Matching Scalar Extensions to Higgs Effective Field Theory

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Based on 2412.00355, 2503.00707, 2602.14418, 2607.26184, 2612.xxxxx

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Outline

- Motivation
- Nonlinear framework and power counting
- Matching example: Type-II seesaw
- Systematic analysis of non-decoupling terms
- Summary

Scalar Extensions and Their Low-Energy Effects

$\mathrm{SM} + \Phi, \quad \Phi \sim (\mathbf{1}, 2T+1)_Y$

- A richer Higgs sector
- Neutrino and dark matter mass generation
- Modified electroweak phase transition

$$\mathrm{SM} + \Phi \xrightarrow{E \ll M_\Phi} \mathrm{EFT}\ (W, Z, h, \ldots)$$

Low-energy description

- EFT simplifies calculations through a systematic expansion.
- EFT coefficients encode heavy-scalar effects.
- UV–EFT matching determines these coefficients.

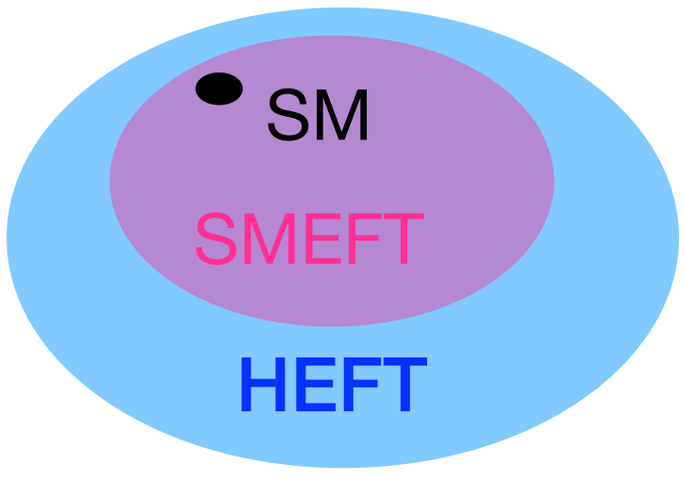
$$\{\lambda_i\}_{\mathrm{UV}} \longrightarrow \{c_i\}_{\mathrm{EFT}}$$

SMEFT and HEFT

SMEFT

- Higgs and Goldstones in **one doublet**
- Expansion in **canonical dimension**

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{d>4,i} \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}$$



R. Alonso, E. Jenkins, A. Manohar
[1511.00724,1605.03602]

SMEFT is a special case of HEFT.

HEFT

• Higgs and Goldstones treated **separately**

• **Derivative expansion** with general functions of h

• **More general** Higgs interactions and potentials

$$\mathscr{L}_{\text{HEFT}} \supset -V(h) + \frac{v^2}{4} F(h)\,\text{Tr}\left[(D_\mu U)^\dagger D^\mu U\right]$$

$$F(h) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \cdots$$

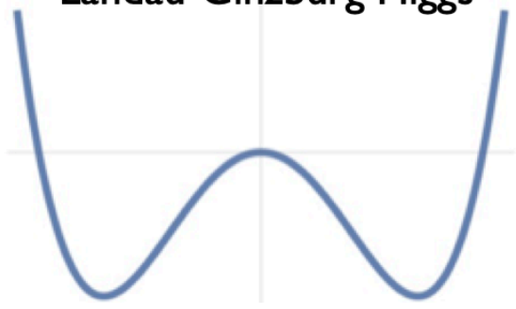
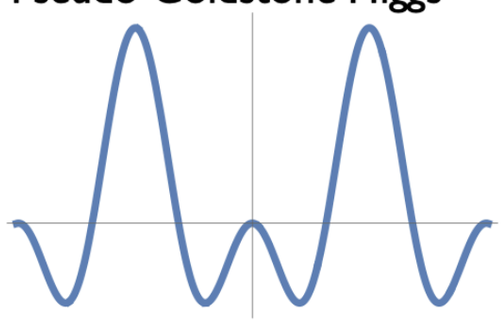
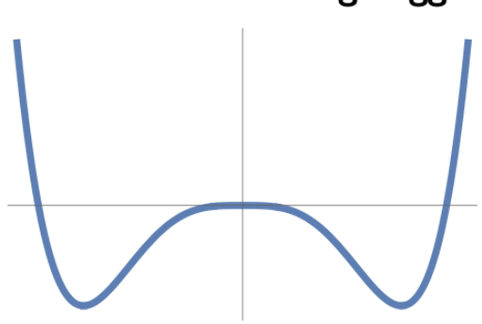
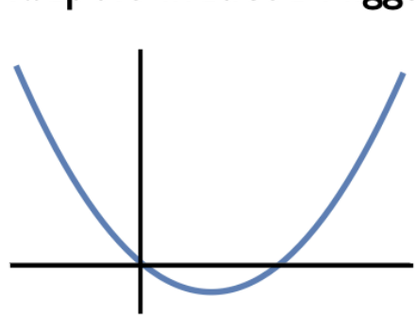
a: hVV coupling
b: hhVV coupling

$$U = \exp\left(\frac{i\pi^a \sigma^a}{v}\right)$$

π^a: electroweak Goldstone fields

Different Higgs Potentials

Agrawal et al., arXiv:1907.02078

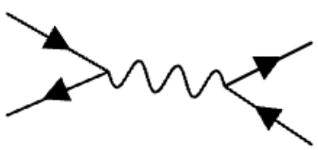
Landau-Ginzburg Higgs	Pseudo-Goldstone Higgs	Coleman Weinberg Higgs	Tadpole-induced Higgs
			
