

The Fate of Chiral Gauge Theories

李浩林

第七届粒子物理前沿研讨会

2026.07.11

Based on

HHL, A. Pastor-Gutierrez, S. Vatani, L.-X. Xu JHEP 12 (2025) 020

HHL, A. Pastor-Gutierrez, S. Vatani, arXiv:2603.19355



Outline

- Why study chiral gauge theory (CGT)
- State of art analysis
- Introduction to functional renormalization group (fRG)
- How to use fRG to diagnose CGT
- IR Phase of Bars-Yankielowicz model
- Summary and outlook

Why Chiral Gauge Theories

- Appealing UV completion
Asymptotic free, scales generated dynamically, conjectured rich IR dynamics,
 - Original SU(5) GUT
[\[Georgi, Glashow '74 Raby, Dimopoulos, Susskind '79 Bars, Yankielowicz '81\]](#)
 - Composite Higgs models
[\[Cacciapaglia, Sannino 1508.00016, Cacciapaglia, Vatani, Zheng 1911.05454\]](#)

Why Chiral Gauge Theories

- Appealing UV completion
Asymptotic free, scales generated dynamically, conjectured rich IR dynamics,
 - Original SU(5) GUT
[\[Georgi, Glashow '74, Raby, Dimopoulos, Susskind '79, Bars, Yankielowicz '81\]](#)
 - Composite Higgs models
[\[Cacciapaglia, Sannino 1508.00016, Cacciapaglia, Vatani, Zheng 1911.05454\]](#)
- SM is a Chiral gauge theory,
no proof of consistency perturbative computation that preserve the chiral symmetry beyond 2-loop

Why Chiral Gauge Theories

- Appealing UV completion
Asymptotic free, scales generated dynamically, conjectured rich IR dynamics,
-- Original SU(5) GUT
[\[Georgi, Glashow '74, Raby, Dimopoulos, Susskind '79, Bars, Yankielowicz '81\]](#)
-- Composite Higgs models
[\[Cacciapaglia, Sannino 1508.00016, Cacciapaglia, Vatani, Zheng 1911.05454\]](#)
- SM is a Chiral gauge theory,
no proof of consistency perturbative computation that preserve the chiral symmetry beyond 2-loop
- Condense Matter Theory: Symmetric mass generation, Luttinger liquid model, Chiral magnetic effect...

Why Chiral Gauge Theories

- Appealing UV completion
Asymptotic free, scales generated dynamically, conjectured rich IR dynamics,
-- Original SU(5) GUT
[\[Georgi, Glashow '74, Raby, Dimopoulos, Susskind '79, Bars, Yankielowicz '81\]](#)
-- Composite Higgs models
[\[Cacciapaglia, Sannino 1508.00016, Cacciapaglia, Vatani, Zheng 1911.05454\]](#)
- SM is a Chiral gauge theory,
no proof of consistency perturbative computation that preserve the chiral symmetry beyond 2-loop
- Condense Matter Theory: Symmetric mass generation, Luttinger liquid model, Chiral magnetic effect...
- Confinement w/o dSB → [Symmetric Mass generation](#) → [Chiral Gauge Theory on Lattice](#)
[\[D. Tong 2104.03997, Wang, You 2204.14271\]](#)
- [Crucial to study the phase of CGTs:](#)
[Conformal?](#) [Confinement?](#) [Dynamical Symmetry Breaking?](#)

Which model

- Benchmark models $SU(N_c)$ with two kinds of left-handed fermions

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \psi^\dagger \bar{\sigma}^\mu D_\mu \psi + \chi^\dagger \bar{\sigma}^\mu D_\mu \chi$$

- Gauge anomaly free restricts the flavor

Georgi-Glashow

	$SU(N_c)$	Flavor
ψ	$\bar{\square}$	$N_c - 4$
χ	$\square$	1

Bars-Yankielowicz

	$SU(N_c)$	Flavor
ψ	$\bar{\square}$	$N_c + 4$
χ	$\square \square$	1

Global Symmetry

$$SU(N_c - 4) \times U(1)$$

't Hooft anomaly can be matched by $\mathcal{B} = \chi\psi\psi$

$$SU(N_c + 4) \times U(1)$$

Which model

- Benchmark models $SU(N_c)$ with two kinds of left-handed fermions

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \psi^\dagger \bar{\sigma}^\mu D_\mu \psi + \chi^\dagger \bar{\sigma}^\mu D_\mu \chi$$

- Gauge anomaly free restricts the flavor

Georgi-Glashow

	$SU(N_c)$	Flavor
ψ	$\overline{\square}$	$N_c - 4$
χ	$\square$	1

Global Symmetry
 $\times N_f \quad SU(N_c - 4) \times U(1) \quad \text{JHEP 12 (2025) 020}$

Bars-Yankielowicz

	$SU(N_c)$	Flavor
ψ	$\overline{\square}$	$N_c + 4$
χ	$\square \square$	1

$SU(N_c + 4) \times U(1) \quad 2603.19355$

State of Art Analysis

- No efficient Monte Carlo Lattice simulations (fermion doubler)

[Nielsen,Ninomiya'80'81]

- Most Attractive Channel $\rightarrow \langle \chi\psi \rangle$ condensate for $N_c=5$ and $N_c \geq 7$

[Raby,Dimopoulos,Susskind '79, Dimopoulos,Raby,Susskind '80, Eichten,Feinberg '82]

- Large N_c analysis $\rightarrow$ If confinement, no dSB

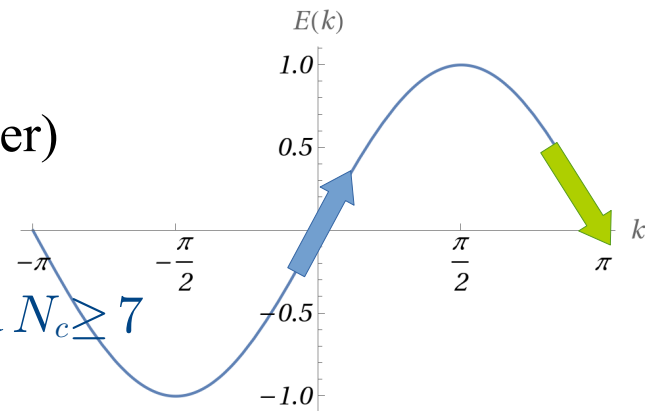
[Eichten,Peccei,Preskill,Zeppenfeld'86 Karasik,Önder,Tong 2208.07842]]

- Anomaly mediated SUSY breaking $\rightarrow$ conformal for $N_c \leq 13$

[Csáki,Murayama,Telem 2105.03444]

- Anomaly matching of higher forms symmetry $\rightarrow \langle \chi\chi \rangle, \langle \chi\psi \rangle$ color-flavor locked phase

[Bolognesi,Konishi,Luzio[2101.02601]



State of Art Analysis

- No efficient Monte Carlo Lattice simulations (fermion doubler)

[Nielsen,Ninomiya'80'81]

- Most Attractive Channel $\rightarrow \langle \chi \psi \rangle$ condensate for $N_c = 5$ and $N_c \geq 7$

[Raby,Dimopoulos,Susskind '79, Dimopoulos,Raby,Susskind '80, Eichten,Feinberg '82]

