MDMs and EDMs

Maxim Pospelov
U of Minnesota and FTPI

Thursday: **MDMs** (Muon and electron g-2, theory and experiment).

Friday: **EDMs** (Electric Dipole Moments).

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Thursday lecture

1. Introduction: different ways to look for new physics
2. Review of the Dirac equation, $g = 2$. Derivation of $g - 2 = \alpha/\pi$ at one loop level.
3. Main principles of the muon g-2 experiment (from a theorist's perspective).
4. Standard Model calculation of muon g-2. Crux of the matter – hadronic vacuum polarization.
5. Sensitivity of muon g-2 to new physics.
6. Electron g-2, determination of alpha
7. Potential way of reducing errors in the future.
8. Other exotic effects with spins, so-called "Lorentz violation", "axion wind", and mass-spin coupling.

Anomalous magnetic moment and EDM of an elementary fermion

Anomalous magnetic moment:

$$\frac{1}{2}\overline{\psi}F_{\mu\nu}\sigma^{\mu\nu}\psi$$

- Induced by "any" interactions.
- The leading effect is large (~O(1) for hadrons, O(10^{-3}) for leptons)
- Can be made into a precision tool *if* the accuracy of measurements is matched by accuracy of calculations

Electric dipole moment:

$$\frac{1}{2}\overline{\psi}\tilde{F}_{\mu\nu}\sigma^{\mu\nu}\psi$$

- Tilde flips E and B.
- Only CP-violating interactions induce it; very special, *not* common.
- Is a precision tool "*automatically*" as SM (apart from strong CP caveat) does not induce large EDMs

Late 1960s Lagrangian (before SM was formulated)

$$\mathcal{L}_{1960s} = -\frac{1}{4}(F_{\rm EM})^2 + \sum_{\psi} \bar{\psi}(\gamma_\mu D_\mu^{\rm EM} - m - \mu(\sigma F^{\rm EM}))\psi$$

$$-\frac{G_F}{\sqrt{2}}\left[\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e \bar{\nu}_\mu \gamma_\alpha(1-\gamma_5)\mu\right] - \frac{G_F}{\sqrt{2}}\left[\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e \bar{p}\underline{\gamma_\alpha(g_V - g_A\gamma_5)}n\right] \times \cos\theta_C$$

$$+(\text{similar terms for strange particles}) \times \sin\theta_C$$

- Pretty complicated, with every new phenomenon requiring ~ extra terms added by hand.

- What to do with resonances? (Notwithstanding beautiful flavor SU(3) symmetries uncovered in the 1960s.) *Quarks were not real.*

- Promising theoretical constructions were built using spontaneous symmetry breaking mechanism by Brout, Englert, Higgs et al, and applied to Glashow's SU(2)*U(1) by Weinberg and Salam.

SM Lagrangian as an EFT

******New orthodoxy*******: Standard Model Lagrangian includes all terms of canonical dimension 4 and less, consistent with three generations of quarks and leptons and the SU(3)*SU(2)*U(1) gauge structure at classical and quantum levels.

- Higgs is finally discovered. Alternatives (e.g. strong coupling at a TeV) are mostly dead/severely constrained.
- CP violation in the quark sector comes CKM
- Neutrinos contain intriguing clues (Masses and oscillations were not part of the 1967 Weinberg-Salam model).
- Problems: Strong CP problem, dark matter problem, neutrino mass problem, and more conceptual problems (gauge hierarchy).

SM as an Effective Field Theory

Typical BSM model-independent approach is to include all possible BSM operators + light new states explicitly.

$$\mathcal{L}_{\rm EFT} = m_H^2 H^\dagger H + \mathcal{L}_{\rm SM,\,dim\,4}(A,\psi,H) + \sum_{n\geq 5} \frac{1}{\Lambda^{n-4}} O^{(n)}_{\rm SM,\,dim\,\mathit{n}}(A,\psi,H)$$

$$+ \mathcal{L}_{\rm portals}(\psi, A, H; \varphi_{\rm DS}) + \mathcal{L}_{DS}(\varphi_{\rm DS}).$$

Higher dimensional operators include many terms, e.g. $\frac{1}{\Lambda^2}(\bar{e}e)(\bar{q}q)$

Light states and portal couplings also have great variety

Portal	Coupling
Vector: Dark Photon, A'	$-\frac{\varepsilon}{2\cos\theta_W} F'_{\mu\nu} B^{\mu\nu}$
Scalar: Dark Higgs, S	$(\mu S + \lambda_{\rm HS} S^2) H^\dagger H$
Fermion: Heavy Neutral Lepton, N	$y_N LHN$
Pseudo-scalar: Axion, a	$\frac{a}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu},\ \frac{a}{f_a} G_{i,\mu\nu}\tilde{G}_i^{\mu\nu},\ \frac{\partial_\mu a}{f_a}\overline{\psi}\gamma^\mu\gamma^5\psi$

How to look for New Physics ?

1. High energy colliders.

$$\frac{1}{\Lambda^2}(\bar{e}e)(\bar{q}q) \rightarrow \sigma \propto \frac{E^2}{\Lambda^4} \rightarrow \Lambda > 10\,\mathrm{TeV}$$

2. Precision measurements, especially when a symmetry is broken

$$\frac{1}{\Lambda_{\mathrm{CP}}^2}(\bar{e}i\gamma_5 e)(\bar{q}q) \rightarrow \mathrm{EDM}, \frac{1}{\Lambda_{\mathrm{CP}}^2} < 10^{-10} G_F \rightarrow \Lambda_{\mathrm{CP}} > 10^7\,\mathrm{GeV}$$

3. Intensity frontier experiments where abnormal to SM appearance of FIPs (or sometimes disappearance, e.g. NA64) can be searched.

$$pp \rightarrow \pi, K, B \rightarrow HNL + X \rightarrow HNL \text{ decay to SM}$$

4. DM searches: *Atom + DM* → *visible energy*
   DM + DM → *visible energy*

Precision g-2 of electrons and muons belongs to an interesting class of tools where we can measure & calculate same quantity to high precision.

Definition of g-factors

Consider a particle at rest, in the external magnetic field,

$$H = -\mu \mathbf{B} \cdot \frac{\mathbf{S}}{S}; \quad \boldsymbol{\mu} = \mu \frac{\mathbf{S}}{S}; \quad \boldsymbol{\mu} \equiv g \frac{e}{2m} \mathbf{S}$$

For electrons, we knew since 1920s that $H = +|e|/(2m_e)\,(\boldsymbol{B}\,\boldsymbol{\sigma})$, and therefore

$$g = 2$$

(from now on we'll switch to notations where *e* is positive, i.e. positron charge, and we will also use the system of units where hbar=c=1.)

Other particles are trickier ($g_{proton} = 5.6$, for example).

The derivation of g=2 for electrons and positrons comes from the Dirac equation. Taus are cool but useless for g-2 – too short lived.*

Dirac Equation, v/c expansion, g=2

Let's start from the Dirac equation for a free fermion:

$$(\gamma_\mu p_\mu - m)\psi = 0$$

We generalize it to include the EM field in the *minimal* way by promoting four-momentum operator to a covariant momentum:

$$(\gamma_\mu P_\mu - m)\psi = 0 \qquad P_\mu = p_\mu + \frac{e}{c}A_\mu = i\partial_\mu + \frac{e}{c}A_\mu; \quad A_\mu = (\Phi, -\mathbf{A})$$

(We keep c for now for the sake of *1/c* expansion). Wave function ψ has a fast oscillation, and to remove it, we transform $\psi \to \psi \exp(imc^2 t)$

Then, in bi-spinor components, $\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$, the Dirac equation is rewritten as

$$(i\frac{\partial}{\partial t} + e\Phi)\varphi = c\boldsymbol{\sigma} \cdot (\mathbf{p} + \frac{e}{c}\mathbf{A})\chi$$

$$(i\frac{\partial}{\partial t} + e\Phi + 2mc^2)\chi = c\boldsymbol{\sigma} \cdot (\mathbf{p} + \frac{e}{c}\mathbf{A})\varphi$$

Dirac Equation, v/c expansion, g=2

Using that mc^2 is much larger than any other energy involved, we solve for χ the second equation and substitute into first.

$$\chi = \frac{1}{2mc}\boldsymbol{\sigma}\cdot(\mathbf{p}+\frac{e}{c}\mathbf{A})\varphi$$

$$i\frac{\partial}{\partial t}\varphi = \frac{1}{2m}\left[\boldsymbol{\sigma}\cdot(\mathbf{p}+\frac{e}{c}\mathbf{A})\right]^2\varphi - e\Phi\varphi$$

Developing the product of two sigma-matrices, $\sigma_i\sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$ we get:

$$\left[\boldsymbol{\sigma}\cdot(\mathbf{p}+\frac{e}{c}\mathbf{A})\right]^2 = \left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2 + i\sigma_k\epsilon_{ijk}[p_i, A_j] = \left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2 + \frac{e}{c}(\boldsymbol{\sigma}\cdot\mathbf{B})$$

Then we get a familiar Schrodinger (Pauli) equation

$$i\frac{\partial}{\partial t}\varphi = \left\{\frac{1}{2m}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2 - e\Phi + \frac{e}{2mc}(\boldsymbol{\sigma}\cdot\mathbf{B})\right\}\varphi$$

$$\boldsymbol{\mu} = \frac{e}{2mc}\boldsymbol{\sigma} = \frac{e}{2mc}\boldsymbol{S}\times\mathbf{2}$$

Prediction of ***g* = *2*** is the *second* most famous prediction of Dirac eq.

Why muons and electrons?

$\boxed{p}$ $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Magnetic moment $\mu = 2.7928473446 \pm 0.0000000008\ \mu_N$

Proton magnetic moment is measured extremely precisely, but theoretical calculation, state of the art lattice, is ~ few% at best

Electromagnetic form factors of the nucleon from $N_f = 2+1$ lattice QCD

Dalibor Djukanovic,[1,2] Georg von Hippel,[3] Harvey B. Meyer,[1,3] Konstantin Ottnad,[3] Miguel Salg,[3,*] and Hartmut Wittig[1,3]

For the proton, we obtain as our final values $\langle r_E^2 \rangle^p = (0.672 \pm 0.014\ \text{(stat)} \pm 0.018\ \text{(syst)})\ \text{fm}^2$, $\langle r_M^2 \rangle^p = (0.658 \pm 0.012\ \text{(stat)} \pm 0.008\ \text{(syst)})\ \text{fm}^2$, and $\mu_M^p = \underline{2.739} \pm 0.063\ \text{(stat)} \pm 0.018\ \text{(syst)}$. The

$$\left(\gamma_\mu P_\mu - m - \frac{1}{2}\mu_{\text{anom}}(F\sigma)\right)\psi = 0$$ Dirac equation for nucleons

The main reason to use electrons and muons is the possibility of calculating 8 significant figures for muon g-2, and even more for electron!

