Theoretical perspective on Electric Dipole Moments

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Recent Review: "Electric Dipole Moments and New Physics"
Maxim Pospelov, Adam Ritz, e-Print: 2509.23531 [hep-ph]
To appear as part of Encyclopedia of Particle Physics
update on 2005 Annals of Physics review

Outline

- Intro: background on EDMs
- CKM CP violation and EDMs.
- Strong CP problem and EDMs, “theta signal”.
- EDMs and BSM.
- Outlook.

New reality after ~15 years of LHC

- No new physics at the LHC thus far, but there is a Higgs boson.

- From Supersymmetry point of view, the Higgs mass of 125 GeV implies very heavy colored superpartners. This is likely to be outside of the LHC reach (though some hopes of discovering electro-weak-ino remain)

- Further progress in BSM searches via quark flavor often hinges on better treatment of QCD.

- Role of *indirect* observables increases. EDMs (and lepton flavor violation) are among the most clear possibilities.

Purcell and Ramsey (1949) ("How do we know that strong interactions conserve parity?" $\longrightarrow |d_n| < 3 \times 10^{-18} e\mathrm{cm}$.)

$$H = -\mu \mathbf{B} \cdot \frac{\mathbf{S}}{S} - d\mathbf{E} \cdot \frac{\mathbf{S}}{S}$$

$d \neq 0$ means that both P and T are broken. If CPT holds then CP is broken as well.

CPT is based on locality, Lorentz invariance and spin-statistics = very safe assumption.

search for EDM = search for CP violation, if CPT holds

Relativistic generalization

$$H_{\mathrm{T,P-odd}} = -d\mathbf{E} \cdot \frac{\mathbf{S}}{S} \to \mathcal{L}_{\mathrm{CP-odd}} = -d\frac{i}{2}\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu},$$

corresponds to dimension five effective operator and naively suggests $1/M_{\text{new physics}}$ scaling. Due to $SU(2) \times U(1)$ invariance, however, it scales as m_f/M^2.

Current limits translate to multi-TeV sensitivity to M.

Anomalous magnetic moment and EDM of an elementary fermion

Anomalous magnetic moment:

$$\frac{1}{2}\overline{\psi}F_{\mu\nu}\sigma^{\mu\nu}\psi$$

- Induced by "any" interactions.
- The leading effect is large (~O(1) for hadrons, O(10^{-3}) for leptons)
- Can be made into a precision tool *if* the accuracy of measurements is matched by accuracy of calculations

Electric dipole moment:

$$\frac{1}{2}\overline{\psi}\tilde{F}_{\mu\nu}\sigma^{\mu\nu}\psi$$

- Tilde flips E and B.
- Only CP-violating interactions induce it; very special, *not* common.
- Is a precision tool "*automatically*" as SM (apart from strong CP caveat) does not induce large EDMs

EDMs and New Physics

- Any EDM observable $\sim$

  $\sim$ [some dimensionless QCD/atomic/nuclear matrix elements] $\times$

  SM mass scale (m_e, m_q, $\langle H \rangle$ etc) $\times$ (CP phase)$_{\mathrm{NP}}$ / $\Lambda_{\mathrm{NP}}{}^2$

$$\mathcal{L}_{\mathrm{EFT}} \sim \frac{1}{\Lambda_1^2} e^{i\phi_1} H\bar{E}\sigma_{\mu\nu}B_{\mu\nu}L + \frac{1}{\Lambda_2^2} e^{i\phi_2} H\bar{E}\sigma_{\mu\nu}\tau^a W^a_{\mu\nu}L + (h.c.) \longrightarrow d \propto \sin\phi \frac{\langle H \rangle}{\Lambda^2_{\mathrm{BSM}}}.$$

With some amount of work all matrix elements can be fixed. For the flavor blind NP, $d_i \sim m_i$. *Unfortunately, we have no idea where actually* Λ_{NP} *is:* 100 GeV, 1 TeV, 10 TeV, 100 TeV, 1000 TeV … GUT scale … M_{Pl}

After the LHC did not find the abundance of new states immediately above EW scale, "guessing EDMs" became even more difficult. *What shall we put in the denominator*? E.g. (TeV)^2 or (PeV)^2?

Current experimental limits

Schiff's theorem forces us to group EDMs into atomic (or molecular) paramagnetic (= open electron shells), diamagnetic (= closed electron shells) and neutron EDMs.

Class	EDM	Current Bound
Paramagnetic	HfF^+	$\lvert d_e^{\rm equiv}\rvert < 4.1 \times 10^{-30}\, e\,{\rm cm}$ [16]
	ThO	$\lvert d_e^{\rm equiv}\rvert < 1.1 \times 10^{-29}\, e\,{\rm cm}$ [17]
Diamagnetic	^{199}Hg	$\lvert d_{\rm Hg}\rvert < 7.4 \times 10^{-30}\, e\,{\rm cm}$ [18]
Nucleon	n	$\lvert d_n\rvert < 1.8 \times 10^{-26}\, e\,{\rm cm}$ [19]

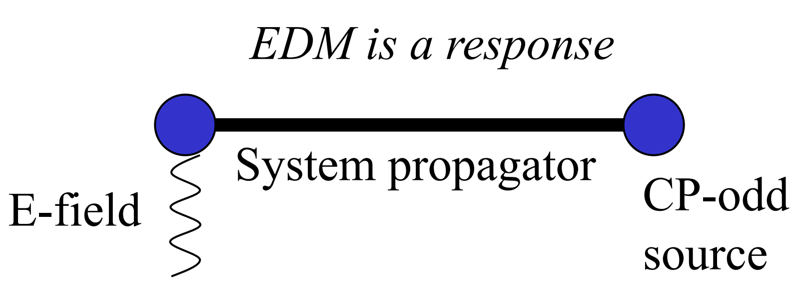
- In the last ~ 12 years improved by a factor of ~ 400, which for a fixed model translates to a factor of 20 improvement to $\Lambda_{\rm NP}$.

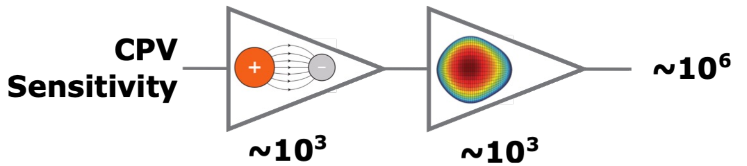
$\lvert d_e\rvert < 1.6 \times 10^{-27} e$ cm (Tl) → $\lvert d_e\rvert < 4.1 \times 10^{-30}\, e$ cm (HfF+), 1.1×10^{-29}(ThO)

- Limits are not on one single Wilson coefficient but on a linear combination of those (with a semileptonic operator C_S being very important)

Future experimental directions

- Neutron EDMs with more intense ultra-cold neutron sources. (New source at TRIUMF)
- New installments of d_e experiments, *possibly* giving between 1 and 2 order of magnitude improvement. Diatomic molecules is the way to go since modest lab electric field is enhanced inside such molecules.
- Exploring nuclei with enhanced Schiff moment due to octupole deformation. Nice connection to rare isotope facilities. (^{225}Ra, ^{223}Fr etc.). Bottleneck is to have enough cold molecules to reduce stat errors. The Cold molecule Nuclear Time-Reversal EXperiment (CeNTREX).
- Proton EDM experiment in the [purposedly built] large storage ring with the goal of 10^{-29} e cm. New technique based on "frozen spin" configurations.
- EDMs of particles with "self-analyzing" decays: muons, charmed baryons, radioactive nuclei (Fermilab, J-PARC, PSI, CERN)

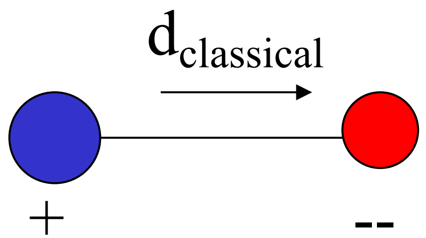




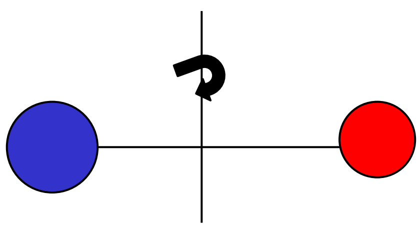
Jadbabaie et al, 2605.12767

A small comment on classical EDMs

- Fundamental EDMs are connected to spin, classical EDMs are not.
- A diatomic molecule (like ThO) will have a classical EDM.



- However, in a quantum state with fixed angular momentum classical EDM average to zero, exactly. States with +M and –M projection of angular momentum remain degenerate (at B=0).



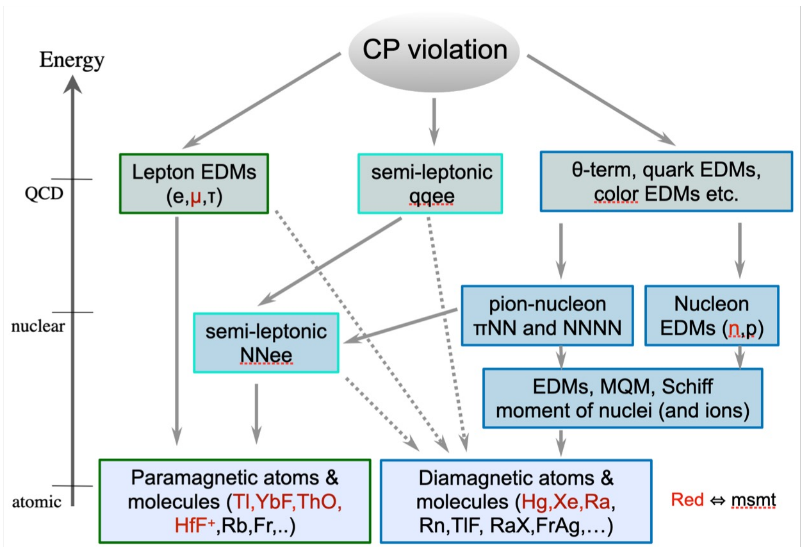
$$< d_{classical} > = 0$$

- If there is fundamental CP-violation, the electric field will induce splitting between +M and –M states, e.g. Zeeman effect but with electric field. EDM experiments are looking for E coupling to spin

Converting an EDM to scale sensitivity

BSM physics and EDMs

$$\mathcal{L}_{eff}^{1\mathrm{GeV}} = \frac{g_s^2}{32\pi^2}\,\theta_{QCD} G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}$$
$$-\frac{i}{2}\sum_{i=e,u,d,s} d_i\,\overline{\psi}_i(F\sigma)\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\widetilde{d}_i\,\overline{\psi}_i g_s(G\sigma)\gamma_5\psi_i$$
$$+\frac{1}{3}w\; f^{abc}G^a_{\mu\nu}\widetilde{G}^{\nu\beta,b}G_\beta{}^{\mu,c} + \sum_{i,j=e,d,s,b} C_{ij}\,(\bar{\psi}_i\psi_i)(\bar{\psi}_j i\gamma_5\psi_j) + \cdots$$



- One needs hadronic, nuclear, atomic matrix elements to connect Wilson coefficients to observables

- Extremely high scales [10-100 TeV] can be probed if new physics generating EDMs violates CP maximally.

