INSTITUTO GALEGO DE FÍSICA DE ALTAS ENERXÍAS IGFAE

# Radiative transitions of charmonium on the light front

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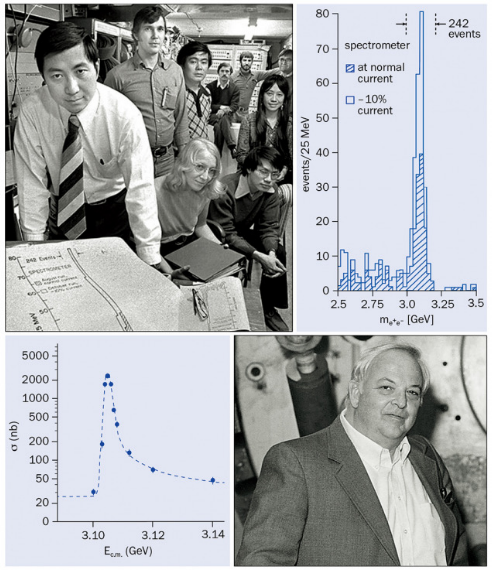
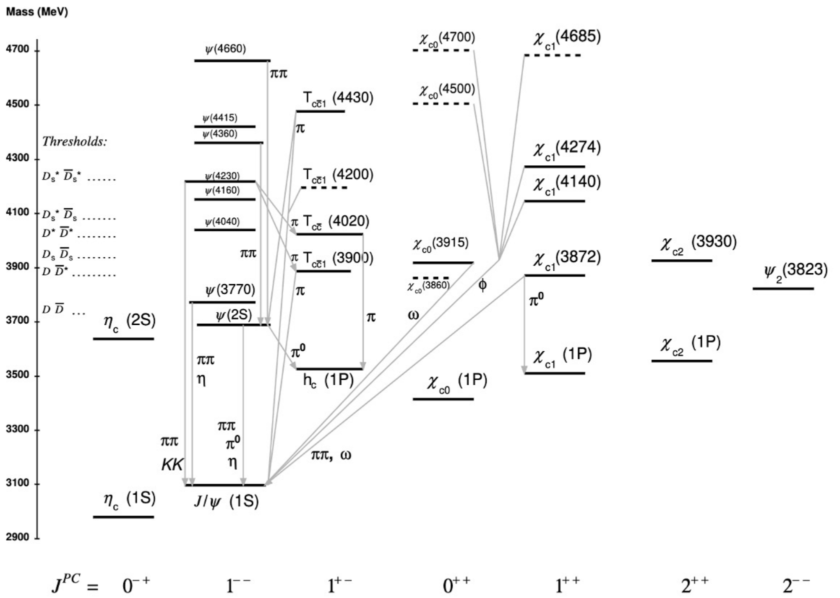
# Charmonium

Charmonium, the bound state of a charm quark and its antiparticle, is a system that sits at a very interesting crossroads in QCD.

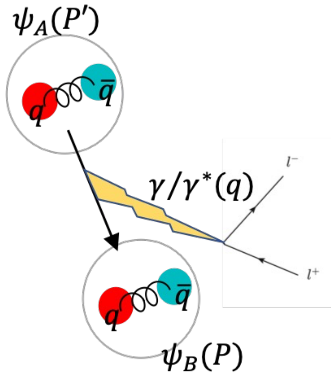
- Theoretically a hard problem: multiscale, multi-physics: $\Lambda_{QCD} \lesssim \underbrace{\alpha_s^2 m_c}_{ultrasolft} < \underbrace{\alpha_s m_c}_{soft} < \underbrace{m_c}_{hard}$
- Physically a simple system: nonrelativistic ($v_c \ll 1$), perturbative ($\alpha_s \ll 1$): $v_c^2 \sim 0.3, \alpha_s(m_c) \sim 0.3 - 0.6$
- Potential model, NRQCD, Lattice QCD, DSE/BSE, light-front Hamiltonian,…

[Review: Brambilla:2010cs,Yuan:2021wpg]



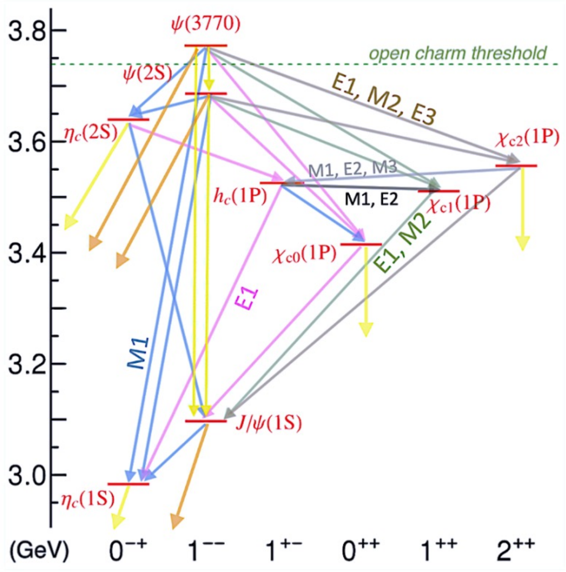



# Radiative transitions




Radiative transitions are processes where a charmonium state decays by emitting one or two photons, $\psi_A \to \psi_B + \gamma^*(Q^2)$, $\psi_A \to \gamma + \gamma^*(Q^2)$, serving as electromagnetic probes to understand the internal structure of QCD bound states

- decay width ($\Gamma_{\psi_A \to \psi_B/\gamma + \gamma}$) → real $\gamma$
- transition form factor (TFF, $F(Q^2)$) → virtual $\gamma^*$




- Golden channel for **hadron identification & discovery** w. J, P, C, gauge symmetry selection rules
- **Clean probes** to charmonium structures -- important for theories

[Review: Eichten:2007qx, Chernyak:2014wra]

- **Extensive measurements** of radiative widths

[Review of particle physics 2025]

- TFFs: **relatively scarce**, $F_{\eta_c\gamma}(Q^2)$, $F_{\chi_{cJ}\gamma}(Q^2)$

[BaBar:2010siw, Belle:2017xsz]

# Physical pictures

The TFF interpolates between two distinct physical regimes

$$iM_{H\to\gamma\gamma} = \varepsilon_\mu \varepsilon_\nu^* \int \mathrm{d}^4 x e^{iq\cdot z} \langle 0 | J^\mu(z) J^\nu(0) | H \rangle$$

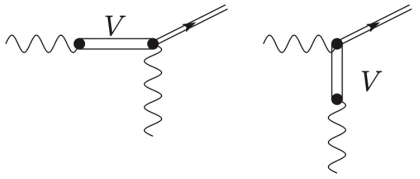
- **Low $Q^2$ -- Vector Meson Dominance (VMD):** The soft photon cannot resolve quark structure, amplitude is dominated by intermediate vector meson states

$$\mathcal{M} \sim \frac{\Psi_V(r=0)}{M_V^2 + Q^2}$$

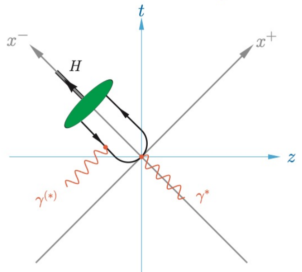
- **High $Q^2$ -- Light-Cone Dominance**: The hard photon resolve the individual quarks, amplitude factorizes into a hard kernel and distribution amplitude

