sets the transition dynamics, the relic abundance and the portal structure — and every step runs through one object: the finite-temperature effective potential.

Part II asks how far that object can be trusted, *before* Parts III–VI use it.

What composes the strongly coupled sector?

- Dark Yang-Mills theories
- Pure gluons $\Rightarrow$ confinement-deconfinement phase transition
- Gluons + Fermions
 - Fermions in fundamental representation $\Rightarrow$ chiral phase transition
 - Fermions in adjoint rep. $\Rightarrow$ confinement & chiral phase transition
 - Fermions in 2-index symmetric rep. $\Rightarrow$ confinement & chiral phase transition
- Gluons + Fermions + Scalars (not explored yet)



T.D. Lee / Nuclear Physics A590 (1995) 11c-28c

How to describe the strongly coupled sector?

● Pure gluons

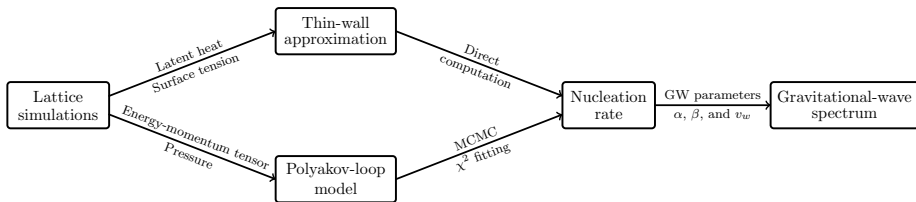
- **Polyakov loop model** (Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005; Kang, Zhu, Matsuzaki, JHEP 09 (2021) 060; Gao, Sun and White, arXiv:2405.00490.)
- **Matrix Model** (Halverson, Long, Maiti, Nelson, Salinas, JHEP **05** (2021) 154)
- **Holographic QCD model** (Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRD **105** (2022) 066020; Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRL **128** (2022) 131101)

● Gluons + Fermions

- **Polyakov loop improved Nambu-Jona-Lasinio model** (Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003; Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
- **Linear sigma model** (Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
- **Polyakov Quark Meson model** (Pasechnik, Reichert, Sannino, Z-W W, JHEP **02** (2024) 159)

Procedure of pure gluon case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)



Part I: Building the Effective Potential

Polyakov loop model $\rightarrow$ lattice input $\rightarrow$ fermions $\rightarrow$ beyond mean
field

Polyakov Loop Model for Pure Gluons: I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory (Pisarski, PRD **62** (2000) 111501).
- In a local $SU(N)$ gauge theory, a **global center symmetry $Z(N)$** is used to distinguish confinement phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the Polyakov Loop (thermal Wilson line) (Polyakov, PLB **72** (1978) 477)

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) d\tau \right]$$

The symbol $\mathcal{P}$ denotes path ordering and A_4 is the Euclidean temporal component of the gauge field

- The Polyakov Loop transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for Pure Gluons: II

- Convenient to define the trace of the **Polyakov loop as an order parameter** for the $Z(N)$ symmetry

$$\ell(\vec{x}) = \frac{1}{N} \text{Tr}_c[\mathbf{L}],$$

where Tr_c denotes the trace in the colour space.

- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell \rightarrow e^{i\phi} \ell, \quad \phi = \frac{2\pi j}{N}, \quad j = 0, 1, \dots, (N-1)$$

- The expectation value of ℓ i.e. $\langle \ell \rangle$ has the **important property**:

$$\langle \ell \rangle = 0 \quad (T < T_c, \text{ Confined}); \quad \langle \ell \rangle > 0 \quad (T > T_c, \text{ Deconfined})$$

- At very high temperature, the vacua exhibit a N -fold degeneracy:

$$\langle \ell \rangle = \exp\left(i \frac{2\pi j}{N}\right) \ell_0, \quad j = 0, 1, \dots, (N-1)$$

where ℓ_0 is defined to be real and $\ell_0 \rightarrow 1$ as $T \rightarrow \infty$

Polyakov loop $\Rightarrow$ static-quark free energy?

Physical picture

At finite temperature, Euclidean time is not an infinite line, but a circle of length $\beta = \frac{1}{T}$. Now insert one **infinitely heavy, static** fundamental quark. Since it is static, it does not move in space. It only propagates from $\tau = 0$ to $\tau = \beta$, namely, it wraps once around the thermal circle. The color phase accumulated along this worldline is precisely the **Polyakov loop**.

Translation into free energy

Define the free-energy cost of inserting one static quark by

$$F_Q^{\text{bare}} \equiv -T \ln \frac{Z_Q}{Z}.$$

After removing the trivial heavy-mass term M and the UV self-energy of the static color source, one obtains the **renormalized** relation

$$F_Q = -T \ln \langle \ell \rangle_{\text{ren}}, \quad \boxed{\langle \ell \rangle_{\text{ren}} = e^{-F_Q/T}}.$$

One-sentence summary: The Polyakov loop is the amplitude for a static quark to go once around the thermal circle, and in the thermal ensemble this amplitude is exactly the Boltzmann factor associated with the free-energy cost of inserting that source.

Strictly speaking, the bare Polyakov loop contains the divergent self-energy of the static color source, so the final physical relation should be written in terms of the renormalized quantity $\langle \ell \rangle_{\text{ren}}$.

Summary of Pure Gluon Facts

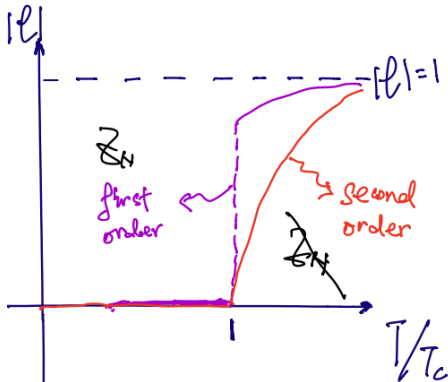
Temperature

Free Gluon
 Z_4 is broken

At $T_{\text{confinement}}$

Confinement
 Glue ball

Z_4 is restored



Second Order for $SU(2)$
 First order $SU(N)$ ($N \geq 3$)

Effective Potential of the Polyakov Loop Model: I

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by (Pisarski, PRD **62** (2000) 111501)

$$V_{\text{PLM}}^{(\text{poly})} = T^4 \left(-\frac{b_2(T)}{2} |\ell|^2 + b_4 |\ell|^4 + \dots - b_3 (\ell^N + \ell^{*N}) \right)$$

$$\text{where } b_2(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3 + a_4 \left(\frac{T_0}{T} \right)^4$$

"..." represent any required lower dimension operator than ℓ^N
i.e. $(\ell \ell^*)^k = |\ell|^{2k}$ with $2k < N$.

- For the $SU(3)$ case, there is also an alternative logarithmic form

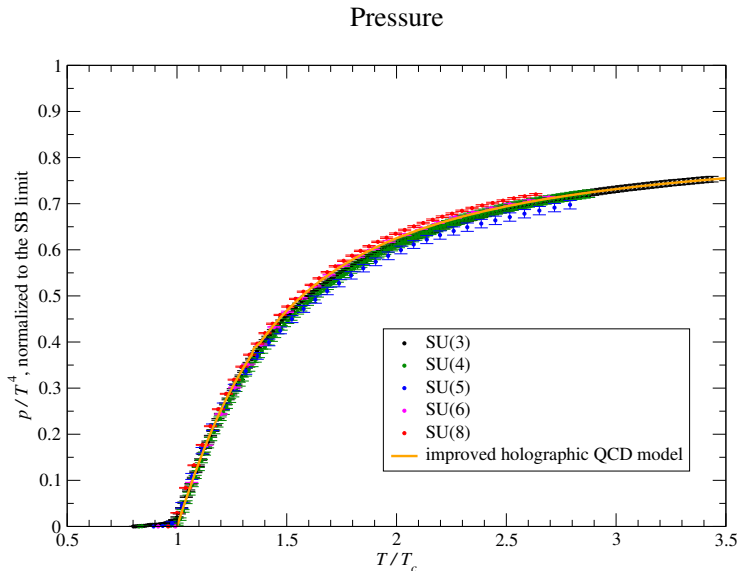
$$V_{\text{PLM}}^{(3\log)} = T^4 \left(-\frac{a(T)}{2} |\ell|^2 + b(T) \ln(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4) \right)$$

$$a(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3, \quad b(T) = b_3 \left(\frac{T_0}{T} \right)^3$$

- The a_i , b_i coefficients in $V_{\text{PLM}}^{(\text{poly})}$ and $V_{\text{PLM}}^{(3\log)}$ are determined by fitting the lattice results

Fitting the Coefficients Using the Lattice Results: I

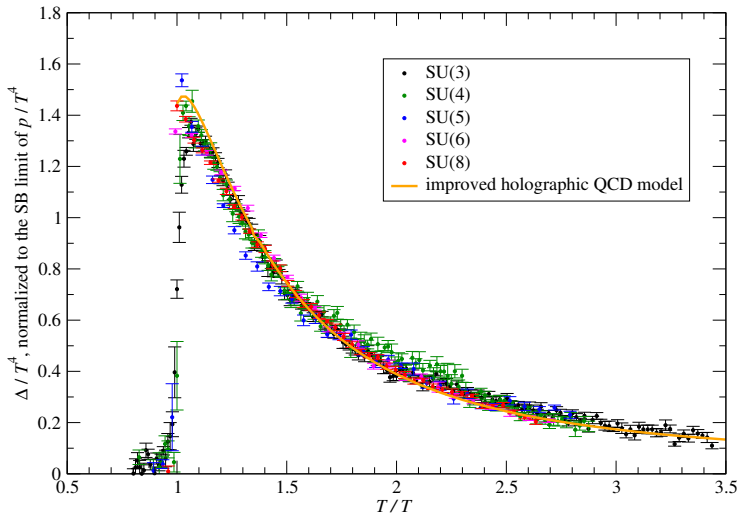
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: II

Marco Panero, Phys.Rev.Lett. 103 (2009) 232001

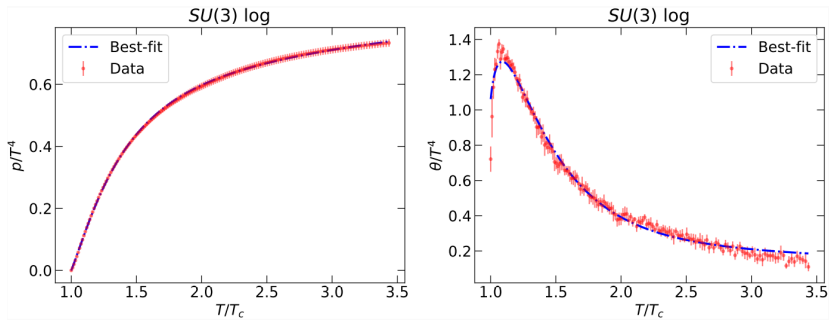
Trace of the energy-momentum tensor



Fitting the Coefficients Using the Lattice Results: III

(Huang, Reichert, Sannino and Z-WW, PRD **104** (2021) 035005)

Fitted to lattice data of pressure and the trace of energy momentum tensor.



Include Fermions: the PNJL Model and PQM Model

K. Fukushima, PLB **591** (2004) 277; Ratti, Thaler Weise, PRD **73** (2006) 014019

B. Schaefer, J. Pawłowski, J. Wambach PRD **76** (2007) 074023; B. Schaefer, M. Wagner, PNP **62** (2009) 391
Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.

Pasechnik, Reichert, Sannino and Z-W W, JHEP **02** (2024) 159.

- The Polyakov-loop-Nambu-Jona-Lasinio (PNJL) model is used to describe phase-transition dynamics in dark gauge-fermion sectors
- The **finite-temperature grand potential** of the PNJL models can be generically written as

$$V_{\text{PNJL}} = V_{\text{PLM}}[\ell, \ell^*] + V_{\text{cond}}[\langle \bar{\psi}\psi \rangle] + V_{\text{zero}}[\langle \bar{\psi}\psi \rangle] + V_{\text{medium}}[\langle \bar{\psi}\psi \rangle, \ell, \ell^*]$$

- The Polyakov quark meson model (PQM) is widely used as an effective theory to study the first order chiral phase transition
- The Lagrangian of the PQM where mesons couple to a spatially constant temporal background gauge field reads

$$\mathcal{L} = \bar{q} (i \not{D} - g (\sigma + i \gamma_5 T^a \pi_a)) q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \pi_a)^2 \\ - V_{\text{PLM}}^{(\text{poly})} + V_{\text{LSM}} + V_{\text{medium}}, \text{ where } \not{D} = \gamma_\mu \partial_\mu - i \gamma_0 A_0$$

- The finite temperature contributions to the potential of linear sigma model are typically via CJT resummation.