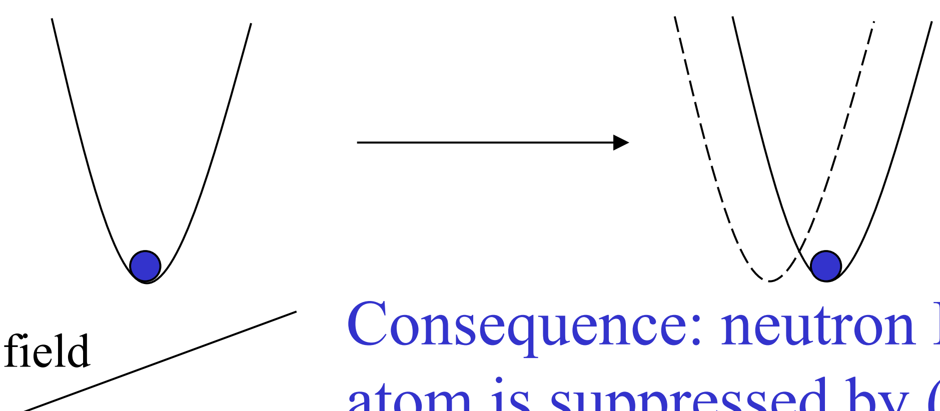
Schiff's screening

- Electric field experienced by the nucleus averages to 0 – applicable to systems bound together by electrostatic force.
- Not an issue for neutrons, Lambda etc.

$$\Delta E = -d\langle i|(\mathcal{E}\cdot s_n)|i\rangle - \sum_{k\neq i}\frac{\langle i|(\mathcal{E}\cdot e r_e)|k\rangle\langle k|(d/e)s_n\cdot\nabla_{r_n}U(r_n-r_e)|i\rangle}{E_i-E_k} - \sum_{k\neq i}\frac{\langle i|(d/e)s_n\cdot\nabla_{r_n}U(r_n-r_e)|k\rangle\langle k|(\mathcal{E}\cdot e r_e)|i\rangle}{E_i-E_k}$$

$$= -d\langle i|(\mathcal{E}\cdot s_n)|i\rangle - d\sum\frac{\langle i|(\mathcal{E}\cdot r_e)|k\rangle\langle k|[(s_n\cdot i p_e),H]|i\rangle + \langle i|[(s_n\cdot i p_e),H]|k\rangle\langle k|(\mathcal{E}\cdot r_e)|i\rangle}{E_i-E_k}$$

$$= -d\langle i|(\mathcal{E}\cdot s_n)|i\rangle - d\langle i|[(\mathcal{E}\cdot r_e),(s_n\cdot i p_e)]|i\rangle = -d\langle i|(\mathcal{E}\cdot s_n)|i\rangle + d\langle i|(\mathcal{E}\cdot s_n)|i\rangle = 0$$



A nonrelativistic point-like charged particle has an equilibrium position at a point where E =0

External E field

Consequence: neutron EDM inside e.g. ^{3}He atom is suppressed by $(R_{nucl}/a_{atom})^2$

Violation of Schiff theorem

- Relativistic effects

- Finite nuclear size

Paramagnetic EDMs

- Schiff theorem *is broken by relativistic [i.e. magnetic] effects*. Relativistically enhanced as $d_{Atom} \sim Z^3 \alpha^2 d_e$. In reality, d_{Atom} is a linear combination of d_e and a semileptonic operator C_S. Using most sensitive results from ThO and HfF+ molecules, one can limit both sources. *Diatomic molecules have strong internal field and can effectively "enhance" modest external electric field.*

- More progress is real (e.g. ACME III). Some other daring proposals want to go down to $d_e \sim 10^{-34}$ e cm.

- Theoretically is the cleanest. Atomic theory is under control at ~ 10% accuracy. In many models - minimum of QCD/nuclear input. SM contributions (θ_{QCD} and δ_{CKM}) were calculated in the last three years. Benchmark CKM value $d_e^{eq} = 1.0 \times 10^{-35}$ e cm. (Ema et al, 2022)

Linear combination of d_e and C_S, d_e^{equiv}

- Paramagnetic EDM (EDM carried by electron spin) is induced by a purely leptonic operator and/or semileptonic operators

$$d_e \times \frac{-i}{2}\,\overline{\psi}\sigma_{\mu\nu}\gamma_5 F_{\mu\nu}\psi \qquad\qquad C_S \times \frac{G_F}{\sqrt{2}}\,\overline{N}N\,\overline{\psi}i\gamma_5\psi$$

In particular, it leads to *s*-*p* mixing:

$$C_S\,\text{operator} \to H = C_S \times \frac{G_F}{\sqrt{2}m_e} \times (\mathbf{S}_e\nabla)\delta(\mathbf{r}_e) \to \psi = [f(r) + \beta g(r)(\vec{\sigma}\vec{n})]\,\frac{\chi}{\sqrt{4\pi}}$$

$$\to \langle e\vec{r}\rangle \sim e\beta\langle\vec{\sigma}\rangle \times \int r^3 dr \times fg$$

- Only a linear combination is limited in any single experiment. ThO 2018 ACME result is:

$$|d_e| < 1.1\times 10^{-29}\ \text{e cm} \qquad \text{at}\ \ C_S = 0$$

$$|C_s^{\text{singlet}}| < 7.3\times 10^{-10} \qquad \text{at}\ \ d_e = 0$$

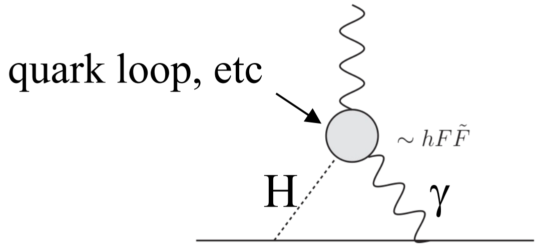
$$d_e^{\text{equiv}} = d_e + C_S \times 1.5 \times 10^{-20}\, e\,\text{cm} \quad \leftarrow \quad \text{Specific for ThO}$$

$$d_e^{\text{equiv}} = d_e + C_S \times 0.9 \times 10^{-20}\, e\,\text{cm} \quad \leftarrow \quad \text{Specific for Hf F+}$$

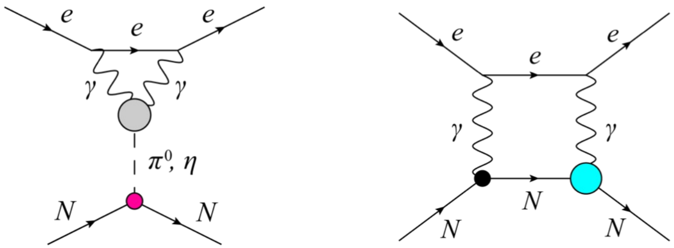
Hydrogen atom EDM – a toy example

Hadronic CP violation → paramagnetic EDMs

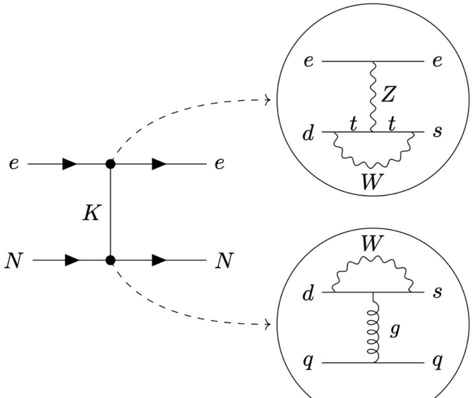
- CP violation in top-Higgs sector – Barr-Zee diagrams, h-γ mediation



- Theta term, light quark, muon EDMs -- γ–γ mediation



- Kobayashi-Maskawa CP-violation – Z (and WW) mediation



EDMs in the Standard Model

- Cabibbo-Kobayashi-Maskawa matrix and nearly maximal CP phase → still EDMs are too small to be observable in the next round of EDM experiments.

- Theta term of QCD: too large EDMs if theta is arbitrary → new naturalness problem because of EDMs. ($d_n \sim \theta\, m_q/m_n^2$, $\theta < 10^{-10}$)

* Importantly, in conventional cosmological scenarios neither of the two sources are enough to deliver required baryon asymmetry → BSM

EDMs from the CKM

CP violation via in CKM matrix

There are two possible sources of CP violation at a renormalizable level: δ_{KM} and θ_{QCD}.

δ_{KM} is the form of CP violation that appears only in the charged current interactions of quarks.

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left(\bar{U}_L \not{W}^+ V D_L \; + \; (\text{H.c.}) \right).$$

CP violation is closely related to flavour changing interactions.

