Spin Polarization and Entanglement of Baryon-Antibaryon Pairs

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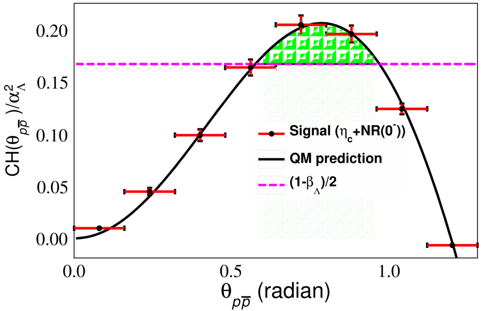
Outline

- Introduction
- Production: $e^+e^- \to B\bar{B}$
- Entanglement in HEP
- Cascade Decay Formalism
- Results
- Summary

- Entanglement: the most profound departure of quantum mechanics from classical physics
- Bell inequality (CHSH): violation rules out local hidden-variable theories – quantum nonlocality confirmed
- Well established in photonic and atomic systems
- In high-energy physics:
  - Entanglement observed in top-quark pair production at the LHC (ATLAS/CMS, 2024)
  - BESIII: test of local realism via entangled $\Lambda\bar{\Lambda}$ system (2025)



$$e^+e^- \to \gamma\eta_c(\to \Lambda\bar{\Lambda}), \qquad \Lambda \to p\pi^-$$

BESIII: Bell inequality test with $\Lambda\bar{\Lambda}$ (Nat. Commun. 16, 4948) von Neumann entropy

$$S(\rho) = -\mathrm{Tr}[\rho \log_2 \rho] = -\sum_i \lambda_i \log_2 \lambda_i$$

where λ_i are the eigenvalues of ρ.

- S quantifies the degree of mixture (our ignorance) of a quantum state
- Pure state $\rho = |\psi\rangle \langle\psi|$: $S = 0$ — the state is fully known
- Mixed state: $S > 0$; the larger S, the more mixed the state

Examples:

$$\rho = |\psi\rangle \langle\psi| \quad \Rightarrow \quad S = 0$$

$$\rho = \frac{1}{2}(|\uparrow\rangle \langle\uparrow| + |\downarrow\rangle \langle\downarrow|) = \frac{1}{2}\mathbb{1} \quad \Rightarrow \quad S = -2 \times \frac{1}{2} \log_2 \frac{1}{2} = 1$$

Entanglement entropy for a pure state $|\psi\rangle_{AB}$, take the partial trace

$$\rho_A = \mathrm{Tr}_B[|\psi\rangle\langle\psi|], \qquad \rho_B = \mathrm{Tr}_A[|\psi\rangle\langle\psi|]$$

The entanglement entropy is the von Neumann entropy of the reduced state:

$$E(\rho) = S(\rho_A) = S(\rho_B)$$

- Product state $|\psi\rangle = |\uparrow\uparrow\rangle$: ρ_A is pure $\Rightarrow\ E = 0$
- Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$: $\rho_A = \frac{1}{2}\mathbb{1} \Rightarrow\ E = 1$

For mixed states, the entanglement of formation is defined by minimizing over all decompositions $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$:

$$E_F(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i E(|\psi_i\rangle)$$

The entanglement of formation E_F can be expressed as

$$E_F(\rho) = H\left(\frac{1+\sqrt{1-\mathcal{C}^2}}{2}\right), \qquad H(x) = -x\log_2 x - (1-x)\log_2(1-x)$$

where $\mathcal{C}$ is called the *concurrence*: since $H(x)$ is monotonically increasing on $[0,1]$, $\mathcal{C}$ is commonly used to quantify the degree of entanglement.
For a pure state, $\mathcal{C}$ is the modulus of the expectation value of Γ:

$$\mathcal{C}(\rho) = \mathcal{C}(|\psi\rangle) = |\langle\psi|\Gamma|\psi\rangle|, \qquad \Gamma = \sigma_y \otimes \sigma_y K$$

where K is the complex conjugation operator. Example: $|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$

$$\mathcal{C}(|\psi\rangle) = 2|ad - bc|$$

- Product state $|\psi\rangle = |\uparrow\uparrow\rangle$: $\mathcal{C} = 0$ (separable)
- Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$: $\mathcal{C} = 1$ (maximally entangled)