Muon g-2 as a probe of New Physics

- Muon anomalous magnetic moment (= (g-2)/2) belongs to a class of observables that do not break any exact or approximate symmetries of the SM (like CP or baryon number). Therefore, there are large contributions from all known forces within SM:

$$a_l^{SM} = a_l^{QED} + ... = \frac{\alpha}{(2\pi)} + ...$$

$$a_l^{NP} \simeq \frac{m_l^2}{\Lambda_{NP}^2}$$

where "New Physics" contribution parametrizes a new contribution not accounted in standard calculations: $\mathcal{L}_{NP} = \frac{m_l}{\Lambda_{NP}^2} \times \overline{\mu} F_{\alpha\beta} \sigma_{\alpha\beta} \mu$

Then the idea is to extract a_l^{NP} from measurement and theory by taking the difference: $a_\mu^{NP} = a_\mu^{experiment} - a_\mu^{SM\ \ theory}$

Muon g-2 as a probe of New Physics

Of all particles muon is ideally suited for such purposes:

- Possible contribution from short-distances physics is likely to be enhanced relative to electron by $(m_\mu/m_e)^2$. (And tau's are useless because they do not live long enough).
- Leptons are elementary, and theoretical calculations can be carried out to high accuracy before running into problems with the complicated nature of strong interactions. (Same program is simply not possible with anomalous magnetic moments of neutrons or protons)
- Nature itself "built in" the muon spin analyzer. The weak decays of muons break parity 100% and as a result the direction of the outgoing decay particle (electron or positron) is correlated with muon spin. + Muons are born polarized.
- 2011 theory: $a_\mu^{SM\ theory} = 116591828 \pm 49 \times 10^{-11}$

$a_\mu^{experiment} = 116592089(63) \times 10^{-11}$ $a_\mu^{NP} = (26.1 \pm 8) \times 10^{-10}$

Derivation of 1-loop QED result

Let's calculate famous 1-loop g-2 correction using external field technique.

We are looking for a finite in the UV correction to the effective Lagrangian

$$\mathcal{L} = -\frac{\mu}{2}\bar{\psi}F_{\alpha\beta}\sigma_{\alpha\beta}\psi \equiv -\frac{\mu}{2}\bar{\psi}(F\sigma)\psi$$

One-loop expression for it is:

$$i\mathcal{M} = i\mathcal{L} = \bar{\psi}(P)\left[\int\frac{d^4q}{(2\pi)^4}(ie)^2\frac{i}{q^2-m_V^2}\gamma_\mu S_f(q+P)\gamma_\mu\right]\psi(P)$$

Covariant momentum satisfies the following properties:

$$[P_\alpha, P_\beta] = ieF_{\alpha\beta},\;\; \not{P}\not{P} = P^2+\frac{e}{2}(F\sigma);\;\; \not{P}\psi(P) = m\psi(P);\;\; P^2\psi(P) = (m^2-\frac{e}{2}(F\sigma))\psi(P)$$

while the propagator in external field takes the following form:

$$S_f(P) = \frac{i}{\not{P}-m} = i\frac{\not{P}+m}{\not{P}+m}\frac{1}{\not{P}-m} = i(\not{P}+m)\frac{1}{\not{P}^{\,2}-m^2} = i(\not{P}+m)\frac{1}{P^2-m^2+\frac{e}{2}(F\sigma)}$$

We've introduced ``fake" photon mass, m_V, and of course at the end of the computation we should be able to take $m_V \rightarrow 0$. The propagator can be expanded in $F\sigma$, and we need to retain 1st and 0th order of expansion.

$$[...] = A + B$$

Derivation of 1-loop QED result

The first order of expansion is easy to evaluate, as we can immediately take $P \rightarrow p$. We also make use of a very useful relation $\gamma_\mu(F\sigma)\gamma_\mu = 0$

$$A = -e^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_V^2)((q+p)^2 - m^2)^2} \gamma_\mu(\not{p} + \not{q}) \frac{e}{2}(F\sigma)\gamma_\mu$$

The integral encountered here can be tackled using standard Feynman parametrization

$$\int_0^1 dx \frac{2x}{(D_1(1-x) + D_2 x)^3} = \frac{1}{D_1 D_2^2}$$

Using it, we can evaluate the loop integral

$$\begin{aligned}
&\int \frac{d^4q}{(2\pi)^4} \frac{(p+q)_\alpha}{(q^2 - m_V^2)((q+p)^2 - m^2)^2} \\
&= \int_0^1 dx \frac{d^4q}{(2\pi)^4} \frac{2x(p+q)_\alpha}{((q+px)^2 + p^2x(1-x) - m^2x - m_V^2(1-x))^3} \\
&= \int_0^1 dx \frac{d^4q}{(2\pi)^4} \frac{2x(1-x)p_\alpha}{(q^2 - m^2x^2 - m_V^2(1-x))^3} \\
&= -\frac{i}{16\pi^2} \int_0^1 dx \frac{x(1-x)p_\alpha}{m^2x^2 + m_V^2(1-x)}
\end{aligned}$$

Derivation of 1-loop QED result

Using the following identity, we simplify the gamma-matrix structure

$$\bar{\psi}(p)\gamma_\mu \not{p}(F\sigma)\gamma_\mu\psi(p) = 2m\psi(p)(F\sigma)\psi(p)$$

With that, we have the following contribution from the first term of expansion in $F\sigma$:

$$\mu_A = -em\frac{\alpha}{2\pi}\int_0^1 dx\frac{x(1-x)}{m^2x^2+m_V^2(1-x)}$$

It cannot be a full answer, as the result IR diverges as we take take $m_V \rightarrow 0$.

We need 0$^{\text{th}}$ order term, that upon the use of the eqs of motion also contribute to μ.

$$B = e^2\int\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2-m_V^2)((q+P)^2-m^2)}\gamma_\mu(\not{P}+\not{q}+m)\gamma_\mu$$

Note that these are ``full" covariant P. We are calculating in the first order in $F\sigma$ and going to use the following relation

$$f(P^2)\psi(P) = \sum_n (n!)^{-1}f^{(n)}(0)\left(P^2\right)^n\psi(P) = f(m^2)\psi(p) - \frac{e(F\sigma)}{2}\psi(p)\times\frac{\partial f(P^2)}{\partial(P^2)}\,|_{P^2\to m^2}$$

and one can show that the loop integral $f(p^2)$ can be calculated by taking $P \rightarrow p$.

Derivation of 1-loop QED result

Using these relations we evaluate B, and we get the following contribution to the anomalous magnetic moment:

$$\mu_B = +em\frac{\alpha}{2\pi}\int_0^1 dx \frac{x(1-x)(1+x)}{m^2x^2+m_V^2(1-x)}$$

Notice that it is also IR divergent. But now we need to combine A and B, and we get perfectly finite (and very much celebrated) result:

$$\mu = \mu_A+\mu_B = \frac{\alpha}{2\pi}\times em\int_0^1 dx\frac{x^2(1-x)}{m^2x^2+m_V^2(1-x)} \to \frac{\alpha}{2\pi}\times\frac{e}{m}\int_0^1 dx(1-x) = \frac{\alpha}{2\pi}\times\frac{e}{2m}$$

The last combination here is the Bohr magneton, and therefore anomalous magnetic moment in units of Bohr magneton is $\alpha/(2\pi)$. Correspondingly $g - 2 = \alpha/\pi$.

Notice that it is customary to introduce "a", defined as $a = (g-2)/2$.

This is a first brick in a long long theoretical road to obtain *a* to many significant figures!

Main principles of g-2 experiment

- We need polarized muons. But remember that in $\pi \rightarrow \mu$ weak decays, μ^+ are born with their spins exactly opposite to their momentum.
- The weak decays of muons have a ``built-in" polarimetry: the direction of highest energy positrons are predominantly in the direction of muon spin. Therefore parity-asymmetric nature of weak interactions is used both at production and decay.
- Precession of spin relative to momentum is relatively slow and directly proportional to the anomalous magnetic moment.
- We need to know A. how to achieve uniform O(ppm) level magnetic field, B. Measure its value rather precisely, C. Know the Larmor frequency of muons very precisely, D. Have lots of muons with roughly same direction of their spins.

Main principles of g-2 experiment

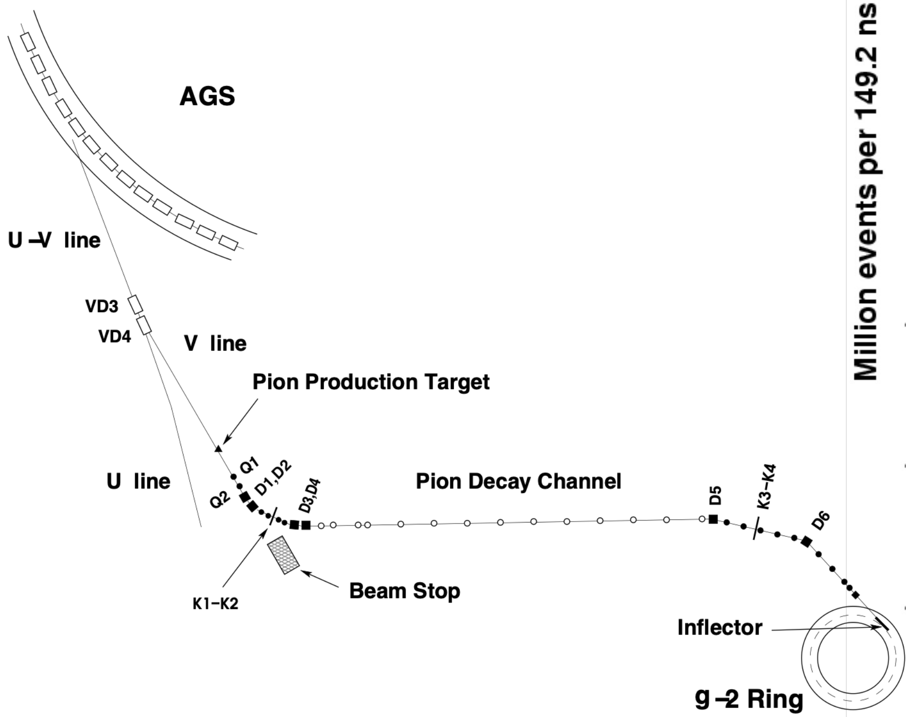
- Both momentum and g=2 intrinsic magnetic moment precess at the same frequency ω_L.
- Precession of spin relative to the momentum in the storage ring is due to g-2 $\neq$ 0. It is in the same plane and a little faster.