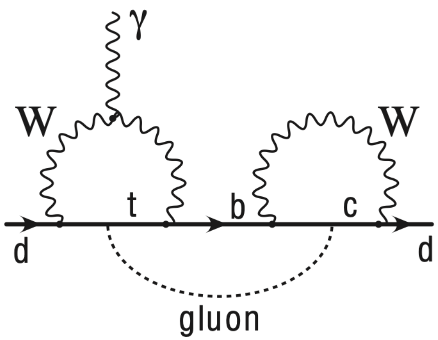
$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

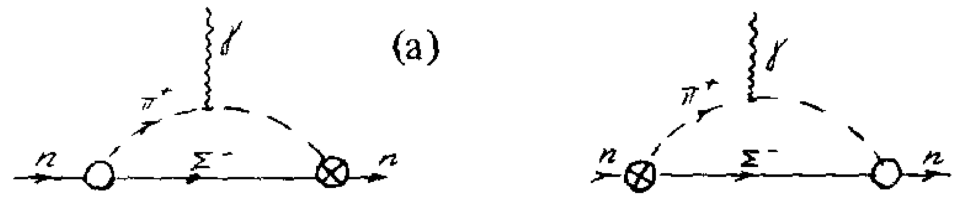
CKM model of CP violation is independenly checked using nutral K and B systems. *No other sources of CP are needed to describe observables!*

CP violation disappear if any pair of the same charge quarks is degenerate or some mxing angles vanish.

$$J_{CP} = \text{Im}(V_{tb} V_{td}^* V_{cd} V_{cb}^*) \times$$
$$(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2)(y_s^2 - y_d^2)$$
$$< 10^{-15}$$

EDMs from the CKM phase





CKM phase generates tiny EDMs:

$$d_d \sim \mathrm{Im}(V_{tb}V_{td}^*V_{cd}V_{cb}^*)\alpha_s m_d G_F^2 m_c^2 \times \text{loop suppression}$$
$$< 10^{-33} e\text{cm}$$

- Quark EDMs identically vanish at 1 and 2 loop levels, $\mathrm{EW}^2=0$ (Shabalin, 1981). 3-loop EDMs, $\mathrm{EW}^2\mathrm{QCD}^1$ are calculated by Khriplovich; Czarnecki, Krause. d_e vanishes at EW^3 level (Khriplovich, MP, 1991) < 10^{-38} e cm. Recent calculations (Yamaguchi, Yamanaka) estimate $d_e \sim 6\times10^{-40}$ e cm

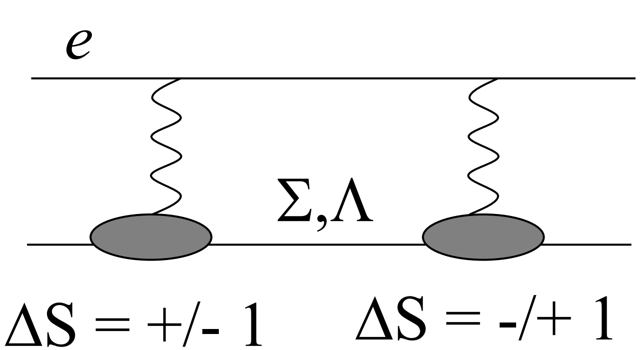
- Long distance effects give neutron EDM ~ 10^{-32} e cm; quite uncertain. Combination of $\Delta S = +1$ and $\Delta S = -1$ transition. Khriplovich, Zhitnitsky; Gavela et al, ... C.-Y. Seng.

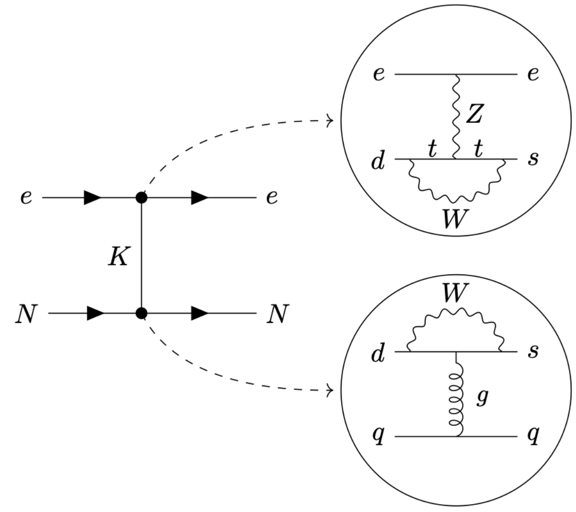
CKM CP-violation and paramagnetic EDMs

- The $d_e(\delta)$ result is small to the point of being not even remotely interesting (e.g. 10 orders of magnitude below current bounds)
- Semileptonic (C_S) operator is more important. MP and Ritz (2012) estimated two-photon mediated EW^2EM^2 effects and found that C_S is induced at the level equivalent to ~ 10^{-38} e cm

$$C_S \times \frac{G_F}{\sqrt{2}}\, \overline{N}N\, \overline{\psi} i\gamma_5 \psi$$

It turns out that there are much larger contributions at EW^3 order





Semi-leptonic Electroweak Penguin

- The upper part: EW penguin (EW^2) $\mathcal{L}_{\mathrm{EWP}} = \mathcal{P}_{\mathrm{EW}} \times \bar{e}\gamma_\mu\gamma_5 e \times \bar{s}\gamma^\mu(1-\gamma_5)d + (h.c$

$$\mathcal{L}_{Uee} = -\frac{if_0^2}{2}\mathcal{P}_{\mathrm{EW}} \times \bar{e}\gamma_\mu\gamma_5 e \times \mathrm{Tr}\left[h^\dagger\left(\partial^\mu U\right)U^\dagger\right] + (h.c.),$$

In the leading order, the dominant diagram is K_S exchange.

$$\mathcal{L}_{Kee} = -2\sqrt{2}f_0 m_e \bar{e}i\gamma_5 e\left(K_S \times \mathrm{Im}\mathcal{P}_{\mathrm{EW}} + K_L \times \mathrm{Re}\mathcal{P}_{\mathrm{EW}}\right)$$

- Lower part: EW[1] B-B-M coupling is related by flavor *SU*(3) to the *s*-wave amplitudes of the non-leptonic hyperon decays. Theory fit to decay amplitudes is [surprisingly] good (~5-10%):

$$\mathcal{L}_{\mathrm{SP}} = -a\mathrm{Tr}(\bar{B}\{\xi^\dagger h\xi, B\}) - b\mathrm{Tr}(\bar{B}[\xi^\dagger h\xi, B]) + (h.c.).$$

contains $2^{1/2} f_0^{-1}((b-a)\bar{p}p + 2b\bar{n}n)K_S$

(EW penguins are "famous" for $B_s \rightarrow \bar{\mu}\mu$, Real part of $K_L \rightarrow \bar{\mu}\mu$, and most recently $K^+ \rightarrow \pi^+\nu\nu$)

LO kaon exchange result

- Using EW penguin and strong penguin below,

$$\mathcal{L}_{KNN} \simeq -\frac{\sqrt{2}\,G_F \times [m_{\pi^+}]^2 f_\pi}{|V_{ud}V_{us}|f_0} \times 2.84(0.7\bar{p}p + \bar{n}n)$$
$$\times\left(\mathrm{Re}(V_{ud}^* V_{us})K_S + \mathrm{Im}(V_{ud}^* V_{us})K_L\right).$$

We calculate C_S

$$C_S \simeq \mathcal{J} \times \frac{N + 0.7Z}{A} \times \frac{13[m_{\pi^+}]^2 f_\pi m_e G_F}{m_K^2} \times \frac{\alpha_{\mathrm{EM}} I(x_t)}{\pi \sin\theta_W^2}$$

$$\mathcal{J} = \mathrm{Im}(V_{ts}^* V_{td} V_{ud}^* V_{us}) \simeq 3.1 \times 10^{-5}$$

That has the following LO scaling

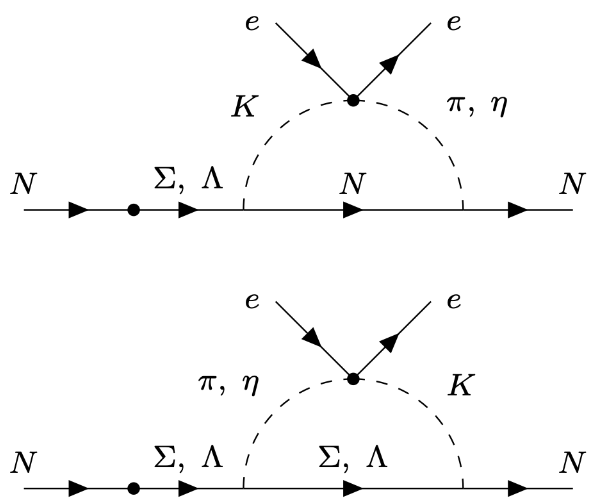
$$G_F C_S \propto \mathcal{J} G_F^3 m_t^2 m_e m_s^{-1} \Lambda_{\mathrm{hadr}}^2$$

Numerically, it is

$$C_S(\mathrm{LO}) \simeq 5 \times 10^{-16}.$$

NLO kaon-pion loop

- We calculate leading order corrections that have $(m_s)^{-1/2}$ scaling



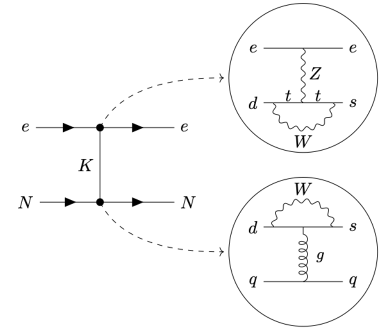
- The loop itself is proportional to $\sim m_K$, but there is a baryonic pole that brings $1/m_s$.

The NLO brings positive contribution of ~ 30%.

$$\frac{C_{S,NLO}(p)}{C_{S,LO}(p)} = \frac{m_K^3(0.77D^2 + 2.7DF - 2.3F^2)}{24\pi f_0^2(m_{\Sigma^+} - m_p)}$$

$$\frac{C_{S,NLO}(n)}{C_{S,LO}(n)} = \frac{m_K^3}{24\pi f_0^2}\left(\frac{(a/b+3)}{2\sqrt{6}(m_\Lambda - m_n)}\right. \times(-0.44D^2 + 3.2DF + 1.3F^2)$$
$$\left. + \frac{a/b - 1}{2\sqrt{2}(m_{\Sigma^0} - m_n)}\left(-0.53D^2 - 1.9DF + 1.6F^2\right)\right).$$

Final result



- Combining $(m_s)^{-1}$ and $(m_s)^{-1/2}$ effects, we get

$$\boxed{\begin{gathered} C_S(\mathrm{LO}+\mathrm{NLO}) \simeq 6.9 \times 10^{-16} \\ \Longrightarrow \;\; d_e^{\mathrm{equiv}} \simeq 1.0 \times 10^{-35}\, e\, \mathrm{cm}. \end{gathered}}$$

- The result EW^3 much larger than the $\mathrm{EW}^2\mathrm{EM}^2$ estimate by ~1000.