Beyond Mean Field: the CJT / 2PI Effective Potential

(J. Cornwall, R. Jackiw, E. Tomboulis, PRD 10 (1974) 2428; G. Amelino-Camelia, PRD 47 (1993) 2356;

Pasechnik, Reichert, Sannino and Z-W W, JHEP 02 (2024) 159.)

- Generalised effective action $\Gamma(\phi, G)$: a functional of the background ϕ and the full propagator G — the generating functional of **two-particle-irreducible (2PI)** vacuum graphs
- Equivalent to summing the infinite class of **daisy and super-daisy** graphs $\Rightarrow$ strongly coupled models beyond mean field; PQM + CJT can **bridge perturbative and non-perturbative regimes**

- Finite-temperature effective potential:

$$V_{\text{CJT}}(\phi, G) = V_0(\phi) + \frac{1}{2} \sum_i \int_{\beta} \ln G_i^{-1}(\phi; k) + \frac{1}{2} \sum_i \int_{\beta} [D^{-1}(\phi; k)G(\phi; k) - 1] + V_2(\phi, G)$$

- Hartree: $V_2 =$ one “double bubble”, $V_2 \propto [\int_{\beta} G]^2$. Stationarity $\delta V_{\text{CJT}}/\delta G_i = 0$ is the gap equation:

$$\frac{1}{2} G_i^{-1}(\phi; k) = \frac{1}{2} D_i^{-1}(\phi; k) + 2 \frac{\delta V_2(\phi, G)}{\delta G_i(\phi; k)}$$

Self-consistent dressing: the propagator comes from a gap equation, the vertices stay at tree level. **How much does the answer depend on how we dress?** In our strongly coupled theory, the uncertainty of the self-consistent truncation is difficult to quantify. Part II therefore turns to a scalar-model laboratory, where we can hold the zero-temperature inputs fixed and isolate the sensitivity to thermal resummation.

CJT Gap Equation: Explicit PQM Spectrum

(Pasechnik, Reichert, Sannino and Z-W W, arXiv:2309.16755.)

- Using the gapped equation, the thermal mass is given by ($R_i \equiv M_i/T$):

$$\begin{aligned}
 M_\sigma^2 = m_\sigma^2 + \frac{T^2}{4\pi^2} & \left[\left(3\lambda_\sigma - \delta_{4,N_f} \frac{3}{2}c \right) I_B(R_\sigma) \right. \\
 & + \left((N_f^2 - 1)(\lambda_\sigma + 2\lambda_a) + \delta_{4,N_f} \frac{15}{2}c \right) I_B(R_a) \\
 & \left. + \left(\lambda_\sigma + \delta_{4,N_f} \frac{3}{2}c \right) I_B(R_\eta) + \left((N_f^2 - 1)\lambda_\sigma - \delta_{4,N_f} \frac{15}{2}c \right) I_B(R_\pi) \right],
 \end{aligned}$$

- CJT improved finite temperature effective potential:

$$\begin{aligned}
 V_{\text{FT}}^{\text{LSM}}(\sigma) &= \frac{T^4}{2\pi^2} \sum_i \left[J_B(R_i^2) - \frac{1}{4} (R_i^2 - r_i^2) I_B(R_i^2) \right], \\
 I_B(R^2) &= 2 \frac{dJ_B(R^2)}{dR^2} = \int_0^\infty dx \frac{x^2}{\sqrt{x^2 + R^2}} \frac{1}{e^{\sqrt{x^2 + R^2}} - 1}, \\
 J_B(R^2) &= \int_0^\infty dx x^2 \ln \left(1 - e^{-\sqrt{x^2 + R^2}} \right).
 \end{aligned}$$

The $-\frac{1}{4}(R_i^2 - r_i^2)I_B$ term is the 2PI subtraction of double-counted daisies — exactly what “full dressing” (Part II) forgets.

Part II: Calibrating the Effective Potential

We have just built V_{eff} with a self-consistent dressing.

Before computing anything with it: **how much does the answer depend on *how* we dressed it?**

In a strongly coupled sector the prescription cannot be varied
— so we go to a laboratory where it can.

(Dolan–Jackiw, PRD **9** (1974) 3320; Carrington, PRD **45** (1992) 2933; Arnold–Espinosa, PRD **47** (1993) 3546)

1. One self-energy, two pieces. A soft ring with p petals is

every petal is the *same* tadpole $\Pi = \frac{\lambda}{2} I^{(1)}$ and costs

$$\delta D_1(k) = -D_0(k) \Pi D_0(k) \text{ — only the modes inside differ:}$$

2. Both blow up where the transition is. $m^2(\phi) = V_0''$ is smallest at the barrier (near-conformal: $\propto \phi^2 \rightarrow 0$); the zero mode's $-\frac{T}{12\pi}(m^2)^{3/2}$ — the cubic that *builds* it — turns imaginary once $m^2 < 0$.

$$\bigcirc + \textcircled{\bigcirc} + \textcircled{\bigcirc\bigcirc} + \textcircled{\bigcirc\bigcirc\bigcirc} + \cdots = \textcircled{\bigcirc\bigcirc\bigcirc\bigcirc} = J(m^2 + \Pi_0)$$

Zhi-Wei Wang 王志伟 (UESTC 电子科技大学)

26 / 88

Near-Conformal: When Resummation *Is* the Potential

The self-consistent mass always splits as

$$\mathcal{M}^2 = \underbrace{m^2(\phi)}_{\text{tree curvature}} + \Pi_{\text{th}}[\mathcal{M}^2]$$

- Away from conformality m^2 dominates: resummation is a *correction*.
- As $m^2 \rightarrow 0$, $\mathcal{M}^2 \rightarrow \Pi_{\text{th}}[\mathcal{M}^2]$ — the barrier, the strength and the supercooling are **generated entirely by the resummed self-energy**.

The prescription is then not a correction to the answer.
It *is* the answer.

The same condition, realised twice

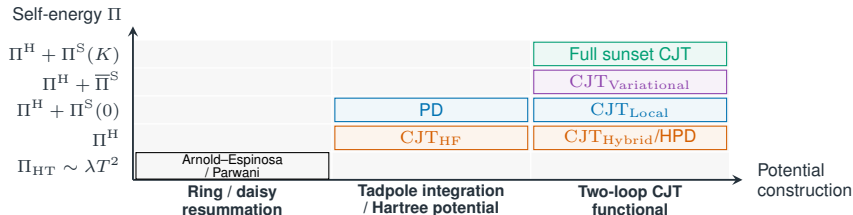
	Dark sector	2HDM
near-conf. arises via	$m_\sigma \rightarrow 0$ in the PQM model (<i>Part III</i>)	Gildener–Weinberg flat direction (<i>C2HDM</i>)
what we observe	strongest (<i>Part III</i>)	100% of points obey $\Omega^{\text{PD}} > \Omega^{\text{Par}} > \Omega^{\text{AE}}$
off conformality	generic N_f , explicit mass scales	no global ordering (58.6 / 34.0 / 7.3 %)

$\Rightarrow$ “clean hierarchy when conformal, no ordering when not”
will appear on *both* sides. It is a property of near-conformal dynamics — not of the 2HDM.

Thermal Resummation: Gap and Potential Construction

(Ji-Wei Li, ZW Wang, Yu-Bo Zhao, Consistent Multi-Field Thermal Resummation at Two Loop)

(a) Classification of thermal resummation prescriptions



(b) Diagram content of the resummation prescriptions

Scheme	Effective-potential diagrams	Tadpole diagrams	Self-energy diagrams
Arnold-Espinosa	+		
Parwani	+		
Hartree	+		
PD	$\int_{\gamma} d\phi_i$		+
HPD	+ +	+ + $\mathcal{R}_i^H$	
Local	+ +	+ + $\mathcal{R}_i^L$	+
Variational	+ +	+	+
Full sunset	+ +	+	+

— tree propagator

• zero mode

— Hartree / HPD

— PD / Local

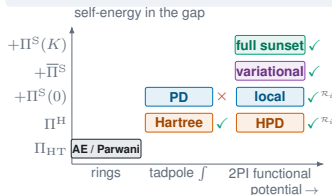
— Variational

— Full sunset

× nonzero modes

One functional derives the whole landscape

Its **variation** gives the gap, its **value** the potential, its **gradient** the tadpole. Each prescription evaluates it at a chosen propagator or removes identifiable terms, so what every scheme misses is *computed, not guessed*.



Functional $\Rightarrow$ integrable

Stationary $\Rightarrow$ no $\mathcal{R}_i$

Schemes evaluated from the functional are path independent ($\checkmark$); PD, whose tadpole has a curl, is not ($\times$). At a stationary propagator (full sunset, variational, Hartree), tree + loop + lollipop is the exact gradient; otherwise $\mathcal{R}_i$ is added (HPD, local). Variational CJT = PD + exact ζ + lollipop.

In multi-field PD, path dependence is a curl, and the curl has one origin.

PD differentiates before it dresses, so $d\tau = -\frac{1}{2} d\mathcal{I} \wedge d\Pi$. It vanishes for Hartree, making HPD integrable with mixing, and survives for the zero-momentum sunset, making PD path dependent.

Momentum overlap is derived, not matched.

Holding the sunset bubble at $K = 0$ factorizes the two coupled loops of the sunset tadpole and overweights it. Varying the functional over constant mass matrices reweights the bubble by its two adjacent propagators, $\bar{\Pi}_{AB}^S = \int_K G_A \Pi_{AB}^S G_B / \int_K G_A G_B$, which determines ζ_{ABCD} in every channel, mixing included (one field: 2/3).

HPD is hybrid CJT.

Introduced as a tadpole integral, it is the full two-loop functional at the Hartree propagator, $V_{HPD} = V_{CJT}[\varphi, G_H]$: a closed form at every point, path independent with mixing, and free of the overlap problem by construction, since the sunset never enters the gap and is kept as a full two-loop vacuum graph.

N mixing scalars, general quartic potential · full thermal loop functions, no high- T expansion
Paper II: error budget and phase transitions

2PI/CJT and Partial Dressing: the *Same* Tadpole

(CJT, PRD **10** (1974) 2428; Pasechnik, Reichert, Sannino and Z-W W, JHEP **02** (2024) 159)

Dark sector — forced non-perturbative

CJT/2PI (Part I):

$$V(\phi, G) = V_0 + \frac{1}{2} \sum_i \int_\beta \left[\ln G_i^{-1} + D^{-1} G - 1 \right] + V_2$$

Hartree gap equation (V_2 = double bubble):

$$\frac{1}{2} G_i^{-1} = \frac{1}{2} D_i^{-1}(\phi) + 2 \frac{\delta V_2}{\delta G_i} \equiv k^2 + M_i^2(\phi, T)$$

Stationarity $\delta V / \delta G = 0 \Rightarrow$ the implicit

G -dependence drops out on differentiation:

$$\frac{dV_{\text{eff}}}{d\phi} = \frac{\partial V}{\partial \phi} \Big|_G = V'_0 + \frac{1}{2} \int_\beta G_M \partial_\phi D^{-1}(\phi)$$

and $\partial_\phi D^{-1} = \partial_\phi m^2$ is the **tree-level** vertex:

$$\frac{dV_{\text{eff}}}{d\phi} = V'_0(\phi) + J'(M^2) \partial_\phi m^2$$

Perturbative sector — free choice

Partial dressing — replace at the *tadpole* level:

$$\partial_a V_{\text{PD}} = \partial_a V_0 + \sum_A J'(\mathcal{M}_A^2) \partial_a m_A^2 \Big|_{u \rightarrow U}$$

Full dressing — substitute into the potential:

$$\partial_a V_{\text{FD}} = \partial_a V_0 + \sum_A J'(\mathcal{M}_A^2) \partial_a \mathcal{M}_A^2$$

$\Rightarrow$ the vertex is dressed too $\Rightarrow$ daisy / super-daisy miscounting beyond one loop.

