Origin of the small nucleon gravitational form factor $B_N(Q^2)$

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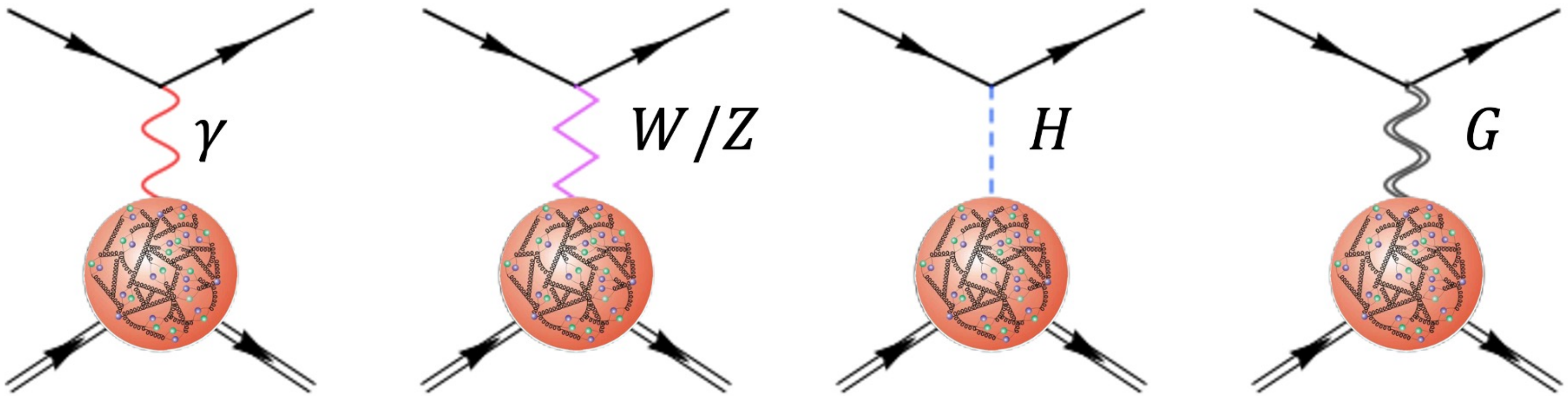
Energy-momentum tensor

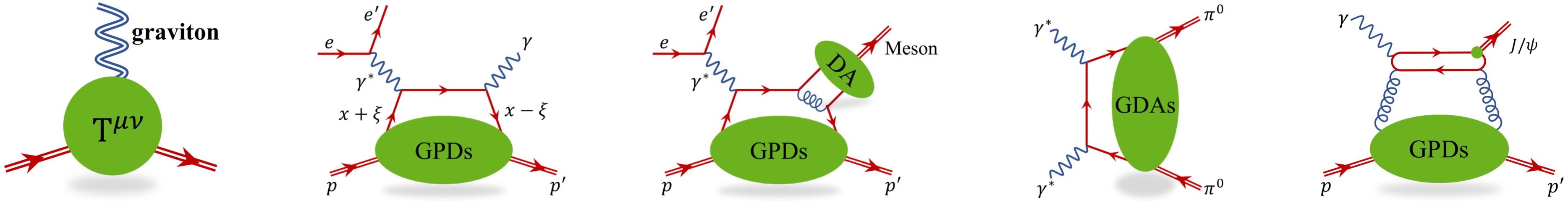


Hadron matrix elements and gravitational form factors (GFFs): [Kobzarev:1962wt, Pagels:1966zza]

$$\langle p', s'|\hat{T}_i^{\mu\nu}(0)|p, s\rangle = \frac{1}{2M}\bar{u}_{s'}(p')\left[2P^\mu P^\nu A_i(q^2) + iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho J_i(q^2) + \frac{1}{2}(q^\mu q^\nu - g^{\mu\nu}q^2)D_i(q^2) + 2g^{\mu\nu}\bar{c}_i(q^2)\right]u_s(p)$$

where $P = (p + p')/2,\ q = p' - p.$

How to access gravitational form factors



[Kumano:2017lhr, Duran:2022xag, Burkert:2023wzr]

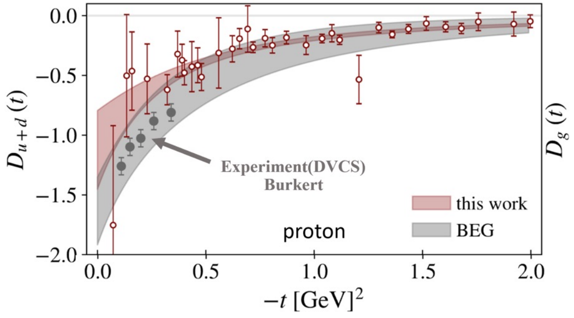
- Deeply virtual Compton scattering
- Deeply virtual meson production
- Two-photon pair production
- J/ψ threshold photoproduction

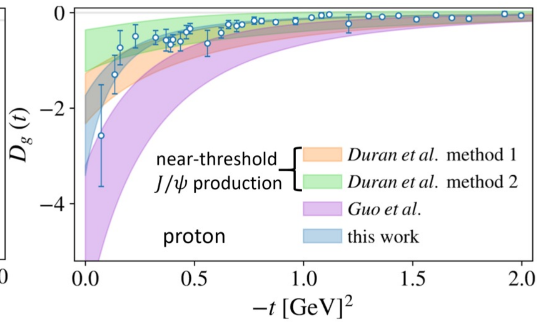
Ji's sum rule:

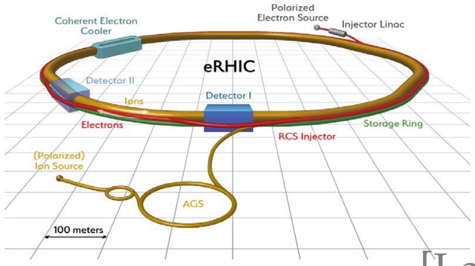
$$\int_{-1}^{1} dx x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t), \quad \int_{-1}^{1} dx x E^{q,g}(x,\xi,t) = B^{q,g}(t) - \xi^2 D^{q,g}(t)$$

[Ji:1996nm]

Here, $H^{q,g}$ and $E^{q,g}$ are generalized parton distributions







[Lattice'23: Hackett:2023nkr]

Form factors and conserved physical quantities

The structure of hadrons can be probed by the other fundamental forces [Polyakov:2018zvc]

