

Probing the nature of tensor mesons via B semi-leptonic decays

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- 1 Introduction
- 2 The CLFQM general calculation procedure
- 3 $B \rightarrow T$ transition form factors
- 4 Semi-leptonic decays $B \rightarrow T \ell \nu_\ell$

Introduction

- The light P-wave tensor mesons with $J^P = 2^+$ form a 3P_2 nonet, which involves the iso-vector mesons $a_2(1320)$, iso-doublet states $K_2^*(1430)$ and two iso-singlet $f_2(1270)$ and $f_2(1525)$.
- The charmed tensor mesons are $D_2^*(2460)$ and $D_{s2}^*(2573)$. In 1989, the neutral and charged $D_2^*(2460)$ was firstly observed by TPS and ARGUS collaborations in $D\pi$ invariant mass distribution. In 1994, $D_{s2}^*(2573)$ was found by CLEO collaboration in the $D^0 K^+$ invariant mass spectrum.
- The semi-leptonic $B \rightarrow T \ell \nu_\ell$ decays play an important role not only in understanding the dynamic mechanisms of weak decay, but also studying the properties of tensor mesons.

Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions describing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at $q^2 = 0$ is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transition form factors.

The CLFQM general calculation procedure

The CLFQM general calculation procedure

The Bauer-Stech-Wirble (BSW) form factors for the $B \rightarrow T$ transitions are defined as follows,

$$\langle T(p'', \varepsilon^{\mu\nu}) | V_\mu | B(p') \rangle = -i \frac{2\varepsilon_{\mu\nu\alpha\beta}}{m_B + m_T} e^{*\nu} P^\alpha q^\beta V^{BT}(q^2), \quad (1)$$

$$\begin{aligned} \langle T(p'', \varepsilon^{\mu\nu}) | A_\mu | B(p') \rangle &= 2m_T \frac{e^* \cdot q}{q^2} q_\mu A_0^{BT}(q^2) + (m_B + m_T) \left[e_\mu^* - \frac{e^* \cdot q}{q^2} q_\mu \right] A_1^{BT}(q^2) \\ &\quad - \frac{e^* \cdot q}{m_B + m_V} \left[P_\mu - \frac{m_B^2 - m_T^2}{q^2} q_\mu \right] A_2^{BT}(q^2), \end{aligned} \quad (2)$$

where $P = p' + p''$, $q = p' - p''$, $e^{*\nu} = \frac{\varepsilon^{*\mu\nu} \cdot p'_\mu}{M_T}$ with $\varepsilon^{\mu\nu}$ being the polarization tensor

of tensor meson. The four-momentum of the initial (final) meson is

$$p' = p_1 + p_2 \quad (p'' = p_1' + p_2).$$

The CLFQM general calculation procedure

In the Isgur-Scora-Grinstein-Wise (ISGW) model, the general expression for the $B \rightarrow T$ transitions is parametrized as

$$\begin{aligned} \langle T(p'', \varepsilon) | (V - A)_\mu | B(p') \rangle = & \quad ih(q^2) \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*\nu\alpha} p'_\alpha P^\rho q^\sigma - k(q^2) \varepsilon_{\mu\nu}^* p'^\nu \\ & - b_+(q^2) \varepsilon_{\alpha\beta}^* p'^\alpha p'^\beta P_\mu - b_-(q^2) \varepsilon_{\alpha\beta}^* p'^\alpha p'^\beta q_\mu. \quad (3) \end{aligned}$$

The relations between these two different sets of form factors are

$$V^{BT}(q^2) = m_B(m_B + m_T)h(q^2), \quad (4)$$

$$A_1^{BT}(q^2) = \frac{m_B}{m_B + m_T} k(q^2), \quad (5)$$

$$A_2^{BT}(q^2) = -m_B(m_B + m_T)b_+(q^2), \quad (6)$$

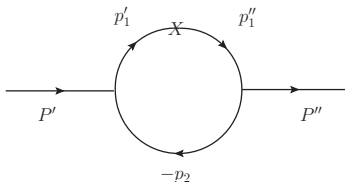
$$A_0^{BT}(q^2) = \frac{m_B}{2m_T} [k(q^2) + (m_B^2 - m_T^2)b_+(q^2) + q^2 b_-(q^2)]. \quad (7)$$

The CLFQM general calculation procedure

The decay amplitude at the lowest order for the general $P \rightarrow T$ transition can be written as

$$\mathcal{A}^{PT} = -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p'_1 \frac{H'_P(iH''_T)}{N'_1 N''_1 N_2} S_\mu^{PT}, \quad (8)$$

where $N'_1{}^{(\prime\prime)} = p'_1{}^{(\prime\prime)2} - m_1{}^{(\prime\prime)2}$ and $N_2 = p_2^2 - m_2^2$ arise from the quark propagators.