$$\mathcal{M} \sim \int dx\, T_H(x, Q^2)\phi(x,\mu)$$

$Q^2 \to 0$ intermediate $Q^2$? $Q^2 \to \infty$

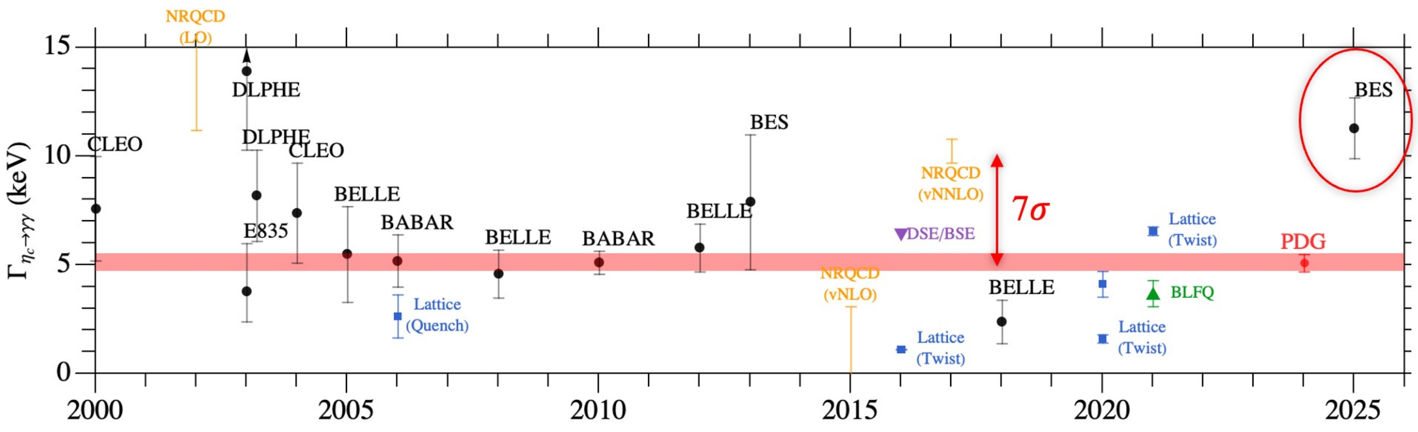


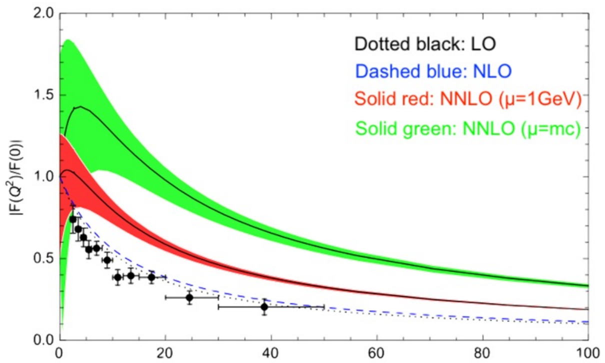

- **Intermediate $Q^2$ (experimentally accessible!)**: Neither limit adequate, non-perturbative methods needed



# Theories to access the TFFs

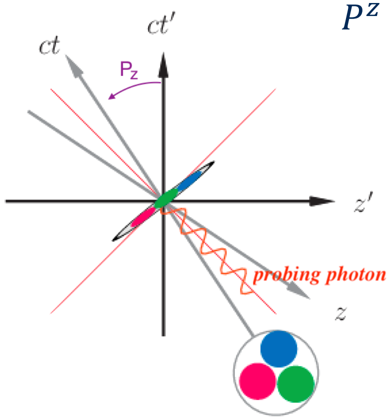
- Potential model: large relativistic corrections [Babiarz:2019sfa]
- NRQCD: discrepancy at NNLO -- a crisis for NRQCD? [Feng:2015uha, Feng:2017hlu]
  $v_c^2 \sim 0.3, \alpha_s(m_c) \sim 0.3 - 0.6$, non-perturbative & relativistic effects
- Lattice QCD: tremendous progress in widths
  $a\, m_c \sim 0.5, O(a^2)$ effects
  QCD + QED: challenge for simulating $\gamma^*$ on the lattice
  [Dudek:2006ut, Dudek:2006ej, Dudek:2009kk, Chen:2011kpa, Becirevic:2012dc, Donald:2012ga, CLQCD:2016ugl, CLQCD:2020njc, Liu:2020qfz, Zou:2021mgf, Meng:2021ecs, Colquhoun:2023zbc, Li:2023zig, Delaney:2023fsc, Meng:2024axn,…]
- Relativistic approaches: DSE, LFQM, LCSR, LF Hamiltonian, … [Chen:2016bpj, Ryu:2018egt, Guo:2019xqa]



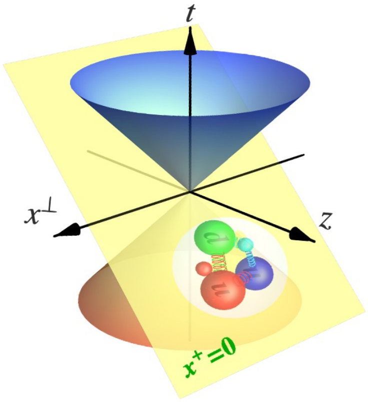
# Physics on the light front

- Infinite momentum frame $\xrightarrow{\neq}$
- Light front quantization




$P^z \to \infty$

the constituents become approximately collinear, hadron admits a simple partonic interpretation




time: $x^+ = x^0 + x^3$

Hamiltonian: $P^- = P^0 - P^3$

$$\frac{1}{2}P^-|\psi(x^+)\rangle = i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle$$

[Review: Brodsky:1997de]

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- charmonium are solved from eigenvalue equations

$$P^-_{QCD}|\phi\rangle = P^-_\phi|\phi\rangle \Longleftrightarrow \underbrace{(P^-_{QCD}P^+ - \vec{P}^2_\perp)}_{H_{\rm LC}}|\psi_h(P,j,m_j)\rangle = M_h^2|\psi_h(P,j,m_j)\rangle$$

*boost invariant*

- transition amplitudes can be evaluated directly with obtained wavefunctions

$$\langle \mathcal{P}(P')|J^\mu_q(0)|\mathcal{V}(P,m_j)\rangle = \sum_{s,\bar{s},s'}\int_0^1 \frac{\mathrm{d}x}{2x^2(1-x)}\int\frac{\mathrm{d}^2\vec{k}_\perp}{(2\pi)^3}\boxed{\psi^{(m_j)}_{s\bar{s}/\mathcal{V}}(\vec{k}_\perp,x)}\boxed{\psi^*_{s'\bar{s}/\mathcal{P}}(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)}$$
$$\times\,\bar{u}_{s'}(xP^+,\vec{k}_\perp+x\vec{P}_\perp+\vec{q}_\perp)\gamma^\mu u_s(xP^+,\vec{k}_\perp+x\vec{P}_\perp).$$

# LF wavefunction representation of TFF

The pseudoscalar transition amplitude is parameterized as $\mathcal{M}^{\mu\nu} = 4\pi\alpha_{\text{em}}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2)$

Single-tagged transition form factor:

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2} + \cdots\cdots$$

higher Fock sector contributions

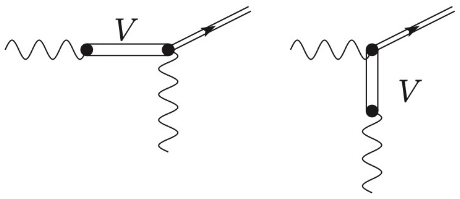
$Q \to 0$

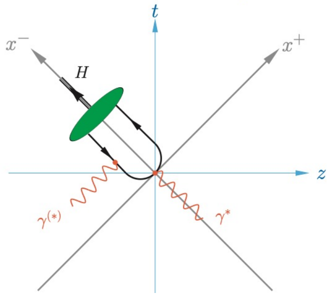
$$\sum_V \frac{e_f^2 f_V}{1+\frac{M_P}{M_V}}\frac{g_V(0)}{M_V^2+Q^2}$$

