Hexaquark Spectroscopy in QCD Sum Rules

From Light Baryoniums to Heavy $\Lambda_Q\bar{\Sigma}_Q$ States

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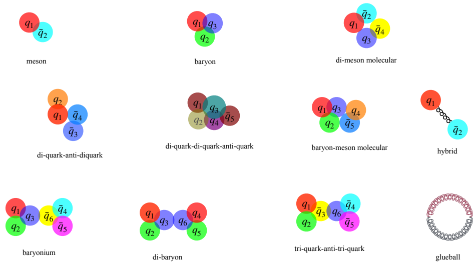
Outline

Section 1

Review

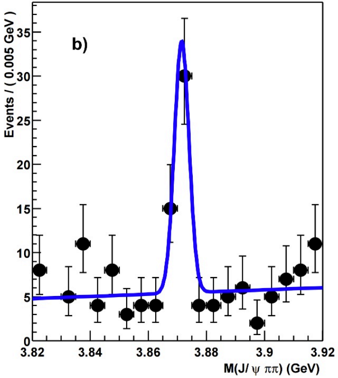
Hadronic States

- Color confinement and Asymptotic freedom – Hadrons – Non-perturbative effects
- Traditional Hadrons: Mesons $(q\bar{q})$; Baryons (qqq).
- QCD allows exotic states beyond Quark Model.

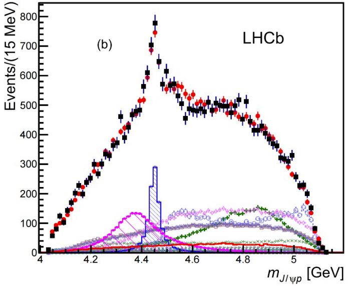


- $X(2370)$: A glueball candidate.

Observation of Multi-quark States



$X(3872)$:
$M_{X(3872)} = (3871.69 \pm 0.17)$ MeV [Belle, 2003]



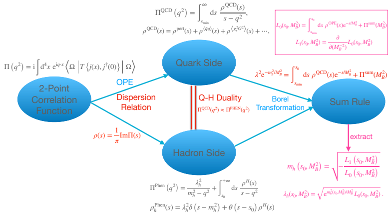
$P_c(4380)^+, P_c(4450)^+$:
$M \approx 4380, 4450$ MeV [LHCb, 2015]

Hexaquark States

- Experimental candidates: $X(1835)$, $X(1860)$, $X(1880)$, $X(2075)$, $X(2085)$, $Y(4260)$, $Y(4660)$, etc.
- [E.Fermi, C.N.Yang, 1949]: Pions may be $N\bar{N}$ bound states.
- Dibaryon molecular states ($\mathscr{B}\mathscr{B}$) :
  - Deuteron: $J^P = 1^+$, $E_B = 2.225\mathrm{MeV}$ pn dibaryon bound state.
  - Dihyperon states: [R.L.Jaffe. 1977] etc. ;
  - Diproton states: [P.J.Mulders, et.al. 1980];
  - $\Lambda_c\Lambda_c$: [Z.G.Wang et.al. 2021]
- Baryon-antibaryon bound states $\rightarrow$ Baryonium ($\mathscr{B}\bar{\mathscr{B}}$, $\mathscr{B}\bar{\mathscr{B}}'$, $\mathscr{B}$ needless to be color singlet):
  - Hidden-charm: [C.F.Qiao, 2005/2007], [H.X.Chen, et.al. 2006], [Z.G.Wang, et.al. 2021], etc. ;
  - $\Lambda_Q\bar{\Lambda}_Q$: [B.D.Wan, L.Tang and C.F.Qiao. 2019];
  - Light Baryoniums: [B.S.Zou, H.C.Chiang. 2004], [S.L.Zhu, C.S.Gao. 2006], [Z.G.Wang, et.al. 2006], [B.D.Wan, S.Q.Zhang and C.F.Qiao. 2021], etc.
- Compact hexaquark states: [Z.G.Wang. 2022], etc.

QCD Sum Rules

- Various methods: Quark Model, Lattice QCD, QCDSR, AdS/QCD, etc.
- [SVZ, 1979]: First principle, analytical, perturbative & non-perturbative.



[SVZ, 1979]

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QCD AND RESONANCE PHYSICS. THEORETICAL FOUNDATIONS

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Received 24 July 1978

QCD AND RESONANCE PHYSICS. APPLICATIONS

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1. First principle, analytical.
2. Sum of perturbative + non-perturbative.
3. Proven for conventional hadrons, tetraquarks, pentaquarks.

QCDSR for $X(3872)$ and P_c States

$$M_{X(3872)} = (3871.69 \pm 0.17)\ \text{MeV}$$

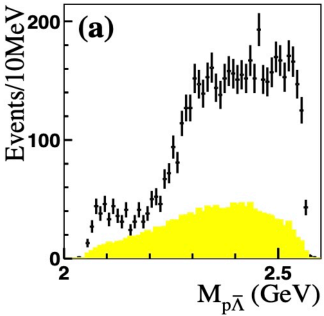
Ref	Configuration	QCDSR results	State
W.Chen, et.al 2013	$\xi \bar{c}Gc + \sqrt{1-\xi^2}\bar{D}D^*$	3.88GeV	molecular
Lee, et.al 2009	$D^0\bar{D}^{*0} - \bar{D}^0 D^*$	$(3.88 \pm 0.06)\text{GeV}$	molecular
Matheus, et.al 2007	$(q_a c_b)(\bar{q}_d \bar{c}_e)$	$(3925 \pm 127)\text{MeV}$	compact

$$M_{4380} = (4380 \pm 8 \pm 29)\text{MeV},\ \ M_{4450} = (4449.8 \pm 1.7 \pm 2.5)\text{MeV}$$