Parwani: truncated full dressing, all modes,

$\bar{m}^2 = m^2 + \Pi(T)$. **Arnold–Espinosa**: zero-mode ring only — counts correctly at $O(g^3)$ but dresses nothing else. Both are what the GW literature uses.

Partial Dressing versus Full Dressing

PD: dress the propagator, keep the tree-level vertex $\partial_a m_A^2$.

FD: substitute $m^2 \rightarrow \mathcal{M}^2(\phi, T)$ in V_1 ; then differentiation also acts on the self-energy: $\partial_a \mathcal{M}_A^2 = \partial_a m_A^2 + \partial_a \Pi_A$.

Dress the tadpole, then integrate: avoid the daisy/super-daisy overcounting of naive full dressing.

Multi-Field Implementation of Partial Dressing

The obstacle is *field-dependent scalar mixing*: the gap equation needs $\partial_a m_A^2$ and $\partial_a \partial_b m_A^2$, not just the eigenvalues.

Feynman–Hellmann gives the first derivative,

$$\partial_a m_A^2 = [u^T (\partial_a m^2) u]_{AA},$$

and the second follows exactly as second-order perturbation theory in QM,

$$\partial_a \partial_b m_A^2 = [u^T \partial_a \partial_b m^2 u]_{AA} + \sum_{\substack{B \neq A \\ m_B^2 \neq m_A^2}} \frac{2 \text{sym}[\mathcal{V}_{AB}^a \mathcal{V}_{BA}^b]}{m_A^2 - m_B^2},$$

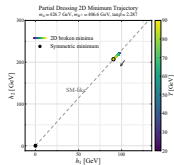
$$\mathcal{V}^a \equiv u^T (\partial_a m^2) u$$

(for degenerate eigenvalues the second term is set to zero).

Iteration. $\partial_a m^2$ and $\partial_a \partial_b m^2$ are computed **once and never updated**; only $(\mathcal{M}_A^2, U)$ are re-diagonalised until convergence.

Potential from the tadpole, straight-line contour $\gamma_a(t) = \phi_a t$:

$$V_{\text{PD}}(\phi, T) = V_0(\phi) + \int_0^1 dt \phi_a T_{\text{PD},a}(\phi t, T)$$



2D consistency check of the neutral PD effective potential.

This is what turns partial dressing from “possible in principle” into “runnable over a full 2HDM scan”.

The Controlled Laboratory: C2HDM and NC2HDM

Why leave the dark sector at all?

In a strongly coupled dark sector there is **no second prescription to compare against** — 2PI is what we have.

The *size* of the ambiguity simply cannot be measured there.

Why the 2HDM.

- Minimal scalar extension, yet it already has field-dependent mixing, genuine first-order transitions, and a complete set of zero-temperature constraints.
- C2HDM**: Gildener–Weinberg flat direction at tree level; a finite-counterterm prescription preserves the EW vacuum and the CP-odd / charged mass blocks.
- NC2HDM**: explicit mass scales already at tree level.

The controlled variable

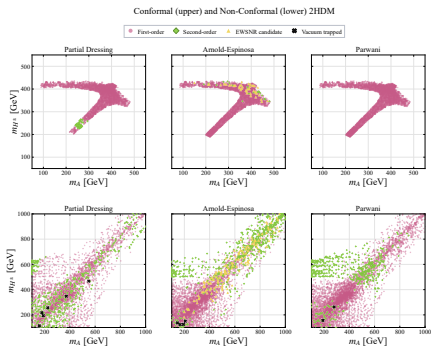
One sample of accepted zero-temperature points

PD | AE | Parwani

Any difference between panels comes from the **finite-temperature treatment alone**.

And the conformal member of this pair is the *perturbative counterpart* of the near-conformal dark dynamics we meet in Part III — so we test the same physical condition in a place where the prescription *can* be varied.

Result I: the Prescription Changes the *Thermal History*

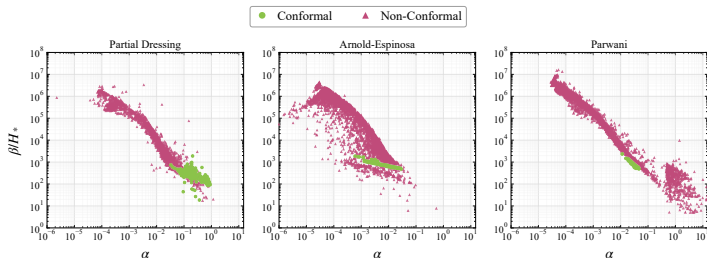


Conformal (upper). First-order dominance under *all three* schemes: the main region is **robust**. Scheme dependence lives only in the sub-dominant classes.

Non-conformal (lower). Second-order regions in PD/Parwani at light masses; AE gives a broad EWSNR band — traceable to its cubic daisy term (negative curvature $\propto \bar{m}T$, $\bar{m} \propto T$).

The ambiguity is not merely quantitative: the *same* zero-temperature point can be assigned a *different thermal history*.

Result II: Transition Strength and Supercooling



Completed transitions in the $(\alpha, \beta/H_*)$ plane; columns: PD, AE, Parwani. α = energy budget of the source, β/H_* = inverse duration; larger α and smaller β/H_* $\Rightarrow$ louder signal.

Conformal: a stable hierarchy

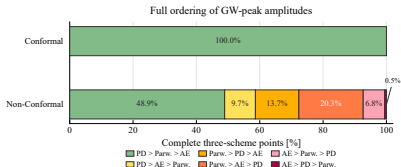
- PD extends to **larger α , smaller β/H_***
- supercooling reaches $T_p/T_c \simeq 0.50$ (sizeable, not extreme)
- ordering **PD > Parwani > AE**

Non-conformal: no global ordering

- PD and Parwani often comparable
- neither dominates universally
- largest α and strongest supercooling can come from *different* schemes in different regions

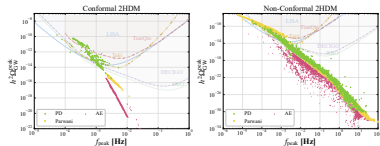
Companion projections ($m_A - m_{H^\pm}$ coloured by $\alpha(T_p)$; T_c versus T_p) are kept as skip-slides.

Result III: GW Spectra → Back to the Dark Sector



Conformal: 100% of common points obey $\Omega_{\text{peak}}^{\text{PD}} > \Omega_{\text{peak}}^{\text{Parw}} > \Omega_{\text{peak}}^{\text{AE}}$.

Non-conformal: PD largest in 58.6%, Parwani 34.0%, AE 7.3%.



$(f_{\text{peak}}, h^2 \Omega_{\text{peak}})$ for completed FOEWPTs. Detector curves are

order-of-magnitude visual references only, not discovery contours.

- 1 Thermal resummation is a **genuine theoretical uncertainty**: it moves the peak amplitude *and* the peak frequency, and can even reassign the thermal history.
- 2 Its impact is **largest near conformality**, because there the barrier is generated entirely by the resummed self-energy (two slides back). **Part III**: the strongest dark-sector signal lives exactly there.
- 3 The dark sector was already doing this right, out of necessity. Next: carry the multi-field PD machinery to $V_{\text{eff}}(\mathcal{H}, \mathcal{A})$ (Part V) and to **dark gauge theories with scalars** — the “not explored yet” entry of the opening slides.

Part III: Bubble Nucleation and Gravitational Waves

now with an error bar on the potential

Bubble Profile of Confinement Phase Transition

(Huang, Reichert, Sannino and Z-WW, PRD **104** (2021) 035005)

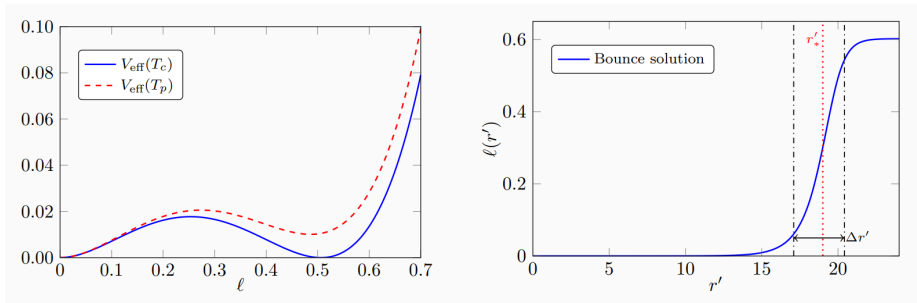


Figure: The bubble radius is indicated by r'_* and the wall width by $\Delta r'$. Inside of the bubble ($r' \ll r'_*$) lying the **confinement phase**, the Z_N symmetry is unbroken and $\langle \ell \rangle = 0$, while outside of the bubble ($r \gg r'_*$) lying the **deconfinement phase**, the Z_N symmetry is broken and $\langle \ell \rangle > 0$.

Gravitational-wave spectrum

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

- Contributions from bubble collision and turbulence are subleading
- The GW spectrum from sound waves is given by

$$h^2 \Omega_{\text{GW}}(f) = h^2 \Omega_{\text{GW}}^{\text{peak}} \left(\frac{f}{f_{\text{peak}}} \right)^3 \left[\frac{4}{7} + \frac{3}{7} \left(\frac{f}{f_{\text{peak}}} \right)^2 \right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\text{peak}} \simeq 1.9 \cdot 10^{-5} \text{ Hz} \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \left(\frac{T}{100 \text{ GeV}} \right) \left(\frac{\tilde{\beta}}{v_w} \right)$$

- The peak amplitude

$$h^2 \Omega_{\text{GW}}^{\text{peak}} \simeq 2.65 \cdot 10^{-6} \left(\frac{v_w}{\tilde{\beta}} \right) \left(\frac{\kappa_{sw} \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \Omega_{\text{dark}}^2$$

- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the non-participating SM d.o.f.

$$\Omega_{\text{dark}} = \frac{\rho_{\text{rad,dark}}}{\rho_{\text{rad,tot}}} = \frac{g_{*,\text{dark}}}{g_{*,\text{dark}} + g_{*,\text{SM}}}$$

The Efficiency Factor κ

- The efficiency factor for the sound waves κ_{SW} consist of κ_v as well as an additional suppression due to the length of the sound-wave period τ_{SW}

$$\kappa_{\text{SW}} = \sqrt{\tau_{\text{SW}}} \kappa_v$$

- τ_{SW} is dimensionless and measured in units of the Hubble time (H.-K. Guo, Sinha, Vagie and White, JCAP **01** (2021) 001)

$$\tau_{\text{SW}} = 1 - 1/\sqrt{1 + 2 \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta} \bar{U}_f}} \Rightarrow \tau_{\text{SW}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta} \bar{U}_f} \text{ for } \beta \gg 1$$

where $\bar{U}_f$ is the root-mean-square fluid velocity

$$\bar{U}_f^2 = \frac{3}{v_w(1+\alpha)} \int_{c_s}^{v_w} d\xi \xi^2 \frac{v(\xi)^2}{1-v(\xi)^2} \simeq \frac{3}{4} \frac{\alpha}{1+\alpha} \kappa_v$$

- τ_{SW} is suppressed for large β occurring often in strongly coupled sectors
- κ_v was numerically fitted to simulation results depends α and v_w . At the Chapman-Jouguet detonation velocity it reads

$$\kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98 + \alpha}}$$

GW Signatures for Arbitrary N in the Pure Gluon Case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

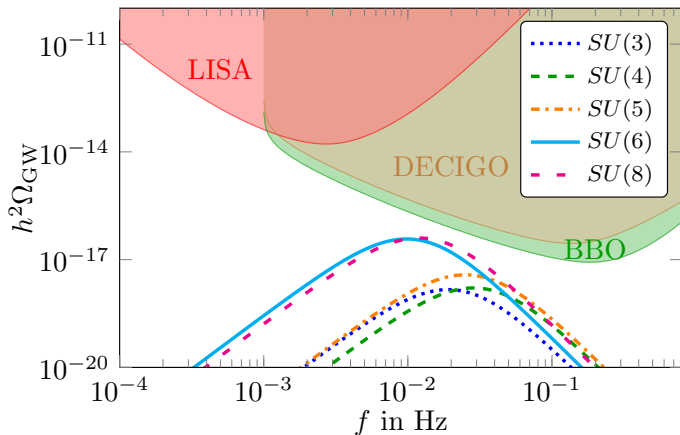


Figure: The dependence of the GW spectrum on the number of dark colours is shown for the values $N = 3, 4, 5, 6, 8$. All spectra are plotted with the bubble wall velocity set to the Chapman-Jouguet detonation velocity and with $T_c = 1$ GeV.