$Q \to \infty$

$$e_f^2 f_P \int_0^1 \mathrm{d}x \frac{\phi_P(x,\mu)}{x(1-x)Q^2 + m_f^2}$$

$Q^2 \to 0$ intermediate $Q^2$ $Q^2 \to \infty$

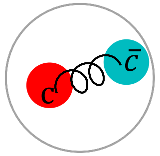


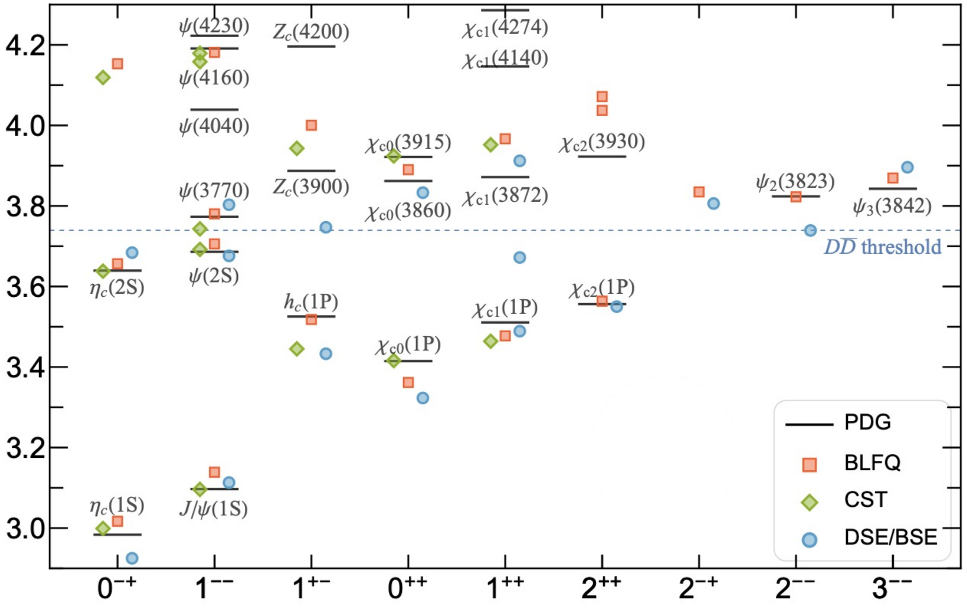
# Charmonium from LFQCD

- The quantum many-body exponential wall
- Wilson's program: low-energy effective Hamiltonian from similarity RG [Wilson:1994fk]]
- BLFQ: starting from semi-classical confinement from AdS/QCD [Brodsky:2003px, Vary:2009gt]

$$H_{\text{LFQCD}} \xrightarrow{\text{SRG}} H_{\text{eff}} = \sum_i \frac{p_{i\perp}^2 + m_i^2}{x_i} + U_i^{(0)} + \sum_{ij} \alpha_s U_{ij}^{(1)} + \sum_{ij} \alpha_s^2 U_{ij}^{(2)} + \cdots$$




$$\underbrace{\kappa^4 x(1-x)\vec{r}_\perp^{\,2} - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}} + \underbrace{V_g}_{\text{one-gluon exchange}}$$



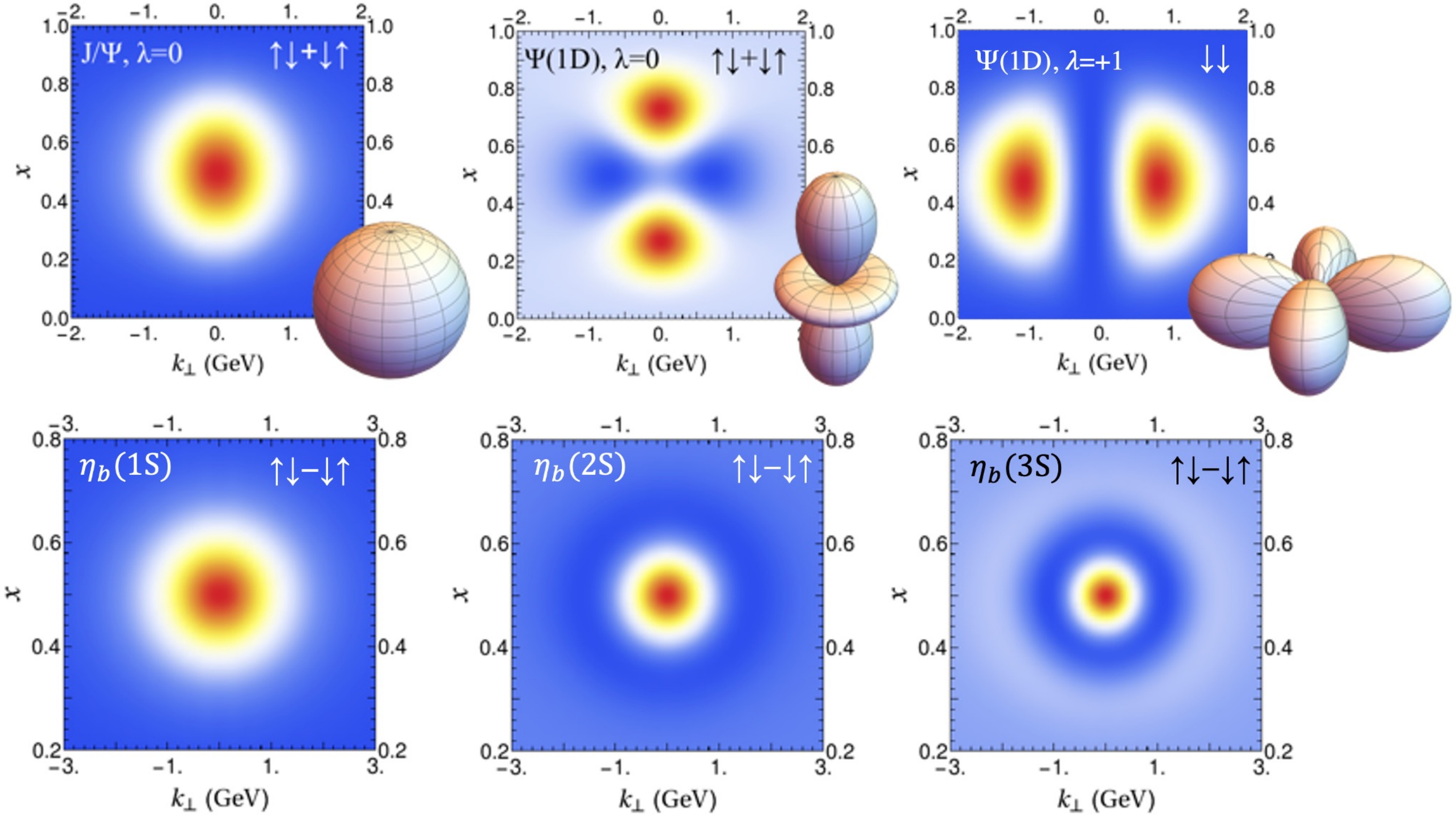

free parameters: $\kappa, m_q$

BLFQ: [Li:2017mlw] ←Y. Li, P. Maris, J. P. Vary
CST: [Leitao:2017mlx]
DSE: [Fischer:2014cfa]

# Light-front wavefunctions of charmonium

angular
excitation
s

radial
excitation
s




→ $\langle\psi|\hat{O}|\psi\rangle$ parameter free

[Li:2017mlw, LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

# Timeline: $c\bar{c}$ TFFs on the light front

| | | |
|---|---|---|
| 2018 | Radiative transitions between 0-+ and 1-- heavy quarkonia on the light front, ML, Y. Li, P. Maris and J. P. Vary, PRD 98, 034024 (2018) | --------- M1 TFF |
| 2019 | Frame dependence of transition form factors and Dalitz decays in light-front dynamics, ML, Y. Li, P. Maris and J. P. Vary, PRD 100, 036006 (2019) | --------- M1 timelike TFF |
| 2022 | Two-photon transitions of charmonia on the light front, Y. Li, ML, and J. P. Vary, PRD 105, L071901(2022) | --------- diphoton TFF |
| | Light-front wavefunction of mesons by design, ML, Y. Li, G. Chen, T. Lappi, and J. P. Vary, EPJC 82, 1045 (2022) | --------- $c\bar{c}$ wavefunction & diphoton TFF |
| 2024 | Shedding light on charmonium, Z. Wang, ML, Y. Li and J. P. Vary, PRD 109, 3 (2024) | --------- E1 TFF |
| 2025 | Relativistic dynamics of charmonia in strong magnetic fields, L. Wen, ML, Y. Zhou, Y. Li and J. P. Vary, PRD112, 014019 (2025) | --------- $c\bar{c}$ in B field |
| 2026 | Meson production in high energy collisions, X. Cao, ML, Y. Li, W. Qian, in progress | --------- $\gamma^* \to c\bar{c}$ |

