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第九届强子谱与强子结构研讨会

arXiv: 2606.04690

The $\Sigma(1380)$ and other elusive resonances in the reaction $\Lambda_c^+ \to \eta\pi^+\Lambda$ via coupled-channel dynamics

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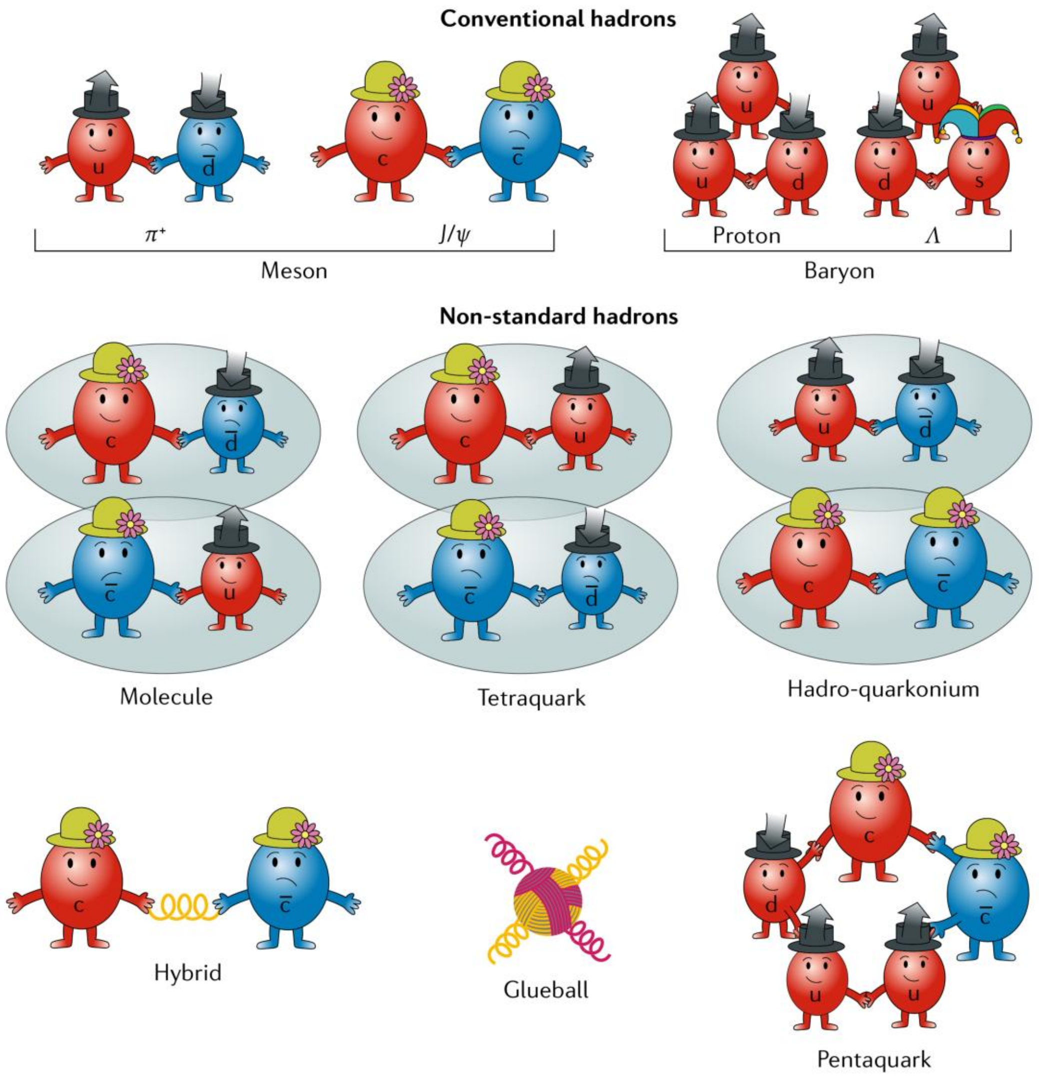
2026年8月20日@新乡

Background



➢Conventional hadrons:

Meson

Baryon

➢Non-standard hadrons:

Molecule

Tetraquark

Pentaquark

Hybrid

Glueball

Nature Rev. Phys. 1 , 480 (2019)

Low-lying baryons with $J^P=1/2^-$

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Workman R L, *et al.*, Review of Particle Physics (2024).

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Λ_c^+	$1/2^+$	****	Λ_b^0	$1/2^+$	***
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	****	Σ^0	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***	$\Lambda_b(5912)^0$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***	$\Lambda_b(5920)^0$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Lambda_c(2765)^+$		*	$\Lambda_b(6070)^0$	$1/2^+$	***
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1580)$	$3/2^-$	*	$\Lambda_c(2860)^+$	$3/2^+$	***	$\Lambda_b(6146)^0$	$3/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	***	$\Sigma(1620)$	$1/2^-$	*	$\Lambda_c(2880)^+$	$5/2^+$	***	$\Lambda_b(6152)^0$	$5/2^+$	***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1660)$	$1/2^+$	***	$\Lambda_c(2910)^+$		*	Σ_b	$1/2^+$	***
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1670)$	$3/2^-$	****	$\Lambda_c(2940)^+$	$3/2^-$	***	Σ_b^*	$3/2^+$	***
$N(1700)$	$3/2^-$	***	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1750)$	$1/2^-$	***	$\Sigma_c(2455)$	$1/2^+$	****	$\Sigma_b(6097)^+$		***
$N(1710)$	$1/2^+$	****	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1775)$	$5/2^-$	****	$\Sigma_c(2520)$	$3/2^+$	***	$\Sigma_b(6097)^-$		***
$N(1720)$	$3/2^+$	****	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1780)$	$3/2^+$	*	$\Sigma_c(2800)$		***	Ξ_b^-	$1/2^+$	***
$N(1860)$	$5/2^+$	**	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1880)$	$1/2^+$	**	Ξ_c^+	$1/2^+$	***	Ξ_b^0	$1/2^+$	***
$N(1875)$	$3/2^-$	***	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1900)$	$1/2^-$	**	Ξ_c^0	$1/2^+$	****	$\Xi'_b(5935)^-$	$1/2^+$	***
$N(1880)$	$1/2^+$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1910)$	$3/2^-$	***	$\Xi_c'^+$	$1/2^+$	***	$\Xi_b(5945)^0$	$3/2^+$	***
$N(1895)$	$1/2^-$	****	$\Delta(2200)$	$7/2^-$	***	$\Sigma(1915)$	$5/2^+$	****	$\Xi_c'^0$	$1/2^+$	***	$\Xi_b(5955)^-$	$3/2^+$	***
$N(1900)$	$3/2^+$	****	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1940)$	$3/2^+$	*	$\Xi_c(2645)$	$3/2^+$	***	$\Xi_b(6087)^0$	$3/2^-$	***
$N(1990)$	$7/2^+$	**	$\Delta(2350)$	$5/2^-$	*	$\Sigma(2010)$	$3/2^-$	*	$\Xi_c(2790)$	$1/2^-$	***	$\Xi_b(6095)^0$	$3/2^-$	***
$N(2000)$	$5/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(2030)$	$7/2^+$	****	$\Xi_c(2815)$	$3/2^-$	***	$\Xi_b(6100)^-$	$3/2^-$	***
$N(2040)$	$3/2^+$	*	$\Delta(2400)$	$9/2^-$	**	$\Sigma(2070)$	$5/2^+$	*	$\Xi_c(2882)$		*	$\Xi_b(6227)^-$		***
$N(2060)$	$5/2^-$	***	$\Delta(2420)$	$11/2^+$	****	$\Sigma(2080)$	$3/2^+$	*	$\Xi_c(2923)$		**	$\Xi_b(6227)^0$		***
$N(2100)$	$1/2^+$	***	$\Delta(2750)$	$13/2^-$	**	$\Sigma(2100)$	$7/2^-$	*	$\Xi_c(2930)$		**	$\Xi_b(6327)^0$		***
$N(2120)$	$3/2^-$	***	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2110)$	$1/2^-$	*	$\Xi_c(2970)$	$1/2^+$	***	$\Xi_b(6333)^0$		***
$N(2190)$	$7/2^-$	****				$\Sigma(2230)$	$3/2^+$	*	$\Xi_c(3055)$		***	Ω_b^-	$1/2^+$	***
$N(2220)$	$9/2^+$	****	Λ	$1/2^+$	****	$\Sigma(2250)$		**	$\Xi_c(3080)$		***	$\Omega_b(6316)^-$		***
$N(2250)$	$9/2^-$	****	$\Lambda(1380)$	$1/2^-$	**	$\Sigma(2455)$		*	$\Xi_c(3123)$		*	$\Omega_b(6330)^-$		***
$N(2300)$	$1/2^+$	**	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2620)$		*	Ω_c^0	$1/2^+$	***	$\Omega_b(6340)^-$		***
$N(2570)$	$5/2^-$	**	$\Lambda(1520)$	$3/2^-$	****	$\Sigma(3000)$		*	$\Omega_c(2770)^0$	$3/2^+$	***	$\Omega_b(6350)^-$		***
$N(2600)$	$11/2^-$	***	$\Lambda(1600)$	$1/2^+$	****	$\Sigma(3170)$		*	$\Omega_c(3000)^0$		***			
$N(2700)$	$13/2^+$	**	$\Lambda(1670)$	$1/2^-$	****				$\Omega_c(3050)^0$		***	$P_{c\bar{c}}(4312)^+$		*
			$\Lambda(1690)$	$3/2^-$	****	Ξ^0	$1/2^+$	****	$\Omega_c(3065)^0$		***	$P_{c\bar{c}s}(4338)^0$	$1/2^-$	*
			$\Lambda(1710)$	$1/2^+$	*	Ξ^-	$1/2^+$	****	$\Omega_c(3090)^0$		***	$P_{c\bar{c}}(4380)^+$		*

These exotic properties of the low-lying excited baryons with the quantum numbers of spin-parity $J^P = 1/2^-$ are difficult to explain in the simple quenched quark model.