- Large N_c

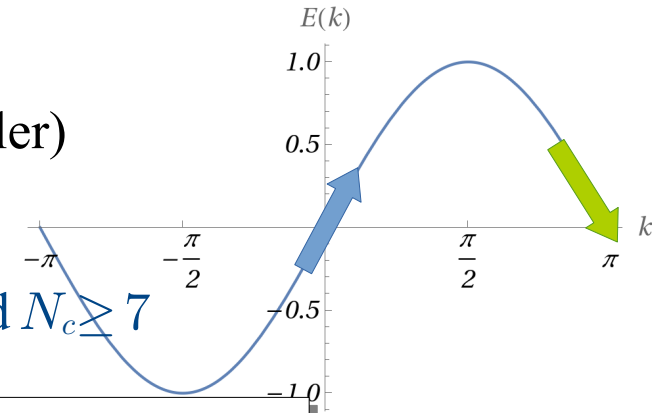
[Eichten,Peccei

- Anomaly

[Csáki,Murayan

- Anomaly

[Bolognesi,Kor



In conclusion:
No good understanding of the
IR dynamics of chiral gauge theory

or locked phase

The tool: Functional renormalization group method

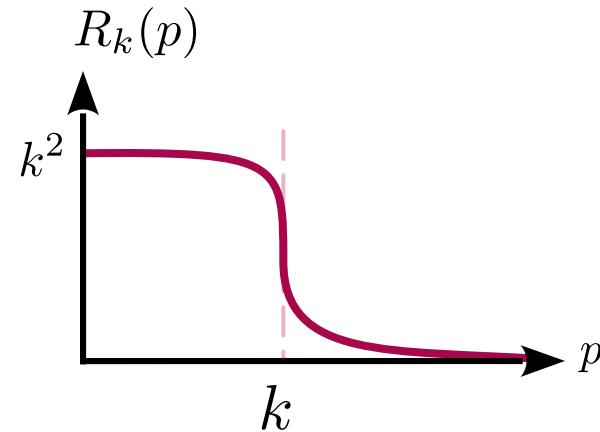
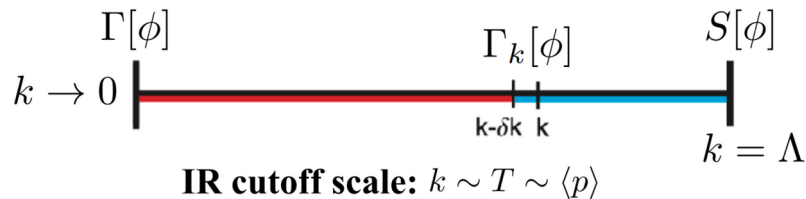
- Functional Renormalization Group:
a non-perturbative method widely used in QCD

See Dupuis, et.al [2006.04853] for a review

- Basic idea: effective average action: $\Gamma_k[\phi]$

$$\int [\mathcal{D}\phi]_{p>k} = \int \mathcal{D}\phi \exp(-\Delta S_k[\phi]) \quad \Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi]$$



The tool: Functional renormalization group method

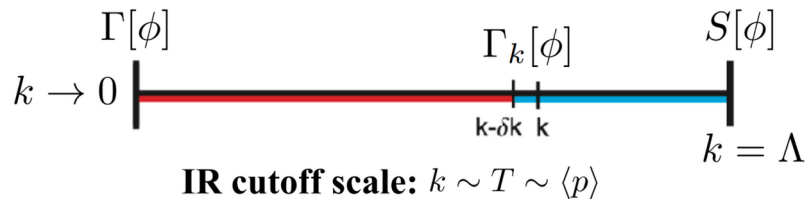
- Functional Renormalization Group:
a non-perturbative method widely used in QCD

See Dupuis,et.al [2006.04853] for a reievw

- Basic idea: effective average action: $\Gamma_k[\phi]$

$$\int [\mathcal{D}\phi]_{p>k} = \int \mathcal{D}\phi \exp(-\Delta S_k[\phi]) \quad \Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi]$$



Chiral symmetry intact by fermion regulator

$$R_{\psi/\chi, k}(p^2) = i Z_{\psi/\chi} \bar{\sigma}_\mu p_\mu r_{\psi/\chi}(x)$$

$$r_{\psi/\chi}(x) = (1/\sqrt{x} - 1) \theta(1 - x)$$

The tool: Functional renormalization group method

Master formula:

Wetterich '93

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \right]$$

$$\partial_t \equiv k \partial_k$$

$$\partial_t \Gamma_k^{(1)} = -\frac{1}{2} \text{STr} \left[\Gamma_k^{(3)} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Both UV and IR finite

$$\partial_t \Gamma_k^{(2)} = -\frac{1}{2} \text{STr} \left[\Gamma_k^{(4)} - 2 \Gamma_k^{(3)} \Gamma_k^{(2)} \Gamma_k^{(3)} \right] \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

⋮ n-pt depend on $m \leq n+2$ -pt vertex functions

Infinite tower of flow equation, needs to truncate $\Gamma_k^{(i > n_{\max})} = 0$

We choose $n_{\max} = 4$, enough to capture the fermion condensates

The tool: Functional renormalization group method

Truncated effective action:

$$\Gamma_k = \int \mathcal{L}_{kin} + \mathcal{L}_{gh} + \mathcal{L}_{gf} + Z_\psi^2 \sum_{i=1}^2 \lambda_i \mathcal{O}_i^\psi + Z_\chi^2 \sum_{i=4}^5 \lambda_i \mathcal{O}_i^\chi + Z_\psi Z_\chi \sum_{i=6}^7 \lambda_i \mathcal{O}_i^{\psi\chi}$$

$$\lambda_i = \lambda_i(p_m)|_{p_m \rightarrow 0}$$

$$\left\{ \begin{array}{l} \mathcal{O}_1^\psi = (\psi^\dagger \bar{\sigma}^\mu \psi)(\psi^\dagger \bar{\sigma}^\mu \psi) \\ \mathcal{O}_2^\psi = (\psi^\dagger{}^{f_1} \bar{\sigma}^\mu \psi_{f_2})(\psi^\dagger{}^{f_2} \bar{\sigma}^\mu \psi_{f_1}) \\ \mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi) \\ \mathcal{O}_6^{\chi\psi} = (\psi^\dagger \bar{\sigma}^\mu \psi)(\chi^\dagger \bar{\sigma}^\mu \chi) \\ \mathcal{O}_7^{\chi\psi} = (\psi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \psi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi) \end{array} \right.$$

Using Young Tensor Method

[HLL,Pastor-Gutierrez,Vatani,Xu
2507.21208]

And prove it's valid for any N_c

[HLL,Pastor-Gutierrez,Vatani,Xu 2507.21208]

The tool: Functional renormalization group method

$$\partial_t \text{---}^{-1} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---}$$

$$\partial_t \text{---}^{-1} = \text{---} \text{---} \text{---} - 2 \text{---} \text{---} \text{---} - \frac{1}{2} \text{---} \text{---} \text{---}$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} = - \text{---} \text{---} \text{---} + 2 \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} = + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} - 2 \text{---} \text{---} \text{---} - \text{---} \text{---} \text{---} + \text{perm.}$$

$$\partial_t \text{---} \propto \alpha_g^2 \text{---} + \alpha_g \lambda_{4\psi} \text{---} + \lambda_{4\psi}^2 \text{---}$$

Finally we solve a coupled differential equations

$$G_{AA,k}^{-1}(p) = \tilde{Z}_{A,k} (p^2 + m_{\text{gap},k}^2) \Pi_{\mu\nu}^{\perp}(p)$$

$$\{\bar{m}_{\text{gap}}, \tilde{Z}_A, Z_c, Z_\psi, Z_\chi \quad \text{2-pt}$$

$$g_{A^3}, g_{A^4}, g_{A\bar{c}c}, g_{A\chi^\dagger\chi}, g_{A\psi^\dagger\psi} \quad \text{3-pt}$$

$$\{\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_4, \bar{\lambda}_5, \bar{\lambda}_6, \bar{\lambda}_7, \}_k \quad \text{4-pt}$$

**Physics are in the solutions of
correlation functions.
Confinement,
Dynamical Symmetry breaking**

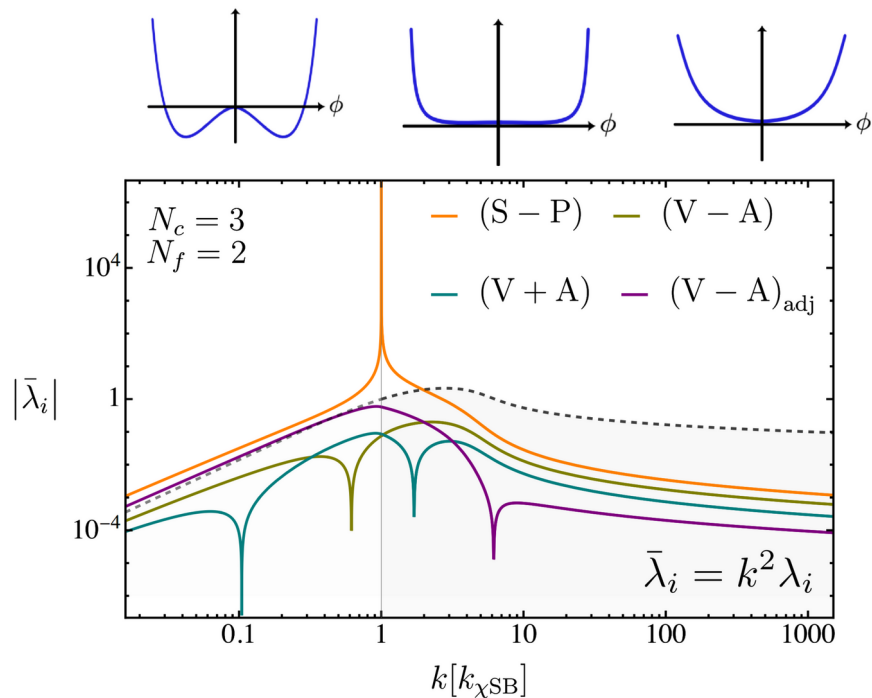
The tool: Functional renormalization group method

➤ How to diagnose Symmetry breaking?