- ***Still small but perhaps not hopeless.***

- The result is under "best possible" theoretical control, and can be improved on the lattice

$$\langle N | i(\bar{s}\gamma_\mu(1-\gamma_5)d - \bar{d}\gamma_\mu(1-\gamma_5)s) | N \rangle_{\mathrm{EW}^1} = \frac{f_S}{m_N} i q_\mu \bar{N} N + \frac{f_T}{m_N} q_\nu \bar{N} \sigma_{\mu\nu}\gamma_5 N.$$

Ema, Gao, MP, PRL2022

EDMs from the theta QCD

Strong CP problem

Energy of QCD vacuum depends on θ-angle:

$$E(\bar{\theta}) = -\frac{1}{2}\bar{\theta}^2 m_* \langle \bar{q}q \rangle + \mathcal{O}(\bar{\theta}^4, m_*^2)$$

where $\langle \bar{q}q \rangle$ is the quark vacuum condensate and m_* is the reduced quark mass, $m_* = \frac{m_u m_d}{m_u + m_d}$. In CP-odd channel,

$$d_n \sim e\frac{\bar{\theta} m_*}{\Lambda_{\rm had}^2} \sim \bar{\theta} \cdot (6 \times 10^{-17})\, e\,\mathrm{cm}$$

Strong CP problem = naturalness problem = Why $|\bar{\theta}| < 10^{-9}$ when it could have been $\bar{\theta} \sim O(1)$? $\bar{\theta}$ can keep "memory" of CP violation at Planck scale and beyond. Suggested solutions

- Minimal solution $m_u = 0 \leftarrow$ apparently can be ruled out by the chiral theory analysis of other hadronic (CP-even) observables.
- $\bar{\theta} = 0$ by construction, requiring either exact P or CP at high energies + their spontaneous breaking. Tightly constrained scenario.
- Axion, $\bar{\theta} \equiv a(x)/f_a$, relaxes to $E = 0$, eliminating theta term. $a(x)$ is a very light field. Not found so far.

"Theta Signal"

$$\mathcal{L}_{eff}^{1\mathrm{GeV}} = \frac{g_s^2}{32\pi^2}\,\theta_{QCD} G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}$$
$$-\frac{i}{2}\sum_{i=e,u,d,s} d_i\,\overline{\psi}_i(F\sigma)\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\tilde{d}_i\,\overline{\psi}_i g_s(G\sigma)\gamma_5\psi_i$$
$$+\frac{1}{3}w\; f^{abc}G^a_{\mu\nu}\widetilde{G}^{\nu\beta,b}G_\beta{}^{\mu,c} + \sum_{i,j=e,d,s,b} C_{ij}\,(\bar{\psi}_i\psi_i)(\bar{\psi}_j i\gamma_5\psi_j) + \cdots$$

- Is theta term of QCD removed by PQ (axion) symmetry?

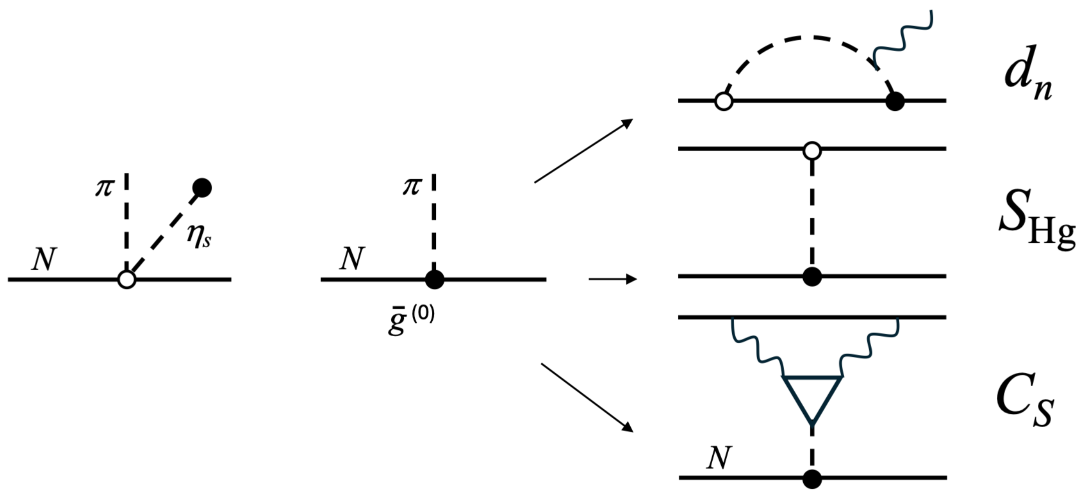
Yes → Focus on higher dim BSM operators

No → theta signal

E.g. discrete symmetries

EDM dependence on θ_{QCD}

- If theta term of QCD is *not* removed by PQ (axion) symmetry:



$$d_n \simeq 2 \times 10^{-16}\,\bar{\theta}\, e\mathrm{cm} \quad \rightarrow \quad |\bar{\theta}| < 1 \times 10^{-10}$$

$$S_{\mathrm{Hg}} \simeq 2 \times 10^{-3}\,\bar{\theta}\, e\mathrm{fm}^3 \quad \rightarrow \quad |\bar{\theta}| < 1.5 \times 10^{-10}$$

$$C_S \simeq 0.03\,\bar{\theta} \quad \rightarrow \quad |\bar{\theta}| < 2 \times 10^{-8}$$

theta signal

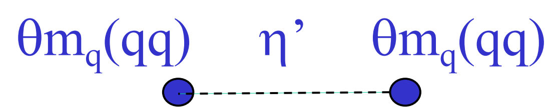
Current sensitivity of paramagnetic (electron-spin-related) EDMs is ~ two orders of magnitude less sensitive than neutron EDM. Flambaum, MP, Ritz, Stadnik, 2019.

Vacuum energy/Axion mass and connection to $U(1)_A$

- There are multiple derivations of the the axion mass (aka topological susceptibility) result. The simplest one is using chiral transformation to "move" theta term in front of the quark mass.

$$\mathcal{L}_{QCD} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

$$\rightarrow m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)\theta + m_*(\bar{u}u + \bar{d}d)\theta^2/2 + \ldots$$



m_* is the reduced quark mass, $m_u m_d/(m_u+m_d)$. The expectation value of the second term over the vacuum here is the vacuum energy dependence on the theta angle (and upon the rescaling the axion mass squared.)

We assume that U(1) problem is solved somehow, and the mass of the singlet is lifted. Otherwise, pole diagram with the singlet will cancel theta dependence.

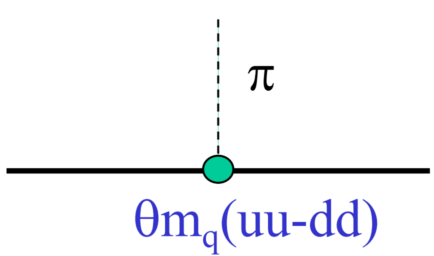
Linear in $G\tilde{G}$ matrix elements

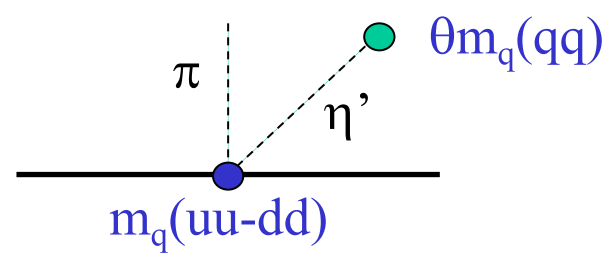
- Consider a matrix element of $\langle H_1\ \pi \,|\, GGdual \,|H_2\rangle$ operator, for the states of a soft pion, where H are arbitrary in- and out- states.

- Chiral PT / current algebra / soft pion theorem allow to "reduce" the pion so that

$\langle H_1\ \pi \,|\, GGdual \,|\, H_2 \rangle \rightarrow i\,(F_\pi)^{-1} \langle H_1 \ \,|\, m_q(uu\text{-}dd) \,|\, H_2 \rangle$. If H_1, H_2 are nucleons, we get a scalar-isovector matrix element, part of the n-p mass splitting.

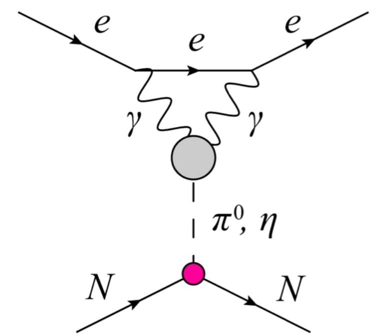
- In *our world* with light quarks $m_\pi^2 = B\, m_q$ while $m_{\eta'}^2 = B\, m_q + m_0^2$, and heavy mass of η' requires m_0^2 to be large and m_q independent in the limit of large m_q. In an *imaginary world*, where eta-prime is light and $m_0^2 = 0$, there is a second diagram that cancel the first one (SVZ 1980, MP, Ritz 1999)





LO chiral contribution

- t-channel pion exchange gives



$$\mathcal{L} = \theta \times \frac{1}{m_\pi^2} \times 0.017 \times 3.5 \times 10^{-7} (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p)$$

$$= (\bar{e} i \gamma_5 e)(\bar{n}n - \bar{p}p) \times \frac{3.2 \times 10^{-13}\theta}{\mathrm{MeV}^2}.$$

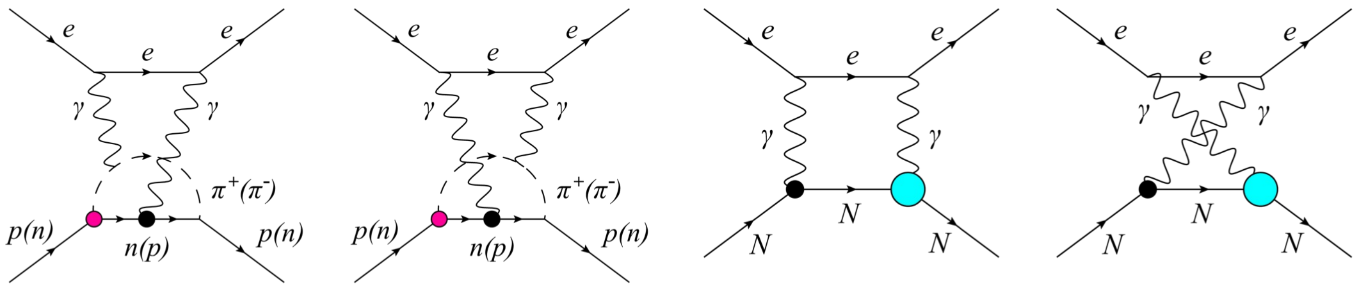
implying $|\theta| < 8.4 \times 10^{-8}$ sensitivity. However, adding exchange of η_8,

$$1 \to 1 - \frac{1}{3}\frac{f_\pi^2 m_\pi^2}{f_\eta^2 m_\eta^2} \times \frac{m_d - m_u}{m_d + m_u} \times \frac{A \times \sigma_N}{\frac{m_d - m_u}{2}\langle p|\bar{u}u - \bar{d}d|p\rangle \times (N - Z)}$$

$$1 \to 1 - 0.88 \simeq 0.12.$$

The effect can completely cancel within error bars on nucleon sigma term σ_N.