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

- The measurement is performed at "magic" $\gamma = 29.3$ that minimizes the influence of electric field.
- Well-measured ratios of muon and proton magnetic moments is used as an additional input $\lambda = \omega_L/\omega_p = 3.18334539(10)$

$$a_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\widetilde{\omega}_p}{\omega_L/\widetilde{\omega}_p - \omega_a/\widetilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}},$$

BNL E821 experiment



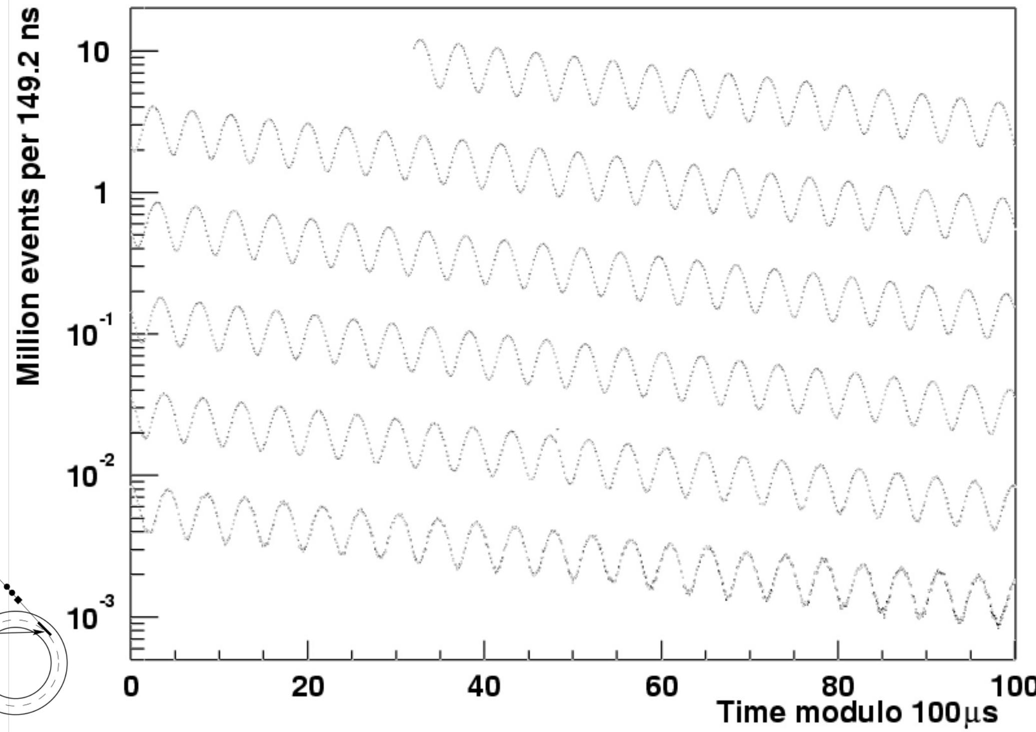


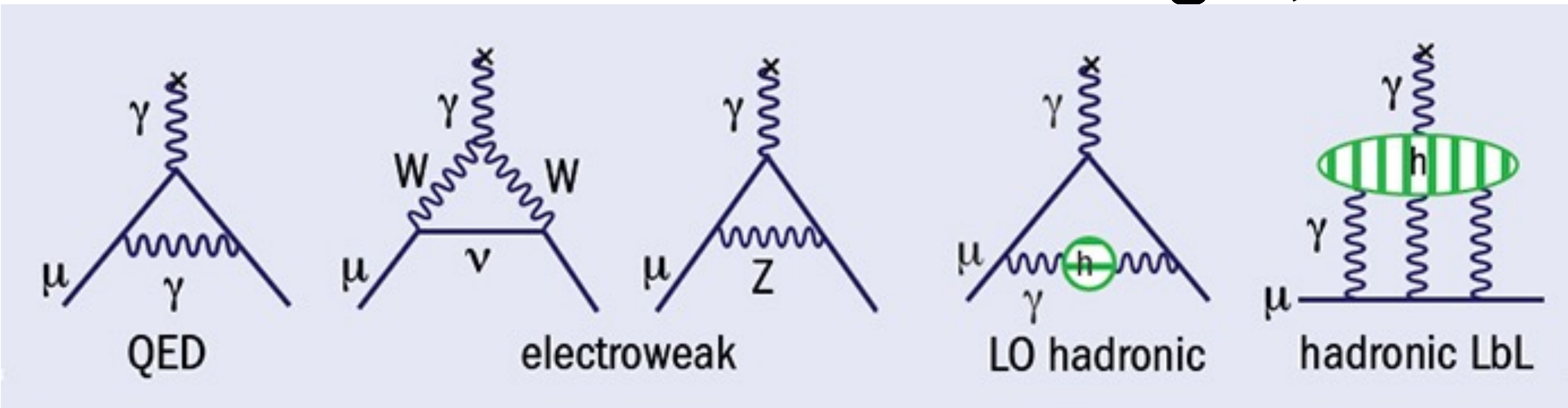
FIG. 2: Distribution of electron counts versus time for the 3.6 billion muon decays in the data-taking period. The data is wrapped around modulo 100 μs.

$a_\mu^{\text{experiment}} = 116592089(63)\times 10^{-11}$,

Consistent for positively and negatively charged muons

Run Period	Polarity	Electrons [millions]	Systematic ω_p [ppm]	Systematic ω_a [ppm]	Final Relative Precision [ppm]
R97	μ^+	0.8	1.4	2.5	13
R98	μ^+	84	0.5	0.8	5
R99	μ^+	950	0.4	0.3	1.3
R00	μ^+	4000	0.24	0.31	0.73
R01	μ^-	3600	0.17	0.21	0.72

Theoretical status of muon g-2, SM



The history of theoretical calculations goes very far in the past. Back to Schwinger's result, $a_\mu^{\text{1-loop QED}} = \alpha\,/\,(2\,\pi)$

Currently, the QED calculations are carried out to four-loop order,

$$a_\mu^{\mathrm{QED}} = 116584718.08(15)\cdot 10^{-11}$$

Electroweak calculations are also under control,

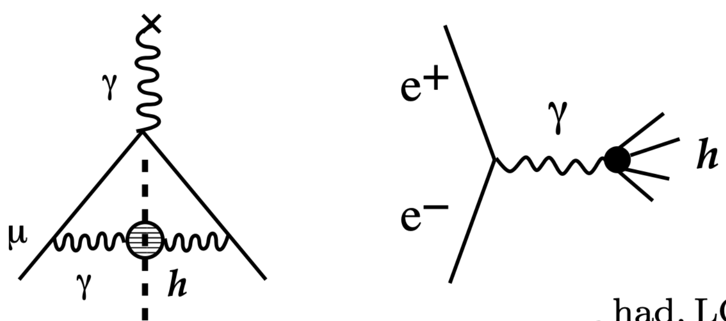
$$a_\mu^{\mathrm{EW}} = (154 \pm 2)\cdot 10^{-11}$$

[the errors on EW and QED are well below the discrepancy]

Theoretical status of muon g-2

Hadronic contributions: A. vacuum polarization B. “light-by-light”

For A. unitarity + analyticity comes to rescue:



$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} \mathrm{d}s\,\sigma^0_{\mathrm{had}}(s)K(s)$$

$$a_\mu^{\mathrm{had,\,LO\,VP}} = (694.91 \pm 3.72_{\mathrm{exp}} \pm 2.10_{\mathrm{rad}})\cdot 10^{-10}\,.$$

For B. some hadronic model calculations are necessary at this points (loops of heavy quarks + loops of light mesons, e.g. pions)

