第九届强子谱和强子结构研讨会
2026.8.17-21，河南新乡

Unified study of scalar, vector and tensor two-meson form factors in resonance chiral theory

Zhi-Hui Guo (郭志辉)

Hebei Normal University (河北师范大学)

arXiv: 2605.14875

in collaboration with 郝津，段春贵，J.A.Oller, J.Ruiz de Elvira

Introduction

- **Form factor(FF): a key object to probe hadrons**

FF can quantify the response of hadrons with EM/ EW external currents.

- **Different types of two-meson (P、Q) FFs**

Scalar FF (Higgs)

$$F^{mn}_{S,PQ}(s) = \langle 0|\bar{q}^m q^n|PQ\rangle$$

Vector FF (W/Z/γ)

$$\langle 0|\bar{q}^m \gamma^\mu q^n|P(p_1)Q(p_2)\rangle = (p_2 - p_1)^\mu F^{mn}_{V+,PQ}(s) + (p_2 + p_1)^\mu F^{mn}_{V-,PQ}(s)$$

Anti-symmetric Tensor FF (BSM new physics)

$$\langle 0|\bar{q}^m \sigma^{\mu\nu} q^n|P(p_1)Q(p_2)\rangle = i\frac{\Lambda_2^T}{F_\pi^2}(p_1^\mu p_2^\nu - p_1^\nu p_2^\mu)F^{mn}_{T,PQ}(s)$$

➢ Key inputs for important reactions allowed by SM

$$P \to Q\ell\nu \qquad D_{(s)}^{(*)}/B_{(s)}^{(*)} \to PQ\ell\nu/\ell^+\ell^-$$

$$\tau \to PQ\ell\nu \qquad D_{(s)}^{(*)}/B_{(s)}^{(*)} \to P_1P_2\gamma^{(*)}$$

... ...

$$e^+e^- \to PQ \qquad J/\psi \to P_1P_2\gamma^{(*)}$$

➢ Critical inputs to constrain the BSM new physics

(1) axion/axion-like particle production in meson decays

(2) CPV in tau/B/D decays involving light mesons

(3) SM precision tests for new physics: rare decays, ...

... ...

- **We focus on the two-meson FFs in the low and intermediate energy regions (below ~1.2 GeV)**
  - **low energy: chiral symmetry**
  - **intermediate region: resonance dyanmics**
  - **asymptotic region: pQCD behavoir**
- **Theoretical frameworks**
  - **Resonance chiral theory**
  - **U(3) chiral EFT: QCD $U_A(1)$ anomlay (η/η')**

Resonance chiral theory (RχT)

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

Resonances : $R \xrightarrow{G} h\,R\,h^\dagger, \quad h \in H$

pNGB and external sources : $X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu}$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-{\Gamma_\mu}^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$\begin{aligned}
\mathcal{L}_{2V} &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \\
\mathcal{L}_{2A} &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle\,, \\
\mathcal{L}_{2S} &= c_d\langle\, S u_\mu u^\mu\,\rangle + c_m\langle\, S\chi_+\,\rangle\,, \\
\mathcal{L}_{2P} &= i d_m\langle\, P\chi_-\,\rangle.
\end{aligned}$$

Operators beyond minimal
[Cirigliano, *et al.*, '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA}\langle[V^{\mu\nu}, A_{\mu\nu}]\chi_-\rangle + \ldots\,,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$\begin{aligned}
\mathcal{L}_{VVP} &= d_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots\,, \\
\mathcal{L}_{VJP} &= \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots\,,
\end{aligned}$$

$$d_4\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\rangle$$

QCD dynamics in RχT

- Low energy QCD: implemented from the construction of RχT
- Intermediate energy: explicit resonance states
- High energy information: to match the same physical objects in RχT and QCD, $\langle J(x_n) \cdot \cdot J(0) \rangle^{\mathrm{R}\chi\mathrm{T}} = \langle J(x_n) \cdot \cdot J(0) \rangle^{\mathrm{QCD}}$.

  For example: $\pi\pi$ vector form factor

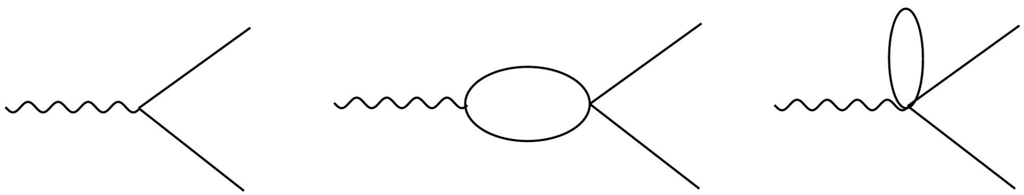
$$
\begin{aligned}
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{R}\chi\mathrm{T}} &= 1 + \frac{F_V G_V}{F^2} \frac{q^2}{M_V^2 - q^2}, \\
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{QCD}} &\to 0, \qquad \text{for } q^2 \to \infty
\end{aligned}
$$

  This leads to

$$
[\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{R}\chi\mathrm{T}} = [\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{QCD}} \Longrightarrow F_V G_V = F^2
$$

Form factors in RχT

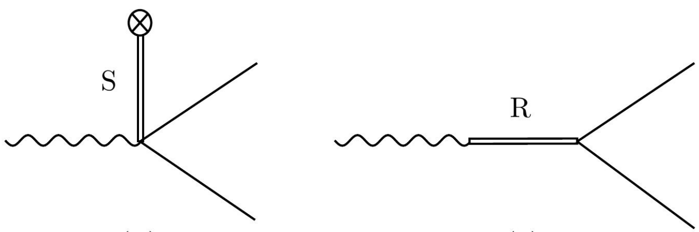
- $$\mathcal{L}_0 = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{3}M_0^2 \ln^2 \det u \qquad \mathcal{L}_T^{(4)} = -i\Lambda_2^T \langle t_+^{\mu\nu} u_\mu u_\nu\rangle$$