Ref	Configuration	QCDSR results	State
Chen, Zhu, et.al 2015	$\bar{D}^*\Sigma_c$	$4.37^{+0.19}_{-0.12}\text{GeV}$	molecular
Chen, Zhu, et.al 2015	$\sin\theta\bar{D}\Sigma_c^* + \cos\theta\bar{D}^*\Lambda_c$	$4.47^{+0.20}_{-0.13}\text{GeV}$	molecular
Z.G.Wang 2016	$\bar{c}_a(u_j d_k)(u_m c_n)$	$(4.38 \pm 0.13)\text{GeV}$	compact
Z.G.Wang 2016	$\bar{c}_a(u_j d_k)(u_m c_n)$	$(4.44 \pm 0.14)\text{GeV}$	compact

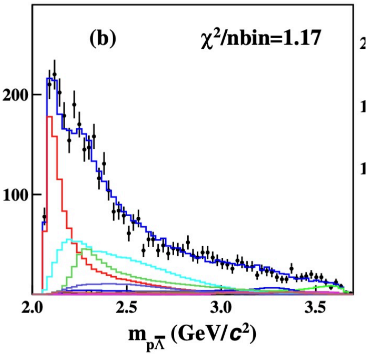
QCDSR accurately predicts masses of $X(3872)$ and P_c states!

$X(2075)$ and $X(2085)$



$$M_{X(2075)} = (2075 \pm 12 \pm 5)\ \text{MeV},\ J = 1$$
[BES, 2003]

$p\bar{\Lambda}/p\bar{\Sigma}$ states in constituent quark models [Huang, Ping, Wang, 2011].



$$M_{X(2085)} = (2086 \pm 4 \pm 6)\ \text{MeV},$$
$J^P = 1^+$ [BESIII, 2023]

Further structures should be investigated!

QCDSR for Light Baryoniums

$$M_{X(1835)} = (1833.7 \pm 6.7 \pm 2.1)\ \text{MeV} \quad [BESIII, 2005]$$
$$M_{X(1880)} = (1882.1 \pm 1.7 \pm 0.7)\ \text{MeV} \quad [BESIII, 2023]$$

Ref	Configuration	QCDSR results	State
Z.G.Wang 2006	$p\bar{p}$, $J^{PC} = 0^{-+}$	$1.9(1)$ GeV	$X(1835)$
B.D.Wan 2021	$p\bar{p}, 0^{-+}/1^{--}$	$1.81(10)$ GeV	$X(1880)$
B.D.Wan 2021	$\Lambda\bar{\Lambda}$, $J^{PC} = 1^{--}$	$2.34(12)$ GeV	$\Lambda\bar{\Lambda}$

Configurations $\mathscr{B}\bar{\mathscr{B}}'$ and compact hexaquarks should be studied!
$\Rightarrow$ $X(2075)$ and $X(2085)$ in $p\bar{\Lambda}$ final state.

$\Lambda_c\bar{\Sigma}_c$ States

- BESIII results: [BESIII, 2025]

We search for a possible $\Lambda_c\bar{\Sigma}_c$ bound state, denoted as $H_c^{\pm}$, via the $e^+e^- \to \pi^+\pi^-\Lambda_c^+\bar{\Lambda}_c^-$ process for the first time. This analysis utilizes 207.8 and 159.3 pb^{-1} of e^+e^- annihilation data at the center-of-mass energies of 4918.02 and 4950.93 MeV, respectively, collected with the BESIII detector at the BEPCII collider. No statistically significant signal is observed. The upper limits of the product of Born cross section and branching fraction $\sigma(e^+e^- \to \pi^+H_c^- + c.c.) \times \mathcal{B}(H_c^- \to \pi^-\Lambda_c^+\bar{\Lambda}_c^-)$ at a 90% confidence level are reported at each energy point and for various H_c mass hypotheses (4715, 4720, 4725, 4730, and 4735 MeV/c^2) and widths (5, 10, or 20 MeV), with the upper limits ranging from 1.1 pb to 6.4 pb.

- Near-threshold bound states or resonances?
- Different theoretical methods should be further examined.

QCDSR for Hidden-heavy States

[S.Q.Zhang and C.F.Qiao, 2025]

State	J^{PC}	QCDSR
Λ_c-$\bar{\Lambda}_c$	0^+	5.00 [560], 5.19±0.24 [561], $5.11^{+0.15}_{-0.12}$ [562]
	0^-	$4.66^{+0.10}_{-0.06}$ [562]
	1^+	4.89 [560], $4.99^{+0.10}_{-0.09}$ [562]
	1^-	4.78±0.23 [561], $4.68^{+0.08}_{-0.08}$ [562]
	2^+	5.15 [560]
	2^-	4.83 [560]
	3^+	5.68 [560]
	3^-	5.04 [560]
Σ_c-$\bar{\Sigma}_c$	0^+	$5.23^{+0.07}_{-0.07}$ [563]
	0^-	$4.88^{+0.08}_{-0.08}$ [563]
	1^+	$5.31^{+0.07}_{-0.07}$ [563]
	1^-	$4.88^{+0.09}_{-0.08}$ [563]
Λ_b-$\bar{\Lambda}_b$	0^+	11.84±0.22 [561]
	0^-	
	1^+	
	1^-	11.72±0.26 [561]

560 : [W.Chen, H.X.Chen, S.L.Zhu et.al, EPJC, 2016]

561 : [B.D.Wan, L.Tang and C.F.Qiao. EPJC, 2019]

562 : [X.W.Wang, Z.G.Wang and G.L.Yu. EPJA, 2021]

563 : [X.W.Wang, Z.G.Wang. AHEP, 2021]

- No prior $\Lambda_Q\bar{\Sigma}_Q$ results in QCDSR!
- $\Lambda_c\bar{\Sigma}_c$ masses may exceed 5 GeV. $\Lambda_b\bar{\Sigma}_b$ masses may exceed 12 GeV.

