



四川大學
SICHUAN UNIVERSITY

Baryon Structures from Dynamical Coupled-Channel Analyses

第九届强子谱与强子结构研讨会

王宇飞

四川大学物理学院

2026 年 8 月 15 日

海纳百川
有容乃大





Table of Contents

Part I: Introduction

- ▶ Part I: Introduction
- ▶ Part II: Baryon Transition Form Factors
- ▶ Part III: Compositeness of the P_c States
- ▶ Part IV: Conclusion & Outlook



Hadron structures

Part I: Introduction

- Structures of matters $\rightarrow$ fundamental task of physics
- Structures of hadrons $\rightarrow$ extremely difficult
 - Not direct observables
 - Non-perturbative nature of QCD at low & middle energies
 - Uncertainties of the hadron spectroscopy
- A bridge between the data and the structures?
World data $\rightarrow$??? $\rightarrow$ Spectra $\rightarrow$ Indications of the structures
- One possible option: comprehensive data-driven models
 $\rightarrow$ Jülich-Bonn/Jülich-Bonn-Washington Model
- Two studies:
 - Baryon transition form factors from the proton
[Y.F. Wang *et al.* (Jülich-Bonn-Washington Collaboration), *Phys. Rev. Lett.* 133, 101901 (2024)]
 - Compositeness of the P_c states
[Y.F. Wang *et al.*, *PRD* 112, 074010 (2025)]

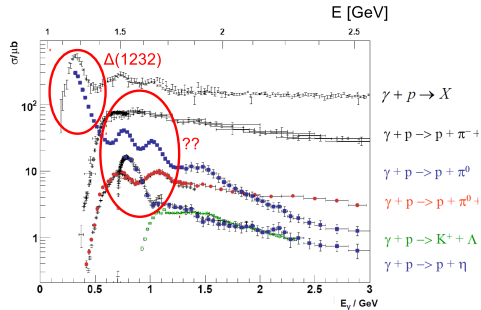




Partial Wave Analyses

Part I: Introduction

- Medium energy $\rightarrow$ involved dynamics & line-shapes
- Extraction of resonances
 $\rightarrow$ **partial wave analyses (PWA)**
- PW decomposition $T \sim T^{JLS} \rightarrow$ fit to the data
 $\rightarrow$ meaningful outputs
- Various methods
 - Unitary isobar models
 $\rightarrow$ unitary amplitudes + BW
[MAID, Yerevan/JLab, KSU]
 - K -matrix Unitarization
 $\rightarrow$ on-shell intermediate states
[GWU/SAID, BnGa, Gießen]
 - Dynamical coupled-channel (DCC) approaches
 $\rightarrow$ interaction potentials + scattering equations
(off-shell intermediate states)
[ANL-Osaka (EBAC), Dubna-Mainz-Taipeh]
& Jülich-Bonn Model
 - ...



[spectra: PDG 2000. quark model calculations: Löring *et al.*, EPJA 10, 395 (2001)]



The Jülich-Bonn Model

Part I: Introduction

Hadron dynamics

Lippmann-Schwinger-like equation

$$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) +$$

$$\sum_{\kappa} \int_0^{\infty} p^2 dp V_{\mu\kappa}(p'', p, z) G_{\kappa}(p, z) T_{\kappa\nu}(p, p', z)$$

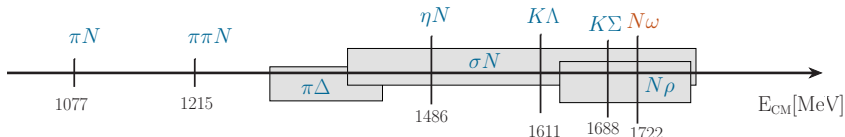
- One-dimensional: time-ordered perturbation theory + *JLS* basis [Jacob & Wick, *Annals Phys.* 7, 404 (1959)]
- Effective Lagrangians $\rightarrow$ SU(3), chiral, CP...
- Effective three-body channels: ρN , σN , $\pi\Delta$