For mixed states, the concurrence is defined by minimizing over all decompositions:

$$\mathcal{C}(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i \mathcal{C}(|\psi_i\rangle), \qquad \rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|$$

Example: mix two maximally entangled Bell states

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle), \qquad |\psi_2\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \qquad C(|\psi_1\rangle) = C(|\psi_2\rangle) = 1$$

with probabilities p and $1-p$:

$$\rho = p|\psi_1\rangle\langle\psi_1| + (1-p)|\psi_2\rangle\langle\psi_2|, \qquad 0 \leq p \leq 1$$

Intuitively, a mixture of two maximally entangled states should be strongly entangled. Is it?

The eigen decomposition of ρ can be written as

$$\rho = |\phi_1\rangle \langle\phi_1| + |\phi_2\rangle \langle\phi_2| \,, \qquad |\phi_1\rangle = \sqrt{p}\, |\psi_1\rangle \,, \quad |\phi_2\rangle = \sqrt{1-p}\, |\psi_2\rangle$$

Any orthogonal rotation gives another decomposition:

$$|x_1\rangle = |\phi_1\rangle \,, \quad |x_2\rangle = -\, |\phi_2\rangle \,, \qquad |u_1\rangle = \cos\theta\, |x_1\rangle - \sin\theta\, |x_2\rangle \,, \quad |u_2\rangle = \sin\theta\, |x_1\rangle + \cos\theta\, |x_2\rangle$$

The concurrence of this decomposition

$$C(|u_1\rangle) + C(|u_2\rangle) = |p\cos^2\theta - (1-p)\sin^2\theta| + |p\sin^2\theta - (1-p)\cos^2\theta|$$

is minimized at $\theta = \pi/4$, giving

$$\mathcal{C}(\rho) = |2p - 1|$$

- $p = 1/2$: $\mathcal{C}(\rho) = 0$ — a 1:1 mixture of two maximally entangled states is separable!
- In general, mixing reduces entanglement

For a general two-qubit state, the concurrence can be computed directly

$$\mathcal{C}(\rho) = \max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right)$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the eigenvalues of the matrix

$$R = \rho\, \sigma_y \otimes \sigma_y\, \rho^*\, \sigma_y \otimes \sigma_y$$

W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998)

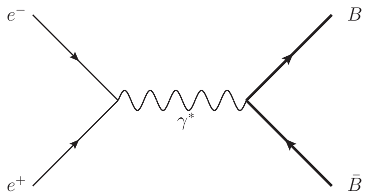
The CHSH inequality:

$$|\,\langle A_1 \otimes B_1 \rangle + \langle A_1 \otimes B_2 \rangle + \langle A_2 \otimes B_1 \rangle - \langle A_2 \otimes B_2 \rangle\,| \leq 2$$

The concurrence constrains the maximal violation of the CHSH inequality:

$$\mathcal{B}[\rho] \leq 2\sqrt{1 + \mathcal{C}^2(\rho)}$$

F. Verstraete and M. M. Wolf, Phys. Rev. Lett. 89, 170401 (2002)



$\gamma^* B_1 \bar{B}_2$ vertex

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + \frac{iF_2(q^2)}{2M}\sigma^{\mu\nu}q_\nu$$

where F_1 and F_2 are the Dirac and Pauli form factors. Electric and magnetic form factors

$$G_E = F_1 + \tau F_2, \quad G_M = F_1 + F_2, \quad \tau = \frac{q^2}{4M^2}$$

The angular distribution parameter α and the relative phase $\Delta\Phi$

$$\alpha = \frac{|G_M|^2 - \tau|G_E|^2}{|G_M|^2 + \tau|G_E|^2}, \qquad \frac{G_E}{G_M} = \left|\frac{G_E}{G_M}\right| e^{i\Delta\Phi}$$