Divergent 4-fermi flow
 $(\psi_1 \psi_2)(\psi_1 \psi_2)$



Fermion condensate
 $\langle \psi_1 \psi_2 \rangle \neq 0$



Exact field redefinition Stratonovich'57 Hubbard'59

$$\begin{array}{c} \vec{p}_1 \\ \downarrow \\ \bullet \\ \uparrow \\ \vec{p}_2 \end{array} \begin{array}{c} \vec{p}_4 \\ \downarrow \\ \bullet \\ \uparrow \\ \vec{p}_3 \end{array} \Big| (\phi) = \sum_{\phi \in \{\sigma, \pi^a\}} \begin{array}{c} \downarrow \\ \bullet \\ \uparrow \end{array} \begin{array}{c} \downarrow \\ \bullet \\ \uparrow \end{array} \Big| \begin{array}{l} (p_1 + p_3)^2 = 0 \\ (p_2 + p_4)^2 = 0 \end{array}$$

$$\lambda_i \propto 1/m_{\phi_i}^2 \qquad m_{\phi_i}^2 = \partial_{\phi_i^2} V(\phi_1)$$

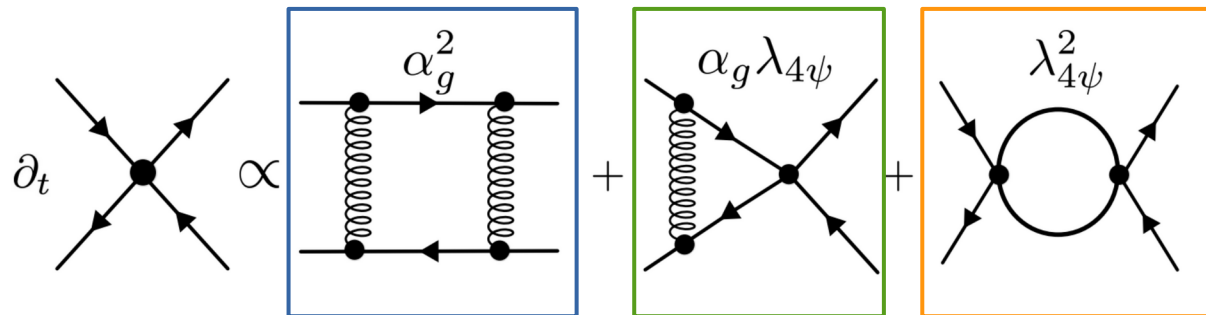
$$\lambda_i \rightarrow \infty \qquad m_{\phi_i}^2 \rightarrow 0 \qquad \xi \rightarrow \infty$$

$$\langle \phi \rangle \sim \langle \bar{\psi} \psi \rangle \neq 0$$

See [2006.04853] for detailed discussion

The tool: Functional renormalization group method

4-fermion flows:



$$\bar{\lambda} = k^2 \lambda$$

$$\partial_t \bar{\lambda}_i \propto 2 \bar{\lambda}_i + \mathbf{c}_{A,i} \cdot \alpha_g^2 + \mathbf{c}_{B,ij} \cdot \alpha_g \bar{\lambda}_j + \mathbf{c}_{C,ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \dots$$

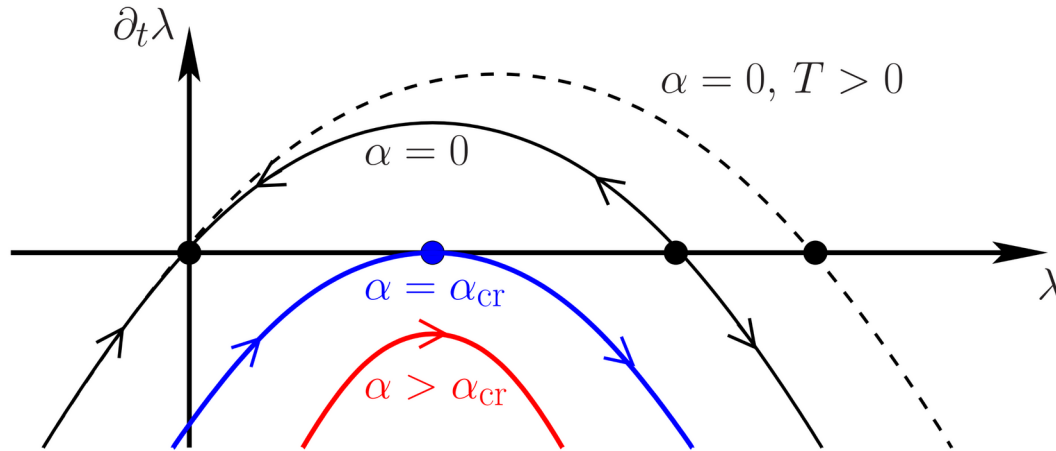


Canonical scaling

The tool: Functional renormalization group method

Focus on one $\bar{\lambda}_i$, the flow is a **quadratic function** of $\bar{\lambda}_i$:

$$\partial_t \bar{\lambda}_i \propto 2 \bar{\lambda}_i + \mathbf{c}_{A,i} \cdot \alpha_g^2 + \mathbf{c}_{B,ij} \cdot \alpha_g \bar{\lambda}_j + \mathbf{c}_{C,ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \dots$$



Necessary condition for dSB:

1. resonant structure (naive)

$$\frac{\mathbf{c}_{A,i}}{\mathbf{c}_{C,iii}} > 0$$

2. α_{SB}^{crit} is reached

two scenarios can prevent α_{SB}^{crit} being reached:

confinement, **conformal**

The tool: Functional renormalization group method

➤ How to diagnose Confinement?

Features of confinement:

- No Asymptotic colored states
- Massive spectrum of bound states (glueballs)
- Existence of gluon mass gap:
 - Wilson area law

Constraints on correlation functions

- Gluon mass gap in 2-pt functions

$$\Gamma_k^{AA}(p) = Z_{A,k}(p)(p^2 + m_{\text{gap},k}^2) = \hat{Z}_{A,k}(p)p^2$$

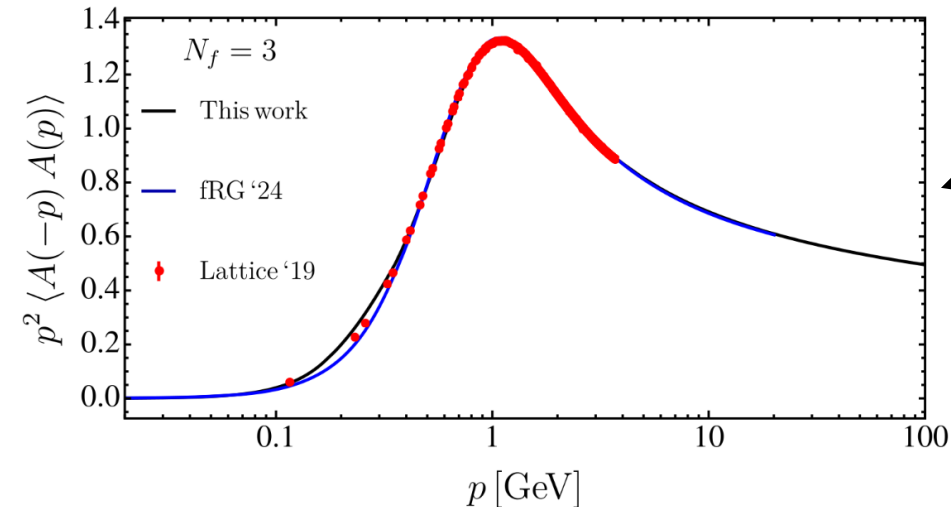
- Kugo-Ojima relation: [\[Kugo, Ojima'79
Nakanishi, Ojima'90 Kugo
hep-th/9511033\]](#)
colorless asymptotic states

$$\lim_{p \rightarrow 0} Z_c(p^2) \sim (p^2)^\kappa \quad \lim_{p \rightarrow 0} Z_A(p^2) \sim (p^2)^{-2\kappa}$$

The tool: Functional renormalization group method

➤ How to diagnose Confinement?