Section 2

Theoretical Framework

Configurations

- $\mathscr{B}\bar{\mathscr{B}}'$ molecular states:

$$[3_c] \otimes [3_c] \otimes [3_c] = [10_c] \oplus [8_c] \oplus [8_c] \oplus [1_c],$$

$[1_c] - [1_c]$ forms the $\mathscr{B}\bar{\mathscr{B}}'$ states. Choose $p\bar{\Lambda}$, $p\bar{\Sigma}$, $\Lambda_Q\bar{\Sigma}_Q$.

- Compact hexaquark states:

$$[\bar{3}_c] \otimes [3_c] \otimes [3_c] = [15_c] \oplus [\bar{6}_c] \oplus [3_c] \oplus [3_c],$$

$qq\bar{q} \to [3_c]$, then $[3_c] \otimes [\bar{3}_c] = [8_c] \oplus [1_c]$. Structures $[3_c]_{qqq}$-$[\bar{3}_c]_{qqs/qsq}$ have same components as $p\bar{\Lambda}/p\bar{\Sigma}$.

Interpolating Currents

The chiral limit $m_u = m_d \to 0$ is taken to simplify the currents.

- Color singlet Baryonic currents

$$\eta_{1\mathscr{B}} = \varepsilon_{abc}[q_a^{iT}\mathcal{C}q_b^j]\gamma_5 q_c^k \ ,$$

$$\eta_{2\mathscr{B}} = \varepsilon_{abc}[q_a^{iT}\mathcal{C}\gamma_5 q_b^j] q_c^k \ ,$$

$(i, j, k) =$ $(u, d, u)/(u, d, s)/(u, s, d)/(u, d, Q)/(u, Q, d)$ for $p/\Lambda/\Sigma/\Lambda_Q/\Sigma_Q$.

- Baryonium currents $J^P = 0^+, 0^-, 1^-, 1^+$

$$j^{0^-} = \mathrm{i}\bar{\eta}_{\mathscr{B}'}\gamma_5\eta_{\mathscr{B}}, \quad j^{0^+} = \bar{\eta}_{\mathscr{B}'}\eta_{\mathscr{B}},$$

$$j_\mu^{1^-} = \bar{\eta}_{\mathscr{B}'}\gamma_\mu\eta_{\mathscr{B}}, \quad j_\mu^{1^+} = \bar{\eta}_{\mathscr{B}'}\gamma_\mu\gamma_5\eta_{\mathscr{B}}.$$

- $[3_c]$ triquark currents

$$\eta_1^a = \mathrm{i}[\bar{q}_b^i q^{a,j}]\gamma_5 q^{b,k}, \ \eta_2^a = \mathrm{i}[\bar{q}_b^i\gamma_5 q^{a,j}]q^{b,k},$$

$(i, j, k) = (u, d, u)/(u, d, s)/(u, s, d)$ for $[3_c]_{udu/uds/usd}$.

- Compact hexaquark currents

$$j_{1(\mu)} = \bar{\eta}_{a;\bar{u}ds}\Gamma_{(\mu)}\eta^a_{\bar{u}du},$$

$$j_{2(\mu)} = \bar{\eta}_{a;\bar{u}sd}\Gamma_{(\mu)}\eta^a_{\bar{u}du},$$

$\Gamma_{(\mu)} = \mathbb{1}, \mathrm{i}\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$ for $0^+, 0^-, 1^-, 1^+$.

All masses evaluated by **two independent currents** (Type-1/Type-2).

Quark Side

- 2-Point correlation functions

$$\Pi(q^2) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x} \langle\Omega| \mathbb{T}\{j(x), j^\dagger(0)\} |\Omega\rangle \,,$$

$$\Pi_{\mu\nu}(q^2) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x} \langle\Omega| \mathbb{T}\{j_\mu(x), j_\nu^\dagger(0)\} |\Omega\rangle \,,$$

$$\Pi(q^2) = -\frac{1}{3}\big(g^{\mu\nu} - \tfrac{q^\mu q^\nu}{q^2}\big)\Pi_{\mu\nu}(q^2),$$

Dispersion Relation $\Downarrow$ $\rho(s) = \dfrac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$

$$\Pi^{\mathsf{QCD}}_{X,J^P}(q^2) = \int_{s_{\min}}^{\infty} \mathrm{d}s\, \frac{\rho^{\mathsf{OPE}}_{X,J^P}(s)}{s-q^2}$$

$s_{\min} = m_s^2$ (light) / $(2m_Q)^2$ (hidden-heavy).