Differential cross section (θ: scattering angle; α from the angular distribution)

$$\frac{d\sigma}{d\Omega} \propto 1 + \alpha\cos^2\theta$$

Polarization ($\Delta\Phi$ from the polarization)

$$P_y = \frac{\sin\theta\cos\theta}{1 + \alpha\cos^2\theta}\sqrt{1-\alpha^2}\sin\Delta\Phi$$

Spin wavefunctions of the $B_1\bar{B}_2$ system, in terms of α, θ, $\Delta\Phi$

$$|\psi_1\rangle = \frac{1}{2}\begin{pmatrix} \sin\theta\sqrt{1-\alpha}e^{i\frac{\Delta\Phi}{2}} \\ (1+\cos\theta)\sqrt{1+\alpha}e^{-i\frac{\Delta\Phi}{2}} \\ (1-\cos\theta)\sqrt{1+\alpha}e^{-i\frac{\Delta\Phi}{2}} \\ \sin\theta\sqrt{1-\alpha}e^{i\frac{\Delta\Phi}{2}} \end{pmatrix}$$

$$|\psi_2\rangle = \frac{1}{2}\begin{pmatrix} -\sin\theta\sqrt{1-\alpha}e^{i\frac{\Delta\Phi}{2}} \\ (1-\cos\theta)\sqrt{1+\alpha}e^{-i\frac{\Delta\Phi}{2}} \\ (1+\cos\theta)\sqrt{1+\alpha}e^{-i\frac{\Delta\Phi}{2}} \\ -\sin\theta\sqrt{1-\alpha}e^{i\frac{\Delta\Phi}{2}} \end{pmatrix}$$

The spin density matrix

$$\rho_{B\bar{B}} = \frac{1}{2(1+\alpha\cos^2\theta)}\left(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|\right)$$

In the Pauli basis (Bloch representation)

$$\rho = \frac{1}{4}\Theta_{\mu\nu}\sigma_\mu \otimes \sigma_\nu$$

Θ in block form

$$\Theta = \frac{1}{A}\begin{pmatrix} A & P_2^T \\ P_1 & C \end{pmatrix}$$

The polarization vectors P_1/A, P_2/A and the spin correlation matrix C:

$$A = 1 + \alpha\cos^2\theta, \quad P_1 = P_2 = \begin{pmatrix} 0 \\ \beta\sin\theta\cos\theta \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} \sin^2\theta & 0 & \gamma\sin\theta\cos\theta \\ 0 & -\alpha\sin^2\theta & 0 \\ \gamma\sin\theta\cos\theta & 0 & \alpha + \cos^2\theta \end{pmatrix}$$

with $\beta = \sqrt{1-\alpha^2}\sin\Delta\Phi$ and $\gamma = \sqrt{1-\alpha^2}\cos\Delta\Phi$.

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Bell nonlocality and entanglement in $e^+e^- \to Y\bar{Y}$ at BESIII

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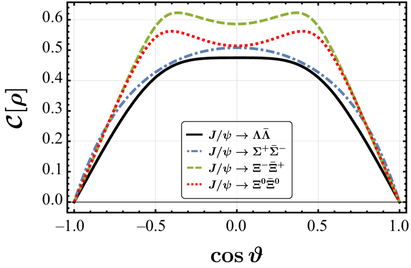
Concurrence: $$\mathcal{C} = \frac{\left|1+\alpha-\sqrt{(1+\alpha\cos 2\theta)^2-\beta^2\sin 2\theta}\right|}{2(1+\alpha\cos^2\theta)}$$



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Entanglement Autodistillation from Particle Decays

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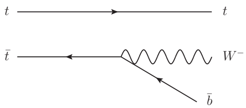