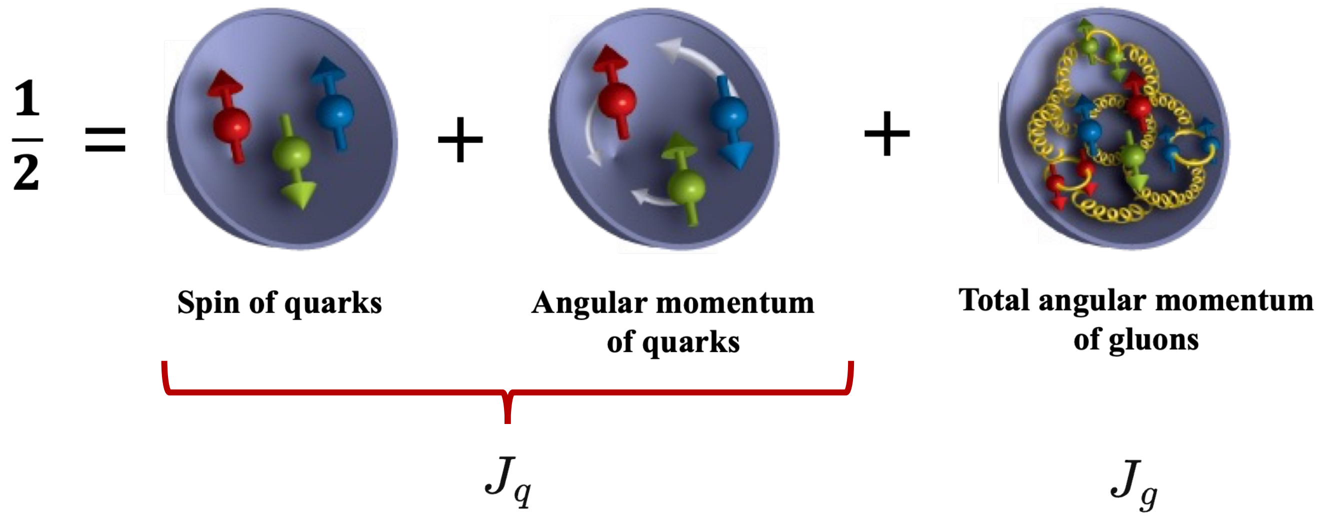
em:	$\partial_\mu J^\mu_{\rm em} = 0$	$\langle N' \vert J^\mu_{\bf em} \vert N\rangle$	$\longrightarrow$	Q	$= 1.602176487(40) \times 10^{-19}\mathrm{C}$
				μ	$= 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\bf weak} \vert N\rangle$	$\longrightarrow$	g_A	$= 1.2694(28)$
				g_p	$= 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\bf grav} = 0$	$\langle N' \vert T^{\mu\nu}_{\bf grav} \vert N\rangle$	$\longrightarrow$	m	$= 938.272013(23)\,\mathrm{MeV}/c^2$
				J	$= \frac{1}{2}$
				$\boldsymbol{D}$	$=$ **?**

Momentum conservation and angular momentum conservation [Cotogno:2019xcl, Lorce:2019sbq]

$$A(0) = 1, \qquad J(0) = \frac{1}{2}$$

Proton spin decomposition

[Ji:1996ek]



Total angular momentum and momentum differ only by the AGM [Teryaev:1999su]

$$2J_{q,g}(0) = A_{q,g}(0) + B_{q,g}(0)$$

anomalous gravitomagnetic moment (AGM)

with

[Ji:1996ek]

$$\left.\begin{aligned} A_q(0) &= \sum_f \int_0^1 x\left[q_f(x) + \bar{q}_f(x)\right]\mathrm{d}x \equiv \langle x \rangle_q, \quad q_f = u, d, s, \cdots \\ A_g(0) &= \int_0^1 x g(x)\mathrm{d}x \equiv \langle x \rangle_g \end{aligned}\right\} \text{Momentum fraction}$$

Anomalous gravitomagnetic moment

Electromagnetic current coupled to photon

[Rosenbluth:1950yq]

$$\langle p', s'|J^\mu|p, s\rangle = \bar{u}_{s'}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)\right]u_s(p)$$

Dirac F.F.

Pauli F.F.

$$F_1(0) = \text{ charge}, F_2(0) = \text{ anomalous magnetic moment}$$

[Polyakov:2018zvc]

$$2J_i = A_i + B_i$$

Energy-momentum tensor coupled to graviton

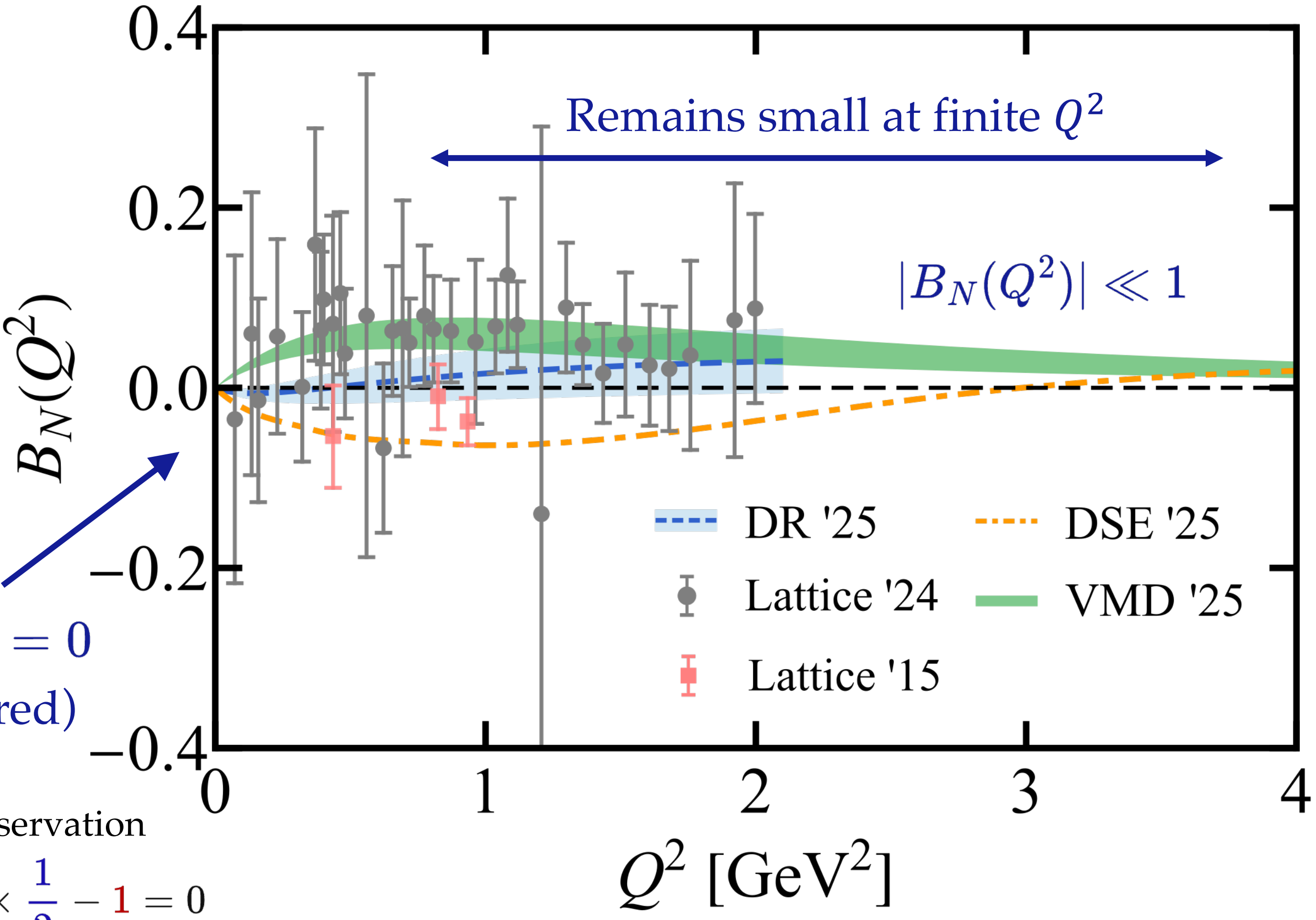
$$\langle p', s'|\hat{T}_i^{\mu\nu}(0)|p, s\rangle = \bar{u}_{s'}(p')\left[\frac{\gamma^{\{\mu}P^{\nu\}}}{2}A_i(q^2) + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho}{4M}B_i(q^2) + \cdots\right]u_s(p)$$