A Landscape of GW Signatures with Pure Gluon

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

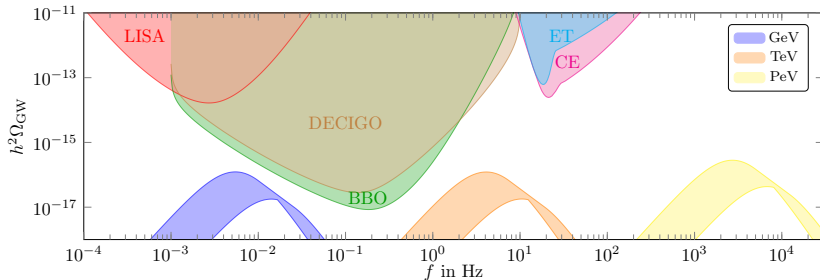


Figure: We display the GW spectrum of the $SU(6)$ phase transition for different confinement scales including $T_c = 1$ GeV, 1 TeV, and 1 PeV. We compare it to the power-law integrated sensitivity curves of LISA, BBO, DECIGO, CE, and ET.

Landscape of GW spectrum with three Dirac fermions

(Reichert, Sannino, Z-W W and Zhang, JHEP 01 (2022) 003, arXiv:2109.11552.)

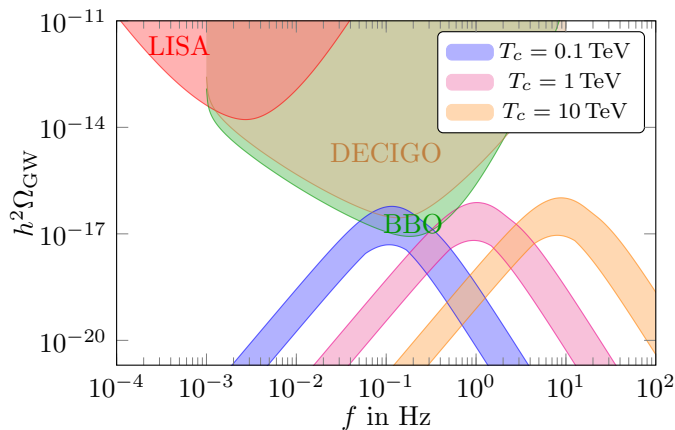


Figure: Gravitational-wave spectrum with three Dirac fermions in the fundamental representation for different critical temperatures. The band comes from varying wall velocity $c_s \leq v_w \leq 1$.

$\alpha - \beta$ Phase diagram via PQM Model

(Pasechnik, Reichert, Sannino and Z-W W, JHEP 02 (2024) 159.)

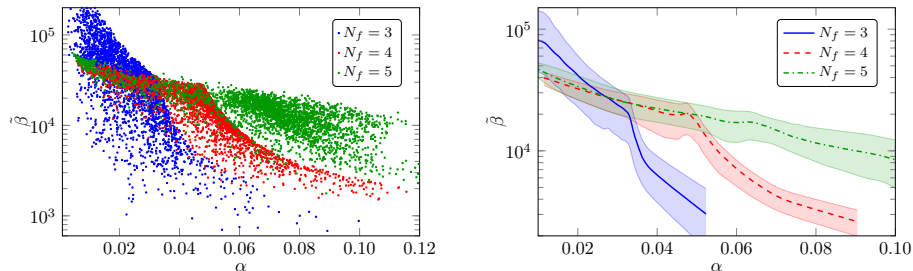


Figure: We show the range of α and $\tilde{\beta}$ values of the LSM for $N_f = 3, 4, 5$. In the left panel, we show the actual distribution of theory points, while in the right panel, we display the averaged values. On average, the LSM produces stronger GW signals with increasing N_f due to the larger α values. Nonetheless, the strongest GW signals are achieved with the LSM for $N_f = 3$, corresponding to the sparse blue dots at small $\tilde{\beta}$ in the left panel.

Strongest GW Signal at Small $m_\sigma \rightarrow$ Near Conformal

(Pasechnik, Reichert, Sannino and Z-W W, JHEP 02 (2024) 159.)

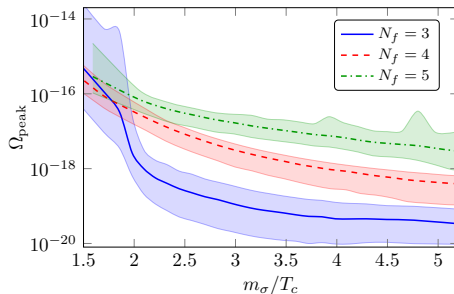


Figure: We show the averaged values of the peak amplitude Ω_{peak} as a function of physical observables m_σ in units of T_c in the LSM for $N_f = 3, 4, 5$. The sigma meson mass has the strong correlation with the peak amplitude: smaller values of m_σ lead to a larger Ω_{peak} . **The strongest signal can almost reach LISA sensitivity.**

Part II, realised in the dark sector: at $m_\sigma \rightarrow 0$ the tree-level curvature vanishes and $\mathcal{M}^2 \rightarrow \Pi[\mathcal{M}^2]$ — the strongest signal comes from the regime where the dressing matters most.

Understanding from Thin Wall Approximation

- The three-dimensional Euclidean action S_3 can be written as a function of the latent heat L and the surface tension σ

$$S_3 = \frac{16\pi}{3} \frac{\sigma(T_c)^3}{L(T_c)^2} \frac{T_c^2}{(T_c - T)^2},$$

- The ratio $S_3(T_p)/T_p$ is typically a number $\mathcal{O}(150)$ for phase transitions around the electroweak scale and the inverse duration $\tilde{\beta}$ follows as

$$\tilde{\beta} = T \left. \frac{d}{dT} \frac{S_3(T)}{T} \right|_{T=T_p} \approx \mathcal{O}(10^3) \frac{T_c^{1/2} L}{\sigma^{3/2}}.$$

- $\tilde{\beta}$ stems from the competition between the surface tension and latent heat. $L \sim N^2$ while σ can be either $\sigma \sim N$ or $\sigma \sim N^2$ with limited data up to $SU(8)$
- How to construct models with smaller latent heat and larger surface tension to enhance the gravitational wave signals?

Part IV: Glueball Dark Matter Production

Motivation: Why Glueball Dark Matter?

- A pure dark Yang–Mills sector is arguably the **simplest confining dark sector**.
- Once the theory confines, it inevitably produces a tower of **glueball bound states**.
- In the absence of dark fermions, the glueballs are **cleanly defined**: there are no mesons, baryons, or fermionic bound states that can mix with them.
- This makes the low-energy spectrum theoretically **simple, predictive**, and directly tied to the confinement scale.
- Lattice studies indicate that the scalar 0^{++} glueball is the **lightest state** in the spectrum.
- Therefore, in a sufficiently secluded dark sector, the lightest 0^{++} glueball naturally becomes the leading **dark-matter candidate**.

Minimal logic

pure $SU(N)_D$ Yang-Mills

↓ confinement

glueball tower

0^{++} lightest

⇒ DM candidate

Take-home message

Glueball dark matter is **minimal, clean, predictive** and **rich**: confinement alone is enough to generate a well-defined dark-matter candidate.

Rigorous Dark Gluon-gluon Dynamics

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- In the literature, for glueball dark matter production, only ϕ^5 interaction is considered, making the $3 \rightarrow 2$ annihilation the only relevant process for DM formation
- However, since glueball is strongly coupled (e.g. $g_4 \sim 10^3$, $g_5 \sim 10^4$), this naive calculation is not rigorous. **A non-perturbative method is required.**
- The dark gluon-gluon dynamics can be effectively described by considering the dimension-4 glueball field $\mathcal{H} \propto \text{tr}(G^{\mu\nu} G_{\mu\nu})$:

$$V[\mathcal{H}, \ell] = \frac{\mathcal{H}}{2} \ln \left[\frac{\mathcal{H}}{\Lambda^4} \right] + T^4 \mathcal{V}[\ell] + \mathcal{H} \mathcal{P}[\ell] + V_T[\mathcal{H}] .$$

- We keep the lowest order in $\mathcal{P}[\ell]$ to satisfy the symmetry:

$$\mathcal{P}[\ell] = c_1 |\ell|^2 ,$$

where c_1 is determined by the lattice results (**jumping of gluon condensate**).

Thermal evolution of the dark gluon-gluon system

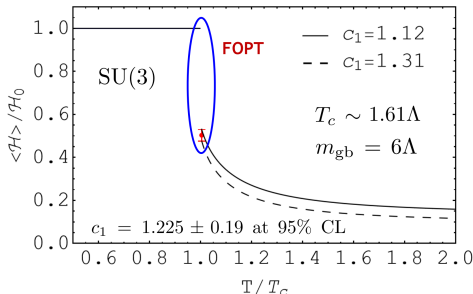
(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- Introducing canonically normalized glueball field, we introduce ϕ as $\mathcal{H} = 2^{-8}c^{-2}\phi^4$ and the effective Lagrangian reads:

$$\mathcal{L} = \frac{c}{2} \frac{\partial_\mu \mathcal{H} \partial^\mu \mathcal{H}}{\mathcal{H}^{3/2}} - V[\mathcal{H}, \ell] \Rightarrow \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V[\phi, \ell],$$

$$V[\phi, \ell] = \frac{\phi^4}{28c^2} \left[2 \ln \left(\frac{\phi}{\Lambda} \right) - 4 \ln 2 - \ln c \right] + \frac{\phi^4}{28c^2} \mathcal{P}[\ell] + T^4 \mathcal{V}[\ell], \quad \mathcal{P}[\ell] = c_1 |\ell|^2$$

$$\mathcal{V}[\ell] = T^4 \left(-\frac{b_2(T)}{2} |\ell|^2 + b_4 |\ell|^4 + \dots - b_3 \left(\ell^N + \ell^{*N} \right) \right),$$



Cosmological evolution of the dark glueball field

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- The glueball field is considered homogeneous and evolves in expanding FLRW universe, with the E.O.M.

$$\ddot{\phi} + 3H\dot{\phi} + \partial_{\phi}V[\phi, T] = 0,$$

- The “classical” E.O.M. describes the mean-field evolution of the effective composite field $\phi \sim \mathcal{H}^{1/4} \sim (\text{Tr } G_{\mu\nu} G^{\mu\nu})^{1/4}$, not necessarily a coherent state with large glueball occupation number.
- The time variable is found in terms of the photon temperature:

$$t = \frac{1}{2} \sqrt{\frac{45}{4\pi^3 g_{*,\rho}(T_{\gamma})}} \frac{m_P}{T_{\gamma}^2}, \quad T_{\gamma} = \xi_T T$$

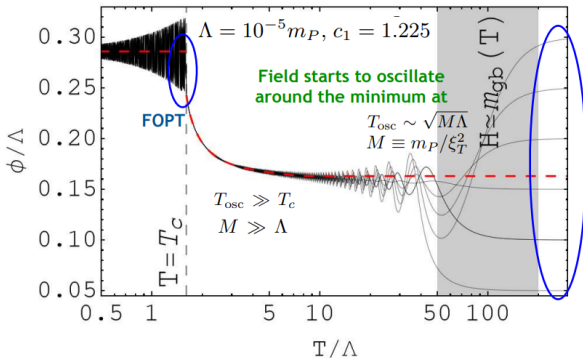
where ξ_T denotes the visible-to-dark sector temperature ratio and $m_P = 1.22 \times 10^{19}$ GeV is the Planck mass and $g_{*,\rho}$ is the number of energy-related degrees of freedom.