Recent evaluations give

$$a_\mu^{\mathrm{had,\,l-by-l}} = (10.5 \pm 2.6)\cdot 10^{-10}$$

$$a_\mu^{\mathrm{had,\,l-by-l}} = (11.6 \pm 4.0)\cdot 10^{-10}$$

γ^* → hadrons cross section

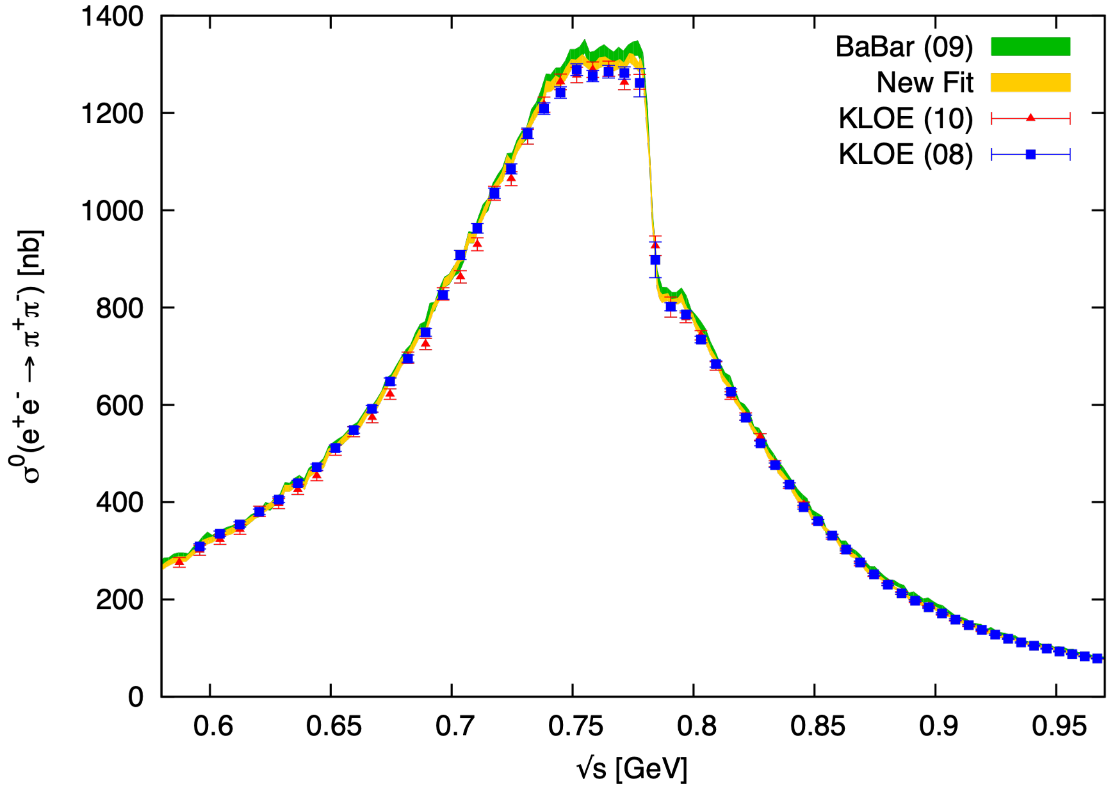
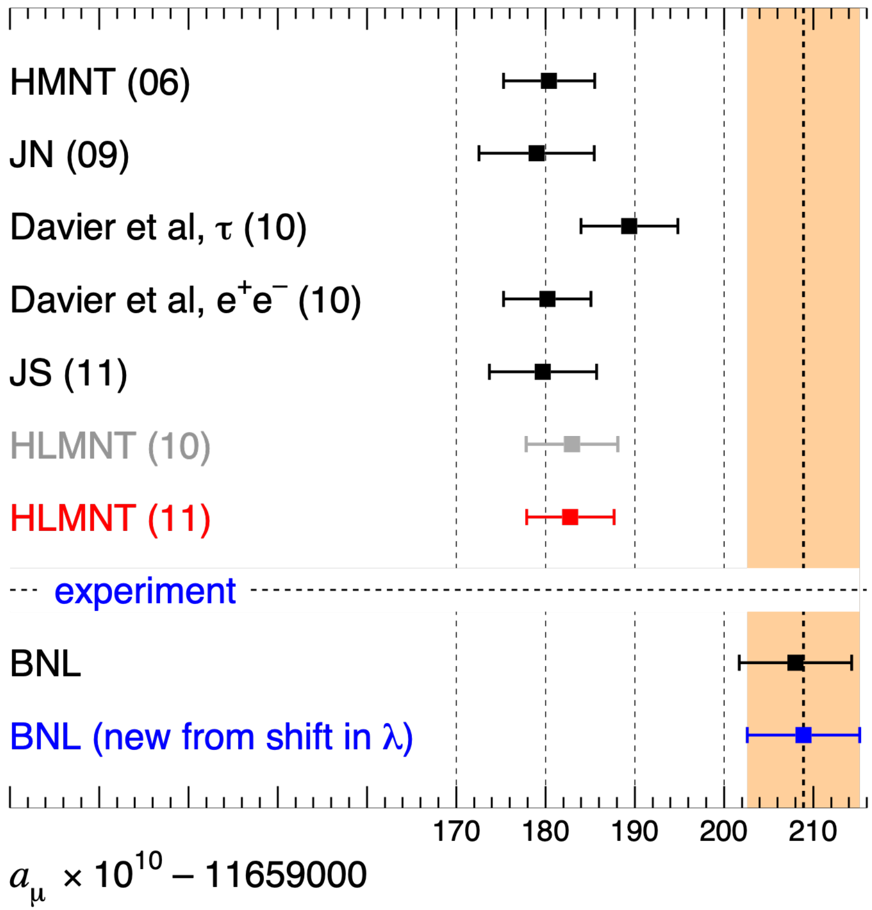


Figure 1: Fit with all data in the 2π channel: light (yellow) band. Radiative return data from BaBar [5] are shown by the darker (green) band, whereas the KLOE [3, 4] data are displayed by the markers as indicated in the plot.

From Hagiwara et al. (2011)

Summary of the pre-2021 discrepancy

From Hagiwara et al. (2011)



There is a theoretical "deficit" of 3×10^{-9}, and the tension is $\sim 3.5\ \sigma$

What are the logical possibilities behind the discrepancy?

1. *Statistical fluctuation or unaccounted errors*. Only one experimental group got the result [could be an error].

2. *SM calculators could be overly optimistic about their error bars*. After all, HVP is needed to O(1%) accuracy, and HLbL is calculated within models, not ab-initio QCD.

3. *Yet undiscovered "New Physics" supplies* $+3\times10^{-9}$. What kind of new physics could it be?

Any combination of the above.

New experiment at Fermilab

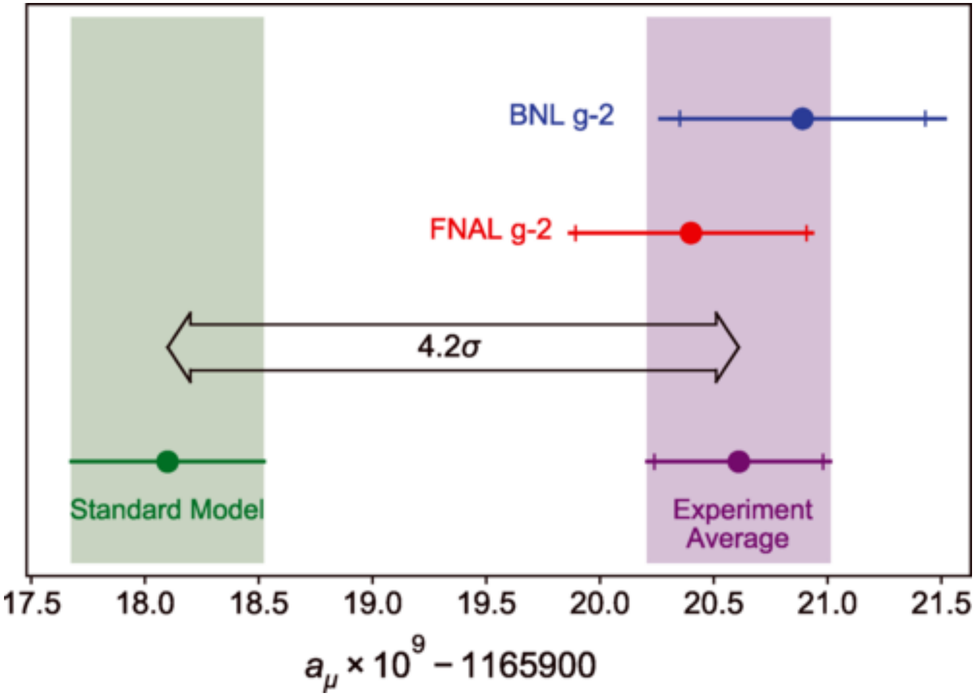
+ Novel experiment at JPARC, to be built.

"Repeat" of the BNL experiment with the same storage ring, but much enhanced intensities, and improved monitoring of the magnetic field.

Goal: *shrink the size of the experimental errors by a factor of ~4.*

+ New efforts for the lattice QCD calculations of hadronic LbL and vacuum polarization contribution.

First results, 2021



Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

B. Abi *et al.* (Muon g−2 Collaboration)

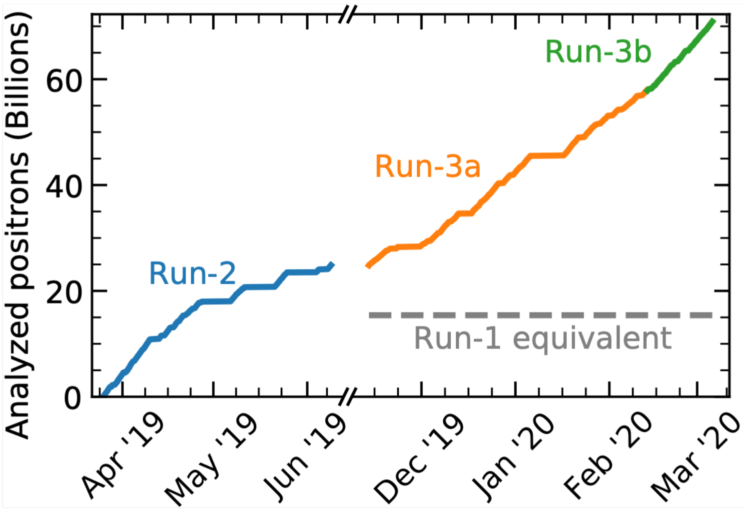
Phys. Rev. Lett. **126**, 141801 – Published 7 April 2021

Data published in 2021 more than doubles the number of events at BNL. Combined experimental error is down by ~Sqrt(2). A lot more progress is anticipated as the data sample will become 10 times larger.

Significance of the discrepancy between experiment and data-driven theory of muon g-2 extraction grew to 4.2 σ

But….

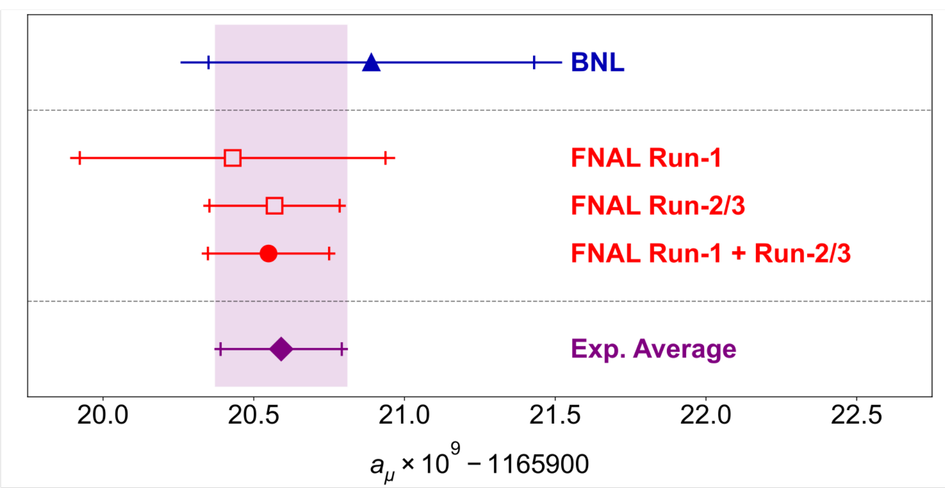
New g-2 results, 2023



- Data taking have finished at Fermilab. Total sample O(5) times more data compared to Run I. BNL results confirmed and improved

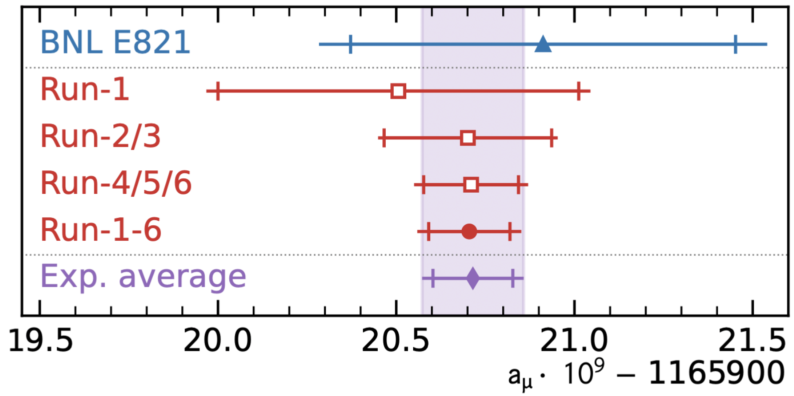
- New results are consistent with old. Experimental error shrank by a factor of 2.

