



Temporal Evolution and Spatial Distribution of a Dynamically Generated Nucleon Resonance

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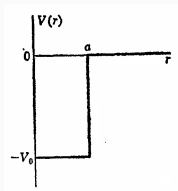
Y. Zhuge and Z.-W. Liu, arXiv:2606.07411

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Brief Review of a Resonance with Quantum Mechanics

Bound states can be well described with quantum mechanics



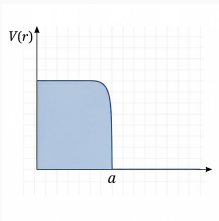
- Schrödinger equation provides
 - mass of bound state
 - wavefunction of bound state
- One can extract everything from the wavefunction.
- The wavefunction has a direct probability interpretation.

A resonance can also be described with Schrödinger equation

- A state χ_k with energy $E = \frac{\hbar^2 k^2}{2m}$ in s wave satisfies:

$$\chi_k''(r) + [k^2 - \frac{2m}{\hbar^2} V(r)] \chi_k(r) = 0$$

The wavefunction $\chi_k(r)$ can be solved with proper boundary and continuity conditions.



- One can extract everything from the wavefunction.
 - The **scattering S matrix** can be obtained by

$$\chi_k(r) = A(k)[e^{-ikr} - \mathbf{S}(k)e^{+ikr}], \quad r > a.$$

- The mass and width can be got from the pole of scattering S matrix.

$$S(k) \xrightarrow{k \rightarrow k_{\text{pole}}} \infty$$
$$E_{\text{pole}} = m_{\text{pole}} - i\Gamma_{\text{pole}}/2$$

There are two main problems for the characterization of resonance

Complex pole energy $E_{\text{pole}} = m_{\text{pole}} - i\Gamma_{\text{pole}}/2$ can be obtained by

- searching for the poles of S matrix;
- or directly solving Schrödinger equation with the complex-scaling method $r \rightarrow re^{i\theta}$.

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There are two main problems for the characterization of resonance:

- One is related to the experiment.
- The other is from the theory itself.

Complex pole energy $E_{\text{pole}} = m_{\text{pole}} - i\Gamma_{\text{pole}}/2$ cannot be directly measured

- For the peaks in experiments:
central value $\neq m_{\text{pole}}$, width $\neq \Gamma_{\text{pole}}$
- The peaks of some resonances are not obvious.
- Some peaks are not originated from resonances.
- The line shapes of invariant spectra would be more complicated.

The direct probability interpretation is lost

- When $k \rightarrow k_{\text{pole}}$, $\chi_k(r)$ seems divergent because of $S(k_{\text{pole}}) \rightarrow \infty$

$$\chi_k(r) = A(k)[e^{-ikr} - S(k)e^{+ikr}]$$

- But one can use different normalization to make the above divergence disappear.

$$\chi_k(r) \cong \frac{1}{S(k)}e^{-ikr} - e^{+ikr} \rightarrow -e^{+ikr}$$

- However there is an **unavoidable divergence** as $r \rightarrow \infty$ at the pole $k_{\text{pole}} = k_R - ik_I$.

$$\chi_{k_{\text{pole}}}(r) \rightarrow -e^{+ik_{\text{pole}}r} = -e^{+ik_R r} e^{k_I r} \rightarrow \infty$$

The resonance is always tangled with the emitted scattering states.

How to interpret $\chi_{k_{\text{pole}}}(r) \xrightarrow{r \rightarrow \infty} \infty$?

- It is not the resonance that spreads infinitely far away.
- The wavefunction obviously contains the **information of the emitted scattering states**.

$$\chi_k(r) = A(k)[e^{-ikr} - S(k)e^{+ikr}]$$

- The emitted particles fly infinitely apart.

**Representations for the $\Lambda(1405)$
Dynamically Generated by
 $\pi\Sigma\text{-}\bar{K}N$**

The $\Lambda(1405)$ is closely related to the $\pi\Sigma\text{-}\bar{K}N$ system

- The $\Lambda(1405)$ is located just below the $\bar{K}N$ threshold.
- It was observed mainly through the $\pi\Sigma$ channel.
- It is widely interpreted as a $\bar{K}N$ quasi-bound state or a dynamically generated meson-baryon resonance.