- E.O.M. in terms of the dark sector temperature:

$$\frac{4\pi^3 g_{*,\rho}}{45 m_P^2} \xi_T^4 T^6 \frac{d^2 \phi}{dT^2} + \frac{2\pi^3}{45 m_P^2} \frac{dg_{*,\rho}}{dT} \xi_T^4 T^6 \frac{d\phi}{dT} + \partial_{\phi}V[\phi, T] = 0$$

Cosmological Evolution of the Dark Glueball Field

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)



- Field starts to oscillate around the minimum of the potential when $H \simeq m_{\text{gb}}$ with temperature $T_{\text{OSC}} \sim \sqrt{M\Lambda}$
- In early times in deconfined regime, for different initial conditions the field evolution follows the minimum (red dashed line).
- First order phase transition washes out any dependence on initial conditions.

Glueball Relic Density

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26)

- Energy stored in these oscillations around $\phi_{\min} \approx 0.28\Lambda$ is the relic DM abundance, $\Omega h^2 = \rho/\rho_c$ (critical density $\rho_c = 1.05 \times 10^4 \text{ eV cm}^{-3}$)

$$\rho = \frac{2\pi^3}{45} g_{*,\rho}(T) \frac{T^6}{M^2} \left(\frac{d\phi}{dT} \right)^2 + V[\phi].$$

- Then the relic density today is calculated (f denotes as final):

$$\Omega h^2 = \frac{\Lambda}{\rho_c/h^2} \left\langle \frac{\tilde{\rho}}{\tilde{T}^3} \right\rangle_f T_f^3 \left(\frac{T_{\gamma,0}}{\zeta_T T_f} \right)^3 = 0.12 \zeta_T^{-3} \frac{\Lambda}{\Lambda_0},$$

with dilution factor $(T_{\gamma,0}/\zeta_T T_f)^3$ to consider the Universe expansion

- Below freeze-out temperature, the predicted glueball relic density is

$$0.12 \zeta_T^{-3} \frac{\Lambda}{137.9 \text{ eV}} \lesssim \Omega h^2 \lesssim 0.12 \zeta_T^{-3} \frac{\Lambda}{82.7 \text{ eV}}, \quad 1.035 < c_1 < 1.415$$

for $\zeta_T^{-1} = 0.1$, the glueball dark matter mass $M_{\text{gl}} \sim 6\Lambda$ is $\sim 0.1 \text{ MeV}$

- It is more than a factor of 10 difference compared to the old calculations

$$\Omega h^2 \sim 0.12 \zeta_T^{-3} \frac{\Lambda}{5.45 \text{ eV}}$$

Glueball Dark Matter Parameter Space (no portal)

(Carenza, Pasechnik, Salinas, Z-W W, Phys. Rev. Lett. **129** (2022) no.26, 26;

(Carenza, Ferreira, Pasechnik and Z-W W, Phys. Rev. D **108** (2023) no.12, 12)

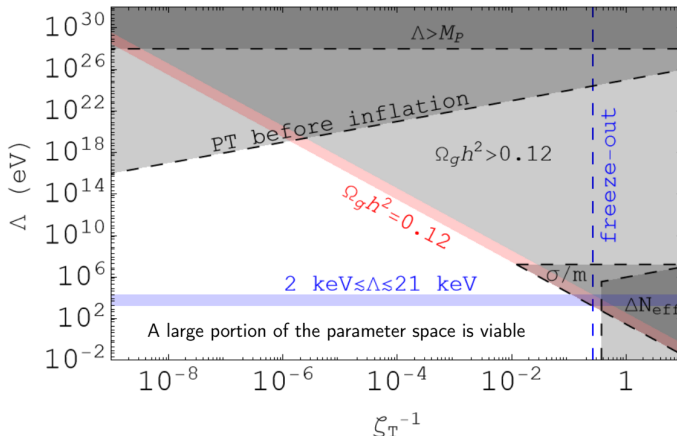


Figure: The red diagonal band corresponds to the full-DM condition, $\Omega_g h^2 \simeq 0.12$. Points below the band are underabundant: glueballs remain viable, but only as a subdominant DM component.

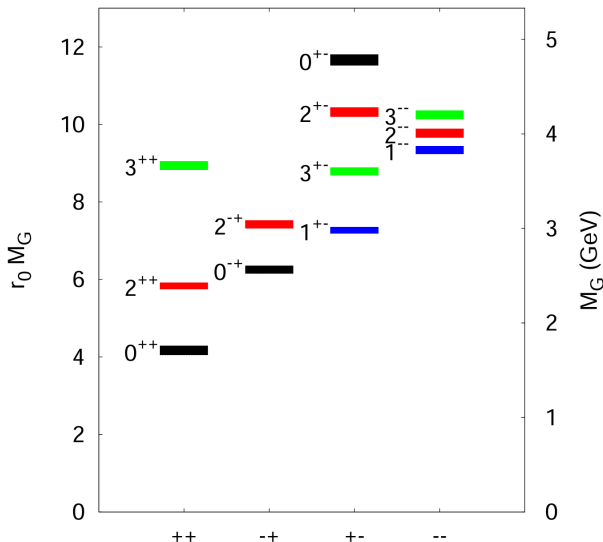
Part V: Glueball Axion-Like Particles (GALPs)

What is a GALP?

GALPs are composite pseudoscalar glueball states—or a mixed glueball mass eigenstate—with ALP-like photon couplings and misalignment-like relic production, but—unlike ordinary axions—their first-order confinement transition washes out the sensitivity to the initial field value. It is composite, naturally heavy, and does not solve the strong-CP problem.

Glueball Spectrum

(Y. Chen *et al.* "Glueball spectrum and matrix elements on anisotropic lattices," Phys. Rev. D **73** (2006) 014516)



Adding the θ term

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- We can add the θ term to incorporate also the pseudoscalar glueball state $\mathcal{A} \equiv 0^{-+}$

$$\mathcal{L}_{\text{SU(N)}} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} + \frac{\theta}{4}G_{\mu\nu}^a \tilde{G}^{\mu\nu a},$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c.$$

- We obtain the glueball EFT Lagrangian as (here $\mathcal{H}$ is ϕ before):

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu \mathcal{H} \partial^\mu \mathcal{H} + \frac{1}{2}\partial_\mu \mathcal{A} \partial^\mu \mathcal{A} - V_{\text{eff}}(\mathcal{H}, \mathcal{A}),$$

where $\mathcal{H} \equiv 0^{++}$ is the standard scalar glueball state while $\mathcal{A} \equiv 0^{-+}$ denotes the pseudoscalar glueball state.

- Specifically, the 0^{++} and 0^{-+} glueball fields are defined in terms of gauge-invariant operators of the lowest dimension as

$$\mathcal{H}^4 \equiv -\frac{\beta(g)}{2g}G_{\mu\nu}^a G^{\mu\nu a}, \quad \mathcal{A} \mathcal{H}^3 \equiv G_{\mu\nu}^a \tilde{G}^{\mu\nu a}$$

in terms of β -function $\beta(g)$.

$V_{\text{eff}}(H, A)$ Determines GALP Phenomenology

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- The two-globall effective potential is **not arbitrary**: its structure is constrained by the **trace anomaly** together with the nonzero θ term.
- $V_{\text{eff}}(H, A)$ fixes the vacuum (η_0, a_0) , the $0_{++} - 0_{-+}$ mass matrix, and the scalar-pseudoscalar mixing after confinement.
- Expanding around the vacuum defines the IR couplings: masses, mixing angle, cubic/quartic self-couplings, and transition amplitudes involving η and a .
- In this sense, $V_{\text{eff}}(H, A)$ plays for GALPs the same role that $V[H, \ell]$ played earlier for scalar glueball production: it is the **IR engine** behind the phenomenology.
- A two-field scalar potential with mixing — the natural next target for the multi-field partial dressing of Part II.

Trace-anomaly input

$$\partial_\mu D^\mu = -\Theta^\mu_\mu \propto -H^4$$

Potential structure

$$\begin{aligned} V_{\text{eff}} \simeq & c_0 \mathcal{H}^4 \ln \left(\frac{\mathcal{H}}{\Lambda} \right) + \frac{\theta}{4} \mathcal{A} \mathcal{H}^3 \\ & + \mathcal{H}^4 f \left(\frac{\mathcal{A}}{\mathcal{H}} \right) + c_1 \mathcal{H}^4 + c_2 \mathcal{H}^2 \mathcal{A}^2 \\ & + c_3 \mathcal{A}^4 + c_4 \mathcal{H} \mathcal{A}^3, \end{aligned}$$

Physical combinations of $c_i, f^{(n)}$ are lattice-matchable; θ can NOT

What it fixes

- vacuum (η_0, a_0)
- mass matrix and mixing
- conversion / decay channels

$\implies$ one-component
or

two-component GALP DM



GALP relic abundance: one vs two-component DM

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- The GALP relic interpretation is **not unique**: it depends on whether the heavier glueball is depleted before late times.
- **One-component benchmark**: if in-medium decay / conversion is efficient, the heavier state is transferred into the lighter one, and the late-time relic density is effectively that of the **lightest glueball**.
- **Two-component benchmark**: if both η and a remain cosmologically stable, the scalar-glueball relic estimate can be approximately extended to a **two-component DM** system.
- In the extended EFT analysis, a realistic simplified benchmark is to assume comparable present-day contributions from η and a . This is especially plausible when θ is **non-negligible**, so that the pseudoscalar sector is dynamically relevant.

Physical question

What survives after confinement and thermal evolution?

Case A: one-component DM

$$a_{\text{heavy}} \rightarrow \eta_{\text{light}}$$

Late-time DM is dominated by the lightest glueball state.

Note that: A one-component limit requires an efficient depletion channel, e.g. $a \rightarrow \eta\eta$ if $m_a > 2m_\eta$. The radiative transition $a \rightarrow \eta\gamma$ is forbidden in vacuum; photon-portal processes such as $a \rightarrow \eta\gamma\gamma$ are higher-order and typically negligible.

Case B: two-component DM

$$\eta + a$$

Both species survive and contribute to the present relic density.

$$\Omega_{\text{DM}} = \Omega_\eta + \Omega_a$$

large mixing+both states survive+no efficient depletion $\rightarrow$ comparable abundances
 $\Omega_\eta \sim \Omega_a$

Why a Heavy Portal Does Not Modify IR Glueball Production

- The portal fermions Ψ are **UV mediators**, not light IR flavors.
- Around $T \sim M_\Psi$, $\Psi\bar{\Psi}$ annihilation / decoupling fixes the dark-visible temperature ratio

$$\zeta_T^{-1} \equiv \frac{T_{\text{dark}}}{T_\gamma}, \quad \Omega_g h^2 \propto \zeta_T^{-3} \Lambda_D.$$

- Their finite-temperature contribution to confinement dynamics is Boltzmann suppressed:

$$\delta V_f \propto e^{-m_\Psi/T} \text{Tr } L.$$

Hence the Polyakov-loop potential is close to the **pure Yang–Mills** limit.

- In a sample case,

$$m_\Psi \gtrsim 5T_c \rightarrow e^{-m_\Psi/T} \sim 7 \times 10^{-3}$$

so the late-time IR remains **effectively pure Yang–Mills**.

- Cosmologically, the symmetric $\Psi\bar{\Psi}$ abundance is efficiently depleted; any tiny residual around T_c is further reduced by confinement / bound-state effects.

Portal: two jobs

$$\frac{\tau \alpha^2}{M_\Psi^4} G_D^2 F^2 \implies \text{EFT portal}$$

$$\Psi\bar{\Psi} \text{ annihilation} \implies \zeta_T$$

What changes vs. what doesn't

Modified: thermal history, ζ_T , EFT matching

Unmodified: IR glueball formation mechanism

Main consequence

The portal fermions modify ζ_T , not the IR glueball-production dynamics.

Therefore the previous PRL glueball-production framework can still be used.

The Portal's Two Jobs: Coupling and Thermal History

- To connect the dark Yang–Mills sector to the SM, introduce a heavy Dirac fermion Ψ carrying both dark color and electric charge.
- Integrating out Ψ above the confinement scale generates dim-8 gauge portals between the dark gluon operators and photons.
- After confinement, the same portal induces the effective GALP–photon coupling discussed on the next slide.
- But the portal does **more than coupling**: it also controls the thermal contact between the visible and dark sectors.
- Around $T \sim M_\Psi$, annihilation / decoupling of $\Psi\bar{\Psi}$ fixes the dark-visible temperature ratio

$$\zeta_T^{-1} \equiv \frac{T_{\text{dark}}}{T_\gamma}.$$

- Hence the portal feeds both the EFT coupling *and* the abundance condition used later to obtain the full-DM mass–coupling relation.