(a) (b) (c)

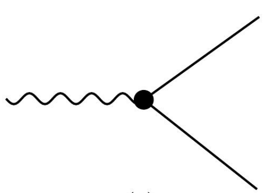
- $$\mathcal{L}_{2,V} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle \qquad \mathcal{L}_{T,V} = F_V^T M_V \langle V_{\mu\nu} t_+^{\mu\nu}\rangle$$

$$\mathcal{L}_S = c_d\langle S_8 u_\mu u^\mu\rangle + c_m\langle S_8\chi_+\rangle + \tilde{c}_d S_1\langle u_\mu u^\mu\rangle + \tilde{c}_m S_1\langle \chi_+\rangle$$



(d) (e)

- $$\mathcal{L}_\Lambda = \Lambda_1 \frac{F^2}{12} D_\mu\psi D^\mu\psi - i\Lambda_2 \frac{F^2}{12}\langle U^\dagger\chi - \chi^\dagger U\rangle\psi \qquad \mathcal{L}_P = id_m\langle P_8\chi_-\rangle + i\tilde{d}_m P_1\langle \chi_-\rangle \qquad \frac{\delta L_8}{2}\langle \chi_+\chi_+ + \chi_-\chi_-\rangle$$



(f)

Unitarity relation for two-meson form factors

$$\mathrm{Im}\, F^{I}_{S,j}(s) = \sum_k T^{IJ}_{jk}(s)^* \,\rho_k(s)\, F^{I}_{S,k}(s)$$

$$\rho_k(s) = \frac{q_k(s)}{8\pi\sqrt{s}}$$

$T^{IJ}(s)$: meson-meson scattering amplitudes

- **Perturbative form factors in RχT do not fulfill this unitarity relation**
- **Elastic FF (single channel) can be exactly solved: Omnes function**
- **Coupled-channel FFs in dispersive formalism: much more involved**

  more subtractions are needed

  phases at s→∞ are needed

  $T^{IJ}(s)$ in a broad energy range are needed

Unitarized form factors in a simple but efficient approach

$$F_S^I(s) = \left[1 + N^{IJ}(s) \cdot g^{IJ}(s)\right]^{-1} \cdot R_S^I(s)$$

$$\mathrm{Im} g_k^{IJ}(s) = -\rho_k(s)$$

$$N^{IJ}(s) = T^{IJ}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}} + T^{IJ}(s)^{(2)} \cdot g^{IJ}(s) \cdot T^{IJ}(s)^{(2)}$$

$$R_S^I(s) = F_S^I(s)^{(2)+\mathrm{Res}+\mathrm{Loop}} + N^{IJ}(s)^{(2)} \cdot g^{IJ}(s) \cdot F_S^I(s)^{(2)}$$

- **Unitarity relation is fulfilled above thresholds**
- **Perturbative FF is exactly reproduced at the order it is calculated**
- **Pure predictions from scattering (FF and T^{IJ} share the same parameters)**
- **Asymptotic 1/*s* behavior, as dictated by perturbative QCD, can be recovered in the chiral and large Nc limits** [Guo, Oller, Ruiz de Elvira, PRD'12]

Two-meson channels coupled to scalar currents

Isospin, Strangeness	Scalar currents	Two-meson channels
$I=0, S=0$	$(\bar{u}u+\bar{d}d)/\sqrt{2}$	$\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$
	$\bar{s}s$	$\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$
$I=1, S=0$	$\bar{u}d$	$\pi\eta$, $K\bar{K}$, $\pi\eta'$
$I=\frac{1}{2}, S=-1$	$\bar{u}s$	$\bar{K}\pi$, $\bar{K}\eta$, $\bar{K}\eta'$

Two-meson channels coupled to vector currents

Isospin, Strangeness	Vector currents	Two-meson channels
$I=0, S=0$	$\frac{\bar{u}\gamma^\mu u+\bar{d}\gamma^\mu d}{\sqrt{2}}$	$K\bar{K}$
	$\bar{s}\gamma^\mu s$	$K\bar{K}$
$I=1, S=0$	$\bar{u}\gamma^\mu d$	$\pi\pi$, $K\bar{K}$
$I=\frac{1}{2}, S=-1$	$\bar{u}\gamma^\mu s$	$\bar{K}\pi$, $\bar{K}\eta$, $\bar{K}\eta'$

- **Same as the tensor currents**

Phenomenological discussions

Parameter inputs [Guo,Oller,PRD'11] [Guo,Oller,Ruiz de Elvira,PRD'12] [Guo,Liu,Meissner,Oller,Rusetsky,PRD'17]

$$c_d = (19.8^{+2.0}_{-5.2})\ \text{MeV}, \quad c_m = (41.9^{+3.9}_{-9.2})\ \text{MeV}, \quad M_{S_8} = (1397^{+73}_{-61})\ \text{MeV},$$
$$M_{S_1} = (1100^{+30}_{-63})\ \text{MeV}, \quad M_\rho = (801.2^{+8.2}_{-6.9})\ \text{MeV}, \quad M_{K^*} = (910.0^{+7.0}_{-9.1})\ \text{MeV},$$
$$G_V = (62.1^{+1.9}_{-2.1})\ \text{MeV}, \quad M_0 = (951^{+50}_{-50})\ \text{MeV}, \quad a_{SL}^{00,\pi\pi} = -1.27^{+0.12}_{-0.12},$$
$$a_{SL}^{\frac{1}{2}0,K\pi} = -1.12^{+0.12}_{-0.17}, \quad a_{SL}^{\frac{1}{2}0,K\eta'} = -1.25^{+1.11}_{-1.23}, \quad a_{SL}^{10,\pi\eta} = 2.0^{+3.3}_{-4.5}, \quad a_{SL}^{00,K\bar{K}} = -0.95^{+0.33}_{-0.16}$$
$$a_{SL}^{\frac{1}{2}0,K\eta} = -0.08^{+0.38}_{-1.04}, \quad \delta L_8 = 0.23^{+0.29}_{-0.19} \times 10^{-3}, \quad \Lambda_2 = -0.37^{+0.19}_{-0.19},$$