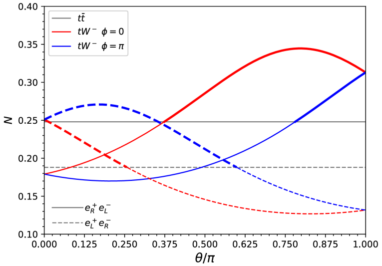

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Quantum entanglement autodistillation in baryon pair decays

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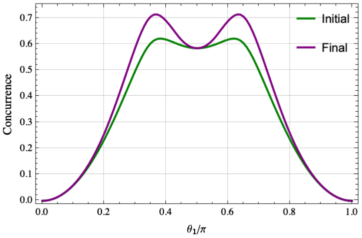
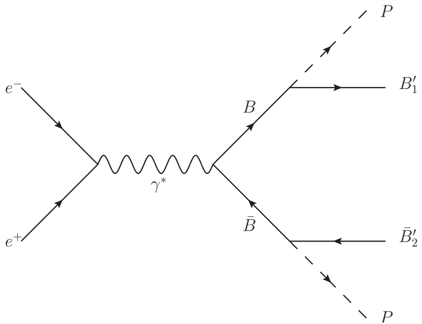
$$J/\psi \to \Xi^-(\to \Lambda\pi^-)\,\bar{\Xi}^+(\to \bar{\Lambda}\pi^+)$$



(b) Concurrence of $\Xi^-\bar{\Xi}^+$ and $\Lambda\bar{\Lambda}$

Motivation

- Is entanglement amplification a general phenomenon in weak decays of $B\bar{B}$ pairs from e^+e^- annihilation?
- Which conditions are required for entanglement amplification?



Weak decay model $B_1 \to B'_1 + P$ and $\bar{B}_2 \to \bar{B}'_2 + P$ The helicity amplitudes

$$
\begin{aligned}
t_{\lambda'_1,\lambda_1} &= \bar{u}(q_1,\lambda'_1)(a + b\gamma^5)u(p_1,\lambda_1) \\
\bar{t}_{\lambda'_2,\lambda_2} &= \bar{v}(p_2,\lambda_2)(\bar{a} + \bar{b}\gamma^5)v(q_2,\lambda'_2)
\end{aligned}
$$

with the angle-independent coefficients

$$
\begin{aligned}
A_+ &= \sqrt{2M}\left(a\sqrt{E'_1 + M'_1} - b\sqrt{E'_1 - M'_1}\right) \\
A_- &= \sqrt{2M}\left(a\sqrt{E'_1 + M'_1} + b\sqrt{E'_1 - M'_1}\right) \\
B_+ &= \sqrt{2M}\left(\bar{a}\sqrt{E'_2 + M'_2} - \bar{b}\sqrt{E'_2 - M'_2}\right) \\
B_- &= \sqrt{2M}\left(\bar{a}\sqrt{E'_2 + M'_2} + \bar{b}\sqrt{E'_2 - M'_2}\right)
\end{aligned}
$$

Decay asymmetry parameters

$$
\alpha_1 = \frac{|A_+|^2 - |A_-|^2}{|A_+|^2 + |A_-|^2}, \quad \alpha_2 = \frac{|B_+|^2 - |B_-|^2}{|B_+|^2 + |B_-|^2}
$$

Relative phases

$$
\frac{A_+}{A_-} = \left|\frac{A_+}{A_-}\right| e^{i\Delta\Phi_1}, \qquad \frac{B_+}{B_-} = \left|\frac{B_+}{B_-}\right| e^{i\Delta\Phi_2}
$$

Decay matrices from the helicity amplitudes

$$(D_1)_{\lambda_1',\lambda_1} = \frac{1}{N_1} t_{\lambda_1',\lambda_1}, \qquad (D_2)_{\lambda_2',\lambda_2} = \frac{1}{N_2} \bar{t}_{\lambda_2',\lambda_2}$$

