Quark helicity PDFs of proton from lattice QCD

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in collaboration with

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第九届强子谱和强子结构研讨会

PRD 114 (2026) 1, 1

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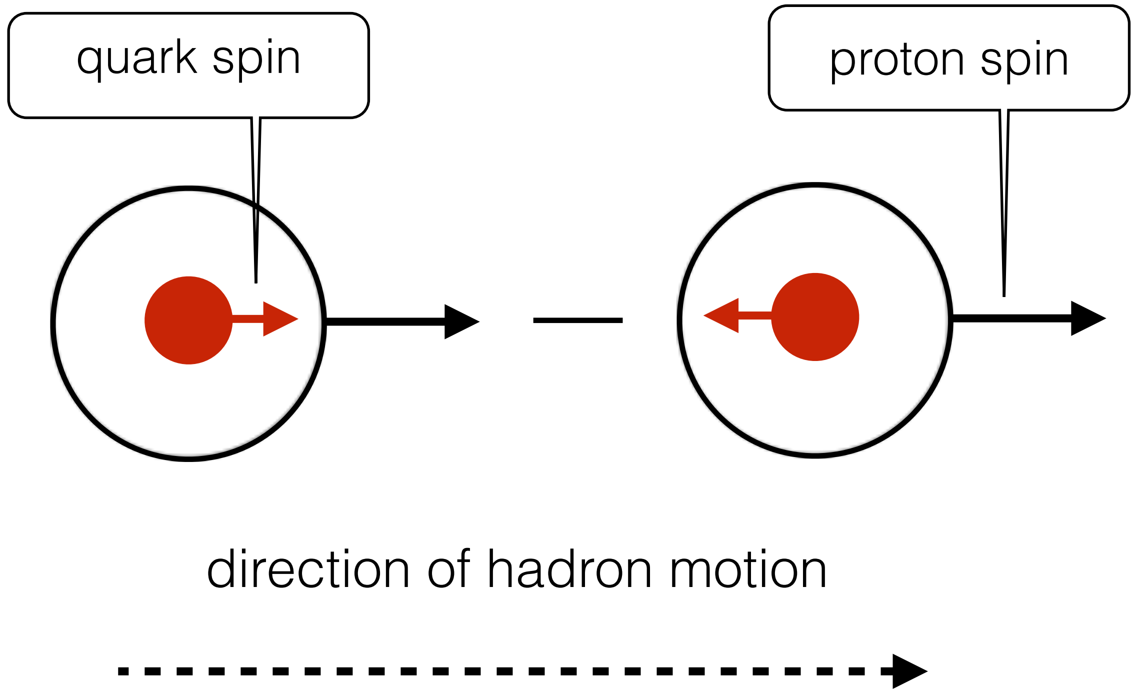
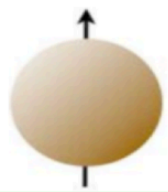
What is quark helicity PDF?

The probability of finding a parton (constituent quark) with momentum fraction x in a longitudinally polarized fast-moving proton



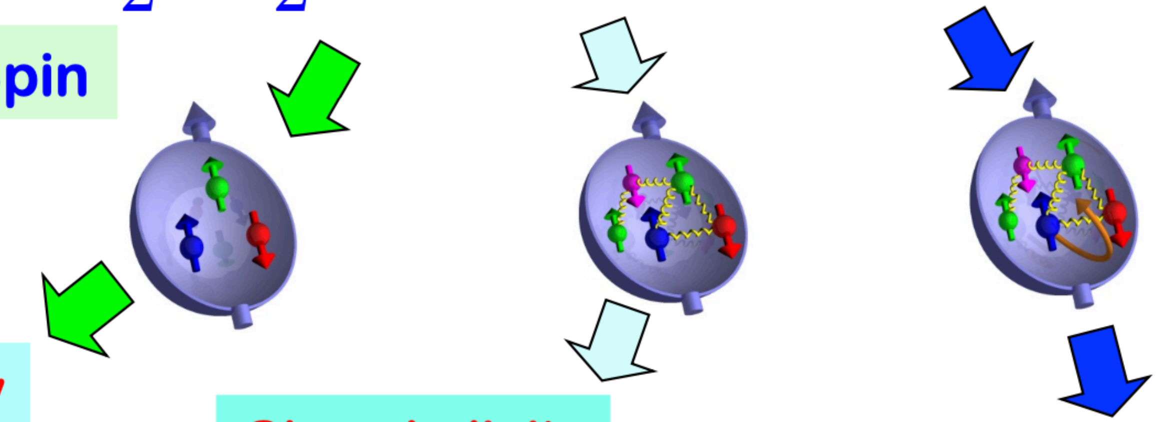
Why is it important?

Spin sum rule: [Jaffe and Manohar, NPB 337, 509 (1990)]



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g) = \sum \langle P, S | \hat{J}^z_f(\mu) | P, S \rangle$$

Proton Spin



Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}\right)$$

$$\sim 30\%$$

Spin "puzzle"

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

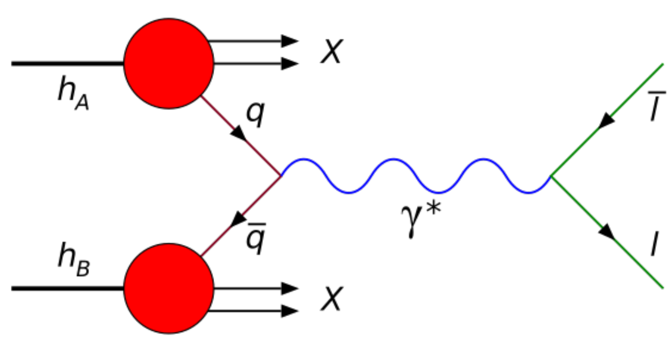
$$\sim 20\% (\text{with RHIC data})$$

Orbital Angular Momentum of quarks and gluons
Little known

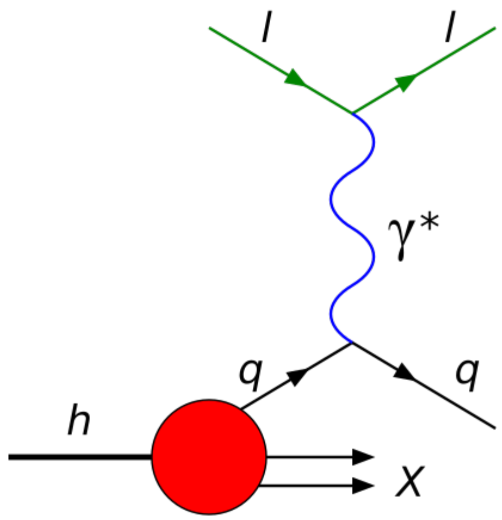
Image courtesy of Jian-Wei Qiu

Quark model assumes spin of proton (1/2) equals spin of u+u+d,
but European Muon Collaboration (EMC) found quarks only contribute < 30%

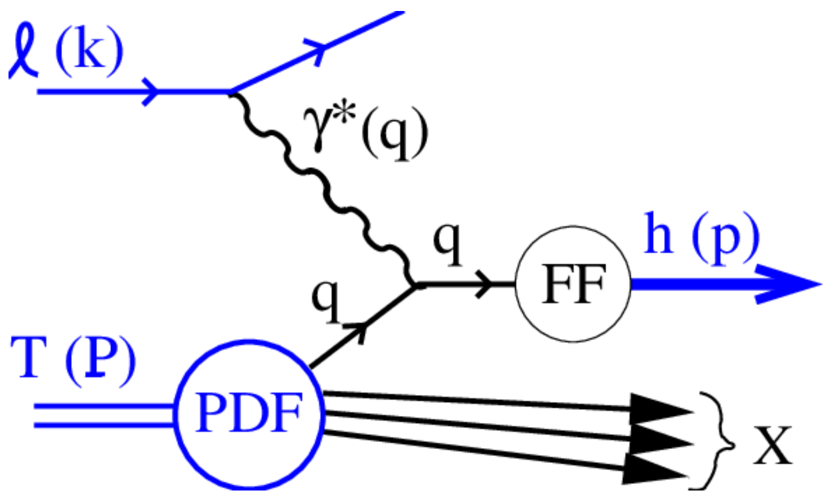
How to access it?