Photo- & electroproduction

Construction from final state interactions

$$M_{\mu\gamma^*}(Q^2) = V_{\mu\gamma^*}(Q^2) + \sum_{\kappa} \int p^2 dp T_{\mu\kappa} G_{\kappa} V_{\kappa\gamma^*}(Q^2)$$

- γ^* : the $\gamma^* N$ channel for electroproduction
- Q^2 : photon virtuality
- $V_{\kappa\gamma^*} \rightarrow$ phenomenologically parameterized
- Constraints: Siegert's theorem, kinematics, etc.
- Photoproduction $\rightarrow Q^2 = 0$





Pole searching

Part I: Introduction

- Resonances $\rightarrow$ poles on the second Riemann sheet
- Analytical continuation $\rightarrow$ contour deformation [Döring et al., NPA 829, 170 (2009)]
- Pole position $z_r = M_r - i\Gamma_r/2$. Coupling strengths $\rightarrow$ normalized residues [PDG, RPP]

$$\tau_{\mu\nu}^{\Pi} \sim \frac{R_{\mu} R_{\nu}}{z_r - z} + \dots, NR_{\mu} \equiv \frac{2R_{\pi N}}{\Gamma_r} \times R_{\mu} \text{ [Indication of the structures!]}$$

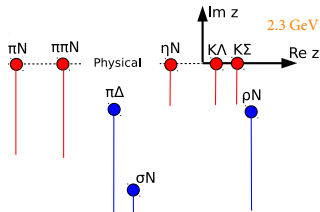
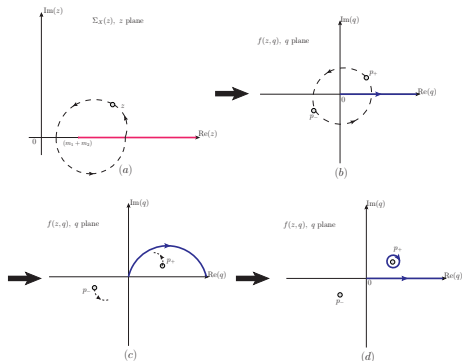




Table of Contents

Part II: Baryon Transition Form Factors

- ▶ Part I: Introduction
- ▶ **Part II: Baryon Transition Form Factors**
- ▶ Part III: Compositeness of the P_c States
- ▶ Part IV: Conclusion & Outlook



Baryon Transition Form Factors

Part II: Baryon Transition Form Factors

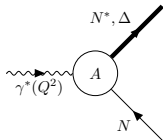
Electromagnetic Probes

- EM interactions
→ clean probes of structures
- Electroproduction
→ additional energy scale Q^2 (photon virtuality)
- Transition form factors (TFFs) → “pictures of hadrons”

[Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]

- Lower Q^2 : meson clouds etc.
- Higher Q^2 : the quark core
- Related to transverse charge densities

[Tiator & Vanderhaeghen, PLB 672, 344 (2009)]



Towards the TFFs

- Predictions at quark level
 - Quark models & Schwinger-Dyson equations
[Segovia *et al.*, Few Body Syst. 55, 1185 (2014)]
[Burkert, Roberts RMP 91, 011003 (2019)]
[Eichmann *et al.*, Prog. Part. Nucl. Phys. 91, 1 (2016)]
 - Lattice QCD [Agadjanov *et al.*, NPB 886, 1199 (2014)]
- Extraction from data
 - Experimental facilities @ JLAB
[Moiseev *et al.*, PRC 93, 025206 (2016)]
 - Unitary isobar models → real-valued, depending on BW parameters
[Drechsel, Kamalov, Tiator, EPJA 34, 69 (2007)]
[Tiator *et al.*, EPJST 198, 141 (2011)]
 - DCC → complex TFFs at the poles
[Kamano, Few Body Syst. 59, 24 (2018)]



