Color gauge invariant theory of diquark interactions

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Based on: Phys. Rev. D 113, 016019

第九届强子谱与强子结构研讨会@河南师范大学

2026年8月20日

LANZHOU UNIVERSITY

提纲

CONTENTS

Background

Part one

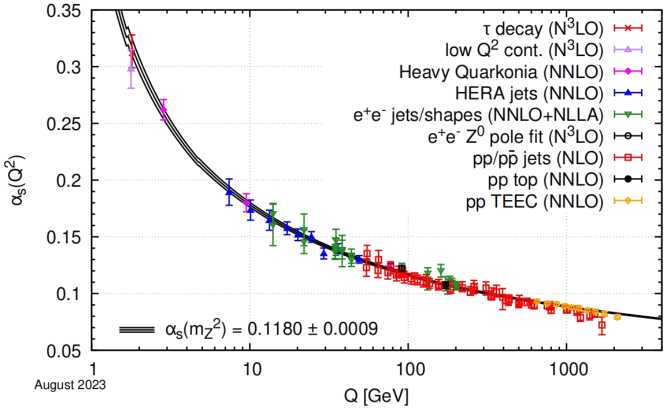
Background

➢ Four fundamental interactions：

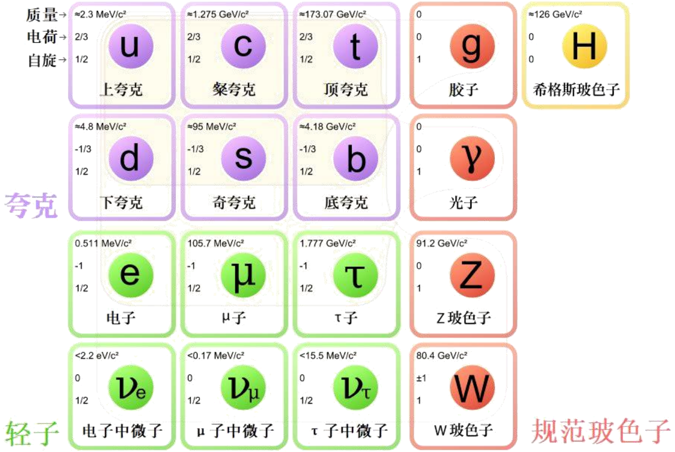
Standard Model

Interaction	Strength	Range	Mediator	Participating particles	Bound states
Strong interaction	1	10^{-15} m	gluon	quarks, gluons	hadrons
Electromagnetic interaction	$1/137$	$F \propto 1/r^2$	photon	electrically-charged particles	atoms
Weak interaction	10^{-5}	$< 10^{-17}$ m	$W^{\pm}, Z^0$	fermions	none
Gravitational interaction	10^{-39}	$F \propto 1/r^2$	graviton?	all particles	solar system, etc.