Drell-Yan



DIS

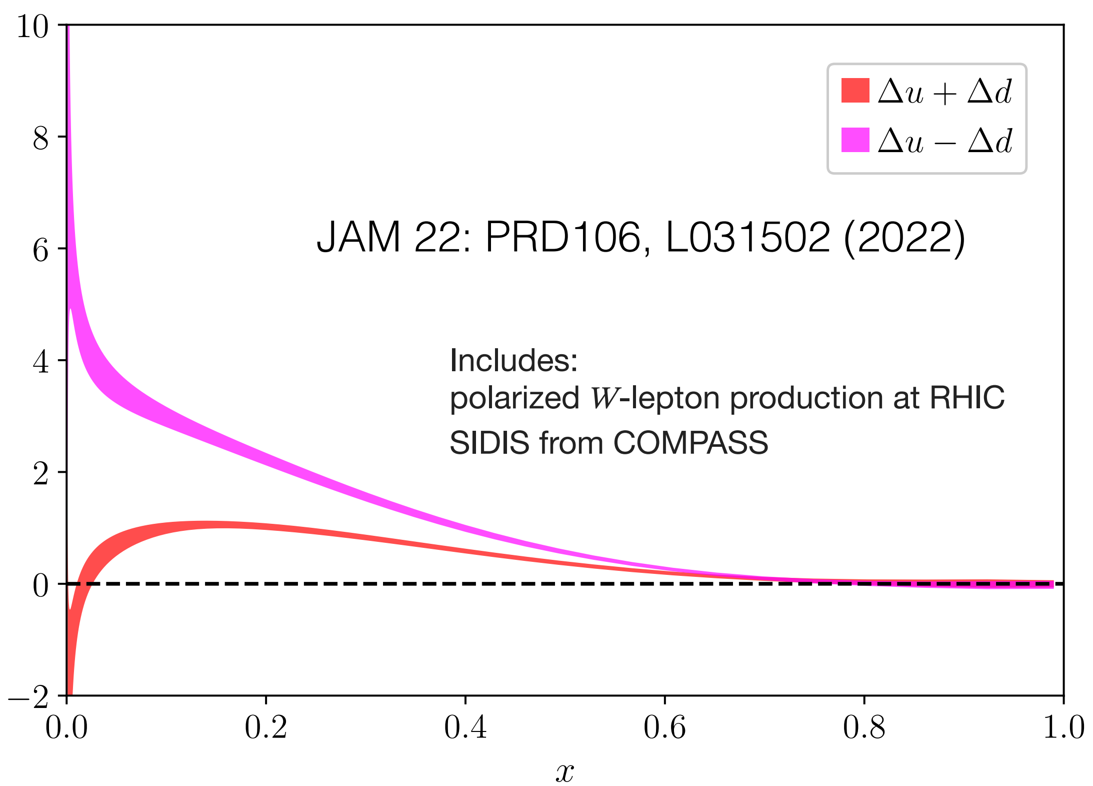


SIDIS

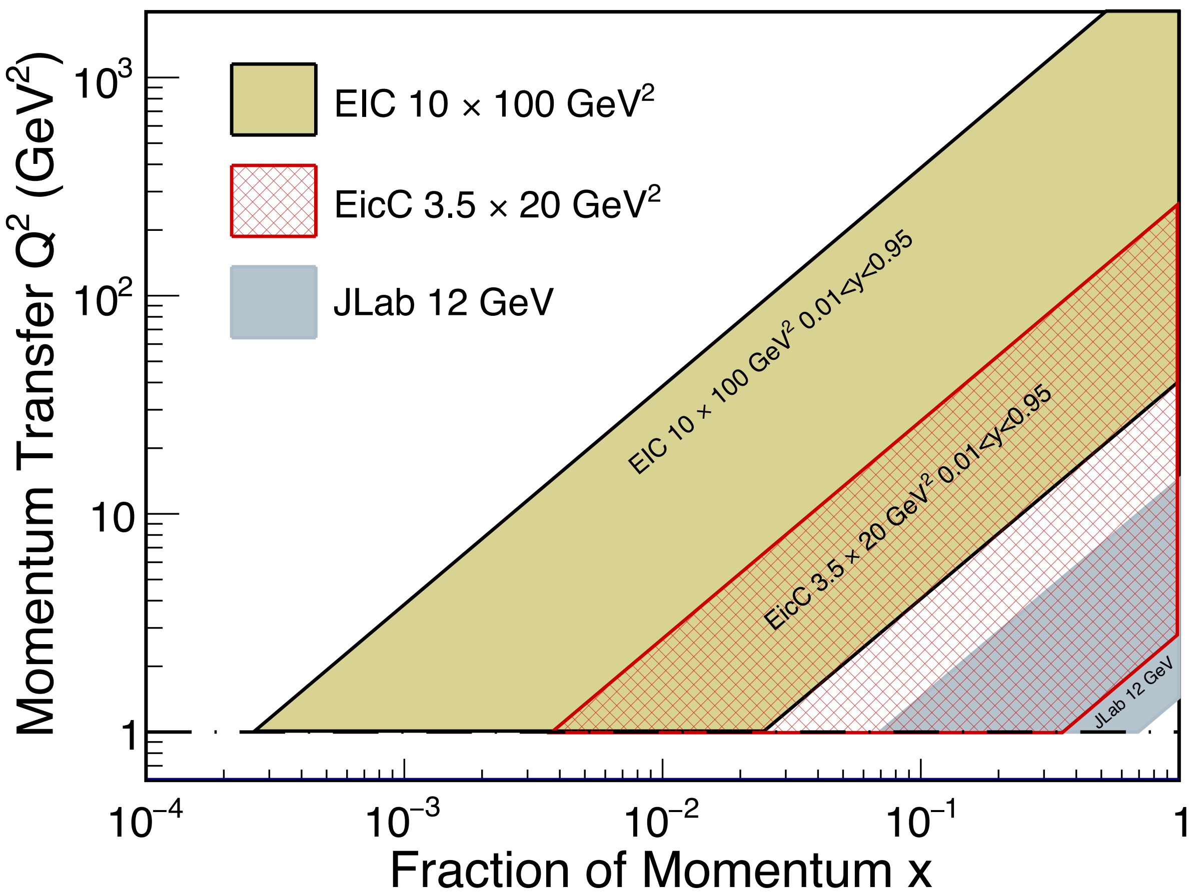
Measure structure functions (via longitudinal spin asymmetries) in processes like polarized lepton/hadron scattering off a polarized target proton

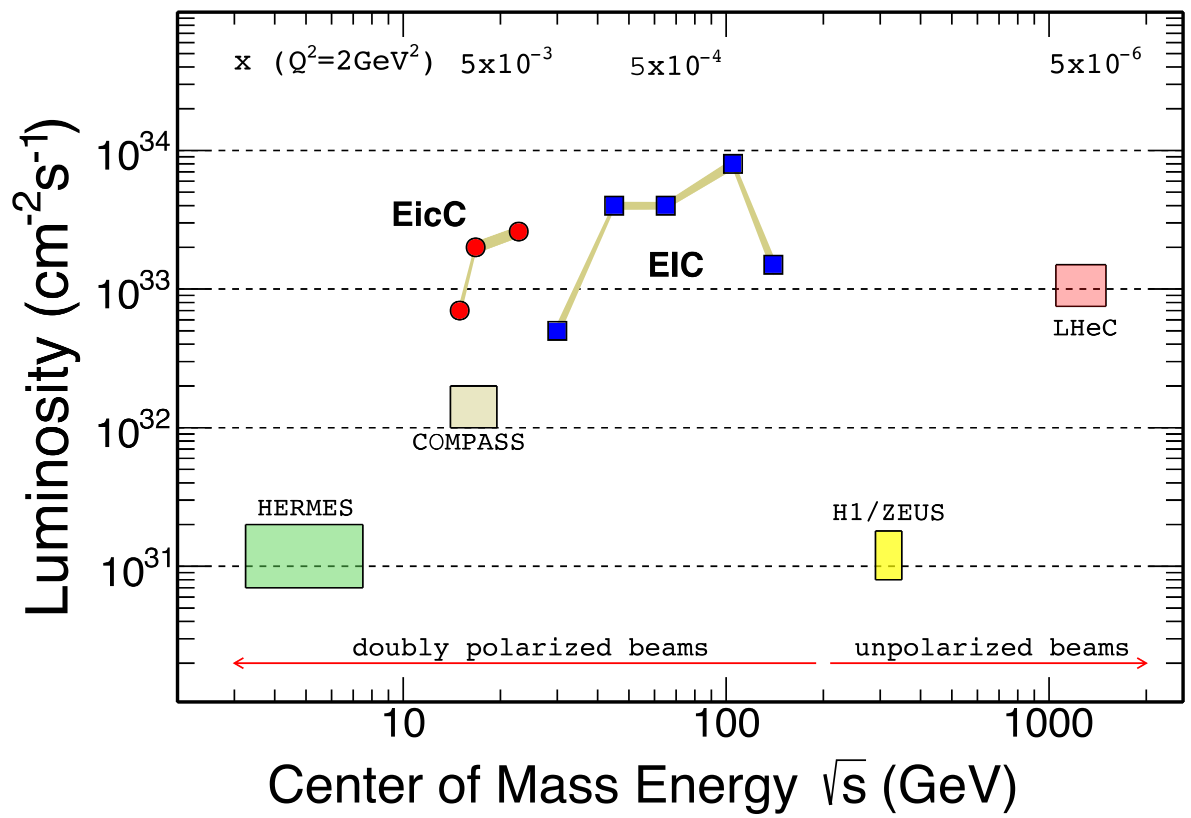
<u>Existing experimental efforts in last 3 decades:</u>
p-p collision at RHIC;
Open-charm muon production at COMPASS;
Neg./pos. pion/kaon production at HERMES…

Global analysis of the experimental data



Why EIC and EICc?





EicC white paper: Frontiers of Physics, Volume 16 Issue (6):64701, 2021

- Unprecedented low-x reach for polarized experiments
- High luminosity provides more precise statistical measurements
- High polarization capacities of electron and nucleon give more access to the spin-dependent observables

Status of LQCD determinations

LQCD provides *ab initio* determination from first-principle!

	m_π [MeV]	a [fm]	P_z [GeV]
LP^3 Coll. $18'$	physical	0.09	3.0
Gao, *et al.*, PRD$20'$	310	0.042	2.77
HadStruc Coll. $22'$	358	0.094	2.5
Holligan and Lin, $24'$	315	0	1.75
This work	physical	0.076	1.53

- Having both continuum limit and physical mass limit is still challenging
- This work: physical quark masses, small lattice spacing, NNLO&NNLL matching

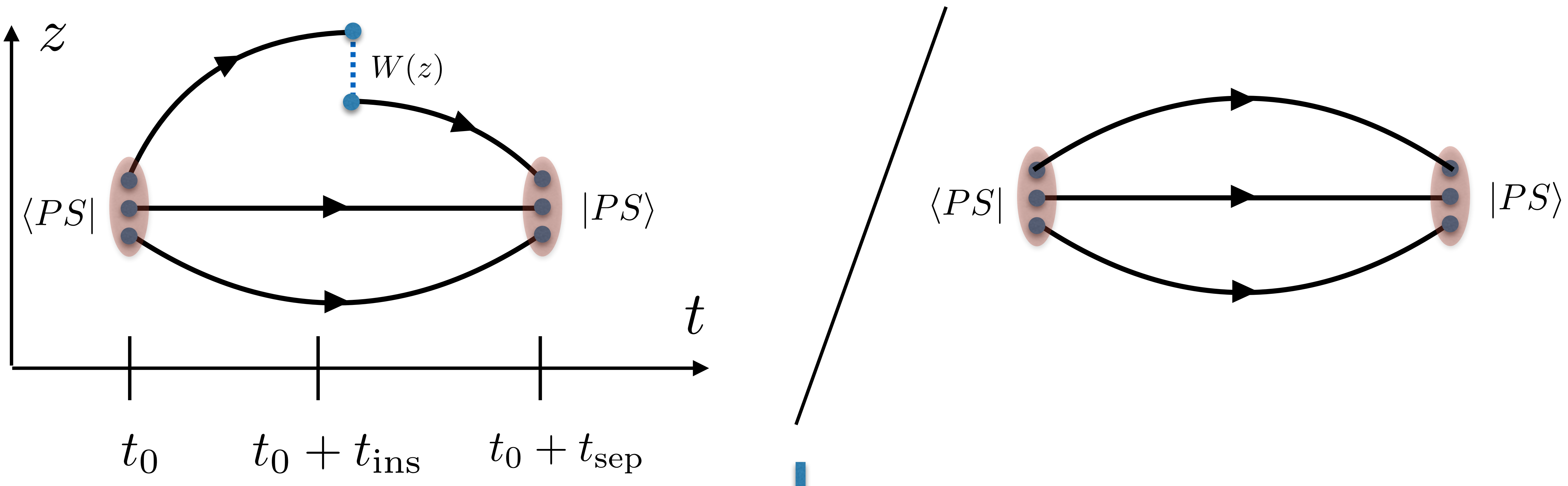
Lattice setup

β	a [fm]	m_π [GeV]	N_σ	N_τ	# conf.
7.13	0.076	0.14	64	64	350

$$m_\pi L = 3.45$$

- Physical light & strange quark mass
- Volume effects under control
- $P_z = \{0,\ 0.25,\ 1.02,\ 1.53\}$ GeV
- Momentum smearing to improve SNR

Ingredient in Lattice QCD: matrix element



$0 \ll t_{\text{ins}} \ll t_{\text{sep}}$

Parton spin in z direction

$$\widetilde{h}_1^B(z, P_z, a) \equiv \frac{1}{2E} \left\langle P, S \left| \bar{\psi}(z)\, \gamma_5 \gamma_z\, W(z,0)\, \psi(0) \right| P, S \right\rangle$$

$f = u - d$

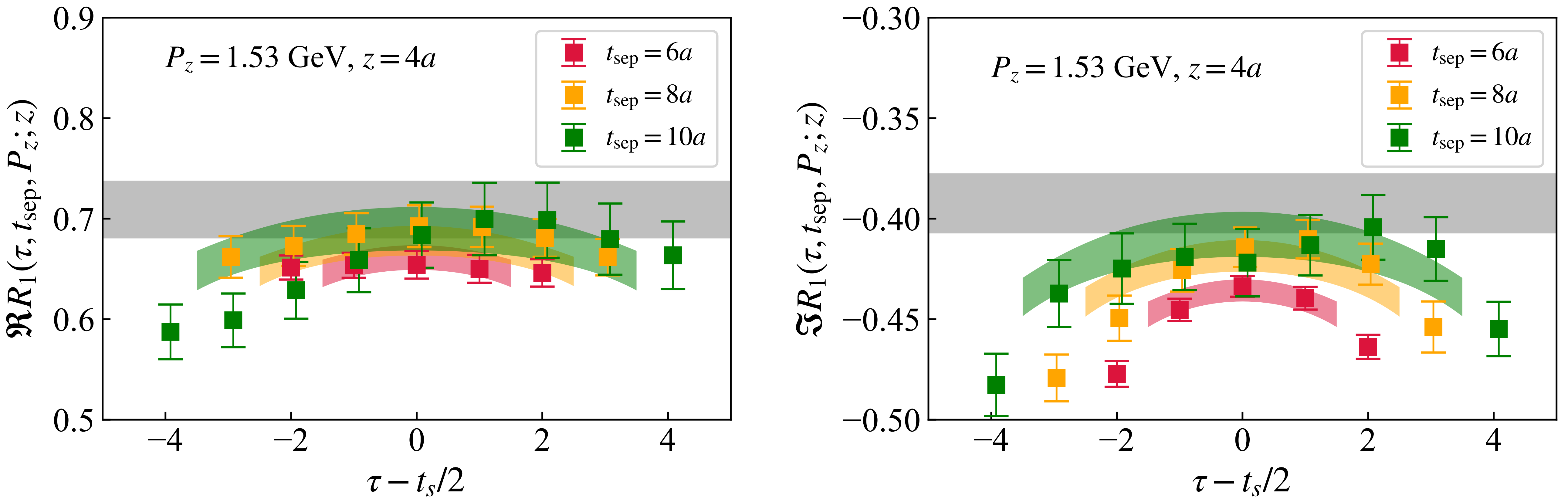
isovector sector: free of disconnected contributions