EW-Geng-Wu-Xie-Zou, CPL. 41 (2024) 101401

$1/2^-$ baryon nonet with strangeness

Zou, EPJA 35 (2008) 325

- Mass pattern : quenched or unquenched ?

uds (L=1) $1/2^-$ ~ $\Lambda^*(1670)$ ~ $[us][ds]\,\bar{s}$

uud (L=1) $1/2^-$ ~ $N^*(1535)$ ~ $[ud][us]\,\bar{s}$

uds (L=1) $1/2^-$ ~ $\Lambda^*(1405)$ ~ $[ud][su]\,\bar{u}$

uus (L=1) $1/2^-$ ~ $\Sigma^*(1390)$ ~ $[us][ud]\,\bar{d}$

Zou et al, NPA835 (2010) 199 ; CLAS, PRC87(2013)035206

- Strange decays of $N^*(1535)$ and $\Lambda^*(1670)$:

$N^*(1535)$ large couplings $g_{N^*N\eta}$, $g_{N^*K\Lambda}$, $g_{N^*N\eta'}$, $g_{N^*N\phi}$

$\Lambda^*(1670)$ large coupling $g_{\Lambda^*\Lambda\eta}$

Report by Bing-Song Zou

**** Existence is certain, and properties are at least fairly explored.

*** Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions, etc. are not well determined.

** Evidence of existence is only fair.

* Evidence of existence is poor.

Search for $\Sigma(1/2^-)$

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Process	$\Sigma(1/2^-)$	Authors	Reference
$\Lambda_c \to \Lambda\pi^+\pi^-$	$\Sigma(1380)$	Wu-Dulat-Zou	PRD 80 (2009) 017503
$K^-p \to \Lambda\pi^+\pi^-$	$\Sigma(1380)$	Wu-Dulat-Zou	PRC 81 (2010) 045210
$\gamma N \to K^+\Sigma(1/2^-)$	$\Sigma(1380)$	Gao-Wu-Zou	PRC 81 (2010) 055203
$\chi_{c0} \to \bar{\Sigma}\Sigma\pi$	$\Sigma(1430)$	EW-Xie-Oset	PLB 753 (2016) 526
$\Lambda_c \to \eta\pi^+\Lambda$	$\Sigma(1380)$	Xie-Geng	PRD 95 (2017) 074024
$\chi_{c0} \to \bar{\Lambda}\Sigma\pi$	$\Sigma(1380)$, $\Sigma(1430)$	EW-Xie-Oset	PRD 98 (2018) 114017
$\Lambda_c \to \Sigma^0\pi^+\pi^0\pi^0$	$\Sigma(1430)$	Dai-Pavao-Sakai-Oset	PRD 97 (2018) 116004
$\Lambda_c \to \Sigma^+\pi^+\pi^0\pi^-$	$\Sigma(1430)$	Xie-Oset	PLB 792 (2019) 450
$\gamma p \to K^+\Sigma(1/2^-)$	$\Sigma(1400)$	Kim-Nam-Hosaka	PRD 103 (2021) 114017
$\gamma n \to K\Sigma(1/2^-)$	$\Sigma(1480)$	Lyu-EW-Xie-Wei	CPC 47 (2023) 053108
$\Lambda_c^+ \to \Lambda\pi^-\pi^+\pi^+$	$\Sigma(1430)$	Li-Song-Oset-Liang-Molina	EPJC 85 (2025) 1086
$\Lambda_c^+ \to \Lambda\pi^-\pi^+\pi^+$	$\Sigma(1430)$	Jun He	PRC 112 (2025) 015205

Evidence of $\Sigma(1/2^-)$

□ $\Lambda_c \to \Lambda\pi^+\pi^-$ **Wu-Dulat-Zou, PRD 80 (2009) 017503**

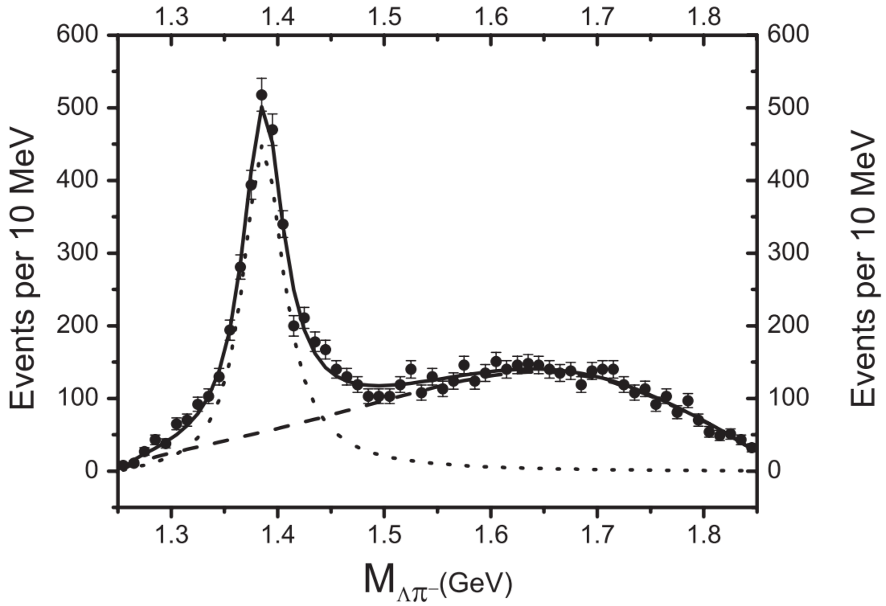
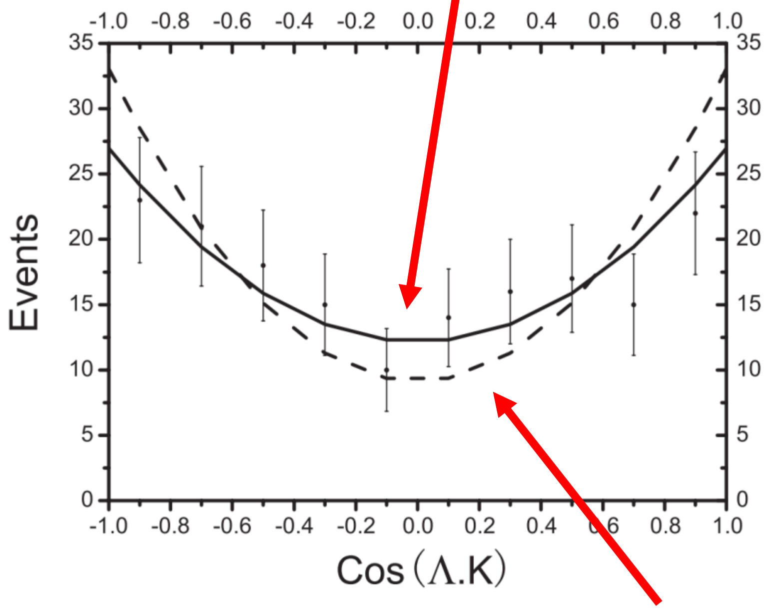


Fig. 1

Fit2



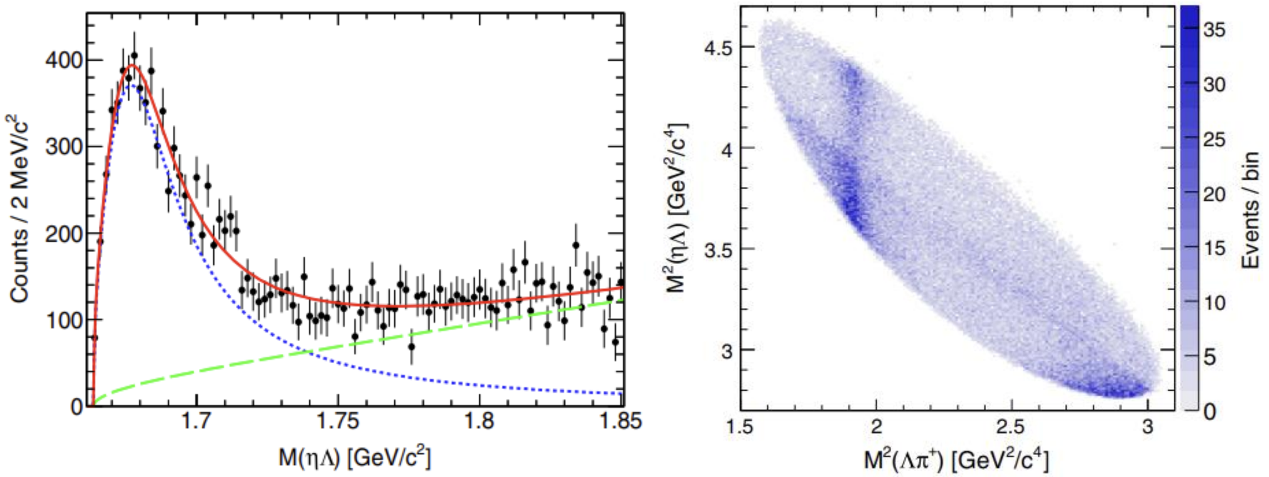
Fit1

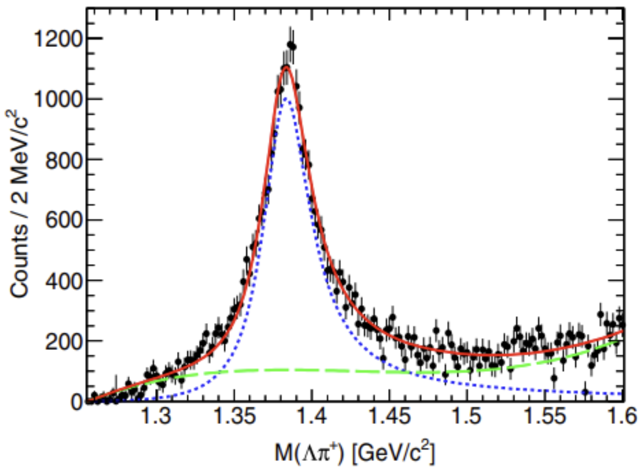
Fig. 2

	$M_{\Sigma^*(3/2)}$	$\Gamma_{\Sigma^*(3/2)}$	$M_{\Sigma^*(1/2)}$	$\Gamma_{\Sigma^*(1/2)}$	χ^2/ndf (Fig. 1)	χ^2/ndf (Fig. 2)
Fit1	1385.3 ± 0.7	46.9 ± 2.5			68.5/54	10.1/9
Fit2	$1386.1^{+1.1}_{-0.9}$	$34.9^{+5.1}_{-4.9}$	$1381.3^{+4.9}_{-8.3}$	$118.6^{+55.2}_{-35.1}$	58.0/51	3.2/9