TFFs in Jülich-Bonn-Washington Model

Part II: Baryon Transition Form Factors

The latest JBW results

- $\gamma^* p$ initial state
- Coupled-channel study of πN , ηN , and $K\Lambda$
[Mai *et al.*, EPJA 59, 286 (2023)]
- Based on the JüBo2017 solution
[Rönchen *et al.*, EPJA 54, 110 (2018)]
- C.M. energy range $z \in [1.13, 1.8]$ GeV
- Virtuality $Q^2 \in [0, 8]$ GeV²
- Orbital angular momentum $L \leq 3$
- Database
 - 10^5 data points for the electroproduction
 - 5×10^4 data points from the photoproduction/hadronic part
- Four fit solutions $\rightarrow$ estimation of uncertainties

Highlights of this study

- TFFs defined at the poles (A, S)

$$H_h = c_I \sqrt{\frac{p_{\pi N}}{\omega_\gamma} \frac{2\pi(2J+1)z_p}{m_N \tilde{R}_{\pi N}}} \tilde{\mathcal{R}}_h$$

- First simultaneous multi-channel study of TFFs
 $\rightarrow$ channel-independent results
- Outputs $\rightarrow$ TFFs of **twelve states** at the poles
- Results for some higher states are given for the first time
- Realistic uncertainties

This study:

[Y.F. Wang *et al.*, (Jülich-Bonn-Washington Collaboration), Phys. Rev. Lett. 133, 101901 (2024)]



Summary of the results

Part II: Baryon Transition Form Factors

Data files on <https://jbw.phys.gwu.edu>

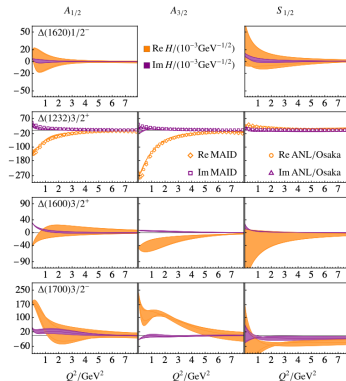
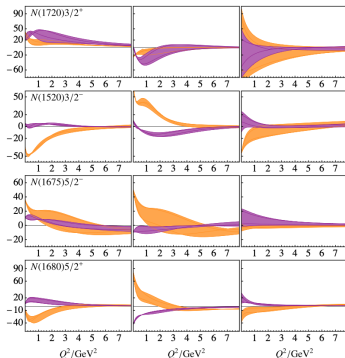
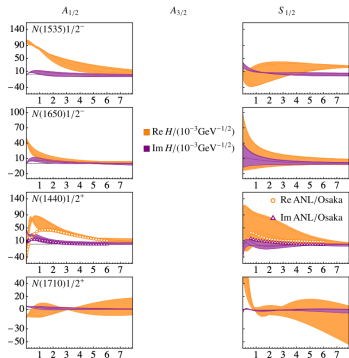




Table of Contents

Part III: Compositeness of the P_c States

- ▶ Part I: Introduction
- ▶ Part II: Baryon Transition Form Factors
- ▶ Part III: Compositeness of the P_c States
- ▶ Part IV: Conclusion & Outlook



Backgrounds

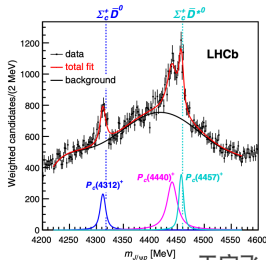
Part III: Compositeness of the P_c States