Features of confinement



Constraints on correlation functions in fRG

- Gluon mass gap in 2-pt functions

$$\Gamma_k^{AA}(p) = Z_{A,k}(p)(p^2 + m_{\text{gap},k}^2) = \hat{Z}_{A,k}(p)p^2$$

- Kugo-Ojima relation:
colorless asymptotic states

[Kugo, Ojima'79
Nakanishi, Ojima'90 Kugo
hep-th/9511033]

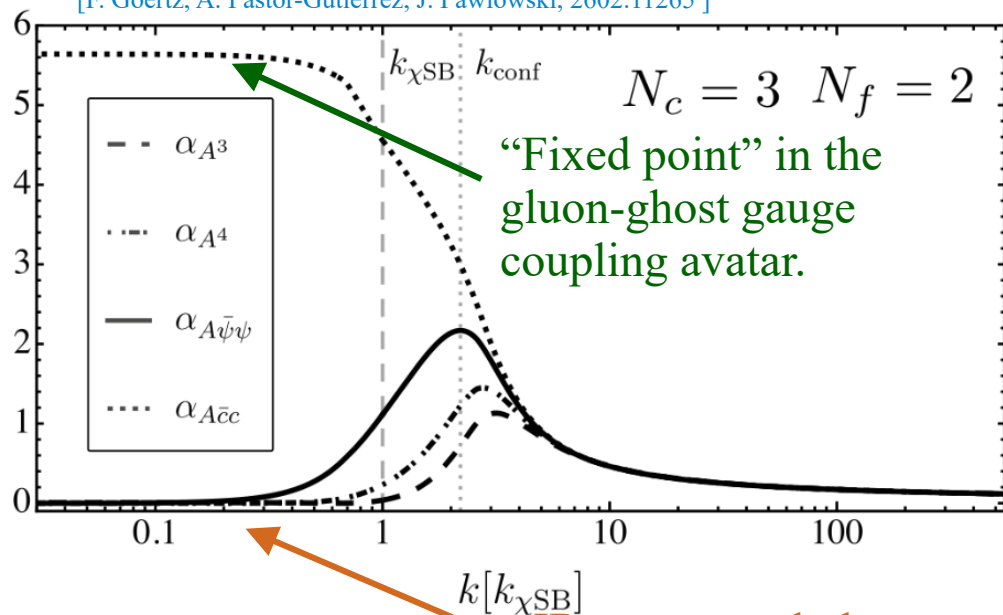
$$\lim_{p \rightarrow 0} Z_c(p^2) \sim (p^2)^\kappa \quad \lim_{p \rightarrow 0} Z_A(p^2) \sim (p^2)^{-2\kappa}$$

The tool: Functional renormalization group method

➤ How to diagnose Confinement?

$$Z_{A,\bar{k}}(p^2)p^2 \approx Z_{A,k}(k^2 + m_{\text{gap},k}^2)|_{k \approx p}$$

[F. Goertz, A. Pastor-Gutiérrez, J. Pawłowski, 2602.11265]



IR suppressed gluon-fermion, gluon-self gauge coupling avatars

Constraints on correlation functions in fRG

Gluon mass gap in 2-pt functions

$$\Gamma_k^{AA}(p) = Z_{A,k}(p)(p^2 + m_{\text{gap},k}^2) = \hat{Z}_{A,k}(p)p^2$$

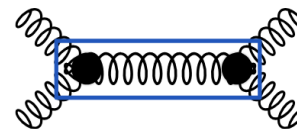
Kugo-Ojima relation:

colorless asymptotic states

[Kugo, Ojima'79
Nakanishi, Ojima'90 Kugo
hep-th/9511033]

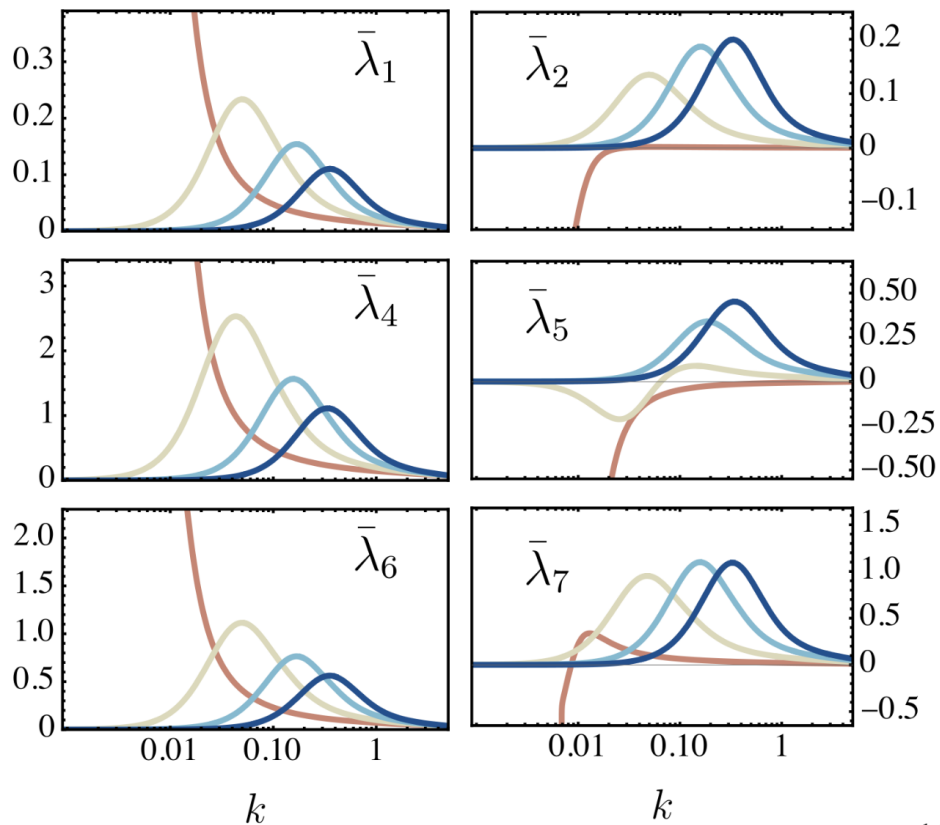
$$\lim_{p \rightarrow 0} Z_c(p^2) \sim (p^2)^\kappa \quad \lim_{p \rightarrow 0} Z_A(p^2) \sim (p^2)^{-2\kappa}$$

$$\alpha_{A^3} = \left[\tilde{\Gamma}_k^{(A^3)} \right]^2 / (4\pi Z_A^3) =$$



Results of BY model

N_c : — 3 — 4 — 5 — 6



$N_c=3$

dominant divergent operators:

$$\mathcal{O}_4^\chi = (\chi^\dagger \bar{\sigma}^\mu \chi)(\chi^\dagger \bar{\sigma}^\mu \chi)$$

