$D_{s1}(2460)$ and other open-charm 1^+ states in relativistic chiral effective field theory

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Based on **ZRL**, X.-Q. Xiao, Z.-H. Guo and D.-L. Yao#, arXiv: *2607.02452* [hep-ph]

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Outline

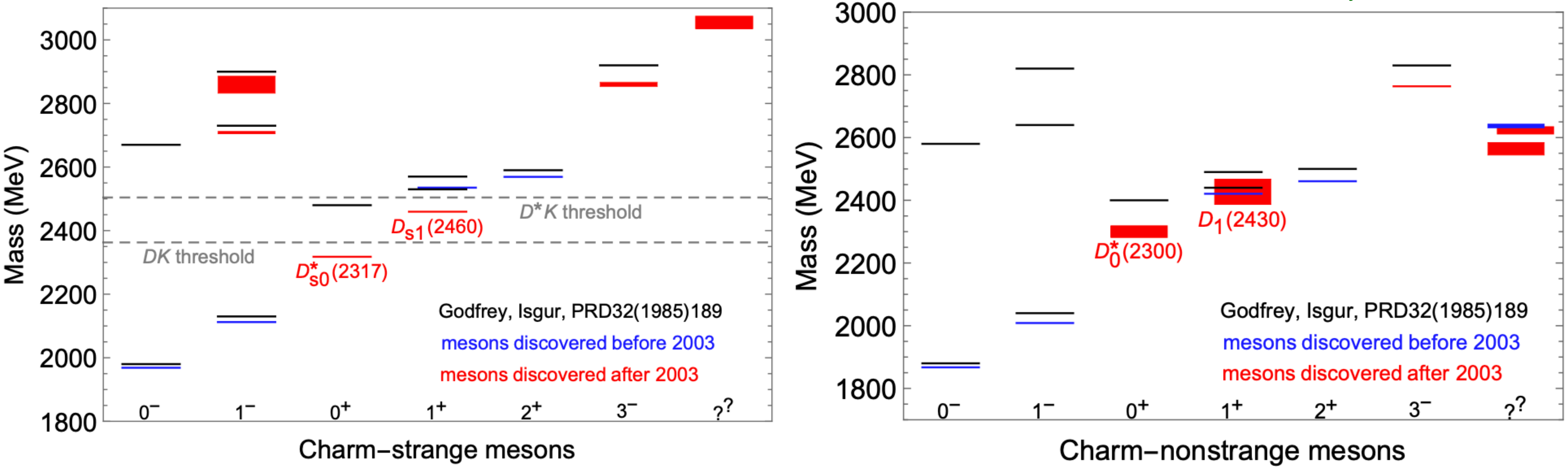
- Introduction
- Chiral potentials and unitarization
- Scattering length and S-matrix parameters
- Poles and their trajectories
- Summary and outlook

Introduction

Puzzles of charmed mesons

- Spectrum of charm-(non)strange mesons

Courtesy of F.-K. Guo



➡ **Puzzle 1:** why are $D^*_{s0}(2317)$ and $D_{s1}(2460)$ so light?

➡ **Puzzle 2:** why $M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^{*\pm}} - M_{D^\pm}$?

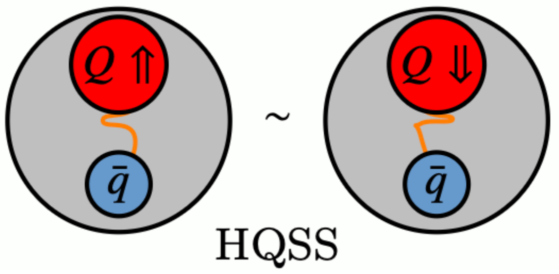
➡ **Puzzle 3:** why $M_{D_0(2300)} \gtrsim M_{D^*_{s0}(2317)}$?

A way out of the puzzles lies in the use of ChEFT!

Status of ChEFT study of charmed mesons

- $P\phi$ **scatterings & 0^+ states:** $\phi \in \{\pi, K, \bar{K}, \eta\}$
  - ➡ Up to NLO: [Kolomeitsev & Lutz, PLB 2004], [Guo, et al., PLB 2006, EPJA 2009], [Gamermann, et al., PRD 2007] [Liu, et al. PRD 2012], [Altenbuchinger, et al. PRD 2014], [Guo, et al., PRD 2018], ….
  - ➡ NNLO: [Liu, et al., PRD 2009], [Huang, et al., PRD 2022] (Heavy Baryon formalism) [Geng, et al., PRD 2010], [Yao, et al., JHEP 2015], [Du, et al., EPJC 2017] (EOMS scheme)
- $P^*\phi$ **scatterings & 1^+ states:** via heavy quark spin symmetry (HQSS)

$P\phi$ **scattering**



$P^*\phi$ **scattering**

$\epsilon^\dagger \cdot \epsilon = -1$

$j_l = \frac{1}{2}$	Exp. mass
$\begin{pmatrix} D^+(0^-) \\ D^{+*}(1^-) \end{pmatrix}$	$\begin{pmatrix} 1869.59 \\ 2010.26 \end{pmatrix}$
$\begin{pmatrix} D_s^+(0^-) \\ D_s^{+*}(1^-) \end{pmatrix}$	$\begin{pmatrix} 1968.28 \\ 2112.10 \end{pmatrix}$
$\begin{pmatrix} D_{s0}^*(0^+) \\ D_{s1}^*(1^+) \end{pmatrix}$	$\begin{pmatrix} 2317.70 \\ 2459.50 \end{pmatrix}$

- **Our work:** [**ZRL**, X.-Q. Xiao, Z.-H. Guo and D.-L. Yao#, arXiv: 2607.02452 [hep-ph]]
  - ➡ Relativistic kinematics: keeping full form of polarization vectors
  - ➡ Both contact and meson-exchange contributions
  - ➡ U(3) framework: inclusion of η'

Chiral potentials and unitarization

Chiral Effective Lagrangians

- **The Lagrangians up to NLO**

$$\mathscr{L}_{\text{eff}} = \mathscr{L}^{(1)}_{P^*\phi} + \mathscr{L}^{(1)}_{P\phi} + \mathscr{L}^{(1)}_{P^*P\phi} + \mathscr{L}^{(1)}_{P^*P^*\phi} + \mathscr{L}^{(2)}_{P^*\phi}$$

▸ **LO contact interactions**

$$\mathscr{L}^{(1)}_{P^*\phi} = -D_\mu P^*_\alpha D^\mu P^{*\alpha\dagger} + \overline{M}^2_{P^*} P^*_\alpha P^{*\alpha\dagger} \ ,$$

$$\mathscr{L}^{(1)}_{P\phi} = D_\mu P D^\mu P^\dagger - \overline{M}^2_P P P^\dagger$$

✓ Charmed vector mesons

$$P^* = (D^{*0}, D^{*+}, D_s^{*+})^T = (c\bar{u}, c\bar{d}, c\bar{s})^T$$

✓ Pseudo-scalar charmed mesons

$$P = (D^0, D^+, D_s^+)^T = (c\bar{u}, c\bar{d}, c\bar{s})^T$$

✓ Pseudo-scalar Goldstone bosons

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix} + \frac{\mathbf{1}}{\sqrt{3}}\eta_0$$

Chiral Effective Lagrangians

- **The Lagrangians up to NLO**

$$\mathscr{L}_{\mathrm{eff}} = \mathscr{L}^{(1)}_{P^*\phi} + \mathscr{L}^{(1)}_{P\phi} + \mathscr{L}^{(1)}_{P^*P\phi} + \mathscr{L}^{(1)}_{P^*P^*\phi} + \mathscr{L}^{(2)}_{P^*\phi}$$

- LO meson-exchange pieces

$$\mathscr{L}^{(1)}_{P^*P\phi} = ig_0 \left(P^*_\mu u^\mu P^\dagger - P\, u^\mu P^{*\dagger}_\mu \right) \ , \qquad \mathscr{L}^{(1)}_{P^*P^*\phi} = \frac{g_1}{2} \left(P^*_\alpha u_\mu D_\nu P^{*\dagger}_\beta - D_\nu P^*_\alpha u_\mu P^{*\dagger}_\beta \right) \epsilon^{\alpha\beta\mu\nu} \ ,$$

- NLO contact interactions [Eur. Phys. J. A 40, 171 (2009)]

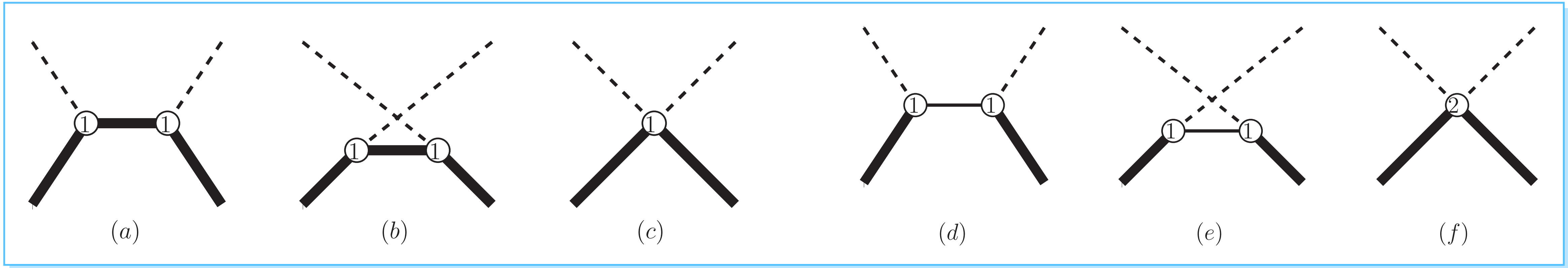
$$\begin{aligned}\mathscr{L}^{(2)}_{P^*\phi} = & -P^*_\alpha \big(-h^*_0 \langle \chi_+ \rangle - h^*_1 \chi_+ \big) P^{*\alpha\dagger} - P^*_\alpha \big(h^*_2 \langle u_\mu u^\mu \rangle - h^*_3 u_\mu u^\mu \big) P^{*\alpha\dagger} \\ & - D_\mu P^*_\alpha \big(h^*_4 \langle u_\mu u^\nu \rangle - h^*_5 \{u^\mu, u^\nu\} \big) D_\nu P^{*\alpha\dagger} \ ,\end{aligned}$$

✓ $h^*_{1,2,3,4,5}$ and $g_{0,1}$ are LECs.

✓ Heavy quark spin symmetry is applied.

NLO potential

- **Feynman diagrams**



- **Chiral potentials:** $V(s,t) = V_{\rm ct}(s,t) + V_{\rm ex}(s,t)$

contact (c+f): $$V_{\rm ct}(s,t) = -\epsilon_1\cdot\epsilon_3^*\left\{\mathscr{C}_{\rm LO}\frac{s-u}{4F_0^2} - \frac{1}{F_0^2}\left[4h_0^*\mathscr{C}_0 - 2h_1^*\mathscr{C}_1 + 2\mathscr{C}_{24}\mathscr{H}_{24} - 2\mathscr{C}_{35}\mathscr{H}_{35}\right]\right\}$$

exchange (a+b+d+e): $$V_{\rm ex}(s,t) = \frac{\mathscr{C}_S}{3F_0^2}\left[g_0^2\mathscr{G}_P(s,t) - g_1^2\mathscr{G}_{P*}(s,t)\right] + \frac{\mathscr{C}_U}{3F_0^2}\left[g_0^2\mathscr{G}_P(u,t) - g_1^2\mathscr{G}_{P*}(u,t)\right]$$

NLO potential

- **The coefficients:**
  - Classified by strangeness and isospin (S,I): 7 channels in total
  - $\eta - \eta'$ mixing:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