$V(H):\ -m^2H^\dagger H+\lambda(H^\dagger H)^2+\frac{c_6\lambda}{\Lambda^2}(H^\dagger H)^3$	$-a\sin^2\left(\frac{\sqrt{H^\dagger H}}{f}\right)+b\sin^4\left(\frac{\sqrt{H^\dagger H}}{f}\right)$	$\lambda(H^\dagger H)^2+\epsilon(H^\dagger H)^2\log\left(\frac{H^\dagger H}{\mu^2}\right)$	$-\kappa^3\sqrt{H^\dagger H}+m^2H^\dagger H$
			Non-analytic at H = 0

HEFT allows a general Higgs potential.

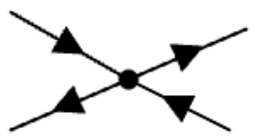
Matching Methods

Diagrammatic approach (amplitude)

- Direct connection to physical processes
- Match a set of amplitudes to determine the coefficients



$$\mathcal{A}_{\rm UV}\Big|_{E\ll M} = \mathcal{A}_{\rm EFT}$$



Carmona, Lazopoulos, Olgoso & Santiago, *Matchmakereft*, 2112.10787

Gómez-Ambrosio,Quezada-Calonge, *Matcha*, 2606.11170

Functional method

- Derive effective interactions at the action level
- A systematic treatment of different scalar models

$$\Gamma^{\rm 1LPI}_{\rm UV}[\phi] = \Gamma^{\rm 1PI}_{\rm EFT}[\phi]$$

Henning, Lu & Murayama, arXiv:1412.1837
Drozd, Ellis, Quevillon & You, arXiv:1512.03003
Cohen, Lu & Zhang, STrEAM, arXiv:2012.07851
Fuentes-Martín, König, Pagès, Thomsen & Wilsch, Matchete, arXiv:2212.04510

We use functional matching.

Challenges in UV–HEFT Matching

Field separation

- Goldstone modes can be mixtures of fields from several multiplets.

  Example: SM + a real triplet $G^+ = c_\theta H^+ + s_\theta \Sigma^+,\ |\tan\theta| = \dfrac{2v_\Sigma}{v_H}$

- Collect them into U and separate them from physical scalars.

$$U = \exp\left(\frac{i\pi^a \sigma^a}{v}\right)$$

Power counting

- Choice of independent parameters

$$\mathscr{P}_{\text{phys}} = \{m^2_{\phi^\pm}, m^2_K, m_h, v, \sin\gamma, \xi = v_\Sigma/v_H\}$$

$$\mathscr{P}_{\text{UV couplings}} = \{Z_1, Z_2, Z_3, Y_1, Y_2, Y_3\}$$ or a mixture of both

- What scales, and what stays fixed?

Dawson et al., “Is HEFT unique?”, arXiv:2311.16897

The Real Triplet Model

SM Higgs doublet H+ a real triplet Σ

$$H \sim (\mathbf{1}, \mathbf{2})_{1/2}, \qquad \Sigma \sim (\mathbf{1}, \mathbf{3})_0$$

$$\mathscr{L}_{\text{scalar}} = (D_\mu H)^\dagger D^\mu H + \text{Tr}\left[(D_\mu \Sigma)^\dagger D^\mu \Sigma\right] - V(H, \Sigma)$$

$$V(H, \Sigma) = Y_1^2 H^\dagger H + Z_1 (H^\dagger H)^2 + Y_2^2 \text{Tr}(\Sigma^2) + Z_2 [\text{Tr}(\Sigma^2)]^2$$
$$+ Z_3 (H^\dagger H) \text{Tr}(\Sigma^2) + 2 Y_3 H^\dagger \Sigma H$$

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H \end{pmatrix}, \qquad \langle \Sigma \rangle = \frac{v_\Sigma}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Both VEVs contribute to electroweak symmetry breaking.

Nonlinear Framework

Song & Wan, arXiv:2412.00355

A common Goldstone matrix U for both multiplets

$$U = \exp\left(\frac{i\pi^a\sigma^a}{v}\right)$$

$$H = U\mathfrak{h}, \qquad \mathfrak{h} = \begin{pmatrix} \chi^+ \\ (v_H + h_0 + i\chi^0)/\sqrt{2} \end{pmatrix}$$

$$\Sigma = U\Phi U^\dagger, \qquad \Phi = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix}$$

$V(H,\Sigma) = V(\mathfrak{h},\Phi)$ U drops out of the potential.