Higher orders



Limit on theta term from ThO (electron EDM) experiment:

$$C_{SP}(\bar{\theta}) \approx \left[0.1_{\mathrm{LO}} + 1.0_{\mathrm{NLO}} + 1.7_{(\mu d)}\right] \times 10^{-2}\bar{\theta} \approx 0.03\,\bar{\theta}$$

$$|\bar{\theta}|_{\mathrm{ThO}} \lesssim 3 \times 10^{-8}$$

A factor of ~100 weaker than neutron EDM

* Improved by a factor of ~ 2 in 2022.

System	$\lvert d_p \rvert$ $(e \cdot \mathrm{cm})$	$\lvert \bar{g}^{(1)}_{\pi NN} \rvert$	$\lvert \tilde{d}_u - \tilde{d}_d \rvert$ (cm)	$\lvert \bar{\theta} \rvert$
ThO	$\mathbf{2 \times 10^{-23}}$	$\mathbf{4 \times 10^{-10}}$	$\mathbf{2 \times 10^{-24}}$	$\mathbf{3 \times 10^{-8}}$
n	—	1.1×10^{-10}	5×10^{-25}	2.0×10^{-10}
Hg	2.0×10^{-25}	1×10^{-12} a	5×10^{-27} a	1.5×10^{-10}
Xe	3.2×10^{-22}	6.7×10^{-8}	3×10^{-22}	3.2×10^{-6}

Dekens et al, 2026, and Mulder et al, 2025, undertook a similar analysis. Differs in detail but close in numerics

More work is required for better treatment of 2-photon contributions

EDMs from the BSM

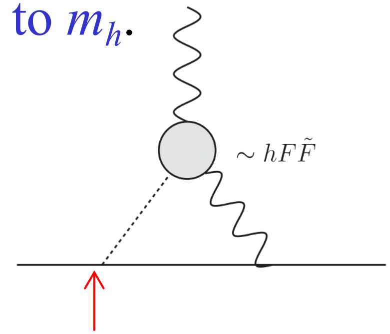
EDMs vs Higgs and electroweak physics

$$\mathcal{L}_{h\gamma\gamma}^{CP-odd} = H^\dagger H \times \left(\frac{g_1^2}{e^2\Lambda_{h\gamma}^2} a_h \times B_{\mu\nu}\widetilde{B}_{\mu\nu} + b_h \times \frac{g_2^2}{e^2\Lambda_{h\gamma}^2} W_{\mu\nu}^a \widetilde{W}_{\mu\nu}^a \right) \to \frac{c_h v_{\mathrm{EW}}}{\Lambda_{h\gamma}^2} \times hF_{\mu\nu}\widetilde{F}_{\mu\nu},$$

At an EFT level, SM Higgs may have CP-odd couplings to matter, e.g. photons, which mixes with an EDM operator upon the RG running from Λ to m_h.

The result is tightly constrained:

$$d_e \simeq c_h \frac{em_e}{4\pi^2\Lambda_{h\gamma}^2} \log\left(\frac{\Lambda_{\mathrm{UV}}^2}{m_h^2}\right) \to \frac{c_h}{\Lambda_{h\gamma}^2} < \frac{1}{(800\,\mathrm{TeV})^2},$$



Assuming h couples to e

CP-odd top-Higgs coupling is very constrained as well:

$$\mathcal{L}_{htQ} = -H^\dagger H \times \frac{y_t \exp(i\phi_t)}{\Lambda_{ht}^2} \times \bar{t}_R Q_{3L} H + (h.c) \to \mathcal{L}_{htt}^{CP-odd} = \sin(\phi_t) \frac{m_t v_{\mathrm{EW}}}{\Lambda_{ht}^2} \times h(\bar{t} i\gamma_5 t),$$

$$d_e^{ht\gamma} = \sin(\phi_t) \times \frac{em_e}{\Lambda_{ht}^2} \times \frac{\alpha}{6\pi^3} g(m_t^2/m_h^2) \to 4.1 \times 10^{-30}\, e\mathrm{cm} \times \left(\frac{10\,\mathrm{TeV}}{\Lambda_{ht}}\right)^2 \times \left(\frac{\sin(\phi_t)}{1/\sqrt{2}}\right).$$

This operator is "useful" for models of EW baryogenesis:

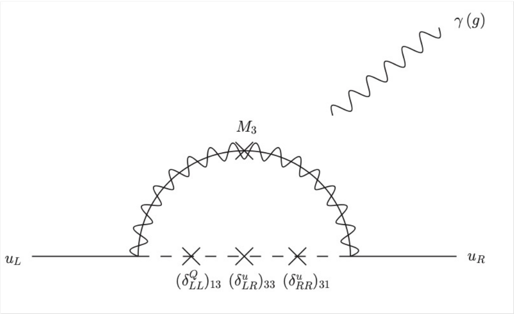
To avoid EDM constraints one needs "special situations", such as closely degenerate levels that enhance CP violation in decays but not in loops.

Importantly, EDMs seem to require additional EW charged scalar states.

EDMs in the LHC era, and beyond

- Higgs mass point to a large scale of SUSY breaking, 10-100 TeV
- The requirements on approximately "flavor-aligned" scalar quark and scalar lepton sector are softened compared to "SUSY at EW scale".
- LR mixing of quarks and leptons can get ~ m_t and m_τ instead of m_u and m_e. This can lead to a significant enhancement (McKeen, MP, Ritz, 2013)

$$\delta m_e \simeq \frac{\alpha}{24\pi\cos^2\theta_W} \times \theta_{13}^2 m_\tau \tan\beta,$$
$$d_e \simeq \frac{e(\delta m_e)}{5} \times \frac{\sin(\phi_{\mathrm{SUSY}})}{m_{\mathrm{SUSY}}^2}.$$



- If correction to the mass is on the order of the mass itself, $\delta m_e \sim m_e$, then effectively there is enhancement:

$$d_e \sim 1.4 \times 10^{-28}\, e\mathrm{cm} \times \left(\frac{100\,\mathrm{TeV}}{m_{\mathrm{SUSY}}}\right)^2 \times \left(\frac{\sin(\phi_{\mathrm{SUSY}})}{1/\sqrt{2}}\right).$$

- Even 100 TeV scale SUSY is *not hopeless* for EDMs if phases are large.

EDMs from dark sectors

- SM can be extended by neutral [“dark sector”] states:

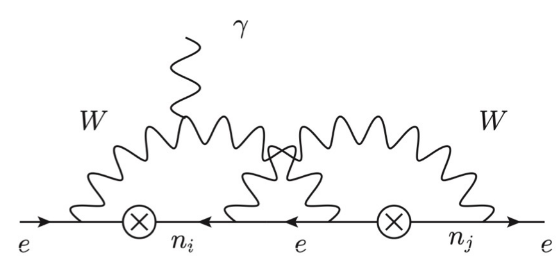
$$\mathcal{L}_{\text{EFT}} = m_H^2 H^\dagger H + \mathcal{L}_{\text{SM, dim}\,4}(A, \psi, H) + \sum_{n \geq 5} \frac{1}{\Lambda^{n-4}} \mathcal{O}^{(n)}_{\text{SM, dim}\,n}(A, \psi, H)$$
$$+ \mathcal{L}_{\text{portals}}(\psi, A, H; \varphi_{\text{DS}}) + \mathcal{L}_{DS}(\varphi_{\text{DS}}).$$

$$\text{Explicitly } CP \text{ conserving}: \quad A'_{\mu\nu} B_{\mu\nu};\ \ (AS + \lambda S^2)(H^\dagger H);\ \ a B_{\mu\nu} \widetilde{B}_{\mu\nu};\ \ \partial_\mu a (\bar{\psi}_i \gamma_\mu \gamma_5 \psi_i).$$
$$\text{Possibly } CP \text{ violating}: \quad e^{i\phi_N} NLH;\ \ a B_{\mu\nu} B_{\mu\nu};\ \ e^{i\delta_{ij}} \partial_\mu a (\bar{\psi}_i \gamma_\mu \gamma_5 \psi_j),\ i \neq j.$$

- Sterile neutrinos allow for EW^2 EDMs

But the effect is very small unless light masses are tuned

$$d_e \leq e \left(\frac{G_F}{16\pi^2} \right)^2 \times m_e m_\nu^2 \times 10 \log \left(\frac{m_N}{m_W} \right) < 10^{-43}\, e\text{cm},$$



- There is no particular EDM consequences from dark photons, or Higgs-like neutral scalars.

An example of an ALP with a CP-violating coupling

- If there are relatively light O(MeV) ALPs with CP-violating couplings, the sensitivity to an effective CP-odd operators can go up above the GUT scale.

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + \left(\frac{i\phi}{\Lambda_P}H\bar{e}_L e_R + \frac{\phi}{\Lambda_S}H\bar{q}_L q_R + h.c.\right)$$

- Upon EW breaking dim=5 operators translate to the pseudoscalar and scalar coupling

$$\to \frac{v_{\rm EW}}{\Lambda_P} \times \phi\bar{e}i\gamma_5 e + \frac{v_{\rm EW}}{\Lambda_S} \times \phi\bar{q}q$$

- The exchange by ϕ scalar creates a C_S operator. Its magnitude scales as

$$C_S \propto \frac{1}{m_\phi^2} \times \frac{v_{\rm EW}^2}{\Lambda_P\Lambda_S}$$

- Let's think of ϕ as an ALP, minimize its mass within BBN limits, and maximize its pseudoscalar coupling to electrons. What is sensitivity to CP violating coupling Λ_S?

$$m_\phi \sim 10\,\text{MeV},\;\; \Lambda_P \sim 10^3 v_{\rm EW} \;\;\to\;\; \Lambda_S > 10^{18}\,\text{GeV}$$

- The sensitivity to the CP-violating scale approaches the Planck scale.