  - LO masses and mixing angle:

$$M_\pi^2 = 2B\hat{m}\ , \quad M_K^2 = B(\hat{m} + m_s)$$

$$M_{\eta'/\eta}^2 = M_K^2 + \frac{M_0^2}{2} \pm \frac{R}{2}$$

$$\sin\theta = -\left(\sqrt{1 + \frac{S^2}{32\Delta^4}}\right)^{-1}$$

(S,I)	Channels	$\mathcal{C}_{\rm LO}$	$\mathcal{C}_S$	$\mathcal{C}_U$	$\mathcal{C}_0$	$\mathcal{C}_1$	$\mathcal{C}_{24}$	$\mathcal{C}_{35}$
$(-1,0)$	$D^*\bar{K} \to D^*\bar{K}$	-1	0	-6	M_K^2	M_K^2	1	-1
$(-1,1)$	$D^*\bar{K} \to D^*\bar{K}$	1	0	6	M_K^2	$-M_K^2$	1	1
$(2,\frac{1}{2})$	$D_s^*K \to D_s^*K$	1	0	6	M_K^2	$-M_K^2$	1	1
$(0,\frac{3}{2})$	$D^*\pi \to D^*\pi$	1	0	6	M_π^2	$-M_\pi^2$	1	1
$(1,1)$	$D_s^*\pi \to D_s^*\pi$	0	0	0	M_π^2	0	1	0
	$D^*K \to D^*K$	0	0	0	M_K^2	0	1	0
	$D^*K \to D_s^*\pi$	1	0	6	0	$-\frac{M_K^2+M_\pi^2}{2}$	0	1
$(1,0)$	$D^*K \to D^*K$	-2	12	0	M_K^2	$-2M_K^2$	1	2
	$D^*K \to D_s^*\eta$	$-\sqrt{3}c_\theta$	$2\sqrt{6}(\sqrt{2}c_\theta + s_\theta)$	$-2\sqrt{3}(c_\theta - \sqrt{2}s_\theta)$	0	$\mathcal{C}_1^{(1,0)\ K\eta}$	0	$\frac{c_\theta+2\sqrt{2}s_\theta}{\sqrt{3}}$
	$D_s^*\eta \to D_s^*\eta$	0	$2(\sqrt{2}c_\theta + s_\theta)^2$	$2(\sqrt{2}c_\theta + s_\theta)^2$	$\mathcal{C}_0^{(1,0)\ \eta\eta}$	$\mathcal{C}_1^{(1,0)\ \eta\eta}$	1	$\frac{2(\sqrt{2}c_\theta+s_\theta)^2}{3}$
	$D^*K \to D_s^*\eta'$	$-\sqrt{3}s_\theta$	$-2\sqrt{6}(c_\theta - \sqrt{2}s_\theta)$	$-2\sqrt{3}(\sqrt{2}c_\theta + s_\theta)$	0	$\mathcal{C}_1^{(1,0)\ K\eta'}$	0	$\mathcal{C}_{35}^{(1,0)\ K\eta'}$
	$D_s^*\eta \to D_s^*\eta'$	0	$\mathcal{C}_S^{(1,0)\ \eta\eta'}$	$\mathcal{C}_U^{(1,0)\ \eta\eta'}$	$\mathcal{C}_0^{(1,0)\ \eta\eta'}$	$\mathcal{C}_1^{(1,0)\ \eta\eta'}$	0	$\mathcal{C}_{35}^{(1,0)\ \eta\eta'}$
	$D_s^*\eta' \to D_s^*\eta'$	0	$2(c_\theta - \sqrt{2}s_\theta)^2$	$2(c_\theta - \sqrt{2}s_\theta)^2$	$\mathcal{C}_0^{(1,0)\ \eta'\eta'}$	$\mathcal{C}_1^{(1,0)\ \eta'\eta'}$	1	$\mathcal{C}_{35}^{(1,0)\ \eta'\eta'}$
$(0,\frac{1}{2})$	$D^*\pi \to D^*\pi$	-2	9	-3	M_π^2	$-M_\pi^2$	1	1
	$D^*\eta \to D^*\eta$	0	$(c_\theta - \sqrt{2}s_\theta)^2$	$(c_\theta - \sqrt{2}s_\theta)^2$	$\mathcal{C}_0^{(0,\frac{1}{2})\ \eta\eta}$	$\mathcal{C}_1^{(0,\frac{1}{2})\ \eta\eta}$	1	$\mathcal{C}_{35}^{(0,\frac{1}{2})\ \eta\eta}$
	$D_s^*\bar{K} \to D_s^*\bar{K}$	-1	6	0	M_K^2	$-M_K^2$	1	1
	$D^*\eta \to D^*\pi$	0	$3(c_\theta - \sqrt{2}s_\theta)$	$3(c_\theta - \sqrt{2}s_\theta)$	0	$M_\pi^2(\sqrt{2}s_\theta - c_\theta)$	0	$c_\theta - \sqrt{2}s_\theta$
	$D_s^*\bar{K} \to D^*\pi$	$-\frac{\sqrt{6}}{2}$	$3\sqrt{6}$	0	0	$-\frac{\sqrt{6}(M_K^2+M_\pi^2)}{4}$	0	$\frac{\sqrt{6}}{2}$
	$D_s^*\bar{K} \to D^*\eta$	$-\frac{\sqrt{6}}{2}c_\theta$	$\sqrt{6}(c_\theta - \sqrt{2}s_\theta)$	$-2\sqrt{3}(\sqrt{2}c_\theta + s_\theta)$	0	$\mathcal{C}_1^{(0,\frac{1}{2})\ \bar{K}\eta}$	0	$\mathcal{C}_{35}^{(0,\frac{1}{2})\ \bar{K}\eta}$
	$D^*\eta' \to D^*\pi$	0	$3(\sqrt{2}c_\theta + s_\theta)$	$3(\sqrt{2}c_\theta + s_\theta)$	0	$-M_\pi^2(\sqrt{2}c_\theta + s_\theta)$	0	$s_\theta + \sqrt{2}c_\theta$
	$D^*\eta \to D^*\eta'$	0	$\mathcal{C}_{(c_\theta,s_\theta)}^{(0,\frac{1}{2})}$	$\mathcal{C}_{(c_\theta,s_\theta)}^{(0,\frac{1}{2})}$	$\mathcal{C}_0^{(0,\frac{1}{2})\ \eta\eta'}$	$\mathcal{C}_1^{(0,\frac{1}{2})\ \eta\eta'}$	0	$\mathcal{C}_{35}^{(0,\frac{1}{2})\ \eta\eta'}$
	$D_s^*\bar{K} \to D^*\eta'$	$-\frac{\sqrt{6}}{2}s_\theta$	$\sqrt{6}(\sqrt{2}c_\theta + s_\theta)$	$2\sqrt{3}(c_\theta - \sqrt{2}s_\theta)$	0	$\mathcal{C}_1^{(0,\frac{1}{2})\ \bar{K}\eta'}$	0	$\mathcal{C}_{35}^{(0,\frac{1}{2})\ \bar{K}\eta'}$
	$D^*\eta' \to D^*\eta'$	0	$(\sqrt{2}c_\theta + s_\theta)^2$	$(\sqrt{2}c_\theta + s_\theta)^2$	$\mathcal{C}_0^{(0,\frac{1}{2})\ \eta'\eta'}$	$\mathcal{C}_1^{(0,\frac{1}{2})\ \eta'\eta'}$	1	$\mathcal{C}_{35}^{(0,\frac{1}{2})\ \eta'\eta'}$

Partial-wave decomposition

- Helicity amplitude $V_{\lambda_3\lambda_1}$ $\Longleftrightarrow$ Covariant amplitude $V_i(s,t)$

$$V_{\lambda_3\lambda_1}(p_2,p_4;\Sigma) = \epsilon^{\mu\dagger}_{\lambda_3}(p_3)\, V_{\mu\nu}(p_2,p_4;\Sigma)\, \epsilon^{\nu}_{\lambda_1}(p_1)$$

$$V^{\mu\nu} = V_1\,(g^{\mu\nu} - \hat\Sigma^\mu\hat\Sigma^\nu) + V_2\,\hat\Sigma^\mu\hat\Sigma^\nu + V_3\,\hat\Sigma^\mu p^\nu_{4\mathrm{T}} + V_4\,p^\mu_{2\mathrm{T}}\hat\Sigma^\nu + V_5\,p^\mu_{2\mathrm{T}}p^\nu_{4\mathrm{T}}$$

- P and T transformation: 5 out of 9 are independent:

$$\mathbb{P}: \quad V_{\lambda_3\lambda_1} = (-1)^{\lambda_3-\lambda_1}V_{-\lambda_3-\lambda_1}$$
$$\mathbb{T}: \quad V_{\lambda_3\lambda_1} = (-1)^{\lambda_3-\lambda_1}V_{\lambda_1\lambda_3}$$

$$\begin{aligned}
\boxed{V_{++}} &= V_{--} = -\frac{1}{2}\left[(z_s+1)V_1 + p_{\mathrm{cm}}\bar p_{\mathrm{cm}}(1-z_s^2)V_5\right]\ , \quad z_s = \cos\theta \\
\boxed{V_{+-}} &= V_{-+} = \frac{1}{2}\left[(z_s-1)V_1 + p_{\mathrm{cm}}\bar p_{\mathrm{cm}}(1-z_s^2)V_5\right]\ , \\
\boxed{V_{+0}} &= -V_{-0} = -\frac{(1-z_s^2)^{\frac12}}{\sqrt{2}m_1}\left[\omega_1 V_1 - p^2_{\mathrm{cm}}V_4 - z_s\omega_1 p_{\mathrm{cm}}\bar p_{\mathrm{cm}}V_5\right]\ , \\
\boxed{V_{0+}} &= -V_{0-} = \frac{(1-z_s^2)^{\frac12}}{\sqrt{2}m_3}\left[\omega_3 V_1 - \bar p^2_{\mathrm{cm}}V_3 - z_s\omega_3 p_{\mathrm{cm}}\bar p_{\mathrm{cm}}V_5\right]\ , \\
\boxed{V_{00}} &= \frac{1}{m_1 m_3}\left[z_s p^2_{\mathrm{cm}}\omega_3 V_4 + z_s\omega_1(\bar p^2_{\mathrm{cm}}V_3 - \omega_3 V_1) + p_{\mathrm{cm}}\bar p_{\mathrm{cm}}(V_2 + z_s^2\omega_1\omega_3 V_5)\right]\ ,
\end{aligned}$$

Partial-wave decomposition

- Partial-wave amplitudes:

$$V_{\lambda_3\lambda_1}(s,t) = \sum_{J=\max\{|\lambda_3|,|\lambda_1|\}}^{\infty} (2J+1) V^J_{\lambda_3\lambda_1}(s) d^J_{\lambda_1\lambda_3}(z_s) \qquad V^J_{\lambda_3\lambda_1}(s) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}z_s V_{\lambda_3\lambda_1}(s,t(s,z_s))\, d^J_{\lambda_1\lambda_3}(z_s)$$

- Helicity basis to ***JLS* basis** (for definite parity)

$$V_{JLS} = \sum_{\lambda_1\lambda_3} \mathcal{U}^{JLS}_{\lambda_3 0} V^J_{\lambda_3\lambda_1} [\mathcal{U}^{JLS}_{\lambda_1 0}]^\dagger \,, \qquad \mathcal{U}^{JLS}_{\lambda_i 0} = \left(\frac{2L+1}{2S+1}\right)^{\frac{1}{2}} \langle L0S\lambda | J\lambda\rangle \langle S_i\lambda_i 00 | S\lambda\rangle \,,$$

▸ **S** wave:

$$V_{J=1,L=0}(s) = \frac{1}{3}\left[2V^1_{++}(s) + 2V^1_{+0}(s) + 2V^1_{+-}(s) + 2V^1_{0+}(s) + V^1_{00}(s)\right] \qquad 1^+$$

▸ **P** wave:

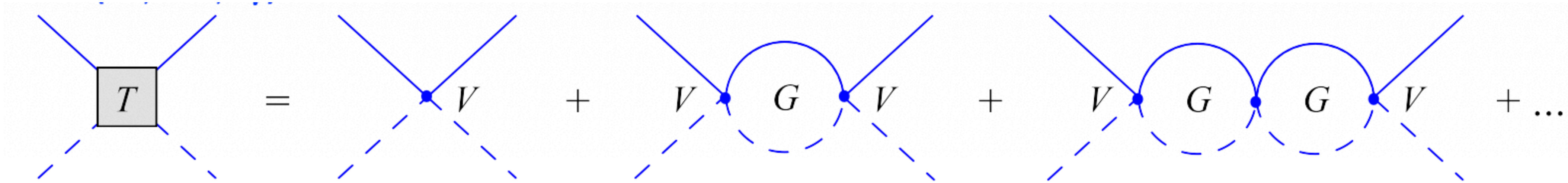
$$V_{J=0,L=1}(s) = V^0_{00}(s) \,, \qquad 0^-$$

$$V_{J=1,L=1}(s) = V^1_{++}(s) - V^1_{+-}(s) \,, \qquad 1^-$$

$$V_{J=2,L=1}(s) = \frac{1}{5}\left[2V^2_{00}(s) + 3\left(V^2_{++}(s) + V^2_{+-}(s)\right) + 2\sqrt{3}\left(V^2_{0+}(s) + V^2_{+0}(s)\right)\right] \qquad 2^-$$