or

$$\langle p', s'|\hat{T}_i^{\mu\nu}(0)|p, s\rangle = \bar{u}_{s'}(p')\left[\frac{P^\mu P^\nu}{M}A_i(q^2) + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho}{2M}J_i(q^2) + \cdots\right]u_s(p)$$

The Smallness of the Nucleon GFF $B_N(Q^2)$

[Cao:2026jzm]



$B_N(0) = 0$
(required)

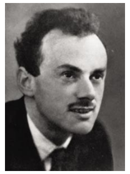
Angular momentum conservation

$B(0) = 2J(0) - A(0) = 2 \times \frac{1}{2} - 1 = 0$

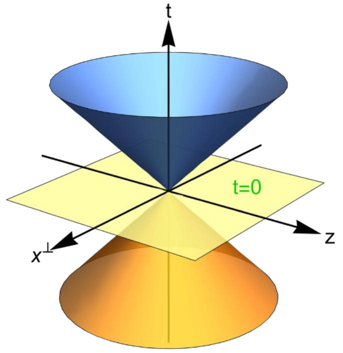
What is the dynamical origin of this persistent suppression?

Light-front quantization

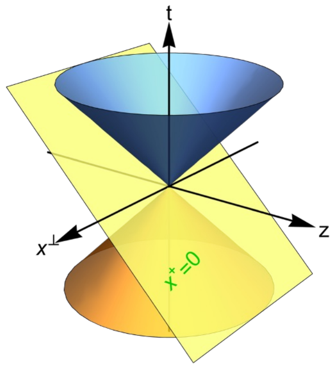
[Dirac:1949cp]



equal time quantization



light-front quantization



- Time: $t \equiv x^0$ $\qquad$ $t \equiv x^+ = x^0 + x^3$
- Hamiltonian: $H \equiv P^0$ $\qquad$ $H \equiv P^- = P^0 - P^3$
- Dispersion relation: $p^0 = \sqrt{\vec{p}^2 + m^2}$ $\qquad$ $p^- = (\vec{p}_\perp^2 + m^2)/p^+$

light-front coordinates

$$x^\pm = x^0 \pm x^3$$

$$\vec{x}^\perp = (x^1, x^2)$$

Light-front quantization is a Hamiltonian method of the quantum field theory

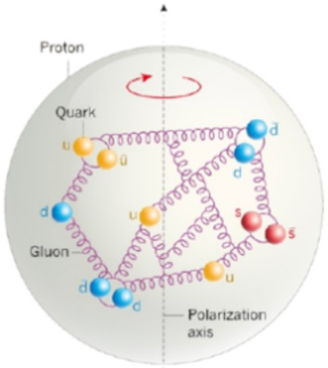
$$(P^+ \hat{P}^- - \vec{P}_\perp^2) \, |\psi_H\rangle = M_h^2 \, |\psi_h\rangle$$

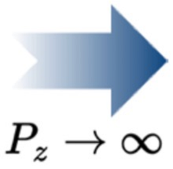
[Review: Brodsky:1997de]

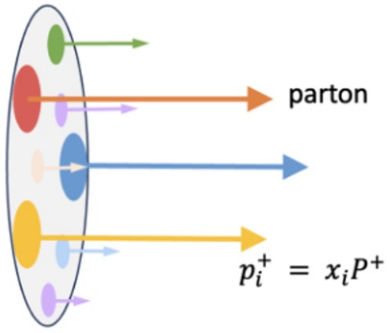
Light-front wave functions

$$|\psi_h(P,j,\lambda)\rangle = \sum_{n=1}^{\infty} \int [\mathrm{d}x_i \mathrm{d}^2 k_{i\perp}]_n \psi_{h/n}(\{\vec{k}_{i\perp}, x_i, \lambda_i\}_n) \big|\{\vec{p}_{i\perp}, p_i^+, \lambda_i\}_n\big\rangle$$

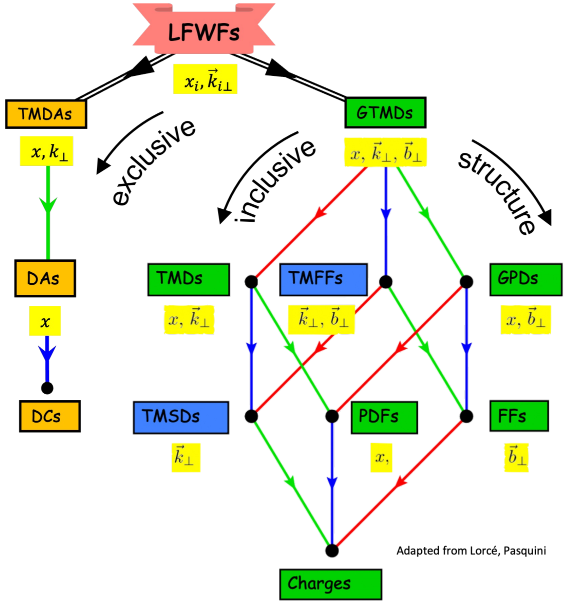
- Light-front physics measures hadron structures in high-energy scattering experiments
- Light-front wave functions (LFWFs) provide full information about the hadron structure
- LFWF representation offers simple physical interpretations