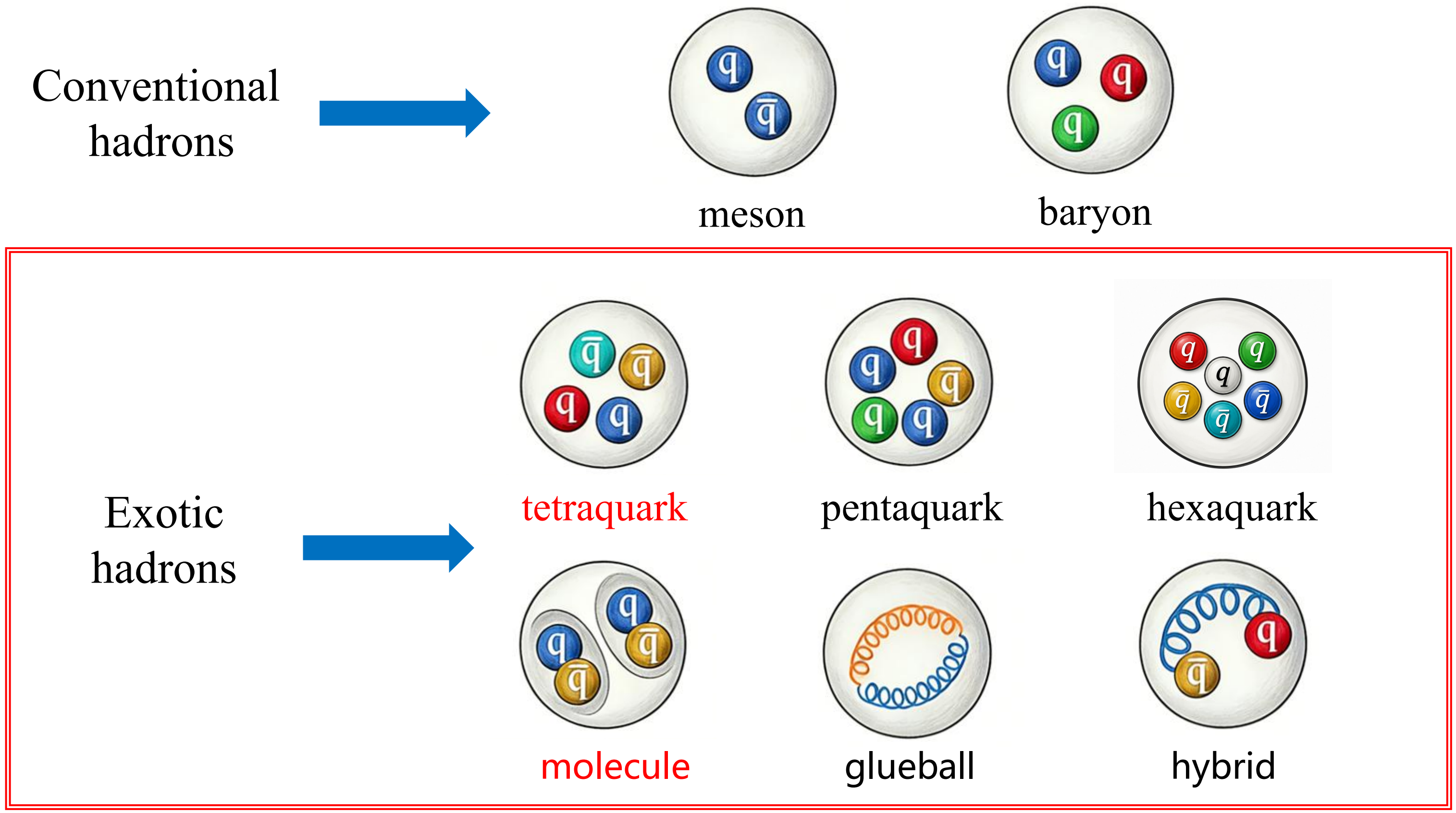
➢ The coupling constant of QCD：



PDG, Phys. Rev. D 110, 030001



Background


Conventional hadrons
meson
baryon
Exotic hadrons
tetraquark
pentaquark
hexaquark
molecule
glueball
hybrid

Background

- Diquark:

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

that it would never have been detected. A search for stable quarks of charge $-\frac{1}{3}$ or $+\frac{2}{3}$ and/or stable di-quarks of charge $-\frac{2}{3}$ or $+\frac{1}{3}$ or $+\frac{4}{3}$ at the highest energy accelerators would help to reassure us of the non-existence of real quarks.

Phys. Lett. 8 (1964) 214-215

Color: $$3 \otimes 3 = \bar{3} \oplus 6$$

GI model: $$H_{ij}^{\mathrm{conf}} = -\left[\frac{3}{4}c + \frac{3}{4}br - \frac{\alpha_s(r)}{r}\right]\mathbf{F}_i \cdot \mathbf{F}_j$$

Factor: $$\langle (qq)_{\bar{3}} | \mathbf{F}_i \cdot \mathbf{F}_j | (qq)_{\bar{3}} \rangle = -\frac{2}{3}$$ attractive

$$\langle (qq)_{6} | \mathbf{F}_i \cdot \mathbf{F}_j | (qq)_{6} \rangle = +\frac{1}{3}$$ repulsive

- Study for diquark-quark structure:

D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B 659, 612-620 (2008);
D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 84, 014025 (2011);
X. Luo, H. X. Chen et al., AAPPS Bull. 36, 9 (2026).
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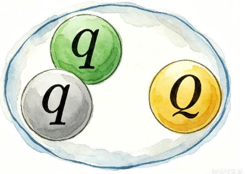
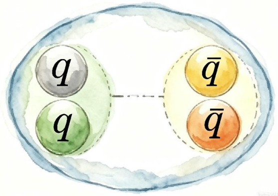


- Study for diquark-antidiquark structure:

V. R. Debastiani and F. S. Navarra, Chin. Phys. C 43, 013105 (2019);
F. S. Navarra, M. Nielsen and S. H. Lee, Phys. Lett. B 649, 166-172 (2007);
L. Maiani et al., Phys. Rev. D 71, 014028 (2005).
……………………



Background

- Our work for the structure of 4-quark particles

The mixture of hadronic molecule and diquark-antidiquark state:

$$Z_b(10610) \Rightarrow B\bar{B}^*/B^*\bar{B}/B^*\bar{B}^*/S\bar{A}/A\bar{S}/A\bar{A}$$

$$Z_b(10650) \Rightarrow B^*\bar{B}^*/A\bar{A}$$

$$Z_{cs}(4000)^+ \Rightarrow D_s^+\bar{D}^{*0}/S_{cu}\bar{A}_{cs}$$

$$X(4500) \Rightarrow D_s^{*+}D_s^{*-}/A_{cq}\bar{A}_{cq}/A_{cs}\bar{A}_{cs}$$

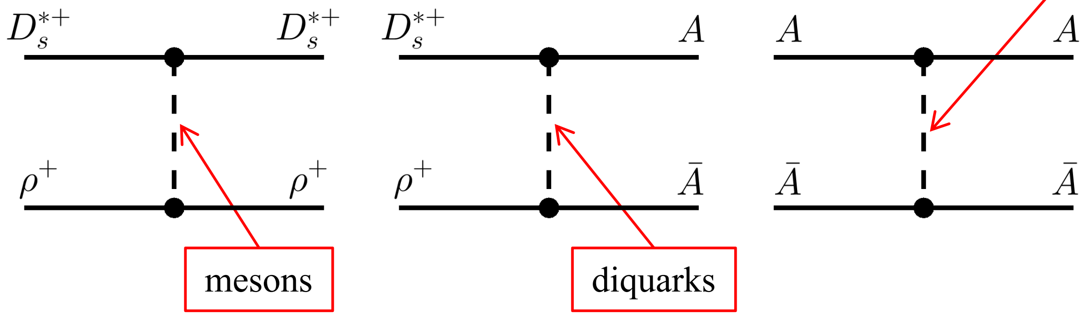
$$Z_c(3900) \Rightarrow D\bar{D}^*/D^*\bar{D}/S_{cq}\bar{A}_{cq}/A_{cq}\bar{S}_{cq}$$

$$T^*_{c\bar{s}0}(2900)^{++} \Rightarrow D_s^{*+}\rho^+/D^{*+}K^{*+}/A_{cu}\bar{A}_{sd}$$