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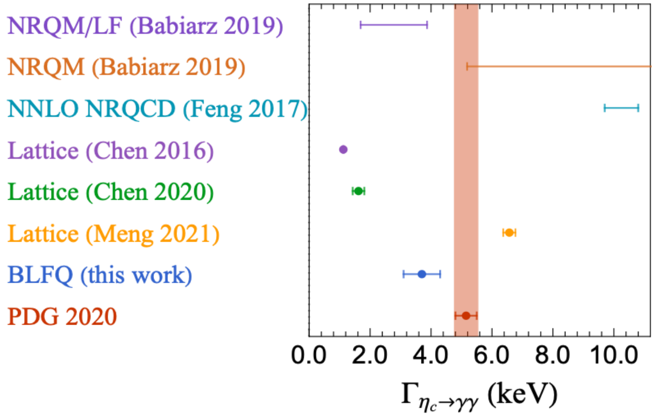
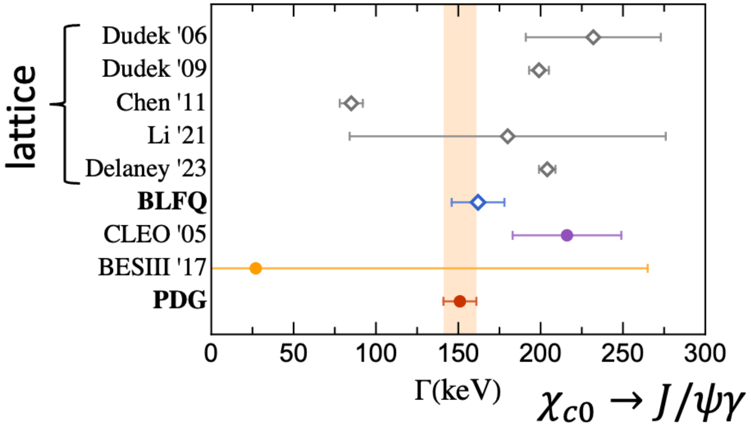
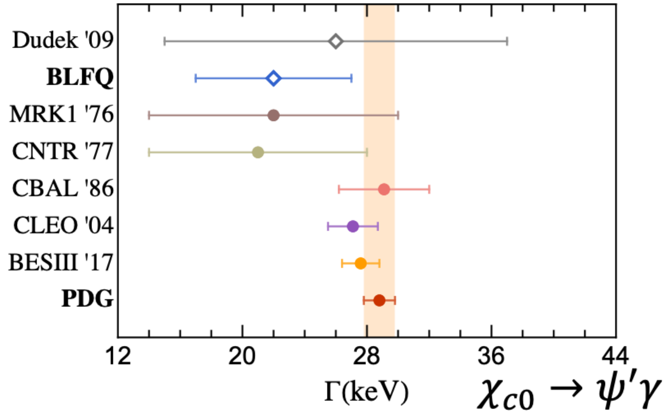
# Radiative widths

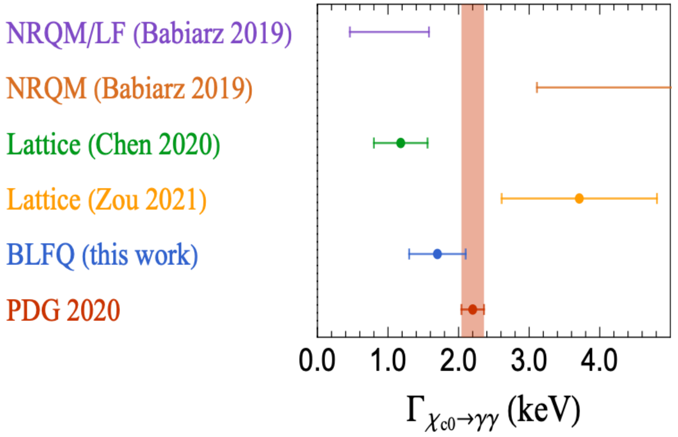
$$\Gamma(i \to \gamma f) = \frac{1}{16\pi}\frac{M_i^2 - M_f^2}{M_i^3}\frac{1}{2J_i+1}\sum_{\lambda_\gamma, s_i, s_f}\left|\langle f|J^\mu(0)|i\rangle\right|^2, \quad f = [c\bar{c}], n\gamma$$

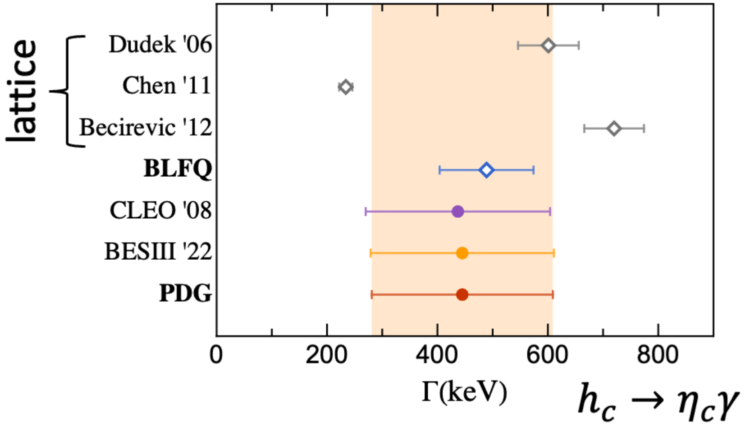
[Li:2021ejv, Wang:2023nhb]