[Lorce:2011kd]



Light-front wave function representation

Brodsky-Hwang-Ma-Schmidt formula

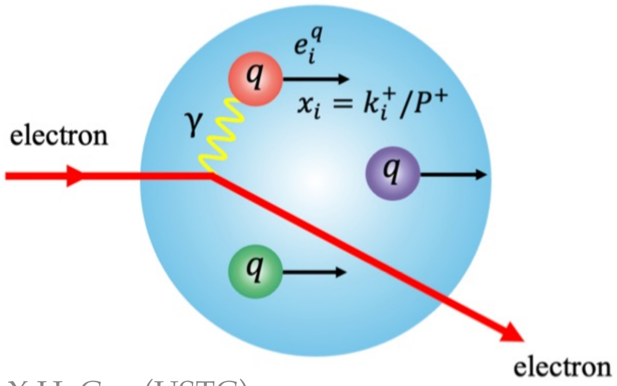
[Brodsky:2000ii]

$$A(q^2) = \langle p', \uparrow | \hat{T}^{++} | p, \uparrow \rangle = \sum_n \int \left[\mathrm{d}x_i \mathrm{d}^2 r_{i\perp} \right]_n \sum_i x_i \mathrm{e}^{\mathrm{i} \vec{r}_{i\perp} \cdot \vec{q}_{i\perp}} \sum_\sigma \tilde{\psi}^{\uparrow\dagger}_{\sigma,n}(\{x_i, \vec{r}_{i\perp}\}) \psi^{\uparrow}_{\sigma,n}(\{x_i, \vec{r}_{i\perp}\})$$

$$\frac{q^1 + iq^2}{M} B(q^2) = \langle p', \downarrow | \hat{T}^{++} | p, \uparrow \rangle = \sum_n \int \left[\mathrm{d}x_i \mathrm{d}^2 r_{i\perp} \right]_n \sum_i x_i \mathrm{e}^{\mathrm{i} \vec{r}_{i\perp} \cdot \vec{q}_{i\perp}} \sum_\sigma \tilde{\psi}^{\downarrow\dagger}_{\sigma,n}(\{x_i, \vec{r}_{i\perp}\}) \psi^{\uparrow}_{\sigma,n}(\{x_i, \vec{r}_{i\perp}\})$$

Vanishing AGM for any composite system

$$\frac{B(0)}{M} = \lim_{\vec{q} \to 0} \frac{\partial}{\partial q^1} \left[\frac{q^1 + iq^2}{M} B(q^2) \right] = \sum_n \int \left[\mathrm{d}x_i \mathrm{d}^2 r_{i\perp} \right]_n \sum_i i x_i r^1_{i\perp} \sum_\sigma \tilde{\psi}^{\downarrow\dagger}_{\sigma,n}(\{x_i, \vec{r}_{i\perp}\}) \psi^{\uparrow}_{\sigma,n}(\{x_i, \vec{r}_{i\perp}\})$$



x_i: Bjorken variable

$r_{i\perp}$: relative transverse position of parton i

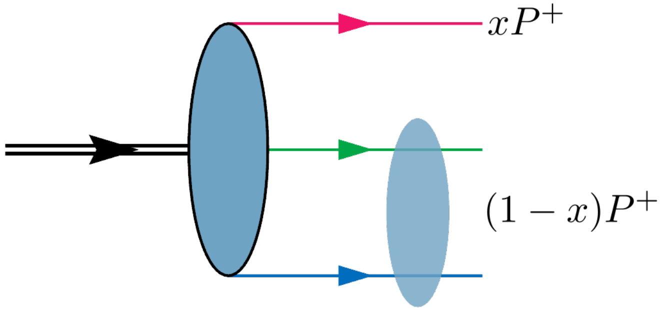
kinematic constraint

$$\sum_i x_i = 1 \qquad \sum_i x_i \vec{r}_{i\perp} = 0$$

Nucleon in light-front holographic QCD

[Brodsky:2014yha]

$$\Psi(x,\vec{\zeta}_\perp) = \sqrt{4\pi}x(1-x)\mathrm{e}^{iL\varphi}X(x)\psi(\zeta_\perp)$$



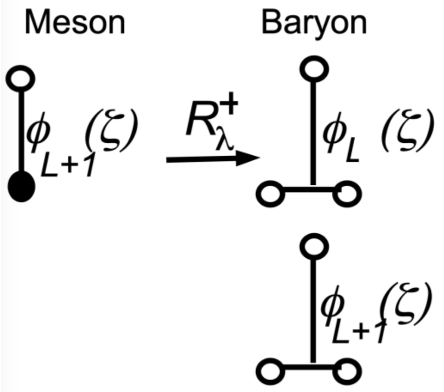
A semiclassical approximation to real QCD

$$H_{\mathrm{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^{\,2} + m_i^2}{x_i} + \mathcal{U}_i + \sum_{ij}\left[V_{ij}^{(\mathrm{QCD})} - \delta_{ij}\mathcal{U}_i\right]$$

Confinement potential from $\mathrm{AdS}_5/\mathrm{QCD}$

$\kappa = 0.54$ GeV from ρ meson mass

$$\Bigg(\underbrace{-\frac{\mathrm{d}^2}{\mathrm{d}\zeta_\perp^2} - \frac{1-4L^2}{4\zeta_\perp^2}}_{\text{kinetic energy}} + \underbrace{\kappa^2\zeta_\perp^2 + 2\kappa L}_{U_{\mathrm{QCD\ conf.}}}\Bigg)\psi = M_h^2\psi$$



[deTeramond:2014asa]

Two solutions corresponding to S wave and P wave

$$\psi_+(\zeta_\perp) \sim \zeta_\perp \exp\left(-\frac{\kappa^2\zeta_\perp^2}{2}\right), \qquad \psi_-(\zeta_\perp) \sim \zeta_\perp^2 \exp\left(-\frac{\kappa^2\zeta_\perp^2}{2}\right)$$

Nucleon gravitational form factor $B_N(Q^2)$

Brodsky-Hwang-Ma-Schmidt formula for a two-body system [Brodsky:2000ii]