The CLFQM general calculation procedure

The expression of trace S_μ^{PT} can be derived using the Lorentz contraction,

$$\begin{aligned}
 S_\mu^{PT} &= \text{Tr}[(\not{p}_1'' + m_1'')\gamma^\mu\gamma_5(\not{p}_1' + m_1')\gamma_5(-\not{p}_2 + m_2)] \\
 &= 2p_{1\mu}[M'^2 + M''^2 - q^2 - 2N_2 - (m_1' - m_2)^2 - (m_1'' + m_2)^2 + (m_1' + m_1'')^2] \\
 &\quad + q_\mu[q^2 - 2M'^2 + N_1' - N_1'' + 2N_2 + 2(m_1' - m_2)^2 - (m_1' + m_1'')^2] \\
 &\quad + P_\mu[q^2 - N_1' - N_1'' - (m_1' + m_1'')^2].
 \end{aligned} \tag{9}$$

The explicit forms of H_P' and H_T'' used in this work are given by

$$H_P' = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}_0'} \varphi', \tag{10}$$

$$H_T'' = (M''^2 - M_0''^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}_0''} \varphi_p''. \tag{11}$$

The CLFQM general calculation procedure

Here φ' and φ_p'' are the light-front wave function for s-wave (initial state) and p-wave (final state) mesons, respectively, which are adopted in the phenomenological Gaussian-type forms,

$$\varphi' = \varphi'(x_2, p'_\perp) = 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp'_z}{dx_2}} \exp\left(-\frac{p_z'^2 + p_\perp'^2}{2\beta'^2}\right), \quad (12)$$

$$\varphi_p'' = \varphi_p''(x_2, p''_\perp) = \sqrt{\frac{2}{\beta^2}} \varphi', \quad \frac{dp'_z}{dx_2} = \frac{e'_1 e_2}{x_1 x_2 M'_0}, \quad (13)$$

where the shape parameter β can be fixed by fitting the corresponding decay constant.

$B \rightarrow T$ transition form factors

Input parameters:

- The constituent quark masses(GeV): $m_b = 4.8, m_c = 1.4, m_s = 0.37, m_{u,d} = 0.25$.
- The masses of the initial and the final mesons(GeV): $m_B = 5.28, m_{B_s} = 5.37, m_{a_2(1320)} = 1.32, m_{f_2(1270)} = 1.27, m_{D_2^*(2460)} = 2.46, m_{D_{s2}^*(2573)} = 2.57, m_{K_2^*(1430)} = 1.43$.
- The CKM matrix element:
 $V_{cb} = (41.1 \pm 1.2) \times 10^{-3}, V_{ub} = (3.82 \pm 0.2) \times 10^{-3}$.

$B \rightarrow T$ transition form factors

Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta_B = 0.555_{-0.060}^{+0.060}, \beta_{B_s} = 0.626_{-0.045}^{+0.045}, \beta_{a_2} = 0.312_{-0.006}^{+0.006},$$

$$\beta_{f_2} = 0.312_{-0.006}^{+0.006}, \beta_{f'_2} = 0.348_{-0.006}^{+0.006}, \beta_{K_2^*} = 0.313_{-0.010}^{+0.010},$$

$$\beta_{D_2^*} = 0.429_{-0.013}^{+0.013}, \beta_{D_{s2}^*} = 0.530_{-0.019}^{+0.019}.$$

- Mean life ($10^{-12}s$):

$$\tau_{B^0} = (1.519 \pm 0.004), \tau_{B_s^0} = (1.520 \pm 0.005), \tau_{B^\pm} = (1.638 \pm 0.004).$$

$B \rightarrow T$ transition form factors

- All the calculations are carried out within the $q^+ = 0$ reference frame, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}. \quad (14)$$

- $F(q^2)$ denotes different form factors, such as $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$.

$B \rightarrow T$ transition form factors

- The $B \rightarrow T$ transition form factors obtained within the CLFQM. The uncertainties mainly arise from the decay constants of the initial and final state mesons.