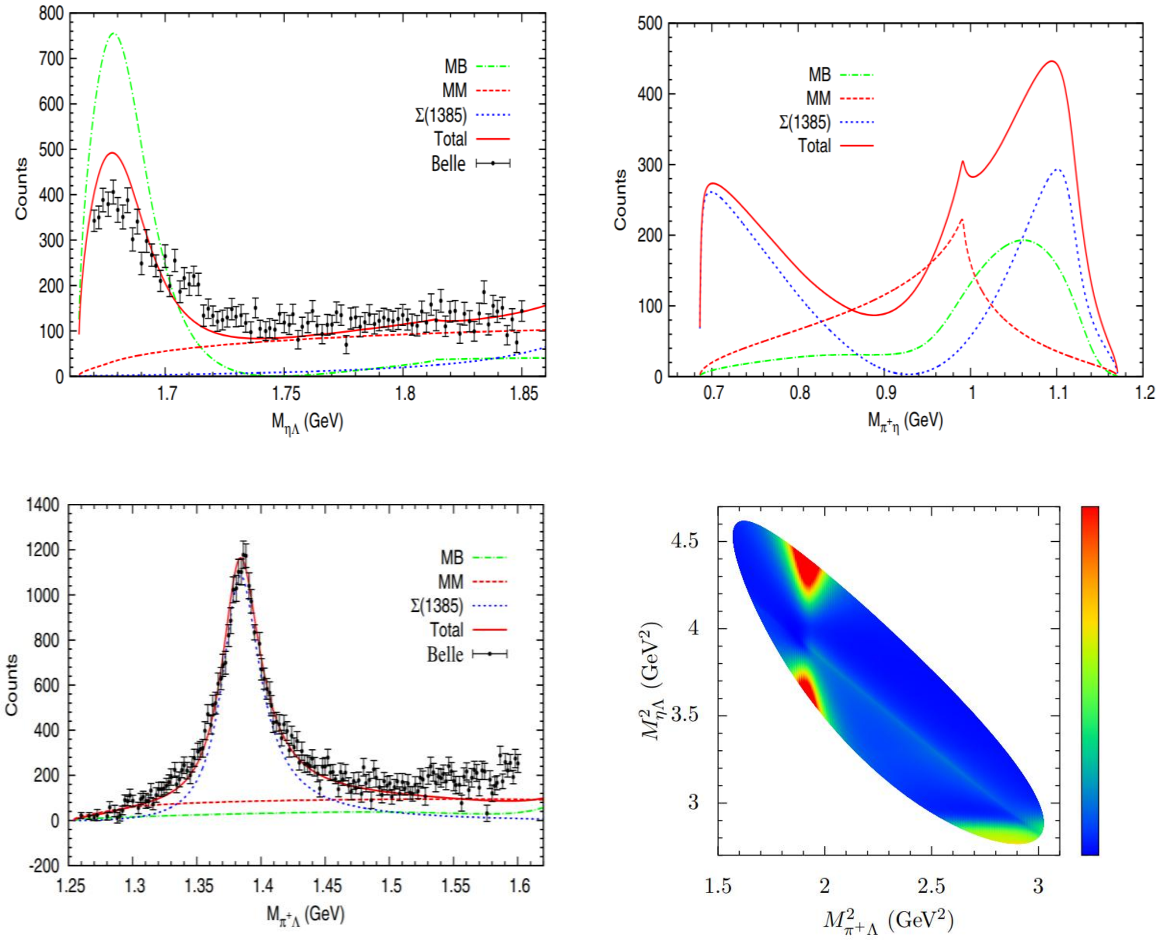
Analyzing the Belle data

- In 2021, the Belle Collaboration measured the process $\Lambda_c^+ \to \Lambda\eta\pi^+$





By regarding the $\Lambda(1670)$ as a molecule state and considering the contributions of $a_0(980)$ and $\Sigma(1385)$, this work could well reproduce the Belle data of the mass distributions.



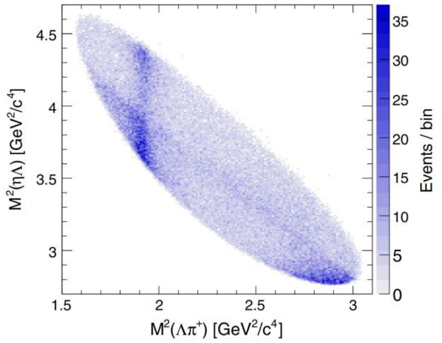
Belle Collaboration, PRD 103 (2021) 052005

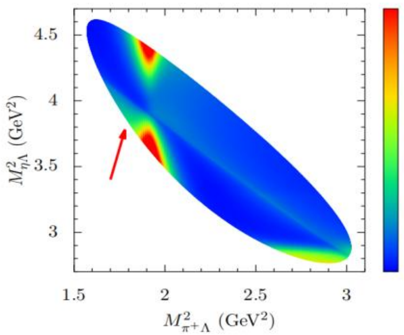
PRD 106 (2022) 056001

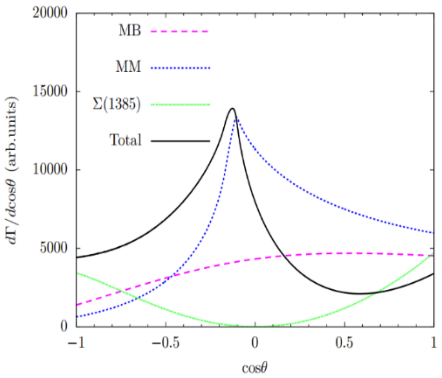
Reanalyzing the Belle data

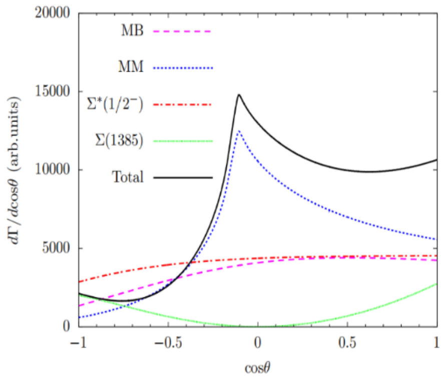
- $\Lambda_c^+ \to \Lambda\eta\pi^+$

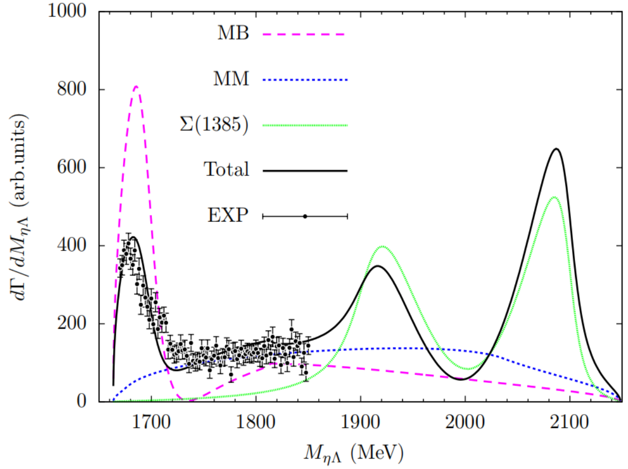
$$
\begin{aligned}
|\mathcal{T}^{\text{Total}}|^2 = & |\mathcal{T}^{MM} + \mathcal{T}^{MB} e^{i\phi} + \mathcal{T}^{\Sigma(1385)} e^{i\phi'} \\
& + \mathcal{T}^{\Sigma^*(1/2^-)} e^{i\phi''}|^2,
\end{aligned}
$$

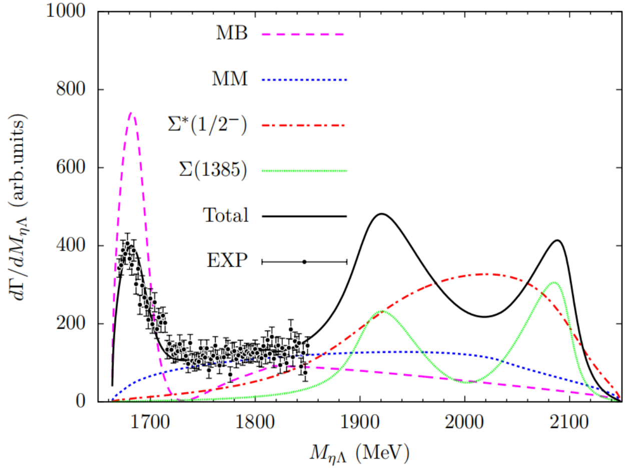


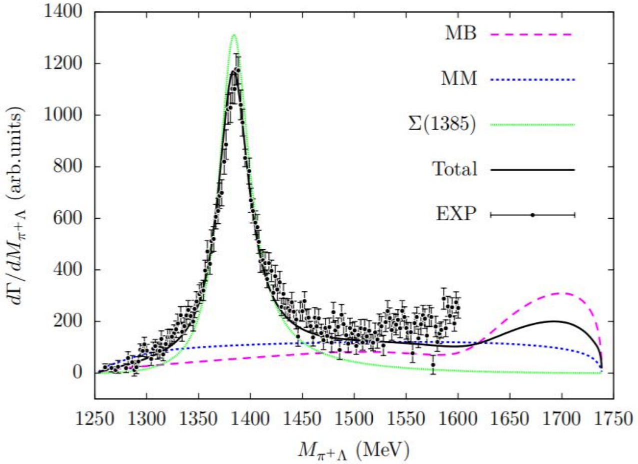


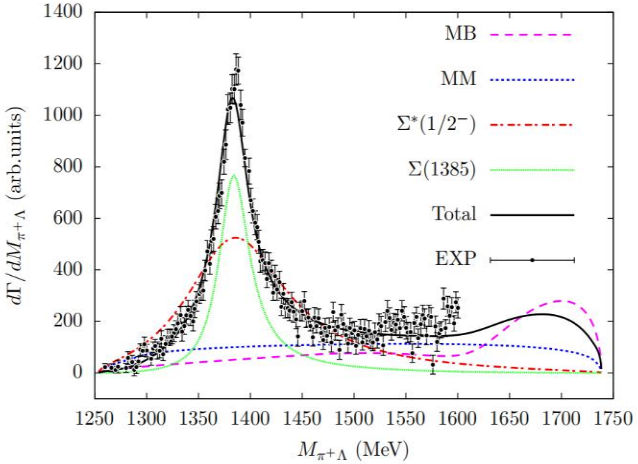












Restricted to $M_{\pi\Lambda} \geq 1440$ MeV and $M_{\eta\Lambda} \geq 1720$ MeV

Phys. Rev. D 110 (2024) 054020

The BESIII measurement

- In 2025, $\Lambda_c^+ \to \Lambda\eta\pi^+$ was subsequently measured by the BESIII Collaboration **BESIII Collaboration, Phys. Rev. Lett. 134 (2025) 021901**

