Introduction to Soft-Collinear Effective Theory

(Matthias Neubert, 17-21 August 2026)

Outline:

- Motivation
- Construction of Soft-Collinear Effective Theory
- Properties of the SCET Lagrangian (Reading Assignm.)
- Matching of the 2-Jet Current
- The Sudakov Form Factor in SCET (Reading Assignm.)
- RG Evolution Equations
- Decoupling of Ultra-Soft Gluons & Factorization
- Sample Application (Reading Assignm.)

Suggested literature:

Review: T. Becher, A. Broggio, A. Ferroglia, 1410.1892 (book!)

Lectures: M. Neubert, hep-ph/0512222 (TASI 2004)

T. Becher, 1803.04310 (Les Houches 2017)

Papers: C. Bauer et al., hep-ph/0011336, 0109045, 0202088

M. Beneke et al., hep-ph/0206152

Tutorials, exercises & discussions

I. Motivation

Multi-scale problems are abundant in physics but difficult to deal with

EFT: modern tool to achieve scale separation in QFT

→ reduce multi-scale problems to a sequence of single-scale problems

→ RGEs allow for systematic resummation of large logarithms of scale ratios; particularly important in QCD, where running of $\alpha_s(\mu)$ is significant and $\alpha_s \ln^n(Q_1/Q_2)$ can be large if $Q_1 \gg Q_2$ $(n=1,2)$

Scale separation is the basis of factorization theorems

→ crucial for separation of short-distance from long-distance physics in QCD ($\sigma_{LHC} = \sigma_{partons} \otimes$ PDFs)

→ in strongly interacting theories, long-distance dofs can be different from short-distance dofs, so scale separation becomes a necessity (quarks & gluons vs. hadrons, electrons vs. Cooper pairs, etc.)

Why should you learn about SCET?

- SCET is the EFT for high-energy processes involving light particles
- very powerful, but one of the most complicated EFTs ever developed
- originally developed to understand factorization theorems in B physics, e.g.:

  $B \to X_s \gamma$, $B \to X_u \ell \bar{\nu}$ (inclusive decays to light part.)

  $B \to D\pi$, $B \to \pi\pi$ (exclusive nonleptonic decays)
- later, SCET found many applications outside flavor physics, in particular: collider physics, dense QCD matter, hadron physics, scattering amplitudes, forward scattering (small-x physics), generalized PDFs, collinear factorization breaking, DM phenomenology, SYM theory, BSM physics, ...

$\hookrightarrow$ series of annual workshops since 2003

Important comment:

The construction of an EFT becomes significantly more complicated in cases where the large scale M remains as a parameter in the EFT, characterizing the large energies of light particles. This is precisely what happens in SCET.

In this case, as we will see, the operators in the effective Lagrangian are non-local on large scales. These non-localities are along light-like directions. They are a characteristic feature of SCET.

* conventional EFTs : based on (Euclidean) operator product expansion (OPE)
  ↳ integrating out heavy particles ($\Delta x \sim 1/M$)

* SCET : "Minkowskian" generalization of the OPE based on the method of regions (Δx lightlike)
  ↳ integrating out "large energies"

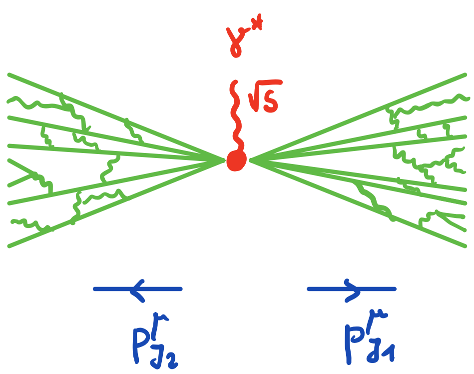
II. Construction of Soft-Collinear Effective Theory

A long-standing problem in QCD was how to systematically account for long-distance effects (including power corrections) in processes which do not admit a local OPE.