Z. H. Cao, W. He and Z. F. Sun, Phys. Rev. D 107, 014017
W. He, D. S. Zhang and Z. F. Sun, Phys. Rev. D 110, 054006
D. S. Zhang, W. He, C. W. Xiao and Z. F. Sun, arxiv: 2410. 05749
Y. Ma, D. S. Zhang, C. Q. Pang and Z. F. Sun, arxiv: 2412. 11144
D. S. Zhang, J. F. Wang and Z. F. Sun, submited to journal

Only considering mesons, not gluon

$T^*_{c\bar{s}0}(2900)^{++}$ ⇒



Lagrangians construction

Part two

Lagrangians construction

- The Lagrangian of free diquark fields:

$$\mathcal{L}_S = \frac{1}{4}\langle\partial_\mu S\partial^\mu S^\dagger\rangle_{CF} - \frac{1}{4}m_S^2\langle SS^\dagger\rangle_{CF}, \qquad \mathcal{L}_{S_Q} = \frac{1}{2}\langle\partial_\mu S_Q\partial^\mu S_Q^\dagger\rangle_C - \frac{1}{2}m_{S_Q}^2\langle S_Q S_Q^\dagger\rangle_C,$$

$$\mathcal{L}_A = -\frac{1}{4}\langle F_{\mu\nu}F^{\dagger\mu\nu}\rangle_{CF} + \frac{1}{2}m_A^2\langle A_\mu A^{\dagger\mu}\rangle_{CF}, \qquad \mathcal{L}_{A_Q} = -\frac{1}{4}\langle F_{Q\mu\nu}F_Q^{\dagger\mu\nu}\rangle_C + \frac{1}{2}m_{A_Q}^2\langle A_{Q\mu}A_Q^{\dagger\mu}\rangle_C.$$

Where $F_{(Q)\mu\nu} = \partial_\mu A_{(Q)\nu} - \partial_\nu A_{(Q)\mu}$ is strength tensor of axis vector diquark, the flavour and color matrix of diquark can be written as:

$$S_F = \begin{pmatrix} 0 & S_{ud} & S_{us} \\ -S_{ud} & 0 & S_{ds} \\ -S_{us} & -S_{ds} & 0 \end{pmatrix}, \quad A_F = \begin{pmatrix} A_{uu} & \frac{1}{\sqrt{2}}A_{ud} & \frac{1}{\sqrt{2}}A_{us} \\ \frac{1}{\sqrt{2}}A_{ud} & A_{dd} & \frac{1}{\sqrt{2}}A_{ds} \\ \frac{1}{\sqrt{2}}A_{us} & \frac{1}{\sqrt{2}}A_{ds} & A_{ss} \end{pmatrix}, \quad X_C = \begin{pmatrix} 0 & X_{rg} & X_{rb} \\ -X_{rg} & 0 & X_{gb} \\ -X_{rb} & -X_{gb} & 0 \end{pmatrix}$$

$$S_{QF} = (S_{Qu}, S_{Qd}, S_{Qs}), \qquad A_{QF} = (A_{Qu}, A_{Qd}, A_{Qs}).$$

Only conside the color antitriplet diquark

X stand for diquark field. Under $SU(3)_c$ symmetry, the diquark field transform by:

$$\boxed{\begin{aligned} S_C &\to U S_C U^T, \\ A_{C\mu} &\to U A_{C\mu} U^T, \\ S_{QC} &\to U S_{QC} U^T, \\ A_{QC\mu} &\to U A_{QC\mu} U^T, \end{aligned}}$$

Lagrangians construction

- The color gauge invariant Lagrangians:

① Introduce a gauge field: gluon field

$$G_\mu \equiv G^a_\mu t^a, \quad \boxed{G_\mu \to U(G^a_\mu t^a + \tfrac{i}{g}\partial_\mu)U^\dagger}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu, \quad \boxed{G^a_{\mu\nu} \to U G^a_{\mu\nu} U^\dagger}$$

② Replace partial derivative with covariant derivative:

$$\begin{aligned} D_\mu S &= \partial_\mu S - iV_\mu S - iS V^T_\mu - igG_\mu S - igS G^T_\mu, \\ D_\mu A_\nu &= \partial_\mu A_\nu - iV_\mu A_\nu - iA_\nu V^T_\mu - igG_\mu A_\nu - igA_\nu G^T_\mu \\ D_\mu S_Q &= \partial_\mu S_Q - iS_Q \alpha^T_{\parallel\mu} - igG_\mu S_Q - igS_Q G^T_\mu, \\ D_\mu A_{Q\nu} &= \partial_\mu A_{Q\nu} - iA_{Q\nu}\alpha^T_{\parallel\mu} - igG_\mu A_{Q\nu} - igA_{Q\nu}G^T_\mu. \end{aligned}$$

$$\boxed{\begin{aligned} D_\mu S &\to U D_\mu S U^T, \\ D_\mu A_\nu &\to U D_\mu A_\nu U^T, \\ D_\mu S_Q &\to U D_\mu S_Q U^T, \\ D_\mu A_{Q\nu} &\to U D_\mu A_{Q\nu} U^T. \end{aligned}}$$