The spin density matrix of the final $B_1' \bar{B}_2'$ system

$$\rho_{B_1' \bar{B}_2'} = \frac{A}{W} (D_1 \otimes D_2)\, \rho_{B\bar{B}}\, (D_1 \otimes D_2)^{\dagger}$$

with the normalization factor

$$W = A + \alpha_1 P_1^T R_1 + \alpha_2 P_2^T R_2 + \alpha_1 \alpha_2 R_1^T C R_2$$

which is proportional to the angular distribution of the final state. The unit vectors

$$R_1^T = (x_1, y_1, z_1), \qquad R_2^T = (x_2, y_2, z_2)$$

give the momentum directions of B_1' and $\bar{B}_2'$ in the rest frames of B_1 and $\bar{B}_2$.

Cascade Decay Formalism

The two physical quantities of interest: Polarization degree of the B_1' baryon

$$d_1' = \frac{1}{W}\left[\Big(\alpha_1(A + \alpha_2 P_2^T R_2) + R_1^T(P_1 + \alpha_2 C R_2)\Big)^2 + (1-\alpha_1^2)(P_1 + \alpha_2 C R_2)^T(1 - R_1 R_1^T)(P_1 + \alpha_2 C R_2)\right]^{1/2}$$

Concurrence of the $B_1'\bar{B}_2'$ system

$$\mathcal{C}' = \frac{A}{W}\sqrt{(1-\alpha_1^2)(1-\alpha_2^2)}\,\mathcal{C}$$

The polarization degree and the concurrence
are independent of the weak decay phases

The entanglement evolution factor

$$Z = \frac{A}{W}\sqrt{(1-\alpha_1^2)(1-\alpha_2^2)}$$

$Z > 1$: entanglement amplification; $Z < 1$: entanglement reduction. Key findings (1)–(3):

(1) If $\alpha_1 = \alpha_2 = 0$ and $\Delta\Phi_1 = \Delta\Phi_2 = 0$, then

$$P_1' = P_1; \qquad P_2' = P_2; \qquad C' = C$$

the decay acts as a global spin rotation, all observables are unchanged.

(2) If $\alpha_1 = \alpha_2 = 0$, then

$$d_1' = d_1; \qquad d_2' = d_2; \qquad \mathcal{C}' = \mathcal{C}$$

the phases only reorient the polarization; the degrees are unchanged.

(3) If $|\alpha_1| = 1$, then $d_1' = 1$; if $|\alpha_2| = 1$, then $d_2' = 1$. For both situations,

$$\mathcal{C}' = 0$$

maximal parity violation fully polarizes the daughter baryon and destroys the entanglement.

Key findings (4)–(6):

(4) If $\alpha_2 = 0$, $|\alpha_1| \in (0,1)$, and $d_1 \le \left(1 - \sqrt{1-\alpha_1^2}\right)/|\alpha_1|$, then for all R_1,

$$d_1' \ge d_1; \qquad \mathcal{C}' \le \mathcal{C}$$

the polarization is enhanced while the entanglement is reduced.

(5) If $|\alpha_1|, |\alpha_2| \in (0,1)$, and $d_1 = d_2 = 0$, then for all R_1 and R_2,

$$Z \le 1$$

no amplification is possible for unpolarized initial states.

(6) If $|\alpha_1| = |\alpha_2| \in (0,1)$, and $d_1 = d_2 > 0$, then there exist R_1 and R_2 such that

$$Z > 1$$

Entanglement amplification always arises in
charge-conjugate decays for polarized initial states

Analytic expression of the maximum amplification factor $Z_{\max}$

$$Z_{\max} = \begin{cases} \dfrac{2(1+\alpha\cos^2\theta)}{1+\alpha+\sqrt{(1+\alpha\cos 2\theta)^2-\beta^2\sin^2(2\theta)}}, & \text{for} \quad \dfrac{|\beta\sin 2\theta|}{1+\alpha\cos 2\theta} < \dfrac{2|\alpha_B|}{1+\alpha_B^2}; \\ \dfrac{(1+\alpha\cos^2\theta)(1-\alpha_B^2)}{1+\alpha\cos^2\theta-\alpha_B^2\alpha\sin^2\theta-|\alpha_B\beta\sin 2\theta|}, & \text{for} \quad \dfrac{|\beta\sin 2\theta|}{1+\alpha\cos 2\theta} \geq \dfrac{2|\alpha_B|}{1+\alpha_B^2} \end{cases}$$

where α_B is the decay asymmetry parameter of the daughter decays.