The on-shell unitarization approach

- **Bethe-Salpeter equation (BSE) with on-shell approximation**



$$T_{JL}^{(S,I)}(s) = V_{JL}^{(S,I)}(s) \cdot \left[1 - G(s) \cdot V_{JL}^{(S,I)}(s)\right]^{-1} \qquad V_{JL}^{(S,I)}(s) = \begin{pmatrix} [V_{JL}^{(S,I)}]_{1\to 1} & \cdots [V_{JL}^{(S,I)}]_{1\to j} & \cdots \\ \vdots \quad \ddots & \vdots \quad \ddots & \\ [V_{JL}^{(S,I)}]_{i\to 1} & \cdots [V_{JL}^{(S,I)}]_{i\to j} & \cdots \\ \vdots \quad \ddots & \vdots \quad \ddots & \end{pmatrix}$$

▸ **Two-point loop functions:** $G = \{\cdots, g_i(s), \cdots\}$, $\sigma_i(s) \equiv \lambda^{1/2}(s, M_{P_i^*}^2, M_{\phi_i}^2)$

$$g_i(s) = \frac{1}{16\pi^2}\Bigg\{ a_i(\mu) + \ln\frac{M_{P_i^*}^2}{\mu^2} + \frac{s - M_{P_i^*}^2 + M_{\phi_i}^2}{2s}\ln\frac{M_{\phi_i}^2}{M_{P_i^*}^2} + \frac{\sigma_i(s)}{2s}\Big[\ln(s - M_{\phi_i}^2 + M_{P_i^*}^2 + \sigma_i(s))$$

$$-\ln(-s + M_{\phi_i}^2 - M_{P_i^*}^2 + \sigma_i(s)) + \ln(s + M_{\phi_i}^2 - M_{P_i^*}^2 + \sigma_i(s)) - \ln(-s - M_{\phi_i}^2 + M_{P_i^*}^2 + \sigma_i(s))\Big]\Bigg\}$$

Scattering length and S-matrix parameters

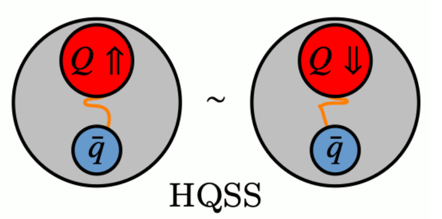
Parameter setup

- Values of LECs in contact term:
  - by mass difference:

$$h_1^* = \frac{M_{D_s^*}^2 - M_{D^*}^2}{4(M_K^2 - M_\pi^2)} = 0.47$$

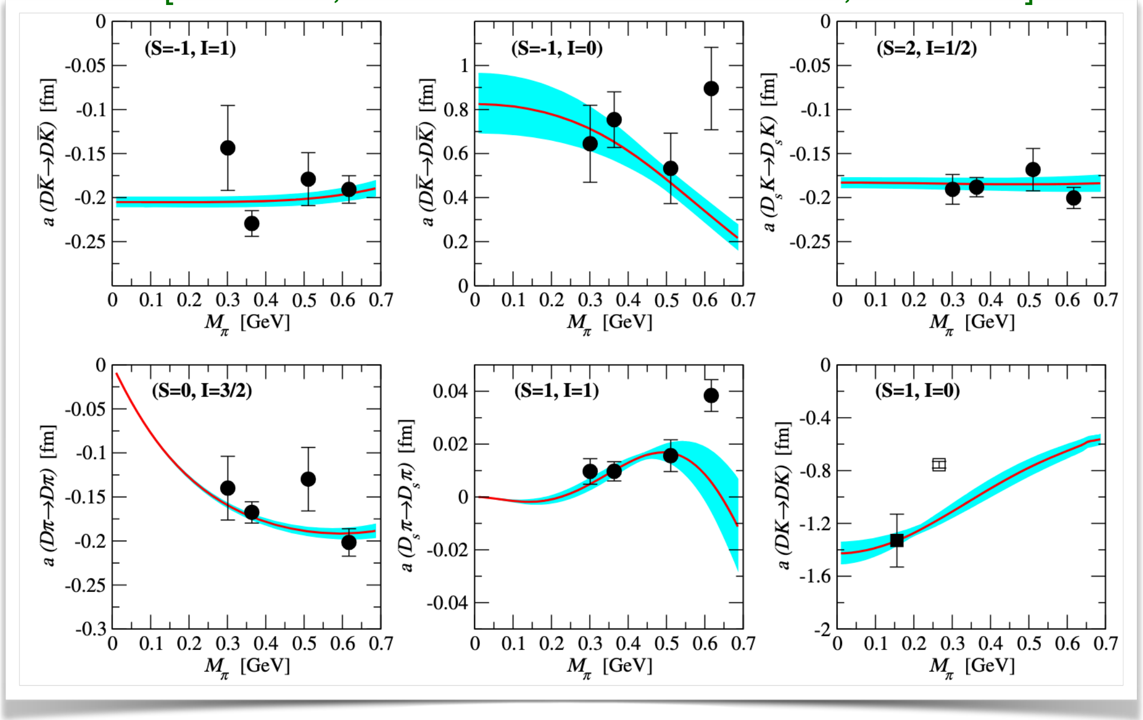
  - via HQSS:

$$h_i^* = h_i \ , \quad i = 0,2,3,4,5$$



LECs	h_0^*	h_1^*	h_2^*	h_3^*	h_4^*	h_5^*
Value	0.033	0.47	$0.08^{+0.31}_{-0.34}$	$3.79^{+0.41}_{-0.41}$	$-0.06^{+0.08}_{-0.07}$	$-0.48^{+0.05}_{-0.05}$

[Z.-H. Guo, Ulf-G. Meissner and D.-L. Yao#, PRD 2015]



- Values of LO coupling in meson-exchange contributions
  - by decay width:

$$\Gamma(D^{*+}(2010) \to D^0\pi^+) = \frac{g_0^2}{12\pi F_\pi^2}\frac{|\vec{q}_\pi|^3}{M_{D^{*+}}^2} \longrightarrow g_0 = 1095.0 \pm 15.8 \text{ MeV}$$

  - via HQSS: $g_1 M_D^* = g_0$

Scattering length

- Prediction of S-wave scattering lengths

$$a_{JL}^{(S,I)}(P_i^*\phi_i \to P_i^*\phi_i) = -\frac{1}{8\pi\sqrt{s_{\rm th}^i}} \lim_{s\to s_{\rm th}^i}\left[\frac{\mathcal{T}^{(S,I)}_{JL,P_i^*\phi_i\to P_i^*\phi_i}(s)}{p_{i,\rm cm}^{2L}}\right]$$

(S,I)	Process	CT	CT + Ex	HMChPT [75]
$(-1,0)$	$D^*\bar{K}\to D^*\bar{K}$	$0.41(2)$	$0.41(2)$	$0.29+i\,5.2\times10^{-6}$
$(-1,1)$	$D^*\bar{K}\to D^*\bar{K}$	$-0.18(1)$	$-0.18(1)$	$-0.19-i\,1.7\times10^{-6}$
$(2,\frac{1}{2})$	$D_s^*K\to D_s^*K$	$-0.17(1)$	$-0.17(1)$	-0.14
$(0,\frac{3}{2})$	$D^*\pi\to D^*\pi$	$-0.10(1)$	$-0.10(1)$	$-0.13-i\,3.6\times10^{-4}$
$(1,1)$	$D_s^*\pi\to D_s^*\pi$	$-0.001(1)$	$-0.004(1)$	-0.039
	$D^*K\to D^*K$	$0.01(1)+i\,0.07(1)$	$0.01(1)+i\,0.05(1)$	$-0.022+i\,0.10$
$(1,0)$	$D^*K\to D^*K$	$-0.86^{+0.09}_{-0.14}$	$-0.85^{+0.09}_{-0.14}$	$0.76-i\,5.2\times10^{-6}$
	$D_s^*\eta\to D_s^*\eta$	$-0.22(1)+i\,0.04(1)$	$-0.22(1)+i\,0.04(1)$	$0.18+i\,0.19$
	$D_s^*\eta'\to D_s^*\eta'$	$-0.24(2)+i\,0.03(1)$	$-0.26(2)+i\,0.04(1)$	-
$(0,\frac{1}{2})$	$D^*\pi\to D^*\pi$	$0.35(1)$	$0.36^{+0.01}_{-0.02}$	$0.27-i\,3.6\times10^{-4}$
	$D^*\eta\to D^*\eta$	$-0.03(2)+i\,0.04(1)$	$-0.05(2)+i\,0.04(1)$	$0.051+i\,0.094$
	$D_s^*\bar{K}\to D_s^*\bar{K}$	$-0.12(3)+i\,0.18^{+0.04}_{-0.03}$	$-0.14^{+0.03}_{-0.02}+i\,0.16^{+0.04}_{-0.03}$	$0.35+i\,0.27$
	$D^*\eta'\to D^*\eta'$	$-0.16(2)+i\,0.01(1)$	$-0.16(2)+i\,0.01(1)$	-

The meson-exchange diagrams contribute negligibly in all channels!

Scattering length

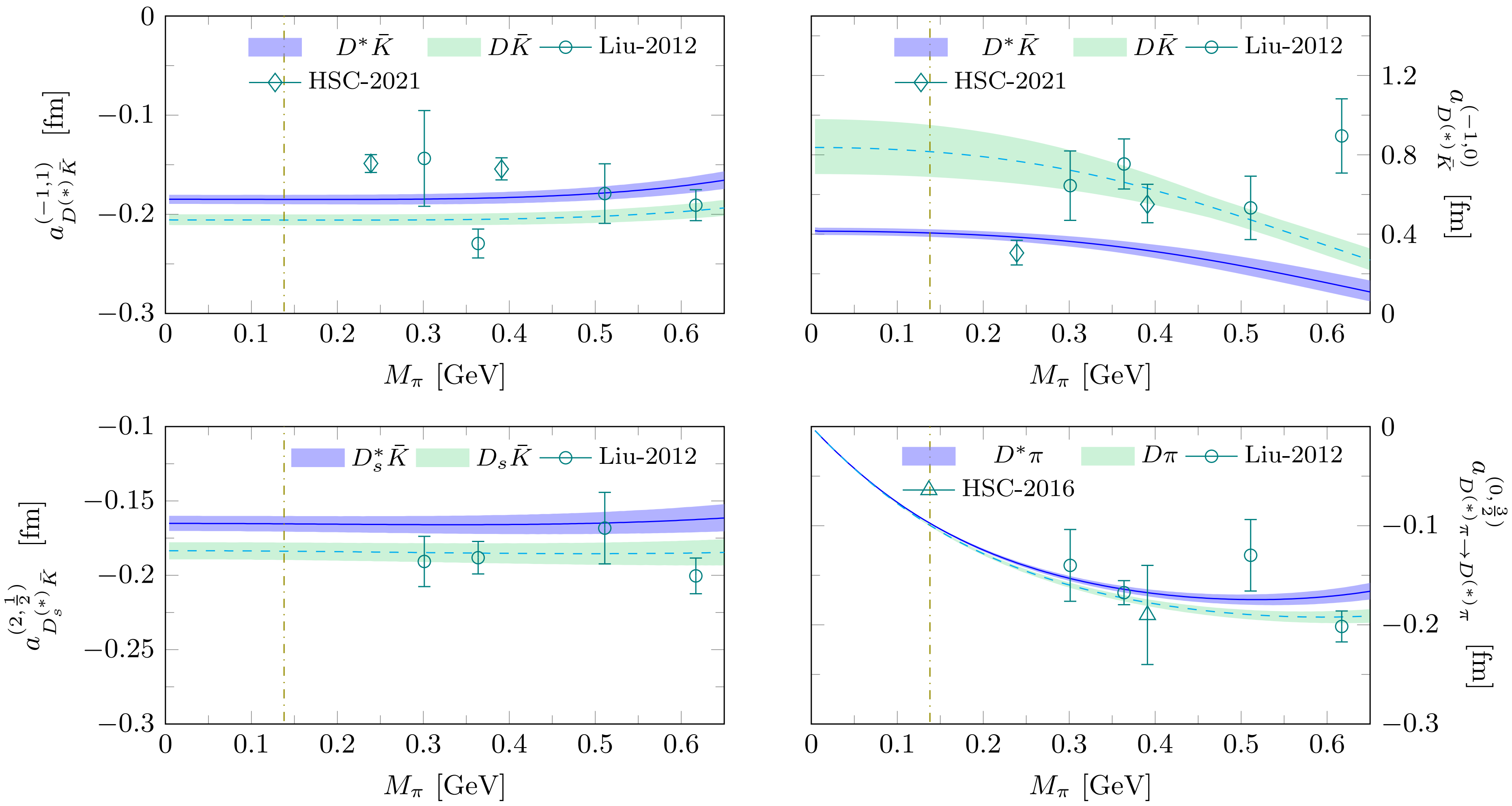
- Prediction of P-wave scattering length