P_c as exotic states

- **Exotic hadrons** since $X(3872)$ [Belle, PRL 91, 262001 (2003)]
→ deviating much from conventional quark models
- Possible pictures
 - Hadronic molecules [Guo *et al.*, RMP 90, 015004 (2018)]:
bound states formed by hadron-level interactions (meson exchanges etc.)
 - compact multi-quark states
 - hybrid states, kinematic cusps, etc.
- The P_c states
 - Discovered in $\Lambda_b \rightarrow J/\psi p K^-$ by LHCb [LHCb, PRL 115, 072001 (2015)]
 $P_c(4380)$ & $P_c(4450)$
 - Updated results [LHCb, PRL 122, 222001 (2019)]
 $P_c(4312)$, $P_c(4440)$, $P_c(4457)$
 $P_c(4380)$ insignificant

Interpretations

- $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules [Liu *et al.*, PRL 122, 242001 (2019)][Du *et al.*, PRL 124, 072001 (2020)][Yao *et al.*, Rept. Prog. Phys. 84, 076201 (2021)]
- Compact or cusps [Wang, Int. J. Mod. Phys. 35, 2050003 (2020)][Nakamura, PRD 103, 111503 (2021)][Burns & Swanson, PRD 106, 054029 (2022)]





Composition of P_c

Part III: Compositeness of the P_c States

Jülich-Bonn model for P_c

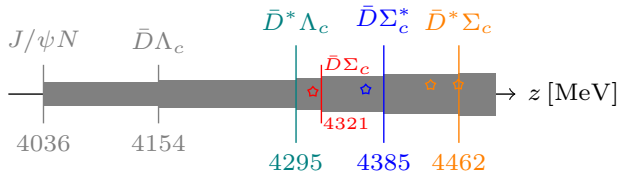
This study:

[Y.F. Wang *et al.*, PRD 112, 074010 (2025)]

- Previous studies
[Wang *et al.*, EPJC 82, 497 (2022)][Shen *et al.*, EPJC 84, 764 (2024)]
- Three solutions $\rightarrow$ updated LHCb data (175)
- P_c as S -wave states near $\bar{D}^{(*)}\Sigma_c^{(*)}$ thresholds:
 $P_c(4312) 1/2^-$, $P_c(4440) 1/2^-$, $P_c(4380) 3/2^-$,
 $P_c(4457) 3/2^-$
- Criteria $\rightarrow$ poles, & residues

Compositeness criteria

- Dynamical generation $\rightarrow$ model dependent
- Origin of compositeness criteria: Weinberg's study of deuteron [Weinberg, PR 137, B672 (1965)]
- Modern extensions
 - Pole-counting rule[Morgan, NPA 543, 632 (1992)]
 - Spectral density functions
[Baru *et al.*, PLB 586, 53 (2004)]
 - Gamow wave functions
[Sekihara, PRC 95, 025206 (2017)]





Criterion I

Part III: Compositeness of the P_c States

Weinberg's criterion

- Weinberg's criterion on deuteron $|d\rangle$

$$a = -\frac{2(1-Z)}{2-Z}R + \mathcal{O}(L)$$

$$r = -\frac{Z}{1-Z}R + \mathcal{O}(L)$$

Z : "elementariness". $R \sim 4.3$ fm: deuteron radius.

$L \sim m_\pi^{-1}$ interaction range.

a : scattering length. r : effective range.

- $Z = 1 - \int d\alpha |\langle \alpha | d \rangle|^2$. α : two-nucleon continuum
- Experimental results $\rightarrow Z \simeq 0$
- Conditions: S -wave, near-threshold, **stable**
- Model-independent [van Kolck, Symmetry 14, 1884 (2022)]

Pole-counting rule

- Near-threshold single-channel amplitudes:
 $T \sim (rp^2/2 - ip + 1/a)^{-1}$
- One bound state p_- and one virtual state p_+ :
 $|p_+| + |p_-| \geq \frac{2}{|r|}$
- **A molecule** $\rightarrow$ quantum mechanical potentials
 $\rightarrow |r|$ being the interaction range
 $\rightarrow$ **only one pole inside $1/|r|$**
- **A genuine state** $\rightarrow |r|$ extremely large,
 $|p_+| \simeq |p_-|$ [Castillejo et al., PR 101, 453 (1956)]
 $\rightarrow$ **two nearby poles**
- Can be extended to coupled-channel scatterings
- **Only for S -wave near threshold states**
- Applications [Meng et al., PRD 92, 034020 (2015)]
[Gong et al., PRD 94, 114019 (2016)][Cao et al., CPC 45, 103102 (2021)]