- Contract with full propagators: OPE up to dim-13 (light) or dim-12 (heavy):

$$\rho^{\mathsf{OPE}}_{X,J^P} = \rho^{\mathsf{pert}} + \sum_{n=3}^{D} \rho^{\langle\mathcal{O}_n\rangle}.$$

Hadron Side

- Phenomenological framework

$$\rho^{\text{Phen}}_{X,J^P}(s) = \lambda^2_{X,J^P}\delta(s - m^2_{X,J^P}) + \theta(s - s_0)\rho_{X,J^P}(s),$$

$$\lambda_{X,J^P} \equiv \langle\Omega|\, j(x)\,|H_0\rangle\,,$$
$$\lambda_{X,J^P}\epsilon^\mu \equiv \langle\Omega|\, j^\mu(x)\,|H_0\rangle\,.$$

$$\Pi^{\text{Phen}}_{X,J^P}(q^2) = \frac{\lambda^2_{X,J^P}}{m^2_{X,J^P} - q^2} + \int_{s_0}^{\infty} \mathrm{d}s \frac{\rho_{X,J^P}(s)}{s - q^2},$$

pole of ground state

s_0: continuum threshold

Quark-Hadron duality ⇓ Borel transformation

$$\lambda^2_{X,J^P}\mathrm{e}^{-m^2_{X,J^P}/M_B^2} = \int_{s_{\min}}^{s_0} \mathrm{d}s\, \rho^{\text{OPE}}_{X,J^P}(s)\mathrm{e}^{-s/M_B^2},$$

which is the sum rule .

- Extract the mass and decay constant

$$L_{X,J^P,0}(s_0, M_B^2) = \int_{s_{\min}}^{s_0} \mathrm{d}s\, \rho^{\text{OPE}}(s)\, \mathrm{e}^{-s/M_B^2},$$

$$L_{X,J^P,1}(s_0, M_B^2) = \frac{\partial}{\partial(M_B^{-2})} L_{X,J^P,0}(s_0, M_B^2).$$

$$m_{X,J^P}(s_0, M_B) = \sqrt{-\frac{L_{X,J^P,1}(s_0, M_B)}{L_{X,J^P,0}(s_0, M_B)}},$$

$$\lambda_{X,J^P}(s_0, M_B) = \sqrt{\mathrm{e}^{m^2_{X,J^P}/M_B^2} L_{X,J^P,0}(s_0, M_B)}.$$

2 additional parameters s_0 and M_B are inserted!

Criteria & Numerical Setup

Criteria:

- **Pole dominance:** $R^{\mathrm{PC}} \geqslant 30\%/15\%$ (heavy/light).
- **OPE convergence:** $R^{\mathrm{OPE}} \lesssim 10\%$.
- M_B^2 **stability:** Borel Window $\geqslant 0.5$ GeV^2.

$\sqrt{s_0} \sim m_X + \delta$, $\delta = 0.4 - 0.8$ (light) / $0.5 - 0.9$ GeV (heavy).

Parameters ($\mu = 2$ GeV):

$\langle \bar{q}q \rangle = -(0.24)^3$ GeV^3,

$\langle \bar{s}s \rangle = 1.15 \langle \bar{q}q \rangle$,

$\langle g_s^2 G^2 \rangle = 0.88$ GeV^4,

$\langle g_s^3 G^3 \rangle = 0.045$ GeV^6,

$m_0^2 = 0.8$ GeV^2,

$m_s = 95$ MeV,

$\bar{m}_c = 1.273$ GeV, $\bar{m}_b = 4.183$ GeV.

Section 3

Numerical Results

Light Baryoniums: $p\bar{\Lambda}$ and $p\bar{\Sigma}$

Current	J^P	State	$\sqrt{s_0}(\mathrm{GeV})$	$M_B^2(\mathrm{GeV}^2)$	$M_X(\mathrm{GeV})$	$\lambda_X(10^{-5}\mathrm{GeV}^8)$	$R^{\mathrm{PC}}(\%)$	$R^{\mathrm{OPE}}(\%)$
Type-1	0^+	$p\bar{\Lambda}$	2.9 ± 0.1	$2.4 - 2.9$	1.96 ± 0.03	3.54 ± 0.10	$18 - 55$	$3.8 - 4.2$
		$p\bar{\Sigma}$	2.9 ± 0.1	$2.4 - 2.9$	1.98 ± 0.04	3.62 ± 0.12	$18 - 54$	$3.8 - 4.3$
Type-2	0^-	$p\bar{\Lambda}$	2.8 ± 0.1	$1.6 - 2.2$	2.00 ± 0.20	3.57 ± 0.89	$18 - 60$	$1.7 - 2.9$
		$p\bar{\Sigma}$	2.8 ± 0.1	$1.6 - 2.1$	1.99 ± 0.18	3.52 ± 0.83	$21 - 60$	$2.2 - 3.5$
	1^-	$p\bar{\Lambda}$	2.8 ± 0.1	$1.7 - 2.2$	2.03 ± 0.17	3.52 ± 0.81	$17 - 52$	$2.5 - 4.2$
		$p\bar{\Sigma}$	2.8 ± 0.1	$1.7 - 2.2$	2.03 ± 0.17	3.52 ± 0.80	$18 - 53$	$1.6 - 2.7$

- 6 possible baryonium states with $J^P = 0^-, 0^+, 1^-$.
  Other J^P: not coupled.