Job 1: low-energy coupling

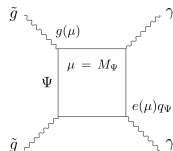
$$\mathcal{L}_{\text{eff}} \supset \frac{\tau^2 \alpha^2}{M_\Psi^4} \left(c_\gamma G^2 F^2 + \tilde{c}_\gamma G \tilde{G} F \tilde{F} \right)$$

$$\Rightarrow g_{a\gamma} \sim \kappa \alpha^2 \Lambda^{-1} \left(\frac{\Lambda}{M_\Psi} \right)^4$$

Job 2: thermal history

$$\Psi\bar{\Psi} \text{ annihilation/decoupling} \Rightarrow \zeta$$

$$\Omega_{\text{GALP}} h^2 \propto \zeta_T^{-3} \Lambda \quad (\text{schematically})$$



The portal sets both the coupling and the cosmological benchmark.

Effective PQ-like Scale

(Carenza, Pasechnik, Z-W W, Phys. Rev. Lett. **135** (2025) 021001)

- $\mathcal{L}_{\text{eff}} \supset \frac{\tau^2 \alpha^2}{M_\Psi^4} \left[c_\gamma G_{\mu\nu}^a G^{\mu\nu a} F_{\alpha\beta} F^{\alpha\beta} + \tilde{c}_\gamma G_{\mu\nu}^a \tilde{G}^{\mu\nu a} F_{\alpha\beta} \tilde{F}^{\alpha\beta} \right].$
- Linearizing around the VEVs (η_0, a_0) and keeping only terms linear in a, η :

$$G_{\mu\nu}^a G^{\mu\nu a} = -\frac{8g\eta_0^3 \eta}{\beta(g)}, \quad G_{\mu\nu}^a \tilde{G}^{\mu\nu a} = \eta_0^3 a + 3a_0 \eta_0^2 \eta.$$

- Turning to the mass basis $\{\eta, a\} \rightarrow \{\phi_1, \phi_2\}$, we obtain:

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{4} \sum_{i=1,2} \left[g_{\phi_i \gamma} \phi_i F_{\mu\nu} F^{\mu\nu} + \tilde{g}_{\phi_i \gamma} \phi_i F_{\mu\nu} \tilde{F}^{\mu\nu} \right].$$

- The effective couplings satisfy $g_{\phi_i \gamma} \sim \tilde{g}_{\phi_i \gamma} \equiv g_{\text{GALP}\gamma}$, where

$$g_{\text{GALP}\gamma} = \kappa \alpha^2 \Lambda^{-1} \left[\frac{\Lambda}{M_\Psi} \right]^4 = 2.45 \times 10^{-7} \text{ GeV}^{-1} \kappa \left[\frac{m_{\text{GALP}}}{\text{GeV}} \right]^3 \left[\frac{M_\Psi}{\text{GeV}} \right]^{-4}.$$

- The PQ-like scale is thus obtained ($f_a \propto g_{\text{GALP}\gamma}^{-1}$):

$$f_a \simeq 4.1 \times 10^3 \text{ GeV} \kappa^{-1} \left[\frac{\Lambda}{\text{GeV}} \right]^{-3} \left[\frac{M_\Psi}{\text{GeV}} \right]^4.$$

GALP DM Mass–Coupling Relation

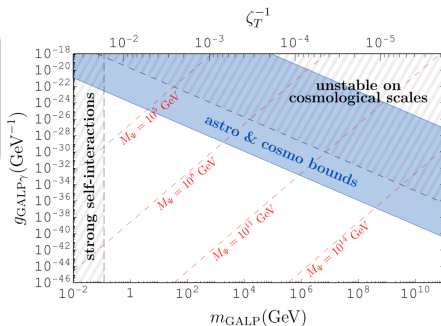
(Carenza, Pasechnik, Z-W W, “Composite Heavy Axionlike Dark Matter” PRL 135 (2025) 021001)

Analytic relation

$$g_{\text{GALP}\gamma} \simeq 2.5 \times 10^{-7} \text{ GeV}^{-1} \kappa \left(\frac{m_{\text{GALP}}}{\text{GeV}} \right)^3 \left(\frac{M_{\Psi}}{\text{GeV}} \right)^{-4}$$

$$f_a^{\text{eff}} \simeq 4.1 \times 10^3 \text{ GeV} \kappa^{-1} \left(\frac{\Lambda}{\text{GeV}} \right)^{-3} \left(\frac{M_{\Psi}}{\text{GeV}} \right)^4$$

- This is the defining relation of the GALP setup.
- For fixed M_{Ψ} , heavier GALPs imply a larger photon coupling.
- For fixed m_{GALP} , increasing M_{Ψ} strongly suppresses $g_{\text{GALP}\gamma}$.
- The full-DM region is selected by cosmological stability and self-interaction bounds.
- GALPs therefore occupy a heavy-mass / tiny-coupling window beyond the standard ALP paradigm.



Take-home:

GALPs naturally realize heavy composite ALP DM with an emergent, effectively super-Planckian PQ-like scale.

Part VI: Glueball Direct Detection

Why Direct Detection Favors C -odd Oddballs

(Li, Pasechnik, Wang and Z-W W, arXiv:2602.18753 (submitted to PRL))

Two benchmark portal scenarios

Scenario I: a single charged fermion portal with a heavy mediator

$\implies$ *radiative decays persist*

and direct detection is strongly suppressed.

Scenario II: a two-flavor vector-like portal with approximate global $SU(2)_D$ and opposite electric charges

$\implies$ *oddball DM can be stable*

while elastic scattering still survives.

Why oddballs can be stable yet detectable

- The relevant DM candidates are the lightest C -odd oddballs, e.g. 1^{+-} and 1^{--} .
- Step 1:** their leading *linear* portal operators involve an **odd number of photons**, not an F^2 -type two-photon structure; hence the usual box-induced $\chi \rightarrow \gamma\gamma$ channel is not the relevant leading decay route.
- Step 2:** in the approximate $SU(2)_D$ limit, $Q_{\psi+} = -Q_{\psi-}$ implies $Q_{\psi+}^{2n+1} + Q_{\psi-}^{2n+1} = 0$, so all **single-oddball odd-photon** amplitudes cancel.
- Therefore radiative cascades such as $1^{+-}, 1^{--} \rightarrow 0^{++} + \gamma$ are forbidden, while the bilinear $\chi\chi\gamma\gamma$ amplitude relevant for elastic scattering still survives.

J^{PC}	Mass/ $m_{0^{++}}$	Decay operator	Scaling	Portal
0^{++}	1	$G^2 F^2$	$\alpha_D^2 \alpha^2 \Lambda_D^9 m_\psi^{-8}$	I
0^{-+}	~ 1.55	$\tilde{G}\tilde{G}\tilde{F}\tilde{F}$	$\alpha_D^3 \alpha^3 \Lambda_D^9 m_\psi^{-8}$	I/II
0^{+-}	~ 2.7	$G^3 F$	$\alpha_D^3 \alpha^3 \Lambda_D^9 m_\psi^{-8}$	
		$G^3 F^3$	$\alpha_D^3 \alpha^3 \Lambda_D^{17} m_\psi^{-16}$	
0^{--}	—	$\tilde{G}G^2 F$	$\alpha_D^3 \alpha^3 \Lambda_D^9 m_\psi^{-8}$	I/II ^a
		$\tilde{G}\tilde{G}^2 \tilde{F}\tilde{F}^2$	$\alpha_D^3 \alpha^3 \Lambda_D^{17} m_\psi^{-16}$	
1^{++}	—	$G^3 F$	$\alpha_D^3 \alpha^3 \Lambda_D^9 m_\psi^{-8}$	I/II ^a
1^{-+}	~ 2.49		$\alpha_D^3 \alpha^3 \Lambda_D^9 m_\psi^{-8}$	
1^{+-}	~ 1.78	$G^3 F$	$\alpha_D^3 \alpha^3 \Lambda_D^9 m_\psi^{-8}$	I/II
		$G^3 F^3$	$\alpha_D^3 \alpha^3 \Lambda_D^{17} m_\psi^{-16}$	
1^{--}	~ 2.44	$\tilde{G}G^2 F$	$\alpha_D^3 \alpha^3 \Lambda_D^9 m_\psi^{-8}$	
		$\tilde{G}\tilde{G}^2 \tilde{F}\tilde{F}^2$	$\alpha_D^3 \alpha^3 \Lambda_D^{17} m_\psi^{-16}$	

Table I of arXiv:2602.18753. C -even states use two-photon operators, while C -odd oddballs use odd-photon operators.

Pomeron Foundation I: Why the Problem is Non-Perturbative

(Li, Pasechnik, Wang and Z-W W, arXiv:2602.18753 (submitted to PRL))

Coulomb-regime elastic scattering and amplitude

In the Galactic DM regime, the coherent process

$$\chi A \rightarrow \chi A$$

is dominated by **two off-shell photons**.

The corresponding elastic amplitude is

$$\mathcal{M}_\chi \propto \int \frac{d^4 k}{(2\pi)^4} \frac{J_A^\mu(k_1) J_A^\nu(k_2) T_{\mu\nu}^\chi(p, q; k)}{(k_1^2 + i\epsilon)(k_2^2 + i\epsilon)}.$$

- J_A^μ : elastic nuclear current, with
 $J_A^\mu(k) = (P + P')^\mu F_A(Q^2)$
- $T_{\mu\nu}^\chi$: doubly-virtual glueball Compton tensor

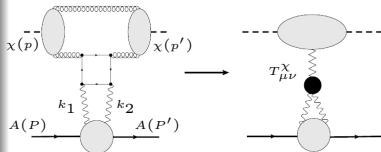


Fig. 2 of arXiv:2602.18753

Why non-perturbative input is necessary

- The **nuclear side** is under control through coherent Coulomb scattering and elastic form factors.
- The **glueball side** is difficult: it contains the long-distance response of a confining dark Yang–Mills state.
- The dim-8 portal matching determines the short-distance piece, but the full scattering amplitude still needs a controlled treatment of **non-perturbative glueball physics**.
- This is exactly why the paper introduces a QCD-inspired dark tensor-Pomeron framework in the next step.

Pomeron Foundation II: Dark Tensor Pomeron as the EFT Bridge

(Li, Pasechnik, Wang and Z-W W, arXiv:2602.18753 (submitted to PRL))

- For $m_{\psi} \gtrsim \frac{1}{2}m_{1+-} \sim 5.5\Lambda_D$, the infrared dynamics is effectively pure Yang–Mills, so the forward vacuum exchange is expected to be **dominated by a gluonic dark Pomeron**.
- In the forward regime, the doubly-virtual glueball Compton tensor is modeled by a **soft spin-2 dark Pomeron** in the Ewerz–Maniatis–Nachtmann (EMN) framework.
- This gives a controlled bridge between **UV portal matching** and the **non-perturbative glueball response** relevant for direct detection.
- Crucially, the long-distance part is not put in by hand. Instead, it is based on QCD soft-scattering phenomenology and then systematically **rescaled** to the dark sector.
- So the direct-detection amplitude is calculable in a way that keeps both **factorization** and **gauge invariance** manifest.