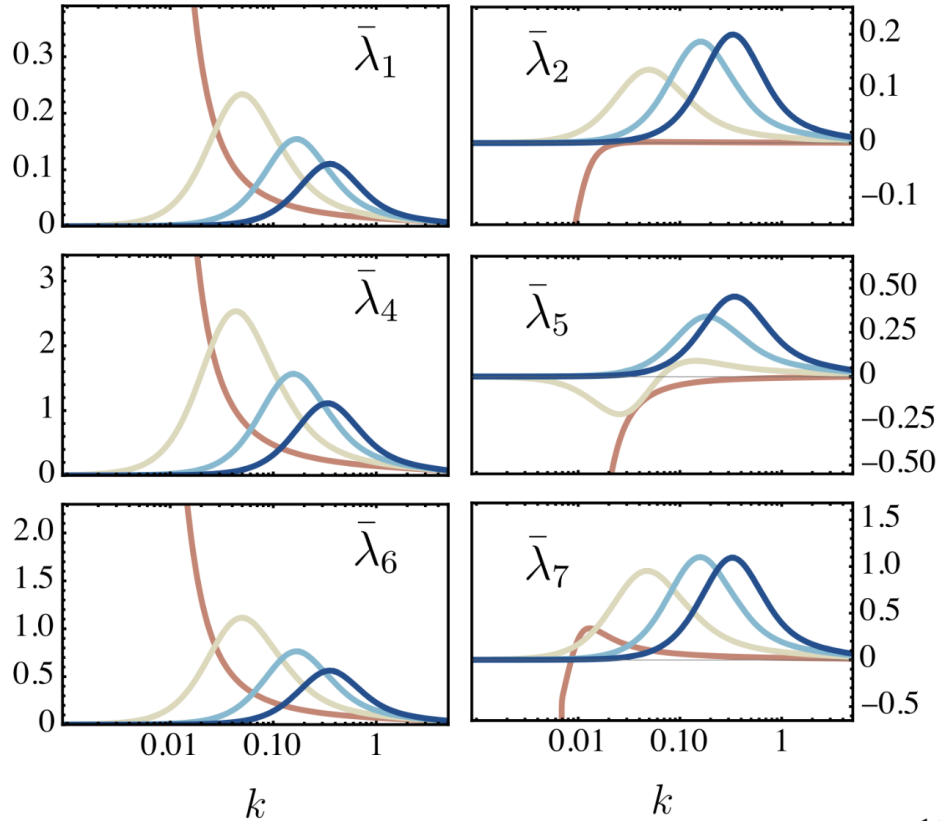
$$\mathcal{O}_5^\chi = (\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{sym}} \chi)$$

-- condensate in IR, dSB

$$\langle \chi \chi \rangle$$

Results of BY model

N_c : — 3 — 4 — 5 — 6

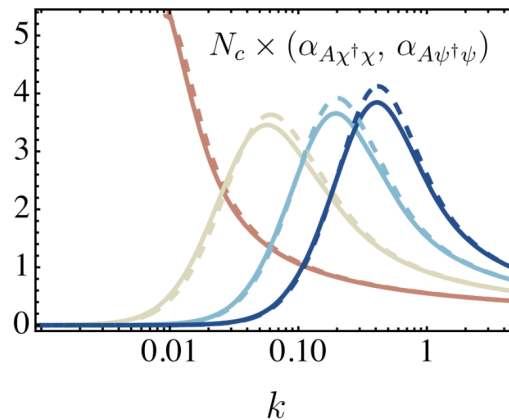
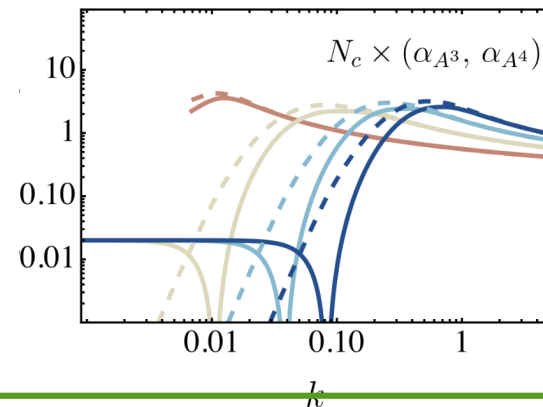
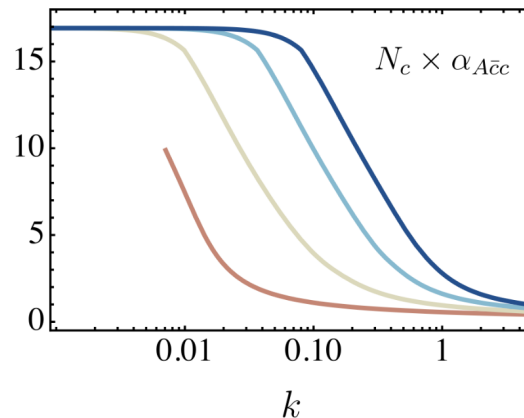
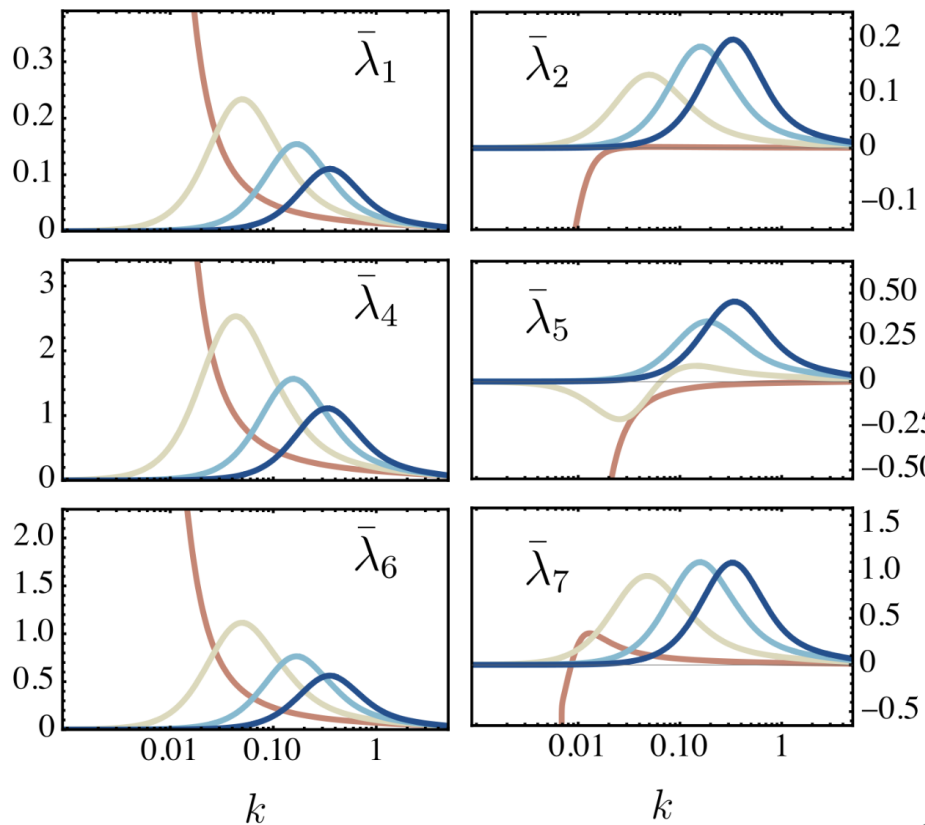