Transitions	References	V	A_0	A_1	A_2	Transitions	References	V	A_0	A_1	A_2
$B \rightarrow a_2$	This work	$0.25^{+0.02+0.00}_{-0.01-0.01}$	$0.21^{+0.03+0.01}_{-0.03-0.01}$	$0.19^{+0.02+0.00}_{-0.01-0.01}$	$0.17^{+0.01+0.00}_{-0.00-0.01}$	$B \rightarrow K_2^*$	This work	$0.27^{+0.01+0.02}_{-0.02-0.01}$	$0.22^{+0.03+0.02}_{-0.03-0.02}$	$0.21^{+0.02+0.02}_{-0.02-0.01}$	$0.19^{+0.01+0.00}_{-0.01-0.01}$
	CLFQM1	$0.24^{+0.04}_{-0.04}$	$0.21^{+0.03}_{-0.03}$	$0.19^{+0.04}_{-0.03}$	$0.17^{+0.03}_{-0.02}$		CLFQM1	$0.28^{+0.05}_{-0.05}$	$0.24^{+0.04}_{-0.04}$	$0.22^{+0.07}_{-0.01}$	$0.20^{+0.04}_{-0.03}$
	CLFQM2	0.28	0.24	0.21	0.19		CLFQM2	0.28	0.25	0.21	0.19
	PQCD	$0.18^{+0.04+0.04}_{-0.03-0.03}$	$0.18^{+0.04+0.04}_{-0.03-0.03}$	$0.11^{+0.02+0.02}_{-0.02-0.02}$	$0.06^{+0.01+0.01}_{-0.01-0.01}$		PQCD	$0.21^{+0.04+0.05}_{-0.04-0.03}$	$0.18^{+0.04+0.04}_{-0.03-0.03}$	$0.13^{+0.03+0.03}_{-0.02-0.02}$	$0.08^{+0.02+0.02}_{-0.02-0.01}$
	LCSR1	$0.27^{+0.018}_{-0.014}$	$0.27^{+0.022}_{-0.015}$	$0.19^{+0.014}_{-0.012}$	$0.14^{+0.008}_{-0.011}$		LCSR1	$0.25^{+0.027}_{-0.015}$	$0.29^{+0.033}_{-0.022}$	$0.18^{+0.020}_{-0.015}$	$0.10^{+0.011}_{-0.009}$
	LCSR2	$0.18^{+0.02}_{-0.02}$	$0.21^{+0.04}_{-0.04}$	$0.14^{+0.02}_{-0.02}$	$0.09^{+0.02}_{-0.02}$		LCSR2	$0.16^{+0.02}_{-0.02}$	$0.25^{+0.04}_{-0.04}$	$0.14^{+0.02}_{-0.02}$	$0.05^{+0.02}_{-0.02}$
	LCSR3	$0.18^{+0.12}_{-0.07}$	$0.30^{+0.06}_{-0.05}$	$0.16^{+0.09}_{-0.05}$	$0.07^{+0.08}_{-0.03}$		LCSR3	$0.22^{+0.11}_{-0.08}$	$0.30^{+0.06}_{-0.05}$	$0.19^{+0.09}_{-0.07}$	$0.11^{+0.05}_{-0.06}$
	3PSR	$0.13^{+0.03}_{-0.03}$	$0.26^{+0.07}_{-0.07}$	$0.11^{+0.04}_{-0.04}$	$0.09^{+0.02}_{-0.02}$		CLFQM3	0.28	-0.26	0.01	0.21
	CLFQM3	0.28	-0.20	0.02	0.17						
$B \rightarrow f_2$	This work	$0.24^{+0.02+0.01}_{-0.02-0.01}$	$0.22^{+0.03+0.01}_{-0.03-0.02}$	$0.19^{+0.02+0.01}_{-0.02-0.00}$	$0.17^{+0.01+0.01}_{-0.01-0.00}$	$B \rightarrow D_2^*$	This work	$0.72^{+0.05+0.02}_{-0.05-0.02}$	$0.69^{+0.06+0.03}_{-0.06-0.02}$	$0.64^{+0.05+0.01}_{-0.05-0.01}$	$0.54^{+0.03+0.00}_{-0.02-0.01}$
	PQCD	$0.12^{+0.02+0.02}_{-0.02-0.02}$	$0.13^{+0.03+0.03}_{-0.02-0.02}$	$0.08^{+0.02+0.01}_{-0.01-0.01}$	$0.04^{+0.01+0.01}_{-0.01-0.00}$		CLFQM 1	$0.75^{+0.13}_{-0.16}$	$0.64^{+0.11}_{-0.11}$	$0.63^{+0.11}_{-0.12}$	$0.58^{+0.12}_{-0.12}$
	LCSR4	0.30	0.22	0.17	0.11		CLFQM3	0.62	0.52	0.53	0.54
	LCSR1	$0.20^{+0.012}_{-0.010}$	$0.18^{+0.015}_{-0.010}$	$0.14^{+0.010}_{-0.009}$	$0.11^{+0.006}_{-0.008}$						
	LCSR2	$0.18^{+0.02}_{-0.02}$	$0.20^{+0.04}_{-0.04}$	$0.14^{+0.02}_{-0.02}$	$0.10^{+0.02}_{-0.02}$						
	LCSR3	$0.11^{+0.07}_{-0.05}$	$0.20^{+0.04}_{-0.04}$	$0.10^{+0.06}_{-0.04}$	$0.04^{+0.05}_{-0.01}$						
	3PSR	$0.12^{+0.04}_{-0.04}$	$0.24^{+0.06}_{-0.06}$	$0.10^{+0.02}_{-0.02}$	$0.09^{+0.02}_{-0.02}$						