Hadronic matrix elements and EDMs

- For the neutron EDM and CP-odd pion-nucleon constants different methods applied: chiral PT, QCD sum rules, lattice QCD.
- Lattice QCD has been useful for the quark tensor and scalar charges inside the nucleon. Helps with $d_n(d_q)$ and C_S. Strange quark is still a problem.
- Even if there is a QCD axion that removes theta term, the matrix element of $d_n(\theta)$ is still needed because of the induced theta term by higher-dimensional operators.

$$\theta_{\text{ind}} = -\frac{\int d^4x \langle T(\frac{\alpha_s}{8\pi}G\tilde{G}(0), \mathcal{L}^{\text{CP-odd}}_{dim\geq 5}(x)\rangle}{\int d^4x \langle T(\frac{\alpha_s}{8\pi}G\tilde{G}(0), \frac{\alpha_s}{8\pi}G\tilde{G}(x)\rangle} \rightarrow -\frac{1}{2}\left(\sum_{q=u,d,s}\frac{\tilde{d}_q}{m_q}\right)\frac{\langle 0|\bar{q}(G\sigma)q|0\rangle}{\langle 0|\bar{q}q|0\rangle} + \cdots$$

- Current estimates of $d_n(\theta)$ via chiral PT and QCD sum rules are consistent. Lattice has not reached the stage where it can show $d_n(\theta) \sim m_q$. *In fact, there is a problem: current choice for the nucleon interpolating current has L $\leftrightarrow$ R transitions for quarks even for $m_q = 0$. We think that the unphysical dependence on theta will persist even in the strict massless quark limit.*
- The most consequential matrix element to calculate on the lattice is the following: $\langle N|\bar{q}g_s(G\sigma)q - m_0^2\bar{q}q|N\rangle$, where $m_0^2 = \langle 0|\bar{q}g_s(G\sigma)q|0\rangle/\langle 0|\bar{q}q|0\rangle$

It determines the strength of the CP-odd pion-nucleon coupling constant induced by chromoelectric EDM operators.

EDMs of heavy flavors

- Among Wilson coefficients of different kind, EDMs of heavy flavours d_i are interesting. i = muon, tau, charm, bottom, top.

- Muon EDM is limited as a bi-product of BNL g-2 experiment. Can be significantly improved in dedicated beam experiments (PSI, Fermilab, J-Parc)

- There is a creative proposal to measure MDMs and limit EDMs of charmed baryons using thin fixed target and bent crystal technology (ALADDIN proposal).

- Heavy flavors contribute to observable EDMs via loops. Top quark EDM is limited indirectly by electron EDM via a two-loop (top-Higgs-gamma) Barr-Zee diagrams. The result is stronger than the direct measurements at LHC.

Muon EDM vs Muon g-2

- Muon anomalous magnetic moment is known precisely. Theory errors on a_μ are at the levels of ~ 10^{-9}. Muon g-2 begins to be sensitive to weak scale corrections due to W, Z loops.

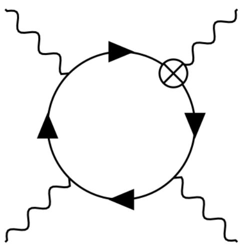
- Let us translate this to ”EDM friendly units”:

$$\Delta\mu_\mu \sim 10^{-9}\, e/(2m_\mu) = 10^{-22}\, e \text{ cm}$$

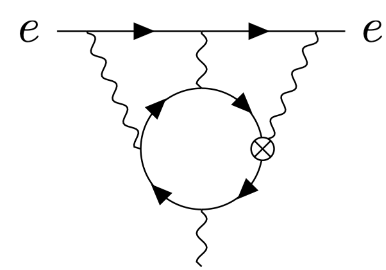
This is the level of muon EDM that would be very interesting. But in some models it can be larger.

Muon EDM inside a loop

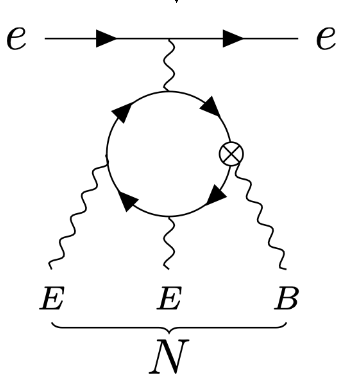
- Muon loop induces E^3B effects, and electron EDM at 3-loops.



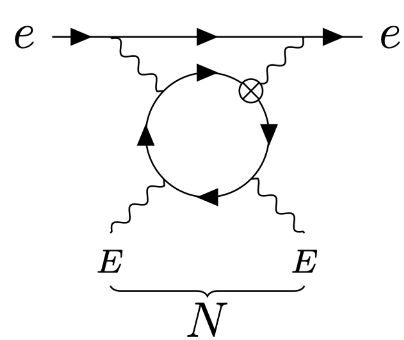
$$\mathcal{L} = -e^4(\tilde{F}_{\alpha\beta}F^{\alpha\beta})(F_{\gamma\delta}F^{\gamma\delta}) \times \frac{d_\mu/e}{96\pi^2 m_\mu^3}$$
$$= -\frac{d_\mu/e}{12\pi^2 m_\mu^3} e^4 (\mathbf{E}\cdot\mathbf{B})(\mathbf{E}\cdot\mathbf{E} - \mathbf{B}\cdot\mathbf{B}),$$



Nuclear Schiff moment



Effective C_S operator



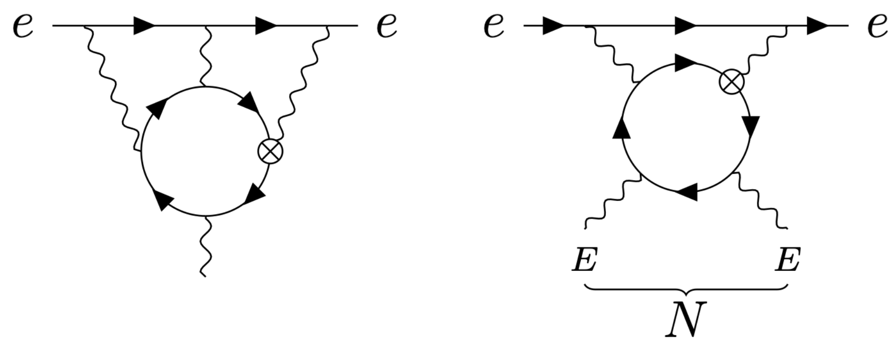
d_{Hg}

$d_e^{equiv}(ThO)$

Paramagnetic EDM calculation

$$\mathcal{L} = -e^4(\tilde{F}_{\alpha\beta}F^{\alpha\beta})(F_{\gamma\delta}F^{\gamma\delta}) \times \frac{d_\mu/e}{96\pi^2 m_\mu^3}$$
$$= -\frac{d_\mu/e}{12\pi^2 m_\mu^3} e^4(\mathbf{E}\cdot\mathbf{B})(\mathbf{E}\cdot\mathbf{E} - \mathbf{B}\cdot\mathbf{B}),$$

Both electron EDM and C_S operator are induced



We have corrected a three-loop result of Grozin, Khriplovich, Rudenko, 2009, and calculated C_S. C_S dominates by a factor of ~5.

$$\frac{G_F}{\sqrt{2}} C_S^{\rm equiv} = \kappa \frac{4Z^2\alpha^4}{\pi A} \times \frac{m_e(d_\mu/e)}{m_\mu^3 R_N} \times \log\left(\frac{m_\mu}{m_e}\right)$$

Equivalent C_S has coherent enhancement due to $\langle E^2 \rangle$ inside the nucleus (a-la Weisskopf EM energy contribution).

New indirect constraints on muon EDM

- Owing to the fact that the electric field inside a large nucleus is not that small $eE \sim Z\,\alpha\, R_N^{-1} \sim 30$ MeV compared to m_μ, effects formally suppressed by higher power of m_μ win over three-loop electron EDM.

- New results:

Hg EDM experiment: $S_{^{199}\mathrm{Hg}}/e \simeq (d_\mu/e) \times 4.9 \times 10^{-7}\,\mathrm{fm}^2, \quad |d_\mu| < 6.4 \times 10^{-20}\, e\,\mathrm{cm}$

ThO EDM experiment: $d_e^{\mathrm{equiv}} \simeq 5.8 \times 10^{-10}\, d_\mu \implies \boxed{|d_\mu| < 1.9 \times 10^{-20}\, e\,\mathrm{cm}.}$

New limit from Boulder HfF: $\sim 9 \times 10^{-21}$

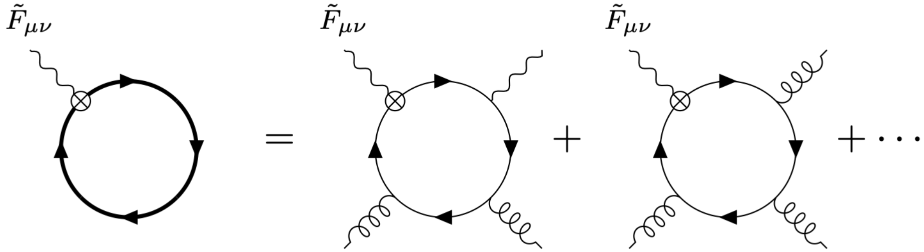
- Factor of 20 improvement over the BNL constraint, $|d_\mu| < 1.8 \times 10^{-19}\, e\mathrm{cm}$,
- New benchmark for the muon beam EDM experiments.

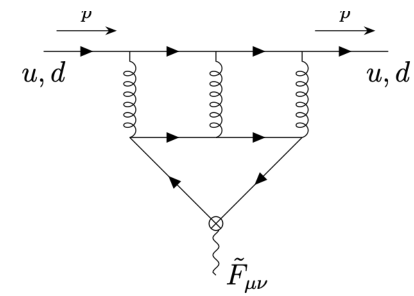
NB: 3-loop contributions calculated by Grozin et al. has been revised

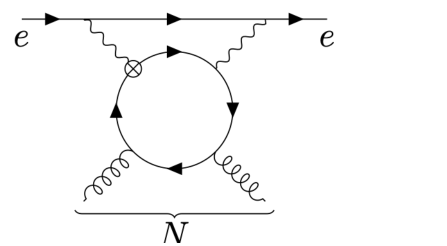
- Tau EDM is constrained by three-loop induced d_e.

Charm and bottom EDMs

Charm loop gives $(\gamma)^2(\text{gluon})^2$ and $(\gamma)^1(\text{gluon})^3$ effective operators

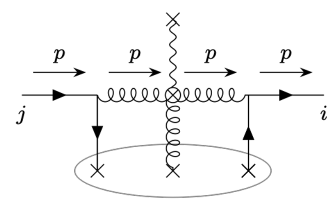






$$\langle N | \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} G^{a\mu\nu} | N \rangle = -\frac{m_N}{9} \bar{N} N,$$

$S_{ij}^{(\cancel{CP})}(p) =$



Nonperturbative 3-gluon induced tensor charge

$d_e^{\text{equiv}}(\text{ThO})$

d_n , d_{Hg}

- All EDMs are induced by charm and bottom EDMs.