The SU(3)$_c$ gauge invariant Lagrangians of diquarks are:

$$\boxed{\begin{aligned} \mathcal{L}'_S &= \frac{1}{4}\langle D_\mu S D^\mu S^\dagger\rangle_{CF} - \frac{1}{4}m_S^2\langle S S^\dagger\rangle_{CF}, \\ \mathcal{L}'_A &= -\frac{1}{4}\langle \widetilde{F}_{\mu\nu}\widetilde{F}^{\dagger\mu\nu}\rangle_{CF} + \frac{1}{2}m_A^2\langle A_\mu A^{\dagger\mu}\rangle_{CF}, \\ \mathcal{L}'_{S_Q} &= \frac{1}{2}\langle D_\mu S_Q D^\mu S_Q^\dagger\rangle_C - \frac{1}{2}m_{S_Q}^2\langle S_Q S_Q^\dagger\rangle_C, \\ \mathcal{L}'_{A_Q} &= -\frac{1}{4}\langle \widetilde{F}_{Q\mu\nu}\widetilde{F}_Q^{\dagger\mu\nu}\rangle_C + \frac{1}{2}m_{A_Q}^2\langle A_{Q\mu}A_Q^{\dagger\mu}\rangle_C. \end{aligned}}$$

$$\begin{aligned} \widetilde{F}_{\mu\nu} &\equiv D_\mu A_\nu - D_\nu A_\mu, \\ \widetilde{F}_{Q\mu\nu} &\equiv D_\mu A_{Q\nu} - D_\nu A_{Q\mu}, \end{aligned}$$

$$\begin{aligned} \widetilde{F}_{\mu\nu} &\to U\widetilde{F}_{\mu\nu}U^T, \\ \widetilde{F}_{Q\mu\nu} &\to U\widetilde{F}_{Q\mu\nu}U^T, \end{aligned}$$

> Lagrangians construction

- Other possible Lagrangians

Consider the symmetry of diquark and gluon, also exist other Lagrangians:

$$\boxed{\begin{aligned}\mathcal{L}_{AAG} &= id_2\langle A^{\mu}G^{T}_{\mu\nu}A^{\dagger\nu}\rangle_{CF}\\ \mathcal{L}_{S_Q A_Q G} &= d_{1Q}\epsilon^{\mu\nu\alpha\beta}\langle D_\mu S_Q G^{T}_{\nu\alpha}A^{\dagger}_{Q\beta} + A_{Q\beta}G^{T}_{\nu\alpha}D_\mu S^{\dagger}_{Q}\rangle_C\\ \mathcal{L}_{A_Q A_Q G} &= id_{2Q}\langle A^{\mu}_{Q}G^{T}_{\mu\nu}A^{\dagger\nu}_{Q}\rangle_C\end{aligned}}$$

Since $\mathcal{L}_{S_{(Q)}S_{(Q)}G}$ is a higher-order contribution, we do not consider it.

Due to the symmetry of diquark, $\mathcal{L}_{SAG} = 0$.

Symmetry of Lagrangian:

SU(3)$_c$

Parity

Charge conjugation

Hermitian

- Diquark-meson interactions

$qq-qq-q\bar{q} \longrightarrow$
$$\mathcal{L}_1 = ik_1\langle S^a\hat{\alpha}^{\mu T}_{\parallel}D_\mu S^{a\dagger}\rangle - ik_1\langle D_\mu S^a\hat{\alpha}^{\mu T}_{\parallel}S^{a\dagger}\rangle + \cdots$$

$qq-Qq-Q\bar{q} \longrightarrow$
$$\mathcal{L}_2 = e_1(iPD_\mu S^a A^{a\mu\dagger}_{c} - iA^{a\mu}_{c}D_\mu S^{a\dagger}P^{\dagger}) + \cdots$$

$Qq-Qq-q\bar{q} \longrightarrow$
$$\mathcal{L}_3 = h_1(iS^a_c\hat{\alpha}^{\mu T}_{\parallel}D_\mu S^{a\dagger}_{c} - iD_\mu S^a_c\hat{\alpha}^{\mu T}_{\parallel}S^{a\dagger}_{c}) + \cdots$$

Z. H. Cao, W. He and Z. F. Sun, Phys. Rev. D 107, 014017
W. He, D. S. Zhang and Z. F. Sun, Phys. Rev. D 110, 054006

One-gluon-exchange potentials

Part three

One-gluon-exchange potentials

- The one-gluon-exchange potentials of diquark-antidiquark systems

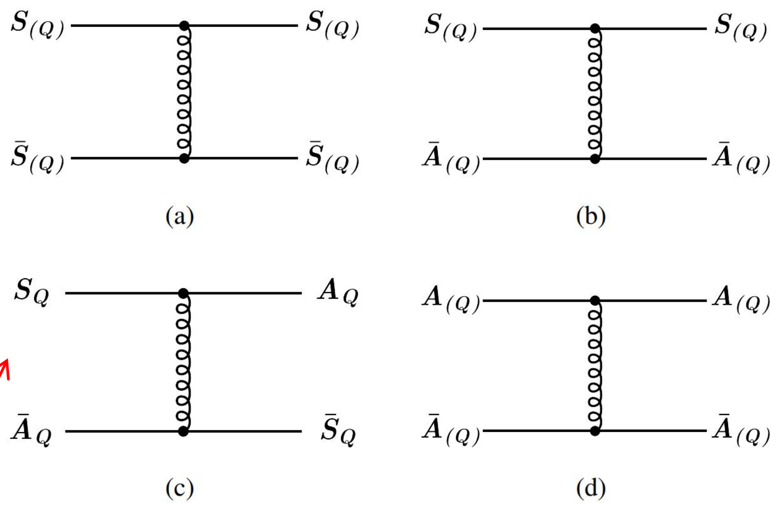
① The gauge invariant Lagrangians of diquarks:

$$
\begin{aligned}
\mathcal{L}'_S &= \frac{1}{4}\langle D_\mu S D^\mu S^\dagger\rangle_{CF} - \frac{1}{4}m_S^2\langle S S^\dagger\rangle_{CF},\\
\mathcal{L}'_A &= -\frac{1}{4}\langle \widetilde{F}_{\mu\nu}\widetilde{F}^{\dagger\mu\nu}\rangle_{CF} + \frac{1}{2}m_A^2\langle A_\mu A^{\dagger\mu}\rangle_{CF},\\
\mathcal{L}'_{S_Q} &= \frac{1}{2}\langle D_\mu S_Q D^\mu S_Q^\dagger\rangle_C - \frac{1}{2}m_{S_Q}^2\langle S_Q S_Q^\dagger\rangle_C,\\
\mathcal{L}'_{A_Q} &= -\frac{1}{4}\langle \widetilde{F}_{Q\mu\nu}\widetilde{F}_Q^{\dagger\mu\nu}\rangle_C + \frac{1}{2}m_{A_Q}^2\langle A_{Q\mu}A_Q^{\dagger\mu}\rangle_C.\\
\mathcal{L}_{AAG} &= id_2\langle A^\mu G^T_{\mu\nu}A^{\dagger\nu}\rangle_{CF},\\
\mathcal{L}_{S_QA_QG} &= d_{1Q}\epsilon^{\mu\nu\alpha\beta}\langle D_\mu S_Q G^T_{\nu\alpha}A^\dagger_{Q\beta} + A_{Q\beta}G^T_{\nu\alpha}D_\mu S_Q^\dagger\rangle_C\\
\mathcal{L}_{A_QA_QG} &= id_{2Q}\langle A_Q^\mu G^T_{\mu\nu}A_Q^{\dagger\nu}\rangle_C.
\end{aligned}
$$

② Breit approximate:

$$\mathcal{V}(q) = \frac{\mathcal{M}(q)}{\sqrt{\prod_i 2m_i \prod_f 2m_f}}$$

S-wave
zero momentum



(a) (b) (c) (d)

③ The one-gluon-exchange potentials in the coordinate space

$$V(r) = \int \frac{d^3q}{(2\pi)^3}e^{i\mathbf{q}\cdot\mathbf{r}}V(q)$$

Fig a: $$V_a^{(Q)}(r) = -\frac{g^2}{3\pi}\frac{1}{r} - \frac{g^2}{3m^2_{S_{(Q)}}}\delta^3(r),$$

One-gluon-exchange potentials

➢ The one-gluon-exchange potentials of diquark-antidiquark systems

$$V_b^{(Q)}(r) = -\frac{g^2}{3\pi}\frac{1}{r}\epsilon_2\cdot\epsilon_4^\dagger - \frac{2g^2}{9m_{S_{(Q)}}m_{A_{(Q)}}}\epsilon_2\cdot\epsilon_4^\dagger\delta^3(r) - \frac{g^2}{12\pi m_{S_{(Q)}}m_{A_{(Q)}}}\frac{1}{r^3}S(\hat{r},\epsilon_2,\epsilon_4^\dagger),$$

$$V_c^{Q}(r) = -\frac{8}{9}d_{1Q}^2\epsilon_2\cdot\epsilon_3^\dagger\delta^3(r) - \frac{d_{1Q}^2}{3\pi}\frac{1}{r^3}S(\hat{r},\epsilon_2,\epsilon_3^\dagger),$$

$$\begin{aligned}V_d^{(Q)}(r) = &-\frac{g^2}{3\pi}\frac{\epsilon_1\cdot\epsilon_3^\dagger\epsilon_2\cdot\epsilon_4^\dagger}{r} + \frac{1}{9m_{A_{(Q)}}^2}\Big[-g^2\epsilon_1\cdot\epsilon_3^\dagger\epsilon_2\cdot\epsilon_4^\dagger - (2d_{2(Q)}g + 2d_{2(Q)}^2 + g^2)\epsilon_1\cdot\epsilon_2\epsilon_3^\dagger\cdot\epsilon_4^\dagger \\ &+ 2(d_{2(Q)}^2 + d_{2(Q)}g)\epsilon_1\cdot\epsilon_4^\dagger\epsilon_2\cdot\epsilon_3^\dagger\Big]\delta^3(r) + \frac{1}{12\pi m_{A_{(Q)}}^2}\frac{1}{r^3}\Big[-g^2\epsilon_2\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_1,\epsilon_3^\dagger) \\ &- g^2\epsilon_1\cdot\epsilon_3^\dagger S(\hat{r},\epsilon_2,\epsilon_4^\dagger) + d_{2(Q)}^2\epsilon_1\cdot\epsilon_2 S(\hat{r},\epsilon_3^\dagger,\epsilon_4^\dagger) + (d_{2(Q)}+g)^2\epsilon_3^\dagger\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_1,\epsilon_2) \\ &- (d_{2(Q)}^2 + d_{2(Q)}g)\epsilon_1\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_2,\epsilon_3^\dagger) - (d_{2(Q)}^2 + d_{2(Q)}g)\epsilon_2\cdot\epsilon_3^\dagger S(\hat{r},\epsilon_1,\epsilon_4^\dagger)\Big].\end{aligned}$$