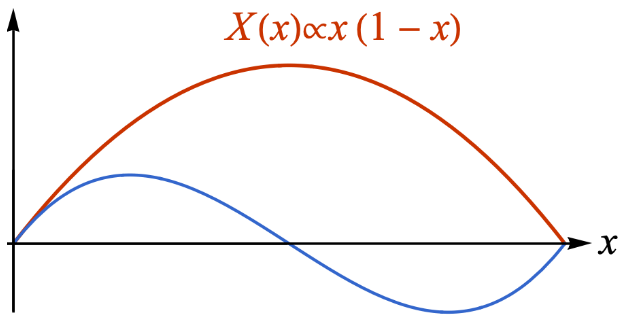
$$\frac{q^1+iq^2}{2M}B_N(q^2)=2\int \mathrm{d}x\mathrm{d}^2\zeta_\perp\sum_\sigma \Psi^{\uparrow *}_\sigma(x,\vec{\zeta}_\perp)\Psi^{\downarrow}_\sigma(x,\vec{\zeta}_\perp)\left[x\mathrm{e}^{-i\sqrt{\frac{1-x}{x}}\vec{q}_\perp\cdot\vec{\zeta}_\perp}+(1-x)\mathrm{e}^{\mathrm{i}\sqrt{\frac{x}{1-x}}\vec{q}_\perp\cdot\vec{\zeta}_\perp}\right]$$

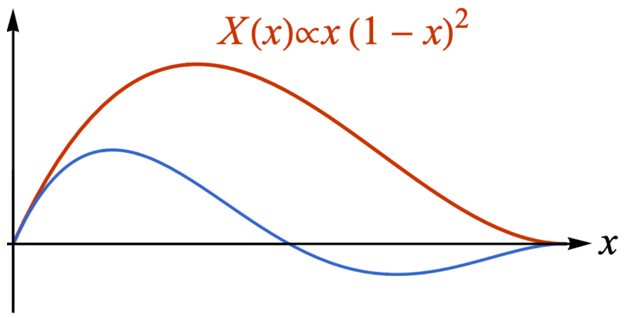
In light-front holographic QCD [Cao:2026jzm]

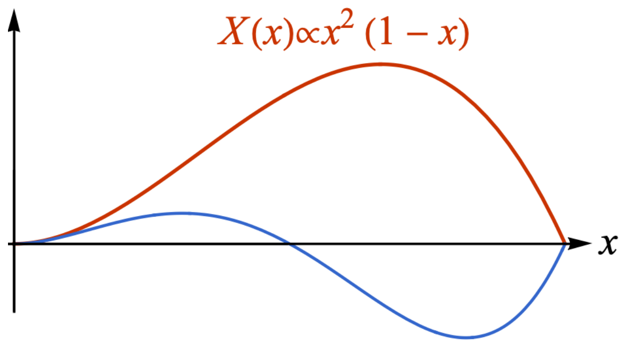
$$B_N(Q^2)=M\int \zeta_\perp\mathrm{d}\zeta_\perp\mathcal{B}^{(0)}(\zeta_\perp,Q^2)\psi_+(\zeta_\perp)\psi_-(\zeta_\perp)$$

$$\mathcal{B}^{(0)}(\zeta_\perp,Q^2)=\frac{8\pi}{Q}\int\mathrm{d}x(1-x)J_1\left(\sqrt{\frac{x}{1-x}}Q\zeta_\perp\right)\underbrace{\left[X_+(1-x)X_-(1-x)-X_+(x)X_-(x)\right]}_{\text{antisymmetric factor}}$$

The asymmetry induces a strong cancellation







Longitudinal modes

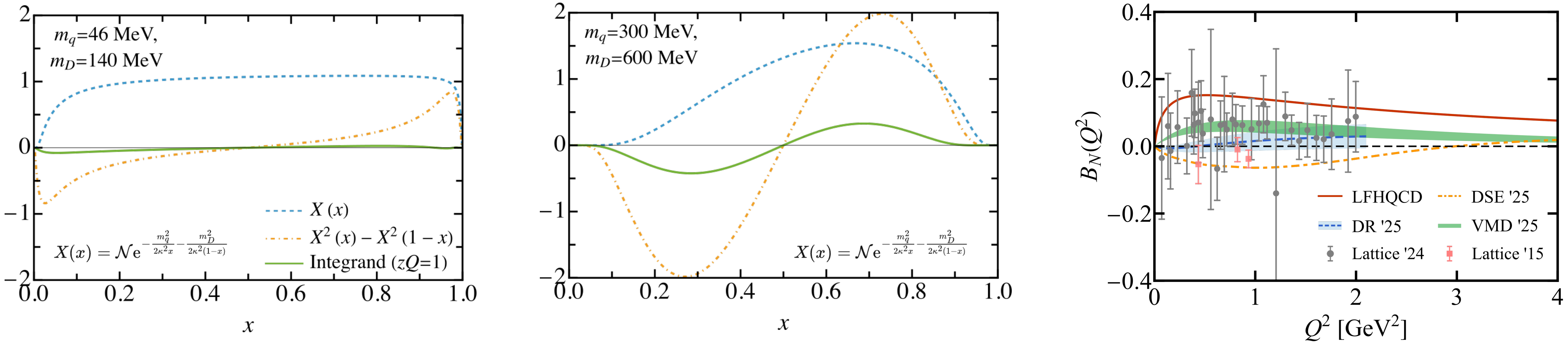
Original LFHQCD contains only the transverse modes (chiral limit) [Abidin:2009hr,Mamo:2021krl,Deng:2025azp]

$$X(x) = 1 \qquad \Rightarrow \qquad B_N(Q^2) = 0$$

BdT light-front wave function ansatz: [Brodsky:2008pg]

$$\Psi(x, \vec{k}_\perp) \sim x(1-x)\mathrm{e}^{-\frac{1}{2\kappa^2}\frac{k_\perp^2}{x(1-x)}} \to x(1-x)\mathrm{e}^{-\frac{1}{2\kappa^2}\left(\frac{k_\perp^2}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_D^2}{1-x}\right)}$$

Two set of parameters from LFHQCD and constituent quark model

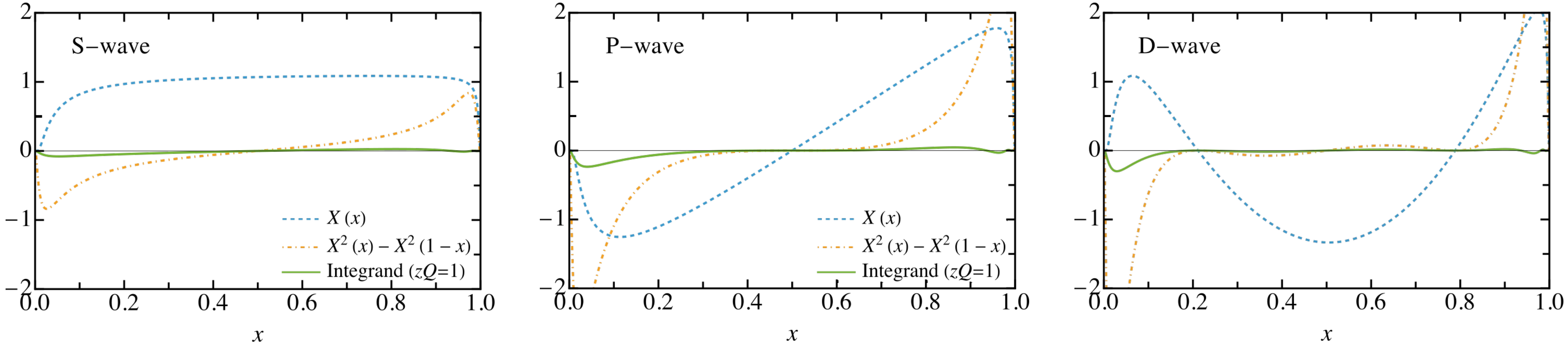


Partial waves

n-th partial wave in longitudinal modes [Li:2021jqb]