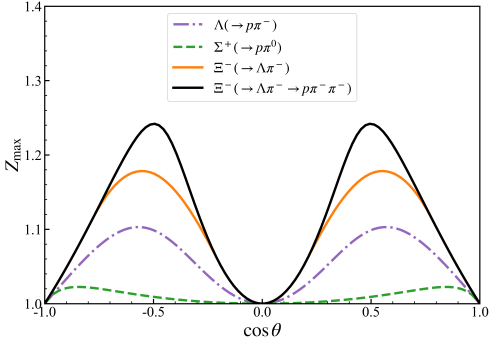
Numerical inputs measured by BESIII:

Table: Parameters for the $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$, $\Sigma^+\bar{\Sigma}^-$, and $\Xi^-\bar{\Xi}^+$ processes and their subsequent decays.

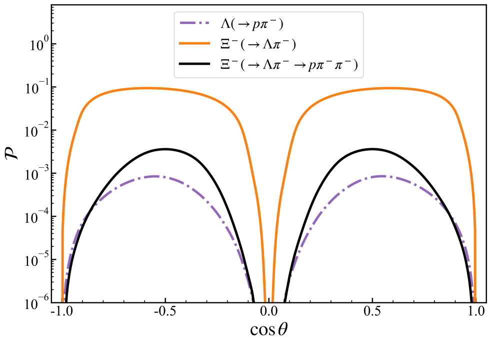
No.	$B\bar{B}$	α	$\Delta\Phi$ (rad)	α_B
1.	$\Lambda\bar{\Lambda}$	0.461±0.09	0.740± 0.014	0.754 ± 0.011
2.	$\Sigma^+\bar{\Sigma}^-$	-0.508± 0.07	-0.270± 0.015	-0.994 ± 0.038
3.	$\Xi^-\bar{\Xi}^+$	0.586± 0.016	1.213± 0.048	-0.374 ± 0.008

Decay modes: $\Lambda \to p\pi^-$, $\Sigma^+ \to p\pi^0$, $\Xi^- \to \Lambda\pi^-$, and the cascade $\Xi^- \to \Lambda(\to p\pi^-)\pi^-$

Numerical results: $Z_{\max}$ and the probability of $Z > 1$



$Z_{\max}$ for different processes



Probability of $Z > 1$

Single-step decays

$$\Xi \to \Lambda : 18\%,\ \Lambda \to p : 10\%,\ \Sigma \to p : 2\%$$

Two-step decays $\Xi \to \Lambda \to p : 24\%$

Maximum probability density

$$\Xi \to \Lambda : 9.4 \times 10^{-2}, \qquad \Lambda \to p : 8 \times 10^{-4}$$

$$\Xi \to \Lambda \to p : 3.6 \times 10^{-3}, \quad \Sigma \to p :\sim 10^{-9}$$

Summary

- We derived the analytic spin density matrix of the final $B_1' \bar{B}_2'$ pair after cascade weak decays, expressed through measurable polarization observables
- Decays with maximal parity violation ($|\alpha| = 1$) produce a fully polarized daughter baryon and destroy the entanglement
- Entanglement amplification ($Z > 1$) is strictly forbidden if the initial $B\bar{B}$ pair is unpolarized
- For polarized initial states—a generic outcome in e^+e^- annihilation—amplification always arises in charge-conjugate decays under CP conservation, as verified for $J/\psi \to \Lambda\bar{\Lambda}$, $\Sigma\bar{\Sigma}$, and $\Xi\bar{\Xi}$

Thanks!