- Notice that this time around, the collaboration refrained from plotting theoretical predictions.



2308.06230, g-2 collaboration

Final g-2 results, 2026



$$a_\mu(\text{exp}) = 116\,592\,0715(145) \times 10^{-12} \quad (124\,\text{ppb}).$$

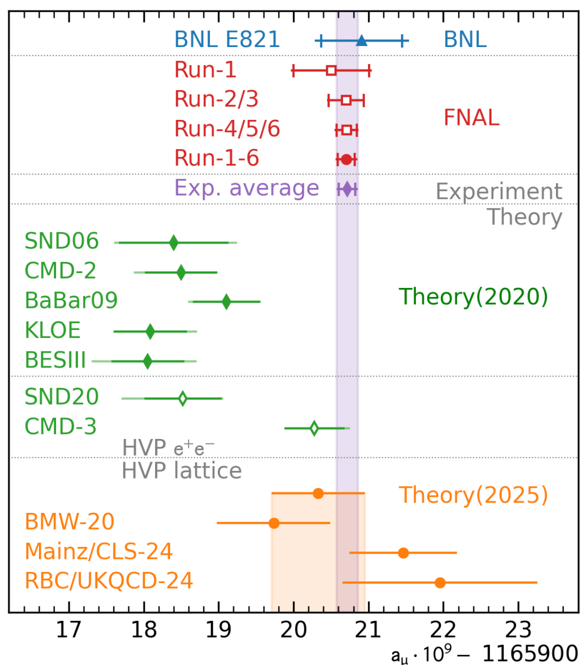


FIG. 38. Comparison of the experimental value of a_μ with the Standard Model predictions.

- Data taking are finished at Fermilab. Goals of the experiment are achieved.
- New results are consistent with old. Experimental error shrank by a factor of ~ 4.

e-Print: [2606.17323] [hep-ex]

- *What is going on with theoretical interpretations?*

Doubts creep in: lattice evaluation oh hadronic VP

BMW collaboration, 2021

19

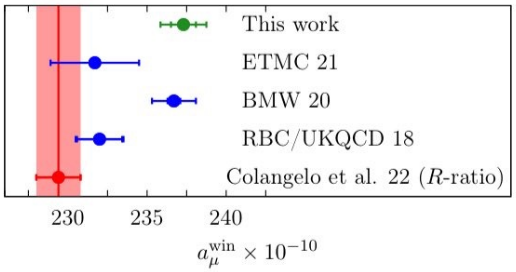
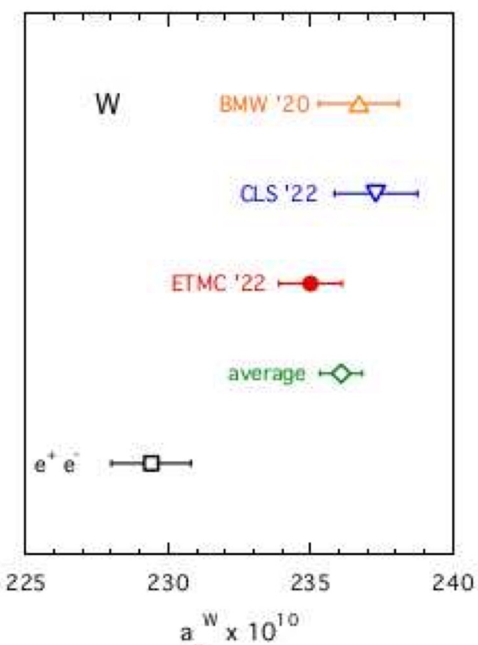


FIG. 8: Comparison of our result for a_μ^{win} including isospin-breaking corrections with the estimates by ETMC [22], BMW [20] and RBC/UKQCD [13]. The estimate based the data-driven method of Ref. [48] is shown in red.

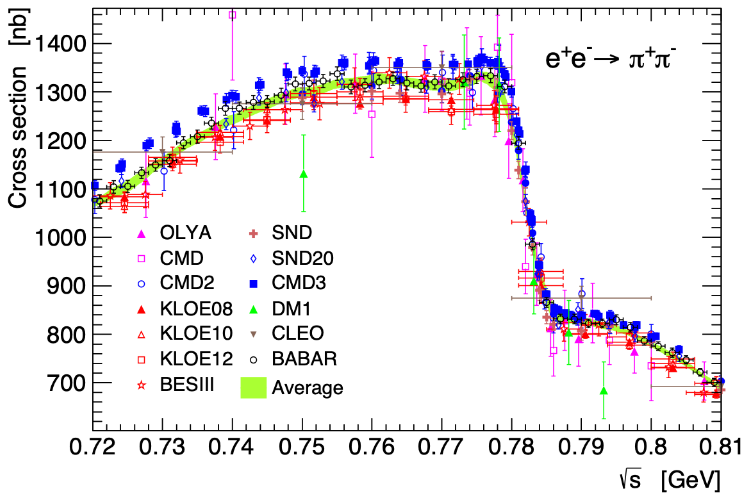


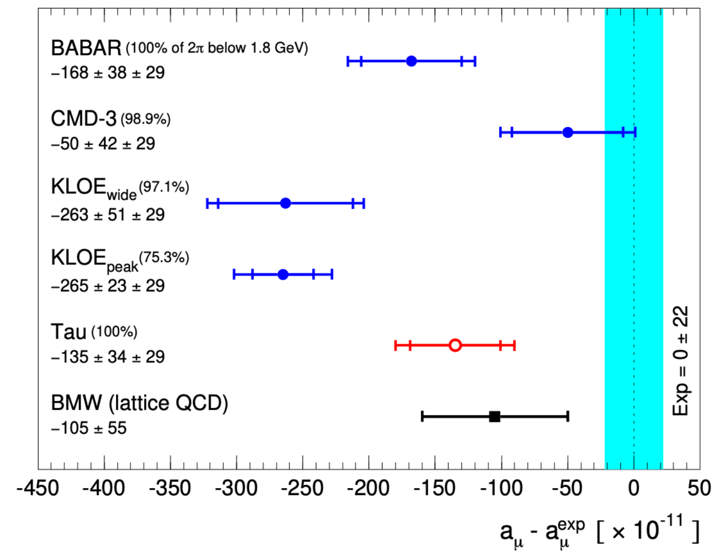
Several Lattice QCD results point to a larger value for the HVP.

There is no meaningful discrepancy between g-2 experiment and lattice determination of g-2.

Are we ready to abandon data-driven method and switch to lattice when 1% accuracy is required?

Doubts grow: new Novosibirsk results (CMD-3)





(From M. Davier et al, 2023)

- The reasons why CMD-3 results are considerably larger (compared to global errors) are not entirely clear.
- They (CMD-3) are consistent with BMW lattice result.
- This invites additional scrutiny for the ``radiative return" method of measuring hadron production cross section.
- One can no longer claim ~5σ discrepancy between theory and experiment for muon g-2.

Hadronic Vacuum Polarization contributions

$$\boxed{a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha}{\pi}\int_{s_{\mathrm{th}}}^{\infty}\frac{ds}{s}\,K(s)\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi_\gamma(s)}$$

$$\boxed{\sigma(e^+e^- \to \mathrm{hadrons}) = \frac{16\pi^2\alpha^2}{s}\,\mathrm{Im}\,\Pi_{\mathrm{em}}(s)\,.}$$

$$\boxed{K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2}}$$

- This is a master formula that allow to connect the Imaginary part of the vacuum polarization with the g-2 via integration with the kernel

- It is easy to recognize that the kernel is indeed similar to a massive photon result ($s = m_V^2$). Moreover, using massive photon formula and the decomposition $\mathrm{Im}\,\Pi \sim \Sigma\,\delta(s - m_{V_i}^2)$ one can derive these formulae above.

Estimates of Hadronic Polarization

- While we need a high-precision result, we can still get a very good estimate following a simple model in D. Greynat and E. de Rafael, [arXiv:2311.11597 [hep-ph]]

$$a_\ell^{\rm HVP} = m_\ell^2 \times \frac{\alpha}{\pi}\int_{t_0}^{\infty}\frac{dt}{t}\int_0^1 dx \frac{x^2(1-x)}{m_\ell^2 x^2 + t(1-x)}\,\frac{{\rm Im}\Pi_{\rm had}}{\pi}$$