$P_\mu = (P_0, 0, 0, P_z)$ Hadron momentum in z direction

$S_\mu = (P_z, 0, 0, \mp P_0)$ Hadron spin in +z/-z direction

Extraction of the bare matrix element

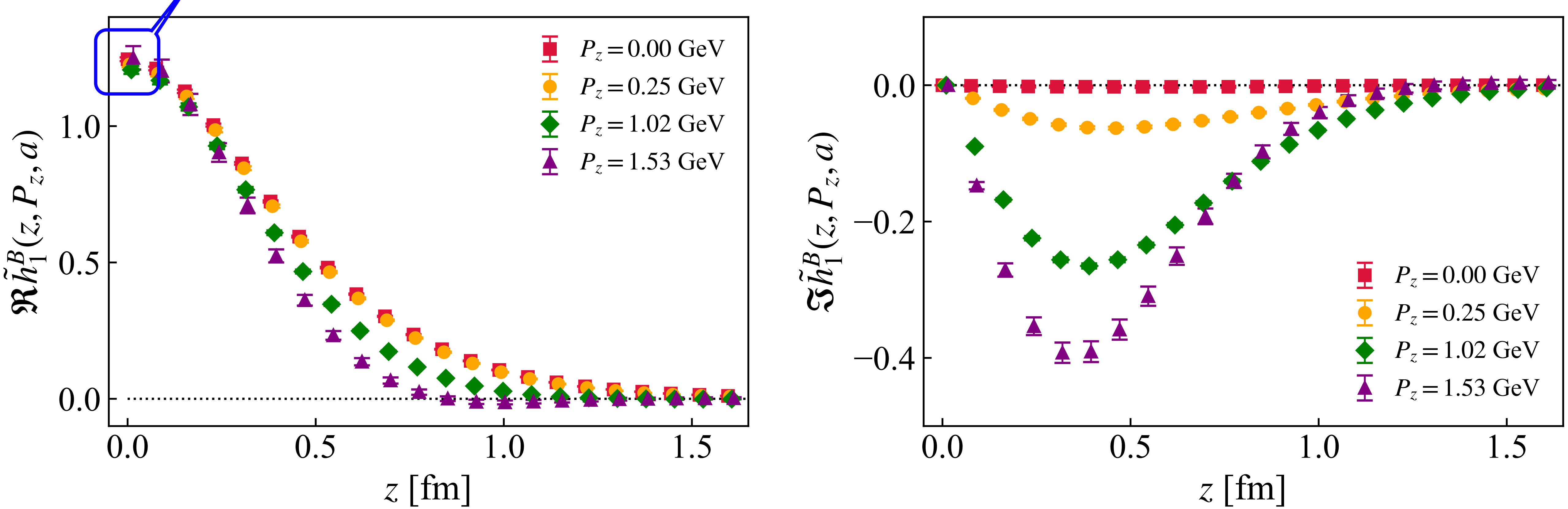
$$R(\tau, t_{\rm sep}, P_z; z) \equiv \frac{C^{\Gamma}_{\rm 3pt}\left(P_z, t_{\rm sep}, \tau, z\right)}{C_{\rm 2pt}\left(P_z, t_{\rm sep}\right)} = \widetilde{h}^B_1(z, P_z, a) \frac{1 + c_1(z, P_z)\left(e^{-\Delta E\, t_{\rm ins}} + e^{-\Delta E(t_{\rm sep} - t_{\rm ins})}\right)}{1 + c_2 e^{-\Delta E\, t_{\rm sep}}} \xrightarrow{t_{\rm sep} \gg \tau \gg 0} \widetilde{h}^B_1$$



- Good description of the data by the the two-state ansatz
- Sizable excited-state contaminations are under control

z and P_z dependence of bare matrix element

$$g_A^{u-d} = Z_A \, \widetilde{h}_1^B(z=0, P_z=0, a) = 1.211(12)$$

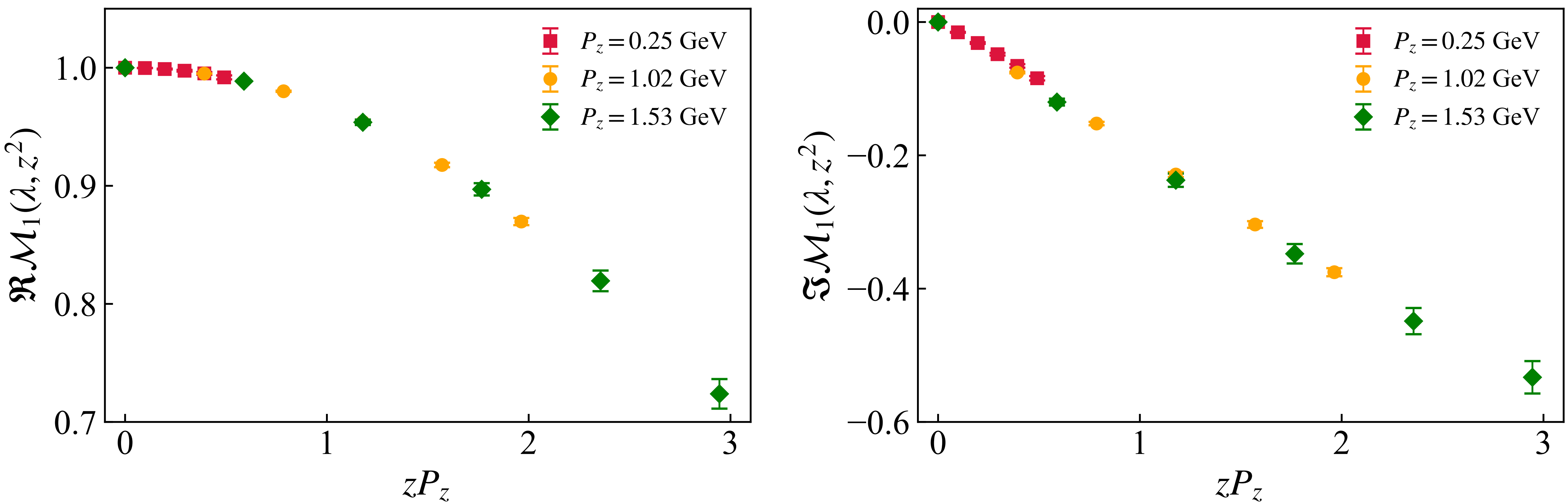


- Consistent results at $z = 0$ from different boosts
- Pronounced momentum dependence for the real&imag parts

Renormalization-group invariant ratios

RI scheme ratios free of linear and logarithmic UV divergences:

$$\mathcal{M}_1(\lambda, z^2) = \frac{\widetilde{h}_1^B(z, P_z)}{\widetilde{h}_1^B(z, P_z^0)} \frac{\widetilde{h}_1^B(0, P_z^0)}{\widetilde{h}_1^B(0, P_z)} = \frac{\widetilde{h}_1^{\overline{\mathrm{MS}}}(\lambda, z^2\mu^2)}{\widetilde{h}_1^{\overline{\mathrm{MS}}}(\lambda^0, z^2\mu^2)}$$



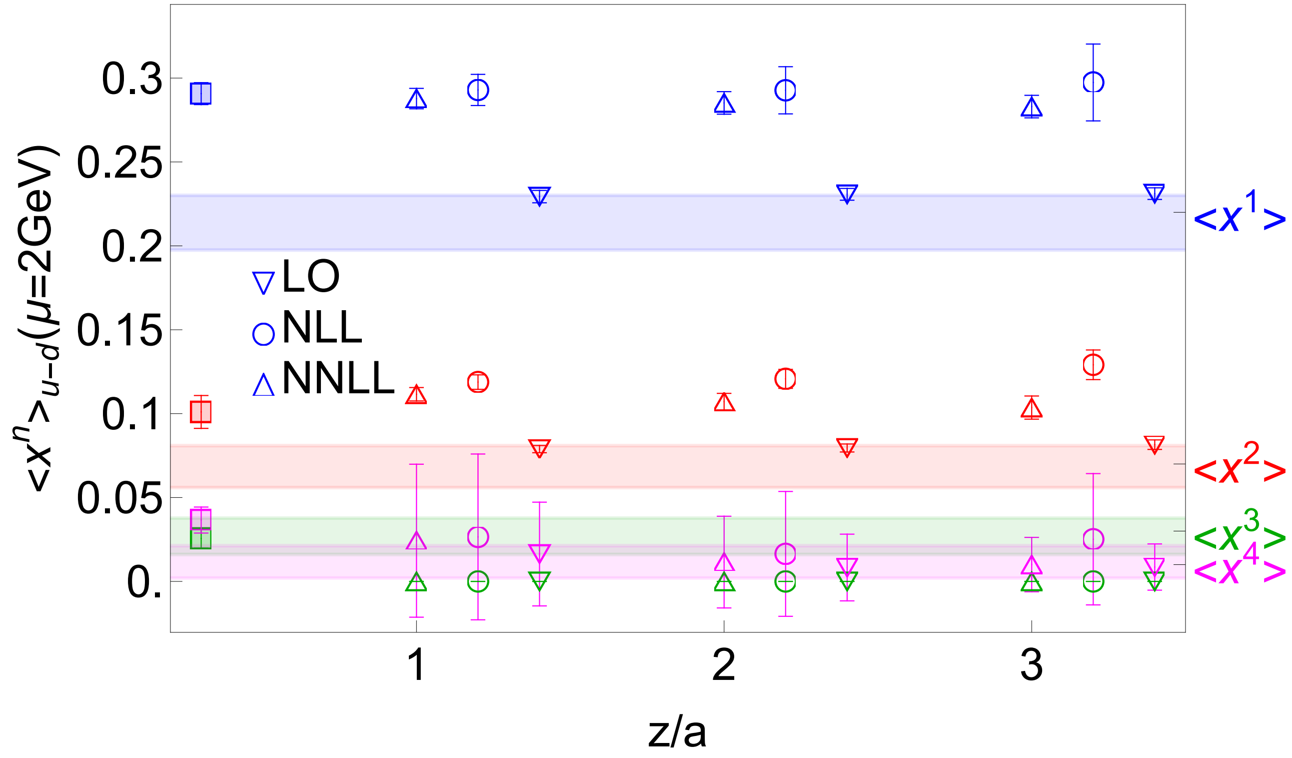
- Normalized to 1 for the real part and 0 for the imaginary part
- Substantial reduction of momentum dependence in the ratio

Mellin moments from reduced ITD

Operator product expansion (OPE) approximation in $\overline{\mathrm{MS}}$ scheme:

$$\mathcal{M}_1(\lambda, z^2) = \frac{\widetilde{h}_1^{\overline{\mathrm{MS}}}(\lambda, z^2\mu^2)}{\widetilde{h}_1^{\overline{\mathrm{MS}}}(\lambda^0, z^2\mu^2)} = \frac{\sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} \boxed{\langle x^n \rangle(\mu)}}{\sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(-i\lambda_0)^n}{n!} \boxed{\langle x^n \rangle(\mu)}} + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2 z^2)$$

$\langle x^n \rangle$ as fit parameters



Points: $\langle x^n \rangle$ from fits of different LQCD data sets

Bands: $\langle x^n \rangle$ from global analysis JAM 22

Quasi-PDF from Hybrid renormalization

$$\widetilde{h}_R(z,P_z)=\begin{cases}\dfrac{\widetilde{h}_1^B(z,P_z,a)}{\widetilde{h}_1^B(z,0,a)}, & |z|\le z_s \\ \dfrac{\widetilde{h}_1^B(z,P_z,a)}{\widetilde{h}_1^B(z_s,0,a)}e^{(\delta m+m_0)(z-z_s)}, & |z|\ge z_s\end{cases}$$