→ OPE provides rigorous framework for an expansion in powers and logarithms of the large scale for euclidean processes

→ processes involving energetic light particles pose new challenges: p^μ has some large components, but $p^2 \approx 0$

Consider $e^+e^- \to 2$ jets:



highly collimated jets of particles: large energy along jet axis, small invariant mass

$$p^\mu_{J_1} = \left(E_1,\ 0,\ 0,\ \sqrt{E_1^2 - m_{J_1}^2}\right)$$

$$p^\mu_{J_2} = \left(E_2,\ 0,\ 0,\ -\sqrt{E_2^2 - m_{J_2}^2}\right) \quad ; \quad E_i \approx \frac{\sqrt{s}}{2}\ ,\quad m_{J_i}^2 \ll s$$

↳ intrinsically Minkowskian process

↳ what is there to integrate out?

Introduce small expansion parameter:

$$\boxed{\lambda \sim \frac{m_J}{Q} \ll 1} \qquad (Q = \sqrt{s})$$

Define two light-like reference vectors along jet directions:

$$n^\mu = (1,0,0,1) \quad , \quad \bar{n}^\mu = (1,0,0,-1)$$

$$n^2 = 0 \; , \quad \bar{n}^2 = 0 \; , \quad n \cdot \bar{n} = 2$$

Decompose 4-vectors in a light-cone basis spanned by n^μ, $\bar{n}^\mu$ and two perpendicular directions:

$$\boxed{p^\mu = (n \cdot p) \frac{\bar{n}^\mu}{2} + (\bar{n} \cdot p) \frac{n^\mu}{2} + p_\perp^\mu}$$

$$\Rightarrow \begin{cases} n \cdot p_{J_1} = E_1 - \sqrt{E_1^2 - m_{J_1}^2} \simeq \dfrac{m_{J_1}^2}{2E_1} \simeq \dfrac{m_{J_1}^2}{\sqrt{s}} \sim \lambda^2 Q \\ \bar{n} \cdot p_{J_1} = E_1 + \sqrt{E_1^2 - m_{J_1}^2} \simeq 2E_1 \simeq \sqrt{s} = 1 \cdot Q \\ p_{J_1}^\perp = 0 \end{cases}$$

Similarly: $\quad n \cdot p_{J_2} \sim 1 \cdot Q \; , \quad \bar{n} \cdot p_{J_2} \sim \lambda^2 Q \; , \quad p_{J_2}^\perp = 0$

Individual partons inside the jets can carry momenta with the same scaling rules, but these can also have transverse components as long as $p_\perp^2 < m_{J_i}^2$. We thus find:

partons inside jet 1: $\quad (n\cdot p_i,\ \bar{n}\cdot p_i,\ p_i^{\perp}) \sim (\lambda^2, 1, \lambda)\, Q$

$\hookrightarrow$ collinear (n-collinear) particles

partons inside jet 2: $\quad (n\cdot p_i,\ \bar{n}\cdot p_i,\ p_i^{\perp}) \sim (1, \lambda^2, \lambda)\, Q$

$\hookrightarrow$ anti-collinear ($\bar{n}$-collinear) particles

$\hookrightarrow$ note: $\quad p_i^2 = (n\cdot p_i)(\bar{n}\cdot p_i) + p_{\perp,i}^2 \sim \lambda^2 Q^2 \quad$ (both cases)

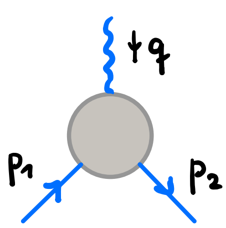
These collinear particles have virtualities much less than the hard scale $Q^2 = s$. But in virtual diagrams we can also exchange hard particles with:

$$p_i^{\mu} \sim Q, \text{ i.e. } \quad (n\cdot p_i,\ \bar{n}\cdot p_i,\ p_i^{\perp}) \sim (1, 1, 1)\, Q$$

$\hookrightarrow$ hard particles

We could try to integrate out these hard quantum fluctuations and construct an EFT built out of collinear and anti-collinear particles. However, one finds that this is not the whole story.