Choose $\chi^\pm, \chi^0$ to cancel bilinear Goldstone–scalar kinetic mixing.

$$\chi^+ = \frac{\sqrt{2}\, v_\Sigma}{v_H}\phi^+, \qquad \chi^0 = 0$$

Goldstone modes are collected in U, separately from the remaining scalar fields.

General Scalar Extensions

Song & Wan, arXiv:2412.00355

Tensor notation

An isospin-T multiplet is a symmetric tensor with 2T indices.

$$X_{i_1\cdots i_n} = U^{a_1}_{i_1}\cdots U^{a_n}_{i_n}\, x_{a_1\cdots a_n}, \qquad n = 2T$$

We consider a VEV $x^0 = \frac{1}{\sqrt{2}}(v_X + \rho_X + i\eta_X)$

Removing bilinear kinetic mixing

$$\chi^+ = \frac{v_X}{v_H}\left[\sqrt{(T-Y)(T+Y+1)}\,(x^-)^* - \sqrt{(T+Y)(T-Y+1)}\,x^+\right],$$

$$\chi^0 = -\frac{2Yv_X}{v_H}\eta_X\,.$$

$x^\pm$: charged components of X

A common framework for UV–HEFT matching.

Primary HEFT

$$\begin{pmatrix} h \\ K \end{pmatrix} = \begin{pmatrix} c_\gamma & -s_\gamma \\ s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} h_0 \\ \phi^0 \end{pmatrix} \qquad s_\gamma \equiv \sin\gamma$$

h: light Higgs K: heavy neutral scalar

$$\mathscr{P} = \{m_K^2, m_{\phi^\pm}^2, m_h, v_{\rm EW}, s_\gamma, \xi\} \qquad v_{\rm EW} = \sqrt{v_H^2 + 4v_\Sigma^2},\ \xi \equiv \frac{v_\Sigma}{v_H}$$

$$m_K^2,\, m_{\phi^\pm}^2 \longrightarrow \frac{m_K^2}{t}, \frac{m_{\phi^\pm}^2}{t}, \qquad m_h,\, v_{\rm EW},\, s_\gamma,\, \xi \textbf{ fixed}$$

- Expand in inverse heavy squared masses.
- Keep the mixing angle and VEV ratio fixed.

Non-decoupling terms are retained.

Why Primary HEFT?

Ge, Song & Wan, arXiv:2602.14418

$$V_{\text{mass}} = \frac{1}{2}\begin{pmatrix} h_0 & \phi^0 \end{pmatrix}\begin{pmatrix} 2Z_1 v_H^2 & v_H(Z_3 v_\Sigma - Y_3) \\ v_H(Z_3 v_\Sigma - Y_3) & 2Z_2 v_\Sigma^2 + \frac{Y_3 v_H^2}{2v_\Sigma} \end{pmatrix}\begin{pmatrix} h_0 \\ \phi^0 \end{pmatrix} + \frac{Y_3(v_H^2 + 4v_\Sigma^2)}{2v_\Sigma}\phi^+\phi^-$$

Express UV couplings in terms of physical masses and mixing angle.

$$Y_3 = \frac{2v_\Sigma}{v_H^2 + 4v_\Sigma^2} m_{\phi^\pm}^2, \qquad Z_1 = \frac{c_\gamma^2 m_h^2 + s_\gamma^2 m_K^2}{2v_H^2},$$

$$Z_2 = \frac{1}{2v_\Sigma^2}\left(s_\gamma^2 m_h^2 + c_\gamma^2 m_K^2 - \frac{v_H^2 m_{\phi^\pm}^2}{v_H^2 + 4v_\Sigma^2}\right), \qquad Z_3 = \frac{s_\gamma c_\gamma (m_K^2 - m_h^2)}{v_H v_\Sigma} + \frac{2m_{\phi^\pm}^2}{v_H^2 + 4v_\Sigma^2}.$$

UV couplings are linear in the heavy squared masses.

$$p_i = p_i^{(0)} + A_i m_K^2 + B_i m_{\phi^\pm}^2, \qquad p_i \in \{Y_1^2, Y_2^2, Y_3, Z_1, Z_2, Z_3\}$$