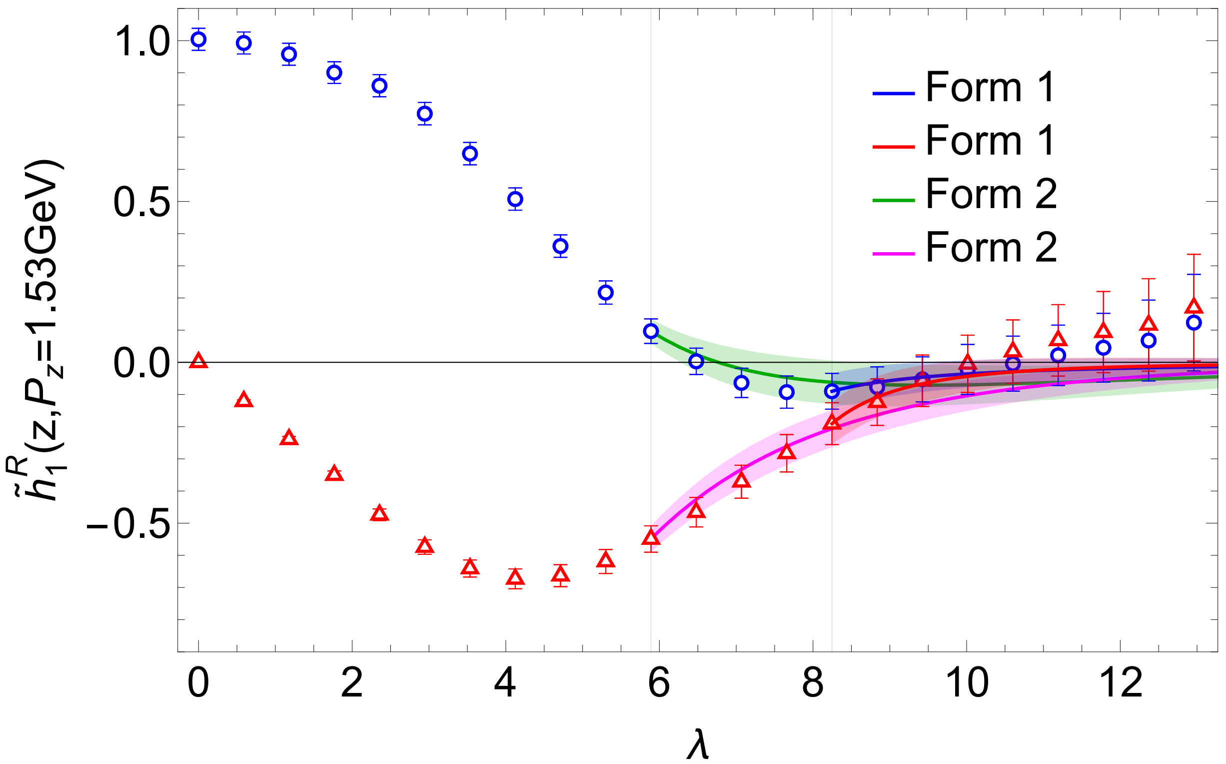
Ratio scheme renormalization ($|z|\le z_s$)

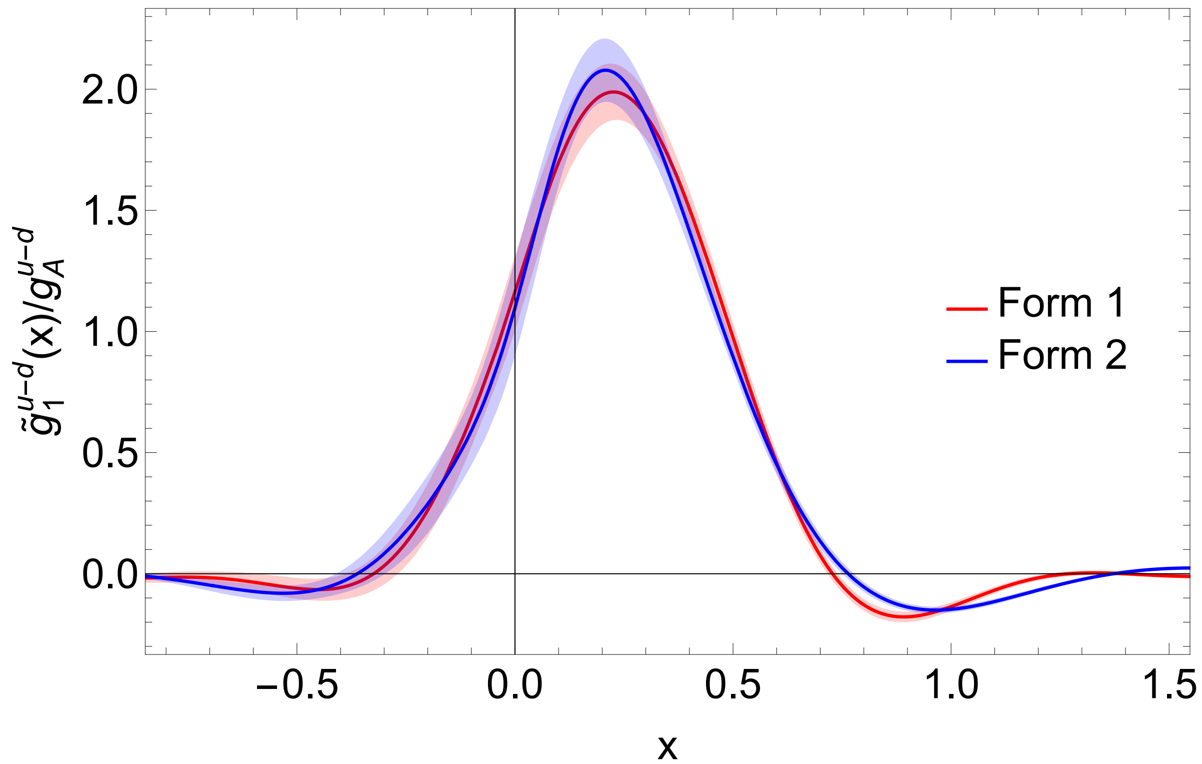
Self renormalization ($|z|\ge z_s$)

Large λ extrapolation:

Form 1: $\widetilde{h}_1^{\rm asy}(\lambda,P_z)=\dfrac{A\,e^{-m_{\rm eff}\lambda/P_z}}{|\lambda|^d}$ (dispersive analysis and Regge behavior)

Form 2: $\widetilde{h}_1^{\rm asy}(z,P_z)=\left(Ae^{i\phi\,{\rm sgn}(z)}+\dfrac{A'e^{i\phi'\,{\rm sgn}(z)}}{|z|}\right)e^{-|z|/\Lambda}$ (next-to-leading asymptotics)





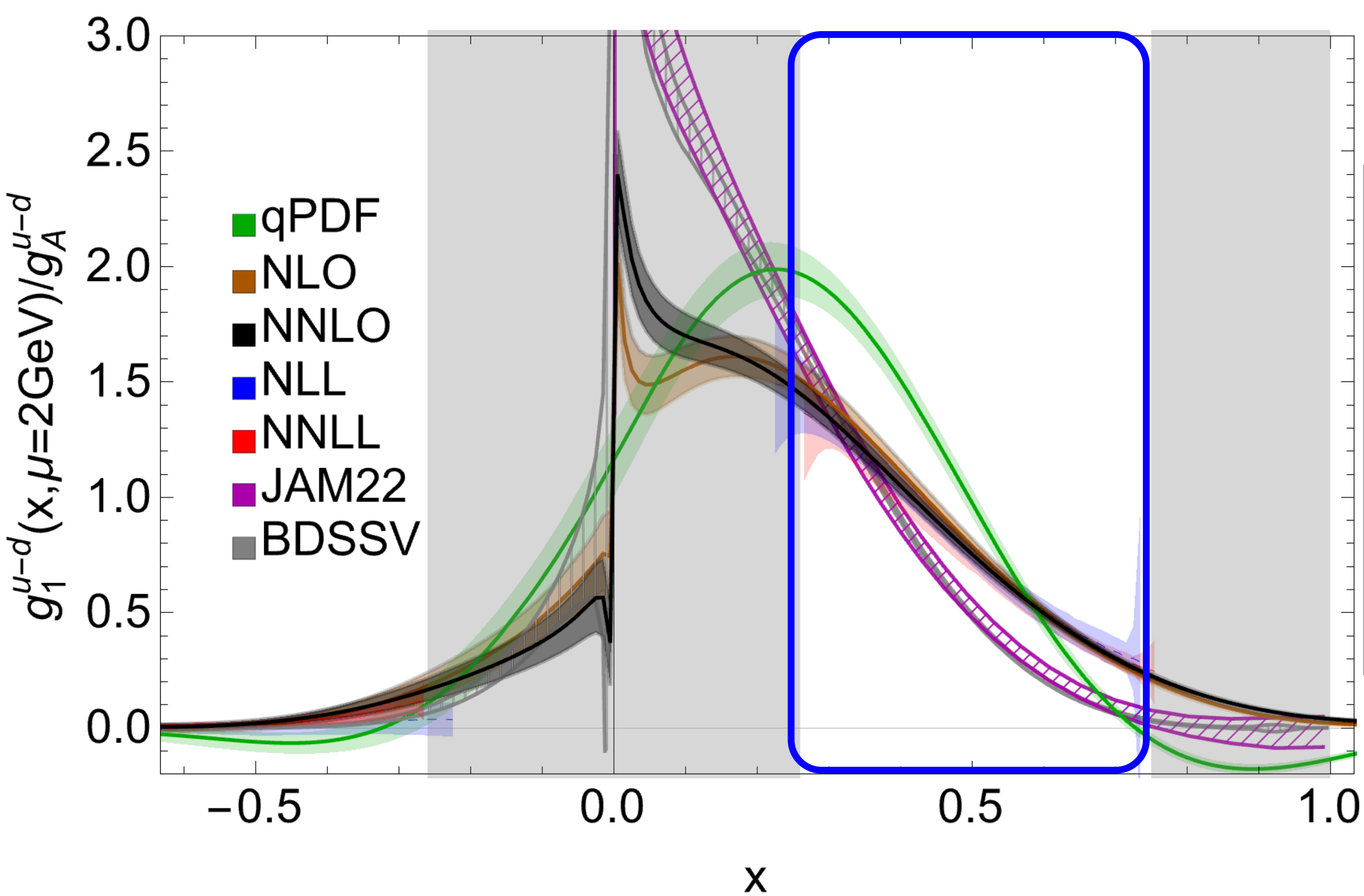
Two extrapolations are consistent in most regions of interest

Quasi PDF to light-cone PDF

Perturbative matching: $$\Delta q(x,\mu) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}y}{|y|}\, \mathcal{C}^{-1}\left(\frac{x}{y}, \mu, z_s, P_z\right) \Delta\tilde{q}(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{\big((1-x)P_z\big)^2}\right)$$

Perturbative expansion: $$\mathcal{C}\left(\frac{x}{y}, \mu, z_s, P_z\right) = \delta\left(\frac{x}{y} - 1\right) + \sum_{n=1}^{\infty} \alpha_s^n\, \mathcal{C}^{(n)}\left(\frac{x}{y}, \mu, z_s, P_z\right)$$