Without Coulomb term

➢ The one-gluon-exchange potential in GI model:

$$V_{ij}(p,r) = \frac{\alpha_s(r)}{r}\boldsymbol{F}_i\cdot\boldsymbol{F}_j - \frac{\alpha_s(r)}{m_i m_j}\left[\frac{8\pi}{3}\boldsymbol{S}_i\cdot\boldsymbol{S}_j\delta^3(r) + \frac{1}{r^3}\left(\frac{3\boldsymbol{S}_i\cdot\boldsymbol{r}\boldsymbol{S}_j\cdot\boldsymbol{r}}{r^2} - \boldsymbol{S}_i\cdot\boldsymbol{S}_j\right)\right]\boldsymbol{F}_i\cdot\boldsymbol{F}_j.$$

$$g = \sqrt{4\pi\alpha_s(r)} \approx 2.7 \quad d_{1Q}^2 \approx \frac{g^2}{16}\frac{1}{m_q^2} \approx 3.1\ \mathrm{GeV}^{-1} \quad d_2^2 \approx g^2\frac{m_{A_{qq}}^2}{m_q^2} \approx 10.5 \quad d_{2Q}^2 \approx \frac{g^2}{4}\frac{m_{A_{Qq}}^2}{m_q^2} \approx 13.3\ (34.1)$$

The form factor of diquark-diquark-gluon vertices

Part four

The form factor of diquark-diquark-gluon vertices

- Other study of diquark-diquark-gluon vertices:

In the study of baryon-meson scattering, consider the diquark-quark structure of baryon.

The effective current of diquark coupling to a gluon are:

$$S\,g\,S \;\rightarrow\; -iG_s\frac{\lambda^\alpha}{2}(q_1+q_2)^\mu,$$

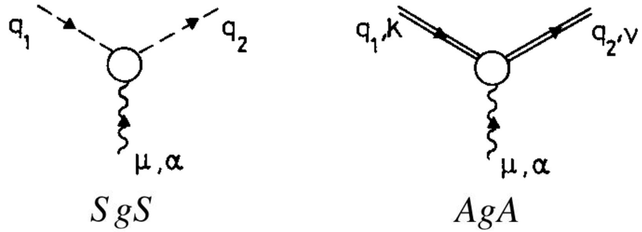
$$A\,g\,A \;\rightarrow\; -i\frac{\lambda^\alpha}{2}[G_1(q_1+q_2)^\mu g^{\kappa\nu} - G_2(q_2^\kappa g^{\mu\nu} + q_1^\nu g^{\mu\kappa}) + G_3(q_1+q_2)^\mu q_1^\nu q_2^\kappa].$$

The coupling constant $G_S = g_s F_S(Q^2)$, for axis-vector diquark: $G_3 = 0, G_1 = G_2 = g_s F_V(Q^2)$.

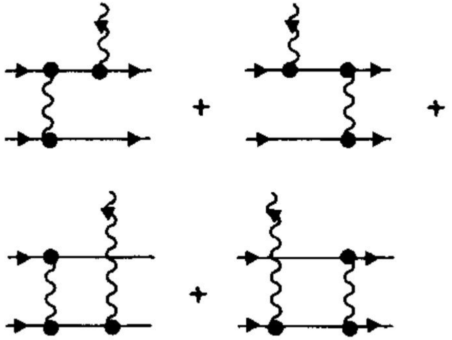
The form factor of diquarks are:

$$F_S(Q^2) = \frac{\alpha_s(Q^2)Q_0^2}{Q_0^2+Q^2}, \quad F_V(Q^2) = \frac{\alpha_s(Q^2)Q_1^2}{Q_1^2+Q^2}.$$

M. Anselmino, P. Kroll and B. Pire
Z. Phys. C 36, 89 (1987)



Vertices of diquark-diquark-gluon



Diquark form factor

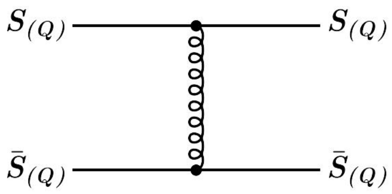
> The form factor of diquark-diquark-gluon vertices

- The one-gluon-exchange potentials after considering form factor:

Form factor:

$$\boxed{F(q^2) = \frac{\alpha_s(q^2)Q_0^2}{Q_0^2 - q^2}.}$$

Fourier transform: $V(r) = \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot r} V(q) F^2(q^2)$,



(a)

The effective potentials in coordinate space:

$$\boxed{\tilde{V}_a^{(Q)}(r) = -\frac{4}{3}g^2\alpha_s^2 Y(Q_0,r) + \frac{g^2\alpha_s^2}{3m_{S_{(Q)}}^2}Z(Q_0,r),}$$

$$\boxed{V_a^{(Q)}(r) = -\frac{g^2}{3\pi}\frac{1}{r} - \frac{g^2}{3m_{S_{(Q)}}^2}\delta^3(r),}$$

$$\tilde{V}_b^{(Q)}(r) = -\frac{4}{3}g^2\alpha_s^2\epsilon_2\cdot\epsilon_4^\dagger Y(Q_0,r) + \frac{2}{9m_{S_{(Q)}}m_{A_{(Q)}}}g^2\alpha_s^2\epsilon_2\cdot\epsilon_4^\dagger Z(Q_0,r) - \frac{2}{9m_{S_{(Q)}}m_{A_{(Q)}}}g^2\alpha_s^2 S(\hat{r},\epsilon_2,\epsilon_4^\dagger)T(Q_0,r),$$