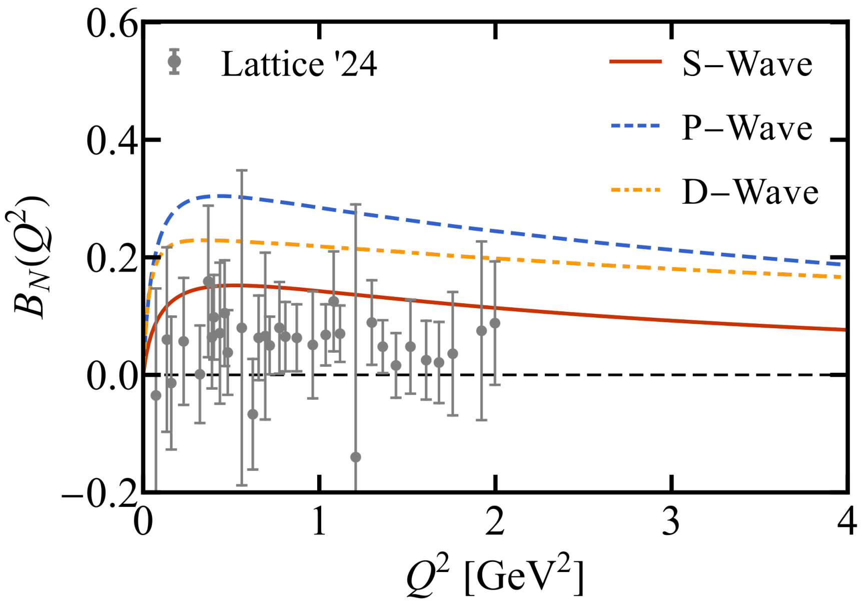
$$X_n(x) = \mathcal{N} e^{-\frac{m_q^2}{2\kappa^2 x} - \frac{m_D^2}{2\kappa^2(1-x)}} P_n(2x-1)$$

The antisymmetric factor grows for higher partial waves, but its integral remains strongly suppressed.



Summary

- $B_N(0) = 0$ is required by angular momentum conservation, yet $B_N(Q^2)$ remains strongly suppressed at finite Q^2
- The suppression originates from asymmetric cancellations in the longitudinal mode of light-front holographic QCD
- The smallness of $B_N(Q^2)$ suggests an S-wave-dominated proton



Thank you!

Based on PRD 113, L071503 (2026)

Backup Slides

Light-front holographic QCD

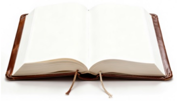
- Light-front holographic QCD is a first approximation to real QCD [Brodsky:2014yha]

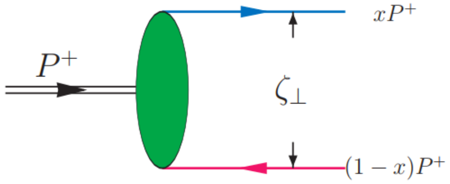
$$H_{\text{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^{\,2} + m_i^2}{x_i} + \mathcal{U}_i + \sum_{ij}\left[V_{ij}^{(\text{QCD})} - \delta_{ij}\mathcal{U}_i\right]$$

$$\Downarrow$$

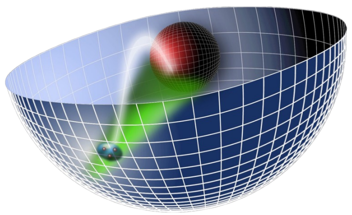
$$\left(\frac{k_\perp^2 + m_q^2}{x} + \frac{k_\perp^2 + m_{\bar{q}}^2}{1-x} + \mathcal{U}\right)\psi(x,\vec{k}_\perp) = M^2\psi(x,\vec{k}_\perp)$$

- The effective potential is from AdS/QCD, a weak-coupling duality of QCD in AdS_5



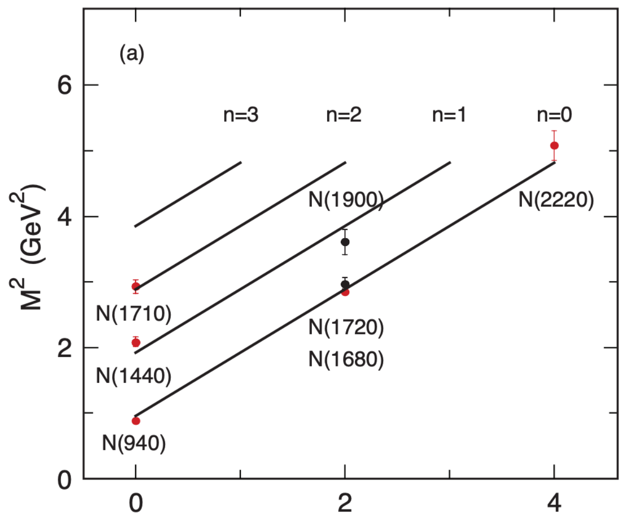


semiclassical LFQCD	$\longleftrightarrow$	semiclassical field theory in AdS_5
$\zeta_\perp = \sqrt{x(1-x)}r_\perp$	$\longleftrightarrow$	fifth coordinate z
LF amplitude	$\longleftrightarrow$	string amplitude
confining potential $\mathcal{U}$	$\longleftrightarrow$	dilation field Φ
$L^2-(J-2)^2$	$\longleftrightarrow$	$(\mu R)^2$
form factors	$\longleftrightarrow$	form factors