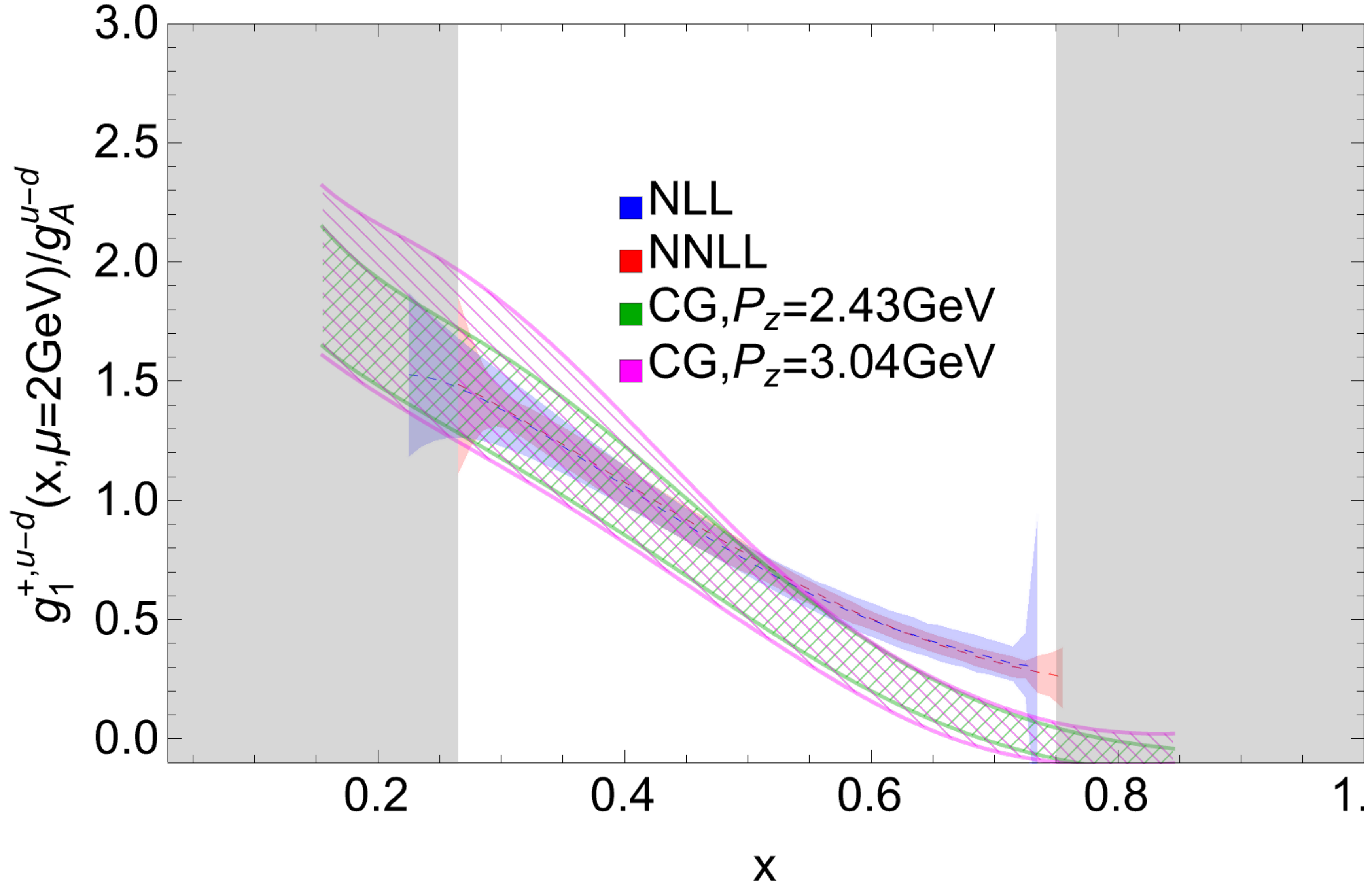
NNLO matching coefficient: PRL. 126, 072001 (2021); PRL. 126, 072002 (2021).
NNLO hybrid scheme corrections: Nucl. Phys. B 991, 116201 (2023).



- RG Resummation is needed when $2xP_z$ becomes small for $x \to 0$
- Threshold Resummation is needed when $2(1-x)P_z$ becomes small for $x \to 1$
- Both resummations applied up to NLL/NNLL
- Both scales are large + scale variations are small → reliable region $x \in [0.25,\ 0.75]$

Comparison with the Coulomb gauge method

$$g_1^+(x) \equiv g_1(x) + g_1(-x) = g_1(x) + \bar{g}_1(x)$$



Results from Coulomb gauge method: arXiv:2602.11283

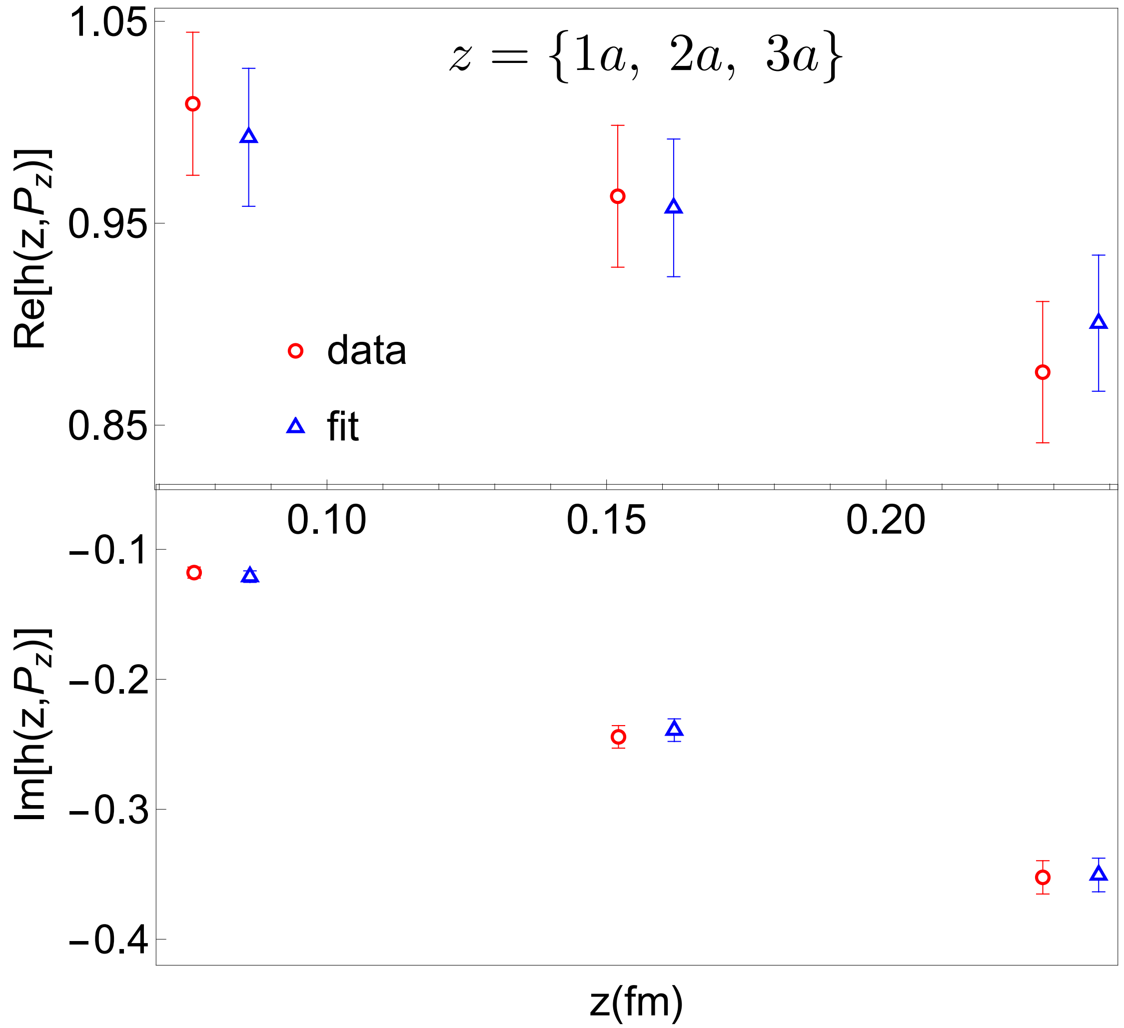
- Good consistency for $x < 0.6$
- Noticeable discrepancies for $x > 0.6$, potentially from the power corrections due to the small momentum $P_z = 1.5$ GeV

Constrain the endpoint regions

- Constrain the endpoint regions via short-distance factorization (SDF)
- Fit the small-z matrix elements convoluted with perturbative matching
- Power-law ansatz on both sides from Regge behavior and power counting rules

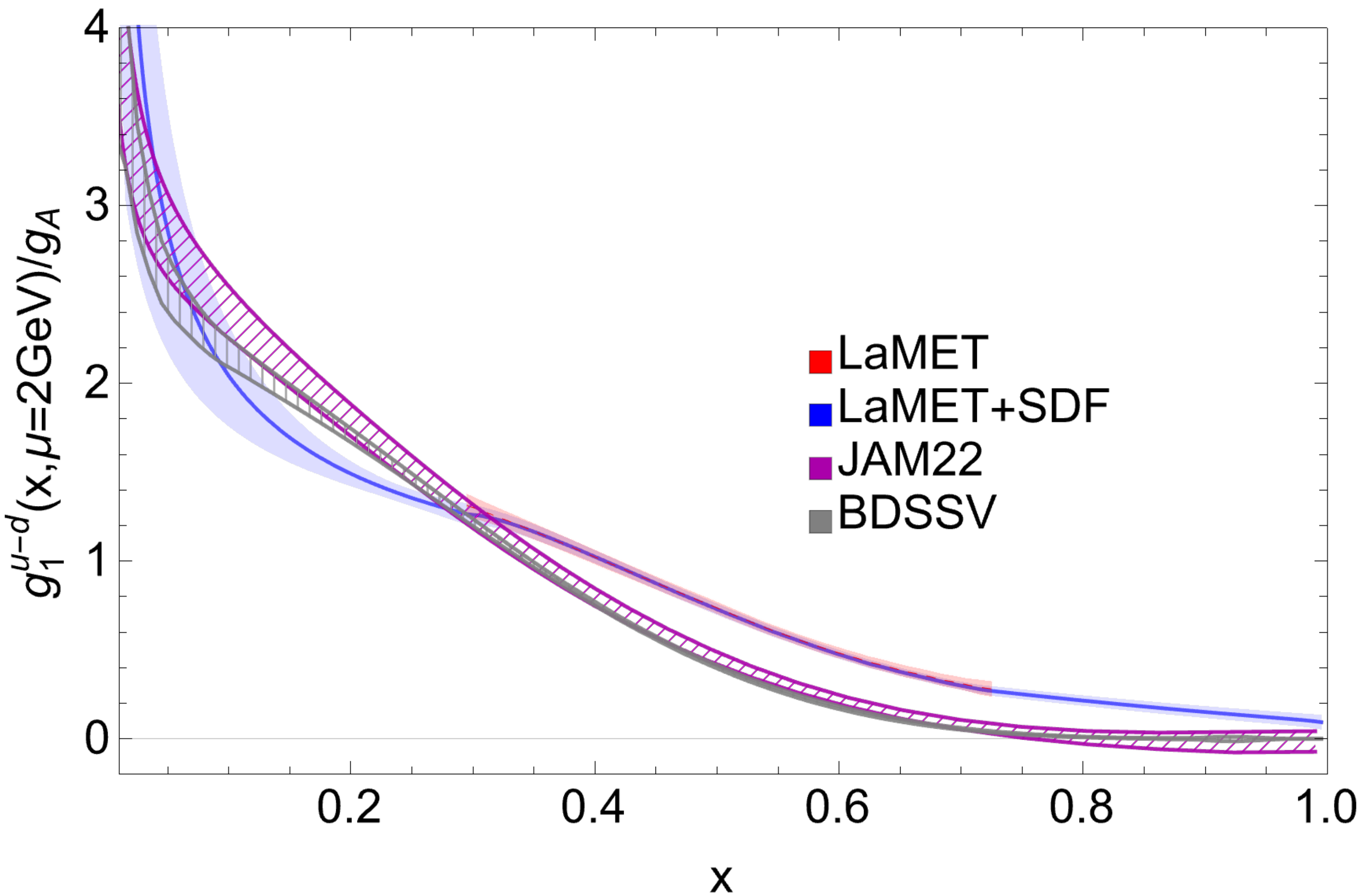
$$g_1(x) = \begin{cases} f(x_0)\dfrac{x^a}{x_0^a}, & x < x_0, \\ f(x), & x_0 < x < 1 - x_0, \\ f(1-x_0)\dfrac{(1-x)^b}{(1-x_0)^b}, & x > 1 - x_0. \end{cases}$$