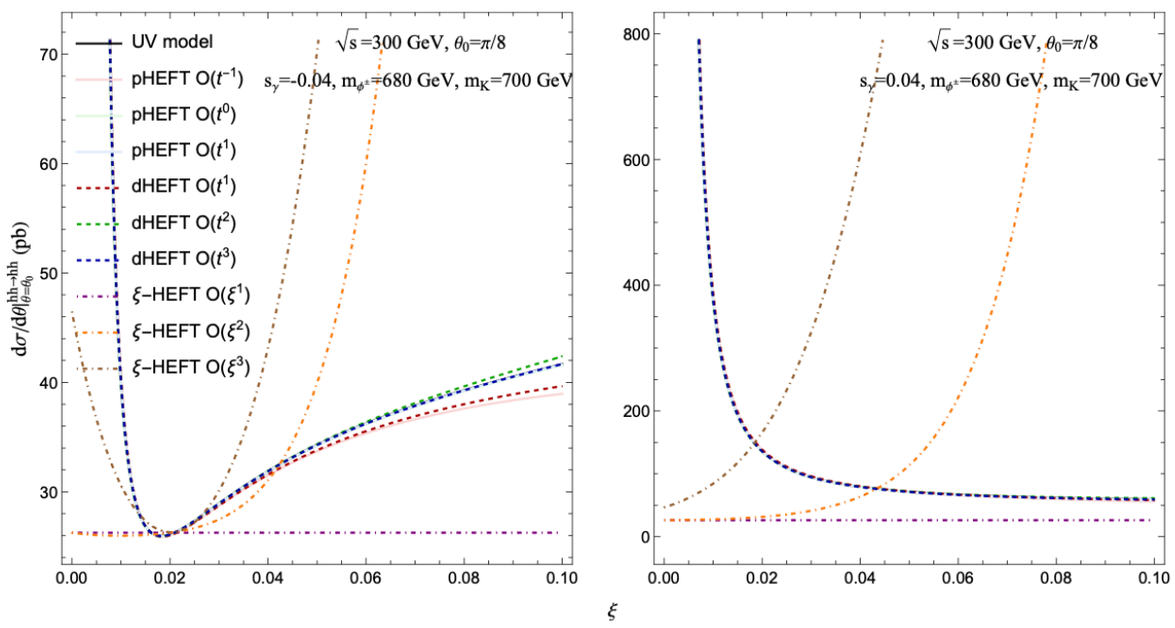
The $1/M^2$ expansion introduces no additional truncations of the original interaction terms.

Comparing Power Countings

Ge, Song & Wan, arXiv:2602.14418

Scheme	Quantities scaled	Inputs kept fixed
pHEFT	$m_K^2, m_{\phi^\pm}^2 \sim t^{-1}$	$m_h, v_{\rm EW}, s_\gamma, \xi$
dHEFT	$m_K^2, m_{\phi^\pm}^2 \sim t^{-1};\ \ s_\gamma, \xi \sim t$	$m_h, v_{\rm EW}$
ξ-HEFT	$\xi \sim t$	Z_1, Z_2, Z_3, Y_3, v_H
Z_2-HEFT	$m_K^2, m_{\phi^\pm}^2 \sim t^{-1}$	$m_h, v_{\rm EW}, Z_2, \xi$
Y_2-HEFT	$Y_2^2 \sim t^{-1}$	Z_1, Z_2, Z_3, Y_1, Y_3

t: expansion parameter



pHEFT ⊃ dHEFT ⊃ **ξ-HEFT**

pHEFT ⊃ **Z_2-HEFT**

pHEFT ⊃ **Y_2-HEFT** ~ SMEFT

The numerical comparison supports pHEFT as a benchmark for other power countings.

Type-II Seesaw Model

We now apply the same framework to a complex triplet with $Y = 1$.

$$H \sim (\mathbf{1}, \mathbf{2})_{1/2}, \qquad \Xi \sim (\mathbf{1}, \mathbf{3})_1 \,.$$

$$\mathscr{L} \supset (D_\mu H)^\dagger D^\mu H + \mathrm{Tr}\left[(D_\mu \Xi)^\dagger D^\mu \Xi\right] - V(H, \Xi) - \left[y_\Delta^{pr} L_L^{pT} C\, i\sigma_2 \Xi L_L^r + \mathrm{h.c.}\right]$$

$$\begin{aligned} V(H, \Xi) = & -m^2 H^\dagger H + M^2 \mathrm{Tr}(\Xi^\dagger \Xi) + \left[\mu H^T i\sigma_2 \Xi^\dagger H + \mathrm{h.c.}\right] \\ & + \lambda_1 (H^\dagger H)^2 + \lambda_2 \left[\mathrm{Tr}(\Xi^\dagger \Xi)\right]^2 + \lambda_3 \mathrm{Tr}(\Xi^\dagger \Xi \Xi^\dagger \Xi) \\ & + \lambda_4 (H^\dagger H)\mathrm{Tr}(\Xi^\dagger \Xi) + \lambda_5 H^\dagger \Xi \Xi^\dagger H \,. \end{aligned}$$

$$\xi \equiv \frac{v_\Xi}{v_H} \qquad \underbrace{h}_{\text{light}} \,, \qquad \underbrace{K,\, \eta,\, H^\pm,\, H^{\pm\pm}}_{\text{heavy}}$$

The complex triplet introduces CP-odd and doubly charged scalar states, together with lepton Yukawa interactions.

Matching Setup

$$\mathscr{P} = \{m_h, m_K, m_\eta, m_+, m_{++}, s_\gamma, v_H, \xi\}, \qquad \xi = \frac{v_\Xi}{v_H}$$

$$m_K^2,\ m_\eta^2,\ m_+^2,\ m_{++}^2 \sim \mathcal{O}(t^{-1}), \qquad m_h,\ v_H,\ \sin\alpha,\ \xi \sim \mathcal{O}(t^0)\,.$$

Integrate out $K, \eta, H^\pm, H^{\pm\pm}$ using their classical equations of motion.