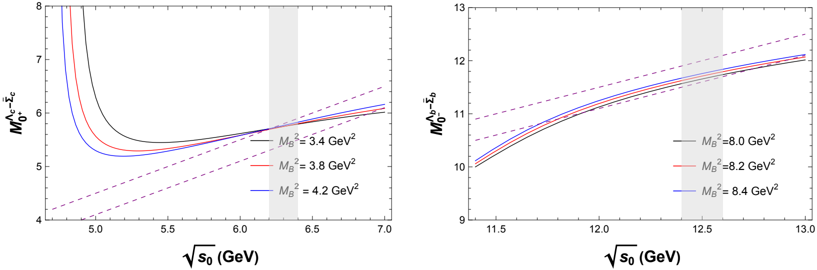
Compact Hexaquark States

Currents	J^P	Configurations	$\sqrt{s_0}$(GeV)	M_B^2(GeV^2)	m_X(GeV)	$\lambda_X(10^{-5}\text{GeV}^8)$	$R^{\text{PC}}(\%)$	$R^{\text{OPE}}(\%)$
Type-I	0^+	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.9 ± 0.1	$2.4 - 2.9$	1.99 ± 0.02	3.21 ± 0.05	$27 - 81$	$3.6 - 4.1$
	1^+	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.9 ± 0.1	$2.3 - 3.0$	1.97 ± 0.03	2.84 ± 0.05	$18 - 90$	$3.5 - 4.3$
		$[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$	2.9 ± 0.1	$2.3 - 2.9$	1.97 ± 0.02	2.83 ± 0.04	$23 - 88$	$3.2 - 3.8$
Type-II	0^-	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.8 ± 0.1	$1.6 - 2.2$	1.84 ± 0.21	2.38 ± 0.56	$17 - 62$	$2.2 - 3.3$
	1^-	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.9 ± 0.1	$1.8 - 2.3$	2.01 ± 0.20	2.84 ± 0.70	$16 - 49$	$2.8 - 4.4$
		$[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$	2.9 ± 0.1	$1.8 - 2.3$	2.02 ± 0.20	2.81 ± 0.69	$16 - 49$	$1.6 - 2.4$

- 6 compact hexaquark candidates with $J^P = 0^-, 0^+, 1^-, 1^+$. $0^+/0^-$ of Type-I/II identical under isospin.
- Other states: no coupling.

Scanning from Threshold

- Thresholds: 4.71 GeV $(\Lambda_c\bar{\Sigma}_c)$, 11.43 GeV $(\Lambda_b\bar{\Sigma}_b)$.

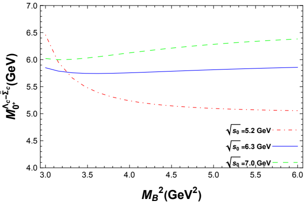


(a) Type-2 0^+ $\Lambda_c\bar{\Sigma}_c$

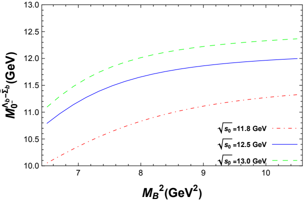
(b) Type-1 0^- $\Lambda_b\bar{\Sigma}_b$

- Unstable behaviors show instability near threshold. Threshold effects may pollute the spectrum.

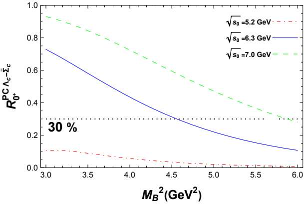
Behaviors at Different s_0



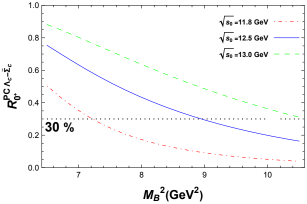
(c) m_X Type-2 0^+ $\Lambda_c\bar{\Sigma}_c$



(d) m_X Type-1 0^- $\Lambda_b\bar{\Sigma}_b$



(e) R^{PC} Type-2 0^+



(f) R^{PC} Type-1 0^-

Results of $\Lambda_Q\bar{\Sigma}_Q$ States

J^P	State	$\sqrt{s_0}$(GeV)	M_B^2(GeV^2)	M_X(GeV)	λ_X(GeV^8)	$R^{\rm PC}$(%)	$\lvert R^{\langle \mathcal{O}_{12}\rangle}\rvert$(%)
0^-	$\Lambda_b\bar{\Sigma}_b$	12.5 ± 0.1	$7.8 - 8.5$	11.68 ± 0.18	$(5.8 \pm 1.7) \times 10^{-3}$	$32 - 51$	$4.8 - 10.4$
1^-	$\Lambda_b\bar{\Sigma}_b$	12.5 ± 0.1	$7.9 - 8.6$	11.70 ± 0.18	$(5.8 \pm 1.7) \times 10^{-3}$	$31 - 50$	$4.5 - 10.0$

TABLE I: The related numerical results of the Type-I currents

J^P	State	$\sqrt{s_0}$(GeV)	M_B^2(GeV^2)	M_X(GeV)	λ_X(GeV^8)	$R^{\rm PC}$(%)	$\lvert R^{\langle \mathcal{O}_{12}\rangle}\rvert$(%)
0^-	$\Lambda_c\bar{\Sigma}_c$	6.3 ± 0.1	$4.1 - 4.7$	$\boxed{5.72 \pm 0.08}$	$(1.7 \pm 0.3) \times 10^{-3}$	$31 - 54$	$0.5 - 1.2$
	$\Lambda_b\bar{\Sigma}_b$	12.6 ± 0.1	$9.1 - 10.5$	11.86 ± 0.10	$(1.4 \pm 0.2) \times 10^{-2}$	$30 - 54$	$0.4 - 1.0$
0^+	$\Lambda_c\bar{\Sigma}_c$	6.3 ± 0.1	$3.3 - 4.3$	$\boxed{5.77 \pm 0.06}$	$(1.4 \pm 0.1) \times 10^{-3}$	$32 - 69$	$1.5 - 8.3$
	$\Lambda_b\bar{\Sigma}_b$	12.5 ± 0.1	$7.5 - 9.3$	11.89 ± 0.07	$(9.2 \pm 1.3) \times 10^{-3}$	$30 - 63$	$1.7 - 9.5$
1^-	$\Lambda_c\bar{\Sigma}_c$	6.4 ± 0.1	$4.2 - 4.9$	$\boxed{5.79 \pm 0.09}$	$(2.0 \pm 0.3) \times 10^{-3}$	$31 - 56$	$0.4 - 1.0$
	$\Lambda_b\bar{\Sigma}_b$	12.7 ± 0.1	$9.1 - 10.6$	11.92 ± 0.11	$(1.5 \pm 0.2) \times 10^{-2}$	$33 - 57$	$0.3 - 1.0$
1^+	$\Lambda_c\bar{\Sigma}_c$	6.4 ± 0.1	$3.3 - 4.5$	$\boxed{5.82 \pm 0.09}$	$(1.4 \pm 0.3) \times 10^{-3}$	$31 - 73$	$1.2 - 8.6$
	$\Lambda_b\bar{\Sigma}_b$	12.6 ± 0.1	$7.6 - 9.5$	11.95 ± 0.07	$(1.0 \pm 0.4) \times 10^{-2}$	$31 - 65$	$1.5 - 8.5$