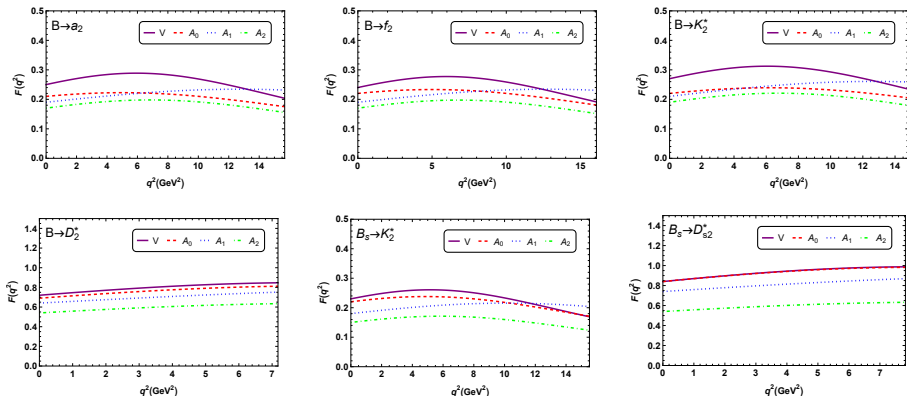
$B \rightarrow T$ transition form factors

- The $B \rightarrow T$ transition form factors obtained within the CLFQM. The uncertainties mainly arise from the decay constants of the initial and final state mesons.

$B_s \rightarrow K_2^*$	This work	$0.23^{+0.02+0.01}_{-0.00-0.02}$	$0.22^{+0.03+0.03}_{-0.02-0.02}$	$0.18^{+0.02+0.01}_{-0.01-0.01}$	$0.16^{+0.00+0.01}_{-0.01-0.00}$	$B_s \rightarrow f_2'$	This work	$0.31^{+0.02+0.01}_{-0.01-0.01}$	$0.31^{+0.03+0.02}_{-0.01-0.01}$	$0.25^{+0.02+0.00}_{-0.00-0.04}$	$0.21^{+0.01+0.01}_{-0.01-0.00}$
CLFQM1	$0.23^{+0.04}_{-0.05}$	$0.18^{+0.04}_{-0.04}$	$0.17^{+0.03}_{-0.04}$	$0.16^{+0.03}_{-0.03}$		CLFQM1	$0.32^{+0.05}_{-0.06}$	$0.26^{+0.05}_{-0.05}$	$0.24^{+0.05}_{-0.05}$	$0.22^{+0.04}_{-0.04}$	
PQCD	$0.18^{+0.03+0.04}_{-0.03-0.03}$	$0.15^{+0.03+0.03}_{-0.02-0.02}$	$0.11^{+0.02+0.02}_{-0.02-0.02}$	$0.07^{+0.01+0.02}_{-0.01-0.01}$		PQCD	$0.20^{+0.04+0.05}_{-0.03-0.03}$	$0.16^{+0.03+0.03}_{-0.02-0.02}$	$0.12^{+0.02+0.03}_{-0.02-0.02}$	$0.09^{+0.02+0.02}_{-0.01-0.01}$	
LCSR1	$0.25^{+0.020}_{-0.014}$	$0.29^{+0.033}_{-0.022}$	$0.18^{+0.020}_{-0.015}$	$0.10^{+0.011}_{-0.009}$		LCSR1	$0.25^{+0.027}_{-0.015}$	$0.32^{+0.035}_{-0.024}$	$0.18^{+0.022}_{-0.015}$	$0.08^{+0.011}_{-0.008}$	
LCSR2	$0.15^{+0.02}_{-0.02}$	$0.22^{+0.04}_{-0.04}$	$0.12^{+0.02}_{-0.02}$	$0.05^{+0.02}_{-0.02}$		LCSR2	$0.15^{+0.02}_{-0.02}$	$0.25^{+0.04}_{-0.04}$	$0.13^{+0.02}_{-0.02}$	$0.03^{+0.02}_{-0.02}$	
3PSR	$0.13^{+0.03}_{-0.03}$	$0.23^{+0.06}_{-0.06}$	$0.10^{+0.02}_{-0.02}$	$0.05^{+0.01}_{-0.01}$							
$B_s \rightarrow D_{s2}^*$	This work	$0.84^{+0.05+0.02}_{-0.02-0.02}$	$0.84^{+0.06+0.02}_{-0.01-0.03}$	$0.74^{+0.04+0.01}_{-0.01-0.02}$	$0.57^{+0.02+0.00}_{-0.01-0.00}$						
CLFQM1	$0.95^{+0.18}_{-0.17}$	$0.76^{+0.13}_{-0.13}$	$0.74^{+0.15}_{-0.14}$	$0.66^{+0.14}_{-0.13}$							
CLFQM4	0.81	0.69	0.68	0.67							

$B \rightarrow T$ transition form factors

图: The q^2 -dependence of the $B_{(s)} \rightarrow a_2, f_2, K_2^*, D_2^*, D_{s2}^*$ transtion form factors.