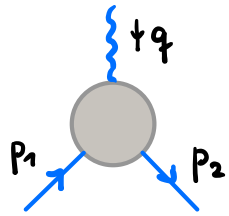
It is instructive at this point to consider a concrete example, the (off-shell) Sudakov form factor:



$|q^2| \gg |p_i^2|$
$(m_i = 0)$

One-loop calculation:

Dropping numerator factors for simplicity one finds:



$$\sim \mu^{2\epsilon} \int d^D k \, \frac{2 p_1 \cdot p_2}{(k^2 + i0)\left[(k+p_1)^2 + i0\right]\left[(k+p_2)^2 + i0\right]}$$

$$= \underbrace{\ln \frac{Q^2}{P_1^2} \ln \frac{Q^2}{P_2^2}}_{\text{Sudakov double log}} + \frac{\pi^2}{3} + \mathcal{O}\left(\frac{P_i^2}{Q^2}\right)$$

finite for $\epsilon \to 0$

with:

$$Q^2 = -q^2 - i0 = -s - i0 < 0 \quad \rightarrow \text{ non-zero imag. part}$$

$$P_i^2 = -p_i^2 - i0 > 0 \quad (\text{off-shell} \rightarrow \text{IR regulators})$$

↪ note that $Q^2 = -(p_2 - p_1)^2 \simeq 2 p_1 \cdot p_2$ up to $\mathcal{O}(\lambda^2)$ terms

In the CMS:

$$(n\cdot q, \bar{n}\cdot q, q_\perp) = (1, 1, 0)\sqrt{s} \quad \text{hard}$$

$$(n\cdot p_1, \bar{n}\cdot p_1, p_1^\perp) \sim (\lambda^2, 1, 0)\sqrt{s} \quad \text{collinear}$$

$$(n\cdot p_2, \bar{n}\cdot p_2, p_2^\perp) \sim (1, \lambda^2, 0)\sqrt{s} \quad \text{anti-collinear}$$

We will now decompose this result into a sum of contributions each depending on only a single scale. The hard contribution (depending on Q^2) arises from hard quantum fluctuations. Contributions from lower scales will later be associated with modes in the EFT.

Method of regions: (Smirnov 1990s)

Systematic method for performing "Taylor expansions" of Feynman graphs F_Γ by decomposing them into "regions":

$$F_\Gamma \sim \sum_\gamma M_\gamma \, F_\Gamma$$

sum over sets γ of subgraphs

Taylor expansion in variables that are small in γ

Practical procedure:

1. determine large and small scales in the graph
2. introduce factorization scales μ_i and divide the loop integrals into regions in which each loop momentum is related to one of these scales
3. perform a Taylor expansion in parameters that are small in a given region
4. after the expansion, ignore the factorization scales and integrate over the entire loop integration domain in each region

For step 4 to be valid, it is essential that F_Γ is defined in <u>dimensional regularization</u>!

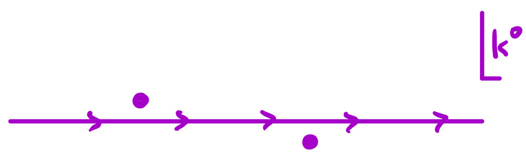
↳ follows from vanishing of scaleless integrals:

$$\boxed{\int d^D k \, \frac{1}{(k^2)^a} \equiv 0}$$

Comment:

A more rigorous treatment uses the notion of "singular surfaces":

- loop integrals (in dim.reg.) are contour integrals in the complex momentum plane
- non-zero contributions result from pinch singularities, where contours cannot be deformed so as to avoid the poles:



- this happens when some propagators go on shell (happens only when some loop momenta match external scales)
- other, off-shell propagators can be expanded about the singular surfaces (no need for hard cutoffs)
- graph F_Γ is then equal to the sum of all singular subgraphs

(see e.g. Beneke, Smirnov 1997)