Tree-level matching onto HEFT through $\mathcal{O}(p^4)$.

Matching Results: Bosonic Sector

$V_\mu = UD_\mu U^\dagger$

$\langle V_\mu V^\mu\rangle \langle V_\nu V^\nu\rangle$
$\langle \boldsymbol{V_\mu V_\nu}\rangle \langle \boldsymbol{V^\mu V^\nu}\rangle$
$\langle V_\mu \sigma_3\rangle \langle V^\mu \sigma_3\rangle \langle V_\nu V^\nu\rangle$
$\langle V_\mu \sigma_3\rangle \langle V_\nu \sigma_3\rangle \langle V^\mu V^\nu\rangle$
$\langle V_\mu \sigma_3\rangle \langle V^\mu \sigma_3\rangle \langle V_\nu \sigma_3\rangle \langle V^\nu \sigma_3\rangle$

Corrections to electroweak processes, including $WW \to WW$.

$h^3 \subset -\frac{1}{3!}V^{(3)}(0)$	$-\frac{s^2 m_\eta^2(2\xi c+s)}{2\xi(4\xi^2+1)v_H} + \frac{m_h^2(s^3-\xi c^3)}{2\xi v_H}$	$-\frac{\lambda}{2}v_{\rm ew} + \epsilon^2 v_{\rm ew}\left[\frac{(1-2\kappa^2)\lambda}{2}\right.$	$\left. -\frac{1-\kappa+2\kappa^2-6\kappa^3}{2\kappa r_\Delta^2}\right]$
$hW_\mu^- W^{+\mu}$ $\subset -\frac{1}{4}F'(0)\langle V_\mu V^\mu\rangle$	$\frac{g^2 v_H}{2}(c-2\xi s)$	$\frac{g^2 v_{\rm ew}}{2} \times [1-\epsilon^2(1-4\kappa+2\kappa^2)]$	
$hZ_\mu Z^\mu$ $\subset -\frac{1}{4}F'(0)\langle V_\mu V^\mu\rangle$ $-\frac{1}{4}G'(0)\langle TV_\mu\rangle\langle TV^\mu\rangle$	$\frac{g^2 v_H}{4c_W^2}(c-4\xi s)$	$\frac{g^2 v_{\rm ew}}{4c_W^2} \times [1-\epsilon^2(1-8\kappa+2\kappa^2)]$	

We reproduce the overlapping results of [2504.02580] and retain higher-order terms in the parameter expansion.

Ge, Song & Wan, arXiv:2607.26184
Liao, Ma & Uchida, arXiv:2504.02580

A Complete LNV Operator Basis

To perform the matching, we first construct a complete $\Delta L = 2$ HEFT operator basis through $\mathcal{O}(p^4)$.

A complex spurion $\hat{S}$ encodes $B - L$ violation, with $X(S) = +1$.

$$\hat{S} \equiv U^* \epsilon S_R U^\dagger, \qquad S_R = S\sigma_-, \qquad X(S) = +1,$$

$$\epsilon = i\sigma_2, \qquad \sigma_- = \frac{\sigma_1 - i\sigma_2}{2}, \qquad X \equiv \frac{B-L}{2}.$$

Class	Label	Operator	Flavor count
		$\mathcal{O}(p^2)$	
L^2 $\frac{n_f(n_f+1)}{2}$	$\mathcal{O}_L^{pr}$	$L_{Lp}^T C \hat{\mathcal{S}} L_{Lr}$	$\frac{n_f(n_f+1)}{2}$
		$\mathcal{O}(p^3)$	
L^2V n_f^2	$\mathcal{O}_{LV}^{pr}$	$L_{Lp}^T C \hat{\mathcal{S}} \gamma^\mu V_\mu U L_{Rr}$	n_f^2
		$\mathcal{O}(p^4)$	
L^2X $2n_f^2 - n_f$	$\mathcal{O}_{LX,B}^{pr}$	$L_{Lp}^T C \hat{\mathcal{S}} \sigma_{\mu\nu} L_{Lr} B^{\mu\nu}$	$\frac{n_f(n_f-1)}{2}$
	$\mathcal{O}_{LX,W}^{pr}$	$L_{Lp}^T C \hat{\mathcal{S}} \sigma_{\mu\nu} W^{\mu\nu} L_{Lr}$	n_f^2
	$\mathcal{O}_{LX,TW}^{pr}$	$L_{Lp}^T C \hat{\mathcal{S}} \sigma_{\mu\nu} L_{Lr} \langle T W^{\mu\nu} \rangle$	$\frac{n_f(n_f-1)}{2}$
	$\mathcal{O}_{LVV,1}^{pr}$	$L_{Lp}^T C \hat{\mathcal{S}} L_{Lr} \langle V_\mu V^\mu \rangle$	$\frac{n_f(n_f+1)}{2}$
	$\mathcal{O}_{LVV,2}^{pr}$	$L_{Lp}^T C \hat{\mathcal{S}} L_{Lr} \langle T V_\mu \rangle \langle T V^\mu \rangle$	$\frac{n_f(n_f+1)}{2}$

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Matching the Type-II seesaw model onto the LNV HEFT basis.