- **Fits to experimental (scattering/event distribution) and lattice (finite-volume spectra) data**

- **F_V = 130 MeV, Λ_2^T = 12 MeV, $F_V^T = F_V/\sqrt{2}$**

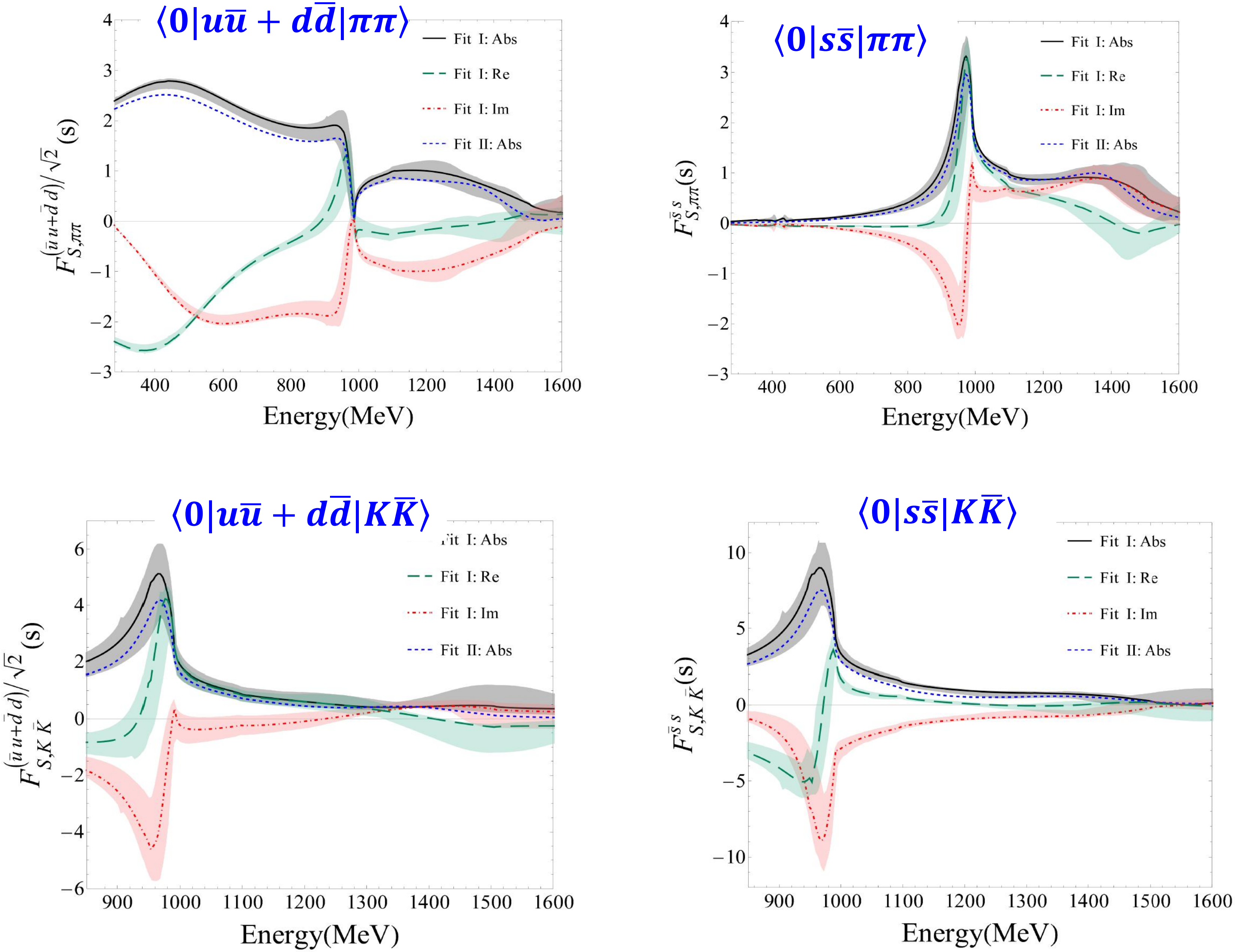
[Miranda,Roig,JHEP'18]