TABLE II: The related numerical results of the Type-II currents

Consistent with the BESIII results! – All J^P masses above thresholds.
Suggest search in $5.5-6.0$ GeV ($\Lambda_c\bar{\Sigma}_c$) and ~ 12 GeV ($\Lambda_b\bar{\Sigma}_b$).

Section 4

Conclusion

Conclusion

Light hexaquarks:

- 6 $p\bar{\Lambda}/p\bar{\Sigma}$ baryonium states $(0^-, 0^+, 1^-)$.
- 6 compact hexaquark candidates $(0^-, 0^+, 1^-, 1^+)$.
- $X(2075)$ within $J^P = 1^-$ range; $X(2085)$ not in 1^+ region.
- $0^+, 0^-$: open-strange hadron candidates near 2 GeV.
- $X(2075)/X(2085)$ possibly not pure states – moleculars and multiquarks coexist.

Heavy $\Lambda_Q\bar{\Sigma}_Q$:

- All $J^P = 0^-, 0^+, 1^-, 1^+$ masses above thresholds.
- Consistent with BESIII; predict $5.5-6.0$ GeV and ~ 12 GeV.
- $\Lambda_b\bar{\Sigma}_b$: hidden-bottom candidates in the 12 GeV region, would be detected in LHCb, BelleII, and others.
- Other researches: see at [Z.G.Wang et.al, 2604.14597]

Nonperturbative hadronization needs further study.

Thanks!

The Full Propagator of Quarks

- The full propagator of light quarks

$$
\begin{aligned}
\mathrm{i}\mathcal{S}_q^{jk}(x) =& \mathrm{i}\delta^{jk}\frac{\not{x}}{2\pi^2x^4} - \delta^{jk}m_q\frac{1}{4\pi^2x^2} - \mathrm{i}t_a^{jk}\frac{G_{\alpha\beta}^a}{32\pi^2x^2}\left(\sigma^{\alpha\beta}\not{x}+\not{x}\sigma^{\alpha\beta}\right) - \delta^{jk}\frac{\langle\bar{q}q\rangle}{12} \\
&+\mathrm{i}\delta^{jk}\frac{\not{x}}{48}m_q\langle\bar{q}q\rangle - \delta^{jk}\frac{x^2}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle + \mathrm{i}\delta^{jk}\frac{x^2\not{x}}{1152}m_q\langle g_s\bar{q}\sigma\cdot Gq\rangle \\
&- t_a^{jk}\frac{\sigma_{\alpha\beta}}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle - \mathrm{i}t_a^{jk}\frac{1}{768}\left(\sigma_{\alpha\beta}\not{x}+\not{x}\sigma_{\alpha\beta}\right)m_q\langle g_s\bar{q}\sigma\cdot Gq\rangle.
\end{aligned}
$$

- The full propagator of heavy quarks

$$
\begin{aligned}
\mathcal{S}_Q^{jk}(p) =& \frac{\mathrm{i}\delta^{jk}\left(\not{p}+m_Q\right)}{p^2-m_Q^2} - \frac{\mathrm{i}}{4}\frac{t_a^{jk}G_{\alpha\beta}^a}{\left(p^2-m_Q^2\right)^2}\left[\sigma^{\alpha\beta}\left(\not{p}+m_Q\right)+\left(\not{p}+m_Q\right)\sigma^{\alpha\beta}\right] \\
&+\frac{\mathrm{i}\delta^{jk}m_Q\left\langle g_s^2G^2\right\rangle}{12\left(p^2-m_Q^2\right)^3}\left[1+\frac{m_Q\left(\not{p}+m_Q\right)}{p^2-m_Q^2}\right] \\
&+\frac{\mathrm{i}\delta^{jk}}{48}\left\{\frac{\left(\not{p}+m_Q\right)\left[\not{p}\left(p^2-3m_Q^2\right)+2m_Q\left(2p^2-m_Q^2\right)\right]\left(\not{p}+m_Q\right)}{\left(p^2-m_Q^2\right)^6}\right\}\left\langle g_s^3G^3\right\rangle
\end{aligned}
\tag{5.1}
$$

The Spectral Density for Light Hexaquark States

- The correlation functions of $p\bar{\Lambda}$ and $p\bar{\Sigma}$ states

$$\Pi^{\text{Type-1}}_{J=0}(q^2) = -\,\mathrm{i}\varepsilon_{abc}\varepsilon_{a_1b_1c_1}\varepsilon_{def}\varepsilon_{d_1e_1f_1}\int[\mathcal{D}x]\,\mathrm{Tr}\left[\mathcal{S}^{cc_1}_{q_1}(-x)\,(\mathbb{1},\mathrm{i}\gamma_5)\,\mathcal{S}^{f_1f}_{u}(x)\,(\mathbb{1},\mathrm{i}\gamma_5)\right]\times$$
$$\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Taa_1}_{u}(-x)\mathcal{C}\mathcal{S}^{bb_1}_{q_2}(-x)\right]\times\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Td_1d}_{u}(x)\mathcal{C}\mathcal{S}^{e_1e}_{d}(x)\right],$$