$$h(z^2, z\cdot P) = \int_{-1}^{1} K(x\omega, z^2, \mu)\, g_1(x,\mu)\, dx$$



Good consistency between the fits and the lattice data

Complete comparison with global fits



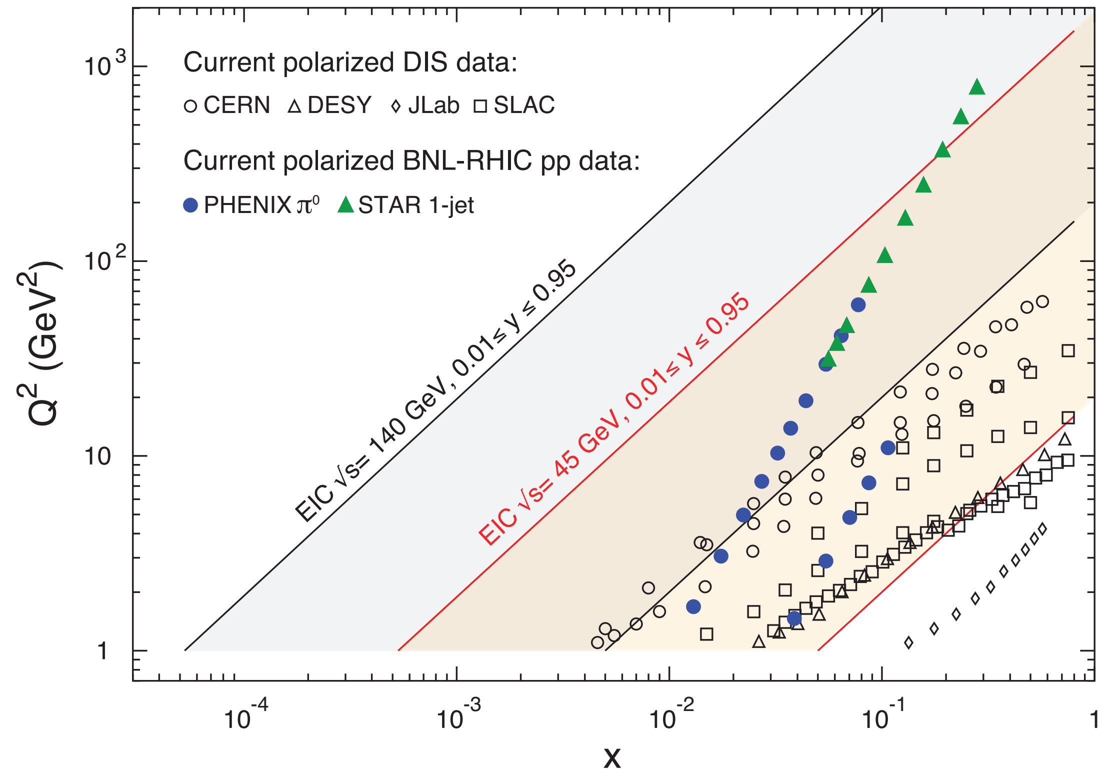
- Smooth connecting between the LaMET determinations and the SDF modeling
- Elevated PDF than the global fitting PDF in mid-x region
- Extra model uncertainties due to finite cutoff et al not quantified & shown
- Further studies on lattice systematics and with higher hadron momentum needed

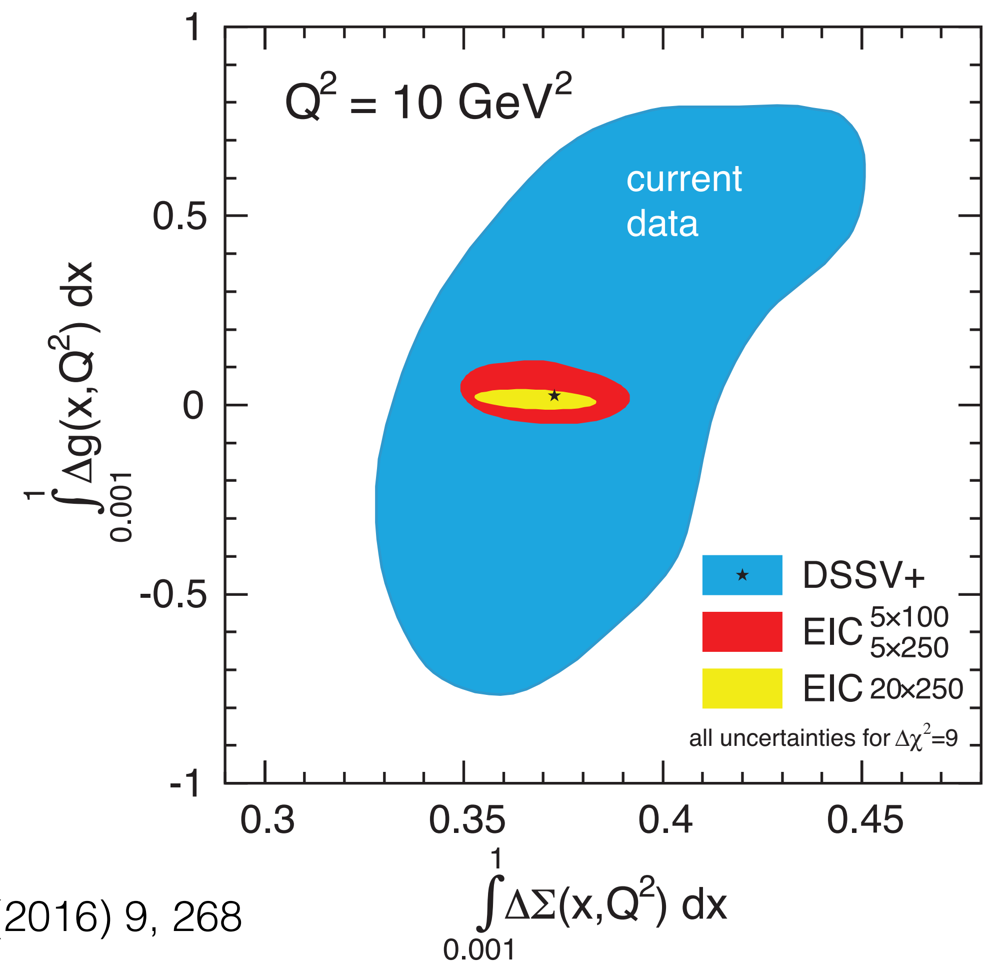
Conclusion

- ☑ Computed quark helicity matrix elements for the isovector sectors
- ☑ Computed the Mellin moments
- ☑ Computed the light cone PDF using the quasi-PDF approach with NNLO matching including NNLL resummations
- ☑ Constrain the endpoint regions using complementary approach

- ☐ Push to larger momentum to suppress the power corrections
- ☐ Add more lattice spacings for the continuum limit

Backup: Why EIC?

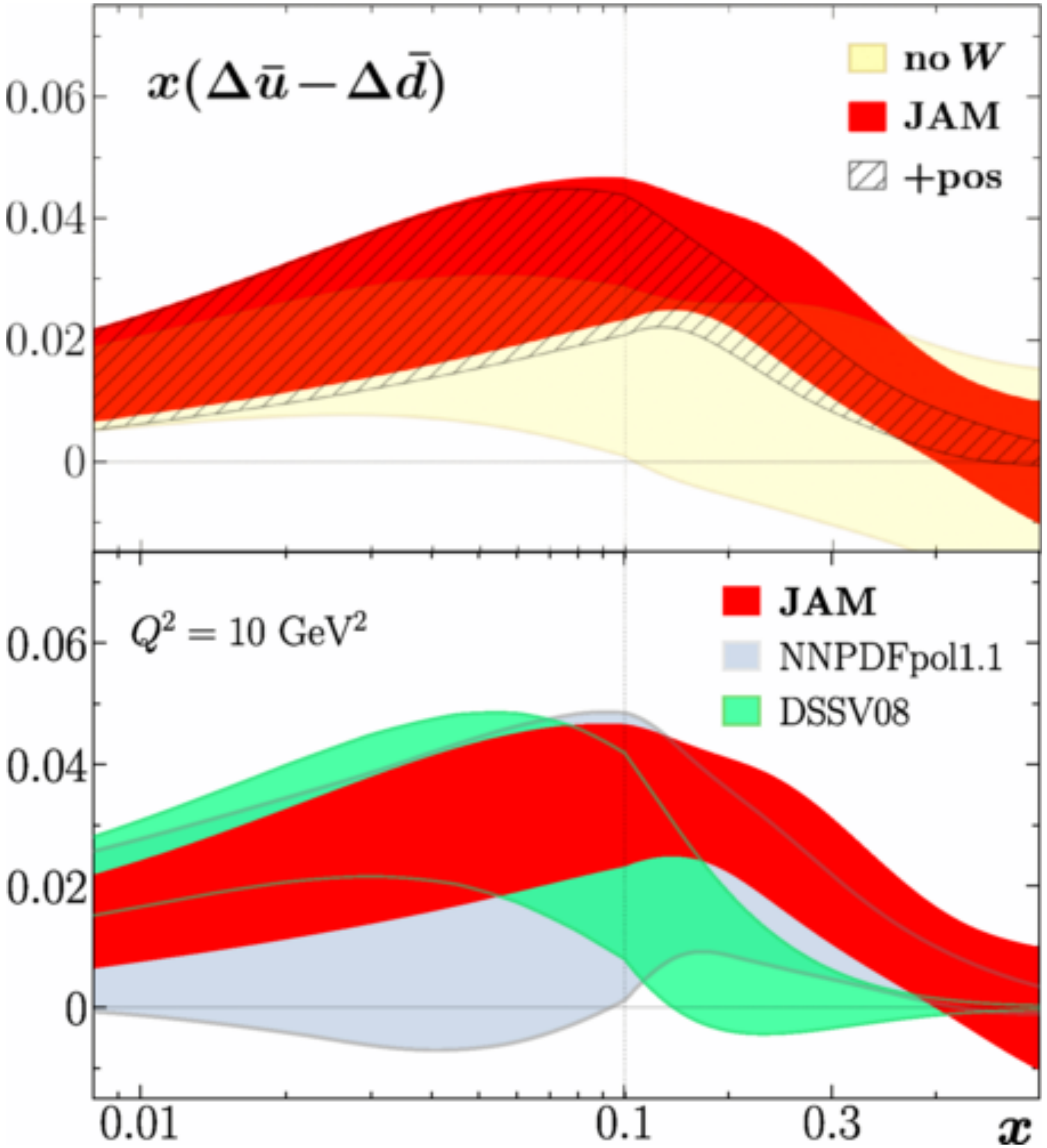




EIC White Paper: EPJA 52 (2016) 9, 268

- High luminosity provides more precise statistical measurements
- High polarization capacities of electron and nucleon give more access to the spin-dependent observables
- Unprecedented low-x reach for polarized DIS experiment

Backup: global helicity PDF



JAM 22: PRD106, L031502 (2022)

Non-zero contribution to spin from antiquarks

Backup: ratio ansatz

$$R = \frac{C_{3\text{pt}}^{\mathcal{P},\Gamma,f}(\vec{p}, t_{\text{sep}}, t_{\text{ins}}, z; \vec{x}, t_0)}{C_{2\text{pt}}^{\mathcal{P}}(\vec{p}, t_{\text{sep}}; \vec{x}, t_0)} = \frac{\sum_{\vec{y},\vec{z}_0} e^{-i\vec{p}\cdot(\vec{y}-\vec{x})}\mathcal{P}\left\langle N(\vec{y}, t_{\text{sep}}+t_0)\mathcal{O}_{\Gamma}^{f}(\vec{z}_0 + z\hat{z}, t_{\text{ins}}+t_0)\bar{N}(\vec{x}, t_0)\right\rangle}{\sum_{\vec{y}} e^{-i\vec{p}\cdot(\vec{y}-\vec{x})}\mathcal{P}\left\langle N(\vec{y}, t_{\text{sep}}+t_0)\bar{N}(\vec{x}, t_0)\right\rangle}$$