(S,I) Process	0^- $[^3P_0]$		1^- $[^3P_1]$		2^- $[^3P_2]$	
	CT	CT + Ex	CT	CT+Ex	CT	CT+Ex
$(-1,0)$ $D^*\bar{K}\to D^*\bar{K}$	$-4.9^{+1.1}_{-1.0}$	$-4.7^{+1.1}_{-1.0}$	$-4.6^{+1.1}_{-1.0}$	$-4.4^{+1.1}_{-1.0}$	$-4.6^{+1.1}_{-1.0}$	$-5.8^{+1.1}_{-1.0}$
$(-1,1)$ $D^*\bar{K}\to D^*\bar{K}$	$4.4^{+0.8}_{-0.7}$	$4.2^{+0.8}_{-0.7}$	$4.1(7)$	$3.9(7)$	$4.1(7)$	$5.3(7)$
$(2,1/2)$ $D_s^*K\to D_s^*K$	$4.2(7)$	$4.0(7)$	$3.9(7)$	$3.6(7)$	$3.9(7)$	$6.1(7)$
$(0,3/2)$ $D^*\pi\to D^*\pi$	$4.2^{+0.8}_{-0.7}$	$-101.8^{+0.8}_{-0.7}$	$4.0(7)$	$106.1(7)$	$4.0(7)$	$-93.0(7)$
$(1,1)$ $D_s^*\pi\to D_s^*\pi$	$-0.4^{+0.8}_{-0.7}$	$-0.5(7)$	$-0.4^{+0.8}_{-0.7}$	$-32.9^{+0.9}_{-0.8}$	$-0.4^{+0.8}_{-0.7}$	$-6.4^{+0.7}_{-0.6}$
$D^*K\to D^*K$	$0.6(7)+i\,1.0(2)$	$0.5(7)+i\,0.9(2)$	$0.5(7)+i\,0.9(2)$	$0.4(7)+i\,0.7(2)$	$0.5(7)+i\,0.9(2)$	$1.4^{+0.7}_{-0.6}+i\,2.0(3)$
$(1,0)$ $D^*K\to D^*K$	$13.3(1.6)$	$5.4^{+1.3}_{-1.2}$	$14.1^{+1.6}_{-1.5}$	$10.7(1.9)$	$13.9^{+1.6}_{-1.5}$	$13.2(1.4)$
$D_s^*\eta\to D_s^*\eta$	$3.5^{+0.7}_{-0.6}+i\,0.2(1)$	$2.6^{+0.7}_{-0.6}+i\,0.04(0)$	$3.4^{+0.7}_{-0.6}+i\,0.4(1)$	$2.8^{+0.7}_{-0.6}+i\,0.02(0)$	$3.4^{+0.7}_{-0.6}+i\,0.4(1)$	$4.3^{+0.7}_{-0.6}+i\,0.2(1)$
$D_s^*\eta'\to D_s^*\eta'$	$-5.5^{+1.7}_{-1.5}+i\,7.9^{+2.4}_{-2.1}$	$-2.9^{+2.6}_{-2.2}+i\,12.6^{+2.8}_{-3.2}$	$-4.7^{+1.7}_{-1.6}+i\,5.3^{+1.7}_{-1.5}$	$-4.8^{+1.7}_{-1.5}+i\,7.1^{+2.3}_{-2.0}$	$-4.9^{+1.8}_{-1.6}+i\,5.2^{+1.8}_{-1.5}$	$-6.4^{+2.0}_{-1.9}+i\,6.6^{+2.3}_{-1.9}$
$(0,1/2)$ $D^*\pi\to D^*\pi$	$7.9(1.0)$	$57.9^{+1.6}_{-1.4}$	$8.4^{+1.1}_{-1.0}$	$-16.1^{+4.5}_{-3.9}$	$8.2(1.0)$	$55.4^{+0.9}_{-0.8}$
$D^*\eta\to D^*\eta$	$0.6^{+0.8}_{-1.1}+i\,8.2^{+1.7}_{-1.6}$	$3.4^{+0.1}_{-0.3}+i\,5.0^{+1.7}_{-1.4}$	$-0.2^{+0.9}_{-1.1}+i\,7.5^{+1.6}_{-1.5}$	$2.4^{+0.2}_{-0.5}+i\,5.9^{+1.7}_{-1.5}$	$-0.4^{+0.9}_{-1.2}+i\,7.6^{+1.6}_{-1.5}$	$-0.6^{+1.2}_{-1.5}+i\,10.8(1.9)$
$D_s^*\bar{K}\to D_s^*\bar{K}$	$1.5(7)+i\,5.1^{+1.0}_{-0.9}$	$3.1(3)+i\,3.0^{+1.1}_{-1.0}$	$0.9^{+0.9}_{-0.8}+i\,5.1(9)$	$2.4^{+0.4}_{-0.5}+i\,4.1(1.0)$	$0.8^{+0.9}_{-0.8}+i\,5.1(9)$	$1.3(8)+i\,5.4(9)$
$D^*\eta'\to D^*\eta'$	$-4.2^{+1.3}_{-1.2}+i\,2.6^{+0.6}_{-0.5}$	$-4.6^{+1.4}_{-1.3}+i\,3.8^{+0.9}_{-0.8}$	$-3.5^{+1.2}_{-1.1}+i\,2.0^{+0.5}_{-0.4}$	$-3.9^{+1.3}_{-1.2}+i\,2.5^{+0.6}_{-0.5}$	$-3.6^{+1.3}_{-1.1}+i\,1.9^{+0.5}_{-0.4}$	$-4.5^{+1.4}_{-1.2}+i\,2.3^{+0.6}_{-0.5}$

The discrepancies between “CT” and “CT+Ex” results (the first five channels) are mainly attributable to the u-channel exchanges.

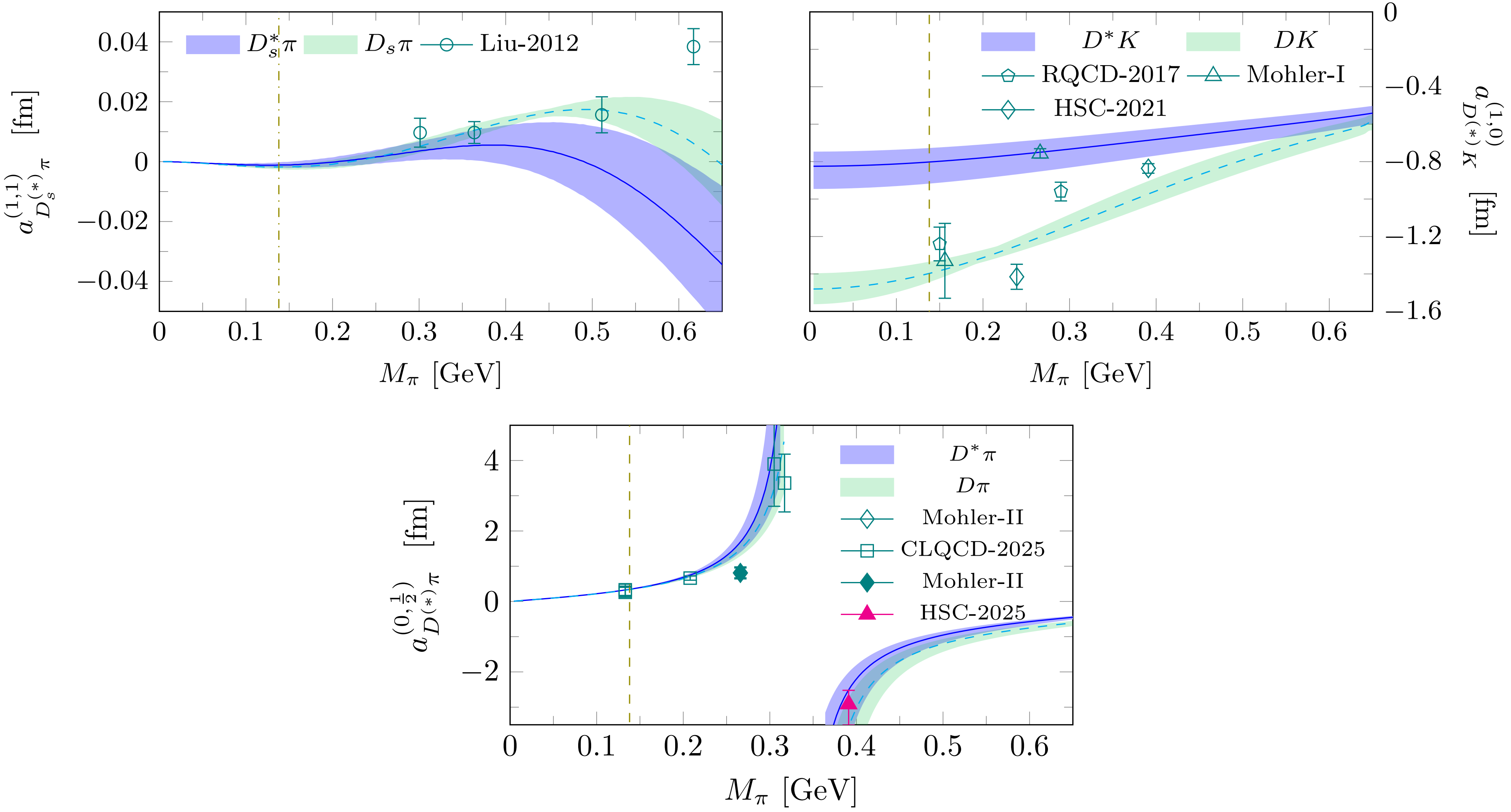
Pion-mass dependence of S-wave scattering lengths

- The four single channels: solid blue ($P^*\phi$) vs dashed green ($P\phi$)



Pion-mass dependence of S-wave scattering lengths

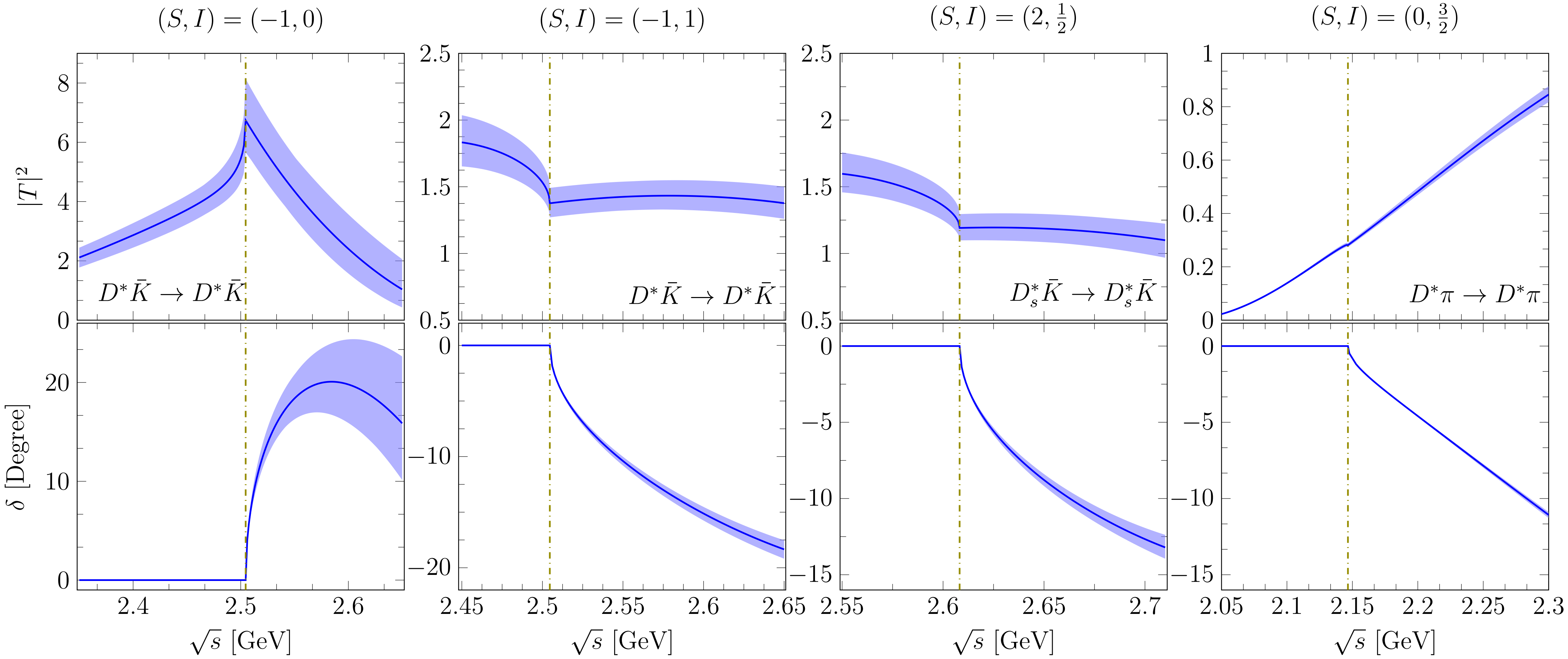
- The three coupled channels: solid blue ($P^*\phi$) vs dashed green ($P\phi$)



Goodness of HQSS: The most recent $P\phi$ and $P^*\phi$ data in $(0,1/2)$ channel are well described!