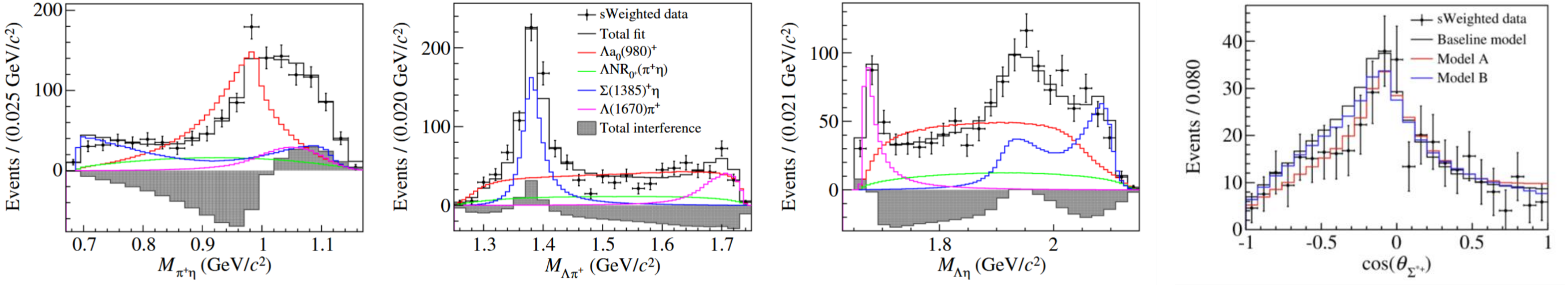
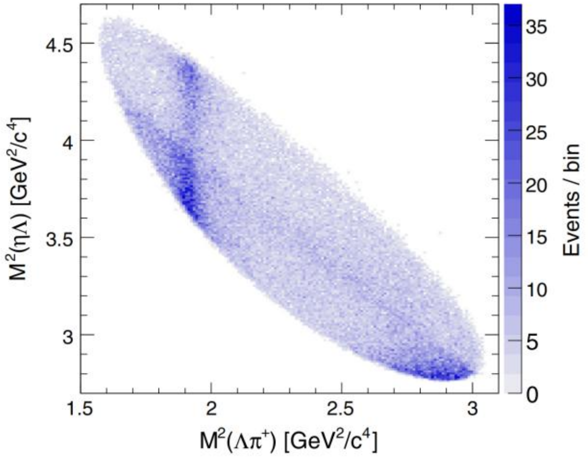


TABLE II. Fit results of FFs and statistical significances for different components in alternative models including $\Sigma(1380)^+$. The total FFs are 115.8% and 119.8% for models A and B, respectively. The uncertainties are statistical only.

Process	Model A	Model B
$\Lambda a_0(980)^+$	$52.9 \pm 4.5(13.4\sigma)$	$50.6 \pm 8.0(11.1\sigma)$
$\Sigma(1385)^+\eta$	$36.6 \pm 2.6(15.8\sigma)$	$31.3 \pm 3.0(14.6\sigma)$
$\Lambda(1670)\pi^+$	$10.7 \pm 1.4(15.0\sigma)$	$9.0 \pm 1.6(11.9\sigma)$
$\Sigma(1380)^+\eta$	$15.5 \pm 4.4(6.1\sigma)$	$17.7 \pm 5.7(3.3\sigma)$
ΛNR_{0^+}	$\cdots$	$11.3 \pm 4.4(4.2\sigma)$



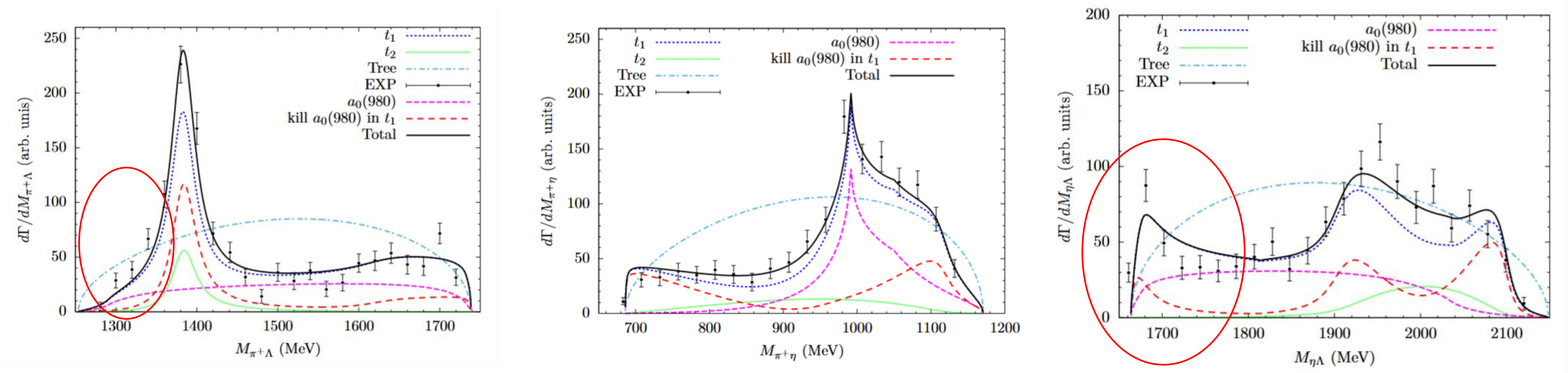
In spite of the small signal seen in the Dalitz plot, the analysis reports a branching fraction of approximately 50% for the $\Lambda a_0^+(980)$ decay mode.

Belle Collaboration, PRD 103 (2021) 052005

Analyzing the BESIII data

□ The invariant mass distributions of the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay

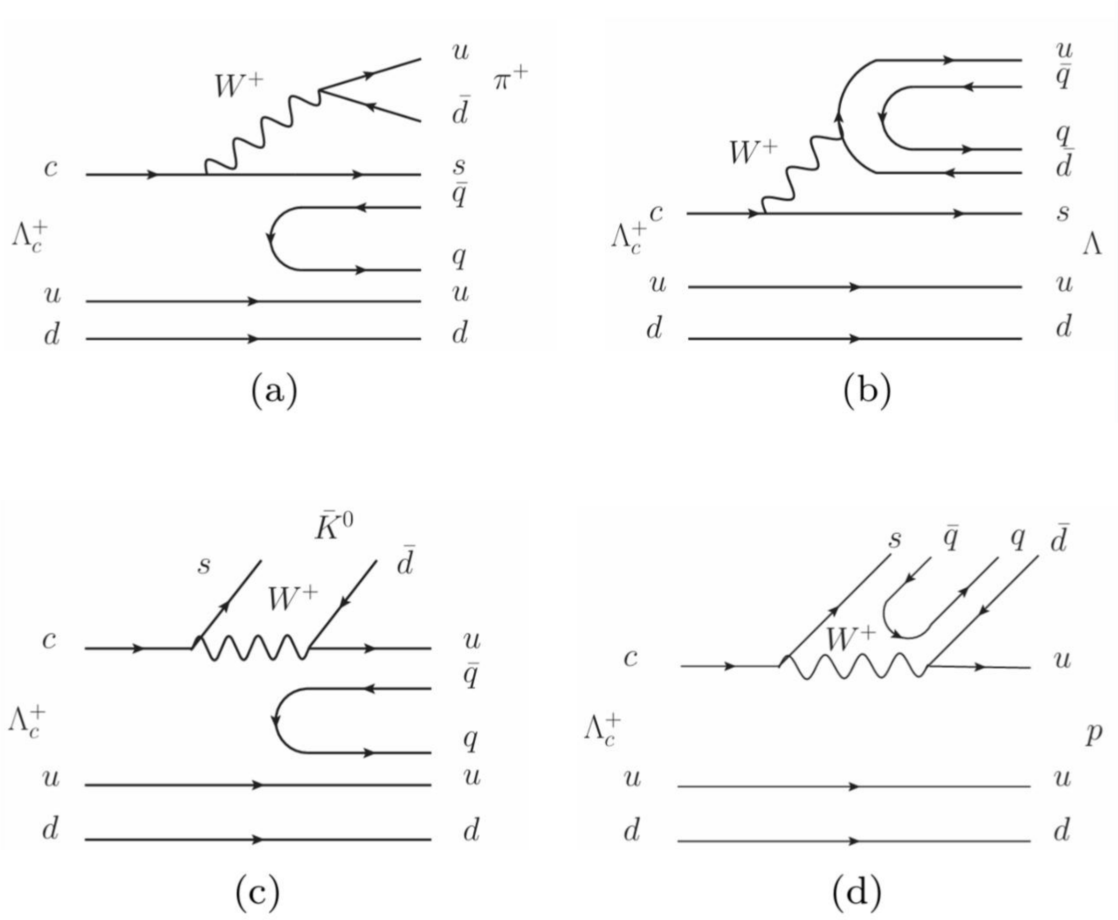
Phys. Rev. D 111 (2025) 016004



Due to the inadequate description in certain regions and the relatively large bin widths of the data, a more refined analysis of this decay process is still needed.