Short $\times$ long factorization

$$T_{\chi}^{\chi} \sim [\gamma^* \gamma^* \rightarrow g_D g_D]_{\text{short}} \times \langle \chi | G_D^2 | \chi \rangle_{\text{long}}$$

$\Downarrow$ forward regime

$$T_{\chi}^{\mu\nu} \sim \Gamma_{\mu\nu\kappa\lambda}^{(\gamma\gamma P_D)} \Delta^{(P_D)\kappa\lambda, \rho\sigma} \Gamma_{\rho\sigma}^{(\chi\chi P_D)} F_{\text{el}}(t)$$

$$k_{1\mu} \Gamma^{\mu\nu\kappa\lambda} = 0, \quad k_{2\nu} \Gamma^{\mu\nu\kappa\lambda} = 0$$

Dark-sector Rescaling

- Pomeron intercept ϵ_D
- Regge slope α'_D
- $\gamma\gamma P_D$ couplings
- $\chi\chi P_D$ couplings
- elastic slope b_{eff}^D

Predicted Xenon Reach

(Li, Pasechnik, Wang and Z-W W, arXiv:2602.18753 (submitted to PRL))

- After the non-perturbative matching, the spin-independent per-nucleon cross section scales as $\sigma_{\text{SI}} \propto \Lambda_D^{2.15} m_\psi^{-8}$.
- Current and next-generation Xenon sensitivity $\sigma_{\text{SI}} \sim 10^{-46} - 10^{-48} \text{ cm}^2$ roughly corresponds to $m_\psi \sim 3 - 30 \text{ GeV}$,
 $\Lambda_D \sim 0.55 - 5.5 \text{ GeV}$.
- Present PandaX already probes approximately $m_\psi \lesssim 20 \text{ GeV}$, while PandaX-xT can extend the reach to $m_\psi \sim 30 \text{ GeV}$.
- Glueball stability simultaneously requires $m_\psi > 5.5 \Lambda_D$.

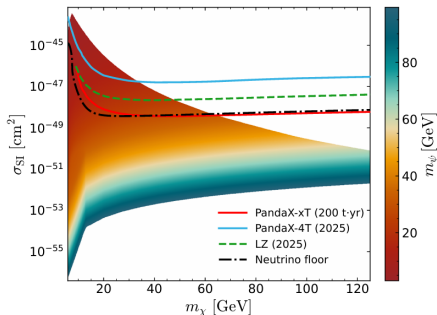


Fig. 3: Direct-detection reach for dark glueball DM.

Why a Light Charged Portal Is Not Immediately Ruled Out

Key physical picture

- LEP2 production is dominantly neutral-current:

$$e^+e^- \rightarrow \gamma^*/Z^* \rightarrow U\bar{U}, D\bar{D},$$

so the primary state is overall electrically neutral.

- Since the produced fermions also carry dark color, the pair is immediately connected by a dark string and forms a compact quirkonium-like bound state, rather than two macroscopic charged tracks.
- The characteristic pullback scale is

$$\sigma \sim \Lambda_{\text{dark}}^2, \quad L_{\text{max}} \sim E_{\text{rel}}/\sigma, \quad t_{\text{pull}} \sim L_{\text{max}}.$$

For $\Lambda_{\text{dark}} \sim 10 \text{ GeV}$, one finds

$$L_{\text{max}} \sim (0.02\text{--}0.2) \text{ fm}, \quad t_{\text{pull}} \lesssim 10^{-24} \text{ s},$$

i.e. bound-state formation is immediate on detector scales.

Collider implication

- The dominant bound states produced from neutral-current events remain electromagnetically neutral.
- Therefore the light charged portal is *not* generically excluded as a pair of free heavy stable charged particles.
- What LEP2 mainly probes is the subsequent dark-shower / glueball topology:
 - prompt 0^{++} decay $\rightarrow$ visible activity,
 - displaced 0^{++} decay $\rightarrow$ LLP-like photons / vertices,
 - escaping $0^{++} \rightarrow \text{mono-}\gamma + \text{MET}.$

Why charged bound states are hard to make

- Charged open-flavor states such as $U\bar{D}$ require extra pair creation from the dark string, which is Schwinger suppressed: $P_{\text{break}} \propto \exp[-\pi m_\psi^2/\sigma]$.
- Charged-current conversion $\bar{U} \rightarrow \bar{D} + W^{*-}$ during the strong cooling stage is also too slow: $P_{\text{CC}}^{\text{during cool}} \sim \Gamma_{\text{weak}}/\Gamma_{\text{cool}} \lesssim 10^{-8} \kappa_W^2$

Light-Portal UV Completion: Model Sketch

- Goal: keep a **light charged portal**

$$m_\psi \sim \mathcal{O}(10\text{--}20 \text{ GeV})$$

without immediate exclusion by LEP/LHC.

- Embed the portal into a **split-mixing singlet–doublet completion**.
- Add two singlets U, D , two vector-like doublets Ψ_U, Ψ_D , one extra scalar doublet Φ , and a Z_2 symmetry.
- The portal Yukawa is **not** attached directly to the SM Higgs H , but is sequestered into the extra scalar doublet Φ .
- After EWSB, only two charged mixings are generated:

$$\psi_{U+} \leftrightarrow U, \quad \psi_{D-} \leftrightarrow D.$$

- Key idea:** the light portal is part of a controlled EW completion, rather than an ad hoc light charged state.

Field content

$$U \sim (\mathbf{1}, +1/2), \quad D \sim (\mathbf{1}, -1/2), \\ \Psi_U \sim (\mathbf{2}, 0), \quad \Psi_D \sim (\mathbf{2}, 0)$$

$$\Phi = \begin{pmatrix} \phi^+ \\ (v_\Phi + \phi + ia)/\sqrt{2} \end{pmatrix}$$

Z_2 assignment

	H	Φ	Ψ_U	Ψ_D	U	D
Z_2	+	−	+	−	−	+

Portal Yukawas

$$\mathcal{L}_Y \supset -y_U \bar{\Psi}_U \tilde{\Phi} U - y_D \bar{\Psi}_D \Phi D + \text{h.c.}$$

$$\delta_U = \frac{y_U v_\Phi}{\sqrt{2}}, \quad \delta_D = \frac{y_D v_\Phi}{\sqrt{2}}$$

Why This Completion Survives LEP/LHC

- The two light charged eigenstates are

$$\psi_+ = c_+ U + s_+ \psi_{U_+}, \quad \psi_- = c_- D + s_- \psi_{D_-}.$$

- Z-phobic window:** the coupling to the Z is controlled by the doublet fractions

$$f_{\pm} \equiv s_{\pm}^2.$$

Hence the portal can stay light without carrying full-strength electroweak couplings.

- No tree-level $W\psi_+\psi_-$:** the W boson acts within each doublet, while ψ_+ and ψ_- come from different mixing blocks.
- Suppressed Higgs coupling:** since the portal Yukawa sits in Φ , the SM-like Higgs coupling is reduced by the small CP-even mixing angle θ_s .
- Oblique parameters:** a positive fermionic ΔT is compensated by a negative scalar contribution for a suitable scalar hierarchy.

Constraint map

Z width:	$\propto f_{\pm}$
W width:	0 at tree level
h width:	$\propto \sin \theta_s$
T parameter:	$\Delta T_{\text{ferm}} + \Delta T_{\text{scal}}$

Collider interpretation

$$e^+e^- \rightarrow \gamma^*/Z^* \rightarrow \psi_{\pm}\bar{\psi}_{\pm}$$

The portal pair first hadronizes into a neutral hidden bound state. The visible signal is then controlled by the $0^{++} \rightarrow \gamma\gamma$ decay length.

prompt:	standard visible activity
displaced:	LLP-like photons/vertices
mono- γ +MET:	if 0^{++} escapes the detector

For the benchmark, $c\tau_{0^{++}} \simeq 5.8$ cm, which lies in the **displaced window**.

Benchmark: displaced regime
 $\Rightarrow$ often the hardest to constrain

The light portal survives by suppressing Z, W, h couplings and placing the signal in the displaced window.

Take-Home Messages

- Dark Yang–Mills confinement can generate a **first-order phase transition** and potentially observable **gravitational-wave signals**.
- The finite- T potential carries a **genuine theoretical uncertainty**: the dressing prescription shifts the GW peak amplitude and frequency — largest in the near-conformal regime where the dark signal is strongest. 2PI (dark sector) and partial dressing (perturbative sector) share the same tadpole.
- The same confining sector also gives a predictive mechanism for **scalar glueball relic formation**, linking early-Universe dynamics to dark matter abundance.
- Adding the pseudoscalar state and a heavy portal leads to **composite heavy ALPs / GALPs** with an emergent effective PQ-like scale.
- A detectable direct-detection window is nontrivial: it requires a **symmetry-protected light portal**, **non-perturbative dark tensor Pomeron** input, and a collider-safe EW completion.
- The viable region is narrow but testable: Xenon searches, displaced-photon / LLP collider searches, and astrophysical GALP probes are complementary.

One dark Yang–Mills sector can leave signatures in cosmology, direct detection, astrophysics, and colliders at the same time.

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超越标准模型的新物理，量子场论



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超越标准模型的新物理，量子场论

SCHOOL OF PHYSICS

Banks-Zaks fixed point: $SU(3)$ Two-loop β -function

- The Two-loop β -function with $SU(3)$

$$\beta(g) = -\frac{\beta_0}{16\pi^2} g^3 - \frac{\beta_1}{(16\pi^2)^2} g^5 + \mathcal{O}(g^7), \text{ where}$$

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f = \frac{33 - 2N_f}{3},$$

$$\beta_1 = \frac{34}{3}N_c^2 - \frac{10}{3}N_cN_f - 2C_FN_f = 102 - \frac{38}{3}N_f, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}.$$

N_f	β_0	β_1	$\alpha_* = \frac{g_*^2}{4\pi}$
9	5	-12	5.24
10	13/3	-74/3	2.21
11	11/3	-112/3	1.23
12	3	-50	0.754
13	7/3	-188/3	0.468
14	5/3	-226/3	0.278
15	1	-88	0.143
16	1/3	-302/3	0.0416

New Progresses in Columbia Plot

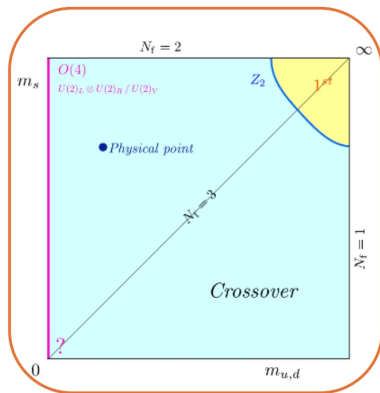
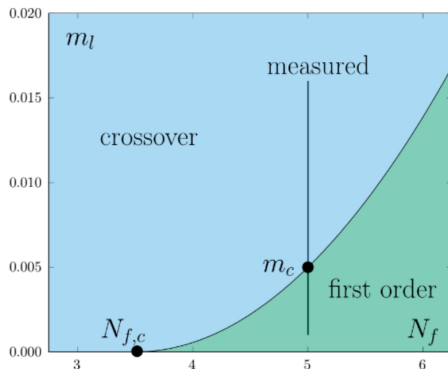


Figure: Left Fig. Columbia plot from JHEP 11 (2021) 141. Staggered fermions used. Right Fig. from PoS **LATTICE2022** (2023) 027.

About Center Symmetry and Confinement

- The standard physical interpretation is that it is related to the free energy of adding an external static color source in the fundamental representation.

$$\ell(\vec{x}) = \exp(-F\beta)$$

- In the confinement phase, Polyakov loop is zero corresponds to infinity free energy to add a color source and the same time center symmetry is unbroken.

Center Symmetry $Z(N)$ at Nonzero Temperature

- The boundary conditions in imaginary time τ the fields must satisfy are:

$$A_\mu(\vec{x}, \beta) = +A_\mu(\vec{x}, 0), \quad q(\vec{x}, \beta) = -q(\vec{x}, 0),$$

where gluons as bosons must be periodic in τ while quarks as fermions must be anti-periodic.

- 't Hooft first noticed that one can consider more general gauge transformations which are only periodic up to Ω_c

$$\Omega(\vec{x}, \beta) = \Omega_c, \quad \Omega(\vec{x}, 0) = 1 \quad \left(\text{here, } \Omega_c = e^{i\phi} I, \phi = \frac{2\pi j}{N} \right).$$

- Color adjoint fields are invariant under this transformation, while those in the fundamental representation are not:

$$A^\Omega(\vec{x}, \beta) = \Omega_c^\dagger A_\mu(\vec{x}, \beta) \Omega_c = A_\mu(\vec{x}, \beta) = +A_\mu(\vec{x}, 0),$$

$$q^\Omega(\vec{x}, \beta) = \Omega_c^\dagger q(\vec{x}, \beta) = e^{-i\phi} q(\vec{x}, \beta) \neq -q(\vec{x}, 0).$$

- Thermal Wilson line transforms like an adjoint field under local $SU(N)$ gauge transformations:

$$L(x) \rightarrow \Omega^\dagger(\vec{x}, \beta) L(\vec{x}) \Omega^\dagger(\vec{x}, 0).$$

Why is the Polyakov loop the exponential of the static-quark free energy?