Type-II Seesaw → HEFT

Operators	Coefficients
$L_{Lp}^T C\hat{\mathcal{S}} L_{Lr}$	$y_\Delta^{pr}\frac{h^2 csm_h^2\left[2c(4\xi^2+1)m_K^2(c\xi+s)+m_\eta^2(-3cs-4\xi+6\xi s^2)\right]}{\sqrt{2}\xi(4\xi^2+1)v_{\mathcal{H}}m_K^4}$
$L_{Lp}^T C\hat{\mathcal{S}} L_{Lr}\, D_\mu h\, D^\mu h$	$-y_\Delta^{pr}\frac{cs\left[c(4\xi^2+1)m_K^2(c\xi+s)+m_\eta^2(-3cs-4\xi+6\xi s^2)\right]}{\sqrt{2}\xi(4\xi^2+1)v_{\mathcal{H}}m_K^4}$
$L_{Lp}^T C\hat{\mathcal{S}} V_\mu L_{Lr}\, D^\mu h$	$-2(y_\Delta^{pr}+y_\Delta^{rp})\frac{c\xi+s}{\sqrt{2}(2\xi^2+1)m_+^2}$
$L_{Lp}^T C\hat{\mathcal{S}} L_{Lr}\,\langle TV_\mu\rangle\, D^\mu h$	$-y_\Delta^{pr}\frac{\sqrt{2}(c\xi+s)}{(4\xi^2+1)m_\eta^2}+2y_\Delta^{pr}\frac{c\xi+s}{\sqrt{2}(2\xi^2+1)m_+^2}$
$L_{Lp}^T C\hat{\mathcal{S}} L_{Lr}\,\langle V_\mu V^\mu\rangle$	$y_\Delta^{pr}\frac{cv_{\mathcal{H}}(2c\xi+s)}{2\sqrt{2}m_K^2}$
$L_{Lp}^T C\hat{\mathcal{S}} L_{Lr}\,\langle TV_\mu\rangle\,\langle TV^\mu\rangle \quad T\equiv U\sigma_3 U^\dagger$	$y_\Delta^{pr}\frac{c^2\xi v_{\mathcal{H}}}{2\sqrt{2}m_K^2}-2y_\Delta^{pr}\frac{\xi v_{\mathcal{H}}}{2\sqrt{2}(2\xi^2+1)m_+^2}$

HEFT → LEFT for $0\nu\beta\beta$

Class	Op. Name	Op. Structure	LEFT Op.	Matching Condition
		$\mathcal{O}(p^2)$		
ψ^2	$\mathcal{O}_L$	$L_{Lp}^T C\hat{\mathcal{S}} L_{Lr}$	$m_{\beta\beta}$	$m_{\beta\beta}^{pr}=-\mathcal{F}_L^{pr}(0)$
		$\mathcal{O}(p^3)$		
$\psi^2 V$	$\mathcal{O}_{LV}$	$L_{Lp}^T C\hat{\mathcal{S}}\gamma^\mu V_\mu U L_{Rr}$	$O_{VL}^{(6)}$	$C_{VL}^{(6)\,prst}\supset -2iV_{st}\mathcal{F}_{LV}^{rp*}(0)$
		$\mathcal{O}(p^4)$		
$\psi^2 W$	$\mathcal{O}_{LX,W}$	$L_{Lp}^T C\hat{\mathcal{S}}\sigma^{\mu\nu}W_{\mu\nu}L_{Lr}$	$O_{VL}^{(6)}$, $O_{VL}^{(7)}$, $O_{VL}^{(9)}$	$C_{VL}^{(6)\,prst}\supset -\frac{4}{g}V_{st}(M_e^\dagger)_{pu}\mathcal{F}_{LX,W}^{ru*}(0)$ $C_{VL}^{(7)\,prst}\supset -\frac{4v}{g}V_{st}\mathcal{F}_{LX,W}^{rp*}(0)$ $C_{1L}^{(9)\,prstuv}\supset -\frac{8v}{g}V_{st}V_{uv}\left[\mathcal{F}_{LX,W}^{rp*}(0)+\mathcal{F}_{LX,W}^{pr*}(0)\right]$