Courtesy of Jun Hua

To obtain the g_V and g_A from the 3pt correlation function, we should do the reduction by inserting,

$$1 = \int \frac{d^3\vec{P}_q}{(2\pi)^3 2E_q}|q\rangle\langle q| \tag{24}$$

thus

$$\begin{aligned} C_{3,\mathcal{O}}^{N\to P}\left(q^2, t, t_{\text{seq}}\right) = \int d^3\vec{x}d^3\vec{y} \int \frac{d^3\vec{P}_N}{(2\pi)^3 2E_N}\frac{d^3\vec{P}_P}{(2\pi)^3 2E_P} \\ T^{\gamma\gamma'}\left\langle 0\left|\psi_\gamma^{(P)}\left(\vec{x}, t_{\text{seq}}\right)\right|P\right\rangle\langle P\left|\mathcal{O}^\mu(\vec{y}, t)\right|N\rangle\left\langle N\left|\bar{\psi}_{\gamma'}^{(N)}(\vec{0}, 0)\right|0\right\rangle. \end{aligned} \tag{25}$$

Consider the reduction for 2pt correlation function:

$$\begin{aligned} C_2^P &= \int d^3\vec{x}T'^{\gamma\gamma'}\left\langle\psi_\gamma^{(P}(\vec{x}, t)\bar{\psi}_{\gamma'}^{(P)}(\vec{0}, 0)\right\rangle \\ &= \int d^3\vec{x}T'^{\gamma\gamma'}\int\frac{d^3\vec{P}_q}{(2\pi)^3 2E_q}\left\langle\psi_\gamma^{(P}(\vec{x}, t)|q\rangle\langle q|\bar{\psi}_{\gamma'}^{(P)}(\vec{0}, 0)\right\rangle. \end{aligned} \tag{26}$$

then the 1/2E*1/2E becomes one 1/2E. due to the “same” condition impose a delta function

The u, d quarks do not differ in chroma, so the 2pt of protons and neutrons is the same. We can easily extract the g_V and g_A by taking the ratio of:

$$R_{V/A}(T, \mu) = \frac{C_3^{V/A}\left(q^2, t, t_{\text{seq}}\right)}{C_2^P(t_{seq})}. \tag{27}$$

Backup: RI-MOM

Euclidean state with momentum squared $-p^2 \gg \Lambda_{\rm QCD}^2$ in a fixed gauge, and then defines $\overline{\rm MS}$ operators as,

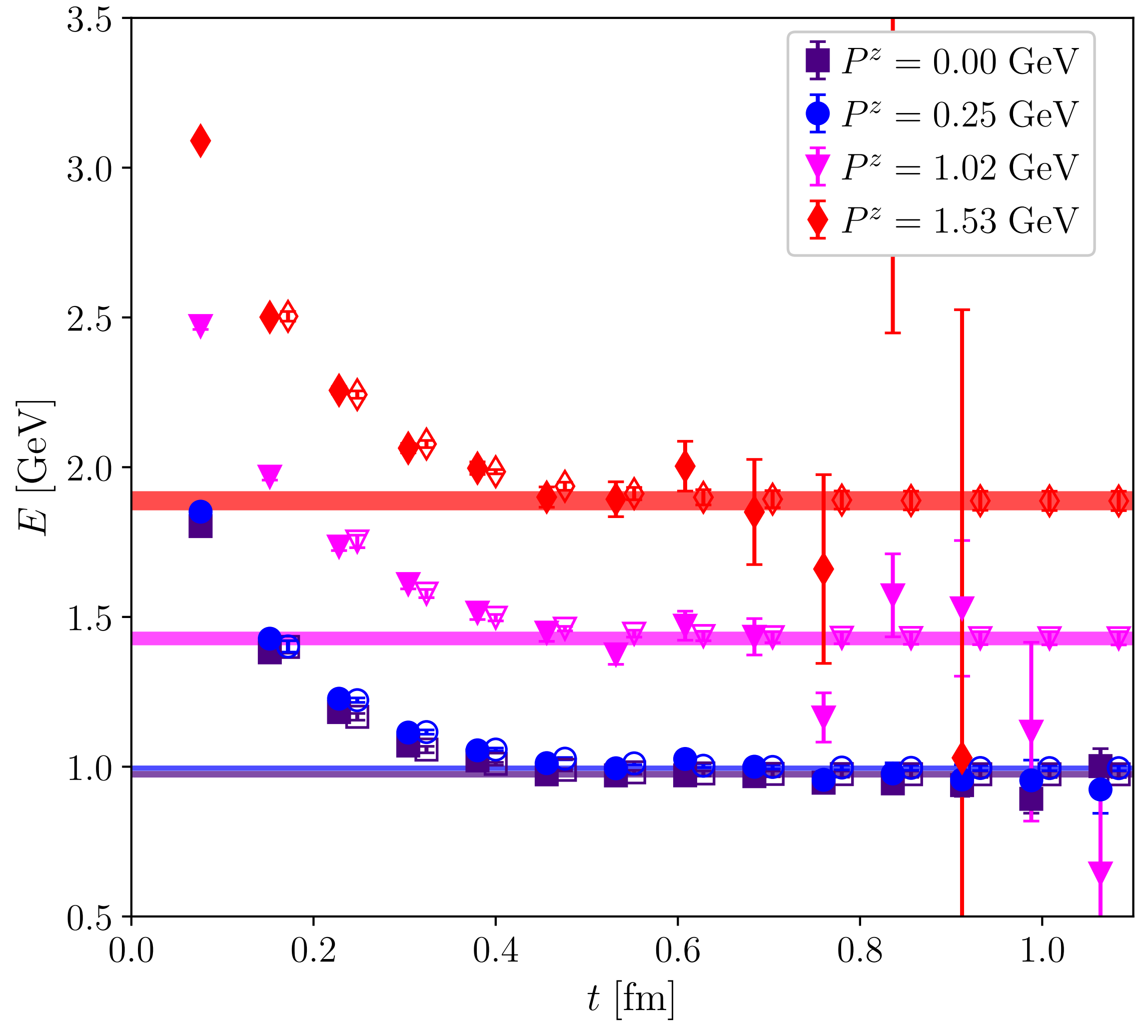
$$O_{\overline{\rm MS}}(z,\mu) \equiv Z_{\overline{\rm MS}}(z,-p^2,\mu)\frac{O(z,a)}{Z(z,-p^2,a)}, \tag{13}$$

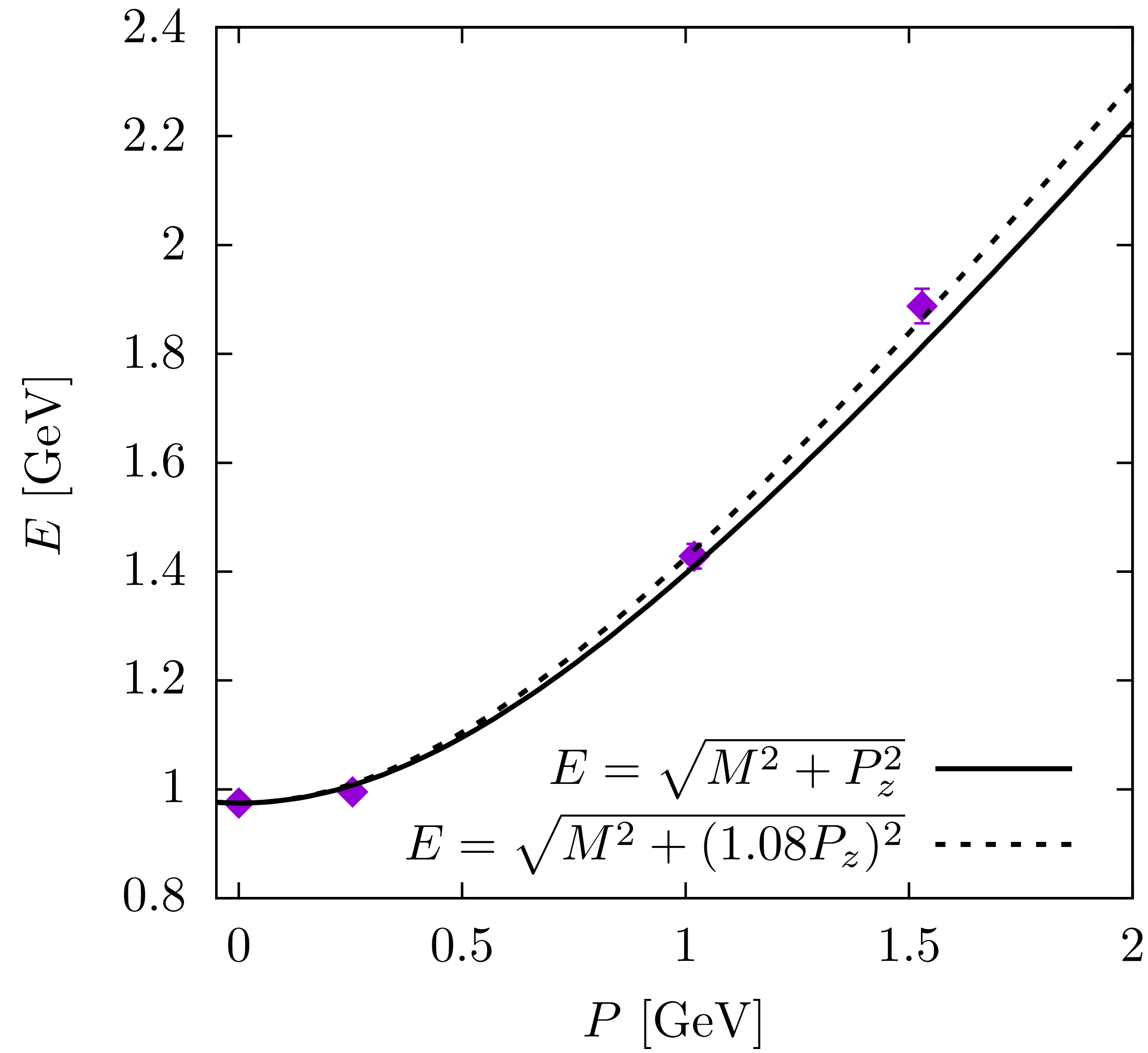
where $Z_{\overline{\rm MS}}$ converts the RI/MOM renormalized result to the $\overline{\rm MS}$ scheme. The gauge and $-p^2$ dependences cancel between two Z-factors. The r.h.s. has a proper continuum limit $a \to 0$ without divergences.