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Hermit $\hat{H}$ vs complex E

Gamow vector with complex scaling method

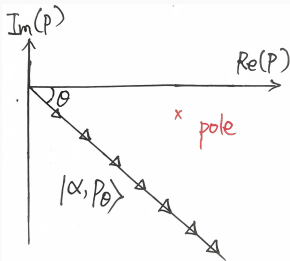
- introduce the two-particle basis with complex-scaling momentum $p \rightarrow p_\theta \equiv p e^{-i\theta}$

$$|\alpha, p_\theta\rangle = |\pi\Sigma, p_\theta\rangle, |\bar{K}N, p_\theta\rangle$$

- solve the Hamiltonian equation in the above basis

$$H|\psi^{\text{Gamow}}\rangle = (H_0 + V)|\psi^{\text{Gamow}}\rangle = E_{\text{pole}}|\psi^{\text{Gamow}}\rangle.$$

- then obtain resonance pole E_{pole} and corresponding Gamow wavefunction $|\psi^{\text{Gamow}}\rangle$.



The Gamow vector is spanned in the complex-momentum space which cannot be measured directly.

Alternatively Gamow wavefunction can be obtained by residues of T matrix



- obtain the T matrix by solving the Lippmann-Schwinger equation, equivalent to the resummation of the above diagrams.
- search for the poles of T matrix on the unphysical Riemann sheet.
- extract the residues $\gamma_\alpha(\mathbf{p}_\theta)$ of T matrix

$$T_{\alpha,\beta}(\mathbf{p}_\theta, \mathbf{p}'_\theta; E) \approx \frac{\gamma_\alpha(\mathbf{p}_\theta) \gamma_\beta(\mathbf{p}'_\theta)}{E - E_{\text{pole}}} + \dots,$$

- finally get the relation between the Gamow wavefunction and residues of T matrix

$$\psi_\alpha^{\text{Gamow}}(\mathbf{p}_\theta) \equiv \langle \alpha, \mathbf{p}_\theta | \psi^{\text{Gamow}} \rangle = \frac{\gamma_\alpha(\mathbf{p}_\theta)}{E_{\text{pole}} - \omega_\alpha(\mathbf{p}_\theta)},$$

Advantage and Disadvantage of Gamow wavefunction

Advantage:

constrained by Hamiltonian eigen equation.

Disadvantage:

- Gamow wavefunctions diverge at infinite distance.
- Complex probability or negative probability.
- Strong tensions between
 - Gamow vector in Complex momentum space;
 - Experimental measurements in Real momentum space.

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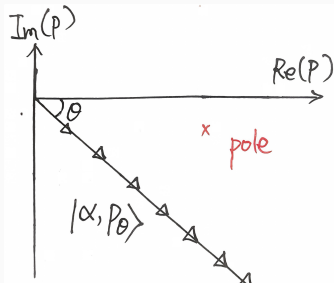
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- (4) describes the behaviors that a resonance not only decays but also **emits**;

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- (3) constrained by Hamiltonian eigen equation;
- (4) describes the behaviors that a resonance not only decays but also **emits**;
- (5) consistent with experimental measurements.

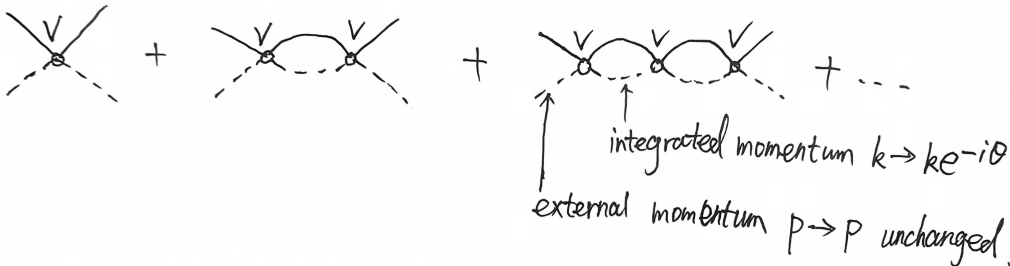
A natural transition to the representation in the real world



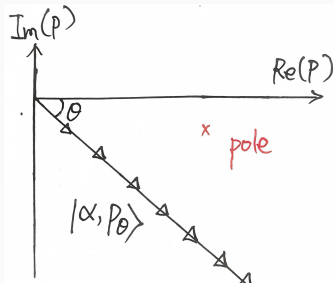
$$\psi_{\alpha}^{\text{Gamow}}(p_{\theta}) = \frac{\gamma_{\alpha}(p_{\theta})}{E_{\text{pole}} - \omega_{\alpha}(p_{\theta})},$$