S-matrix: phase shifts and inelasticity

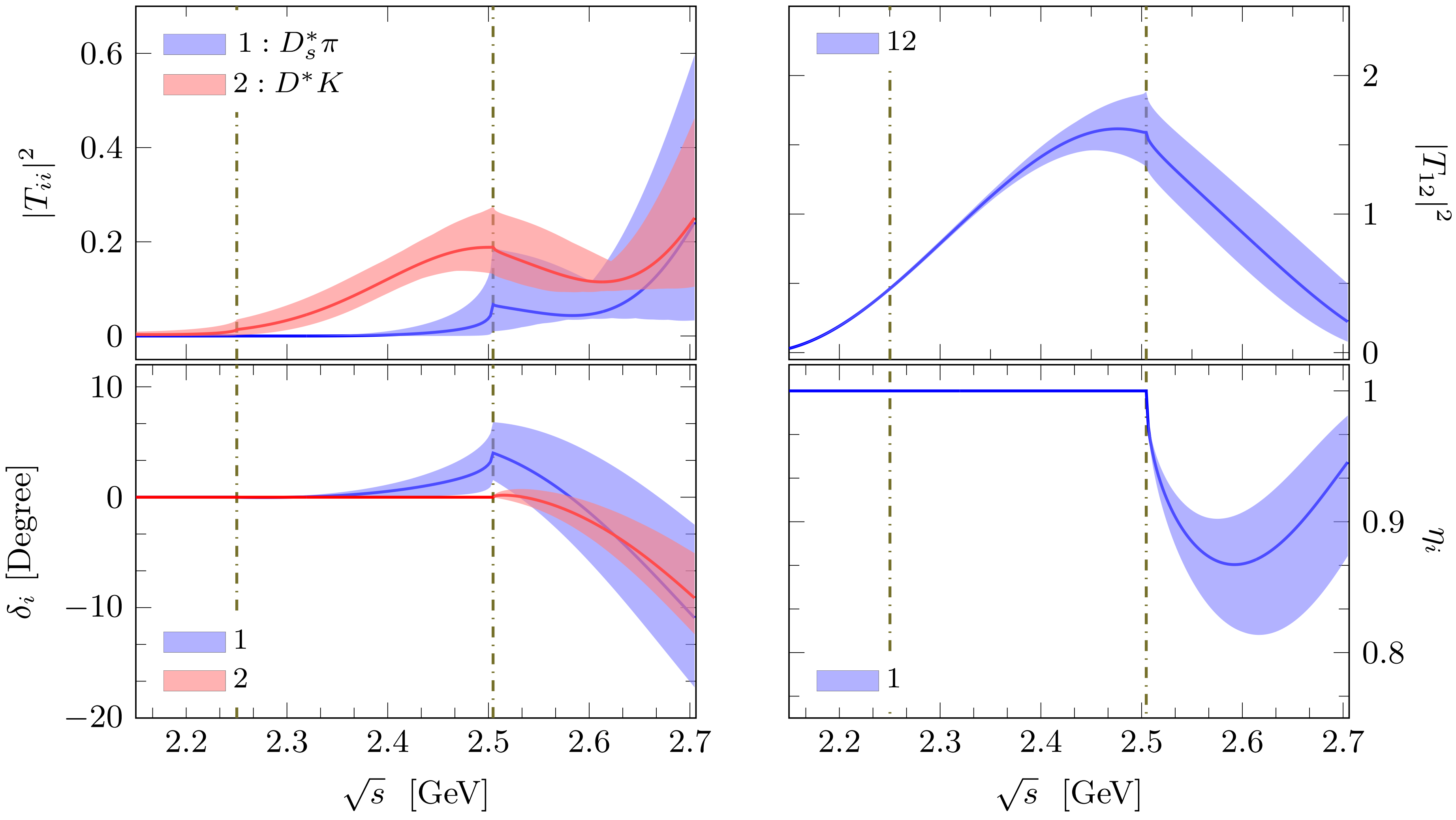
- Single channels:



The cusp structure at the $D^*\bar{K}$ $(I = 0)$ threshold suggests the presence of a possible subthreshold resonance.

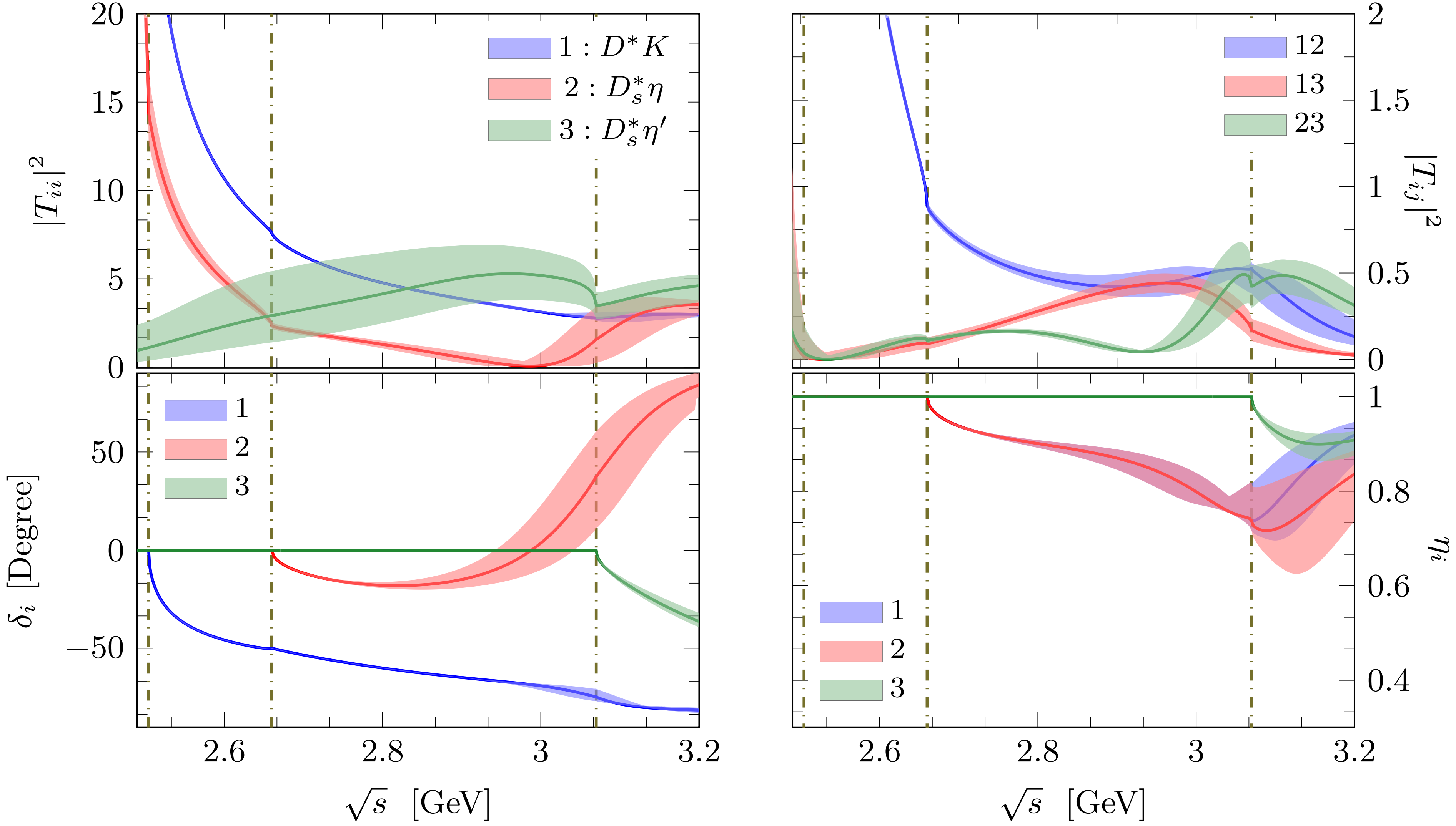
S-matrix: phase shifts and inelasticity

- The coupled channel with $(S, I) = (1,1)$:



S-matrix: phase shifts and inelasticity

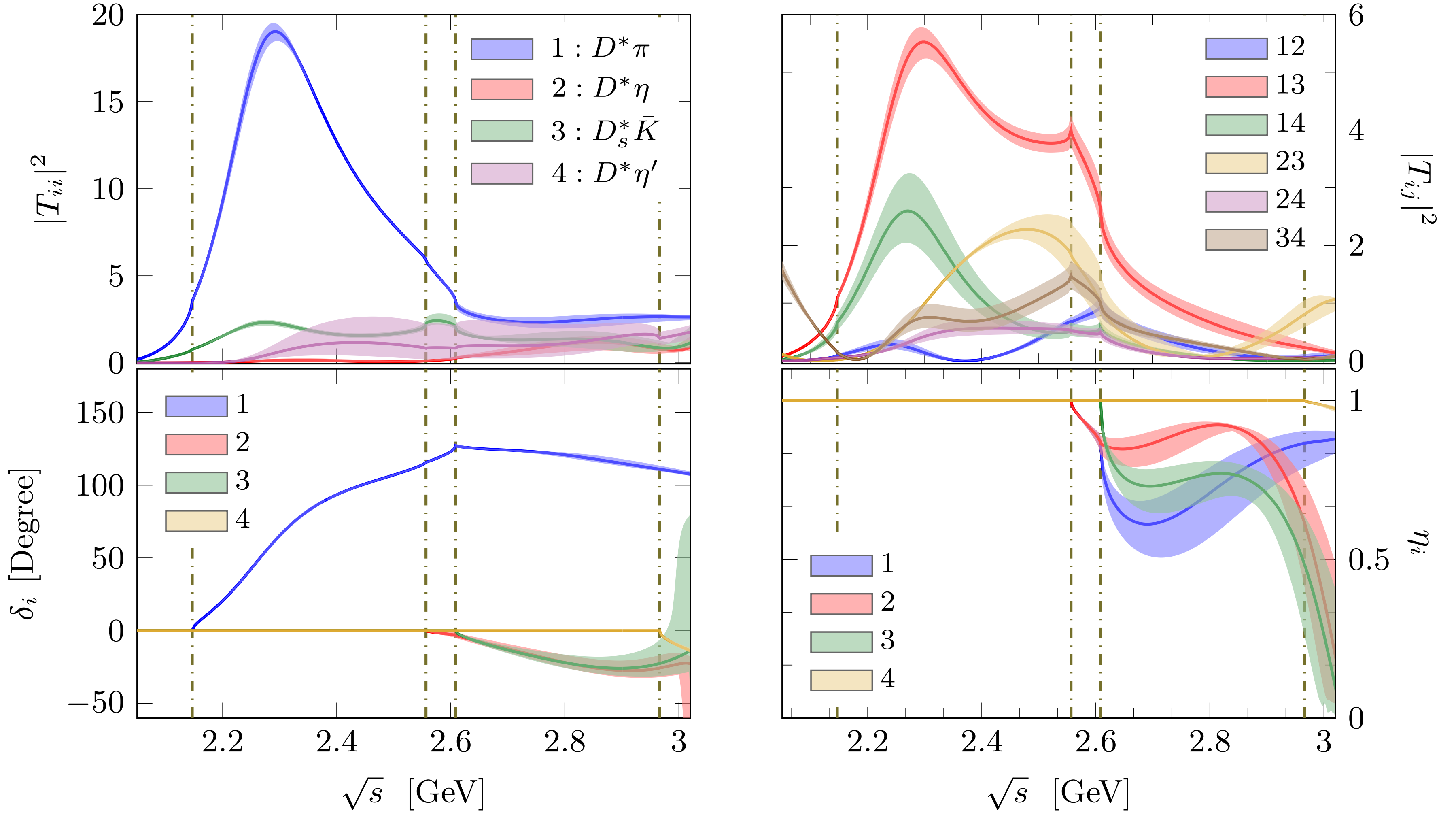
- The coupled channel with $(S, I) = (1,0)$:



Signal of a subthreshold pole — the $D_{s1}(2460)$ bound state

S-matrix: phase shifts and inelasticity

- The coupled channel with $(S, I) = (0, 1/2)$:



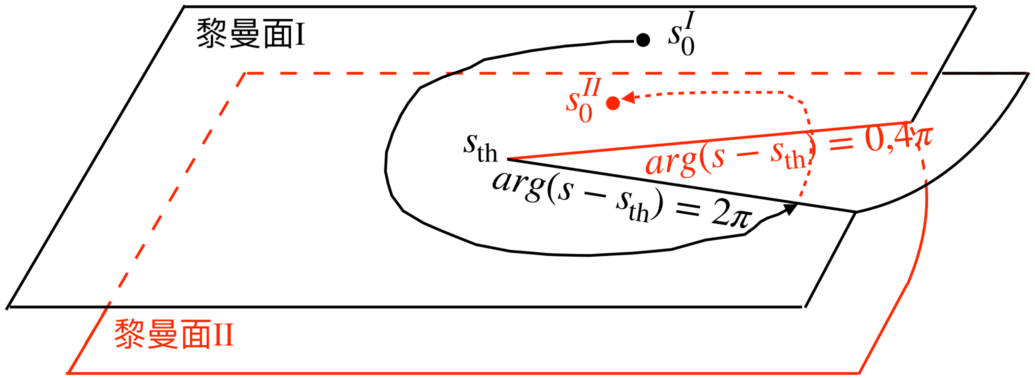
The broad bump of T_{11} between the $D^*\pi$ and $D^*\eta$ thresholds, together with the sharp rise of the phase shift δ_1 past 90 degrees, provide a strong indication of a near-threshold resonance.