$N_c=4,5,6$

- No divergent 4F operators
- no dynamical SB

Results of BY model

N_c : — 3 — 4 — 5 — 6

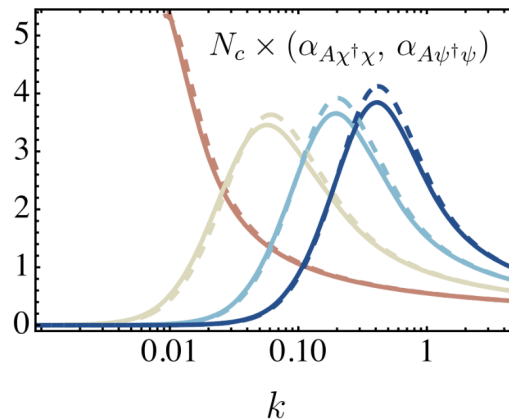
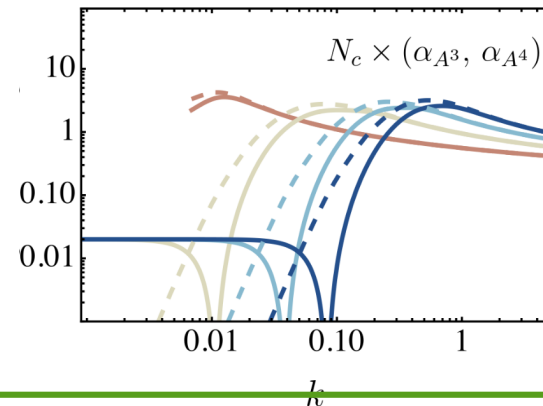
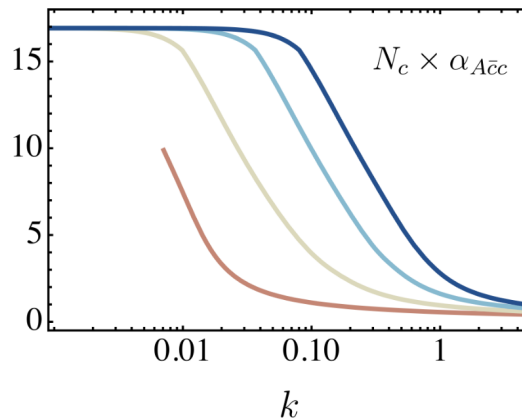
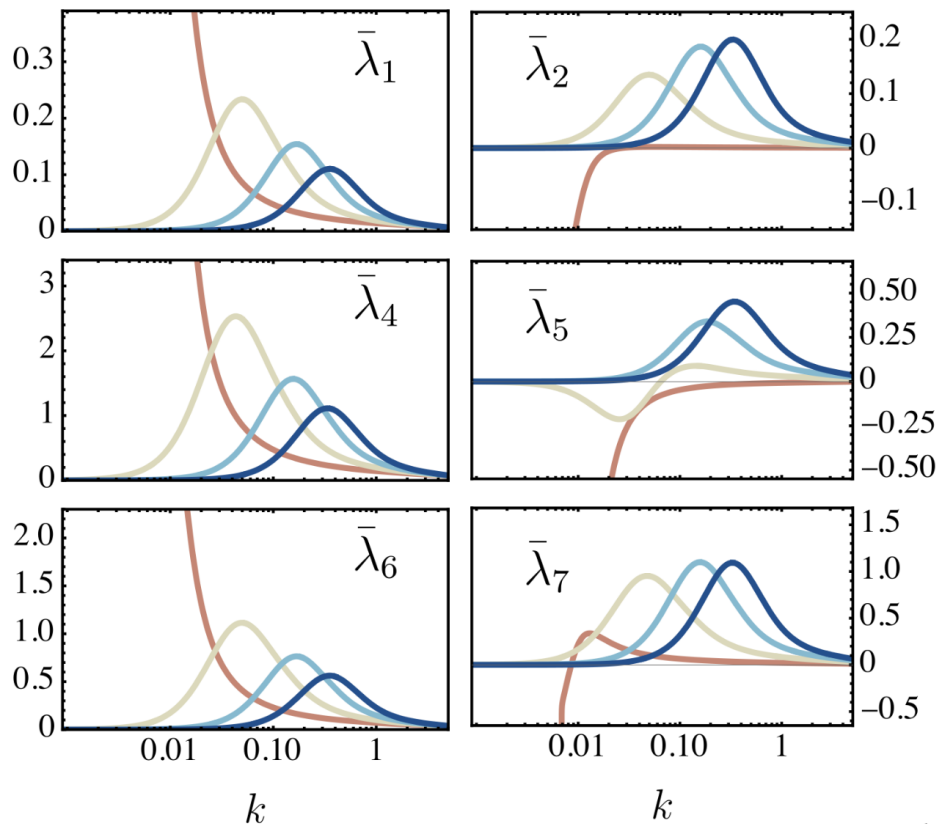


$N_c=4,5,6$

- No divergent 4F operators
 - no dynamical χ SB
- Found Scaling solution satisfies the KO condition
 - confinement in Landau gauge

Results of BY model

N_c : — 3 — 4 — 5 — 6

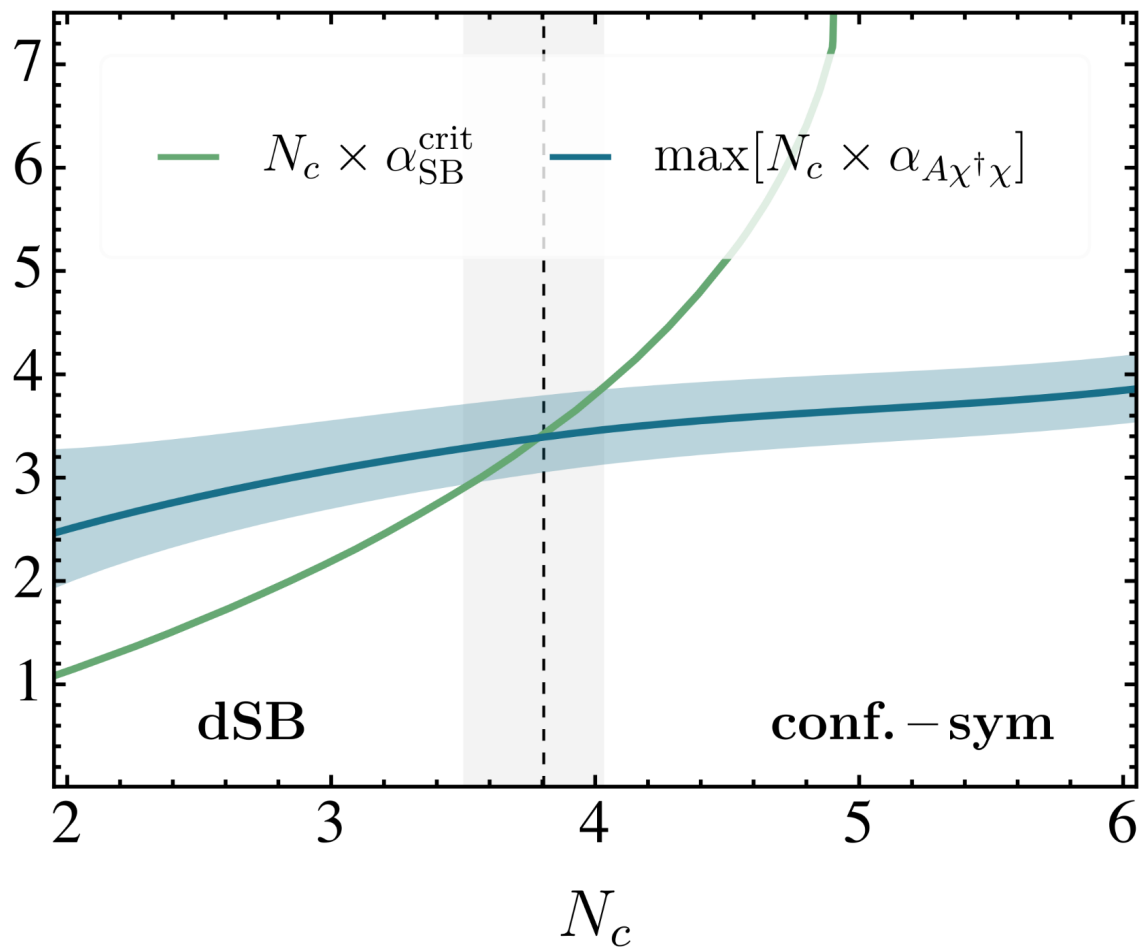


$N_c=4,5,6$

- No divergent 4F operators
 - no dynamical χ SB
- Found Scaling solution satisfies the KO condition
 - confinement in Landau gauge

Evidence for Confinement w/o dynamical symmetry breaking!