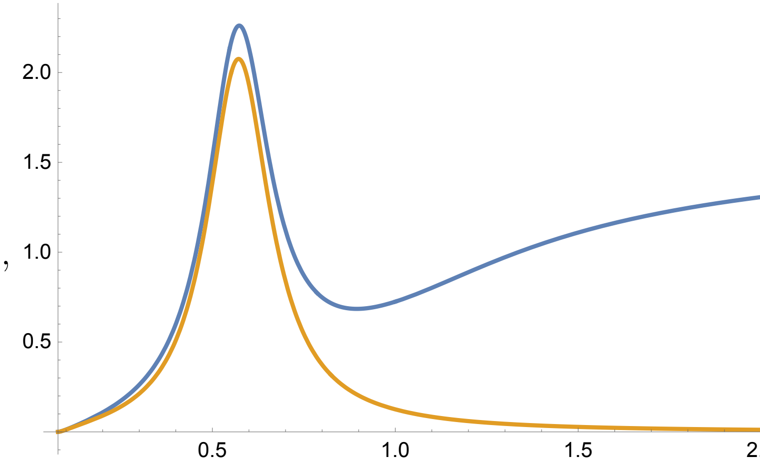
$$\frac{{\rm Im}\Pi_{\rm had}}{\pi} = \frac{\alpha}{\pi}\left(1-\frac{4m_\pi^2}{t}\right)^{3/2}\left[\frac{F_\rho^2(t)}{12} + \sum_q Q_q^2 \Theta(t,t_c,\Delta)\right]\theta(t-t_0).$$

$$F_\rho^2(t) = \frac{1}{(1-t/m_\rho^2)^2 + \Gamma^2(t)/m_\rho^2} \qquad \Gamma^2(t) = \frac{m_\rho^2 t}{96\pi F_\pi^2}\left[\left(1-\frac{4m_\pi^2}{t}\right)^{3/2}\theta(t-4m_\pi^2) + \left(1-\frac{4m_K^2}{t}\right)^{3/2}\theta(t-4m_K^2)\right]$$

$$\Theta(t,t_c,\Delta) = \frac{\frac{2}{\pi}\arctan\left(\frac{t-t_c}{\Delta}\right) - \frac{2}{\pi}\arctan\left(\frac{t_0-t_c}{\Delta}\right)}{1-\frac{2}{\pi}\arctan\left(\frac{t_0-t_c}{\Delta}\right)}$$

$$F_\pi = 0.093\,{\rm GeV},\ m_\rho = 0.77\,{\rm GeV},\ t_0 = (2m_\pi)^2,\ t_c = 1\,{\rm GeV}^2,\ \Delta = 0.51\,{\rm GeV},$$

$$\sum_q Q_q^2 = \frac{5}{3}$$



This model gives $a_\ell^{\rm HVP} = m_\mu^2\left(\frac{\alpha}{\pi}\right)^2 I(m_\mu^2) \simeq 719\times 10^{-10}.$

Recent lattice calculation gives 7132 (61) 10^{-11}

Path forward?

- We need more work from lattice collaborations to scrutinize the BMW results.
- The reasons behind large discrepancies between e.g. KLOE and CMD-3 experiments need to be understood.
- More data on $e^+e^- \rightarrow$ hadrons from BELLE-II and BES-III are expected. More modelling of radiative corrections for ``radiative return” measurements will occur.
- Could one use a new extensive tau data sample to improve on $\tau \rightarrow \pi\pi\nu$?
- One could try to use alternative ways of measuring $\Pi_{hadr}(t)$ using the secondary muon beam at CERN, scattering on electrons. *MUonE* experiment.
- Possibly new measurement with completely independent techniques can come from the JPARC muon g-2 experiment.

New physics and muon g-2

Short distance contributions. Let's distinguish between ``internal" and "external" (occurring on external muon wave function) chirality flips.

$$\mathcal{L}_{\rm ext} = -c_{\rm ext}\frac{1}{2\Lambda_{\rm ext}^2}\bar{\psi}\{\not{p},(F\sigma)\}\psi = -c_{\rm ext}\frac{m_\mu}{\Lambda_{\rm ext}^2}\bar{\psi}(F\sigma)\psi$$

If $c_{\rm ext}$ is on the order of 1-loop coefficient, e.g. $\alpha_W/(4\pi)$, then $\Lambda_{\rm ext} \sim m_W$, as we know that EW contribution ~ O(discrepancy). *Thus, we either have to have a strongly-coupled theory in muon sector, or new physics at 100 GeV, both being unlikely propositions*.

Internal chirality flips could produce a larger result. We still need ``Higgs insertion", but the muon Yukawa suppression can be milder,

$$\mathcal{L}_{\rm int} = -c_{\rm int}\frac{\langle H\rangle}{\Lambda_{\rm int}^2}\bar{\psi}(F\sigma)\psi$$

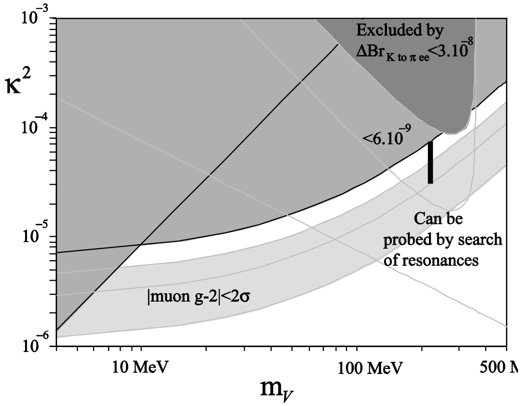
E.g. in SUSY models at large $<H_u>/<H_d>= \tan\beta \sim 50$, $c_{\rm int}$ can be 50 times larger $c_{\rm int} \propto \frac{\alpha_W}{4\pi}\times y_\mu^{\rm SM}\times\tan\beta$. 300-500 GeV sleptons are ok

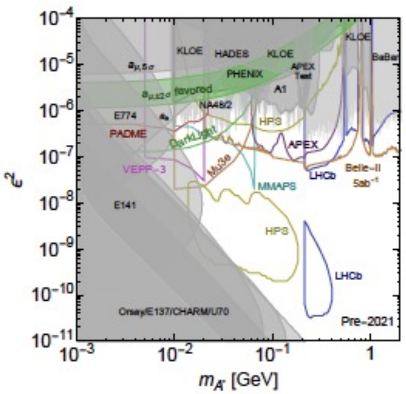
New physics and muon g-2

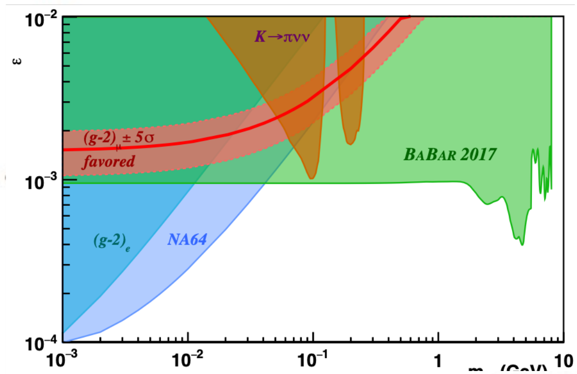
Models with light particles.

Dark photons (pushes g-2 in the positive direction, reducing disrepancy)

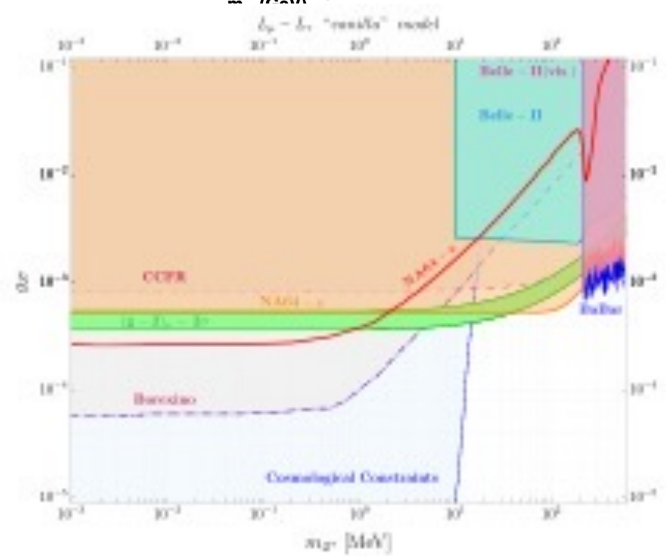
$$\mu_{\rm d.ph.} = \frac{\alpha\varepsilon^2}{2\pi} \times em \int_0^1 dx \frac{x^2(1-x)}{m_\mu^2 x^2 + m_V^2(1-x)} \rightarrow \begin{cases} \frac{\alpha\varepsilon^2}{2\pi} \times \frac{e}{2m_\mu} \text{ at } m_V \ll m_\mu \\ \frac{\alpha\varepsilon^2}{2\pi} \times \frac{em_\mu}{3m_V^2} \text{ at } m_V \ll m_\mu \end{cases}$$







By now both visibly and invisibly decaying dark photons (e.g. to dark matter) are probed much below levels interesting for the muon g-2.



In the “barely alive” category: L_μ-L_τ gauge boson below the dimuon threshold. Invisible decay, hard to produce with e^-.

Electron g-2 and precision α_{EM}

Completely different methods used, ``long lived" electrons in ground states of cyclotron trap, serving as self-magnetometers.

Measurement of the Electron Magnetic Moment

X. Fan,[1,2,*] T. G. Myers,[2] B. A. D. Sukra,[2] and G. Gabrielse[2,†]

[1] *Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2] *Center for Fundamental Physics, Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA*

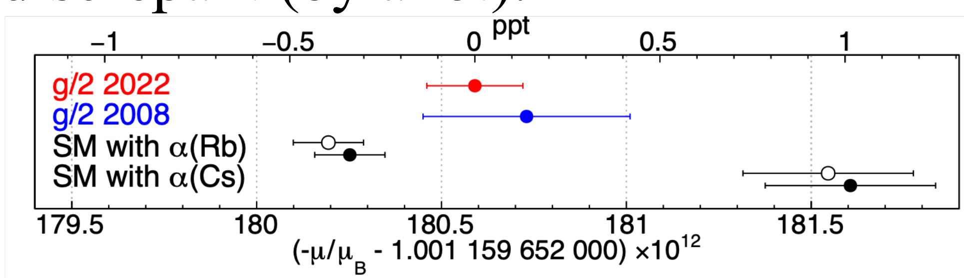
(Dated: December 8, 2022)

2022 results

Upgrades coming

The electron magnetic moment, $-\mu/\mu_B = g/2 = 1.001\,159\,652\,180\,59\,(13)$ [0.13 ppt], is determined 2.2 times more accurately than the value that stood for 14 years. The most precisely determined

To test SM, one needs not only theoretical $(g\text{-}2)_e$, but also independent determination of a. Currently, two results, with Rb and Cs recoil are discrepant (by a lot).



More precise value of α is needed to realize full potential of $(g\text{-}2)_e$

What is "atomic recoil" and why it is needed?

- The following chain of relations is important

$$\boxed{\alpha^2 = \frac{2R_\infty}{c}\frac{m_X}{m_e}\frac{h}{m_X}} \quad \boxed{\alpha^2 = \frac{2R_\infty}{c}\frac{A_r(X)}{A_r(e)}\frac{h}{m_X}.}$$

- Ratio of masses are measured in a Penning trap with accuracy that is more than sufficient. Rydberg constant is measured via Hydrogen spectroscopy.
- Accuracy on the ratio (h/m_{Atom}) *controls* the accuracy of alpha. can be measured very precisely via absorption of photons of known fixed frequency by very cold atoms. Cs and Rb are the two best candidates.
- With on-going investment in the atomic interferometry (Fermilab, UK, CERN), there is a possibility for more accurate measurement of the fine structure constant.