Poles and their trajectories

Dynamically generated poles

- Analyticity continuity:

$$g_i^{\rm II} = g_i(s) - 2\xi_i {\rm Im} g_i(s)\ , \quad \xi_i \in \{0,1\}$$



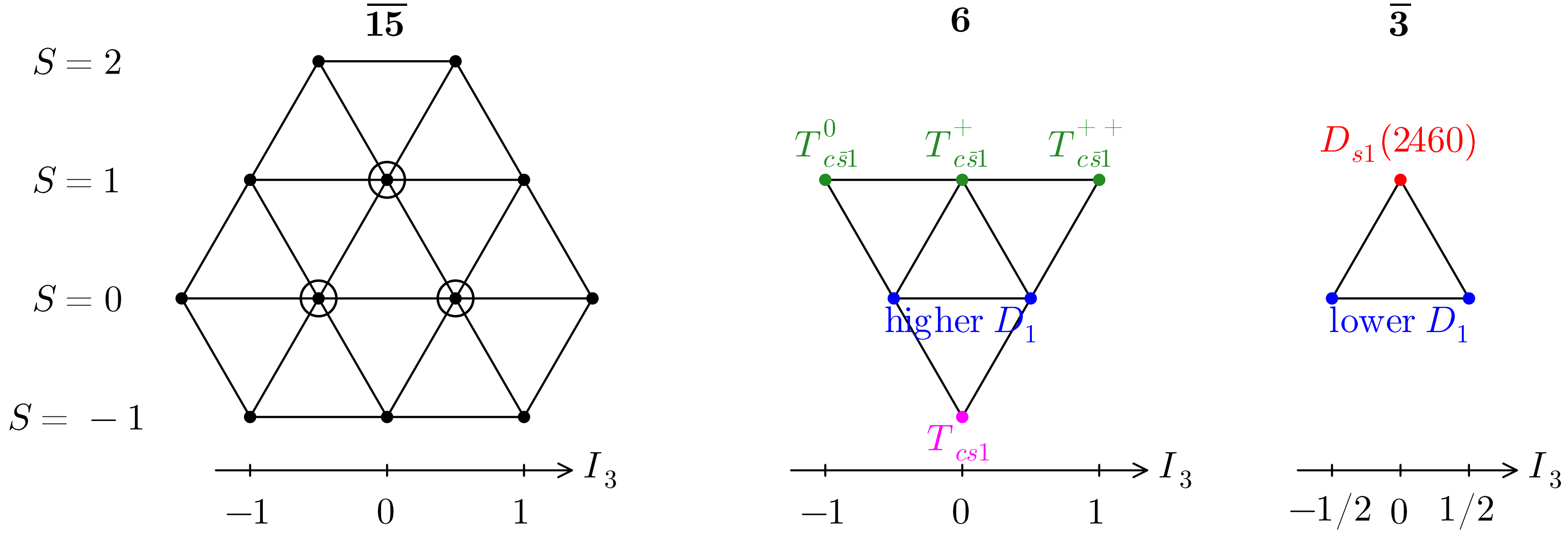
- Notation of Riemann sheets:

$$[\xi_1, \xi_2, \xi_3, \cdots] \quad \Longleftrightarrow \quad n = 1 + \sum_{i=1}^{N} \xi_i 2^{i-1}$$

(S,I)	RS	poles $(M - i\frac{\Gamma}{2})$	|Residues| (GeV) $\lvert g_1\rvert$	$\lvert g_2\rvert$	$\lvert g_3\rvert$	$\lvert g_4\rvert$
$(-1,0)$	II	$2362.1^{+13.6}_{-10.2} - i\,111.0^{+25.1}_{-26.7}$	$11.2^{+1.0}_{-0.6}(D^*\bar{K})$	–	–	–
$(1,1)$	II	$2576.7^{+30.5}_{-20.4} - i\,323.7^{+3.0}_{-5.4}$	$6.3^{+0.6}_{-0.4}\ (D_s^*\pi)$	$11.8^{+0.3}_{-0.3}\ (D^*K)$	–	–
	III	$2347.1^{+15.1}_{-11.5} - i\,227.4^{+11.5}_{-13.3}$	$7.1^{+0.3}_{-0.2}(D_s^*\pi)$	$5.6^{+0.2}_{-0.2}\ (D^*K)$	–	–
$(1,0)$	I	$2455.2^{+3.2}_{-2.7}$	$10.6^{+0.2}_{-0.2}(D^*K)$	$7.0^{+0.0}_{-0.0}(D_s^*\eta)$	$0.6^{+0.6}_{-0.6}(D_s^*\eta')$	–
$(0,\frac{1}{2})$	II	$2255.6^{+3.3}_{-2.8} - i\,112.5^{+2.6}_{-2.9}$	$10.3^{+0.1}_{-0.1}(D^*\pi)$	$2.0^{+0.3}_{-0.3}(D^*\eta)$	$5.2^{+0.1}_{-0.1}\ (D_s^*\bar{K})$	$4.4^{+0.6}_{-0.6}\ (D^*\eta')$
	III	$2558.1^{+31.0}_{-23.4} - i\,207.2^{+7.8}_{-8.4}$	$5.8^{+0.1}_{-0.0}\ (D^*\pi)$	$5.3^{+0.4}_{-0.3}(D^*\eta)$	$12.2^{+0.2}_{-0.2}\ (D_s^*\bar{K})$	$2.3^{+0.4}_{-0.3}\ (D^*\eta')$
	IV	$2307.1^{+16.1}_{-12.1} - i\,186.4^{+20.9}_{-14.5}$	$7.4^{+0.3}_{-0.4}(D^*\pi)$	$5.0^{+0.2}_{-0.0}(D^*\eta)$	$5.0^{+0.7}_{-0.3}(D_s^*\bar{K})$	$4.7^{+2.3}_{-1.2}(D^*\eta')$

SU(3) analysis

- Weight diagrams of flavor SU(3) symmetry: $\overline{3} \otimes 8 = \overline{15} \oplus 6 \oplus \overline{3}$



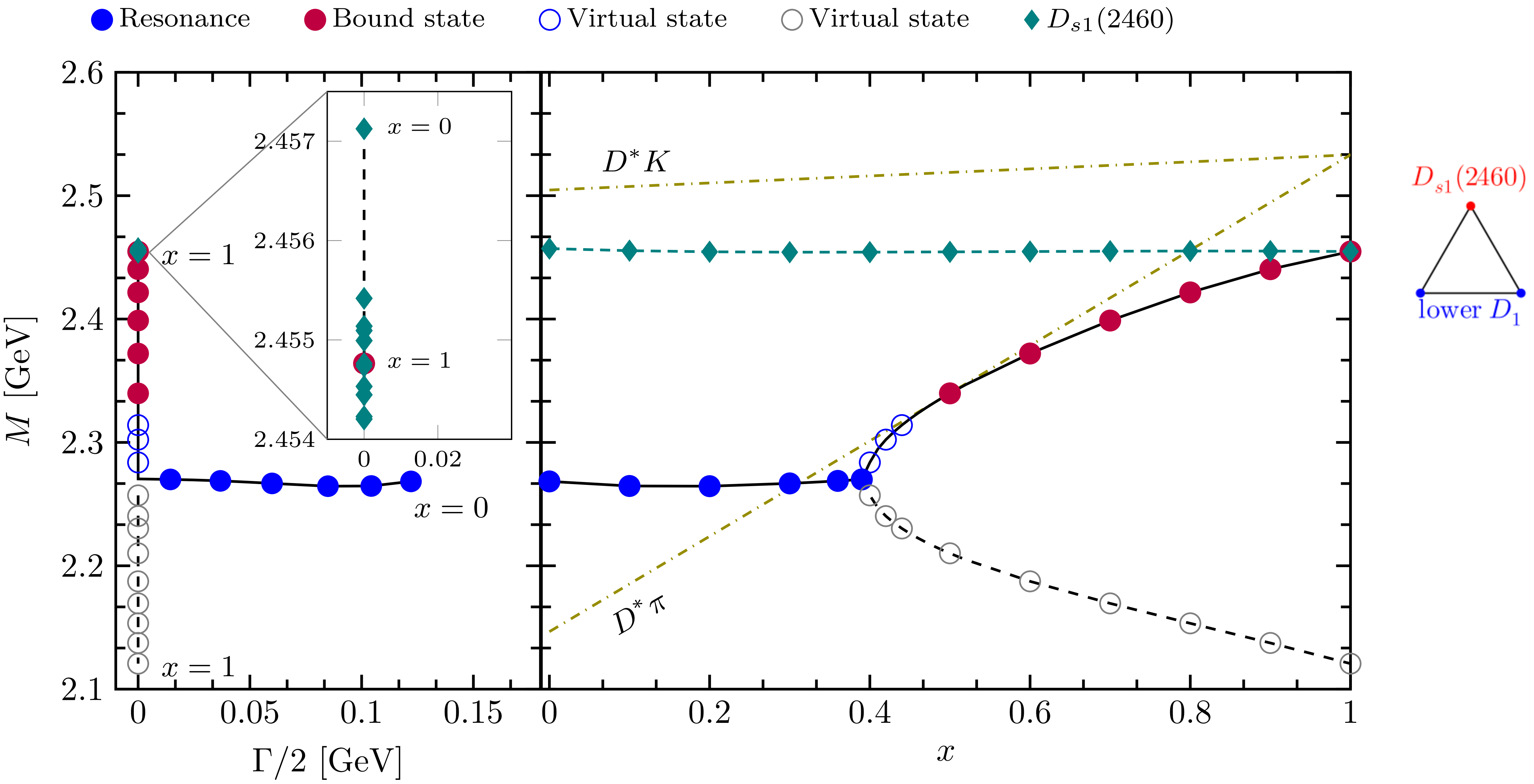
- Physical world vs SU(3) symmetry limit:

$$M_\phi = M_\phi^{\mathrm{phy.}} + x(\overline{M}_0 - M_\phi^{\mathrm{phy.}}) \ , \quad \phi \in \{\pi, K, \bar{K}, \eta\} \ ,$$

$$M_{P*} = M_{P*}^{\mathrm{phy.}} + x(\overline{M}_{P*} - M_{P*}^{\mathrm{phy.}}) \ , \quad P* \in \{D^*, D_s^*\} \ ,$$