Semi-leptonic decays $B \rightarrow T\ell\nu_\ell$

Semi-leptonic Decays $B \rightarrow T\ell\nu_\ell$

The partial decay widths of the semi-leptonic decays $B \rightarrow T\ell\nu_\ell$

$$\frac{d\Gamma}{dq^2} = \sum_{i=L,\pm} \frac{d\Gamma_i}{dq^2}, \quad (15)$$

$$\frac{d\Gamma_{L,\pm}}{dq^2} = \frac{|G_F V_{CKM}|^2 \sqrt{\lambda_T}}{256 m_B^3 \pi^3 q^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 (X_L, X_\pm), \quad (16)$$

where $\lambda_T = \lambda(m_B^2, m_T^2, q^2) = (m_B^2 - m_T^2 - q^2)^2 - 4m_T^2 q^2$, and $X_L, X_\pm$ are defined as

$$X_L = \frac{\lambda_T}{9m_T^2 m_B^2} \left[(2q^2 + m_\ell^2) h_0^2(q^2) + 3\lambda_T m_\ell^2 A_0^2(q^2) \right], \quad (17)$$

$$X_\pm = \frac{2q^2}{3} (2q^2 + m_\ell^2) \frac{\lambda_T}{8m_T^2 m_B^2} \left[(m_B + m_T) A_1(q^2) \mp \frac{\sqrt{\lambda_T}}{m_B + m_T} V(q^2) \right]^2, \quad (18)$$

$$h_0(q^2) = \frac{1}{2m_T} \left[(m_B^2 - m_T^2 - q^2)(m_B + m_T) A_1(q^2) - \frac{\lambda_T}{m_B + m_T} A_2(q^2) \right]. \quad (19)$$

Semi-leptonic decays $B \rightarrow T\ell\nu_\ell$

One can define the forward-backward asymmetry $\mathcal{A}_{FB}$ of the lepton,

$$\mathcal{A}_{FB} = \frac{1}{X_L + X_+ + X_-} \left(\frac{\lambda_T}{6m_T^2 m_B^2} 2m_\ell^2 \sqrt{\lambda_T} h_0(q^2) A_0(q^2) - \frac{\lambda_T}{8m_T^2 m_B^2} 4q^4 \sqrt{\lambda_T} A_1(q^2) V(q^2) \right). \quad (20)$$

The longitudinal polarization fraction is defined as

$$f_L = \frac{\Gamma_L}{\Gamma_L + \Gamma_+ + \Gamma_-}. \quad (21)$$

Semi-leptonic decays $B \rightarrow T\ell\nu_\ell$

- The branching ratios (10^{-5}) of the semi-leptonic decays $B \rightarrow (a_2, f_2, K_2^*)\ell\nu_\ell$.

Reference	This work	PQCD	3PSR	LCSR	RQM
$B^+ \rightarrow a_2^0 e^+ \nu_e$	$4.63^{+0.01+0.55+0.84}_{-0.01-1.46-0.30}$	—	—	$5.60^{+0.9+1.1}_{-0.7-1.0}$	—
$B^+ \rightarrow a_2^0 \mu^+ \nu_\mu$	$4.61^{+0.01+0.56+0.83}_{-0.01-1.45-0.29}$	—	—	$5.50^{+0.9+1.1}_{-0.6-1.0}$	—
$B^+ \rightarrow a_2^0 \tau^+ \nu_\tau$	$1.08^{+0.00+0.15+0.17}_{-0.00-0.30-0.18}$	—	—	$1.10^{+0.1+0.2}_{-0.2-0.2}$	—
$B^0 \rightarrow a_2^- e^+ \nu_e$	$8.56^{+0.02+1.03+1.55}_{-0.02-2.70-0.55}$	$11.60^{+8.1}_{-5.7}$	$8.20^{+0.025}_{-0.025}$	$10.30^{+1.7+2.1}_{-1.2-1.9}$	$12.70^{+5.1}_{-5.1}$
$B^0 \rightarrow a_2^- \mu^+ \nu_\mu$	$8.54^{+0.02+1.03+1.54}_{-0.02-2.69-0.53}$	$11.60^{+8.1}_{-5.7}$	$8.20^{+0.025}_{-0.025}$	$10.20^{+1.7+2.1}_{-1.1-1.8}$	$12.70^{+5.1}_{-5.1}$
$B^0 \rightarrow a_2^- \tau^+ \nu_\tau$	$2.00^{+0.01+0.28+0.31}_{-0.01-0.56-0.33}$	$4.10^{+2.9}_{-2.0}$	$5.10^{+0.017}_{-0.017}$	$2.00^{+0.3+0.4}_{-0.3-0.4}$	$2.80^{+1.1}_{-1.1}$
$B^+ \rightarrow f_2^0 e^+ \nu_e$	$6.23^{+0.02+1.56+1.09}_{-0.02-2.34-0.54}$	$6.90^{+4.8}_{-3.4}$	$7.70^{+0.023}_{-0.023}$	$6.00^{+1.0+1.2}_{-0.7-1.1}$	—
$B^+ \rightarrow f_2^0 \mu^+ \nu_\mu$	$6.19^{+0.02+1.54+1.10}_{-0.02-2.34-0.52}$	$6.90^{+4.8}_{-3.4}$	$7.70^{+0.023}_{-0.023}$	$6.00^{+1.0+1.2}_{-0.7-1.1}$	—
$B^+ \rightarrow f_2^0 \tau^+ \nu_\tau$	$1.44^{+0.00+0.33+0.23}_{-0.00-0.47-0.12}$	$2.50^{+1.8}_{-1.3}$	$5.30^{+0.018}_{-0.018}$	$1.20^{+0.2+0.3}_{-0.1-0.2}$	—
$B^0 \rightarrow \bar{f}_2^- e^+ \nu_e$	$11.54^{+0.03+2.89+2.02}_{-0.03-4.34-0.99}$	—	—	—	—
$B^0 \rightarrow \bar{f}_2^- \mu^+ \nu_\mu$	$11.47^{+0.03+2.85+2.03}_{-0.03-4.33-0.97}$	—	—	—	—
$B^0 \rightarrow \bar{f}_2^- \tau^+ \nu_\tau$	$2.67^{+0.01+0.62+0.42}_{-0.01-0.86-0.23}$	—	—	—	—
$B_s^0 \rightarrow K_2^{*-} e^+ \nu_e$	$7.90^{+0.03+0.90+2.45}_{-0.03-1.29-0.82}$	$7.30^{+4.8}_{-3.3}$	$6.50^{+0.020}_{-0.020}$	$9.60^{+2.2+1.9}_{-1.5-1.8}$	$13.30^{+2.7}_{-2.7}$
$B_s^0 \rightarrow K_2^{*-} \mu^+ \nu_\mu$	$7.87^{+0.03+0.90+2.43}_{-0.03-1.27-0.82}$	$7.30^{+4.8}_{-3.3}$	$6.50^{+0.020}_{-0.020}$	$9.50^{+2.2+1.9}_{-1.4-1.7}$	$13.30^{+2.7}_{-2.7}$
$B_s^0 \rightarrow K_2^{*-} \tau^+ \nu_\tau$	$1.77^{+0.01+0.23+0.46}_{-0.01-0.23-0.21}$	$2.50^{+1.7}_{-1.2}$	$3.50^{+0.011}_{-0.011}$	$1.70^{+0.4+0.4}_{-0.2-0.3}$	$3.60^{+0.7}_{-0.7}$