However, while the RI/MOM approach is justified for local operators, it has potential problems when applied to nonlocal ones. For instance, when z becomes large, $Z_{\overline{\rm MS}}(z,-p^2,\mu^2)$ contains IR logarithms of z and the perturbative calculation of z-dependence is not reliable. Moreover, although the RI/MOM factor $Z(z,-p^2,a)$ helps to cancel the lattice UV divergences, the composite operator at large-z contains non-perturbative physics as well. Therefore, both Z-factors contain non-cancelling non-perturbative effects which alter the IR properties of $O(z)$. Thus, the RI/MOM renormalization scheme is not reliable at large-z. Moreover, when gluon distributions are involved, it requires external off-shell gluon states which bring in potential mixing with gauge-variant operators and make things much more complicated [67].

arXiv:2008.03886

Sanity check of the joint fit





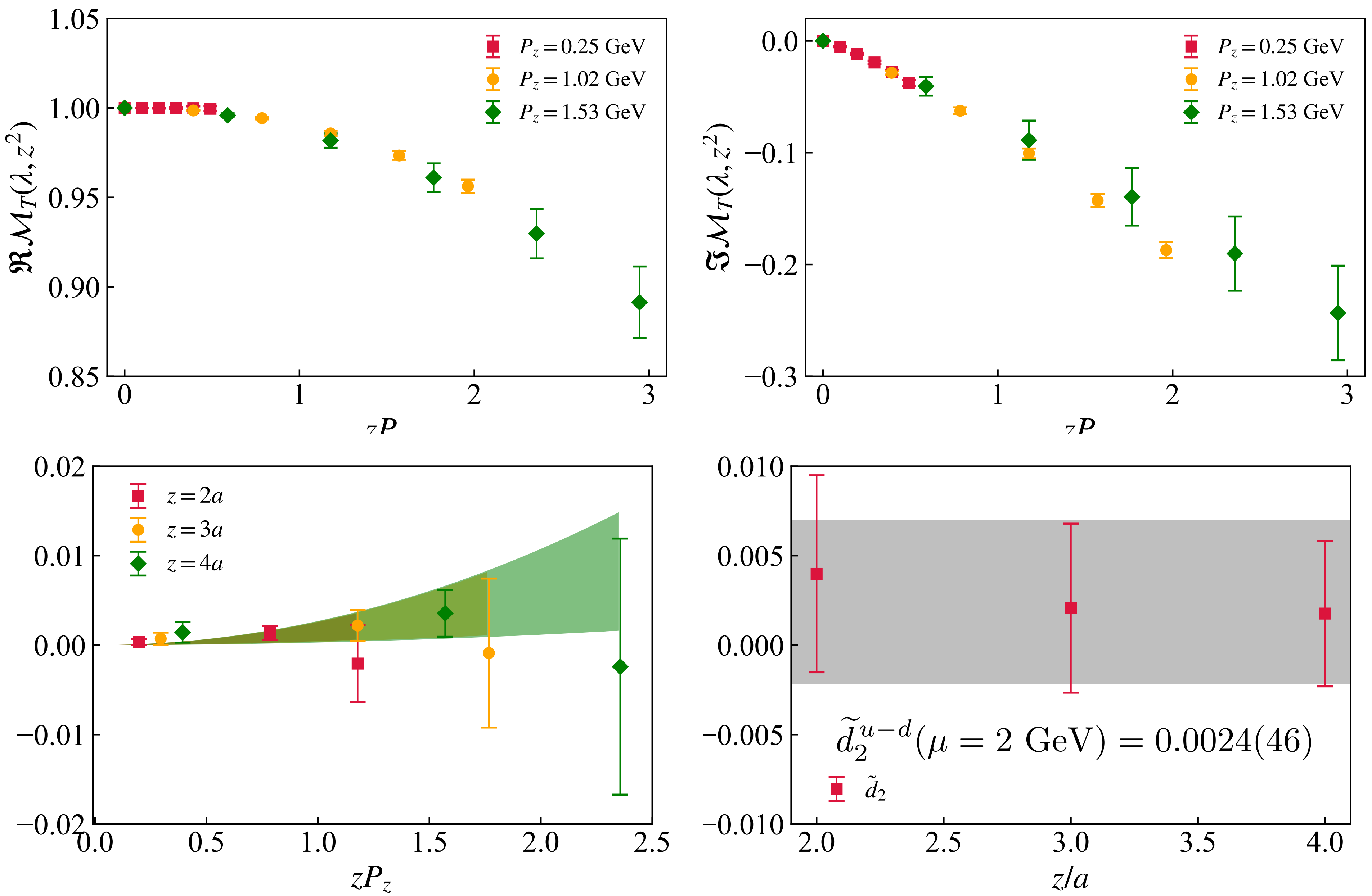
Filled points: effective mass from 2pt
Open points: effective mass from 2pt reconstructed from the joint-fits
Bands: ground-state mass from joint fit of R and 2pt

Good control of lattice discretization effects

Backup: $\tilde{d}_2$-term

$\tilde{d}_2$-term: genuine twist-3 quark–gluon correlation inside the polarized proton, probing how the quark is coupled to the gluon

$$\widetilde{d}_2 = \int_0^1 dx\, x^2 \left[3g_T(x) - g_1(x)\right] \qquad \Gamma = \gamma_5\gamma_x$$



Backup: determination of renormalon

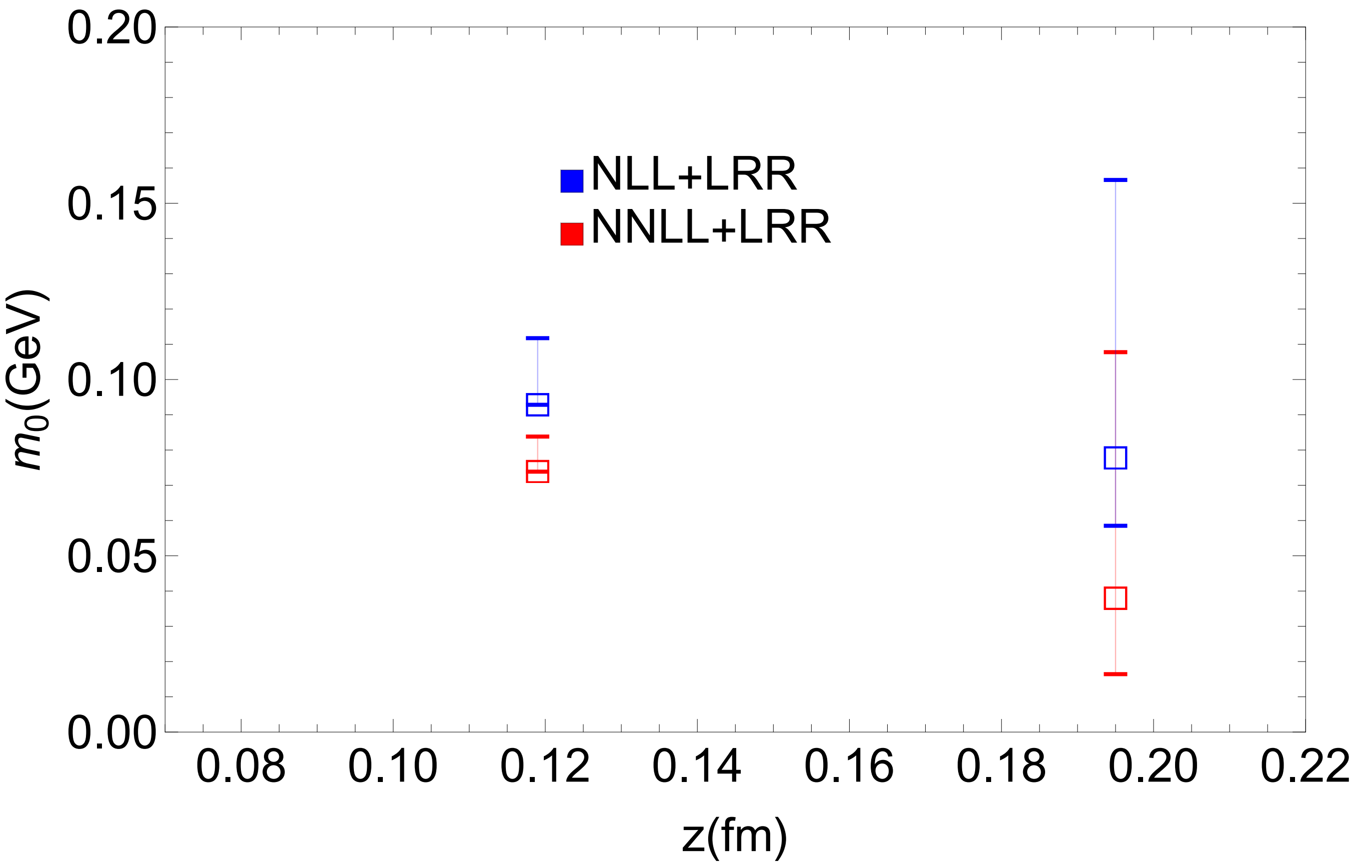
Linear divergence from WL self-energy

$$a\delta m = 0.1597(16)$$

$$\frac{\widetilde{h}_1^B(z,0,a)}{\widetilde{h}_1^B(z',0,a)}\,e^{|z-z'|[\delta m(a)+m_0]} = \frac{C_{0,\mathrm{PV}}^{\mathrm{F.O.+LRR}}(z,\mu_0)\,e^{-\mathcal{I}(\mu_0)}}{C_{0,\mathrm{PV}}^{\mathrm{F.O.+LRR}}(z',\mu_0')\,e^{-\mathcal{I}(\mu_0')}}$$

$$C_{0,\mathrm{PV}}^{\mathrm{F.O.+LRR}}(\mu_0) \equiv C_{0,\mathrm{PV}}^{\mathrm{LRR}}(\mu_0) + \left[C_0^{\mathrm{F.O.}}(\mu_0) - C_{0,\mathrm{PV}}^{\mathrm{LRR}}(\mu_0)\Big|_{\mathrm{F.O.}}\right]$$

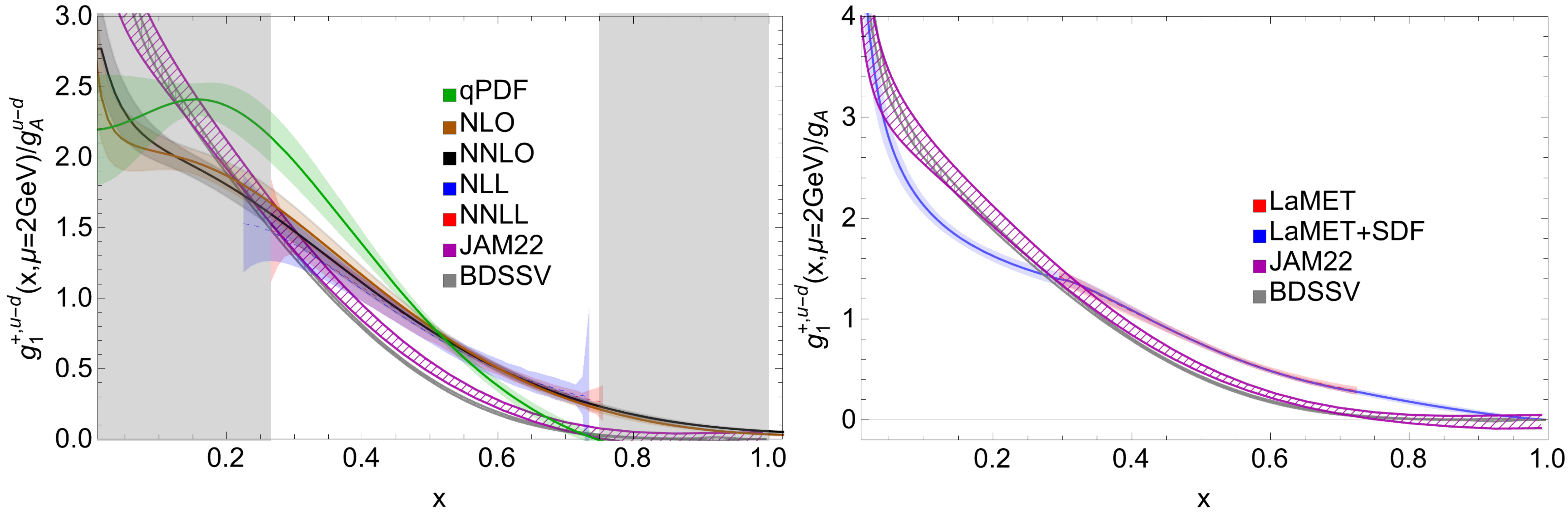
$$\mu_0 = \frac{2\kappa e^{-\gamma_E}}{z}, \qquad \mu_0' = \frac{2\kappa e^{-\gamma_E}}{z'}, \qquad \mathcal{I}(\mu) = \int d\alpha\,\frac{\gamma(\alpha)}{\beta(\alpha)}\bigg|_{\alpha=\alpha_s(\mu)}$$



The size of z-dependence and perturbative convergence is small

Backup: full distributions g+ (valence dist. g-)

$$g_1^+(x) \equiv g_1(x) + g_1(-x) = g_1(x) + \bar{g}_1(x)$$



Backup: iso-scalar u+d