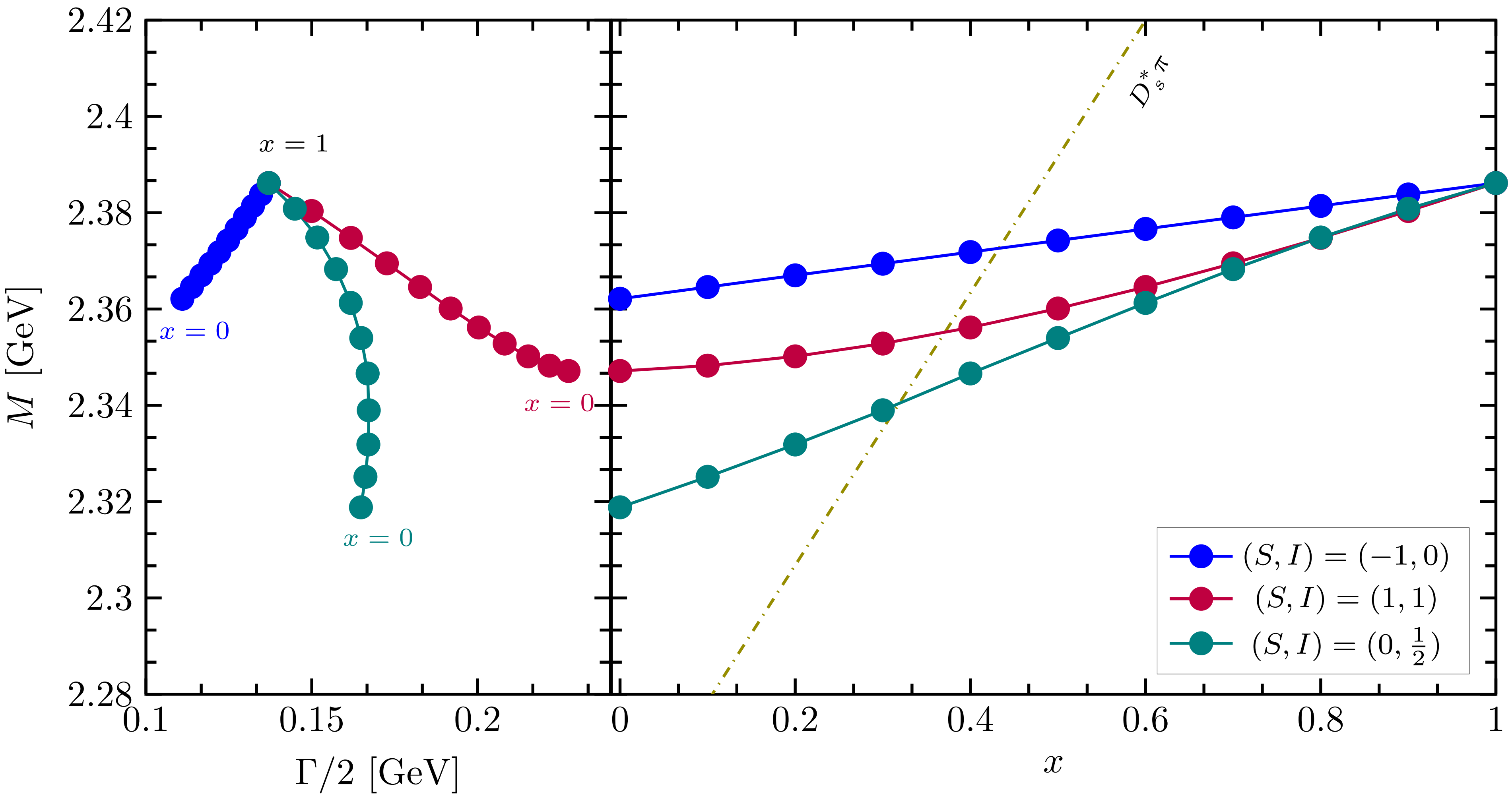
SU(3) analysis

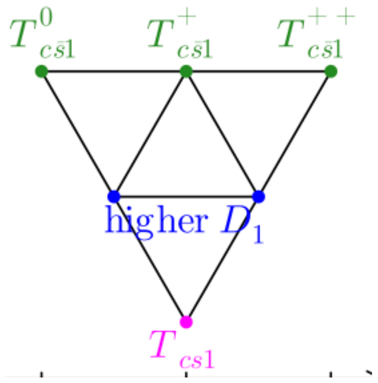
- The SU(3) triplet:



SU(3) analysis

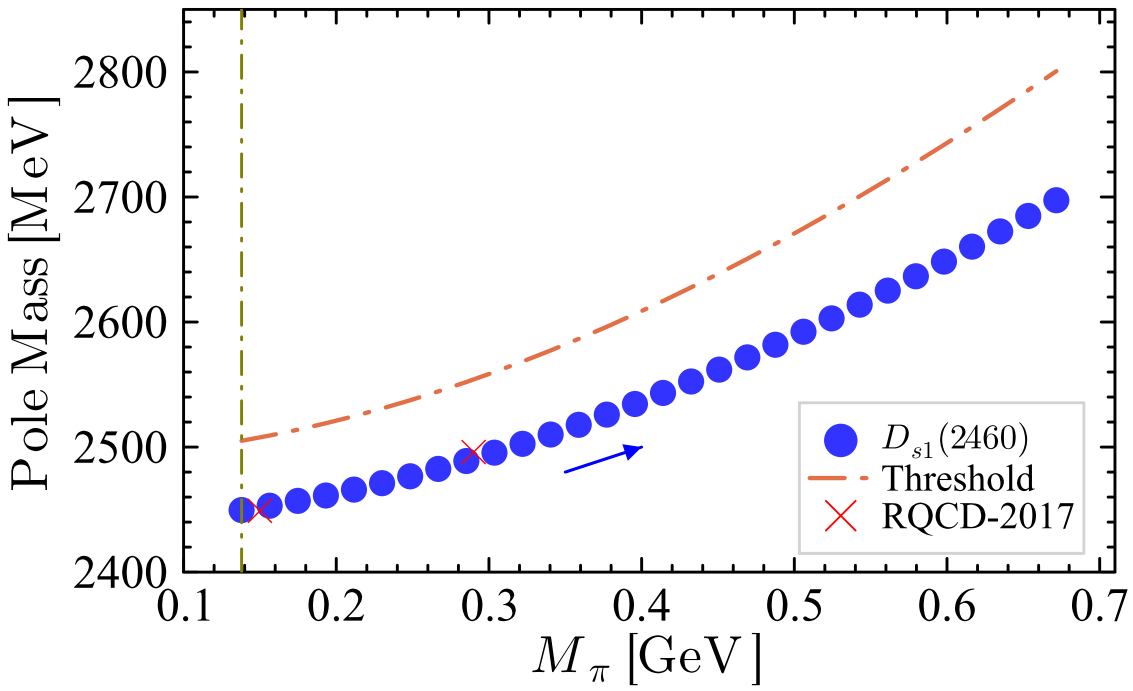
- The SU(3) sextet:





M_π trajectory

- The bound-state pole: (1,0)



- Pion mass dependence

$$M_K^2 = \mathring{M}_K^2 + \frac{1}{2}M_\pi^2$$

$$M_{\eta'/\eta}^2 = M_K^2 + \frac{M_0^2}{2} \pm \frac{R}{2}$$

$$\sin\theta = -\left(\sqrt{1+\frac{S^2}{32\Delta^4}}\right)^{-1}$$

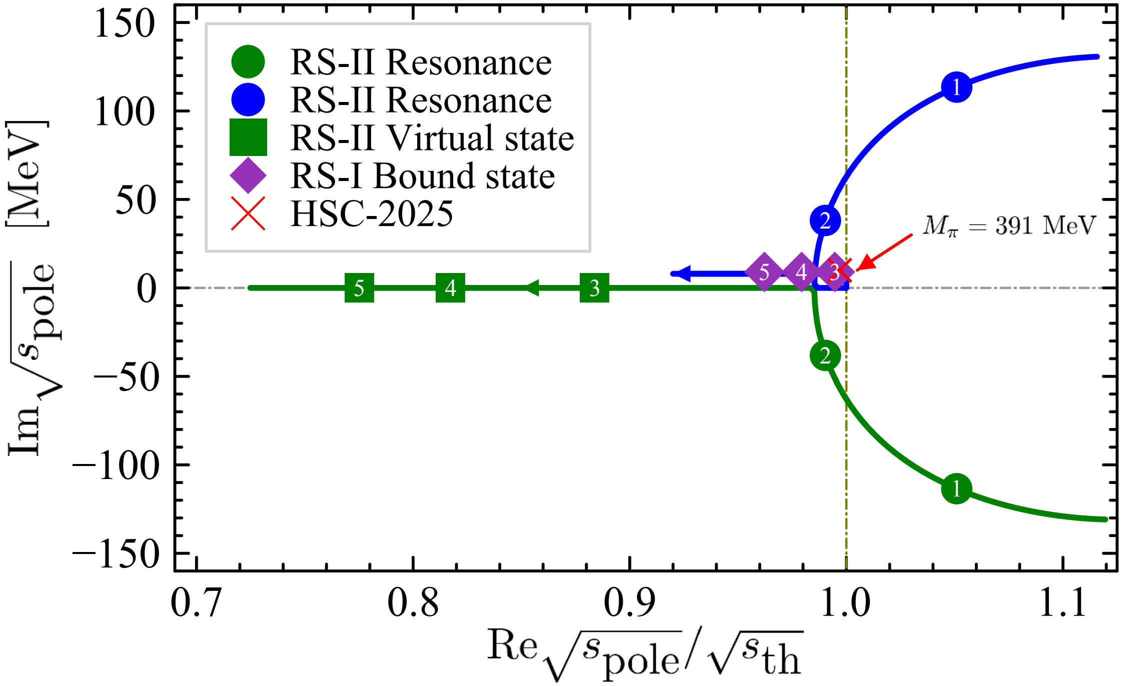
$$M_{D^*}^2 = \mathring{M}_{D^*}^2 + 2(2h_0^* + h_1^*)M_\pi^2$$

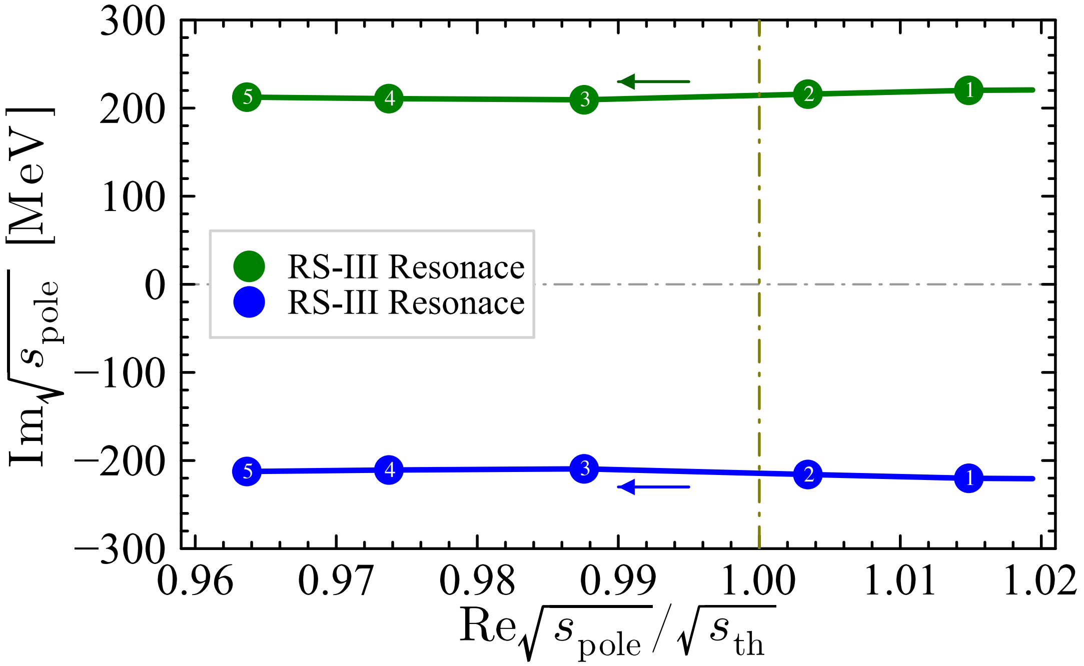
$$M_{D_s^*}^2 = \mathring{M}_{D_s^*}^2 + 4h_0^* M_\pi^2$$

- Remains as a bound state as M_π increase
- In consistent with lattice QCD results

M_π trajectory

- The two poles in (0,1/2)