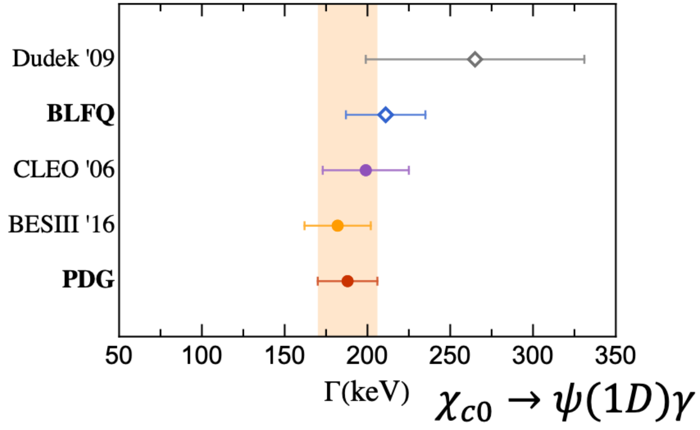


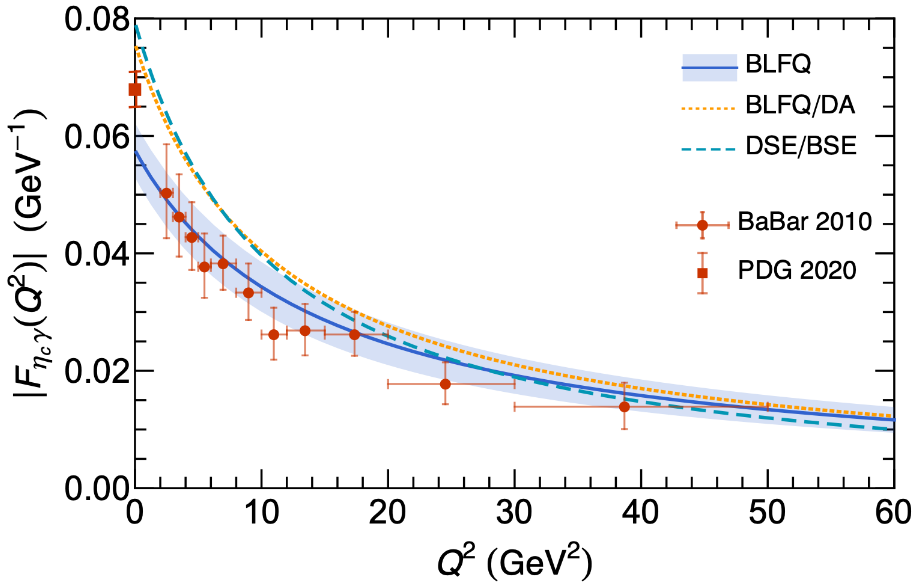
*pseudoscalar* $0^{+-}$

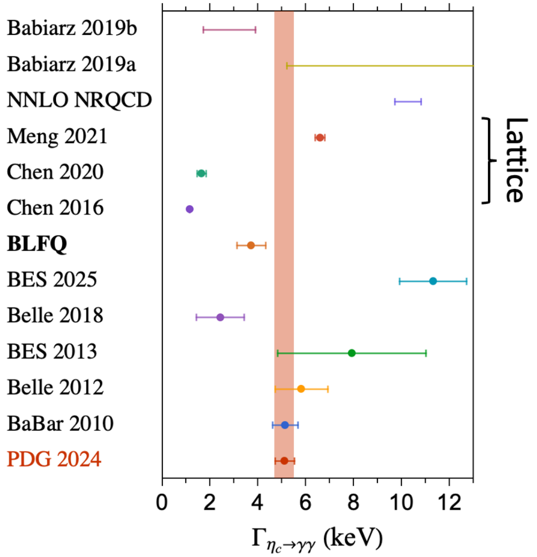
# Two-photon TFF: $\eta_c \to \gamma + \gamma^*$

$$\epsilon^*_\mu(q_1)\mathcal{M}^{\mu\nu} = \langle \gamma(q_1,\lambda_1)|J^\nu(0)|P\rangle = \epsilon^*_\mu(q_1)4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2,q_2^2),$$

Diphoton width: $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha_{\rm em}^2 M_P^3 |F_{P\gamma\gamma}(0,0)|^2$, where, $F_{P\gamma}(Q^2) \equiv F_{P\gamma\gamma}(Q^2,0) = F_{P\gamma\gamma}(0,Q^2)$ is the single-tag TFF.

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int \frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2}$$



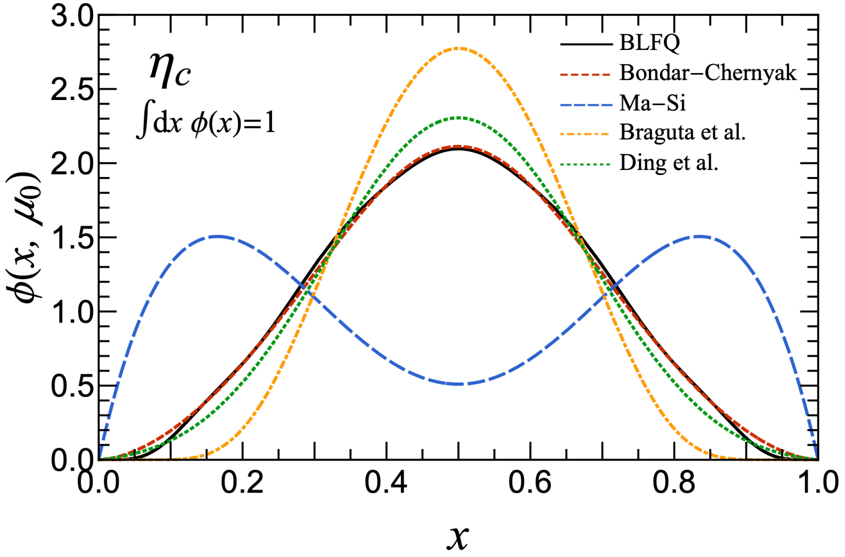



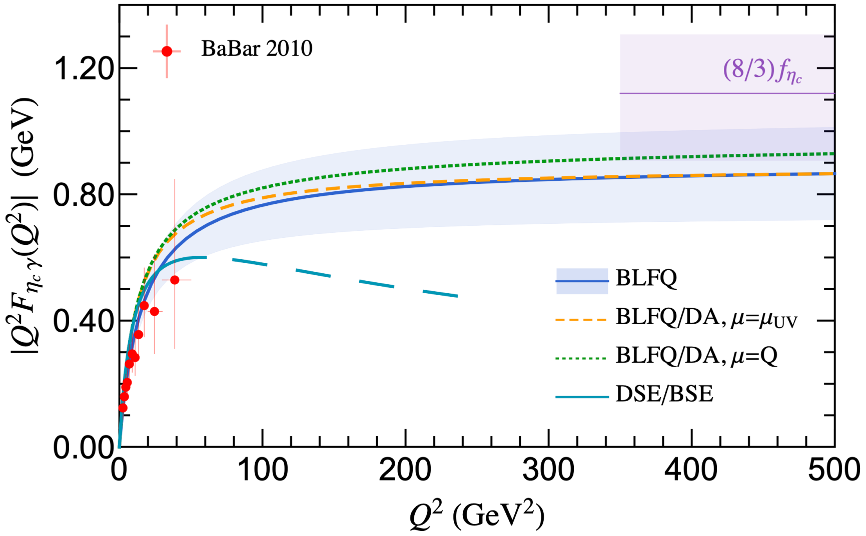

[Chen:2016bp, Babiarz:2019sfa, Meng:2021ecs, Li:2021ejv]

# LCDA and large-$Q^2$ asymptotics

At large-$Q^2$, viz. $Q^2 + \langle m_f^2/x(1-x)\rangle \gg \langle k_\perp^2/x(1-x)\rangle$,

$$F_{P\gamma}(Q^2) \approx e_f^2 f_P \int_0^1 \mathrm{d}x \frac{\phi_P(x,\mu)}{x(1-x)Q^2 + m_f^2} \xrightarrow{Q\to\infty} \frac{6e_f^2 f_P}{Q^2}.$$




[Ma:2004qf, Bondar:2004sv, Braguta:2006wr, Ding:2015rkn, Chen:2016bp, Li:2021ejv] pseudoscalar $0^{+-}$

# Two-photon TFF: $\eta_c(2S) \to \gamma + \gamma^*$

$$\epsilon^*_\mu(q_1)\mathcal{M}^{\mu\nu} = \langle\gamma(q_1,\lambda_1)|J^\nu(0)|P\rangle = \epsilon^*_\mu(q_1)4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2,q_2^2),$$

Diphoton width: $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha^2_{\rm em}M_P^3\left|F_{P\gamma\gamma}(0,0)\right|^2$, where, $F_{P\gamma}(Q^2) \equiv F_{P\gamma\gamma}(Q^2,0) = F_{P\gamma\gamma}(0,Q^2)$ is the single-tag TFF.

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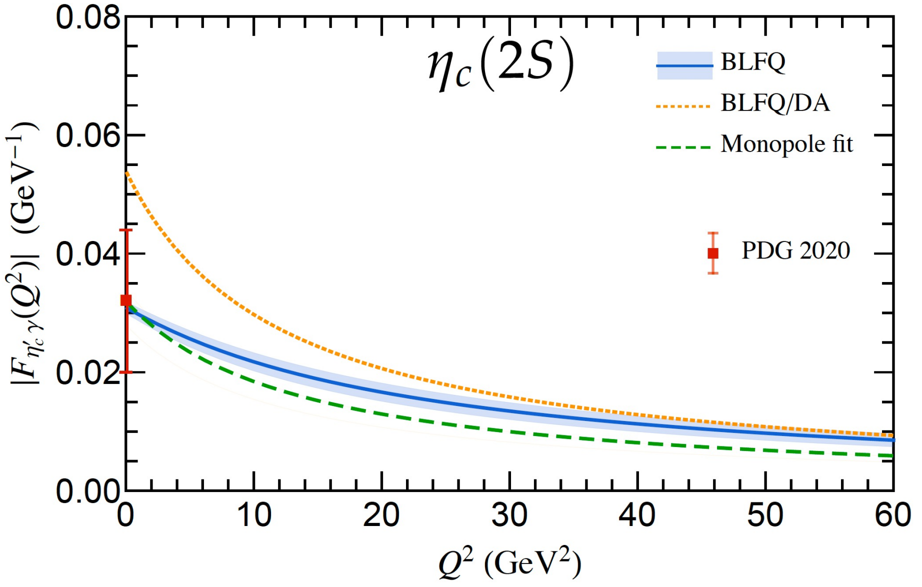



- No experimental measurement yet
- A monopole fit using $\Lambda^2 = M_\psi^2$, is included for comparison.