Pole positions and residues for the resonances appearing in our amplitudes

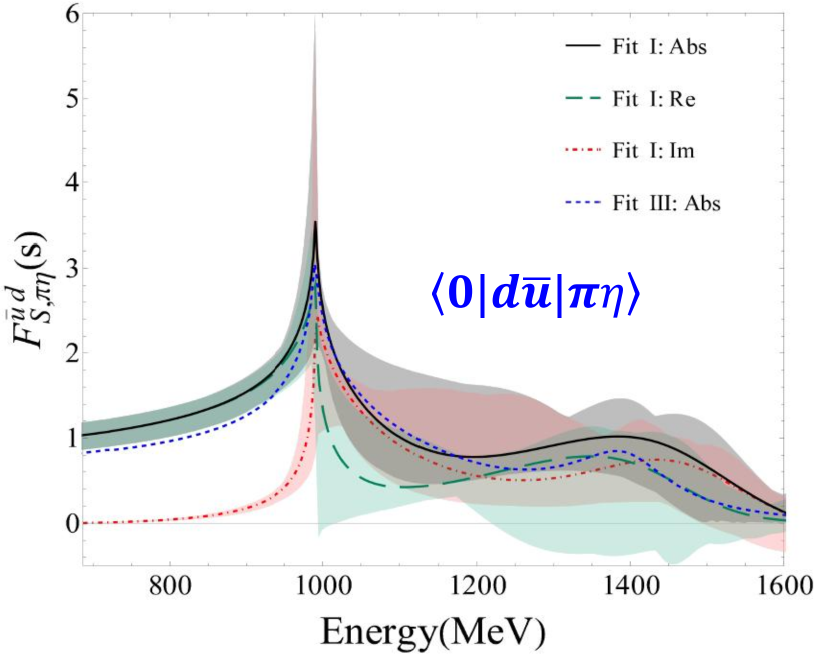
R	M (MeV)	$\Gamma/2$ (MeV)	$\lvert\text{Residues}\rvert^{1/2}$ (GeV)	Ratios	
$f_0(500)$	442^{+4}_{-4}	246^{+7}_{-5}	$3.02^{+0.03}_{-0.04}(\pi\pi)$	$0.50^{+0.04}_{-0.08}(K\bar{K}/\pi\pi)$	$0.17^{+0.09}_{-0.09}(\eta\eta/\pi\pi)$
				$0.33^{+0.06}_{-0.10}(\eta\eta'/\pi\pi)$	$0.11^{+0.05}_{-0.06}(\eta'\eta'/\pi\pi)$
$f_0(980)$	978^{+17}_{-11}	29^{+9}_{-11}	$1.8^{+0.2}_{-0.3}(\pi\pi)$	$2.6^{+0.2}_{-0.3}(K\bar{K}/\pi\pi)$	$1.6^{+0.4}_{-0.2}(\eta\eta/\pi\pi)$
				$1.0^{+0.3}_{-0.2}(\eta\eta'/\pi\pi)$	$0.7^{+0.2}_{-0.3}(\eta'\eta'/\pi\pi)$
$f_0(1370)$	1360^{+80}_{-60}	170^{+55}_{-55}	$3.2^{+0.6}_{-0.5}(\pi\pi)$	$1.0^{+0.7}_{-0.3}(K\bar{K}/\pi\pi)$	$1.2^{+0.7}_{-0.3}(\eta\eta/\pi\pi)$
				$1.5^{+0.4}_{-0.5}(\eta\eta'/\pi\pi)$	$0.7^{+0.2}_{-0.3}(\eta'\eta'/\pi\pi)$
$K_0^*(800)$	643^{+75}_{-30}	303^{+25}_{-75}	$4.8^{+0.5}_{-1.0}(K\pi)$	$0.9^{+0.2}_{-0.3}(K\eta/K\pi)$	$0.7^{+0.2}_{-0.3}(K\eta'/K\pi)$
$K_0^*(1430)$	1482^{+55}_{-110}	132^{+40}_{-90}	$4.4^{+0.2}_{-1.1}(K\pi)$	$0.3^{+0.3}_{-0.3}(K\eta/K\pi)$	$1.2^{+0.2}_{-0.2}(K\eta'/K\pi)$
$a_0(980)$	1007^{+75}_{-10}	22^{+90}_{-10}	$2.4^{+3.2}_{-0.4}(\pi\eta)$	$1.9^{+0.2}_{-0.5}(K\bar{K}/\pi\eta)$	$0.03^{+0.10}_{-0.03}(\pi\eta'/\pi\eta)$
$a_0(1450)$	1459^{+70}_{-95}	174^{+110}_{-100}	$4.5^{+0.6}_{-1.7}(\pi\eta)$	$0.4^{+1.2}_{-0.2}(K\bar{K}/\pi\eta)$	$1.0^{+0.8}_{-0.3}(\pi\eta'/\pi\eta)$
$\rho(770)$	760^{+7}_{-5}	71^{+4}_{-5}	$2.4^{+0.1}_{-0.1}(\pi\pi)$	$0.64^{+0.01}_{-0.02}(K\bar{K}/\pi\pi)$	
$K^*(892)$	892^{+5}_{-7}	25^{+2}_{-2}	$1.85^{+0.07}_{-0.07}(K\pi)$	$0.91^{+0.03}_{-0.02}(K\eta/K\pi)$	$0.41^{+0.07}_{-0.06}(K\eta'/K\pi)$
$\phi(1020)$	$1019.1^{+0.5}_{-0.6}$	$1.9^{+0.1}_{-0.1}$	$0.85^{+0.01}_{-0.02}(K\bar{K})$		

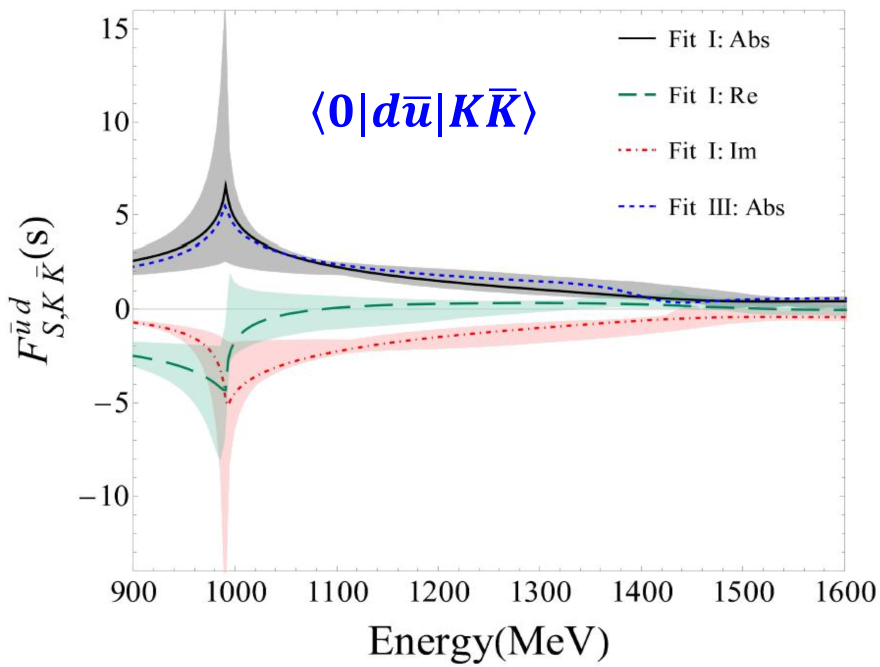
➢ **Question: how such resonances manifest themselves in the form factors ?**