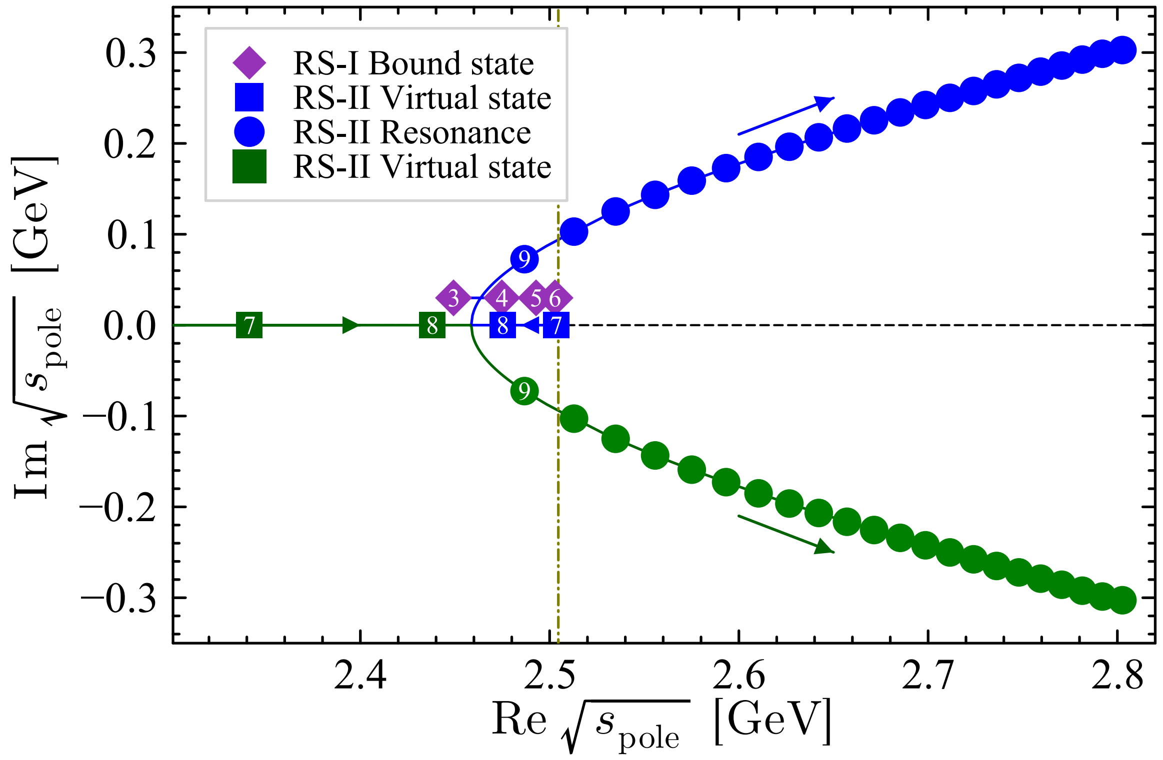




- **Lower pole (left):** a pair of resonances $\longrightarrow$ two virtual states $\longrightarrow$ a bound state + a virtual state
- **Higher pole (right):** always a pair of resonances

N_C trajectory

- The bound-state pole in (1,0)



- N_C dependence

$$M_\pi, M_K, \overline{M}_{P*} \sim \mathcal{O}(1)$$

$$F_0 \sim \mathcal{O}(\sqrt{N_C})$$

$$M_0^2 \sim 1/N_C$$

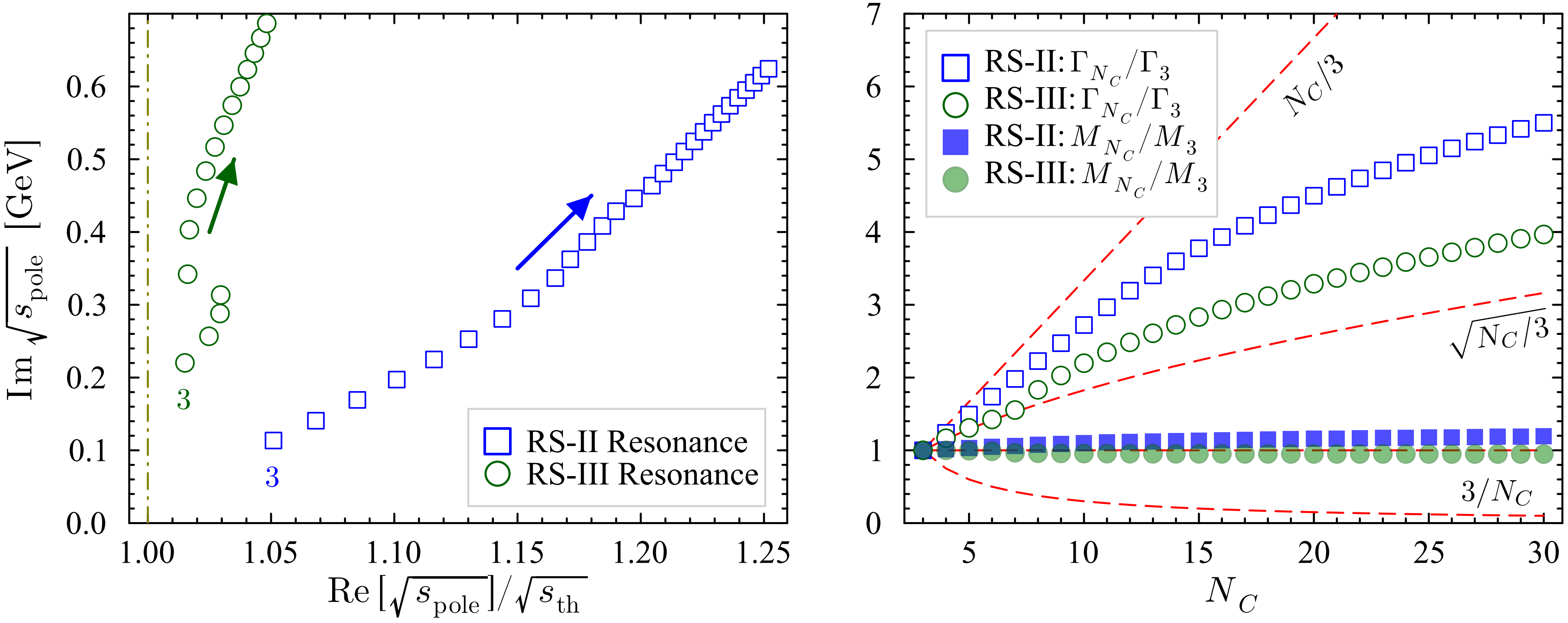
$$h^*_{1,3,5} \sim \mathcal{O}(1) \quad h^*_{2,4,5} \sim \mathcal{O}(1/N_C)$$

$$a(\mu) \sim \mathcal{O}(1)$$

- Evolution of the pole: **bound state** $\longrightarrow$ virtual state $\longrightarrow$ resonance, to complex infinity as $N_C \to \infty$
- $D_{s1}(2460)$ is of non-$\bar{q}q$ nature

N_C trajectory

- The two poles in (0,1/2)



  - Both the two poles of $D_1(2430)$ flow to complex infinity as $N_C \to \infty$;
  - Scale as N_C^α with $1/2 \leq \alpha < 1$;
  - $D_1(2430)$ is of non-$\bar{q}q$ nature

Summary and outlook

Summary and outlook

- **Chiral potentials of $P^*\phi$ interaction up to NLO**
  - Relativistic kinematics: polarization vectors
  - U(3) framework: the contribution of η_0 singlet
  - Both contact and meson-exchange (EX) contributions: the S-wave EX is negligible

- **Systematical Generation of axial-vector charmed mesons**
  - $D_{s1}(2460)$: bound-state pole, member of SU(3) triplet
  - $D_1(2430)$: two resonance poles, lower/higher pole belonging to SU(3) triplet/sextet
  - Inner structure: both of them are of non-$\bar{q}q$ nature

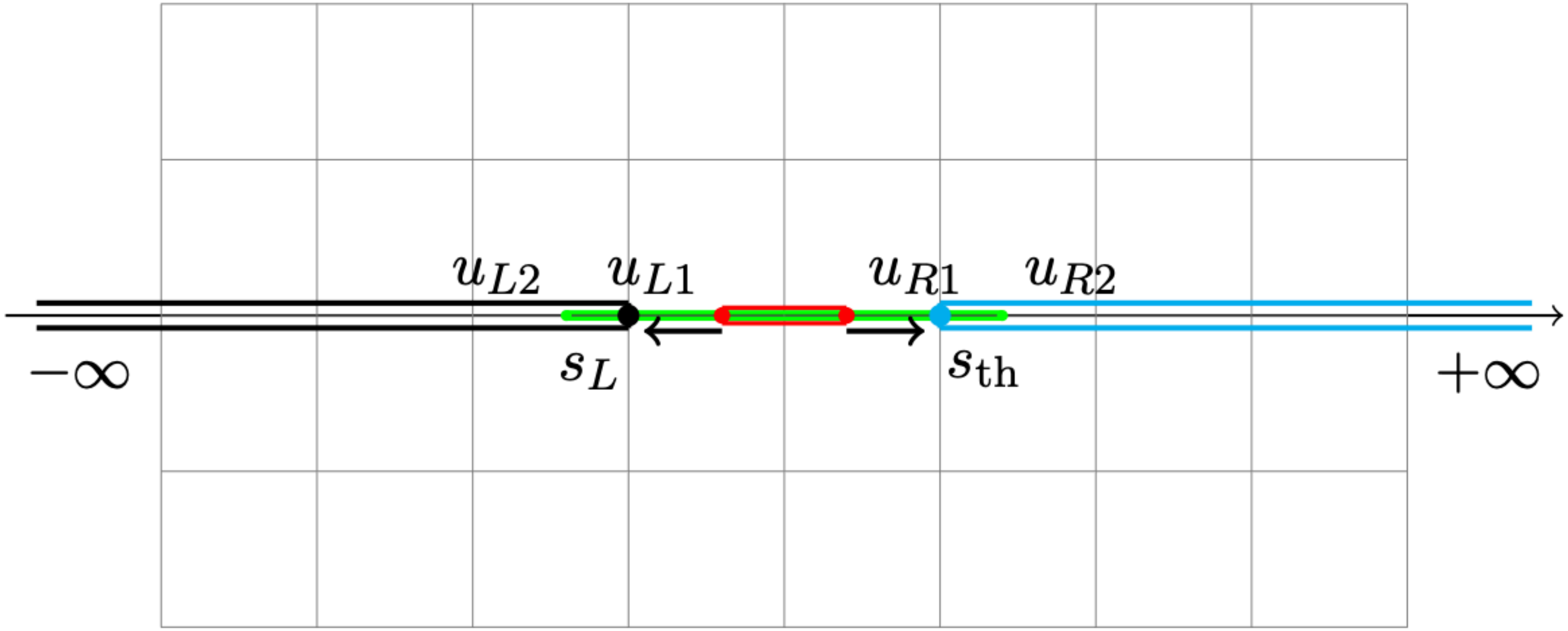
- **Outlook**
  - Correlation functions for femtoscopy studies in experiments
  - Off-shell treatment of the BSE, free of unphysical cuts of on-shell artifact

Thanks!

Backup

Left-hand cut

- The left-hand cut induced by u-channel meson exchange
  - s-channel cuts: $s_L = (M_D^* - M_\pi)^2, \;\; s_R = (M_D^* + M_\pi)^2$
  - Segment cut (green) from u-channel D^* exchange: $u_{L1} = (M_{D^*}^2 - M_\pi^2)^2/M_{D^*}^2 \;, \quad u_{R1} = 2M_\pi^2 + M_{D^*}^2 \;.$
  - Segment cut (red) from u-channel D exchange: $u_{L2} = (M_{D^*}^2 - M_\pi^2)^2/M_D^2 \;, u_{R2} = 2M_\pi^2 + 2M_{D^*}^2 - M_D^2 \;.$



NLO potential

- **Explicit expressions of the potential functions:**
  - NLO Contact

$$\mathscr{H}_{24}(s,t) = 2h_2^* \, p_2 \cdot p_4 + h_4^* \left(p_1 \cdot p_2 p_3 \cdot p_4 + p_1 \cdot p_4 p_2 \cdot p_3\right) ,$$
$$\mathscr{H}_{35}(s,t) = h_3^* \, p_2 \cdot p_4 + h_5^* \left(p_1 \cdot p_2 p_3 \cdot p_4 + p_1 \cdot p_4 p_2 \cdot p_3\right) .$$

  - LO meson exchange

$$\mathscr{G}_P(s,t) = \frac{p_2 \cdot \epsilon_1 p_4 \cdot \epsilon_3^*}{(p_1+p_2)^2 - M_{h^2}} ,$$
$$\mathscr{G}_{P^*}(s,t) = \frac{1}{(p_1+p_2)^2 - M_{h^*}^2} \Big\{ p_1 \cdot \epsilon_3^* \left[p_2 \cdot p_4 p_3 \cdot \epsilon_1 - p_2 \cdot p_3 p_4 \cdot \epsilon_1\right] + p_2 \cdot \epsilon_3^*$$
$$\times \left[p_1 \cdot p_3 p_4 \cdot \epsilon_1 - p_1 \cdot p_4 p_3 \cdot \epsilon_1\right] + \epsilon_1 \cdot \epsilon_3^* \left[p_2 \cdot p_3 p_1 \cdot p_4 - p_1 \cdot p_3 p_2 \cdot p_4\right] \Big\} .$$