[Li:2021ejv]

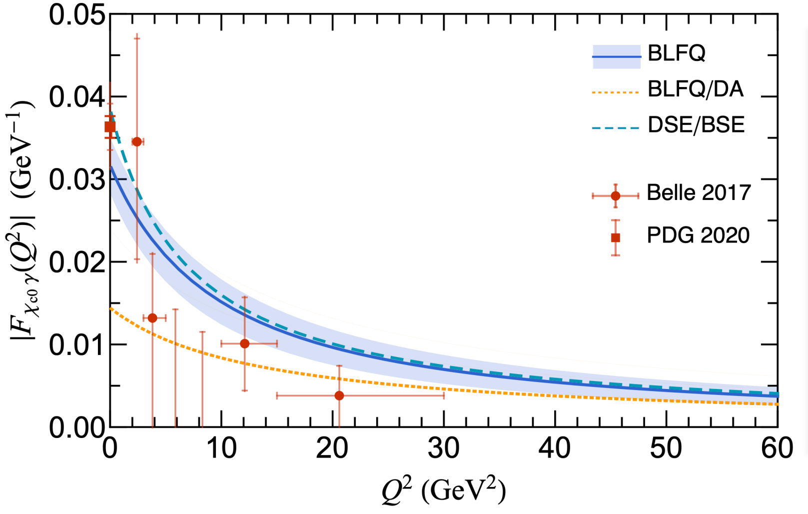
*scalar* $0^{++}$

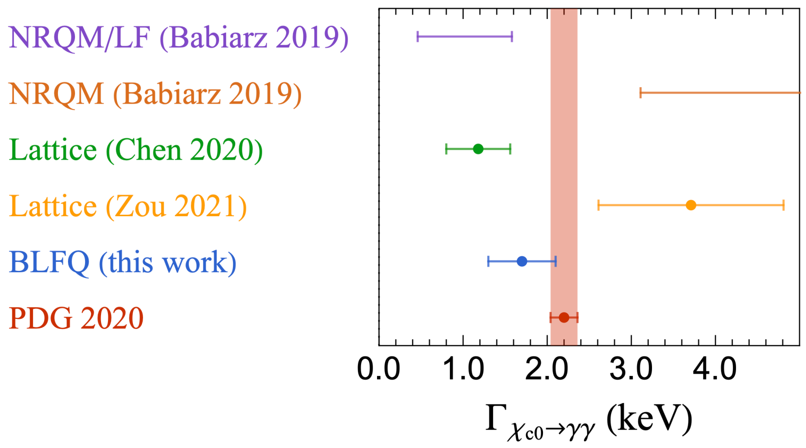
# Two-photon TFF: $\chi_{c0} \to \gamma + \gamma^*$

$$\mathcal{M}^{\mu\nu}_{S\to\gamma\gamma} = 4\pi\alpha_{\rm em}\Big\{\big[(q_1\cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu\big]F_1^S(q_1^2,q_2^2)+ \frac{1}{M_S^2}\big[q_1^2q_2^2g^{\mu\nu} + (q_1\cdot q_2)q_1^\mu q_2^\nu - q_1^2q_2^\mu q_2^\nu - q_2^2q_1^\mu q_1^\nu\big]F_2^S(q_1^2,q_2^2)\Big\}$$

Width $\Gamma_{\gamma\gamma} = \frac{\pi\alpha_{\rm em}^2}{4}M_S^3|F_{S\gamma}(0)|^2$, where $F_{S\gamma}(q^2) = F_1^S(q^2,0) = F_1^S(0,q^2)$ is the single-tag TFF.

- Belle provides the first measurement of the TFF, albeit with limited statistics.



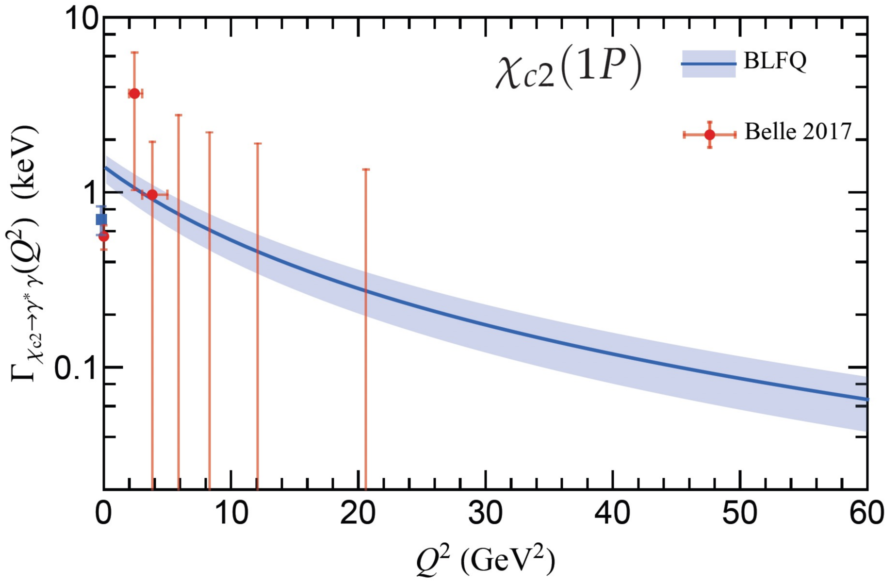

[Belle:2017xsz, Chen:2016bp, Babiarz:2019sfa, Zou:2021mgf, CLQCD:2020njc, Li:2021ejv] *scalar* $2^{++}$

# Two-photon TFF: $\chi_{c2} \to \gamma + \gamma^*$

The two-photon decay width of the tensor is determined by two helicity amplitudes,

$$\Gamma_{T\to\gamma\gamma} = \frac{\pi\alpha_{\text{em}}^2}{5M_T}\left(\left|\mathcal{M}_{++;0}\right|^2 + \left|\mathcal{M}_{+-;2}\right|^2\right)$$

- Belle provides the first measurement of the single-tagged $\Gamma_{T\to\gamma^*\gamma}(Q^2)$, albeit with limited statistics.