$$\Pi^{\text{Type-1}}_{J=1,\mu\nu}(q^2) = -\,\mathrm{i}\varepsilon_{abc}\varepsilon_{a_1b_1c_1}\varepsilon_{def}\varepsilon_{d_1e_1f_1}\int[\mathcal{D}x]\,\mathrm{Tr}\left[\mathcal{S}^{cc_1}_{q_1}(-x)\gamma_\mu\,(\mathbb{1},\gamma_5)\,\mathcal{S}^{f_1f}_{u}(x)\gamma_\nu\,(\mathbb{1},\gamma_5)\right]\times$$
$$\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Taa_1}_{u}(-x)\mathcal{C}\mathcal{S}^{bb_1}_{q_2}(-x)\right]\times\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Td_1d}_{u}(x)\mathcal{C}\mathcal{S}^{e_1e}_{d}(x)\right],$$

$$\Pi^{\text{Type-2}}_{J=0}(x) = -\,\mathrm{i}\varepsilon_{abc}\varepsilon_{a_1b_1c_1}\varepsilon_{def}\varepsilon_{d_1e_1f_1}\int[\mathcal{D}x]\,\mathrm{Tr}\left[\mathcal{S}^{cc_1}_{q_1}(-x)\,(\mathbb{1},\mathrm{i}\gamma_5)\,\mathcal{S}^{f_1f}_{u}(x)\,(\mathbb{1},\mathrm{i}\gamma_5)\right]\times$$
$$\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Taa_1}_{u}(-x)\mathcal{C}\gamma_5\mathcal{S}^{bb_1}_{q_2}(-x)\gamma_5\right]\times\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Td_1d}_{u}(x)\mathcal{C}\gamma_5\mathcal{S}^{e_1e}_{d}(x)\gamma_5\right],$$

$$\Pi^{\text{Type-2}}_{J=1,\mu\nu}(x) = -\,\mathrm{i}\varepsilon_{abc}\varepsilon_{a_1b_1c_1}\varepsilon_{def}\varepsilon_{d_1e_1f_1}\int[\mathcal{D}x]\,\mathrm{Tr}\left[\mathcal{S}^{cc_1}_{q_1}(-x)\gamma_\mu\,(\mathbb{1},\mathrm{i}\gamma_5)\,\mathcal{S}^{f_1f}_{u}(x)\gamma_\nu\,(\mathbb{1},\mathrm{i}\gamma_5)\right]\times$$
$$\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Taa_1}_{u}(-x)\mathcal{C}\gamma_5\mathcal{S}^{bb_1}_{q_2}(-x)\gamma_5\right]\times\mathrm{Tr}\left[\mathcal{C}\mathcal{S}^{Td_1d}_{u}(x)\mathcal{C}\gamma_5\mathcal{S}^{e_1e}_{d}(x)\gamma_5\right],$$

$$\int[\mathcal{D}x]\equiv\int\mathrm{d}^4x.$$

The Spectral Density for Light Hexaquark States

- The correlation functions of triquark-antitriquark states
  - $[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$:

$$\Pi_{1(\mu\nu)}(q^2) = \int [\mathcal{D}x] \ \mathrm{Tr}\left[\mathcal{S}_s^{c_1 c}(-x)\Gamma_2\Gamma_{(\mu)}\Gamma_2\mathcal{S}_q^{bb_1}(x)\Gamma_2\Gamma_{(\nu)}\Gamma_2\right] \times \mathrm{Tr}\left[\mathcal{S}_q^{a_1 a}(-x)\Gamma_1\mathcal{S}_q^{cc_1}(x)\Gamma_1\right] \mathrm{Tr}\left[\mathcal{S}_q^{aa_1}(x)\Gamma_1\mathcal{S}_q^{b_1 b}(-x)\Gamma_1\right];$$

  - $[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$:

$$\Pi_{2(\mu\nu)}(q^2) = \int [\mathcal{D}x] \ \mathrm{Tr}\left[\mathcal{S}_q^{c_1 c}(-x)\Gamma_2\Gamma_{(\mu)}\Gamma_2\mathcal{S}_q^{bb_1}(x)\Gamma_2\Gamma_{(\nu)}\Gamma_2\right] \times \mathrm{Tr}\left[\mathcal{S}_s^{a_1 a}(-x)\Gamma_1\mathcal{S}_q^{cc_1}(x)\Gamma_1\right] \mathrm{Tr}\left[\mathcal{S}_q^{aa_1}(x)\Gamma_1\mathcal{S}_q^{b_1 b}(-x)\Gamma_1\right];$$

- The OPE of spectral density up to dimension 13

$$\begin{aligned}\rho^{\mathrm{OPE}}(s) =& \rho^{\mathrm{pert}}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle G^2\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle^2}(s) + \rho^{\langle G^3\rangle}(s) \\ &+ \rho^{\langle\bar{q}q\rangle\langle G^2\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(s) + \rho^{\langle G^4\rangle}(s) + \rho^{\langle\bar{q}q\rangle^3}(s) + \rho^{\langle\bar{q}Gq\rangle\langle G^2\rangle}(s) \\ &+ \rho^{\langle\bar{q}q\rangle^2\langle G^2\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle^2}(s) + \rho^{\langle\bar{q}q\rangle^2\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle G^4\rangle}(s) \\ &+ \rho^{\langle\bar{q}q\rangle^4}(s) + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\langle G^2\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle^2}(s) + \rho^{\langle\bar{q}Gq\rangle\langle G^4\rangle}(s).\end{aligned}$$

