Monte Carlo Program EKHARA for BELLE2

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Outline

Monte Carlo EKHARA

EKHARA is developed by the same group who wrote PHOKHARA which has been use by us in belle for radiative return processes since we joined belle. It simulates

- $e^+e^- \to e^+e^-\pi^0$
- $e^+e^- \to e^+e^-\eta$
- $e^+e^- \to e^+e^-\eta'$
- $e^+e^- \to e^+e^-\chi_{c0}$
- $e^+e^- \to e^+e^-\chi_{c1}$
- $e^+e^- \to e^+e^-\chi_{c2}$

- $e^+e^- \to e^+e^-\chi_{cJ}$ then $\chi_{cJ} \to \gamma J/\psi$ then $J/\psi \to \mu^+\mu^-$ with $J = 0,1,2$

Only leading terms in α are included.

The use of EKHARA

EKHARA is developed by the same group who has developed PHOKHARA. As PHOKHARA, it is programed in FORTRAN, and written in a way convenient to experimental group.

All we need to do is to

call EKHARA(-1) to initialize
call EKHARA(0) to generate events
call EKHARA(1) to finalize (output the cross section and statistics)

I have written the necessary interface program (in 2019) (very simple) following what Torben Ferber had done for PHOKHARA, in order to implement EKHARA into BELLE2 environment.

The use of EKHARA

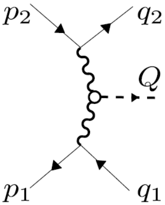
channel_id= 2,3,4,5,6,7,8,9,10,11

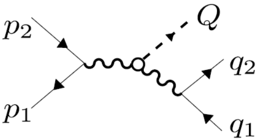
Mode selection (in the file input.dat)

- 1 not implemented for belle2
- 2 one π^0 meson in the final state $e^+e^- \to e^+e^-\pi^0$
- 3 one η meson in the final state $e^+e^- \to e^+e^-\eta$
- 4 one η' meson in the final state $e^+e^- \to e^+e^-\eta'$
- 5 one χ_{c0} in the final state $e^+e^- \to e^+e^-\chi_{c0}$
- 6 one χ_{c1} in the final state $e^+e^- \to e^+e^-\chi_{c1}$
- 7 one χ_{c2} in the final state $e^+e^- \to e^+e^-\chi_{c2}$
- 8 $\gamma\mu^+\mu^-$ in the final state $e^+e^- \to e^+e^-\chi_{c0}(\to \gamma J/\psi(\to \mu^+\mu^-))$
- 9 $\gamma\mu^+\mu^-$ in the final state $e^+e^- \to e^+e^-\chi_{c1}(\to \gamma J/\psi(\to \mu^+\mu^-))$
- 10 $\gamma\mu^+\mu^-$ in the final state $e^+e^- \to e^+e^-\chi_{c2}(\to \gamma J/\psi(\to \mu^+\mu^-))$
- 11 $\gamma\mu^+\mu^-$ in the final state $e^+e^- \to e^+e^-(\chi_{c0}+\chi_{c1}+\chi_{c2})(\to \gamma J/\psi(\to \mu^+\mu^-))$

Monte Carlo Program EKHARA

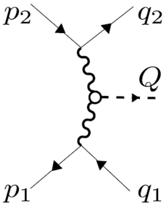
The $\gamma*\gamma*$ process goes through two diagrams, the t channel and s channel.





Monte Carlo Program EKHARA

The matrix element of t channel reads

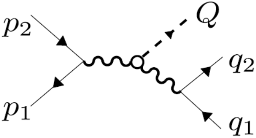


$$\mathcal{M}_t = -\frac{4i\alpha^2}{f_\pi} F((p_1-q_1)^2,(p_2-q_2)^2)\varepsilon_{\mu\nu\alpha\beta}\frac{1}{(p_1-q_1)^2(p_2-q_2)^2} \quad (1)$$

$$(q_1-p_1)^\alpha(q_2-p_2)^\beta(\overline{v}(p_1)\gamma^\mu v(q_1))(\overline{u}(q_2)\gamma^\nu u(p_2)) \quad (2)$$

In the expression of $\mathcal{M}_t$, there are propagators $1/(p_1-q_1)^2$ and $1/(p_2-q_2)^2$ where p_1 and q_1 are the four momentum of the initial state and final state of e^+; while p_2 and q_2 are the four momentum of the initial state and final state of e^-. If either $(p_1-q_1)^2$ or $(p_2-q_2)^2$ is small, (i.e. either electron or positron moves in almost forward direction) t channel amplitude would peak.