Systematic Matching Across Scalar Extensions

Ge, Song & Wan, in progress

Selected

Model	$\{T, Y\}$	Field type	Additional physical scalars
Z_2-symmetric real singlet	$\{0, 0\}$	Real	K
Z_2-symmetric complex singlet	$\{0, 0\}$	Complex	K, A
Two-Higgs-doublet model	$\{\frac{1}{2}, \frac{1}{2}\}$	Complex	$H, A, H^{\pm}$
Real triplet	$\{1, 0\}$	Real	$K, H^{\pm}$
Complex triplet	$\{1, 1\}$	Complex	$K, A, H^{\pm}, \Phi^{\pm\pm}$
Complex quadruplet	$\{\frac{3}{2}, \frac{3}{2}\}$	Complex	$K, A, H^{\pm}, \Phi^{\pm\pm}, \Phi^{\pm\pm\pm}$
Complex quadruplet	$\{\frac{3}{2}, \frac{1}{2}\}$	Complex	$K, A, H^{\pm}, \bar{H}^{\pm}, \Phi^{\pm\pm}$
Real quintuplet	$\{2, 0\}$	Real	$K, H^{\pm}, \Phi^{\pm\pm}$
Complex sextuplet	$\{\frac{5}{2}, \frac{1}{2}\}$	Complex	$K, A, H_1^{\pm}, \bar{H}_1^{\pm}, H_2^{\pm\pm}, \bar{H}_2^{\pm\pm}, \Phi^{\pm\pm\pm}$
Real septet	$\{3, 0\}$	Real	$K, H^{\pm}, \Phi^{\pm\pm}, \Phi^{\pm\pm\pm}$
Complex septet	$\{3, 2\}$	Complex	$K, A, H^{\pm}, \bar{H}^{\pm}, \Phi^{\pm\pm}, \Phi^{\pm 3}, \Phi^{\pm 4}, \Phi^{\pm 5}$

What common low-energy effects arise from these scalar extensions?

As a start, we choose **non-decoupling** terms.

Tree-level bosonic matching, retaining terms that grow or remain finite under the heavy-mass scaling.

Extracting Non-Decoupling Terms

$$\mathscr{L}_{\rm UV} = \mathscr{L}_{\rm kin} - \frac{1}{t}V_{-1}(h,\Phi) - V_0(h,\Phi), \qquad M_A^2 \to \frac{M_A^2}{t}$$

$$\Phi^a_{\rm cl} = \Phi^a_0(h) + t\Phi^a_1 + \cdots, \qquad \left.\frac{\partial V_{-1}}{\partial \Phi^a}\right|_{\Phi=\Phi_0} = 0$$

$$\mathscr{L}^{\rm retained}_{\rm HEFT} = \mathscr{L}_{\rm kin}\Big|_{\Phi=\Phi_0} - \frac{1}{t}V_{-1}(h,\Phi_0(h)) - V_0(h,\Phi_0(h))$$

The leading heavy-field solution determines all retained t^{-1} and t^0 terms.

General Expressions for a, b and c_i

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{2}(\partial h)^2 - \mathcal{V}(h) - \frac{v^2}{4}\mathcal{F}(h)\langle V_\mu V^\mu\rangle + \frac{1}{2}\mathcal{C}(h)\langle V_\mu \sigma_3\rangle\langle V^\mu \sigma_3\rangle$$

$$F(h) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \cdots, \qquad C(h) = v^2\left(c_0 + c_1\frac{h}{v} + c_2\frac{h^2}{v^2} + \cdots\right)$$

a: hVV b: hhVV

custodial symmetry breaking coefficients

$$\Phi_0(h) = q_2 h^2 + \mathcal{O}(h^3), \quad v^2 = v_H^2 + \omega v_X^2, \quad \zeta = \frac{\omega - 4Y^2}{4}, \quad \omega = 2[T(T+1) - Y^2]$$

$$a = \frac{v_H c_\gamma - \omega v_X s_\gamma}{v},$$

$$b = c_\gamma^2 + \omega s_\gamma^2 + 2(v_H s_\gamma + \omega v_X c_\gamma)q_2,$$

$$c_0 = \frac{\zeta v_X^2}{v^2},$$

$$c_1 = -\frac{2\zeta v_X s_\gamma}{v},$$

$$c_2 = \zeta\left(s_\gamma^2 + 2v_X c_\gamma q_2\right).$$

Benchmark	ω	ζ
Real Z_2 singlet, Z2CSE	0	0
2HDM	1	0
Real triplet, $Y = 0$	4	1
Complex triplet, $Y = 1$	2	$-1/2$
Quadruplet, $Y = 1/2$	7	$3/2$
Quadruplet, $Y = 3/2$	3	$-3/2$
Real quintuplet, $Y = 0$	12	3
Complex sextuplet, $Y = 1/2$	17	4
Real septet, $Y = 0$	24	6
Complex septet, $Y = 2$	16	0