Criterion II

Part III: Compositeness of the P_c States

Spectral Density Functions

- Weinberg's Z for bound states: $Z = |\langle \psi_0 | \Psi_B \rangle|^2$
- Lower decay channels: $Z \rightarrow w(E) = -\text{Im}D(E)/\pi \rightarrow$ distribution on energy E
- Narrow resonance $E = E_R - i\Gamma_R/2$:

$$w(E) \simeq \frac{Z_B}{\pi} \frac{\Gamma_R/2}{(E-E_R)^2 + (\Gamma_R/2)^2}$$
- S -wave nearby channels:

$$w(E) \simeq -\frac{1}{\pi} \text{Im}(E - E_0 - \sum_j ig_j^2 p_j/2)^{-1}$$
 j : channels. E_0 and g_j 's: from residues (couplings) and pole positions
- Elementariness $\rightarrow$ deviation of $w(E)$ from "Breit-Wigner":

$$BW(E) = \frac{1}{\pi} \frac{\Gamma_R/2}{(E-E_R)^2 + (\Gamma_R/2)^2}$$
- Applications [Kalashnikova and Nefediev, PRD 80, 074004 (2009)]
 [Baru et al., EPJA 44, 93 (2010)][Hanhart et al., EPJA 47, 101 (2011)]

Complex Compositeness

- Resonances
 - Unphysical Gamow states
 [Gamow, Zeitschrift für Physik 51, 204 (1928)]
 [Civitaresse and Gadella, Physics Reports 396, 41(2004)]
 - Complex eigenstates of the Hamiltonian
 $\hat{H}|\Psi_R\rangle = E_R|\Psi_R\rangle$
 - Non-normalizable [Xiao and Zhou, PRD 94, 076006 (2016), J. Math. Phys. 58, 062110 (2017)]
- Complex elementariness: $Z_R = (\Psi_R^*|\psi_0)\langle\psi_0|\Psi_R\rangle$, **complex-valued**
- Complex compositeness $\rightarrow$ off-shell residues $r(k)$

$$T(E, p_i, p_f) \sim \frac{r(p_i)r(p_f)}{E-E_R}$$

$$X = 1 - Z_R = \int_C \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{r^2(k)}{[\mathcal{E}_R - k^2/(2\mu)]^2}$$
 [Sekihara, PRC 95, 025206 (2017)]
- Naive measure**, e.g. $\tilde{X}_j = \frac{|X_j|}{\sum_n |X_n| + |Z_R|}$

王宇飞 (四川大学)