Semi-leptonic decays $B \rightarrow T \ell \nu_\ell$

- The branching ratios (10^{-3}) of the semi-leptonic decays $B_{(s)} \rightarrow D_{(s)2}^* \ell \nu_\ell$.

Reference	This work	RQM	RQM	BT	GI	ISGW	OPAL
$B^+ \rightarrow \bar{D}_2^{*0} e^+ \nu_e$	$7.08^{+0.02+1.17+0.03}_{-0.02-1.26-0.48}$	—	5.9	$4.5 \sim 8.0$	7.0	7.7	$8.8 \pm 3.5 \pm 1.7$
$B^+ \rightarrow \bar{D}_2^{*0} \mu^+ \nu_\mu$	$7.02^{+0.02+1.16+0.03}_{-0.02-1.25-0.47}$	—	—	$4.5 \sim 8.0$	7.0	7.7	$8.8 \pm 3.5 \pm 1.7$
$B^+ \rightarrow \bar{D}_2^{*0} \tau^+ \nu_\tau$	$0.42^{+0.00+0.05+0.06}_{-0.00-0.02-0.01}$	—	—	—	—	—	—
$B^0 \rightarrow D_2^{*-} e^+ \nu_e$	$6.56^{+0.02+1.08+0.02}_{-0.02-1.17-0.44}$	—	—	$4.5 \sim 8.0$	—	—	$22.5 \pm 6.5 \pm 5.1$
$B^0 \rightarrow D_2^{*-} \mu^+ \nu_\mu$	$6.50^{+0.02+1.07+0.03}_{-0.02-1.16-0.44}$	—	—	$4.5 \sim 8.0$	—	—	$22.5 \pm 6.5 \pm 5.1$
$B^0 \rightarrow D_2^{*-} \tau^+ \nu_\tau$	$0.34^{+0.00+0.06+0.01}_{-0.00-0.06-0.02}$	—	—	—	—	—	—
$B_s^0 \rightarrow D_{s2}^{*-} e^+ \nu_e$	$8.20^{+0.03+0.37+0.47}_{-0.03-0.46-0.28}$	6.7 ± 0.7	7.5	—	—	—	—
$B_s^0 \rightarrow D_{s2}^{*-} \mu^+ \nu_\mu$	$8.20^{+0.03+0.98+0.43}_{-0.06-0.42-0.31}$	—	—	—	—	—	—
$B_s^0 \rightarrow D_{s2}^{*-} \tau^+ \nu_\tau$	$0.42^{+0.00+0.05+0.06}_{-0.00-0.03-0.01}$	0.29 ± 0.03	—	—	—	—	—

The errors come from the lifetime of the $B_{(s)}$ meson, the decay constants of initial and final state mesons, respectively.