By analytical continuation of the residue $\gamma_{\alpha}(p_{\theta}) \rightarrow \gamma_{\alpha}(p)$,

$$|\psi^{\text{phys}}\rangle = \sum_{\alpha} \int dp p^2 \frac{\gamma_{\alpha}(p)}{E_{\text{pole}} - \omega_{\alpha}(p)} |\alpha, p\rangle.$$



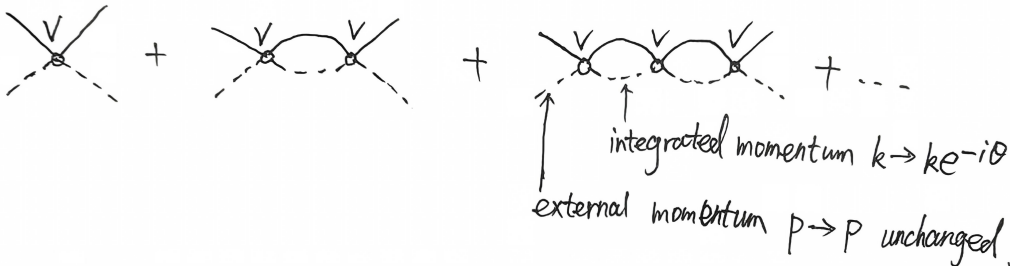
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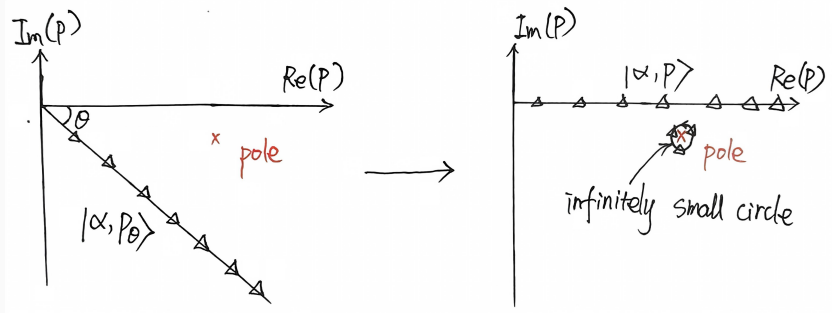
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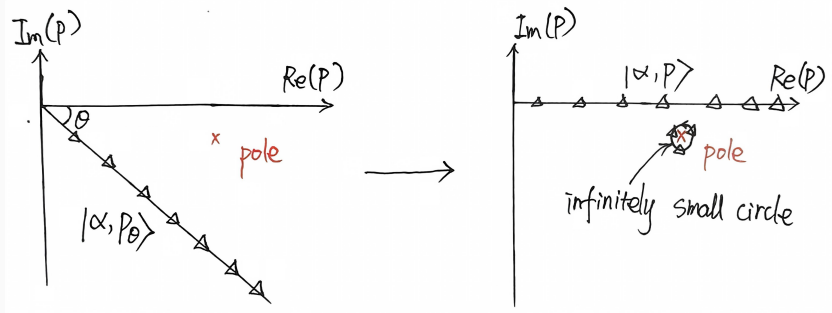
A New Relations between Gamow and Physical Representations



- By introducing a auxiliary vector state spanned around the basis states with the kinetic energy approaching the pole

$$|\psi^{\text{auxiliary}}\rangle = \sum_{\{\alpha \mid \theta_\alpha \neq 0\}} \oint_{C_\epsilon^\alpha \rightarrow p_\alpha^{\text{on}}} dp p^2 \frac{\gamma_\alpha(p)}{E_{\text{pole}} - \omega_\alpha(p)} |\alpha, p\rangle,$$

A New Relations between Gamow and Physical Representations

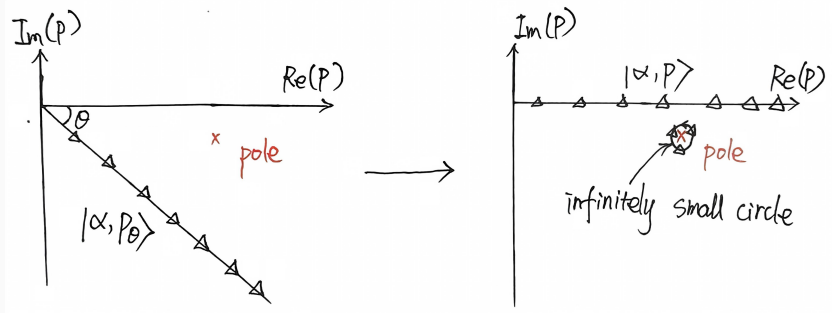


- we can prove

$$H(|\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle) = E_{\text{pole}} (|\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle)$$