The Spectral Density for $\Lambda_Q\bar{\Sigma}_Q$ States

- Using the Wick's theorem,

$$\Pi_{(\mu\nu)}(q^2) = -\,\mathrm{i}\varepsilon_{abc}\varepsilon_{a_1b_1c_1}\varepsilon_{def}\varepsilon_{d_1e_1f_1}\int_X\int_P \mathrm{Tr}\left[\mathcal{S}_d^{aa_1}(-x)\Gamma_1\Gamma_{(\mu)}\Gamma_1\mathcal{S}_Q^{f_1f}(p_1)\Gamma_1\Gamma_{(\nu)}\Gamma_1\right] \times \mathrm{Tr}\left[\mathcal{C}\mathcal{S}_u^{Tcc_1}(-x)\mathcal{C}\Gamma_2\mathcal{S}_Q^{bb_1}(-p_2)\Gamma_2\right] \times \mathrm{Tr}\left[\mathcal{C}\mathcal{S}_u^{Td_1d}(x)\mathcal{C}\Gamma_2\mathcal{S}_d^{e_1e}(x)\Gamma_2\right].$$

$$\int_X\int_P = \int \mathrm{d}^4x\int\frac{\mathrm{d}^4p_1}{(2\pi)^4}\int\frac{\mathrm{d}^4p_2}{(2\pi)^4}$$

- The spectral density can be expressed with the Källén-Lehmann spectral representation $\rho(s) = \dfrac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$,

$$\rho^{\mathrm{OPE}}(s) = \rho^{\mathrm{pert}}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle G^2\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle^2}(s) + \rho^{\langle G^3\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle G^2\rangle} + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle^3}(s) + \rho^{\langle\bar{q}Gq\rangle\langle G^2\rangle}(s) + \rho^{\langle\bar{q}q\rangle^2\langle G^2\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle^2}(s) + \rho^{\langle\bar{q}q\rangle^2\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle^4}(s). \tag{5.2}$$

- The loop integrals of the relevant Feynman diagrams can be evaluated by the Schwinger parametrization method, and the ultraviolet divergences arising from these loop integrals are removed through renormalization in the $\overline{\mathrm{MS}}$ scheme.

Borel Transformation

- Definition

$$B\left[\Pi\left(q^2\right)\right] = \lim_{\substack{q^2, n\to\infty \\ -q^2/n = M_B^2}} \frac{\left(-q^2\right)^{n+1}}{n!} \left(\frac{\partial}{\partial q^2}\right)^n \Pi\left(q^2\right).$$

- Useful Relations

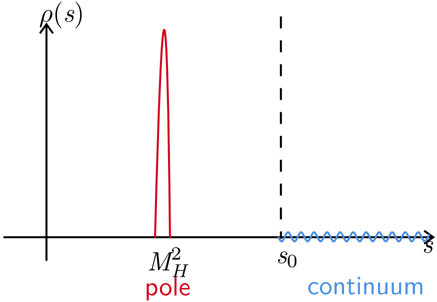
$$\mathcal{B}\left[\left(q^2\right)^k\right] = 0,$$
$$\mathcal{B}\left[\frac{1}{\left(s-q^2\right)^k}\right] = \frac{1}{(k-1)!}\left(\frac{1}{M_B^2}\right)^{k-1} \mathrm{e}^{-s/M_B^2}.$$

- Borel transformation is used for suppressing the contribution of higher excited states and the continuum spectra.

Reliability of QCDSR for Multiquark States

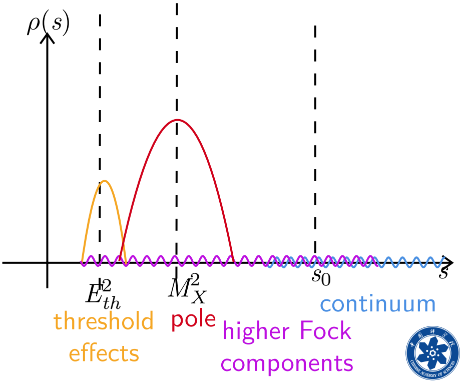
Conventional Hadrons

- Ground state usually well separated from excited states.
- Spectral function often dominated by a clear lowest pole.



Multiquark States

- Ground state and continuum are less clearly separated.
- Pole dominance is usually weaker.



Reliability of QCDSR for Multiquark States

Conventional Hadrons

- Compact quark configurations.
- OPE typically converges rapidly.
- Effective threshold $\sqrt{s_0}$ is usually close to the first excited state.
- QCDSR predictions are often quantitatively successful.

Multiquark States

- More spatially extended configurations.
- Higher-dimensional condensates may be more important.
- Choice of $\sqrt{s_0}$ is more subtle due to limited knowledge of excited spectra.
- Larger systematic uncertainties are expected.

Reliability of QCDSR for Multiquark States

- Existing QCDSR studies of multiquark systems show no obvious contradiction with current experimental observations.
- Typical uncertainties are of order Λ_{QCD}, but still sufficient to distinguish near-threshold states from states well above threshold.
- If QCDSR obtains a near-threshold result, it should be carefully considered. It may be a molecular state, threshold effects, or 2-hadron scattering states. In the $\Lambda_c\bar{\Sigma}_c$, the near threshold result may also contribute to the $c\bar{c}q\bar{q}+\pi$ structure whose masses may lie in 4–5 GeV. [W.Chen, H.X.Chen, S.L.Zhu et.al, EPJC, 2016]
- Therefore, QCDSR remains a semi-quantitative but useful framework for exploring multiquark spectroscopy. The applicability remains an open question deserving further investigation, especially clarifying the interplay between the OPE and the complex spectral structure of such systems.