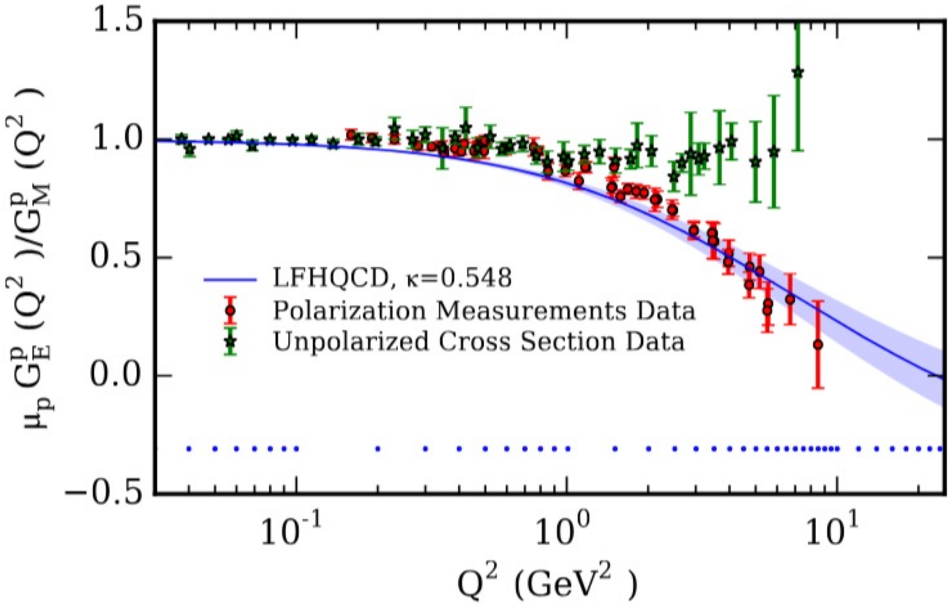


Application of light-front holographic QCD

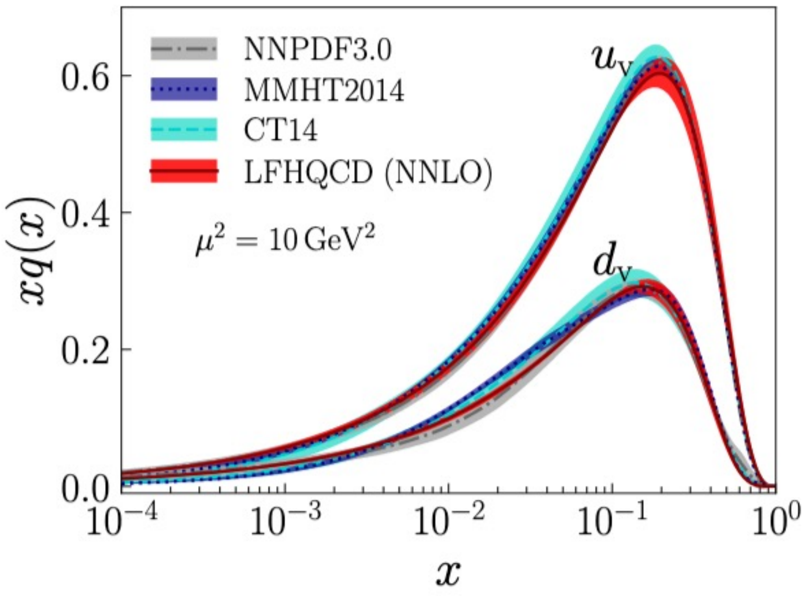
[deTeramond:2014asa, Sufian:2016hwn, deTeramond:2018ecg]



Baryon mass spectrum



Electromagnetic form factor



Parton distribution function

Microscopic interpretation of form factors

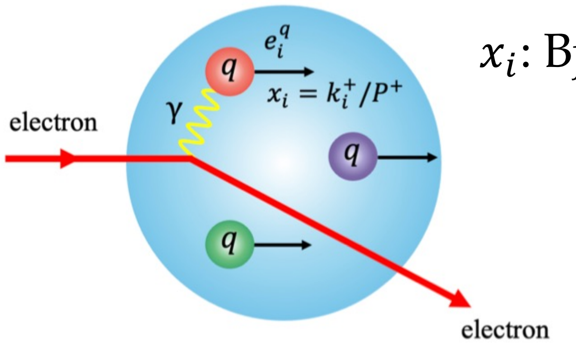
Drell-Yan-West formula for charge form factor: [Drell:1969km, West:1970av, Brodsky:1980zm]

$$\rho_{\rm ch}(r_\perp) = \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n |\tilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})|^2 \sum_j e_j \delta^{(2)}(r_\perp - r_{j\perp}) \equiv \langle \sum_j e_j \delta^{(2)}(r_\perp - r_{j\perp}) \rangle$$

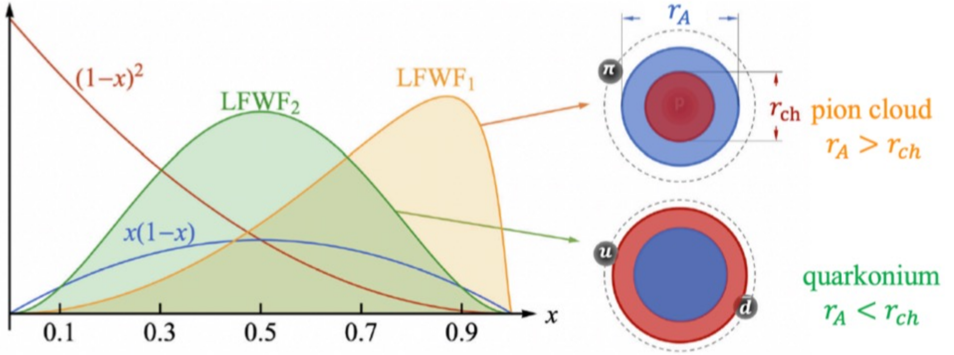
Brodsky-Hwang-Ma-Schmidt formula for gravitational form factor A: [Brodsky:2000ii]

$$\mathcal{A}(r_\perp) = \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n |\tilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})|^2 \sum_j x_j \delta^{(2)}(r_\perp - r_{j\perp}) \equiv \langle \sum_j x_j \delta^{(2)}(r_\perp - r_{j\perp}) \rangle$$

Matter density $\mathcal{A}(r_\perp)$ mainly samples the valance partons $x_j \sim O(1)$; wee parton $x_j \ll 1$ contributions suppressed



x_i: Bjorken variable



$r_{ch}^\pi \sim 1.6 r_A^\pi$

Mechanical properties of hadrons

$D(q^2)$ is related to the pressure and shear forces inside hadrons

[Polyakov:2018zvc]

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}p(r)$$

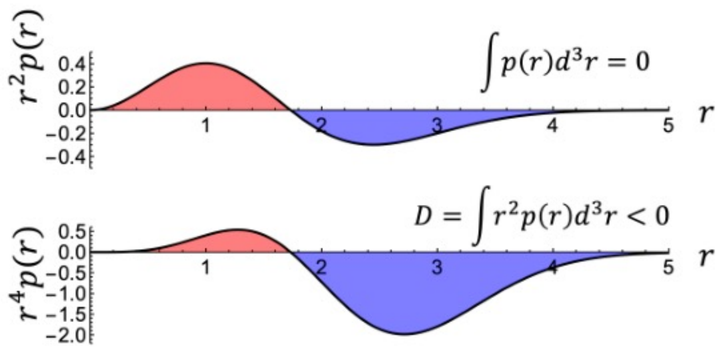
$$\Downarrow$$

$$p(r) = \frac{1}{6M}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}r^2\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r), \qquad s(r) = -\frac{1}{4M}r\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r)$$