[Belle:2017xsz, Li:2021ejv]

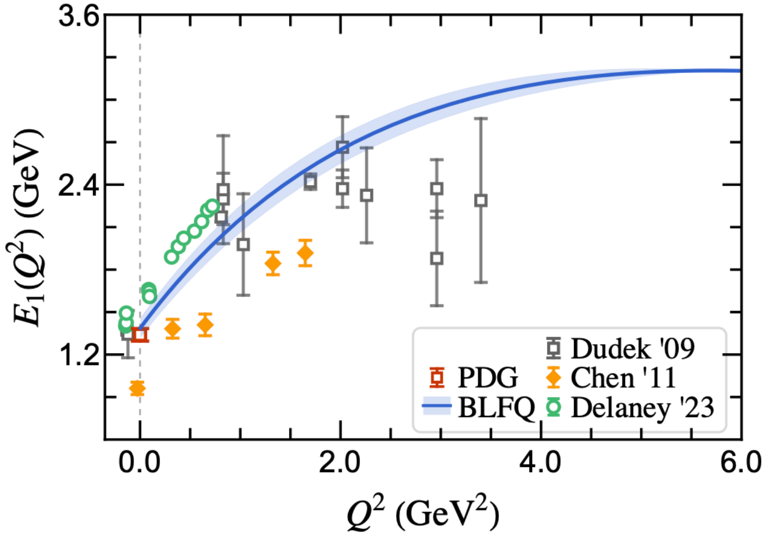
# E1 TFF: $\chi_{c0} \to J/\psi + \gamma^*$

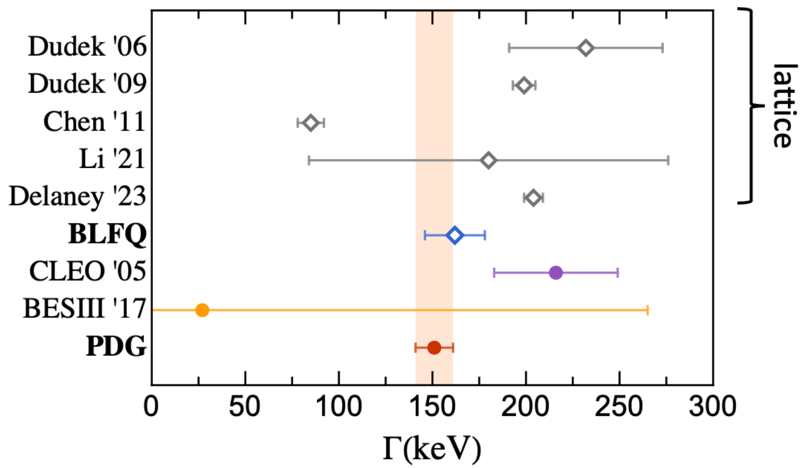
*scalar* $0^{++}$ — *vector* $1^{--}$

$$\langle V(p',\lambda')|J^\mu(0)|S(p)\rangle = E_1(Q^2)\Big[e^{\mu *}_{\lambda'}(p') - \frac{e^*_{\lambda'}\cdot p}{(p\cdot p')^2 - M_S^2 M_V^2}\big(p'^\mu(p\cdot p') - M_V^2 p^\mu\big)\Big]$$
$$+ C_1(Q^2)\frac{M_V}{Q(p\cdot p')^2 - M_S^2 M_V^2}(e^*_{\lambda'}\cdot p)\Big[(p\cdot p')(p+p')^\mu - M_S^2 p'^\mu - M_V^2 p^\mu\Big],$$

Note that there are two TFFs, and $E_1$ is related to the radiative width: $\Gamma = \frac{2\alpha_{\rm em}}{9}\frac{M_S^2 - M_V^2}{M_S^3}|E_1(0)|^2$.

$$E_1(0) = \frac{M_S^2 - M_V^2}{i\sqrt{2}}\sum_{s,\bar s}\int_0^1 \frac{dx}{4\pi}\int d^2 r_\perp (r_x + i r_y)\psi^{(\lambda=+1)*}_{s\bar s/V}(x,\vec r_\perp)\psi_{s\bar s/S}(x,\vec r_\perp).$$




[Dudek:2009kk, Chen:2011kpa, Delaney:2023fsc, Wang:2023nhb]

# E1 TFF: $\psi(2S) \to \chi_{c0} + \gamma^*$

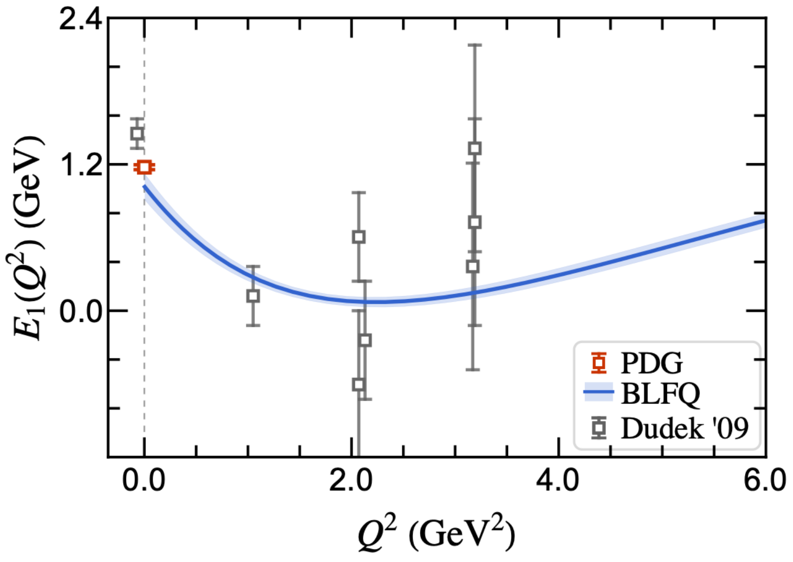
*vector* $1^{--}$

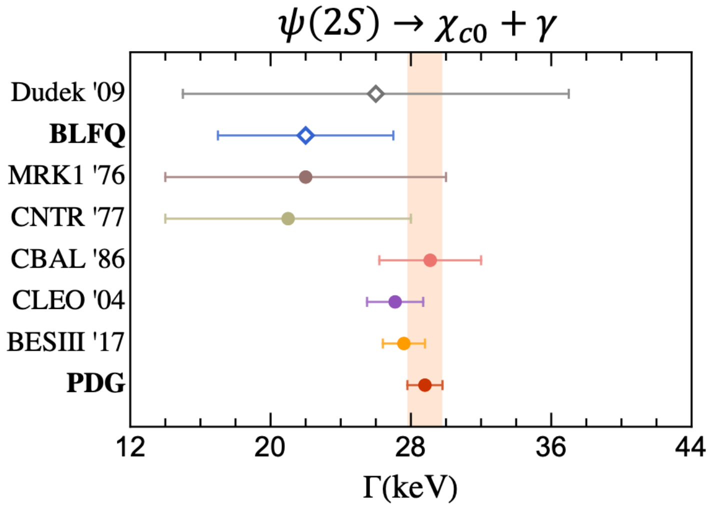
*scalar* $0^{++}$

$$\langle V(p',\lambda')|J^\mu(0)|S(p)\rangle = E_1(Q^2)\left[e^{\mu *}_{\lambda'}(p') - \frac{e^*_{\lambda'}\cdot p}{(p\cdot p')^2 - M_S^2 M_V^2}\left(p'^\mu(p\cdot p') - M_V^2 p^\mu\right)\right]$$
$$+ C_1(Q^2)\frac{M_V}{Q(p\cdot p')^2 - M_S^2 M_V^2}(e^*_{\lambda'}\cdot p)\left[(p\cdot p')(p+p')^\mu - M_S^2 p'^\mu - M_V^2 p^\mu\right],$$

Note that there are two TFFs, and $E_1$ is related to the radiative width: $\Gamma = \frac{2\alpha_{\rm em}}{9}\frac{M_S^2 - M_V^2}{M_S^3}|E_1(0)|^2$.