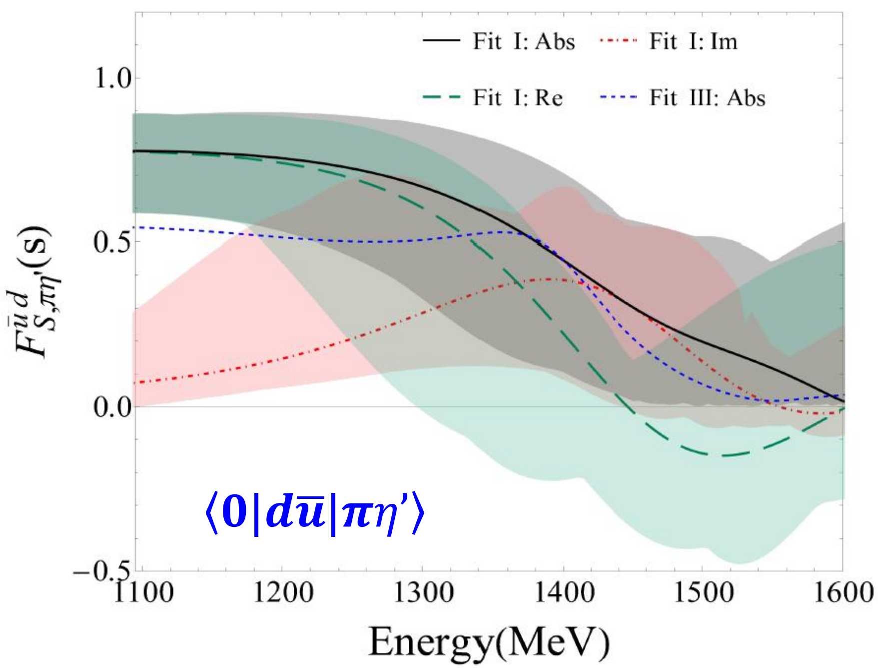
Scalar form factors (I=0)



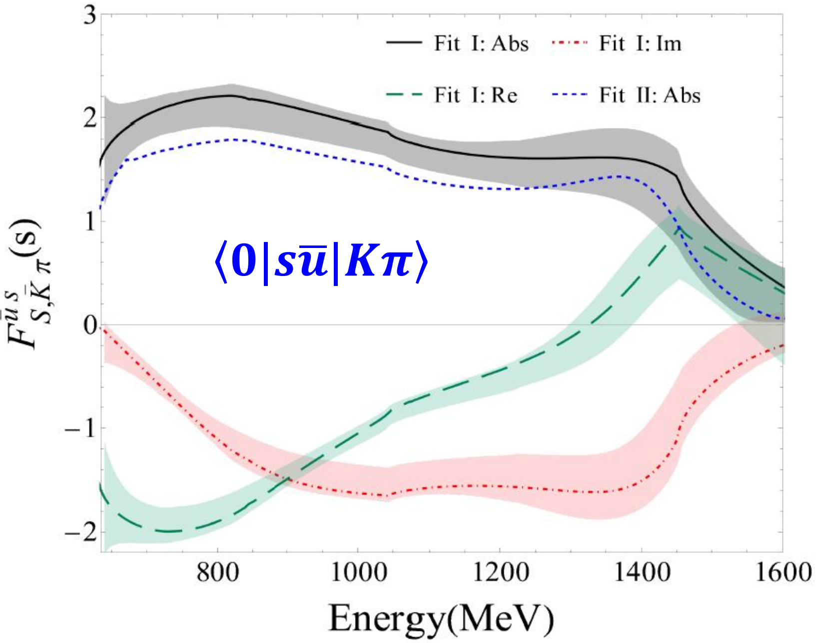
Scalar form factors (I=1)