Formalism

□ Quark level diagram



□ Hadronization

(a) $$\Lambda_c^+ \Rightarrow \pi^+ \left\{ \frac{1}{\sqrt{2}} K^- p + \frac{1}{\sqrt{2}} \bar{K}^0 n + \frac{1}{3} \eta \Lambda \right\}$$

(b) $$\Lambda_c^+ \Rightarrow \frac{\sqrt{6}}{3} \Lambda \left\{ \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} \right) \pi^+ + \pi^+ \left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} \right) + K^+ \bar{K}^0 \right\}$$

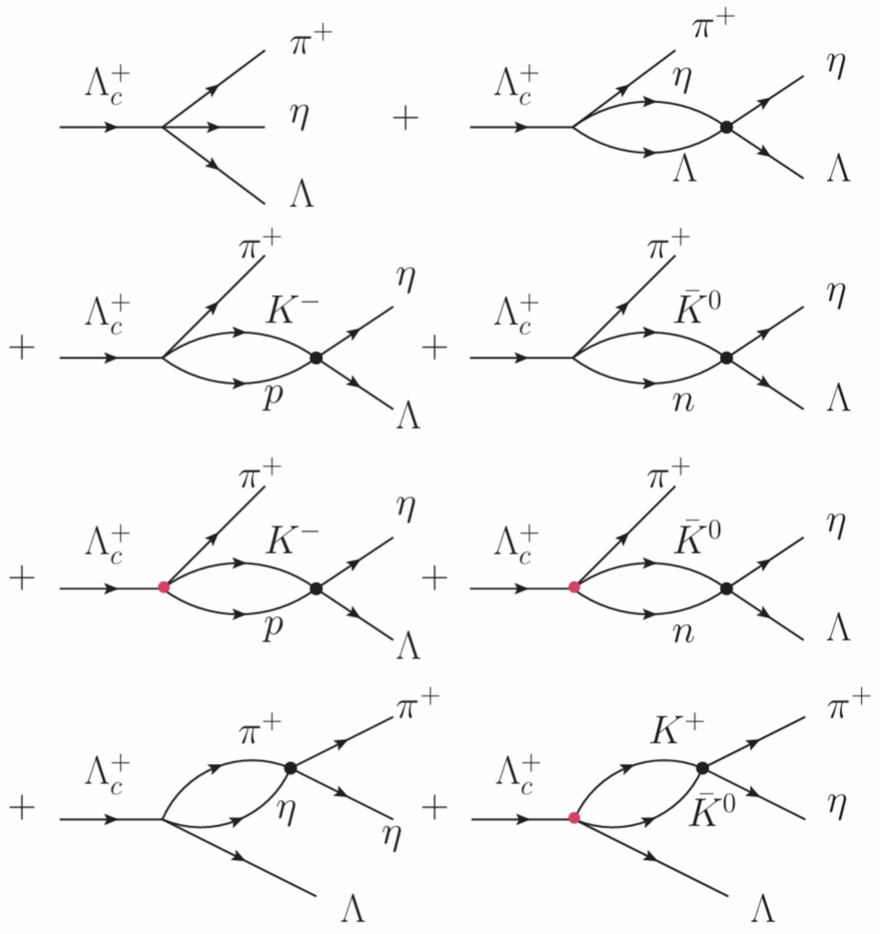
No contributions $\Rightarrow$ $\mathcal{L}_{WPP} \propto W^\mu [P, \partial_\mu P]$

(c) $$\Lambda_c^+ \Rightarrow \frac{\bar{K}^0}{\sqrt{2}} \left\{ \left(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} \right) p + \pi^+ n - \sqrt{\frac{2}{3}} K^+ \Lambda \right\}$$

(d) $$\Lambda_c^+ \Rightarrow \frac{p}{\sqrt{2}} \left\{ K^- \pi^+ - \frac{1}{\sqrt{2}} \bar{K}^0 \pi^0 \right\}$$

Formalism

□ Mechanisms for tree level and rescattering



□ Hadronization

$$\Lambda_c^+ \Rightarrow \frac{1}{3}\pi^+\eta\Lambda + \frac{1}{\sqrt{2}}\pi^+K^-p + \frac{1}{\sqrt{2}}\pi^+\bar{K}^0 n$$
$$+\gamma\left\{\frac{1}{\sqrt{2}}\pi^+\bar{K}^0 n - \frac{1}{\sqrt{3}}\bar{K}^0K^+\Lambda + \frac{1}{\sqrt{2}}\pi^+K^-p\right\}.$$

□ Amplitudes

$$\tilde{\mathcal{T}} = A\left(\mathcal{T}_{\text{Tree}} + \mathcal{T}_{MB} + \mathcal{T}_{MM}\right)$$

$$\mathcal{T}_{\text{Tree}} = h_{\pi^+\eta\Lambda},$$

$$\begin{aligned}\mathcal{T}_{MB} = {} & h_{\pi^+\eta\Lambda}G_{\eta\Lambda}(M_{\text{inv}}(\eta\Lambda))T_{\eta\Lambda,\eta\Lambda}(M_{\text{inv}}(\eta\Lambda)) \\ & +h_{\pi^+\bar{K}N}G_{K^-p}(M_{\text{inv}}(\eta\Lambda))T_{K^-p,\eta\Lambda}(M_{\text{inv}}(\eta\Lambda)) \\ & +h_{\pi^+\bar{K}N}G_{\bar{K}^0n}(M_{\text{inv}}(\eta\Lambda))T_{\bar{K}^0n,\eta\Lambda}(M_{\text{inv}}(\eta\Lambda)), \\ & +\gamma e^{i\phi'}h_{\pi^+\bar{K}N}G_{K^-p}(M_{\text{inv}}(\eta\Lambda))T_{K^-p,\eta\Lambda}(M_{\text{inv}}(\eta\Lambda)) \\ & +\gamma e^{i\phi'}h_{\pi^+\bar{K}N}G_{\bar{K}^0n}(M_{\text{inv}}(\eta\Lambda))T_{\bar{K}^0n,\eta\Lambda}(M_{\text{inv}}(\eta\Lambda)),\end{aligned}$$

$$\mathcal{T}_{MM} = h_{\pi^+\eta\Lambda}G_{\pi^+\eta}(M_{\text{inv}}(\pi^+\eta))T_{\pi^+\eta,\pi^+\eta}(M_{\text{inv}}(\pi^+\eta))$$
$$+\gamma e^{i\phi'}h_{K^+\bar{K}^0\Lambda}G_{K^+\bar{K}^0}(M_{\text{inv}}(\pi^+\eta))T_{K^+\bar{K}^0,\pi^+\eta}(M_{\text{inv}}(\pi^+\eta)).$$

Formalism

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The MB amplitudes

$$
\begin{aligned}
G_l &= i\int \frac{d^4q}{(2\pi)^4}\frac{M_l}{E_l(\vec{q}\,)}\frac{1}{\sqrt{s}-q^0-E_l(\vec{q}\,)+i\epsilon}\frac{1}{q^2-m_l^2+i\epsilon} \\
&= \frac{2M_l}{16\pi^2}\Bigg\{a_l(\mu)+\ln\frac{M_l^2}{\mu^2}+\frac{s+m_l^2-M_l^2}{2s}\ln\frac{m_l^2}{M_l^2} \\
&\quad+\frac{|\vec{q}\,|}{\sqrt{s}}\Big[\ln\Big(s-\big(M_l^2-m_l^2\big)+2|\vec{q}\,|\sqrt{s}\Big) \\
&\quad+\ln\Big(s+\big(M_l^2-m_l^2\big)+2|\vec{q}\,|\sqrt{s}\Big) \\
&\quad-\ln\Big(-s+\big(M_l^2-m_l^2\big)+2|\vec{q}\,|\sqrt{s}\Big) \\
&\quad-\ln\Big(-s-\big(M_l^2-m_l^2\big)+2|\vec{q}\,|\sqrt{s}\Big)\Big]\Bigg\},
\end{aligned}
$$

$a_{K\Xi}$	$a_{\bar{K}N}$	$a_{\pi\Lambda}$	$a_{\eta\Sigma}$	$a_{\pi\Sigma}$	$a_{\eta\Lambda}$
-2.776	-1.84	-1.83	-2.38	-2.00	-2.25

Eur. Phys. J. C 84 (2024) 1253

The scattering amplitudes

$$T=[1-VG]^{-1}V,$$

$$V_{ij}=-C_{ij}\frac{1}{4f^2}(k^0+k'^0)$$

	K^-p	$\bar{K}^0n$	$\pi^0\Lambda$	$\pi^0\Sigma^0$	$\eta\Lambda$	$\eta\Sigma^0$	$\pi^+\Sigma^-$	$\pi^-\Sigma^+$	$K^+\Xi^-$	$K^0\Xi^0$
K^-p	2	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{\sqrt{3}}{2}$	0	1	0	0
$\bar{K}^0n$		2	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$-\frac{\sqrt{3}}{2}$	1	0	0	0
$\pi^0\Lambda$			0	0	0	0	0	0	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$
$\pi^0\Sigma^0$				0	0	0	2	2	$\frac{1}{2}$	$\frac{1}{2}$
$\eta\Lambda$					0	0	0	0	$\frac{3}{2}$	$\frac{3}{2}$
$\eta\Sigma^0$						0	0	0	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$
$\pi^+\Sigma^-$							2	0	1	0
$\pi^-\Sigma^+$								2	0	1
$K^+\Xi^-$									2	1
$K^0\Xi^0$										2