Results I

Part III: Compositeness of the P_c States

- Pole-counting rule: always only one pole
- Effective ranges: at fm level

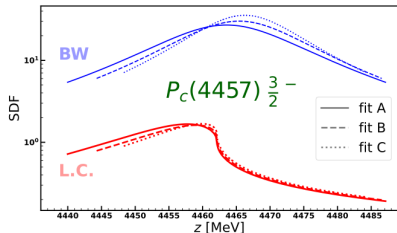
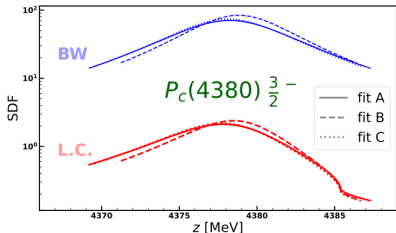
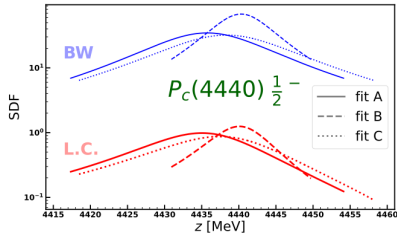
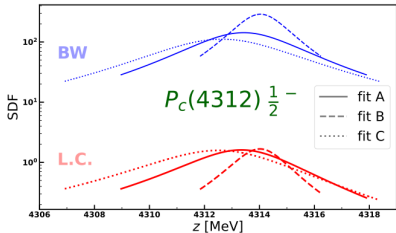
States	Channels	a	r
$P_c(4312) 1/2^-$	$\bar{D}\Sigma_c$	$-2.0 + 0.39i$	$-3.0 + 2.9i$
		$-2.2 + 0.29i$	$-3.0 + 2.9i$
		$-1.9 + 0.40i$	$-3.0 + 2.9i$
$P_c(4440) 1/2^-$	$\bar{D}^*\Sigma_c$	$-1.2 + 0.14i$	$0.36 - 1.9i$
		$-1.3 + 0.099i$	$0.44 - 1.8i$
		$-1.3 + 0.17i$	$0.43 - 1.9i$
$P_c(4380) 3/2^-$	$\bar{D}\Sigma_c^*$	$-1.9 + 0.29i$	$-0.84 - 1.4i$
		$-2.0 + 0.26i$	$-0.82 - 1.3i$
		$-1.9 + 0.27i$	$-0.83 - 1.4i$
$P_c(4457) 3/2^-$	$\bar{D}^*\Sigma_c$	$-1.2 + 0.96i$	$0.78 - 1.6i$
		$-1.2 + 1.1i$	$0.77 - 1.6i$
		$-1.1 + 1.2i$	$0.76 - 1.7i$



Results II

Part III: Compositeness of the P_c States

Spectral density functions: **one/two orders of magnitude smaller than BW!**





Results III

Part III: Compositeness of the P_c States

- Gamow wave functions $\rightarrow$ fairly small elementariness
- Analyses of the compositions $\rightarrow$ the **closest S -wave channels**
 - $P_c(4312) 1/2^-: \bar{D}\Sigma_c$ (80% $\sim$ 81%)
 - $P_c(4440) 1/2^-: \bar{D}^*\Sigma_c$ (72% $\sim$ 77%)
 - $P_c(4380) 3/2^-: \bar{D}\Sigma_c^*$ (80% $\sim$ 81%)
 - $P_c(4457) 3/2^-: \bar{D}^*\Sigma_c$ (66% $\sim$ 71%), $\bar{D}\Sigma_c^*$ (9% $\sim$ 12%)
- Relatively large $\bar{D}\Sigma_c^*$ configuration of $P_c(4457) \rightarrow$ large coupled-channel effects
- Independent of model parameterization
- **Further confirmation of the molecular interpretation!!**
- A similar study on the N^* and Δ states $\rightarrow$ much more complicated
[Y.F. Wang *et al.*, PRC 109, 015202 (2024)]



Table of Contents

Part IV: Conclusion & Outlook

- ▶ Part I: Introduction
- ▶ Part II: Baryon Transition Form Factors
- ▶ Part III: Compositeness of the P_c States
- ▶ Part IV: Conclusion & Outlook



Conclusion & Outlook

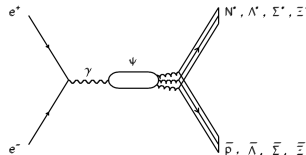
Part IV: Conclusion & Outlook

Conclusions

- The Jülich-Bonn Model
 - Comprehensive dynamical coupled-channel approaches
 - Data driven PWA $\rightarrow$ resonance spectra
 - Connecting experimental observations to hadron structures!
- Study 1: transition form factors
 - Determined by multi-channel data
 - Defined at the poles
 - Realistic uncertainties
 - Outputs for twelve states
- Study 2: compositeness of P_c
 - Generalization of Weinberg's criterion
 - Four P_c states $\rightarrow$ larger $\bar{D}^{(*)}\Sigma_c^{(*)}$ components!