Results of BY model



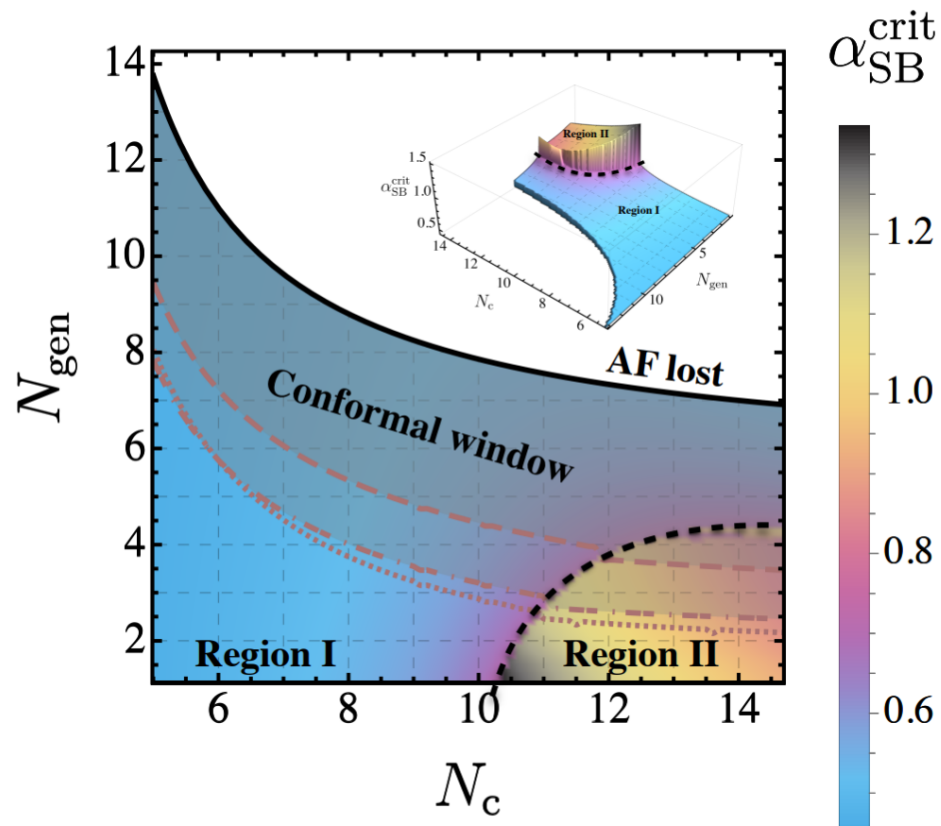
Summary

- We present first fRG study of BY class of chiral gauge theories.
- For $N_c \lesssim 3$ confinement w dSB, $N_c \gtrsim 4$ confinement w/o dSB.

Outlook

- Bosonize the theory to analysis the direction of dSB
- To analysis whether there is massless fermion in IR to match the 't Hooft anomaly
- Can these model help with symmetric mass generation?

Results



- Region I:**

- Weak α_g^{crit} : dynamics derivable within perturbation theory
- Clear dominance of $\mathcal{O}_5 = (\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi)(\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi)$
- Condensate $\langle \chi \chi \rangle \neq 0$

- Region II:**

- strong α_g^{crit} non-perturbative, higher-order effects relevant
- cannot resolve a clear single resonant channel.

- Conformal window:**

$$\alpha_g^{\text{crit}} = \alpha_g^*$$

$$\begin{cases} \alpha_g^*|_{\overline{\text{MS}} \text{ 2-loop}} & \text{---} \\ \alpha_g^*|_{\overline{\text{MS}} \text{ 3-loop}} & \text{---} \\ \alpha_g^*|_{\overline{\text{MS}} \text{ 4-loop}} & \text{---} \end{cases} \quad \text{New!}$$

$$S = \int \bar{\psi} \not{\partial} \psi - \frac{\lambda}{2} \left((\bar{\psi} \psi)^2 - (\bar{\psi} \gamma^5 \psi)^2 \right)$$

$$Z \propto \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^S = \mathcal{N} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\phi \ e^{\frac{m^2}{2} \phi^2} e^S \qquad \phi = (\sigma, \pi)$$

$$\sigma \rightarrow \sigma + y \frac{\bar{\psi} \psi}{\sqrt{2} m^2} \qquad \pi \rightarrow \pi + i \ y \frac{\bar{\psi} \gamma^5 \psi}{\sqrt{2} m^2}$$

$$Z \propto \mathcal{N} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\phi e^{S_{BF}} \qquad S_{FB} = \int \bar{\psi} \partial \psi + \frac{y}{\sqrt{2}} \bar{\psi} \left(\sigma + i \pi \gamma^5 \right) \psi + \frac{m^2}{2} \left(\sigma^2 + \pi^2 \right) \qquad \lambda \equiv \frac{y^2}{2 m^2}$$

$$\sigma = \frac{-y}{\sqrt{2} m^2} \bar{\psi} \psi \qquad \pi = \frac{-i y}{\sqrt{2} m^2} \bar{\psi} \gamma^5 \psi \qquad S_B = \int d^4 x \left\{ m^2 \phi^* \phi - \ln \det \left[i \not{\partial} + h \left(P_L \phi - P_R \phi^* \right) \right] \right\}$$

$$U(1)_A \qquad \begin{pmatrix} \sigma \\ \pi \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$$

$$\partial_t \Gamma[\bar{\phi}] = \frac{1}{2} \text{Tr} G_k \partial_t R_k ,$$

$$\partial_t \Gamma^{(1)}[\bar{\phi}] = -\frac{1}{2} \text{Tr} \Gamma_k^{(3)} \left(G_k \partial_t R_k G_k \right) ,$$

$$\partial_t \Gamma^{(2)}[\bar{\phi}] = -\frac{1}{2} \text{Tr} \left[\Gamma_k^{(4)} - 2 \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right] \left(G_k \partial_t R_k G_k \right) ,$$

$$\partial_t \Gamma^{(3)}[\bar{\phi}] = -\frac{1}{2} \text{Tr} \left[\Gamma_k^{(5)} - 6 \Gamma_k^{(4)} G_k \Gamma_k^{(3)} + 6 \Gamma_k^{(3)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right] \left(G_k \partial_t R_k G_k \right) ,$$

$$\begin{aligned} \partial_t \Gamma^{(4)}[\bar{\phi}] = & -\frac{1}{2} \text{Tr} \left[\Gamma_k^{(6)} - 8 \Gamma_k^{(5)} G_k \Gamma_k^{(3)} - 6 \Gamma_k^{(4)} G_k \Gamma_k^{(4)} + 18 \Gamma_k^{(4)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right. \\ & \left. + 12 \Gamma_k^{(3)} G_k \Gamma_k^{(4)} G_k \Gamma_k^{(3)} - 24 G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \cdot G_k \Gamma_k^{(3)} \right] \left(G_k \partial_t R_k G_k \right) , \end{aligned}$$

$$\begin{aligned}
\Gamma_{\text{gauge},k}[A, c, \bar{c}] &= \frac{1}{2} \int_p A_\mu^a(p) \left[\tilde{Z}_{A,k} (p^2 + m_{\text{gap},k}^2) \Pi_{\mu\nu}^\perp(p) + \frac{1}{\xi} Z_{A,k}^\parallel (p^2 + m_{\text{mSTI},k}^2) \frac{p_\mu p_\nu}{p^2} \right] A_\nu^a(-p) \\
&+ \frac{1}{3!} \int_{p_1, p_2} \tilde{Z}_{A,k}^{3/2} g_{A^3,k} \left[\mathcal{T}_{A^3}^{(1)}(p_1, p_2) \right]_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3} \prod_{i=1}^3 A_{\mu_i}^{a_i}(p_i) + \frac{1}{4!} \int_{p_1, p_2, p_3} \tilde{Z}_{A,k}^2 g_{A^4,k} \left[\mathcal{T}_{A^4}^{(1)}(p_1, p_2, p_3) \right]_{\mu_1 \mu_2 \mu_3 \mu_4}^{a_1 a_2 a_3 a_4} \prod_{i=1}^4 A_{\mu_i}^{a_i}(p_i) \\
&+ \int_p Z_{c,k} \bar{c}^a(p) p^2 \delta^{ab} c^b(-p) + \int_{p_1, p_2} Z_{c,k} \tilde{Z}_{A,k}^{1/2} g_{c\bar{c}A,k} \left[\mathcal{T}_{A\bar{c}c}^{(1)}(p_1, p_2) \right]_\mu^{a_1 a_2 a_3} \bar{c}^{a_2}(p_2) c^{a_1}(p_1) A_\mu^{a_3}(-p_1 - p_2). \quad (13)
\end{aligned}$$