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Formalism

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□ The MM amplitudes

$$\mathcal{T}_{MM} = h_{\pi^+\eta\Lambda} G_{\pi^+\eta}(M_{\rm inv}(\pi^+\eta)) T_{\pi^+\eta,\pi^+\eta}(M_{\rm inv}(\pi^+\eta)) + \gamma e^{i\phi'} h_{K^+\bar{K}^0\Lambda} G_{K^+\bar{K}^0}(M_{\rm inv}(\pi^+\eta)) T_{K^+\bar{K}^0,\pi^+\eta}(M_{\rm inv}(\pi^+\eta)),$$

$K^+\overline{K}^0$ (1) and $\pi^+\eta$ (2)

$$\begin{aligned} V_{11} &= -\frac{s}{4f^2}, \quad (f = 93 \text{ MeV}), \\ V_{12} &= -\frac{1}{3\sqrt{3}f^2}(3s - 2m_K^2 - m_\eta^2), \\ V_{22} &= -\frac{2m_\pi^2}{3f^2}, \end{aligned}$$

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□ The cutoff method

$$\begin{aligned} G(s) = &\frac{1}{16\pi^2 s}\left\{\sigma\left(\arctan\frac{s+\Delta}{\sigma\lambda_1} + \arctan\frac{s-\Delta}{\sigma\lambda_2}\right)\right. \\ &- \left[(s+\Delta)\ln\frac{q_{\max}(1+\lambda_1)}{m_1}\right. \\ &\left.\left. + (s-\Delta)\ln\frac{q_{\max}(1+\lambda_2)}{m_2}\right]\right\}, \end{aligned}$$

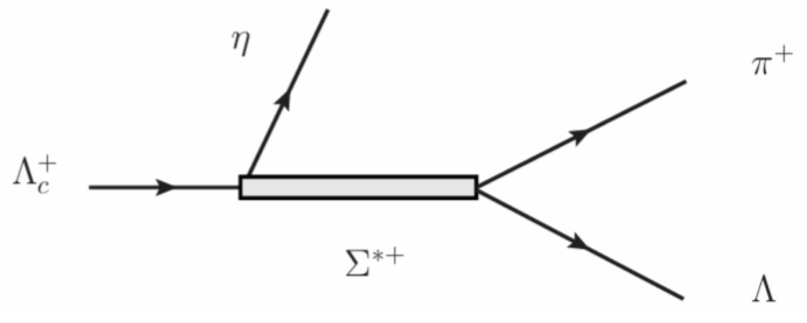
$$\sigma = \left[-\left(s-(m_1+m_2)^2\right)\left(s-(m_1-m_2)^2\right)\right]^{1/2},$$

$$\Delta = m_1^2 - m_2^2,$$

$$\lambda_1 = \sqrt{1+\frac{m_1^2}{q_{\max}^2}}, \quad \lambda_2 = \sqrt{1+\frac{m_2^2}{q_{\max}^2}},$$

Formalism

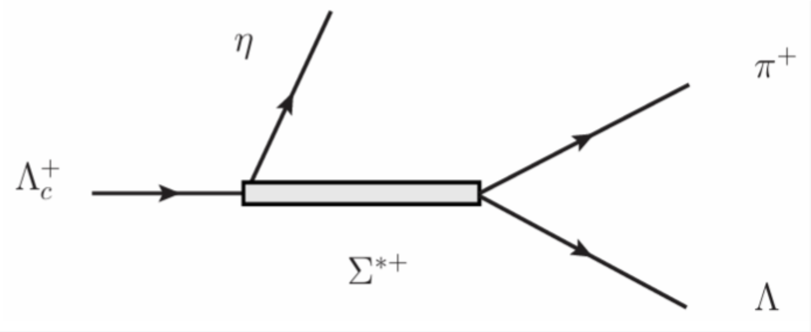
- **The mechanism from the intermediate $\Sigma(1385)$**



$$\begin{aligned}\mathcal{T}_{\Sigma(1385)} &= \frac{\beta e^{i\phi}}{M_\Lambda}\left\langle m' \left| S_i \vec{P}^*_{\pi^+ i} \right| M \right\rangle \left\langle M \left| S_j^\dagger \vec{P}^*_{\eta j} \right| m \right\rangle \mathcal{D} \\ &= \frac{\beta e^{i\phi}}{M_\Lambda}\mathcal{D}\left\langle m' \left| \frac{2}{3}\delta_{ij} - \frac{i}{3}\epsilon_{ijs}\sigma_s \right| m \right\rangle \vec{P}^*_{\pi^+ i}\vec{P}^*_{\eta j} \\ &= \frac{\beta e^{i\phi}}{M_\Lambda}\mathcal{D}\left\langle m' \left| \frac{2}{3}\vec{P}^*_{\pi^+}\cdot\vec{P}^*_{\eta} - \frac{i}{3}\epsilon_{ijs}\sigma_s \vec{P}^*_{\pi^+ i}\vec{P}^*_{\eta j} \right| m \right\rangle,\end{aligned}$$

$$\mathcal{D} = \frac{1}{M_{\text{inv}}(\pi^+\Lambda) - M_{\Sigma(1385)} + i\Gamma_{\Sigma(1385)}/2},$$

- **The mechanism from the intermediate $\Sigma(1380)$**



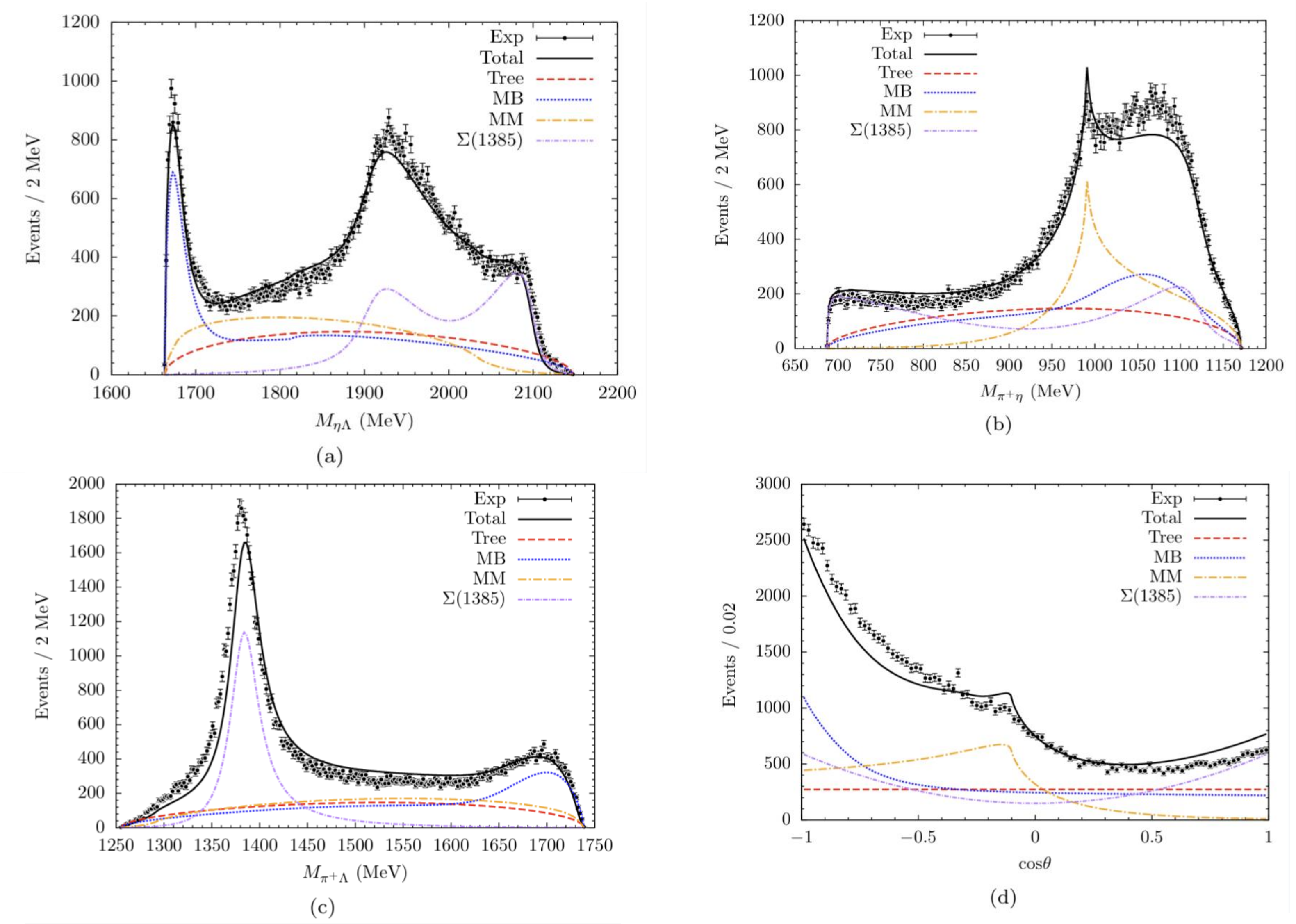
$$\mathcal{T}_{\Sigma(1380)} = \frac{V_{\Sigma(1380)} M_{\Sigma(1380)}}{M_{\text{inv}}(\pi^+\Lambda) - M_{\Sigma(1380)} + i\Gamma_{\Sigma(1380)}/2}$$

$M_{\Sigma(1385)} = 1382.83$ MeV , $\Gamma_{\Sigma(1385)} = 36.2$ MeV
$M_{\Sigma(1380)} = 1380$ MeV , $\Gamma_{\Sigma(1380)} = 120$ MeV

$$\boxed{\mathcal{T}_{\text{Total}} = A\left\{\mathcal{T}_{\text{Tree}} + \mathcal{T}_{MB} + \mathcal{T}_{MM} + \mathcal{T}_{\Sigma(1385)} + \mathcal{T}_{\Sigma(1380)}\right\}}$$

Results-BESIII MC data

□ The invariant mass distributions of the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay without $\Sigma(1380)$