Outlook

- The hyperon spectroscopy in the near future
- JBW model $\rightarrow$ energy extension to 1.95 GeV
- ωN photoproduction underway $\rightarrow$ more modern data!
- Broader applications
 - **BES**: $J/\psi(\psi') \rightarrow \bar{N}N^*$ [BES, PRL 97, 062001 (2006)] [BES, PRL 110, 022001 (2013)] [BESIII, CPC 44, 040001 (2020)]
 - HHaS @ HIAF: π beams for πN interaction [Chen et al., arXiv: 2511.22864 (2025)]



*Thank
you*



Backups

The Jülich-Bonn Model

Backups

A comprehensive coupled-channel model, fitting to a worldwide collection of data

Hadronic part

- The πN elastic scatterings [Schütz *et al.*, PRC 51, 1374 (1995)] [Schütz *et al.*, PRC 49, 2671 (1994)]
- Extended to $\pi\pi N$ and ηN [Schütz *et al.*, PRC 57, 1464 (1998)] [Krehl *et al.*, PRC 62, 025207 (2000)] [Gasparyan *et al.*, PRC 68, 045207 (2003)]
- Extended to $K\Lambda$ and $K\Sigma$ [Döring *et al.*, NPA 851, 58 (2011)] [Rönchen *et al.*, EPJA 49, 44 (2013)]
- Extended to ωN [Wang *et al.*, PRD 106, 094031 (2022)]
- Analytical continuation for searching poles [Döring *et al.*, NPA 829, 170 (2009)]

Photo- & Electroproduction

- Photoproduction
[Rönchen *et al.*, EPJA 50, 101 (2014)] [Rönchen *et al.*, EPJA 51, 70 (2015)] [Rönchen *et al.*, EPJA 54, 110 (2018)] [Rönchen *et al.*, EPJA 558, 229 (2022)]
- Electroproduction (Jülich-Bonn-Washington Model)
[Mai *et al.*, PRC 103, 065204 (2021)] [Mai *et al.*, PRC 106, 015201 (2022)] [Mai *et al.*, EPJA 59, 286 (2023)]
- P_c states [Wang *et al.*, EPJC 82, 497 (2022)] [Shen *et al.*, EPJC 84, 764 (2024)] [Wang *et al.*, PRD 112, 074010 (2025)]

Details of the scattering equation

Backups

The Lippmann-Schwinger-like equation

$$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) + \sum_{\kappa} \int_0^{\infty} p^2 dp V_{\mu\kappa}(p'', p, z) G_{\kappa}(p, z) T_{\kappa\nu}(p, p', z)$$

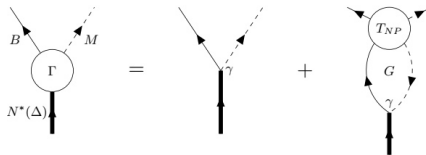
- Reaction channels $\nu \rightarrow \mu$ (after PW and isospin projection, *JLS* basis [Jacob & Wick, *Annals Phys.* 7, 404 (1959)], $J \leq 9/2$)
- Intermediate channel: κ
- CM initial (final) momentum: p' (p''). CM energy: z
- Potential (kernel): V . Amplitude: T ($\rightarrow$ observables)
- Propagator: G ($\pi\pi N$ channel: effective channels $\rho N, \sigma N, \pi\Delta$. E/ω - energy of the baryon/meson.)

$$G_{\kappa}(z, p) = \begin{cases} (z - E_{\kappa} - \omega_{\kappa} + i0^+)^{-1} & \text{(if } \kappa \text{ is a two-body channel) ,} \\ [z - E_{\kappa} - \omega_{\kappa} - \Sigma_{\kappa}(z, p) + i0^+]^{-1} & \text{(if } \kappa \text{ is an effective channel) .} \end{cases}$$