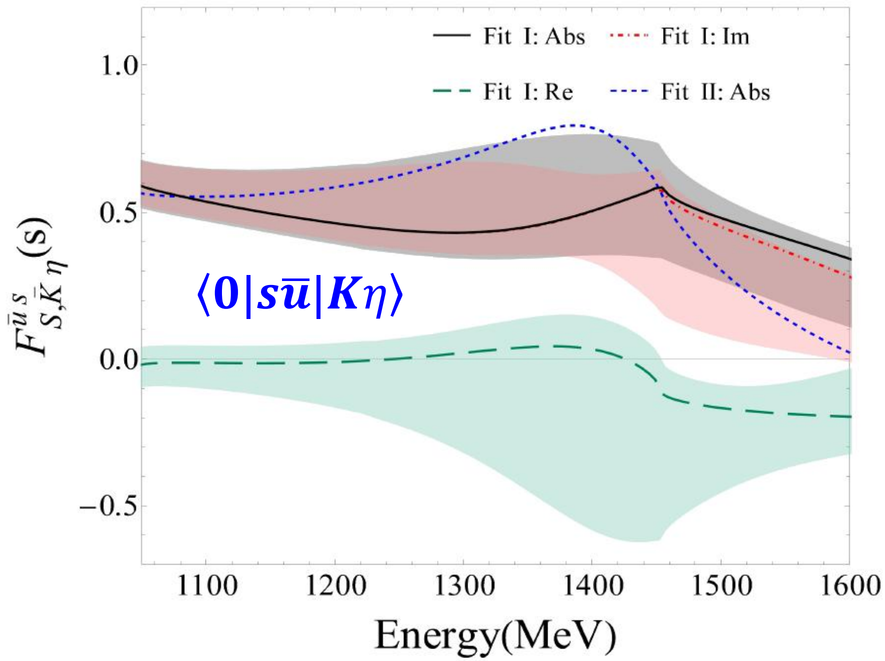


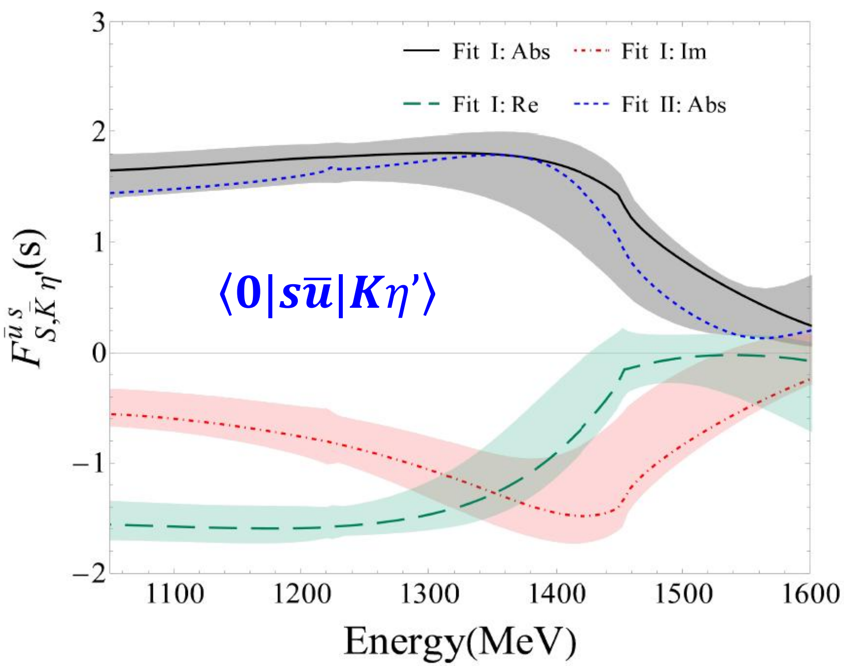




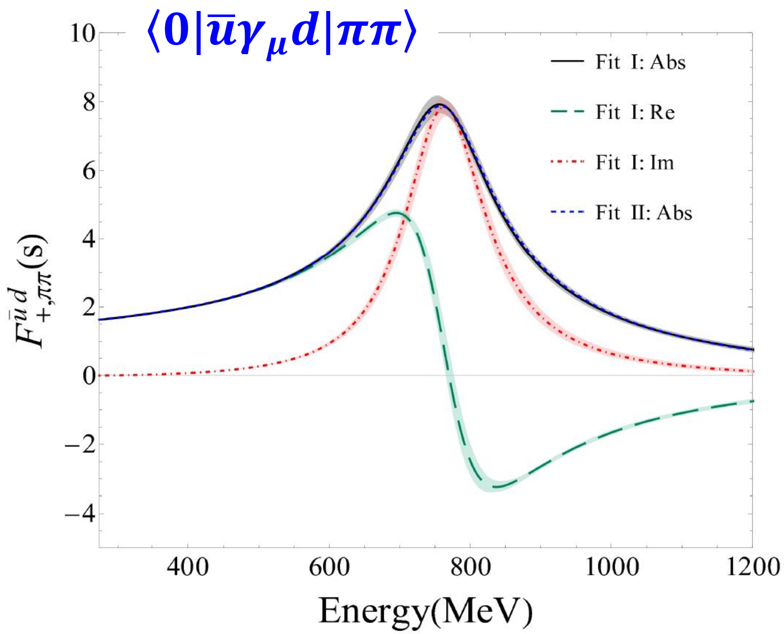
Scalar form factors (I=1/2)