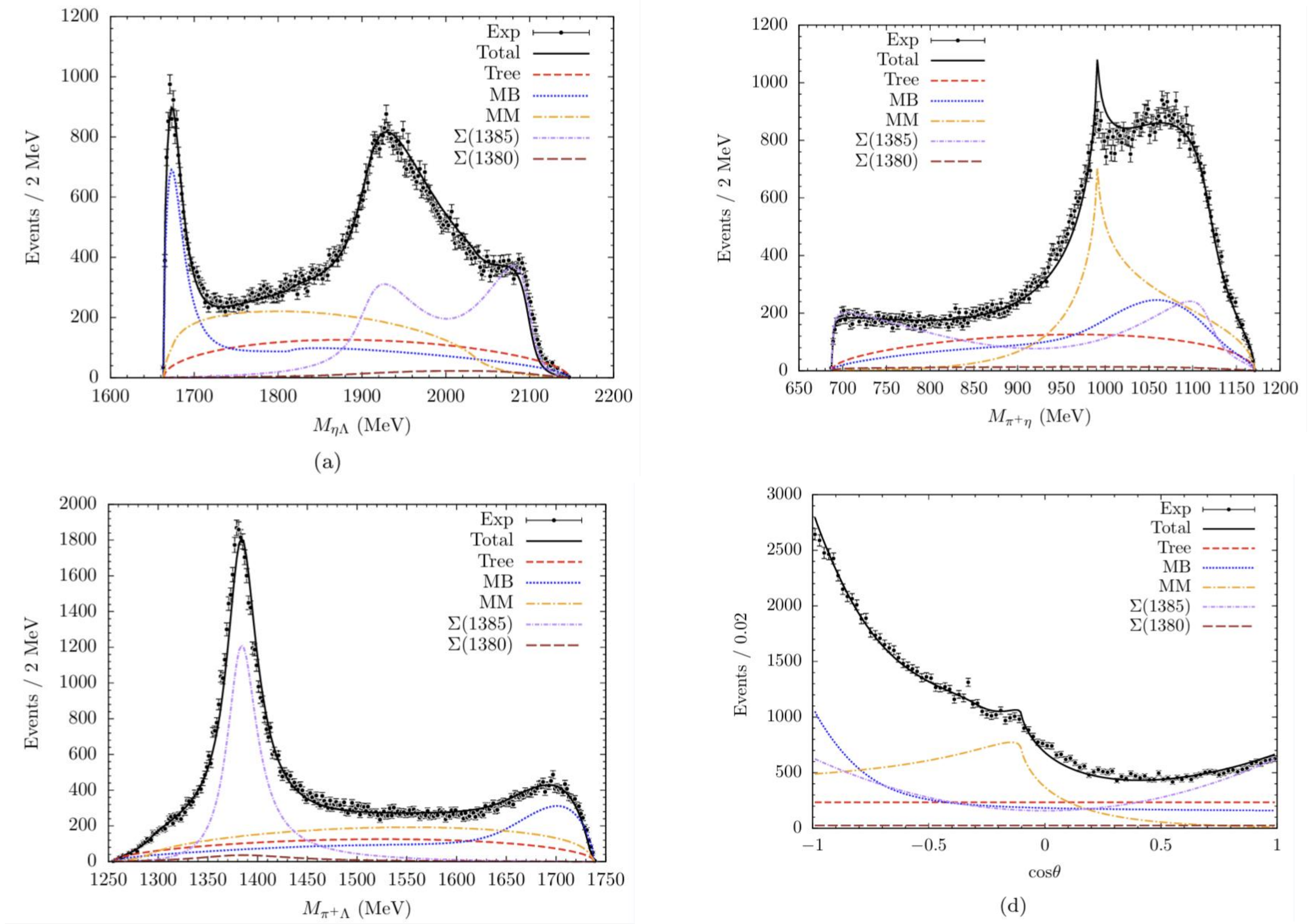


	w/o $\Sigma(1380)$	w $\Sigma(1380)$
A	1.52 MeV^{-1}	1.41 MeV^{-1}
β	0.23	0.25
γ	0.76	0.94
ϕ	0.25π	0.37π
ϕ'	1.42π	1.40π
$V_{\Sigma(1380)}$	–	0.01
ϕ''	–	-0.45π
χ^2/d.o.f.	8.75	2.92

θ is the angle between the π^+ and η particles in the center-of-mass frame of the $\pi^+\Lambda$ system

Results-BESIII MC data

□ The invariant mass distributions of the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay with $\Sigma(1380)$



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Results-Belle data

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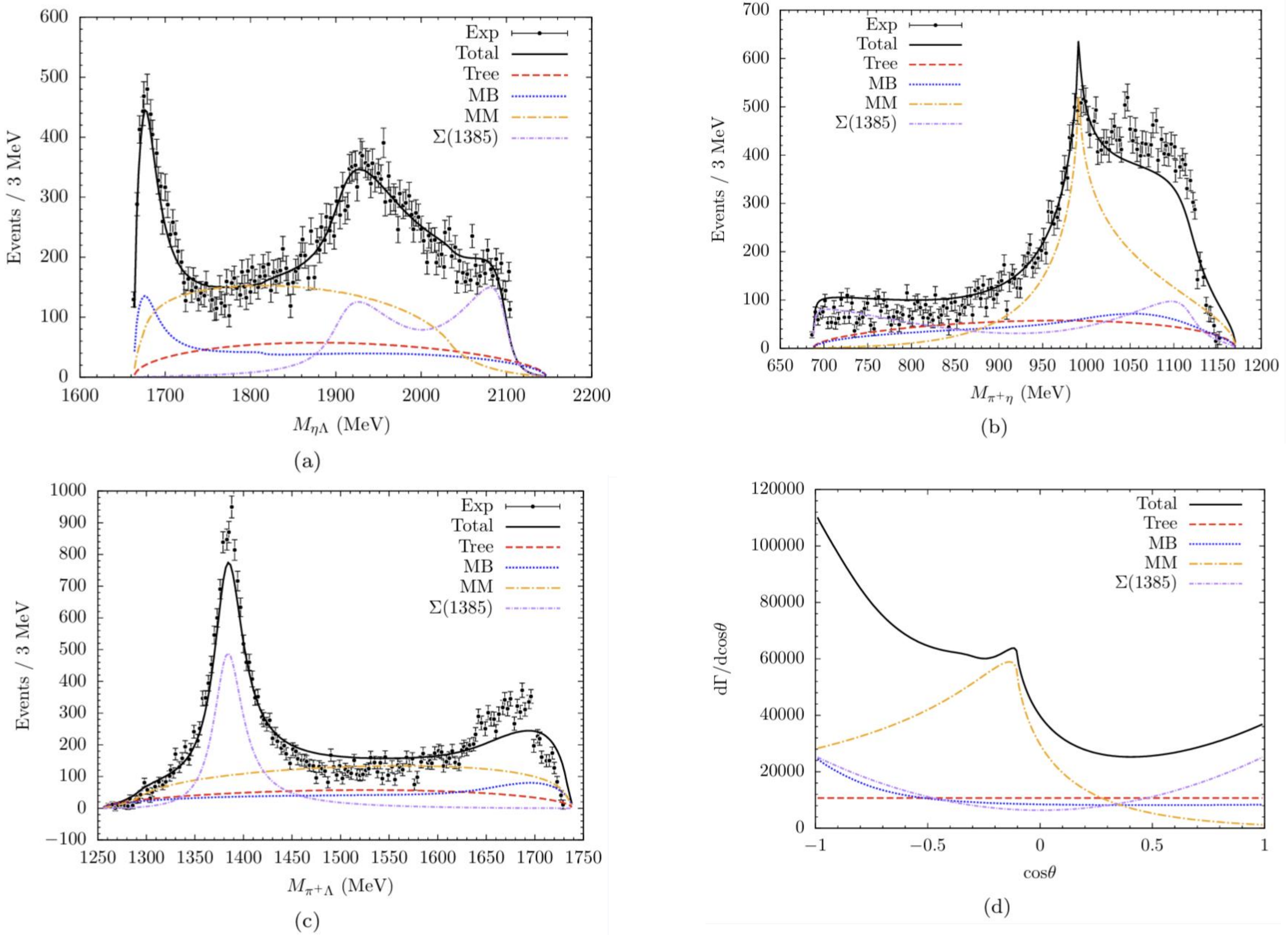
□ The invariant mass distributions of the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay without $\Sigma(1380)$

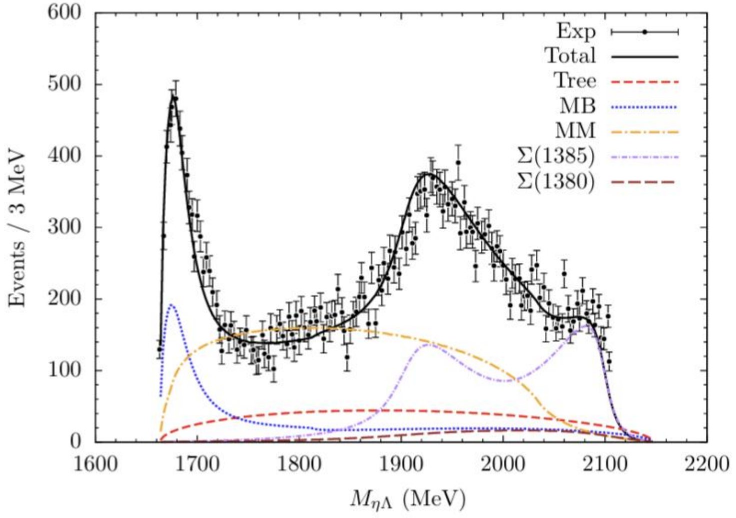


	w/o $\Sigma(1380)$	w $\Sigma(1380)$
A	1.35	1.18
β	0.24	0.28
γ	1.57	1.77
ϕ	-0.05π	0.05π
ϕ'	-1.05π	-0.98π
$V_{\Sigma(1380)}$	–	0.01
ϕ''	–	-0.62π
χ^2/d.o.f.	4.88	3.90