$$\tilde{V}_c^{Q}(r) = \frac{8}{9}d_{1Q}^2\alpha_s^2\epsilon_2\cdot\epsilon_3^\dagger Z(Q_0,r) - \frac{4}{9}d_{1Q}^2\alpha_s^2 S(\hat{r},\epsilon_2,\epsilon_3^\dagger)T(Q_0,r),$$

$$\begin{aligned}\tilde{V}_d^{(Q)}(r) = &-\frac{4}{3}g^2\alpha_s^2\epsilon_1\cdot\epsilon_3^\dagger\epsilon_2\cdot\epsilon_4^\dagger Y(Q_0,r) + \frac{\alpha_s^2}{9m_{A_{(Q)}}^2}\Big[g^2\epsilon_1\cdot\epsilon_3^\dagger\epsilon_2\cdot\epsilon_4^\dagger + (2d_{2(Q)}g + 2d_{2(Q)}^2 + g^2)\epsilon_1\cdot\epsilon_2\epsilon_3^\dagger\cdot\epsilon_4^\dagger - 2(d_{2(Q)}^2\\ &+ d_{2(Q)}g)\epsilon_1\cdot\epsilon_4^\dagger\epsilon_2\cdot\epsilon_3^\dagger\Big]Z(Q_0,r) - \frac{1}{9m_{A_{(Q)}}^2}\alpha_s^2\Big[g^2\epsilon_2\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_1,\epsilon_3^\dagger) + g^2\epsilon_1\cdot\epsilon_3^\dagger S(\hat{r},\epsilon_2,\epsilon_4^\dagger) - d_{2(Q)}^2\epsilon_1\cdot\epsilon_2 S(\hat{r},\epsilon_3^\dagger,\epsilon_4^\dagger)\\ &-(d_{2(Q)}+g)^2\epsilon_3^\dagger\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_1,\epsilon_2) + (d_{2(Q)}^2 + d_{2(Q)}g)\epsilon_1\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_2,\epsilon_3^\dagger) + (d_{2(Q)}^2 + d_{2(Q)}g)\epsilon_2\cdot\epsilon_3^\dagger S(\hat{r},\epsilon_1,\epsilon_4^\dagger)\Big]T(Q_0,r).\end{aligned}$$

Summary and outlook

- Summary

  ① Constructed the color gauge invariant diquark fields and diquark-gluon interaction Lagrangians.

  ② Calculated the one-gluon-exchange potential of diquark-antidiquark systems, obtained the numercial results for the coupling constants.

  ③ We modified the one-gluon-exchange potentials by introducing a form factor.

- Outlook

  ① In the diquark-antidiquark system, considering the confinement potential as well as spin–orbit and color-magnetic terms.

  ② Verify the correctness of the model in diquark-quark systems.

Thanks for your attention!

Appendix

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- The one-gluon-exchange potentials after considering form factor:

Introduce the following form factor:

$$\boxed{F(q^2) = \frac{\alpha_s(q^2)Q_0^2}{Q_0^2 - q^2}.}$$

Through Fourier transform: $V(r) = \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot r} V(q) F^2(q^2)$,

we get the effective potentials in coordinate space:

$$Y(Q_0,r) = \frac{1}{4\pi r}(1-e^{-Q_0 r}) - \frac{Q_0}{8\pi}e^{-Q_0 r},$$

$$Z(Q_0,r) = \nabla^2 Y(Q_0,r) = r^2\frac{\partial}{\partial r}\frac{1}{r^2}\frac{\partial}{\partial r}Y(Q_0,r),$$

$$T(Q_0,r) = r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}Y(Q_0,r).$$

$$\boxed{\widetilde{V}_a^{(Q)}(r) = -\frac{4}{3}g^2\alpha_s^2 Y(Q_0,r) + \frac{g^2\alpha_s^2}{3m_{S_{(Q)}}^2}Z(Q_0,r),}$$

$$\boxed{V_a^{(Q)}(r) = -\frac{g^2}{3\pi}\frac{1}{r} - \frac{g^2}{3m_{S_{(Q)}}^2}\delta^3(r),}$$

$$\widetilde{V}_b^{(Q)}(r) = -\frac{4}{3}g^2\alpha_s^2\epsilon_2\cdot\epsilon_4^\dagger Y(Q_0,r) + \frac{2}{9m_{S_{(Q)}}m_{A_{(Q)}}}g^2\alpha_s^2\epsilon_2\cdot\epsilon_4^\dagger Z(Q_0,r) - \frac{2}{9m_{S_{(Q)}}m_{A_{(Q)}}}g^2\alpha_s^2 S(\hat{r},\epsilon_2,\epsilon_4^\dagger)T(Q_0,r),$$

$$\widetilde{V}_c^{Q}(r) = \frac{8}{9}d_{1Q}^2\alpha_s^2\epsilon_2\cdot\epsilon_3^\dagger Z(Q_0,r) - \frac{4}{9}d_{1Q}^2\alpha_s^2 S(\hat{r},\epsilon_2,\epsilon_3^\dagger)T(Q_0,r),$$