Monte Carlo Program EKHARA



The matrix element for the *s* channel reads

$$\mathscr{M}_s = \frac{4i\alpha^2}{f_\pi} F(s,(q_1+q_2)^2)\varepsilon_{\mu\nu\alpha\beta}\frac{1}{s(q_1+q_2)^2} \quad (3)$$

$$(p_1+p_2)^\alpha (q_1+q_2)^\beta (\overline{v}(p_1)\gamma^\mu u(p_2))(\overline{u}(q_2)\gamma^\nu v(q_1)) \quad (4)$$

In the s channel matrix element, there are the propagators $1/s$ and $1/(q_1+q_2)^2$ (q_1 and q_2 are the four momentum of the e^+ and e^- in the final states) in the amplitude. Unless $(q_1+q_2)^2$ is small (i.e. the invariant mass of the final state electron-positron pair is small), s channel contribution is small compared to t channel, and usually neglected.

Monte Carlo Program EKHARA

For χ_{cJ} final states, the matrix elements are more complicated. But like the case of pseudoscalar final states, there are the same propagators in t channel ($(p_1 - q_1)^2$ and $(p_2 - q_2)^2$) and s channel ($1/s$ and $(q_1 + q_2)^2$) respectively.

So usually for χ_{cJ} final states, the t channel dominates, and s channel is usually neglected.

The use of EKHARA

For π^0, η and η' meson in the final state, there is the control variable sw in the control card card_1pi.dat, which specifies if s channel (sw=1), t channel (sw=2) or both s and t as well as their inteference (sw=3) are included.

In EKHARA, for final states with χ_{cJ}, ony t channel is considered.

The use of EKHARA

For π^0, η and η' meson in the final state, the form factor of $\gamma\gamma\mathscr{P}$ is selected by the variable piggffsw in the control card card_1pi.dat. It could take 1,2,3,4,5,6,7,8,9,10.
piggffsw=1 means constant form factor, so it is unphysical, and is used only for testing the program.
piggffsw=8 (two octets model) or piggffsw=10 (three octets model) are suggested, which are fitted with BaBar data.

We may write our own program for the form factor of $\gamma\gamma\mathscr{P}$ as belle2 accumulates data. The function piggFF(t1_t,t2_t,W2_t) in the file routines_user.inc.for (As EKHARA has done so with BaBar as CLEO data)

Other input parameters in card_1pi.dat for $e^+e^-\mathscr{P}$ and card_chi.dat for $e^+e^-\chi_{cJ}$ final states, have meanings which are self-apparent, like the energy and angular cuts of the final state particles.

Estimated number of events

In the paper by Henryk Czyz and Patrycja Kisza

"Testing χ_c properties in BELLE II"

arXiv:1612.07509v2 [hep-ph] May 2017. Phys. Lett.B771,487(2017)

In the paper, it is estimated that in BELLE II,

140 million χ_{c0},
4.3 million χ_{c1},
142 million χ_{c2}

will be produced in a data sample of $50\mathrm{ab}^{-1}$ of integrated luminosity and with full phase space.

Estimated number of events

If we tag the positron in the reaction $e^+e^- \to e^+e^-\chi_{cJ}$, in the range between 17° and 150°, the expected number of events drops to

6.7 million χ_{c0},
1.4 million χ_{c1},
7.2 million χ_{c2}

If both electron and positron are tagged in the same range (17° and 150°), the expected number of events drops to

249 thousand χ_{c0},
174 thousand χ_{c1},
295 thousand χ_{c2}

The estimated number of events

But the above numbers are not the physics which Xiaolong is interested. For the experimental set up which xiaolong wants to do, we need to run EKHARA for the Monte Carlo simulation.

Summary

The Monte Carlo generator EKHARA is to be used in belle2 to simulate $\gamma * \gamma *$ process. It is similar to PHOKHARA which has been used in belle/belle2 for radiative return measurement in the past 20 years.
For the experimental set up which Xiaolong wants to do, we shall use the program for Monte Carlo simulation.