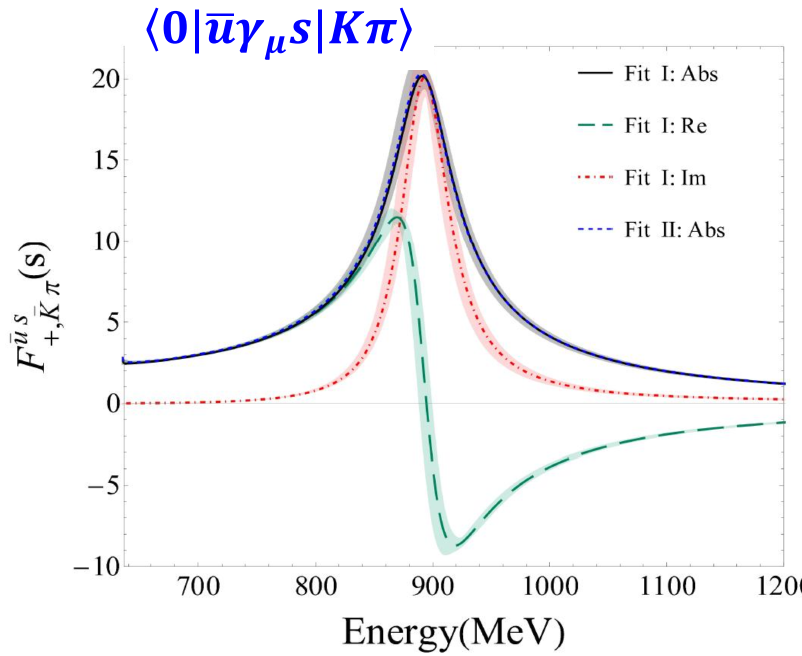


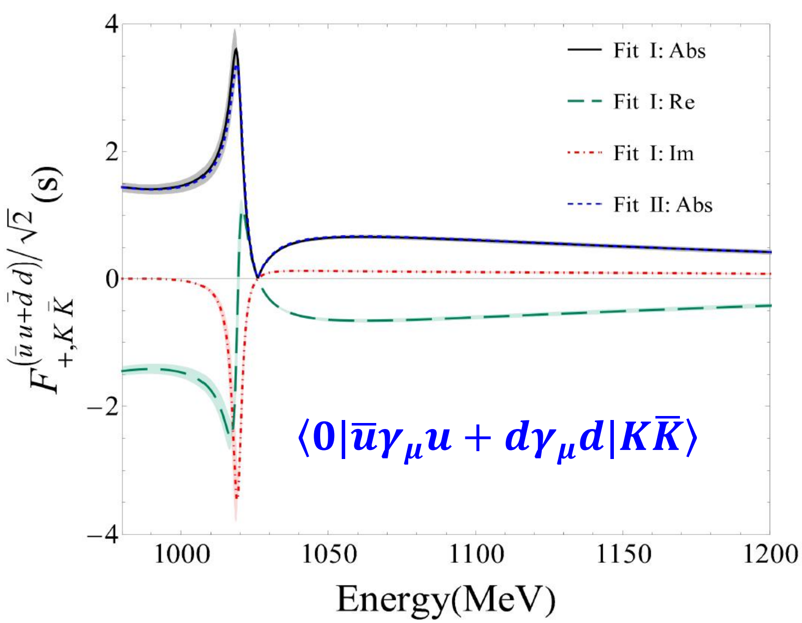


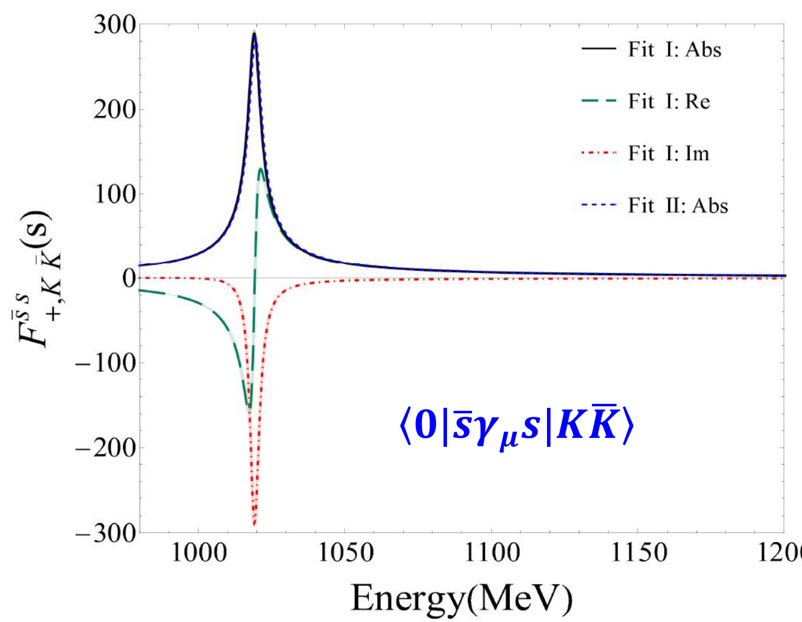


Vector form factors



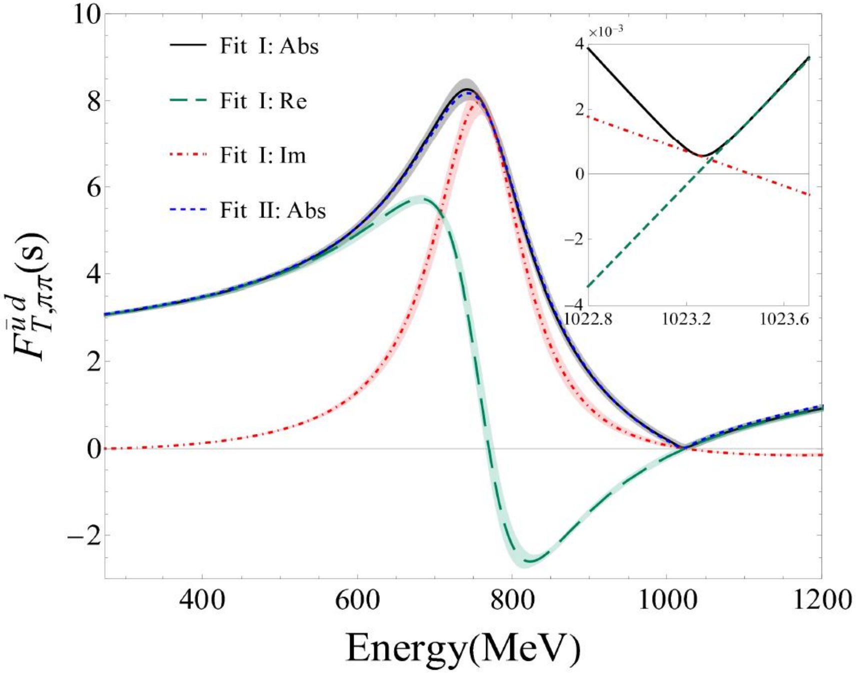




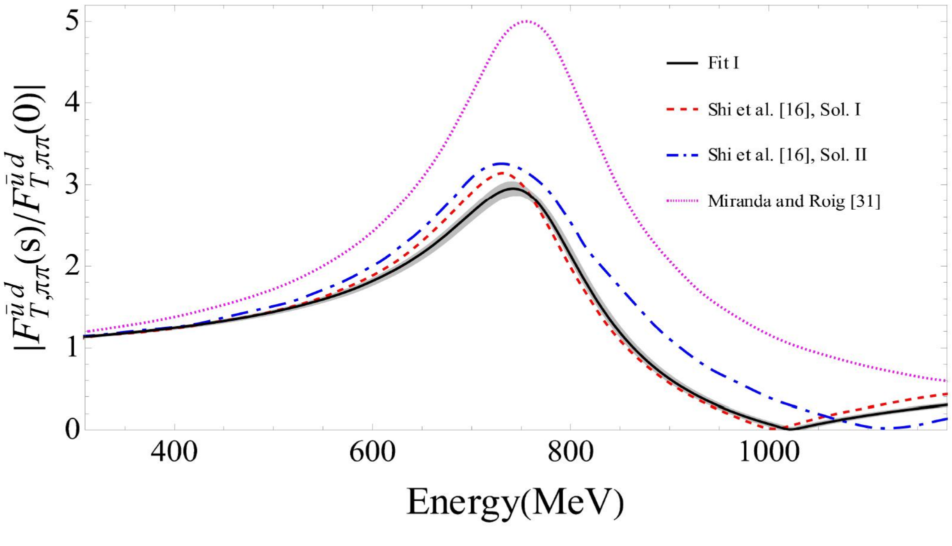


Tensor form factors

$\langle 0|\bar{u}\sigma_{\mu\nu}d|\pi\pi\rangle$



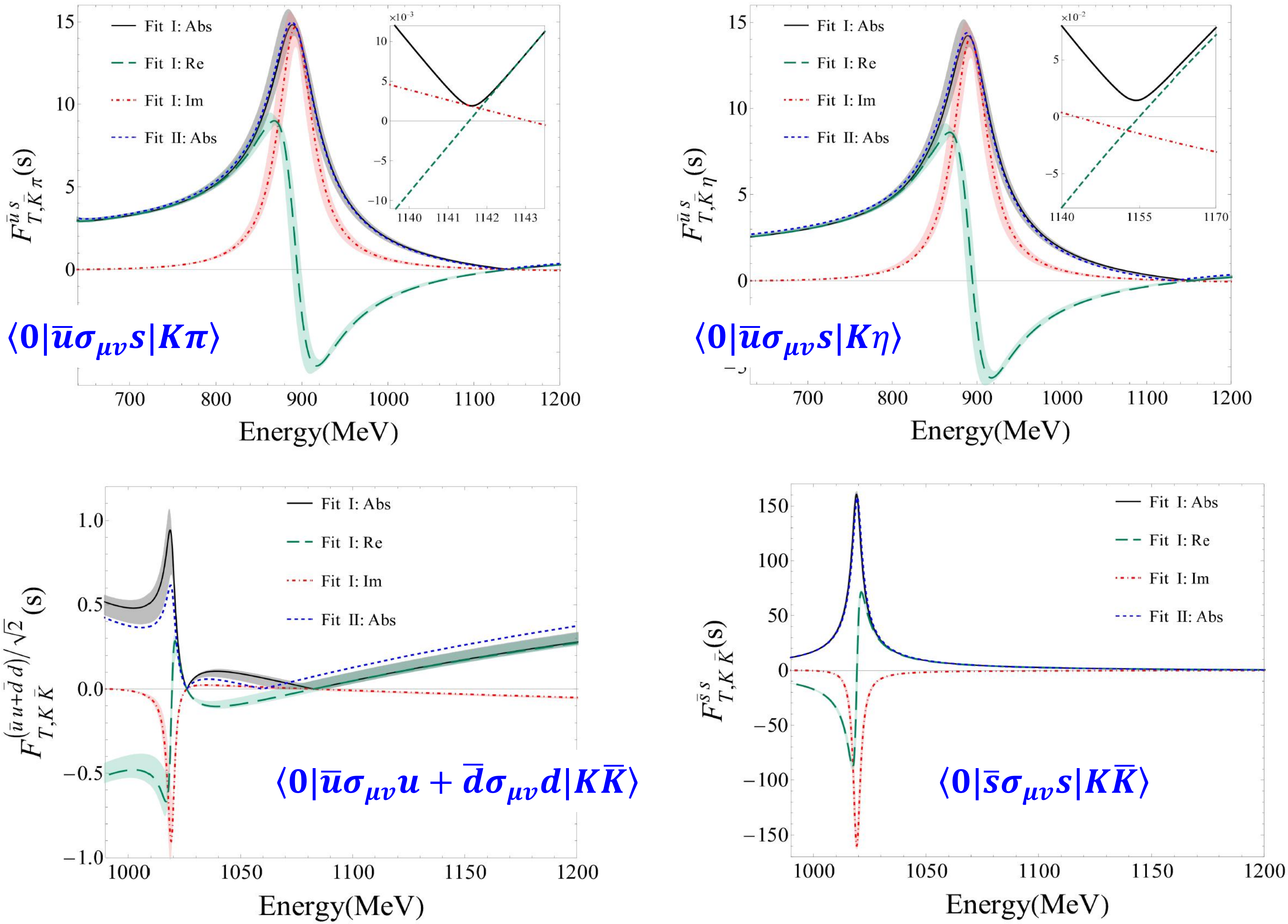
Comparison of normalized tensor FFs from different works



[Miranda, Roig,JHEP'18]

[Shi,et al.,JHEP'21]

Tensor form factors



Summary & Outlook

[Hao,Duan,ZHG,Oller,Ruiz de Elvira, 2605.14875]

- **Scalar form factors below 1.6 GeV**

$\langle 0|u\bar{u} + d\bar{d}\,|\pi\pi, K\bar{K}, \eta\eta\rangle$ $\langle 0|\bar{u}s\,|K\pi, K\eta, K\eta'\rangle$