Hadron stability conditions:

[Perevalova:2016dln]

- Force equilibrium (von Laue condition): $\int d^3 r p(r) = 0$
- Stability conjecture: $D(0) = \int \mathrm{d}^3 r r^2 p(r) < 0$

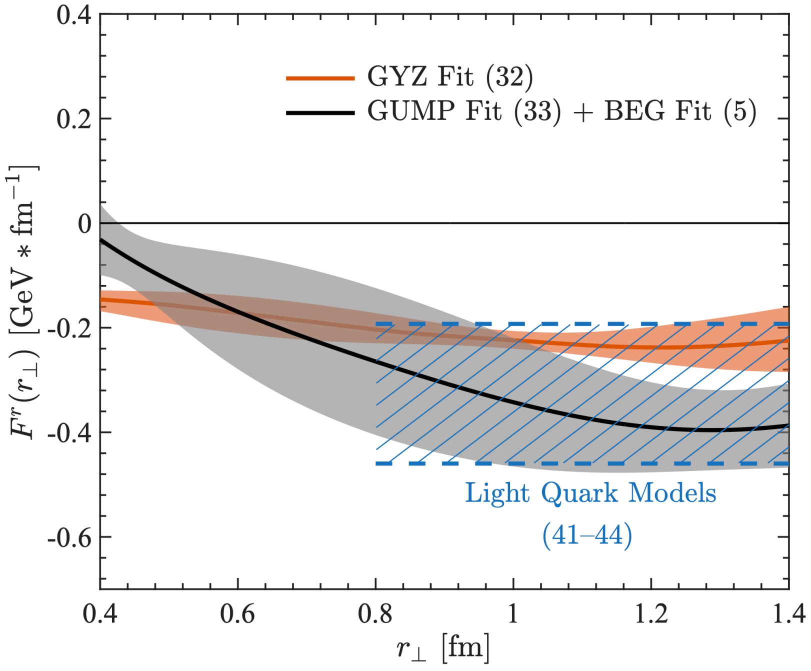


Forces inside the hadron

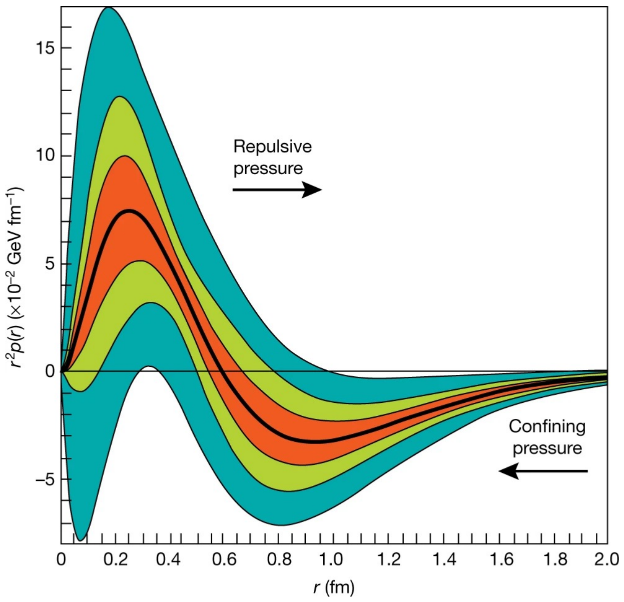
The divergence of EMT defines the force on quarks and gluons [Ji:2025qax, Ji:2026lyj]

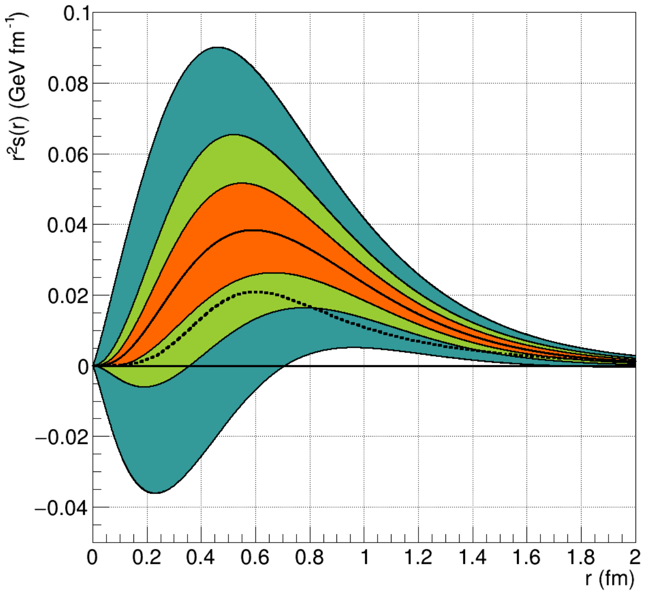
$$\mathcal{F}_q^i \equiv \partial_\mu T_q^{\mu i}$$

Confinement induced by a constant force?



The first measurement of the pressure and shear





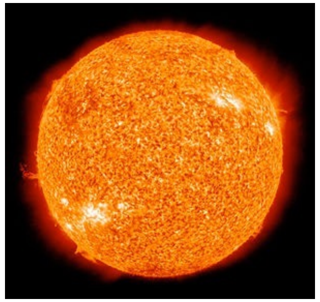
[Burkert:2018bqq]
[Burkert:2021ith]

The proton contains the highest pressure in nature

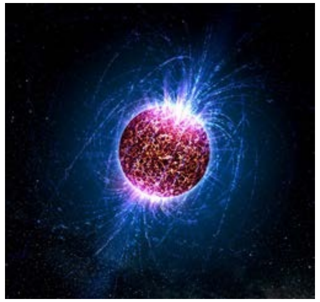
Adapted from Kumano: LC2024

Earth atmosphere
10^5Pa

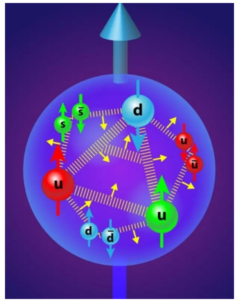
Center of earth
10^{11}Pa



Center of sun
10^{16}Pa



Neutron star
10^{34}Pa



Proton
10^{35}Pa

Big problems related with energy-momentum tensor

- Origin of hadron mass [Burkert:2023wzr]
- Spin decomposition
- Mechanical properties of hadrons