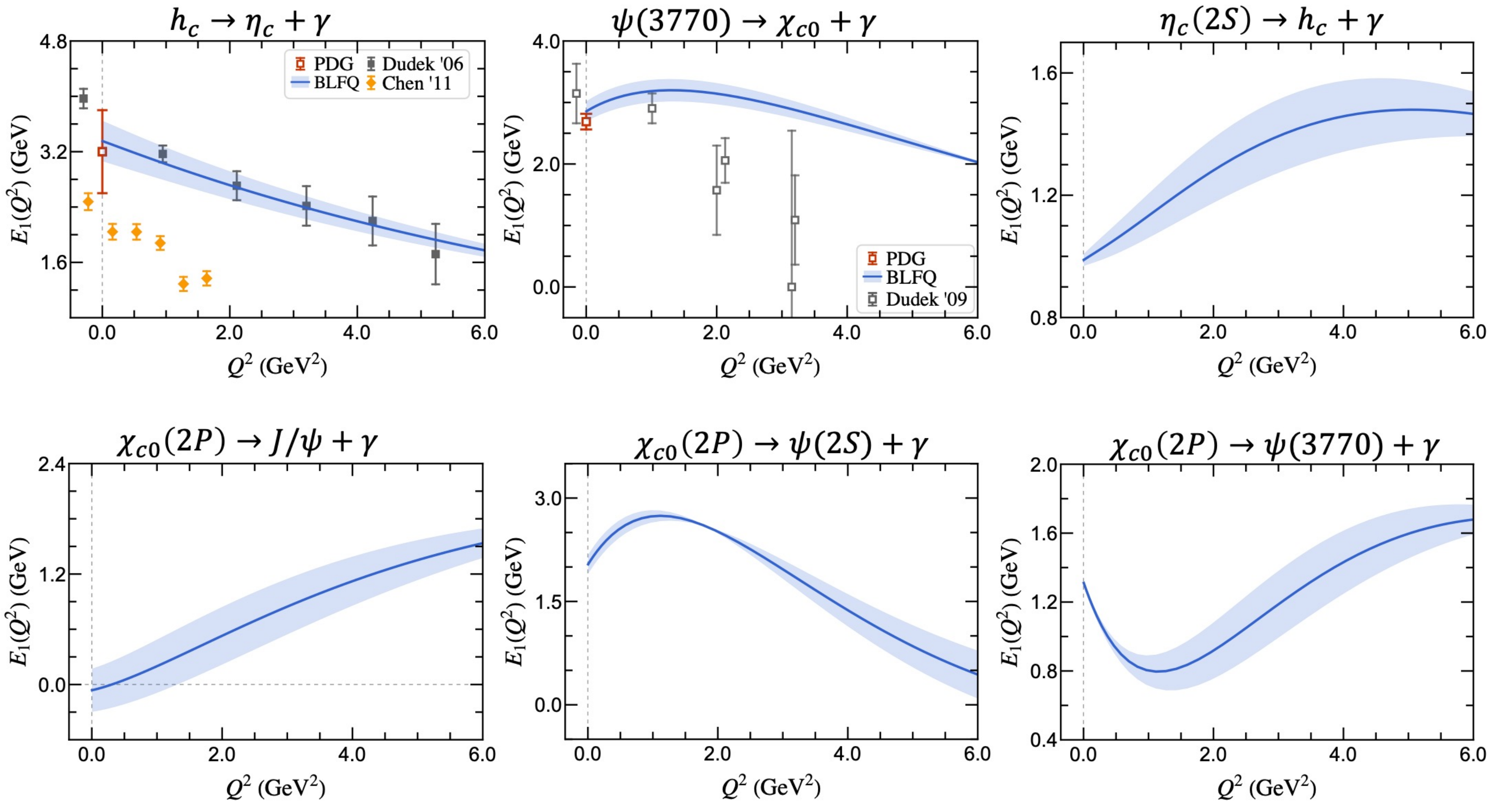
$$E_1(0) = \frac{M_S^2 - M_V^2}{i\sqrt{2}}\sum_{s,\bar{s}}\int_0^1 \frac{dx}{4\pi}\int d^2 r_\perp (r_x + i r_y)\psi^{(\lambda=+1)*}_{s\bar{s}/V}(x,\vec{r}_\perp)\psi_{s\bar{s}/S}(x,\vec{r}_\perp).$$



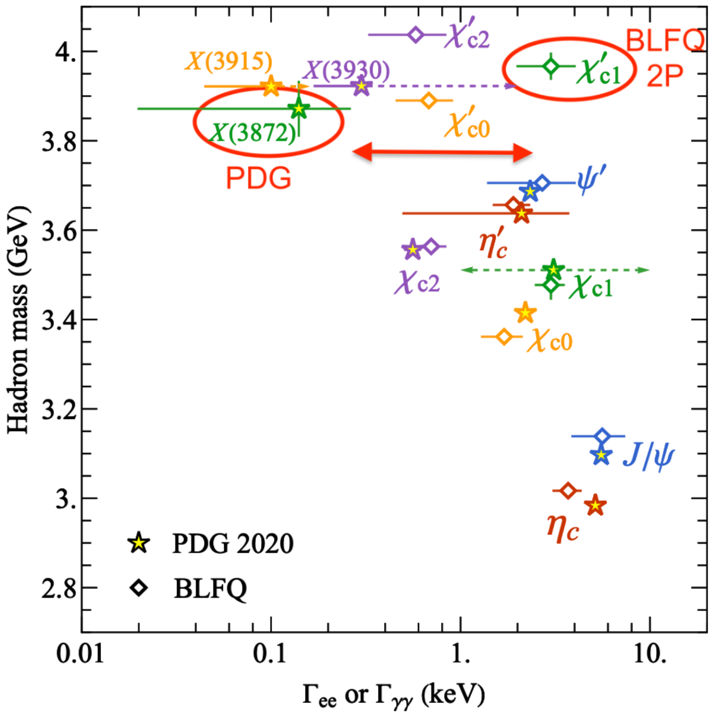

[Dudek:2009kk, Wang:2023nhb]

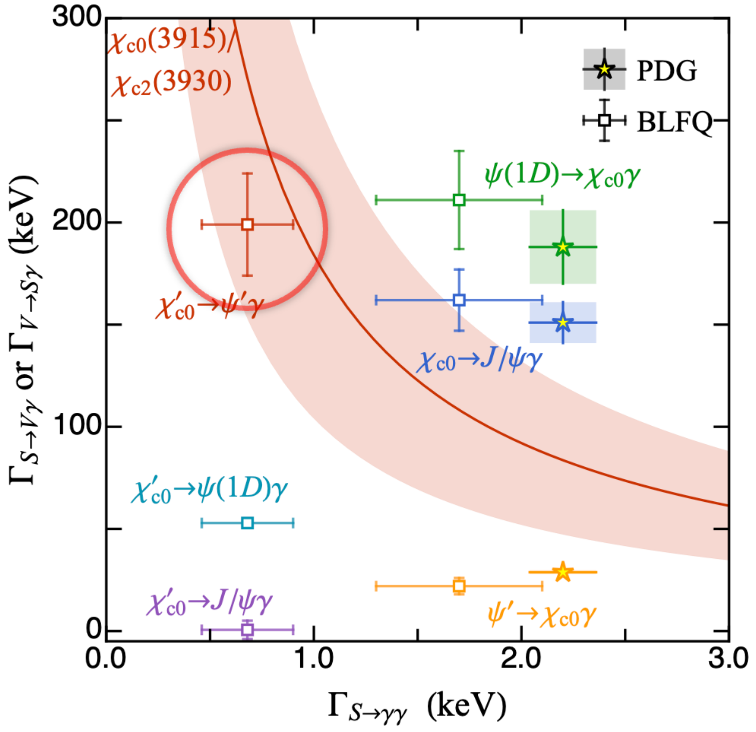
# E1 TFFs




[Dudek:2009kk, Chen:2011kpa, Delaney:2023fsc, Wang:2023nhb]

# Implication to charmonium-like mesons




[Li:2021ejv, Wang:2023nhb]

- $\Gamma_{ee}, \Gamma_{\gamma\gamma} \propto |\Psi(0)|^2$
- $\tilde{\Gamma}_{\gamma\gamma}$ does not support X(3872) as $\chi_{c1}(2P)$ [Belle:2020ndp, cf. Babiarz:2023ebe]
- $\Gamma_{R\to\psi'+\gamma}$ and $\Gamma_{R\to\gamma+\gamma}$ is compatible with $\chi_{c0}(2P)$ [Belle:2021nuv]

# Summary and outlook

- We investigate charmonium radiative transitions using **light-front Hamiltonian** formalism that incorporates **relativity** and **nonperturbative effects**
- High-quality fit to mass spectrum and **parameter-free** predictions on radiative widths and transition form factors, in good agreement with experiments and lattice results whenever available
- Future developments: high Fock sectors (e.g. gluons and sea), nonperturbative Hamiltonian renormalization, hadronization process

*Thank you!*