- $|\psi^{\text{phys}}\rangle$ in the ordinary Hilbert space
- $|\psi^{\text{auxiliary}}\rangle$ in the special complex space closely around the pole

A New Relations between Gamow and Physical Representations



- With respect to the Hamiltonian equation,

$$|\psi^{\text{Gamow}}\rangle \cong |\psi^{\text{phys}}\rangle + |\psi^{\text{auxiliary}}\rangle,$$

which is a new essential relation between $|\psi^{\text{Gamow}}\rangle$ and $|\psi^{\text{phys}}\rangle$ in addition to their analytic connection.

Our framework for the dynamics of $\Lambda(1405)$

- We noticed
 - $\Lambda(1405)$ is mainly a $\bar{K}N$ - $\pi\Sigma$ dynamically generated state.
 - The bare triquark core is important for $\Lambda(1670)$.
- In this work we use a simple separable interaction in the $\pi\Sigma$ and $\bar{K}N$ channels

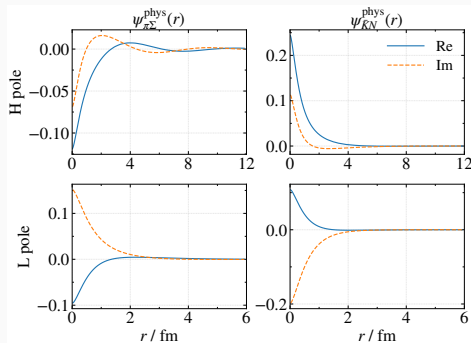
$$V_{\alpha\beta}(p, p') = \frac{3v_{\alpha\beta}}{4\pi^2 f_\pi^2} u_\alpha(p) u_\beta(p'), \quad u_\alpha(p) = \left(1 + \frac{p^2}{\Lambda_\alpha^2}\right)^{-2}.$$

By adjusting the coupling parameters, we obtain

- the higher-mass (H) pole $E_{\text{pole}}^{\text{H}} = 1430 - 22 i \text{ MeV}$,
- the lower-mass (L) pole $E_{\text{pole}}^{\text{L}} = 1338 - 89 i \text{ MeV}$.

Coordinate Wavefunctions of $\Lambda(1405)$

The wavefunctions $\psi_{\alpha}^{\text{phys}}(r) \equiv \langle \alpha, r | \psi^{\text{phys}} \rangle$ in coordinate space can be obtained by Fourier transformation.



In $\Lambda(1405)$, $\pi\Sigma$ and $\bar{K}N$ are confined within finite ranges.

Temporal evolution for the two $\Lambda(1405)$ poles

With

$$|\psi^{\text{phys}}\rangle = \sum_{\alpha} \int dp p^2 \frac{\gamma_{\alpha}(p)}{E_{\text{pole}} - \omega_{\alpha}(p)} |\alpha, p\rangle ,$$

we can calculate the evolution of the states

$$|\psi^{\text{phys}}, t\rangle = e^{-iHt} |\psi^{\text{phys}}\rangle .$$

In detail

- insert the completeness relation with $|\alpha, p\rangle$
- discretize the resulting momentum integrals using Gaussian quadrature
- thus reduce time evolution to a finite-dimensional matrix-vector operation for numerical solutions.

Resonances decay with the scattering states emitted

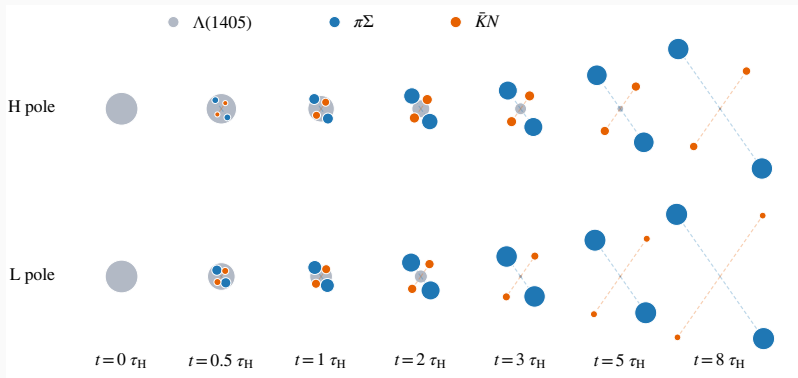
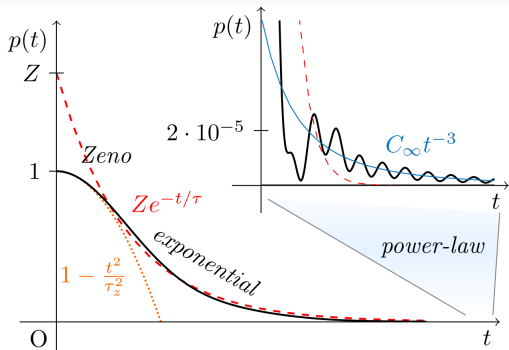