Combining a_μ and a_e

- In the distant future, we could use one g-2 (e.g. electron) to extract HVP and substitute into another (muon). Let us see how it works:

$$a_{\mu-e} \equiv a_\mu - \frac{m_\mu^2}{m_e^2} a_e .$$

- When the errors one α and a_e become a factor of ~ 10^4 smaller than for muons, this becomes very useful because HPV is tiny for this quantity. The hadronic effects are the same and cancel out to ~ 85%.

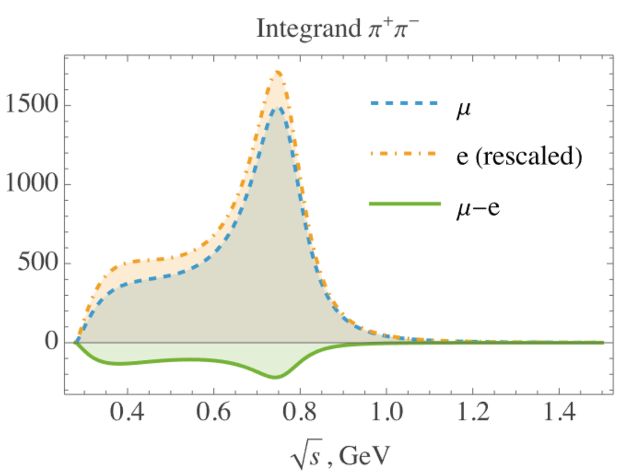


FIG. 4: The integrand of $a_\ell(\times 10^{-10})$ at $\pi^+\pi^-$ channel for muon, rescaled electron and their difference $\mu - e$.

$$\mathcal{L}_{\text{universal}} = \frac{e}{4\Lambda^2} \sum_{\ell=e,\mu} m_\ell \bar{\psi}_\ell \sigma_{\mu\nu} F_{\mu\nu} \psi_\ell ,$$

$$a_{\mu-e}^{\text{BSM}} = m_\mu^2 \left(\frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2} \right)$$

$$\begin{aligned} \mathcal{L}_{d.\,ph.} &= -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(F'_{\mu\nu})^2 + \frac{1}{2} m_{A'}^2 A_\mu'^2 \\ &+ \sum_\ell \bar{\psi}_\ell \gamma_\mu (i\partial_\mu - eA_\mu - e\varepsilon A'_\mu)\psi_\ell . \end{aligned}$$

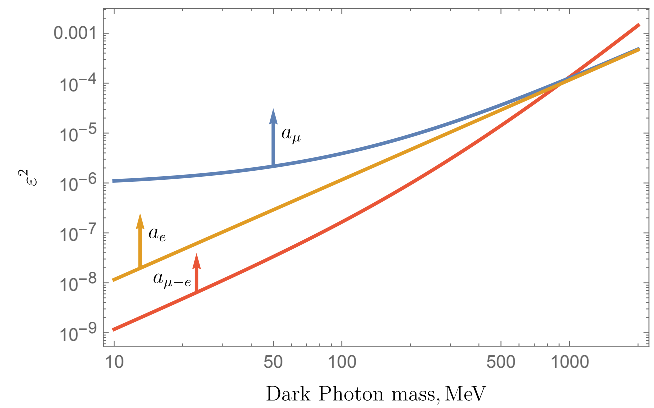


FIG. 6: Exclusion plot for the dark-photon parameters. It is assumed that the only limitation comes from the HVP error, and it is taken to be $\delta a_\mu^{\text{HVP}} = 1. \times 10^{-9}$ and $\delta a_{\mu-e}^{\text{HVP}} = 1. \times 10^{-10}$ for illustration. Above colored lines the parameter space would be excluded. It is easy to see that $a_{\mu-e}$ would provide a deeper reach into ε^2 for all $m_{A'} < 1\,\text{GeV}$.

Understanding QCD via MDM

- Magnetic dipole moments of baryons have always been a textbook example of constituent quark model successful predictions:

$\mu_{\mathrm{n}}/\mu_{\mathrm{p}} = (4/3\ \mu_d - 1/3\ \mu_u)/(4/3\ \mu_u - 1/3\ \mu_d) =$

$(- 4/9 - 2/9\ )/(8/9 + 1/9\ ) = - 2/3 = -0.6666....$

Experimental value: $\mu_{\mathrm{n}}/\mu_{\mathrm{p}} = -0.685$ (Same within 3 %.)

- The magnitude is consistent with $m_Q \sim 300$ MeV.
- Strange baryons magnetic moments fit these patterns as well, with a slightly higher m_Q.
- Here is my ”prediction” based on this pattern for Λ_c :

$\mu\ (\Lambda_c\,) = \mu_c \sim\ 2/3 * 1/(1.5) \sim 0.44$

- Magnetic moments (and other quantities) try to tell us that constituent quarks may be some *emerging degrees of freedom*.
- Measuring MDM of charged baryons with new ALADDIN proposal at CERN is very interesting and can test lattice QCD.

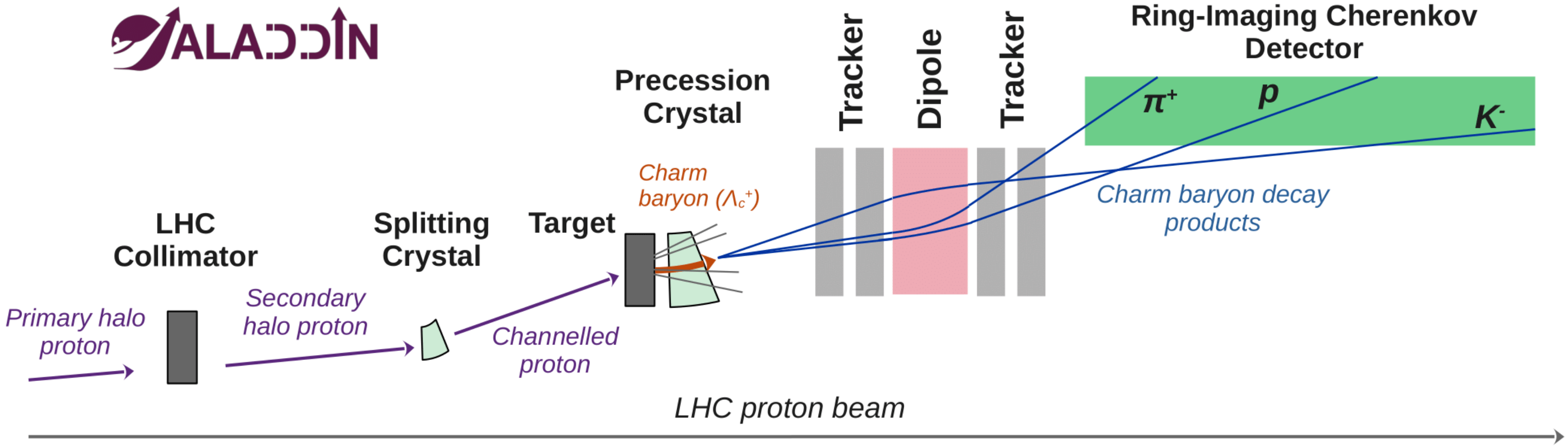
Nucleon MDMs in Naïve Quark Model

- Let us consider 3nonrelativistic quarks, same mass, confined to be in the s-wave, with their spins adding to ½. We will treat them as Dirac fermions. Let us look at the neutron first.
- All three quarks have to be in the color singlet combination, therefore the wavefunction is totally antisymmetric with respect to color indices. Due to identical nature of *d*-quarks, their spin in the s-wave have to add to 1, to satisfy Pauli principle. So, effectively, this is a spin 1 d-quark diaquark + spin ½ up quark → neutron spin.
- $|1/2,1/2\rangle = \sqrt{2/3}\,|1,1\rangle\,|1/2,\text{-}1/2\rangle - \sqrt{1/3}\,|1,0\rangle\,|1/2,1/2\rangle$
- Averaging $\mu_d\, \sigma^d{}_z + \mu_u\, \sigma^u{}_z$ with this wave function we get

  $\mu_\mathrm{n} = (4/3\ \mu_d - 1/3\ \mu_u\ ) = (-\ 4/9 - 2/9\ )e\ (\mu/e) = -\ 2/3\ e\ (\mu/e)$
- $(\mu/e) = 1/(2\ m_q)$. If $m_q = 1/3\ m_{nucleon}$, then $\mu_\mathrm{n} = -\ 2$ Nucleon magneton.

In reality, it is -1.91. *The model works unreasonably well !!! Why?*

ALADDIN proposal at CERN



- In the past, magnetic moments of strange baryons was measured at Fermilab. Baryons come out polarized perpendicular to the collison plane.
- Charmed baryons have a much shorter lifetime, but the *boost* at the LHC is much larger. Travel distance ~ O(10 cm)
- ALADDIN proposal seeks to exploit enormous magnetic fields in the rest frame of Λ_c that is channeled through a bent crystal. We can test QCD/naïve QM predictions
- If successful, we could discuss tau MDM that is chaneled after being produced via the standard decay $D_s \rightarrow \tau\ \nu$.
- Polarimetry is done via weak interaction decay $\Lambda_c \rightarrow$ p K π, sensitive to spin.

“Exotic” topics

- Nucleon/nuclear MDM cannot be calculated super-precisely. But they can be *controlled* super-precisely, and separated from MDM if the new effects do not scale as MDM
- Classic example – an EDM – topic of tomorrow’s lecture.
- Interactions of spins with gradients of (pseudo)scalar fields can be very “visible” if fields are light and evolve on the time scales much longer than nuclear/atomic.
- Precision measurements in this domain are known as searches of “Lorentz breaking”, clock comparison experiments, etc.

Ultralight Frontier

Let us call by ϕ, ϕ_1, ϕ_2, … - ultra-light scalar/vector fields. (Perhaps evolving on cosmic scales or perhaps oscillating around a minimum – contributing to the Ω_{matter}). Let us represent SM field by an electron, and a nucleon.