$$\begin{aligned}\widetilde{V}_d^{(Q)}(r) = &-\frac{4}{3}g^2\alpha_s^2\epsilon_1\cdot\epsilon_3^\dagger\epsilon_2\cdot\epsilon_4^\dagger Y(Q_0,r) + \frac{\alpha_s^2}{9m_{A_{(Q)}}^2}\Big[g^2\epsilon_1\cdot\epsilon_3^\dagger\epsilon_2\cdot\epsilon_4^\dagger + (2d_{2(Q)}g + 2d_{2(Q)}^2 + g^2)\epsilon_1\cdot\epsilon_2\epsilon_3^\dagger\cdot\epsilon_4^\dagger - 2(d_{2(Q)}^2\\ &+d_{2(Q)}g)\epsilon_1\cdot\epsilon_4^\dagger\epsilon_2\cdot\epsilon_3^\dagger\Big]Z(Q_0,r) - \frac{1}{9m_{A_{(Q)}}^2}\alpha_s^2\Big[g^2\epsilon_2\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_1,\epsilon_3^\dagger) + g^2\epsilon_1\cdot\epsilon_3^\dagger S(\hat{r},\epsilon_2,\epsilon_4^\dagger) - d_{2(Q)}^2\epsilon_1\cdot\epsilon_2 S(\hat{r},\epsilon_3^\dagger,\epsilon_4^\dagger)\\ &-(d_{2(Q)}+g)^2\epsilon_3^\dagger\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_1,\epsilon_2) + (d_{2(Q)}^2 + d_{2(Q)}g)\epsilon_1\cdot\epsilon_4^\dagger S(\hat{r},\epsilon_2,\epsilon_3^\dagger) + (d_{2(Q)}^2 + d_{2(Q)}g)\epsilon_2\cdot\epsilon_3^\dagger S(\hat{r},\epsilon_1,\epsilon_4^\dagger)\Big]T(Q_0,r).\end{aligned}$$

> Appendix

State	Λ(GeV)	E(MeV)	r_{RMS}(fm)	P_M(%)	P_D(%)
$Z_b(10610)$	1.0	-1.83	1.64	94.24/0.09/2.85/0.02	2.18/0.04/0.57/0.01
	1.1	-9.96	0.82	84.24/0.06/8.30/0.01	5.15/0.10/2.14/0.01
	1.2	-26.14	0.57	73.78/0.15/13.55/0.00	7.83/0.16/4.53/0.01
$Z_b(10650)$	1.0	-2.60	1.45	96.06/0.31	0.03/0.01/3.57/0.02
	1.1	-9.01	0.88	92.34/0.28	0.09/0.02/7.23/0.04
	1.2	-19.82	0.66	88.30/0.26	0.18//0.03/11.17/0.06

q_{max}		1360	1385	1410
$\bar{D}^{*0}D_s^{+}/\bar{A}_{cs}S_{cu}$	RS-II	$4018 \pm i46$	$4013 \pm i42$	$4009 \pm i39$
	RS-II	$4209 \pm i13$	$4208 \pm i13$	$4207 \pm i12$
$\bar{D}^{*0}D_s^{*+}/\bar{A}_{cs}A_{cu}$	RS-II	4105	4106	4107
	RS-I	4088	4091	4093
$\bar{D}^{0}D_s^{*+}/\bar{S}_{cs}A_{cu}$	RS-II	$4115 \pm i69$	$4112 \pm i66$	$4108 \pm i64$

$Z_{cs}(4000)^+$:

$$m = 3991^{+12}_{-10}{}^{+9}_{-17}\ \mathrm{MeV}$$

$$\Gamma = 105^{+29}_{-25}{}^{+17}_{-23}\ \mathrm{MeV}$$

$Z_{cs}(4220)^+$:

$$m = 4216 \pm 24^{+43}_{-30}\ \mathrm{MeV}$$

$$\Gamma = 233 \pm 52^{+97}_{-73}\ \mathrm{MeV}$$

LHCb, Phys. Rev. Lett 131, 131901 (2023)
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Appendix

q_{max} (MeV)	900	1000	1100
Pole (MeV)	4488.0±i 29.6	4485.8±i 24.9	4482.2±i 21.3

q_{max} (MeV)	900	1000	1100
$g_{D_s^{*+}D_s^{*-}}$ (MeV)	5244	5408	5340
$g_{A_{cq}\bar{A}_{cq}}$ (MeV)	3784	3726	3610
$g_{A_{cs}\bar{A}_{cs}}$ (MeV)	10513	9795	9122

$X(4500)$: $m = 4474 \pm 3 \pm 3$ MeV, $\Gamma = 77 \pm 6^{+10}_{-8}$ MeV

LHCb, Phys. Rev. Lett 127, 082001 (2021)

q_{max} (MeV)	1400	1450	1500
Pole (MeV)	3901±i26	3894±i22	3888±i17
$\lvert g_{D\bar{D}^*}\rvert$ (MeV)	8606	8488	8412
$\lvert g_{S_{cq}\bar{A}_{cq}}\rvert$ (MeV)	15727	15248	14874

$Z_c(3900)$: $m = 3884.6 \pm 0.7 \pm 3.3$ MeV, $\Gamma = 37.2 \pm 1.3 \pm 6.6$ MeV

BESIII, Phys. Rev. D 112, 092013 (2025)

q_{max} [MeV]	1000	1050	1100	1150	1200
Pole [MeV]	$\boxed{2948 \pm i66}$	$2948 \pm i56$	$2948 \pm i48$	$2947 \pm i43$	$2945 \pm i38$
$\lvert g_{D_s^{*+}\rho^+}\rvert$ [MeV]	11620	10540	9415	8557	7919
$\lvert g_{D^{*+}K^+}\rvert$ [MeV]	9421	7834	6287	5179	4310
$\lvert g_{A_{cu}\bar{A}_{sd}}\rvert$ [MeV]	10404	10371	10117	9837	9577

$T^*_{c\bar{s}0}(2900)^{++}$: $m = 2921 \pm 17 \pm 20$ MeV, $\Gamma = 137 \pm 32 \pm 17$ MeV

LHCb, Phys. Rev. Lett 131, 041902 (2023)