Illustration of the time evolution $|\psi^{\text{phys}}, t\rangle = e^{-iHt} |\psi^{\text{phys}}\rangle$ of the resonance components

- The gray area is proportional to the survival probability and decreases with time in units of $\tau_H = 1 / |\Gamma_{\text{pole}}^H|$
- The emitted $\pi\Sigma$ and $\bar{K}N$ move away from the interaction region. Their distances from the center are proportional to their root-mean-square radii.

Survival probability of a resonance



This figure is from A. Crespi, et. al.
Phys. Rev. Lett. **122** (2019), 130401

It is well known that the survival probability of a resonance is

- quadratic at short times (Zeno effect)
- exponential at mediate times
- power law at long times

Experimentally confirmed in:
trapped atoms, Bose-Einstein condensates,
Rydberg atoms, optical waveguide arrays,
cavity quantum electrodynamics, ...

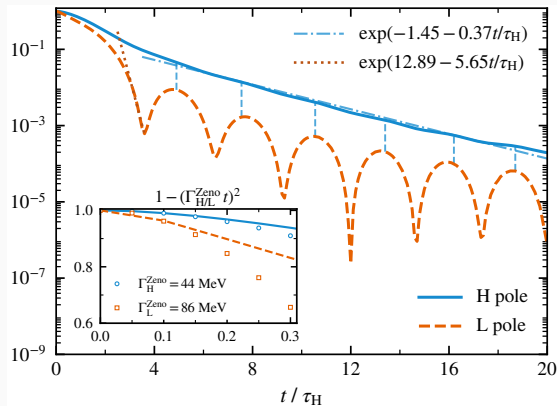
Survival probability of $\Lambda(1405)$ would be more interesting

- strong correlations with the emitted particles
- two poles may interference
- ...

Survival probabilities for the two $\Lambda(1405)$ poles

$$|\psi^{\text{phys}}, t\rangle = C(t) |\psi^{\text{phys}}\rangle + |\chi_{\text{scatt.}}, t\rangle.$$

$$\text{Survival probabilities} = |C(t)|^2.$$

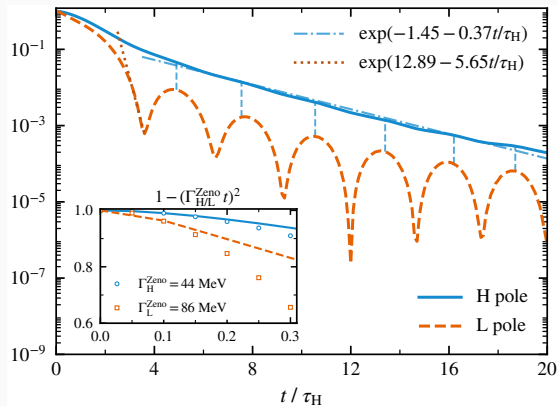


- At very early times,
 - the survival probability follows the expected quadratic behavior, known as the quantum Zeno effect [Wigner:1955zz, Chiu:1977ds, LEVITAN1988267]
 - the fact that $\Gamma_H^{\text{Zeno}} \approx \Gamma_{\text{pole}}^{\text{H}}$ but $\Gamma_L^{\text{Zeno}} < \Gamma_{\text{pole}}^{\text{L}}$ indicates the complicated interference between these two nearby poles and the scattering channels.

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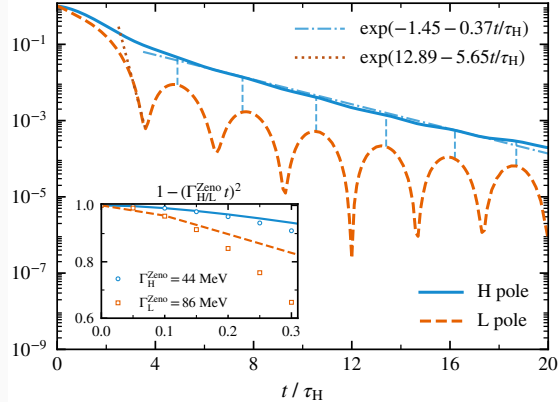


- At late times for the lower-mass pole,
its survival probability shows an obvious damped oscillatory behavior
which is a known quantum phenomenon of resonance.