Interactions can be again be organized as "portals": $O_{\text{DS}}O_{\text{SM}}$.

$$\frac{\partial_\mu \phi}{f_a} \sum_{\text{SM particles}} c_\psi \bar{\psi}\gamma_\mu\gamma_5\psi \quad \text{axionic portal}$$

Torque on spin

$$\frac{\phi}{M_*} \sum_{\text{SM particles}} c_\psi^{(s)} m_\psi \bar{\psi}\psi \quad \text{scalar portal}$$

Shift of atomic ω + extra gr. force

$$\frac{\phi_1^2 + \phi_2^2}{M_*^2} \sum_{\text{SM particles}} c_\psi^{(2s)} m_\psi \bar{\psi}\psi \quad \text{quadratic scalar portal}$$

Shift of atomic ω + extra gr. force

$$\frac{\phi_1 \partial_\mu \phi_2}{M_*^2} \sum_{\text{SM particles}} g_\psi \bar{\psi}\gamma_\mu\psi \quad \text{current} - \text{current portal}$$

Extra force

Scalar/vector fields can be e.g. ultralight DM, with m_ϕ as small as $\sim 10^{-20}$ eV. In that case, it is tempting to use for the *amplitude* of oscillations $(2\rho_{\text{DM}}\, m_\phi^{-2})^{1/2}$. *While observable effects are very intriguing, and can be looked at with e.g. atomic clocks, magnetometers, precision gravitational measurements including atomic interferometers, most models are technically unnatural* (= *unjustifiably light fields*.)

Coherent spin force induced by "gravity"

The strongest limits come from Hg201/Hg199 and Xe129/Xe131 comparison.

$$H_i = -\mu_N g_i(\mathbf{I}\cdot\mathbf{B}) + \Delta E_i(\mathbf{I}\cdot\mathbf{n}),$$

one can formulate the constraints on exotic spin interactions as follows:

$$|\Delta E_{\mathrm{Hg}}| = |\Delta E_{201} - \Delta E_{199}\,\rho_{\mathrm{Hg}}| < 3.0\times 10^{-21}\ \mathrm{eV},$$
$$|\Delta E_{\mathrm{Xe}}| = |\Delta E_{129} - \Delta E_{131}\,\rho_{\mathrm{Xe}}| < 1.7\times 10^{-22}\ \mathrm{eV},$$

Venema et al 1991, Zhang et al 2023

with $\rho_{\mathrm{Hg}} = g_{201}/g_{199} = -0.369139$ and ρ_{Xe} $g_{129}/g_{131} = -3.37337$ the gyromagnetic ratios.

There are also limits from paramagnetic species (Jackson-Kimball's work and earlier papers).

Exotic muon spin force

With Y. Ema and T. Gao, 2308.01356 [hep-ph]

Simplest case: coupling of muon spin to a vertical gradient of a scalar field created by nucleons of the Earth

$$\mathcal{L}_{\text{eff}} = \ldots + \frac{1}{2}(\partial_\nu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - g_S \phi (\bar{n} n + \bar{p} p)$$
$$+ \frac{g_A^{(\mu)}}{2 m_\mu} \partial^\alpha \phi \times \bar{\mu} \gamma_\alpha \gamma_5 \mu - g_P^{(\mu)} \phi \times \bar{\mu} i \gamma_5 \mu .$$

$$\downarrow$$

$$H_{s-m}^{(\mu)} = \frac{g_S (g_A^{(\mu)} + g_P^{(\mu)})}{4\pi \times 2 m_\mu} (\boldsymbol{\sigma}^{(\mu)} \cdot \boldsymbol{\nabla}) \frac{\exp(-m_\phi r)}{r}$$

In the limit of a very light/massless mediator, this gives additional slowly varying field gradient that gives energy splitting for muon polarized up and down. This will correct the precession frequency of muons polarized in the plane perpendicular to a vertical.

Speculative size of the effect

In the limit of a very light/massless mediator, this gives additional slowly varying field gradient that gives energy splitting for muon polarized up and down.

$$H = \Delta E_{(\mu)} (\mathbf{s} \cdot \mathbf{n}) = \Delta E_{(\mu)}\, s_z,$$

On the other hand, we speculate that muon spin precession frequency in the g-2 experiment, ω_a, needs to be "helped" by

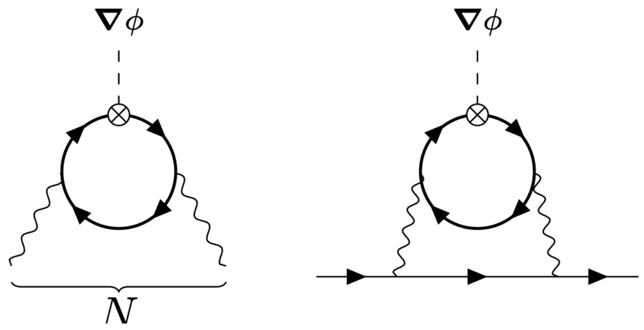
$$\Delta E_{(\mu)} = (\hbar\omega_a) \times \gamma \times (\Delta a_{(\mu)}/a_{(\mu)}),$$

$$\Delta a_{(\mu)} \simeq +2.5 \times 10^{-9}. \longrightarrow \underline{\Delta E_{(\mu)} = 6 \times 10^{-14}\,\mathrm{eV}}$$

Seems small? But this is in the 100 Hz range compared to μHz-nHz sensitivity achieved with ordinary atomic spins.

Muon spin force inducing regular spin force

Effect arises at one- and two-loop level



One loop effect is sensitive to the EM fields inside the nucleus. Two-loop effect is sensitive to the n/p content of nuclear spin and quark spin content of *n* and *p* spin.

Summary: 1. Nuclear model dependence is quite significant. 2. Regardless of that dependence, g_P couplings at the level needed for g-2 are robustly excluded. 3. Axial vector couplings are in tension by a factor of a ~ few, but nuclear/model uncertainties prevent us from placing a firm bound/excluding g-2 needed values.

Details of indirect constraints, one loop

$$\mathcal{L}_{eff} = -\frac{e^2}{16\pi^2 m_\mu}\left(g_P^{(\mu)}\phi F_{\mu\nu}\tilde{F}_{\mu\nu} + \frac{g_A^{(\mu)}\partial_\nu\phi}{3m_\mu^2}\tilde{F}_{\alpha\nu}\partial_\mu F_{\alpha\mu}\right)$$
$$= \frac{e^2\boldsymbol{\nabla}\phi}{4\pi^2 m_\mu}\cdot\left(g_P^{(\mu)}\mathbf{B}A_0 + \frac{g_A^{(\mu)}}{12m_\mu^2}\mathbf{B}\,\nabla^2 A_0\right).$$

Shell model + simplistic core polarization model give the following result:

$$\left|\frac{\Delta E_{\rm Hg}}{\Delta E_{(\mu)}}\right| = 1\times 10^{-6};\quad \left|\frac{\Delta E_{\rm Xe}}{\Delta E_{(\mu)}}\right| = 3\times 10^{-6}\ \text{for}\ g_P^{(\mu)},$$

$$\left|\frac{\Delta E_{\rm Hg}}{\Delta E_{(\mu)}}\right| = 2\times 10^{-8};\quad \left|\frac{\Delta E_{\rm Xe}}{\Delta E_{(\mu)}}\right| = 4\times 10^{-9}\ \text{for}\ g_A^{(\mu)}.$$

The recent Xe experiment [indirectly] excludes a needed size for g_P by three orders of magnitude.

Details of indirect constraints, two loops

Two-loop running is UV sensitive

$$\mathcal{L}_{\rm eff} = \sum_{i=u,d,s,e} \frac{g_A^{(i)}}{2m_\mu} \partial_\alpha \phi \times \bar{\psi}_i \gamma^\alpha \gamma_5 \psi,$$

$$g_A^{(i)} = -g_A^{(\mu)} \times \frac{3}{4} \left(\frac{\alpha}{\pi}\right)^2 Q_i^2 \log(\Lambda_{\rm UV}^2/\Lambda_{\rm IR}^2),$$

Lattice/β-decays give

$$g_A^{(p)} = 0.777 g_A^{(u)} - 0.438 g_A^{(d)} - 0.053 g_A^{(s)};$$
$$g_A^{(n)} = -0.438 g_A^{(u)} + 0.777 g_A^{(d)} - 0.053 g_A^{(s)},$$

Using nuclear models of Xe129 and Xe131 (Klos et al) we get

$$\Delta E_{\rm Xe} = (3-7) \times 10^{-8} \times \Delta E_{(\mu)} \times \frac{\log(\Lambda_{\rm UV}^2/\Lambda_{\rm IR}^2)}{\log(m_\tau^2/m_p^2)}.$$

which is not enough to exclude a g-2 possibility

An alternative way of testing muon spin force is needed!

μSR type measurement

- G-2 measurements are done in the strong magnetic field. There is no need in that if you want to test for an extra effect due to abnormal muon spin force.

$$\Delta\psi = \frac{\Delta E_{(\mu)} t}{\hbar} \implies \Delta\psi(t = \tau_\mu) = 2 \times 10^{-4}$$

- One needs to perform muon spin precession measurements of muons stopped in a non-magnetic material (e.g. liquid He), in the presence of a magnetometer, and at different values of magnetic field, 1 – 100 Gs.

- Ratio of muon spin precession frequency relative to magnetometer will change with the magnetic field

$$r_{\mu/p} = \omega_{(\mu)}/\omega_{(p)}$$

$$-B\frac{dr_{\mu/p}}{dB} = \frac{\Delta E_{(\mu)}}{2\mu_p B} = 3.4 \times 10^{-4} \times \frac{\Delta E_{(\mu)}}{6 \times 10^{-14}\,\mathrm{eV}} \times \frac{10\,\mathrm{Gs}}{B},$$

Conclusions

1. *Standard Model based on SU(3)*SU(2)*U(1) gauge group, three generations of fermions and one Higgs boson has been successfully validated in experiment.*
2. One way of probing the SM is to measure very precisely some quantities that *we can calculate very well*. Muon g-2 is an important example.
3. Current measurements of muon g-2 (BNL, Fermilab) point towards discrepancy at 5 σ level if we stick to the 2020 theoretical prediction. Could be a sign of new physics? However, latest lattice QCD evaluations and CMD-3 two-pion production cross sections results spell some doubt about reliability of 2020 extraction of hadronic HVP. *This become the main issue for the program of testing the SM with the muon g-2.*
4. Short-distance (EW scale and above) as well as light physics can provide ~ +2.5 in units of 10^{-9}, *but space for speculations shrinks*