q_2 structure

Benchmark	Δq_2 in $q_2 = -d_\gamma/2 + \Delta q_2$
Real and complex Z_2 singlets	0
2HDM, fixed m_{12}^2	0
Real triplet, $Y = 0$	$\frac{m_{H^\pm}^2 s_\gamma (3 c_\gamma s_\gamma + 4\xi - 6\xi s_\gamma^2)}{2 m_K^2 v_H \xi (1 + 4\xi^2)}$
Complex triplet, $Y = 1$	$\frac{m_A^2 s_\gamma (3 c_\gamma s_\gamma + 4\xi - 6\xi s_\gamma^2)}{2 m_K^2 v_H \xi (1 + 4\xi^2)}$
Quadruplets, $Y = \frac{1}{2}, \frac{3}{2}$	$\frac{3 m_A^2 (s_\gamma + \xi c_\gamma)^2 (c_\gamma - \xi s_\gamma)}{2 m_K^2 v_H \xi (1 + 4 Y^2 \xi^2)}$
Real quintuplet, $Y = 0$	$-\frac{m_{\Phi^{\pm\pm}}^2 c_\gamma s_\gamma^2}{6 m_K^2 v_H \xi} + \frac{m_{H^\pm}^2 \mathcal{P}_5^{(q)}}{3 m_K^2 v_H \xi (1 + 12\xi^2)}$
Complex sextuplet, $Y = \frac{1}{2}$	$\frac{3 B_6 (s_\gamma + \xi c_\gamma)^2 (c_\gamma - \xi s_\gamma)}{2 m_K^2 v_H \xi}$
Real septet, $Y = 0$	$-\frac{9 m_{H^\pm}^2 s_\gamma (c_\gamma s_\gamma + 4\xi - 6\xi s_\gamma^2)}{m_K^2 v_H \xi (1 + 24\xi^2)}$
Complex septet, $Y = 2$	$\frac{3 m_A^2 \mathcal{R}_7}{2 m_K^2 v_H \xi (1 + 16\xi^2)}$

$$d_\gamma = s_\gamma c_\gamma \left(\frac{c_\gamma}{v_H} + \frac{s_\gamma}{v_X} \right)$$

- Common term from the biquadratic potential.
- Additional interactions contribute to Δq_2.
- Heavy-mass ratios remain in Δq_2.

Higgs Self-Couplings: κ_3 and κ_4

$$\ell_\gamma = \frac{c_\gamma^3}{v_H} - \frac{s_\gamma^3}{v_X}, \qquad V_{-1}(h,0) - V_{-1}(0,0) = E_3 h^3 + \left(\frac{m_K^2 d_\gamma^2}{8} + U_4\right) h^4 + \mathcal{O}(h^5).$$

$$\kappa_3 = v\ell_\gamma + \frac{2vE_3}{m_h^2},$$

$$\kappa_4 = v^2\left(\ell_\gamma^2 + 8d_\gamma q_2 - \frac{16}{3}q_2^2\right) + \frac{v^2}{m_h^2}\left[m_K^2(d_\gamma^2 - 4q_2^2) + 8U_4\right]$$

Benchmark	E_3	U_4
Real and complex Z_2 singlets	0	0
2HDM, fixed m_{12}^2	0	0
Real and complex triplets	$\frac{m_*^2 s_\gamma^2(2\xi c_\gamma + s_\gamma)}{2v_H\xi(1+4\xi^2)}$	$\frac{E_3(2\xi c_\gamma - s_\gamma)}{4v_H\xi}$
Quadruplets, $Y = \frac{1}{2}, \frac{3}{2}$	$\frac{m_A^2(s_\gamma + \xi c_\gamma)^3}{2v_H\xi(1+4Y^2\xi^2)}$	$\frac{E_3(3\xi c_\gamma - s_\gamma)}{4v_H\xi}$
Real quintuplet, $Y = 0$	$\frac{m_{H^\pm}^2 s_\gamma \mathcal{P}_5^{(3)}}{9v_H\xi(1+12\xi^2)} - \frac{m_{\Phi^{\pm\pm}}^2 s_\gamma^3}{18v_H\xi}$	$\frac{m_{H^\pm}^2 s_\gamma \mathcal{P}_5^{(4)}}{6v_H^2\xi^2(1+12\xi^2)} + \frac{m_{\Phi^{\pm\pm}}^2 s_\gamma^4}{24v_H^2\xi^2}$
Complex sextuplet, $Y = \frac{1}{2}$	$\frac{B_6(s_\gamma + \xi c_\gamma)^3}{2v_H\xi}$	$\frac{E_3(\xi c_\gamma - 3s_\gamma)}{4v_H\xi}$
Real septet, $Y = 0$	$-\frac{3m_{H^\pm}^2 s_\gamma^2(s_\gamma + 6\xi c_\gamma)}{v_H\xi(1+24\xi^2)}$	$\frac{E_3(2\xi c_\gamma - 3s_\gamma)}{4v_H\xi}$
Complex septet, $Y = 2$	$\frac{m_A^2 \mathcal{P}_7}{2v_H\xi(1+16\xi^2)}$	$\frac{E_3(6\xi c_\gamma - s_\gamma)}{4v_H\xi}$

κ_3: no explicit q_2 contribution.

κ_4: direct quartic, heavy-field response, and canonical normalization.

Both can retain terms growing with heavy masses.

Summary

- A common nonlinear framework separates Goldstone modes from physical scalars.

- Primary HEFT provides a benchmark for comparing different power countings.

- Type-II seesaw matching and a complete $\Delta L = 2$ HEFT operator basis through $\mathcal{O}(p^4)$.

- Preliminary results and analytic insights into non-decoupling effects across scalar extensions.

Thanks for your attention!