Semi-leptonic decays $B \rightarrow T\ell\nu_\ell$

- For the decays $B \rightarrow (a_2, f_2, K_2^*)\ell\nu_\ell$ induced by the $b \rightarrow u$ transition, their branching ratios lie in the range $10^{-5} \sim 10^{-4}$. While the branching ratios of the decays $B_{(s)} \rightarrow D_{(s)2}^*\ell\nu_\ell$ can reach $10^{-4} \sim 10^{-3}$.
- Due to phase space constraints, the branching ratios of the decays $B \rightarrow T\ell'\nu_{\ell'}$ with $\ell' = e, \mu$ are estimated to be $5 \sim 20$ of those the corresponding decays $B \rightarrow T\tau\nu_\tau$.
- The branching ratios of most considered decays are consistent with the results from other approaches, such as the PQCD, QCDSRs, LCSRs and RQM.
- All the branching ratios of these considered decays are larger than 10^{-5} , indicating promising prospects for measuring these processes in ongoing Belle II and LHCb experiments.

Semi-leptonic decays $B \rightarrow T\ell\nu_\ell$

- The forward-backward asymmetries $\mathcal{A}_{FB}$ for the decays $B \rightarrow T\ell\nu_\ell$.

Decay modes	$B^0 \rightarrow a_2^- e^+ \nu_e$	$B^0 \rightarrow a_2^- \mu^+ \nu_\mu$	$B^0 \rightarrow a_2^- \tau^+ \nu_\tau$
This work	$-0.204^{+0.007+0.016}_{-0.009-0.008}$	$-0.203^{+0.007+0.016}_{-0.001-0.008}$	$0.012^{+0.002+0.007}_{-0.001-0.006}$
PQCD	$-0.186^{+0.003}_{-0.002}$	$-0.186^{+0.003}_{-0.002}$	$0.031^{+0.005}_{-0.005}$
Decay modes	$B^+ \rightarrow f_2^0 e^+ \nu_e$	$B^+ \rightarrow f_2^0 \mu^+ \nu_\mu$	$B^+ \rightarrow f_2^0 \tau^+ \nu_\tau$
This work	$-0.180^{+0.031+0.009}_{-0.001-0.016}$	$-0.178^{+0.031+0.010}_{-0.001-0.016}$	$0.027^{+0.008+0.007}_{-0.007-0.009}$
PQCD	$-0.175^{+0.004}_{-0.004}$	$-0.175^{+0.004}_{-0.004}$	$0.048^{+0.007}_{-0.007}$
Decay modes	$B^0 \rightarrow D_2^{*-} e^+ \nu_e$	$B^0 \rightarrow D_2^{*-} \mu^+ \nu_\mu$	$B^0 \rightarrow D_2^{*-} \tau^+ \nu_\tau$
This work	$-0.146^{+0.020+0.025}_{-0.011-0.019}$	$-0.143^{+0.020+0.025}_{-0.011-0.019}$	$0.117^{+0.002+0.003}_{-0.003-0.000}$
Decay modes	$B_s^0 \rightarrow K_2^{*-} e^+ \nu_e$	$B_s^0 \rightarrow K_2^{*-} \mu^+ \nu_\mu$	$B_s^0 \rightarrow K_2^{*-} \tau^+ \nu_\tau$
This work	$-0.183^{+0.006+0.009}_{-0.006-0.007}$	$-0.182^{+0.006+0.009}_{-0.009-0.007}$	$0.045^{+0.006+0.004}_{-0.004-0.010}$
PQCD	$-0.221^{+0.005}_{-0.005}$	$-0.221^{+0.005}_{-0.005}$	$-0.024^{+0.005}_{-0.005}$
Decay modes	$B_s^0 \rightarrow D_{s2}^{*-} e^+ \nu_e$	$B_s^0 \rightarrow D_{s2}^{*-} \mu^+ \nu_\mu$	$B_s^0 \rightarrow D_{s2}^{*-} \tau^+ \nu_\tau$
This work	$-0.139^{+0.000+0.002}_{-0.002-0.001}$	$-0.135^{+0.000+0.002}_{-0.002-0.001}$	$0.126^{+0.001+0.001}_{-0.001-0.001}$
NRQM	-0.14	-0.12	0.06

Semi-leptonic decays $B \rightarrow T\ell\nu_\ell$

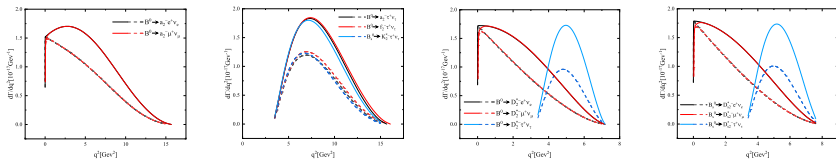
- The longitudinal polarization fractions f_L (%) for the decays $B \rightarrow T\ell\nu_\ell$.