Core formulas

First define the temporal Wilson line around the thermal circle:

$$W(\mathbf{x}) = \mathcal{P} \exp \left(ig \int_0^\beta d\tau A_0(\tau, \mathbf{x}) \right), \quad \ell(\mathbf{x}) = \frac{1}{N_c} \text{Tr} W(\mathbf{x}).$$

For a static heavy quark, the Euclidean action can be written as

$$S_Q = \int_0^\beta d\tau Q^\dagger(\tau, \mathbf{x}) (\partial_\tau + M - ig A_0(\tau, \mathbf{x})) Q(\tau, \mathbf{x}).$$

Because the quark is static, its propagation picks up only two pieces:

$$G_Q(\beta; \mathbf{x}) \propto e^{-\beta M} \mathcal{P} \exp \left(ig \int_0^\beta d\tau A_0(\tau, \mathbf{x}) \right) = e^{-\beta M} W(\mathbf{x}).$$

Taking the thermal trace over color then gives the ratio of partition functions with and without one inserted static quark:

$$\frac{Z_Q(\mathbf{x})}{Z} = e^{-\beta M} \left\langle \frac{1}{N_c} \text{Tr} W(\mathbf{x}) \right\rangle = e^{-\beta M} \langle \ell(\mathbf{x}) \rangle.$$

Fitting the Coefficients Using the Lattice Results: IV

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

Table: The parameters for the best-fit points.

N	3	$3 \log$	4	5	6	8
a_0	3.72	4.26	9.51	14.3	16.6	28.7
a_1	-5.73	-6.53	-8.79	-14.2	-47.4	-69.8
a_2	8.49	22.8	10.1	6.40	108	134
a_3	-9.29	-4.10	-12.2	1.74	-147	-180
a_4	0.27		0.489	-10.1	51.9	56.1
b_3	2.40	-1.77		-5.61		
b_4	4.53		-2.46	-10.5	-54.8	-90.5
b_6			3.23		97.3	157
b_8					-43.5	-68.9

- The PNJL Lagrangian can be generically written as:

$$\mathcal{L}_{\text{PNJL}} = \mathcal{L}_{\text{pure-gauge}} + \mathcal{L}_{4\text{F}} + \mathcal{L}_{6\text{F}} + \mathcal{L}_k$$

- Without losing generality, we consider below massless 3-flavour case in fundamental representation of $SU(3)$ gauge symmetry
- Here, $\mathcal{L}_{4\text{F}}$ is the four-quark interaction which reads:

$$\mathcal{L}_{4\text{F}} = G_S \sum_{a=0}^8 [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma^5\lambda^a\psi)^2], \quad \psi = (u, d, s)^T$$

- Six-fermion interaction $\mathcal{L}_{6\text{F}}$ denotes the Kobayashi-Maskawa-'t Hooft (KMT) term breaking $U(1)_A$ down to Z_3 (generically Z_{N_f} for N_f flavours)

$$\mathcal{L}_{6\text{F}} = G_D [\det(\bar{\psi}_{Li}\psi_{Rj}) + \det(\bar{\psi}_{Ri}\psi_{Lj})]$$

Medium Potential: Finite Temperature Contribution

- In the standard NJL model, the medium effect (finite temperature contribution) is implemented by the grand canonical partition function
- In the PNJL model, we can simply do the following replacement to include the contribution from Polyakov loop

$$V_{\text{medium}} = -2N_c T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \left(\ln \left[1 + e^{-\beta(E-\mu)} \right] + \ln \left[1 + e^{-\beta(E+\mu)} \right] \right) \\ \rightarrow -2T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \text{Tr}_c \left\{ \left(\ln \left[1 + \mathbf{L} e^{-\beta(E-\mu)} \right] + \ln \left[1 + \mathbf{L}^\dagger e^{-\beta(E+\mu)} \right] \right) \right\}$$

- $\mathbf{L}$ is the Polyakov loop:

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) d\tau \right]$$

- In this work, we consider chemical potential $\mu = 0$.

The Constituent Quark Mass and Zero Point Energy: I

(Fukushima, Skokov, PNP 96 (2017) 154)

- In $\mathcal{L}_{6F}$, there is also $\langle \bar{u}u \rangle^2 \bar{u}u$ term contributes to the constituent quark mass of u
- The total constituent quark mass from $\mathcal{L}_{4F}$ and $\mathcal{L}_{6F}$ is:

$$M = -4G_S\sigma - \frac{1}{4}G_D\sigma^2$$

- The expression for the zero-point energy is given by:

$$V_{\text{zero}}[\langle \bar{\psi}\psi \rangle] = -\dim(R) 2N_f \int \frac{d^3p}{(2\pi)^3} E_p, \quad E_p = \sqrt{\vec{p}^2 + M^2}$$

E_p is the energy of a free quark with constituent mass M and three-momentum $\vec{p}$

- The above integration diverges and a regularization is required. We choose a sharp three-momentum cutoff Λ entering the expression for observables and thus also a parameter of the theory.
- Parameters: G_S , G_D , Λ ; Observables: M , f_π , m_σ

The Constituent Quark Mass and Zero Point Energy: II

(Fukushima, Skokov, PPNP **96** (2017) 154)

- The integration can be carried analytically and the result is:

$$V_{\text{zero}}[\langle\bar{\psi}\psi\rangle] = -\frac{\dim(\mathbf{R})N_f\Lambda^4}{8\pi^2}\left[(2+\xi^2)\sqrt{1+\xi^2} + \frac{\xi^4}{2}\ln\frac{\sqrt{1+\xi^2}-1}{\sqrt{1+\xi^2}+1}\right],$$

in which $\xi \equiv \frac{M}{\Lambda}$.

- In $\mathcal{L}_{4F}$, the condensate energy then comes from the combination

$$(\bar{\psi}\lambda^0\psi)^2 + (\bar{\psi}\lambda^3\psi)^2 + (\bar{\psi}\lambda^8\psi)^2 = 2(\bar{u}u)^2 + 2(\bar{d}d)^2 + 2(\bar{s}s)^2$$

- We use the trick is to rewrite $(\bar{u}u)^2$ as

$$\begin{aligned}(\bar{u}u)^2 &= [(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle]^2 = (\bar{u}u - \langle\bar{u}u\rangle)^2 + 2\langle\bar{u}u\rangle(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle^2 \\ &\simeq -\langle\bar{u}u\rangle^2 + 2\langle\bar{u}u\rangle\bar{u}u,\end{aligned}$$

where the $(\bar{u}u - \langle\bar{u}u\rangle)^2$ term is dropped in the spirit of the **mean-field approximation**.

- The $2\langle\bar{u}u\rangle\bar{u}u$ term contributes to the constituent quark mass of u
- The $-\langle\bar{u}u\rangle^2$ term leads to a contribution to the condensate energy
- Similar procedures can be applied to $(\bar{d}d)^2$ and $(\bar{s}s)^2$, and to $\mathcal{L}_{6F}$ gives $\langle\bar{u}u\rangle^3$ and we obtain the total condensate energy:

$$V_{\text{cond}} = 6G_S\sigma^2 + \frac{1}{2}G_D\sigma^3, \quad \sigma \equiv \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle = \frac{1}{3}\langle\bar{\psi}\psi\rangle$$

Bubble Nucleation: Chiral Phase Transition (PNJL)

(Reichert, Sannino, Z-W W and Zhang, JHEP 01 (2022) 003, arXiv:2109.11552)

- Chiral phase transition occurs when including fermions
- $\bar{\sigma}$ is classically nonpropagating in PNJL and it's kinetic term is induced only via quantum fluctuations
- We thus include its wave-function renormalization Z_σ with

$$Z_\sigma^{-1} = - \frac{d\Gamma_{\sigma\sigma}(q^0, \mathbf{q}, \bar{\sigma})}{d\mathbf{q}^2} \Big|_{q^0=0, \mathbf{q}^2=0}, \quad \Gamma_{\sigma\sigma} = -i \sum \text{2 point 1PI } \sigma\sigma \text{ graph}$$

- The three-dimensional Euclidean action and E.O.M. are modified to:

$$S_3(T) = 4\pi \int_0^\infty dr r^2 \left[\frac{Z_\sigma^{-1}}{2} \left(\frac{d\bar{\sigma}}{dr} \right)^2 + V_{\text{eff}}(\bar{\sigma}, T) \right]$$

$$\frac{d^2 \bar{\sigma}}{dr^2} + \frac{2}{r} \frac{d\bar{\sigma}}{dr} - \frac{1}{2} \frac{\partial \log Z_\sigma}{\partial \bar{\sigma}} \left(\frac{d\bar{\sigma}}{dr} \right)^2 = Z_\sigma \frac{\partial V_{\text{eff}}}{\partial \bar{\sigma}}$$

- The associated boundary conditions:

$$\frac{d\bar{\sigma}(r=0, T)}{dr} = 0,$$

$$\lim_{r \rightarrow \infty} \bar{\sigma}(r, T) = 0$$

GW parameters α , β and PNJL observables

(Reichert, Sannino, Z-W W and Zhang, JHEP 01 (2022) 003, arXiv:2109.11552.)

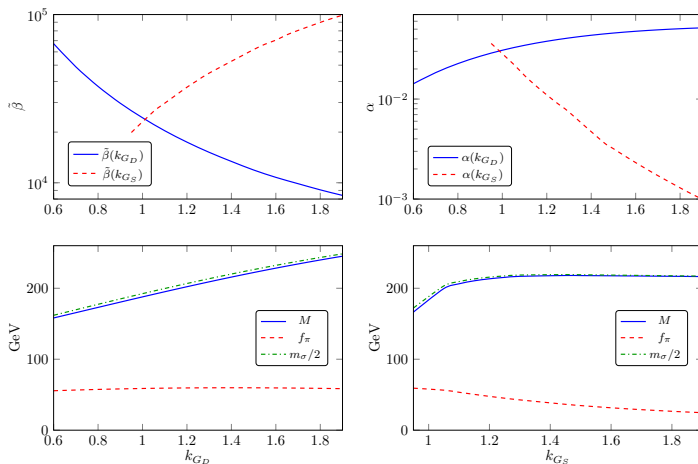


Figure: The GW parameters $\tilde{\beta}$, α with the observables M , f_π , and m_σ as a function of $G_S = k_{GS} \cdot 4.6 \text{ GeV}^{-2}$ and $G_D = k_{GD} \cdot (-743 \text{ GeV}^{-5})$. We use $T_c = 100 \text{ GeV}$, the ratio $\Lambda/T_0 = 3.54$. Below $k_{G_S, \text{crit}} = 0.882$, no chiral symmetry breaking occurs.

Signal to Noise Ratio

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

$$\text{SNR} = \sqrt{\frac{3\text{year}}{s} \int_{f_{\min}}^{f_{\max}} df \left(\frac{h^2 \Omega_{\text{GW}}}{h^2 \Omega_{\text{det}}} \right)^2}$$

$h^2 \Omega_{\text{GW}}$ is the GW spectrum while Ω_{det} is the sensitivity curve of the detector.

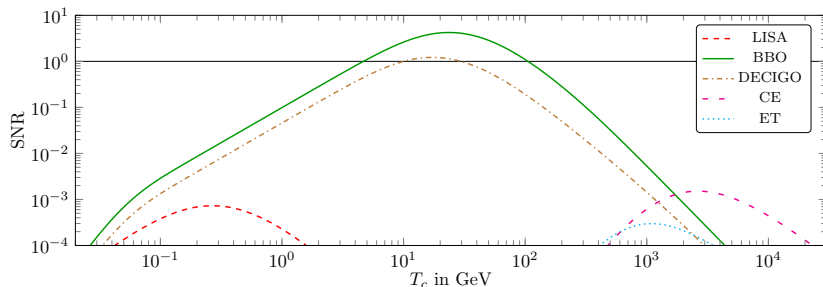


Figure: We display the SNR for the phase transition in a dark $SU(6)$ sector as a function of the confinement temperature T_c from experiments of LISA, BBO, DECIGO, CE, and ET. We assume an observation time of three years.