New indirect constraints on c, b quarks EDMs

- New-ish results (~2022):

Neutron EDM experiment: $|d_c| < 6 \times 10^{-22}\, e\,\mathrm{cm}, \quad |d_b| < 2 \times 10^{-20}\, e\,\mathrm{cm},$

ThO EDM experiment: $|d_c| < 1.3 \times 10^{-20}\, e\,\mathrm{cm}, \quad |d_b| < 7.6 \times 10^{-19}\, e\,\mathrm{cm},$

- Neutron EDM estimates have uncertainty ~ a factor of O(few) due to limitation of QCD sum rule method in this channel. C_S derived limits have *minimal* uncertainty, O(10%).
- Independent of (similar order of magnitude) bounds based on RG running of operators, and contribution to the GGGdual Weinberg operator.
- The strength of these limits on charm EDM points to the conclusion that future charmed baryon EM moment proposal should focus on MDM.

Conclusions

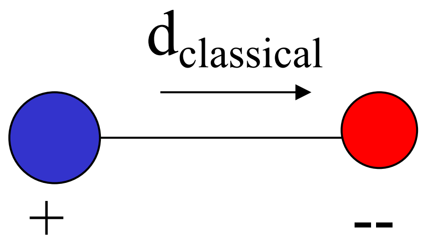
- Searches for EDMs are very important part of the fundamental physics program. Sensitive to ~ 100 TeV scales of New Physics through the best limits. CP-odd couplings of the Higgs are tightly constrained.

- If there is no PQ symmetry → expect theta signal. Theta is constrained by the electron EDM experiments (via C_S) and current sensitivity is behind d_n by ~ two orders of magnitude.

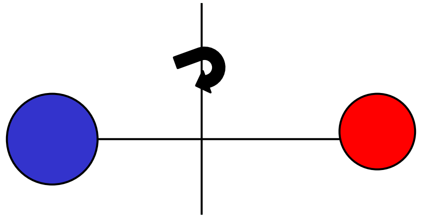
- CKM CP violation is never a background. The *equivalent* d_e (ThO) is predicted to be $+1.0 \times 10^{-35}$ e cm (via C_S). This is 1000 times larger than previously believed, but still 5 orders of magnitude away from observations.

A small comment on classical EDMs

- Fundamental EDMs are connected to spin, classical EDMs are not.
- A diatomic molecule (like ThO) will have a classical EDM.



- However, in a quantum state with fixed angular momentum classical EDM average to zero, exactly. States with +M and –M projection of angular momentum remain degenerate (at B=0).



$$< d_{classical} > = 0$$

- If there is fundamental CP-violation, the electric field will induce splitting between +M and –M states, e.g. Zeeman effect but with electric field. EDM experiments are looking for E coupling to spin.

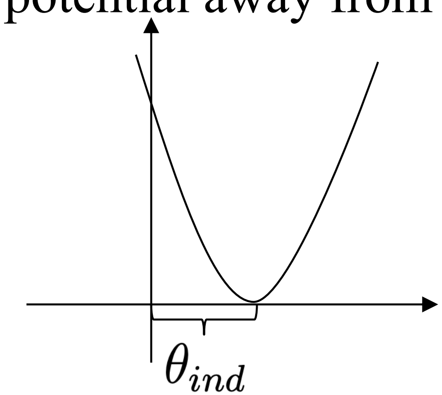
Axion mechanism in the presence of extra CP violation – proper UV decoupling

Imagine that at some scale Λ_{CP} there is some new CP-violating physics with phases δ_{CP}. Integrating it out, we end up with a series of effective CP-odd operators of various dimensions, O_{CP}.

$$\mathcal{L}_{eff} = \frac{\Delta\theta(\delta_{CP})g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \sum_q \overset{O_5}{\tilde{d}_q(-i/2)\overline{q}(G\sigma)\gamma_5 q} + \overset{O_6}{c_W(GG\tilde{G})} + \ldots$$

$\Delta\theta$ is an additive renormalization of theta-term and is unobservable in the axion model.

Higher dimensional operators induce axion tadpoles, leading to the minimum of the potential away from $\theta = 0$.



$$\theta_{ind} \propto \frac{\int d^4x T\langle 0|G\tilde{G}(0), O_5(x)|0\rangle}{\int d^4x T\langle 0|G\tilde{G}(0), G\tilde{G}(x)|0\rangle}$$

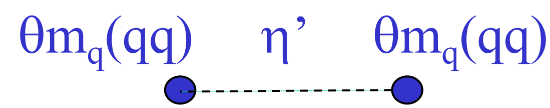
θ_{ind} will have the decoupling properties, i.e. $\theta_{ind} \rightarrow 0$ as $\Lambda_{UV} \rightarrow$ infinity.

Vacuum energy/Axion mass and connection to $U(1)_A$

- There are multiple derivations of the the axion mass (aka topological susceptibility) result. The simplest one is using chiral transformation to “move” theta term in front of the quark mass.

$$\mathcal{L}_{QCD} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

$$\to m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)\theta + m_*(\bar{u}u + \bar{d}d)\theta^2/2 + \ldots$$



m_* is the reduced quark mass, $m_u m_d/(m_u+m_d)$. The expectation value of the second term over the vacuum here is the vacuum energy dependence on the theta angle (and upon the rescaling the axion mass squared.)

We assume that U(1) problem is solved somehow, and the mass of the singlet is lifted. Otherwise, pole diagram with the singlet will cancel theta dependence.

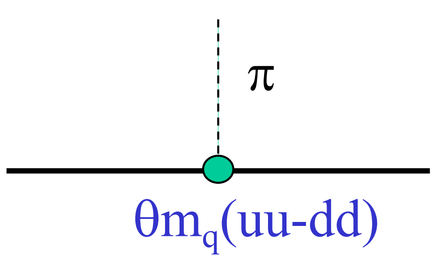
Linear in $G\tilde{G}$ matrix elements

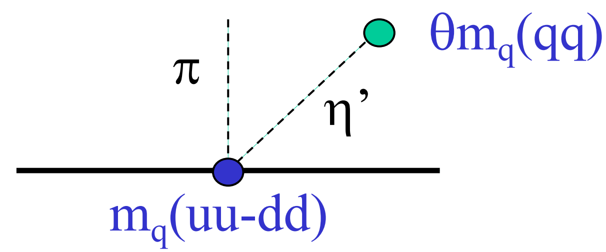
- Consider a matrix element of $\langle H_1\ \pi \,|\, GGdual \,|H_2\rangle$ operator, for the states of a soft pion, where H are arbitrary in- and out- states.

- Chiral PT / current algebra / soft pion theorem allow to "reduce" the pion so that

$\langle H_1\ \pi \,|\, GGdual \,|\, H_2 \rangle \rightarrow i\,(F_\pi)^{-1} \langle H_1\ \,|\, m_q(uu\text{-}dd) \,|\, H_2 \rangle$. If H_1, H_2 are nucleons, we get a scalar-isovector matrix element, part of the n-p mass splitting.

- In *our world* with light quarks $m_\pi^2 = B\, m_q$ while $m_{\eta'}^2 = B\, m_q + m_0^2$, and heavy mass of η' requires m_0^2 to be large and m_q independent in the limit of large m_q. In an *imaginary world*, where eta-prime is light and $m_0^2 = 0$, there is a second diagram that cancel the first one (SVZ 1980, MP, Ritz 1999)





EDMs induced by θ_{QCD}

- Neutron EDM.

$$d_n \simeq e \times g_A \times (15 \times 10^{-3}\theta) \frac{\log(m_N^2/m_\pi^2)}{8\pi^2 F_\pi},$$

Crewther et al showed logarithmic sensitivity to m_π, and numerically this is ~ few 10^{-16} e cm. $\Theta < 10^{-10}$.

- ^{199}Hg EDM. This is the tightest constraint on atomic EDM, the sensitivity to theta is reduced because one has to use Schiff moment of the nucleus. Similar sensitivity to θ, with different systematics.

- Paramagnetic EDMs (aka electron EDM) – coupling of electric field to an unpaired electron spin. *What is the sensitivity to theta?* [See the answer in Flambaum, MP, Ritz, Stadnik, 1912.13129]

Revisiting nonperturbative calculations of d_n

- Use chiral PT, rely on IR enhanced contributions, use some pheno input (or lattice input) to infer π-NN CP-odd couplings. (Crewther, DiVecchia, Veneziano, Witten, ++, 1980++)

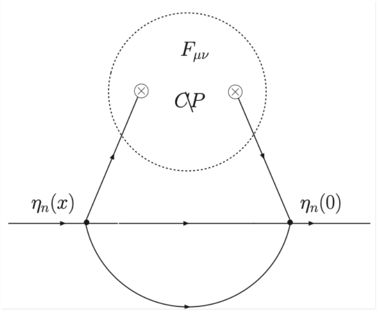
- MP, A. Ritz: 1999-2002: apply QCD sum rules to estimate the OPE coefficients in the external CP-violating and EM backgrounds, including the theta term.

- Preferable direction: set up proper lattice QCD calculations. Various nucleon matrix elements are calculated, but observables that are very sensitive to the quark mass, such as *$d_n(\theta)$ prove to be difficult.*

- *Ema, Gao, MP, Ritz* – PRD2024. Investigate chiral properties of the correlator of nucleon interpolating currents, re-derive $d_n(\theta)$.

Nonperturbative calculations of nucleon (hadronic) observables

$$\Pi(Q^2) = i\int d^4x e^{ip\cdot x}\langle 0|T\{\eta_N(x)\overline{\eta}_N(0)\}|0\rangle_{C\!\!\!/P,F,\pi},$$



- Interpolating η currents can be formulated in terms of 3 quarks with appropriate quantum numbers.

- $\Pi(x)$ can be calculated at short distances, using perturbative QCD + nonperturbative condensates. On the other hand, due to quark-hadron duality, we expect that $\Pi(Q^2)$ has also representation in terms of the hadronic resonances and their matrix elements. QCD sum rules *hopes* to match the two at some intermediate/borderline scale, $Q^2 \sim \text{GeV}^2$.