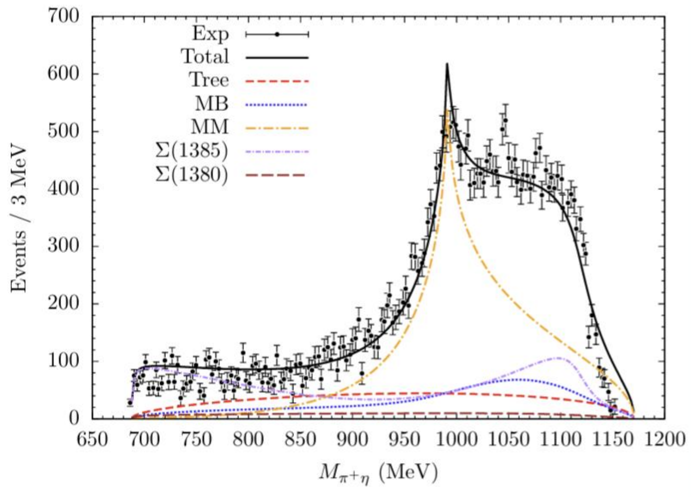
θ is the angle between the π^+ and η particles in the center-of-mass frame of the $\pi^+\Lambda$ system

Results-Belle data

□ The invariant mass distributions of the $\Lambda_c^+ \to \Lambda\eta\pi^+$ decay with $\Sigma(1380)$

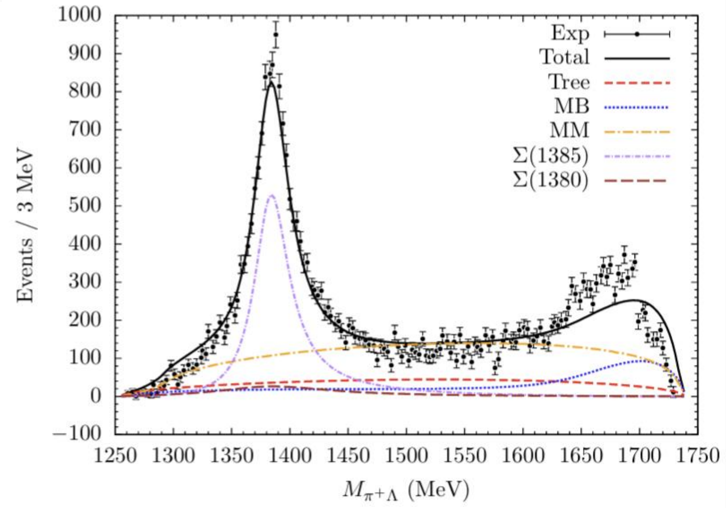


(a)

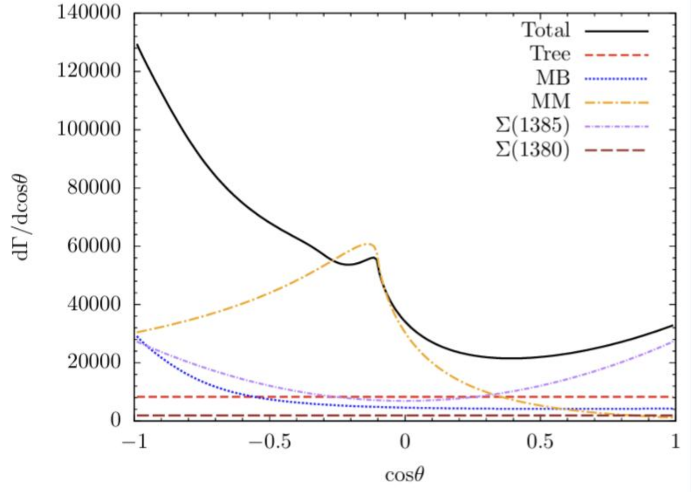


(b)

	w/o $\Sigma(1380)$	w $\Sigma(1380)$
A	1.35	1.18
β	0.24	0.28
γ	1.57	1.77
ϕ	-0.05π	0.05π
ϕ'	-1.05π	-0.98π
$V_{\Sigma(1380)}$	–	0.01
ϕ''	–	-0.62π
χ^2/d.o.f.	4.88	3.90



(c)



(d)

θ is the angle between the π^+ and η particles in the center-of-mass frame of the $\pi^+\Lambda$ system

Summary

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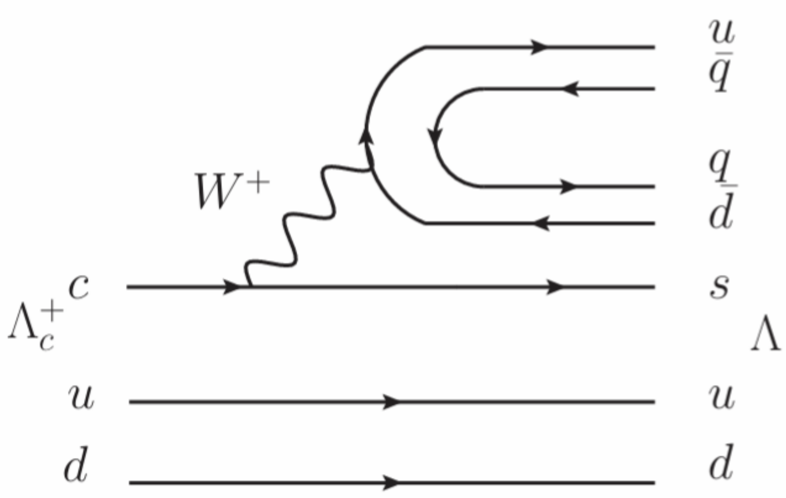
- The inclusion of $\Sigma(1380)$ substantially improves the description of BESIII (MC) and Belle (data) distributions.
- A combined analysis is highly valuable to stringently test the $\Sigma(1380)$ state using Belle and Belle II data.
- This model is suggested for future analyses to help describe coupled-channel resonances (e.g., $a_0(980)$, $\Lambda(1670)$) and potentially reduce systematic uncertainties.

Thank you very much!

Back up

□ Quark level diagram



(b)

$$(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}})\pi^+ + \pi^+(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}}) + K^+\bar{K}^0.$$

□ The structure

$$\mathcal{L}_{WPP} \propto W^\mu[P, \partial_\mu P]$$

$$\Gamma^\mu \propto P_1\partial^\mu P_2 - P_2\partial^\mu P_1 \propto (p_2^\mu - p_1^\mu)$$

Since $\mu = 0$ provides the dominant contribution

$$\Gamma^\mu \propto (p_2^0 - p_1^0)$$

□ For $\eta\pi^+$ and $\pi^+\eta$ terms

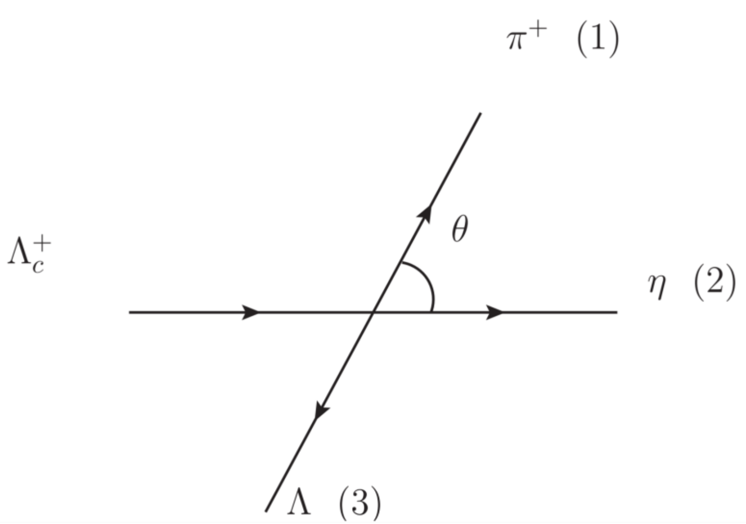
$$\Gamma_{\eta\pi}^\mu \propto (p_\pi^0 - p_\eta^0) \qquad \Gamma_{\pi\eta}^\mu \propto (p_\eta^0 - p_\pi^0)$$

$$\Gamma_{\eta\pi}^\mu + \Gamma_{\pi\eta}^\mu = 0$$

Back up

□ Angular distribution

θ is the angle of the particles π^+ and η in the center-of-mass frame of the $\pi^+\Lambda$ system



$$M^2_{\pi^+\eta} = m^2_{\pi^+} + m^2_\eta + 2\tilde{E}_{\pi^+}\tilde{E}_\eta - 2|\vec{\tilde{p}}_{\pi^+}||\vec{\tilde{p}}_\eta|\cos\theta,$$

$$\tilde{E}_{\pi^+} = \frac{M^2_{\pi^+\Lambda} + m^2_{\pi^+} - m^2_\Lambda}{2M_{\pi^+\Lambda}},$$

$$\tilde{E}_\eta = \frac{M^2_{\Lambda_c} - M^2_{\pi^+\Lambda} - m^2_\eta}{2M_{\pi^+\Lambda}},$$

$$|\vec{\tilde{p}}_{\pi^+}| = \sqrt{\tilde{E}^2_{\pi^+} - m^2_{\pi^+}},$$

$$|\vec{\tilde{p}}_\eta| = \sqrt{\tilde{E}^2_\eta - m^2_\eta}.$$

$$M^2_{\pi^+\eta} + M^2_{\eta\Lambda} + M^2_{\pi^+\Lambda} = M^2_{\Lambda_c} + m^2_{\pi^+} + m^2_\eta + m^2_\Lambda.$$

$$\frac{d^2\Gamma}{dM_{\pi^+\Lambda}d\cos\theta} = \frac{|\vec{p}_\eta||\vec{p}^{\,*}_{\pi^+}|}{(2\pi)^3}\frac{M_\Lambda}{2M_{\Lambda_c^+}}\overline{\sum}|\mathcal{T}_{\text{Total}}|^2,$$

$$\frac{d\Gamma}{d\cos\theta} = \int_{(m_{\pi^+}+M_\Lambda)}^{(M_{\Lambda_c^+}-m_\eta)} \frac{d^2\Gamma}{dM_{\pi^+\Lambda}\,d\cos\theta}\,dM_{\pi^+\Lambda}.$$

Back up

□ Pseudoscalar meson matrix

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \sqrt{\frac{2}{3}}\eta' - \frac{\eta}{\sqrt{3}} \end{pmatrix}$$

□ Baryon matrix

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$