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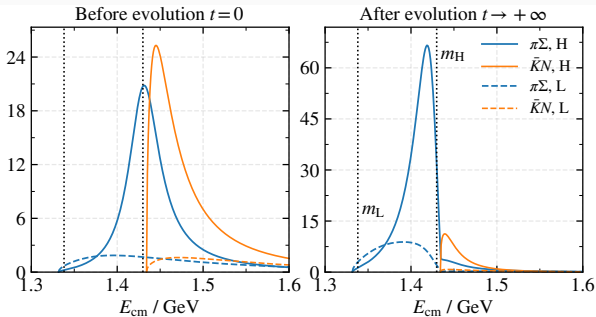
- In the intermediate-time region for the higher-mass pole,
 - we show an exponential fit $\exp(-1.45 - 0.37t/\tau_H)$ over $3.5 \leq t/\tau_H \leq 20$
 - the deviation exhibits weak oscillations whose peaks marked by the vertical dashed lines coincide with the corresponding peaks in the survival probability of the lower-mass pole suggesting these two states are strongly correlated.

The asymptotic state at the infinite time

The asymptotic state $|\psi^{\text{phys}}, t \rightarrow +\infty\rangle$ in the Schrödinger picture

$$|\psi^{\text{phys}}, t \rightarrow +\infty\rangle = \sum_{\alpha} \int dp p^2 \psi_{\alpha}^{\text{phys}}(p) \times \left(e^{-i\omega_{\alpha}(p)t} |\alpha, p\rangle + \sum_{\beta} \int dq q^2 \frac{T_{\beta,\alpha}(q, p; E)}{\omega_{\beta}(q) - \omega_{\alpha}(p) + i\epsilon} e^{-i\omega_{\beta}(q)t} |\beta, q\rangle \right).$$

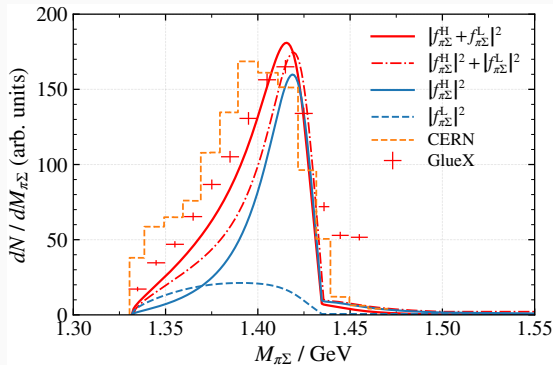
The resulting distribution $|f_{\alpha}(E)|^2 = |\langle \alpha, E | \psi^{\text{phys}}(t = +\infty) \rangle|^2$



For the high-mass pole

- the $\bar{K}N$ component is significant when initially formed
- most of it turns into the $\pi\Sigma$ rather than $\bar{K}N$ scattering states because of
 - energy conservation when $t \rightarrow \infty$
 - the larger mass of $\bar{K}N$ than m_{pole}^H

Comparison with the experimental measurements



Y. Zhuge and Z. W. Liu, arXiv:2606.07411.

We give the full results with both

- the strongest interference scenario $|f_{\pi\Sigma}^H(E) + f_{\pi\Sigma}^L(E)|^2$
- no interference assumption $|f_{\pi\Sigma}^H(E)|^2 + |f_{\pi\Sigma}^L(E)|^2$

The currently observed $\pi\Sigma$ spectrum prefers the strongest interference scenario.

Summary

Summary

- We have developed a physical description $|\psi_{H/L}^{\text{phys}}\rangle$ of the two-pole $\Lambda(1405)$ in the real momentum basis by analytically continuing Gamow wavefunction from complex momentum space.
- The $|\psi_{H/L}^{\text{phys}}\rangle$ is spanned in the ordinary Hilbert space, and thus the probability interpretation can be naturally obtained.
- With respect to the Hamiltonian equation, $|\psi_{H/L}^{\text{phys}}\rangle + |\psi_{H/L}^{\text{auxiliary}}\rangle$ is equivalent to $|\psi_{H/L}^{\text{Gamow}}\rangle$.

With this framework, we suggest the final invariant spectrum should include the convolution of resonance wavefunctions in addition to the scattering T matrix.

Thank you for your attention!