$\langle 0|s\bar{s}\,|\pi\pi, K\bar{K}, \eta\eta\rangle$ $\langle 0|\bar{u}d\,|\pi\eta,\ K\bar{K}, \pi\eta'\rangle$

- **Vector form factors below 1.2 GeV**

$\langle 0|\bar{u}\gamma_\mu u + d\gamma_\mu d|K\bar{K}\rangle$ $\langle 0|\bar{u}\gamma_\mu s|K\pi, K\eta\rangle$

$\langle 0|\bar{s}\gamma_\mu s|K\bar{K}\rangle$ $\langle 0|\bar{u}\gamma_\mu d|\pi\pi, K\bar{K}\rangle$

- **Anti-symmetric tensor form factors below 1.2 GeV**

$\langle 0|\bar{u}\sigma_{\mu\nu}u + \bar{d}\sigma_{\mu\nu}d|K\bar{K}\rangle$ $\langle 0|\bar{u}\sigma_{\mu\nu}s|K\pi, K\eta\rangle$

$\langle 0|\bar{s}\sigma_{\mu\nu}s|K\bar{K}\rangle$ $\langle 0|\bar{u}\sigma_{\mu\nu}d|\pi\pi, K\bar{K}\rangle$

- **Possible applications: future projects (SM & BSM)**

$\tau \to PP'\,\nu_\tau$ $D \to PP'\,l\,\bar{\nu}_l$ ……