- Lattice QCD can perform these calculations "honestly", $x \rightarrow$ large

Nucleon Interpolating Currents

$$\eta_n = j_1^{(n)} + \beta j_2^{(n)}.$$

$$j_1^{(n)} = 2\epsilon_{ijk}\left(d_i^T \mathcal{C}\gamma_5 u_j\right) d_k, \quad j_2^{(n)} = 2\epsilon_{ijk}\left(d_i^T \mathcal{C} u_j\right)\gamma_5 d_k,$$

- $\beta = 0$, $\eta = j_1$, is the so-called QCD current i.e. the current used the most in the lattice QCD community. It takes its origin in the naïve quark model, because it is j_1 that has a nonrelativistic limit.

- $\beta = -1$ can be called "Ioffe current", and it has been used the most in various QCD SR literature of 1980s-1990s.

- $\beta = +1$ found to be the most convenient choice (MP and Ritz) for the neutron EDM calculations created by external sources.

Recap of d_n results (QCD SR, $\beta = 1$)

- Use odd-number of γ-matrices for the SR, and spurious phases of the 2-point functions will never appear

- Simple estimate based on the leading term of the OPE has a strong correspondence with the NQM (according to "Ioffe formula", the coefficient outside the square brackets below = 1).

$$d_n^{\text{est}} = \frac{8\pi^2|\langle\bar{q}q\rangle|}{m_n^3}\left[-\frac{2\chi m_*}{3}e(\bar{\theta}-\theta_{\text{ind}})\right.$$
$$\left. +\frac{1}{3}(4d_d - d_u) + \frac{\chi m_0^2}{6}(4e_d\tilde{d}_d - e_u\tilde{d}_u)\right],$$

- Here, c stands for another vacuum condensate, EM susceptibility of the QCD vacuum, $\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle = F_{\mu\nu}\times eQ_q\langle 0|\bar{q}q|0\rangle \times \chi$

Numerically, $\chi \sim -6\ \text{GeV}^{-2}$.

Technical Note

- There are two types of chiral structures in $\Pi(Q^2)$, with an odd and even number of γ_μ.

- Overall γ_5 - phase of hadronic interpolator current does not affect $\Pi(Q^2)$, $\exp(i\,\alpha\gamma_5)\,\Pi_{odd}(Q^2)\,\exp(i\,\alpha\gamma_5) = \Pi_{odd}(Q^2)$. This was used by (MP, Ritz, 1999) to calculate 3-point function without the need to determine α. The most suitable current for that is $\beta = 1$, as LO OPE is maximized and NLO is minimized.

- Alternative method is to look at $\Pi_{even}(Q^2) \sim \Pi_{(\mu)}(F\sigma) + \Pi_{(d)}(\tilde{F}\sigma)$. However, due to non-invariance of $\Pi_{even}(Q^2)$ under chiral rotations, one has to consider 2-point functions $\Pi_{2\text{-point}}(Q^2)$ and determine α. Physical observables are then $d/\mu \sim \Pi_{(d)}/\Pi_{(\mu)} - \alpha$. The most suitable current for that is $\beta = -1$, but it was not done before.

- Our goal is to evaluate d_n using QCD SR in the chirality-even channel.

Back to basics: QCD + theta term

$$\mathcal{L}_{QCD} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Do a standard iso-singlet quark chiral rotation to eliminate θGGdual.

$$\to m_*(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)\theta + m_*(\bar{u}u + \bar{d}d)\theta^2/2 + \ldots$$

m_* is the reduced quark mass, $m_u m_d (m_u + m_d)$. The expectation value of the second term over the vacuum here is the vacuum energy dependence on the theta angle (and upon the rescaling the axion mass squared.) Expectation value of the second term over nucleon, gives theta-dependence of nucleon mass.

All observables that depend on θ should also depend on m_* and vanish in the chiral limit. Also, observables do not depend on how you distribute θ, putting some parts to quark mass, and some to GGdual.

QCD + theta term + Nucleon Source

$$\mathcal{L} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_{u,d,} \bar{q}(iD_\mu\gamma_\mu - m_q)q + \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \text{Source}\times(j_1+\beta j_2) + h.c.$$

- This is the basis for studying nucleon properties. It is almost QCD, but not quite!

- Let us perform a chiral rotation, as on the previous slide. If this transformation would lead to

$$\text{Source} \times (j_1 + \beta j_2) \;\to\; \text{Source} \times e^{i\alpha\gamma_5} \times (j_1 + \beta j_2)$$

then it is an *innocent transformation*, and the new phase can be reabsorbed into the source. Otherwise, θ dependence will persist even in $m_q \rightarrow 0$ limit.

- This is true only for β=1 and β=-1 current choices. It is specifically not true for the lattice current β=0. It has unphysical L←→R quark transitions.

Chiral transformation of interpolating current

- Under the iso-singlet chiral transformation, the lattice current changes

$$j_a^\beta = j_1^a + \beta j_2^a,$$

we see that for the special choices of $\beta = \pm 1$,

$$j_a^+ = 2\epsilon_{ijk}\left[-\left(q_{iL}^T \mathcal{C} q_{jL}\right) q_{kL}^a + \left(q_{iR}^T \mathcal{C} q_{jR}\right) q_{kR}^a\right],$$
$$j_a^- = 2\epsilon_{ijk}\left[\left(q_{iR}^T \mathcal{C} q_{jR}\right) q_{kL}^a - \left(q_{iL}^T \mathcal{C} q_{jL}\right) q_{kR}^a\right],$$

$$j_a^\beta \to \frac{1+\beta}{2} e^{3i\theta_A\gamma_5} j_a^+ + \frac{1-\beta}{2} e^{-i\theta_A\gamma_5} j_a^- .$$

Only β= *-1* and β= *+1* are self-similar (covariant) under chiral rotation.

We think that away from these two special points one has a big problem in evaluating θ - dependent observables.

Unphysical θ dependence of some correlators

- Under the isosinglet chiral transformation, the lattice current changes

$$j_1 = 2\epsilon^{abc} d^a (d^{bT} C\gamma_5 u^c) \to 2\epsilon^{abc} e^{i\theta\gamma_5} d^a (d^{bT} C\gamma_5 e^{i2\theta\gamma_5} u^c)$$

- This results in a rephasing-invariant theta-dependent pieces in the OPE:

$$\frac{aM^4}{16} \times \left(6(1-\beta^2)e^{i2\theta\gamma_5} + (1-\beta)^2 e^{-i2\theta\gamma_5}\right) \ldots$$
$$= |\lambda|^2 e^{i\alpha\gamma_5}(m_N + i\gamma_5 m_{N5})e^{i\alpha\gamma_5} \times \exp(-m_N^2/M^2),$$

where a is $|\langle \bar{q}q \rangle| \times 4\pi^2$.

- If the correlator $\Pi(M^2)$ is matched to physical observables (e.g. hadron masses, they will acquire θ-dependence in the strict chiral limit.)
- Absolutely same problems will persist in the $d_n(\theta)$ calculation performed with the "lattice current". There will be dependences, in general, on unphysical phases, related to the chirality breaking built into the interpolating current itself.

Repeating $d_n(\theta)$ calculation for Ioffe current

- Original calculation by Ritz and MP was using β = +1, and a channel with odd number of gamma-matrices so that the EDM correlator is insensitive to the phase of the two-point function:

$$\lambda_I^2 \exp\{i\alpha\gamma_5\}\frac{i}{\not{p} - m_n - O}\exp\{i\alpha\gamma_5\} = i\lambda_I^2 \exp\{i\alpha\gamma_5\}\frac{(\not{p}+m_n)O(\not{p}+m_n)}{(p^2-m_n)^2}\exp\{i\alpha\gamma_5\}$$

- For β = -1, one needs to use even number of gamma matrices, and evaluate both two- and three-point function.
- *Assuming* that ground state (i.e. the neutron) contributes dominantly to $\Pi(M^2)$, after some calculations, we derive the sum rule result for this channel:

$$d_n = -\theta\mu_n\left[\frac{3}{4\pi^2}\frac{m_*}{\chi\langle\bar{q}q\rangle}\left(\log(M^2/\Lambda_{IR}^2) - \frac{Q_d}{Q_u}\right) - \frac{2m_*}{m_n}\right].$$

- The results, β = -1 and β = +1, are A. having the same sign, B. consistent, C. predict EDM at O(10^{-16} e cm θ) level, D. Same sign as the chiral estimate answer, a little smaller but comparable value.

Bonus: magnetic moment estimate using β = +1

- Original calculation by Ioffe and Smilga (1985) was using β = -1 for the MDM. Let us repeat the same calculation for β = +1. Do we get sensible answers? We do!
- Saturating the OPE by the leading order terms for 2- and 3-point functions, neglecting higher resonances etc (most naïve approach), we get:

$$\mu_n = \frac{2}{m_n} \times \left(\frac{4}{3}e_d - \frac{1}{3}e_u\right); \quad \frac{\mu_n}{\mu_p} = -\frac{2}{3}.$$

- Calculating the NLO terms we get improved result:

$$\frac{\mu_n}{e/(2m_n)} = -\frac{8}{3} \times \frac{1 + \frac{b}{24M^4}}{1 + \frac{b}{4M^4}} \simeq -2.05 \text{ at } M = m_n.$$

where $$b \equiv (2\pi)^2 \langle (\alpha_s/\pi) G^a_{\mu\nu} G^a_{\mu\nu} \rangle \sim 1.2\,\mathrm{GeV}^4.$$

[Actual answer is -1.91] *Not worse than Ioffe and Smilga result.*

Questions/suggestions/lessons for lattice QCD

- *Tensor charges of light quarks*. ✓ Are we sure about the strange-quark EDM results?
- *Tensor charges of heavy quarks*. (I.e. charm, bottom EDMs etc.) Amounts to calculation of $<N|\, O_{[\mu\nu]}\, |N>$ where $O_{[\mu\nu]} = d^{abc}(G^a G^b G^c)_{\mu\nu}$. This should be far easier than theta, Weinberg operator, Color EDM.
- I would like to see *different Lorentz structures of Π(x)* to be explored. Some of them are far less susceptible to the chiral rotation of the source.
- I understand that calculations of neutron EDM with theta term being distributed between GGdual and gamma-5 mass are difficult. Yet I would like to see the *vanishing of θ-dependence for any observable in the $m_q \to 0$ limit* (mass, EDMs etc).
- Please use the hadron *interpolating currents* such that they do not break axial U(1) symmetry. Failure to do so will guarantee to have unphysical pieces of theta-dependence in all observables.