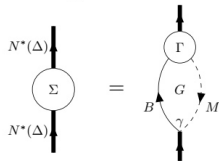
Details of the scattering equation

Backups

- **Separating the amplitude** $\rightarrow$ with/without s -channel poles $T = T^P + T^{NP}$
- Reconstruction of the amplitude $\rightarrow T^{NP} = V^{NP} + \sum \int p^2 dp V^{NP} G T^{NP}$,
 $T_{\mu\nu}^P(p'', p', z) = \sum_{i,j} \Gamma_{\mu,i}^a(p'') D_{ij}(z) \Gamma_{\nu,j}^c(p')$, $(D^{-1})_{ij} = \delta_{ij}(z - m_i^b) - \Sigma_{ij}(z)$
 - $\Gamma(\gamma)$: the dressed (bare) vertices (a - annihilation, c - creation)
 - Σ : coupled-channel self-energy functions of the s -channel states



(a) The vertex.



(b) The self energy.

Details of the scattering equation

Backups

Potentials → field-theoretical construction

Parameters → determined by fits

The NP part

- Tree-level potentials
 - t -channel + u -channel + contact
 - Stemming from effective Lagrangians → SU(3) flavour symmetry, CP conservation, chiral symmetry
 - Established by time-ordered perturbation theory (TOPT) → stationary perturbation in Schrödinger picture
 - TOPT+partial wave → one-dimensional integral $\int p^2 dp$
 - **Regulators** for every vertex → to make the integral converge: $F(q) \sim \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + q^2} \right)^n$
 m : the mass of the exchanged particle. Λ : cut-off (fit parameter)
- Beyond tree-level → correlated two-pion exchanges [Schütz *et al.*, PRC 49, 2671 (1994)] [Schütz *et al.*, PRC 51, 1374 (1995)]

The P part

- Stemming from effective Lagrangians with CP conservation (tree-level bare vertices)
- **Phenomenological contact terms** → $D \sim (1 - \Sigma)^{-1}$ [Rönchen *et al.*, EPJA 51, 70 (2015)]
- Renormalization of the nucleon mass

Formulae & Extraction

Backups

Original definition

[Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]

$$A_h = \sqrt{\frac{2\pi\alpha}{K}} \langle R, h | \epsilon_+ \cdot J | N, h-1 \rangle$$

$$S_{\frac{1}{2}} = \frac{|\mathbf{q}|}{Q} \sqrt{\frac{2\pi\alpha}{K}} \langle R, \frac{1}{2} | \epsilon_0 \cdot J | N, \frac{1}{2} \rangle$$

- A, S : helicity transition amplitudes
- $h = 1/2, 3/2$: the helicity
- α : fine structure constant
- $\epsilon(J)$: virtual photon polarization vector (current)
- $\mathbf{q}$: 3-momentum of the virtual photon
- $M_R(m_N)$: mass of the excitation state R (nucleon)
- $K = (M_R^2 - m_N^2)/(2M_R)$

At the pole

[Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]

$$H_h = C_I \sqrt{\frac{p_{\pi N}}{\omega_0} \frac{2\pi(2J+1)z_p}{m_N \tilde{R}}} \tilde{\mathcal{H}}_h$$

- H is either A or S
- C_I : isospin factor, $C_{1/2} = -\sqrt{3}$ and $C_{3/2} = \sqrt{2/3}$
- $p_{\pi N}$: πN c.m. momentum
- ω_0 : photon energy at $Q^2 = 0$
- $z_p = M_R - i\Gamma_R/2$ the pole position
- $\tilde{R}, \tilde{\mathcal{H}}$: the residues of $\pi N, \gamma^* N$ channels
- Understanding: the $|R\rangle \rightarrow |R\rangle$ defined from the Gamow state
- **Complex-valued**