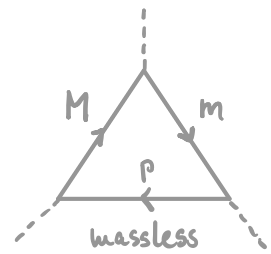
Problem Set 1 (16 August 2026)

Exercise 1:

Perform the region analysis for the integral:

$$I = \mu^{2\epsilon} \int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 (p^2 - M^2)(p^2 - m^2)} \quad ; \quad M^2 \gg m^2$$



(zero ext. momenta)

Consider the five regions:

i) $|p^2| \gg M^2$ ii) $|p^2| \sim M^2$ iii) $M^2 \gg |p^2| \gg m^2$

iv) $|p^2| \sim m^2$ v) $|p^2| \ll m^2$

In each case, Taylor expand the integrand an keep the leading term only.

Useful integrals:

$$\int \frac{d^D p}{(2\pi)^D} \frac{1}{(p^2)^a} = 0$$

$$\int \frac{d^D p}{(2\pi)^D} \frac{1}{p^2 - M^2} \frac{1}{(p^2)^a} = - \frac{i}{(4\pi)^{D/2}} \Gamma\left(1 - \frac{D}{2}\right) (M^2)^{\frac{D}{2} - 1 - a}$$

Exercise 2:

Evaluate the collinear contribution

$$I_c = i\,\pi^{-D/2}\,\mu^{2\epsilon}\int d^Dk\,\frac{2\,p_{1-}\cdot p_{2+}}{(k^2+i0)\left[(k+p_1)^2+i0\right]\left[2k\cdot p_{2+}+i0\right]}$$

to the Sudakov form factor.

Hint: Combine the first two propagators using the Feynman parameterization:

$$\frac{1}{A_1^a\,A_2^b} = \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}\int_0^1 dx\,\frac{x^{a-1}\,(1-x)^{b-1}}{(xA_1+(1-x)A_2)^{a+b}}$$

Then use the parametrization:

$$\frac{1}{A^a\,B^b} = \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}\int_0^\infty d\lambda\,\frac{\lambda^{b-1}}{(A+\lambda B)^{a+b}}$$

where B is the third propagator in I_c.

Exercise 3 :

Evaluate the integral

$$I = \int_1^\infty dx \, \frac{x^{-\epsilon}}{x + \mu x^3} \quad ; \qquad 0 < \mu \ll 1 \qquad (\epsilon > -2)$$

using the method of regions, dropping terms of $\mathcal{O}(\mu^2)$.

Hint: Rewrite the integral in the form:

$$I = \int_{-\infty}^\infty dx \, \frac{x^{-\epsilon}}{x + \mu x^3} \, \theta(x - x_0) \quad ; \quad x_0 = 1$$

Which regions give leading contributions to I ?