Decay modes	$B^0 \rightarrow a_2^- e^+ \nu_e$	$B^0 \rightarrow a_2^- \mu^+ \nu_\mu$	$B^0 \rightarrow a_2^- \tau^+ \nu_\tau$
This work	$69.53^{+0.09+2.34}_{-2.60-2.41}$	$69.56^{+0.06+2.33}_{-2.56-2.37}$	$62.47^{+0.77+1.90}_{-2.32-6.99}$
PQCD	$73.3^{+0.4}_{-0.5}$	$73.3^{+0.4}_{-0.5}$	$69.9^{+0.6}_{-0.6}$
LCSR	$71.8^{+1.3}_{-1.0}$	$71.8^{+1.3}_{-1.0}$	$59.6^{+1.3}_{-1.0}$
Decay modes	$B^0 \rightarrow \bar{f}_2^- e^+ \nu_e$	$B^0 \rightarrow \bar{f}_2^- \mu^+ \nu_\mu$	$B^0 \rightarrow \bar{f}_2^- \tau^+ \nu_\tau$
This work	$72.58^{+2.59+1.60}_{-3.20-0.24}$	$72.58^{+2.57+1.61}_{-3.22-0.21}$	$64.74^{+2.12+1.36}_{-2.84-0.10}$
PQCD	$74.9^{+0.6}_{-0.7}$	$74.9^{+0.6}_{-0.7}$	$71.9^{+0.8}_{-0.8}$
LCSR	$68.9^{+1.8}_{-1.4}$	$68.8^{+1.8}_{-1.4}$	$56.7^{+1.7}_{-1.3}$
Decay modes	$B^0 \rightarrow D_2^{*-} e^+ \nu_e$	$B^0 \rightarrow D_2^{*-} \mu^+ \nu_\mu$	$B^0 \rightarrow D_2^{*-} \tau^+ \nu_\tau$
This work	$70.65^{+0.25+0.06}_{-0.31-0.16}$	$70.60^{+0.28+0.05}_{-0.29-0.19}$	$55.98^{+0.24+0.21}_{-0.24-0.08}$
Decay modes	$B_s^0 \rightarrow K_2^{*-} e^+ \nu_e$	$B_s^0 \rightarrow K_2^{*-} \mu^+ \nu_\mu$	$B_s^0 \rightarrow K_2^{*-} \tau^+ \nu_\tau$
This work	$71.86^{+1.69+0.11}_{-2.51-1.02}$	$71.89^{+0.13+1.87}_{-1.92-0.58}$	$64.54^{+0.96+1.20}_{-0.99-0.00}$
PQCD	$68.3^{+0.5}_{-0.5}$	$68.3^{+0.5}_{-0.5}$	$64.1^{+0.4}_{-0.3}$
LCSR	$77.6^{+0.0}_{-0.1}$	$77.5^{+0.1}_{-0.1}$	$65.8^{+0.1}_{-0.0}$
Decay modes	$B_s^0 \rightarrow D_{s2}^{*-} e^+ \nu_e$	$B_s^0 \rightarrow D_{s2}^{*-} \mu^+ \nu_\mu$	$B_s^0 \rightarrow D_{s2}^{*-} \tau^+ \nu_\tau$
This work	$71.52^{+5.51+0.36}_{-0.23-0.08}$	$71.29^{+0.01+0.56}_{-0.43-0.12}$	$56.31^{+0.19+4.06}_{-0.06-0.17}$

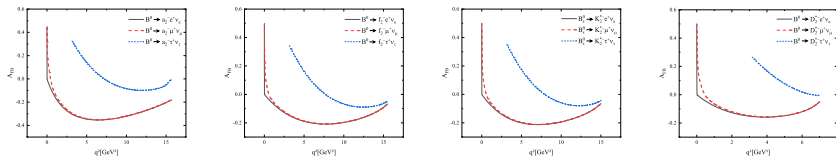
Semi-leptonic decays $B \rightarrow T \ell \nu_\ell$

- The q^2 dependence of the differential decay widths $d\Gamma/dq^2$ (solid lines) and $d\Gamma_L/dq^2$ (dashed lines) for the decays

$$B \rightarrow a_2 \ell' \nu_{\ell'}, B_{(s)} \rightarrow (a_2, f_2, K_2^*) \tau \nu_\tau, B \rightarrow D_2^* \ell \nu_\ell \text{ and } B_s \rightarrow D_{s2}^* \ell \nu_\ell.$$



- The q^2 dependence of the $\mathcal{A}_{FB}$ for the decays $B \rightarrow a_2 \ell \nu_\ell$, $B \rightarrow f_2 \ell \nu_\ell$, $B_s \rightarrow K_2^* \ell \nu_\ell$, $B \rightarrow D_2^* \ell \nu_\ell$.



Summary

- All the branching ratios of these decays are larger than 10^{-5} , in which the maximum value can reach up to 10^{-3} , which are likely to be detected by the present LHCb and Belle II experiments.
- The decays $B \rightarrow T\ell'\nu_{\ell'}$ are dominated by the longitudinal polarization, where the polarization fractions are close to 70%. Those of the decays $B \rightarrow T\tau\nu_\tau$ are a little smaller, where the least values coming from the decays $B_{(s)} \rightarrow D_{(s)2}^*\tau\nu_\tau$ are about 56%.
- There exists obvious disparity about the $\mathcal{A}_{FB}$ for the decays $B \rightarrow T\tau\nu_\tau$ among different theoretical predictions, which remains to be clarified with further investigation.

Thank you for your attention!