$$\begin{aligned}
\Gamma_{\text{gauge-fermion},k} [A_\mu, \psi^\dagger, \psi, \chi^\dagger, \chi] &= \\
&\int_p Z_{\psi,k} \psi^{\dagger, i, f}(p) \bar{\sigma}_\mu \partial_\mu \psi^{i, f}(-p) - \text{i} \int_{p_1, p_2} \tilde{Z}_{\psi,k} \tilde{Z}_{A,k}^{1/2} g_{A\psi^\dagger\psi,k} \psi^{\dagger, i_1 f_1}(p_2) \left[\mathcal{T}_{A\psi^\dagger\psi}^{(1)}(p_1, p_2) \right]_\mu^{a i_1 i_2 f_1 f_2} A_\mu^a(-p_1 - p_2) \psi^{i_2 f_2}(p_1) \\
&+ \int_p Z_{\chi,k} \chi^{\dagger, \alpha}(p) \bar{\sigma}_\mu \partial_\mu \chi^\alpha(-p) - \text{i} \int_{p_1, p_2} Z_{\chi,k} \tilde{Z}_{A,k}^{1/2} g_{A\chi^\dagger\chi,k} \chi^{\dagger, \alpha_1}(p_2) \left[\mathcal{T}_{A\chi^\dagger\chi}^{(1)}(p_1, p_2) \right]_\mu^{a \alpha_1 \alpha_2} A_\mu^a(-p_1 - p_2) \chi^{\alpha_2}(p_1)
\end{aligned} \quad (15)$$

$$\{g_{A^3}, g_{A^4}, g_{A\bar{c}c}, g_{A\chi^\dagger\chi}, g_{A\psi^\dagger\psi}, \bar{m}_{\text{gap}}, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_4, \bar{\lambda}_5, \bar{\lambda}_6, \bar{\lambda}_7, \tilde{Z}_A, Z_c, Z_\psi, Z_\chi\}.$$

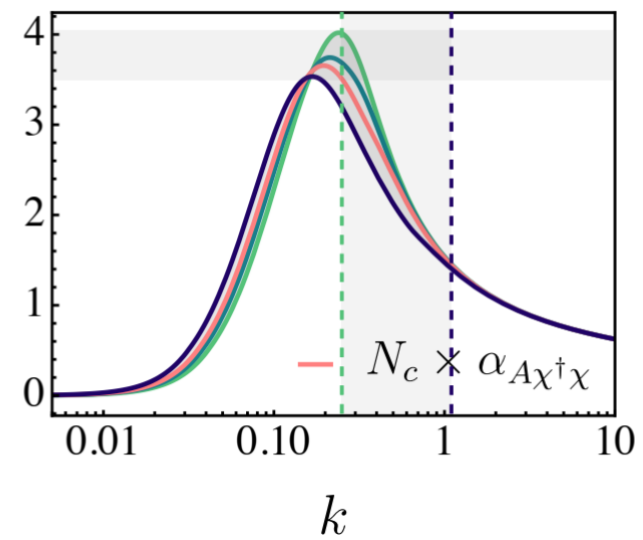
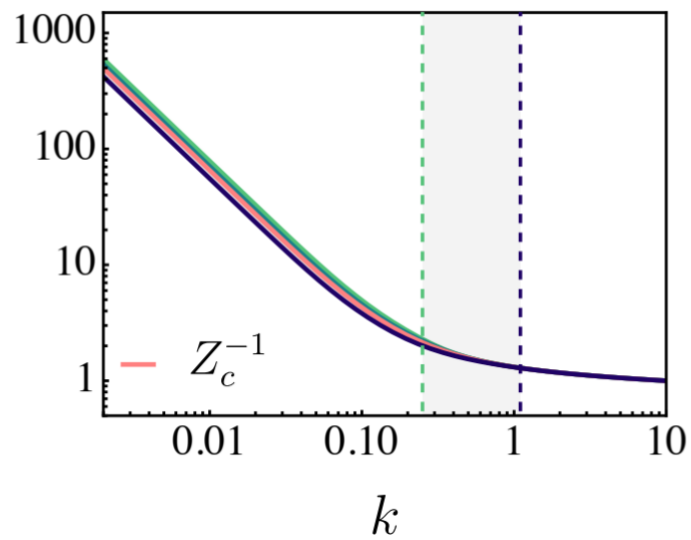
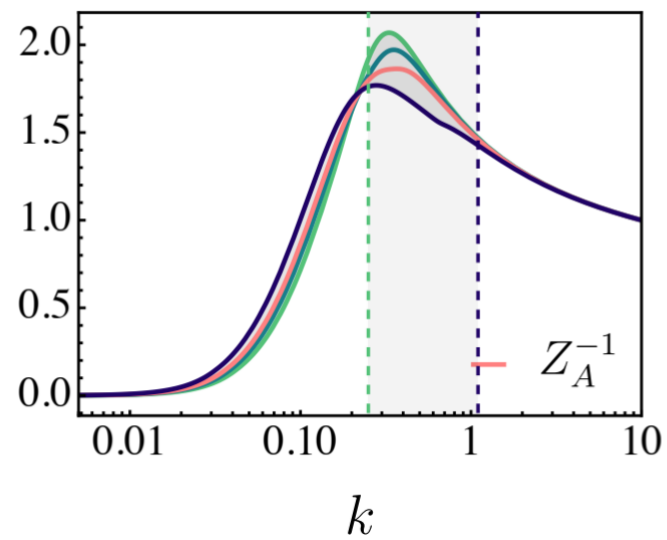
- Insertion of mass-like regulator: Slavnov-Taylor identities (STIs) for gauge invariance $\rightarrow$ modified STIs (mSTIs)

$$\int_x \frac{\delta \Gamma}{\delta \mathcal{Q}_i(x)} \frac{\delta \Gamma}{\delta \Phi_i(x)} = 0 \quad \longrightarrow \quad \int_x \frac{\delta \Gamma_k}{\delta \mathcal{Q}_i(x)} \frac{\delta \Gamma_k}{\delta \Phi_i(x)} = \text{Tr} R^{ij} G_{jl} \frac{\delta^2 \Gamma_k[\Phi, \mathcal{Q}]}{\delta \Phi_l \delta \mathcal{Q}^i} \quad \begin{array}{l} \Phi_i = \{A_\mu, c, \bar{c}, \dots\} \\ \mathcal{Q}^i : \text{BRST sources of the } \Phi_i \text{ fields} \end{array}$$

- In the physical limit $k \rightarrow 0$ and $R_k \rightarrow 0$ and mSTI $\rightarrow$ STI and no gauge symmetry breaking is present (*)
- “easy” approach: physics at k -dependent correlation functions
 - Minimise mSTI and STI deviations for $k \neq 0$
 - Hands-on approach: include problematic power-law corrections to the flow of the gauge mass gap only in the confinement regime

$$\partial_t \bar{m}_{\text{gap}}^2 = (-2 + \eta_A) \bar{m}_{\text{gap}}^2 + \underbrace{\overline{\text{Flow}}_{AA}(p^2 = 0)}$$

- Upgrade with full momentum dependencies



't Hooft anomaly can be matched by $\mathcal{B} = \chi\psi\psi$

$$Z_{A,\bar{k}}(p^2)p^2 \approx Z_{A,k}(k^2 + m_{\text{gap},k}^2)|_{k \approx p},$$

$$G_{A,k}(p) = \frac{1}{Z_{A,k}(p^2)p^2 + R_{A,k}(p)}$$

Gluon propagator dressing: $p^2 \langle A(-p)A(p) \rangle = p^2 \left[\Gamma_k^{(AA)}(p^2 = 0) \right]_{k=c p}^{-1}$

Goertz,APG,Pawlowski[2412.12254]

