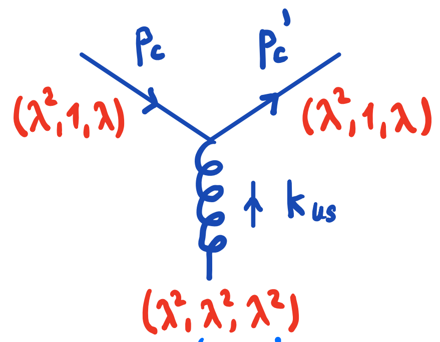
Coupling to ultra-soft gluons:

Besides the collinear Lagrangian, the leading-order SCET Lagrangian contains interactions of collinear fields with the component $n \cdot A_{us}$ of the ultra-soft gluon field:

$$\mathcal{L}_{c+us} = \bar{\xi}_n \frac{\bar{n}}{2} g_s \, n \cdot A_{us} \, \xi_n + (\text{pure glue terms})$$

Let us look at the structure of these interactions in more detail:



power suppressed compared with collinear momenta

In the method of regions it is important that we expand the Feynman integrands consistently to leading order in λ. This means that we should expand:

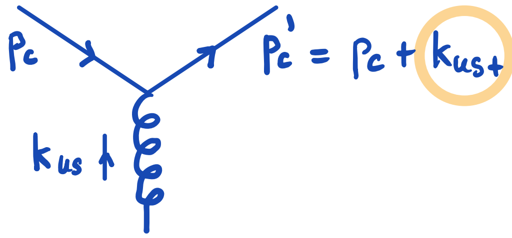
$$p_c'^{\mu} = p_c^{\mu} + k_{us}^{\mu} = (\overset{\lambda^2}{n \cdot p_c} + \overset{\lambda^2}{n \cdot k_{us}}) \frac{\bar{n}^{\mu}}{2} + (\overset{1}{\bar{n} \cdot p_c} + \overset{\lambda^2}{\bar{n} \cdot k_{us}}) \frac{n^{\mu}}{2} + \overset{\lambda}{p_{c\perp}^{\mu}} + \overset{\lambda^2}{k_{us\perp}^{\mu}}$$

$$= (n \cdot p_c + n \cdot k_{us}) \frac{\bar{n}^{\mu}}{2} + \bar{n} \cdot p_c \frac{n^{\mu}}{2} + p_{c\perp}^{\mu} + \text{higher orders}$$

must expand away!

$$\longrightarrow p_c^{\mu} + k_{us+}^{\mu} \; ; \quad k_+^{\mu} \equiv n \cdot k \, \frac{\bar{n}^{\mu}}{2}$$

This implies that 4-momentum is not conserved at the vertex:



To implement this rule at the Lagrangian level, we must perform a <u>multipole expansion</u> of the ultra-soft fields whenever they interact with collinear fields:

$$x^\mu = \underbrace{n\cdot x \frac{\bar{n}^\mu}{2}} + \underbrace{\bar{n}\cdot x \frac{n^\mu}{2}} + x_\perp^\mu$$

$$\equiv \underset{1}{x_+^\mu} + \underset{\lambda^{-2}}{x_-^\mu} + \underset{\lambda^{-1}}{x_\perp^\mu}$$

(since $x\cdot p_c \sim 1$)

$\leftarrow$ in interactions with collinear fields

$$\Rightarrow \quad \phi_{us}(x) = \phi_{us}(x_-) + \overset{\lambda^{-1}}{x_\perp}\cdot\overset{\lambda^{2}}{\partial_\perp}\, \phi_{us}(x_-)$$
$$+ \Big(\underset{1}{x_+}\cdot\underset{\lambda^2}{\partial_-} + \underset{\lambda^{-2}}{\frac{x_\perp^\mu x_\perp^\nu}{2}}\, \underset{\lambda^2}{\partial_\mu^\perp}\, \underset{\lambda^2}{\partial_\nu^\perp}\Big)\phi_{us}(x_-) + \ldots$$

$\hookrightarrow$ generates series of higher-order terms in λ

At leading order in λ, the correct form of the effective Lagrangian thus contains:

$$\boxed{\mathcal{L}_{c+us}(x) = \bar{\xi}_n(x)\,\frac{\bar{n}}{2}\, g_s\, n\cdot A_{us}(\overset{!}{x_-})\,\xi_n(x) + (\text{pure glue terms})}$$

For the vertex shown above, the action $\int d^4x\, \mathcal{L}_{c+us}(x)$ generates:

$$\int d^4x\; e^{i(p_c'\cdot x - p_c\cdot x - \overbrace{k_{us}\cdot x_-}^{\frac{1}{2}\bar{n}\cdot x\, n\cdot k_{us} = x\cdot\left(\frac{\bar{n}}{2}\, n\cdot k_{us}\right) = x\cdot k_{us+}})}$$

$$= \int d^4x\; e^{i(p_c'\cdot x - p_c\cdot x - k_{us+}\cdot x)} = (2\pi)^4\, \delta^{(4)}(p_c' - p_c - k_{us+}) \quad ✓$$

Leading-order SCET Lagrangian:

Collecting our results, and adding back the anti-collinear sector, we obtain:

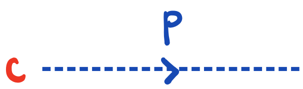
$$\begin{aligned}
\mathcal{L}_{SCET} = {} & \left.\begin{aligned} & \bar{\xi}_n \frac{\bar{\not{n}}}{2}\, i n\cdot D_c\, \xi_n(x) \\ & + (\bar{\xi}_n\, i\not{D}_c^\perp W_c)(x)\, \frac{\bar{\not{n}}}{2}\, i\int_{-\infty}^{0} dt\, (W_c^\dagger\, i\not{D}_c^\perp\, \xi_n)(x+t\bar{n}) \end{aligned}\right\} && \mathcal{L}_c \\
& + \left[\text{same with } n\leftrightarrow\bar{n},\ c\leftrightarrow\bar{c}\right] && \mathcal{L}_{\bar{c}} \\
& + \bar{q}_{us}\, i\not{D}_{us}\, q_{us}(x) && \mathcal{L}_{us} \\
& + \bar{\xi}_n(x)\, \frac{\bar{\not{n}}}{2}\, g_s\, n\cdot A_{us}(x_-)\, \xi_n(x) && \mathcal{L}_{c+us} \\
& + \bar{\xi}_{\bar{n}}(x)\, \frac{\not{n}}{2}\, g_s\, \bar{n}\cdot A_{us}(x_+)\, \xi_{\bar{n}}(x) && \mathcal{L}_{\bar{c}+us} \\
& + (\text{pure glue terms}) && \text{same structure as above}
\end{aligned}$$

Note the important fact that ultra-soft quarks do not interact with collinear fields at leading order!

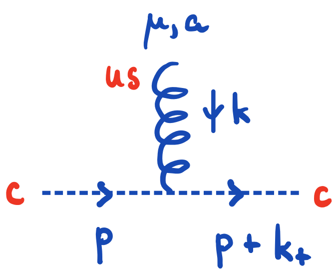
In principle, it is possible to go to higher orders in the expansion in λ ($\rightarrow$ a topic of intensive current research), but we will focus on the leading terms in this course.

Feynman rules:

$$\frac{\not{n}\not{\bar{n}}}{4}\,\not{p}\,\frac{\not{\bar{n}}\not{n}}{4} = \frac{\not{n}}{2}\,\bar{n}\cdot p$$

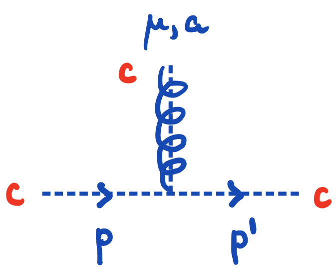


$$= \frac{\not{n}}{2}\,\frac{i\,\bar{n}\cdot p}{n\cdot p\;\bar{n}\cdot p + p_\perp^2 + i0} = P_n\,\frac{i\not{p}}{p^2+i0}\,\bar{P}_n$$

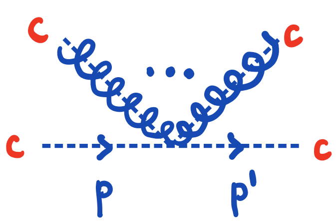


$$= i g_s t^a\, n^\mu\, \frac{\not{\bar{n}}}{2}$$

(momentum not conserved)



$$= i g_s t^a \left(n^\mu + \frac{\gamma_\perp^\mu\,\not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}'_\perp\,\gamma_\perp^\mu}{\bar{n}\cdot p'} - \bar{n}^\mu\,\frac{\not{p}'_\perp\,\not{p}_\perp}{\bar{n}\cdot p'\;\bar{n}\cdot p} \right) \frac{\not{\bar{n}}}{2}$$



= complicated

(any number of collinear gluons due to W_c)

plus pure glue and ghost vertices

please study part IV. Properties of the SCET Lagrangian as a reading assignment to learn more about SCET

V. Matching of the 2-Jet Current

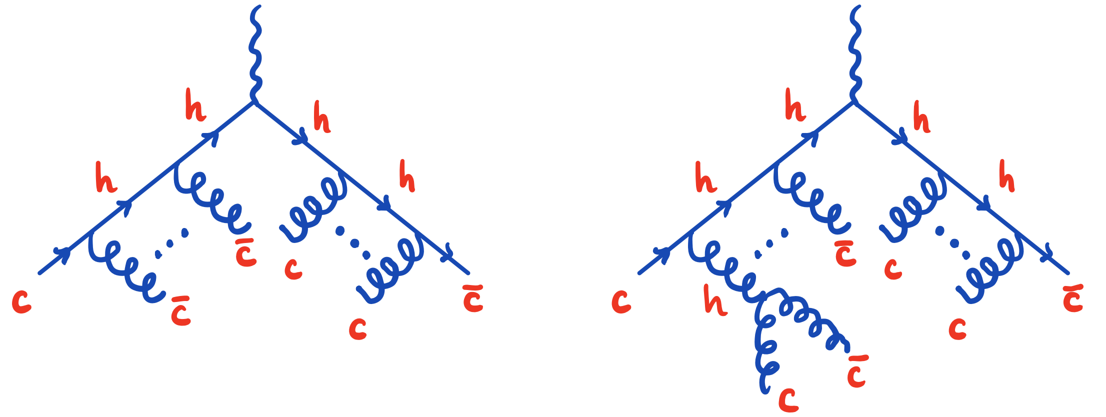
As an important application of our formalism, we now reconsider the process $e^+e^- \to \gamma^* \to 2$ jets. What is the proper representation of the vector current $\bar\psi\gamma^\mu\psi$ in SCET? The naive guess

$$\bar\psi\gamma^\mu\psi \;\longrightarrow\; \underset{\substack{\uparrow\\ \text{anti-collinear}\\ \text{quark}}}{\bar\xi_{\bar n}}\;\gamma_\perp^\mu\;\underset{\substack{\uparrow\\ \text{collinear}\\ \text{quark}}}{\xi_n}$$

is not gauge invariant, because ξ_n and $\xi_{\bar n}$ transform under <u>different</u> sets of residual gauge transformations (see reading assignment). A gauge-invariant current operator is:

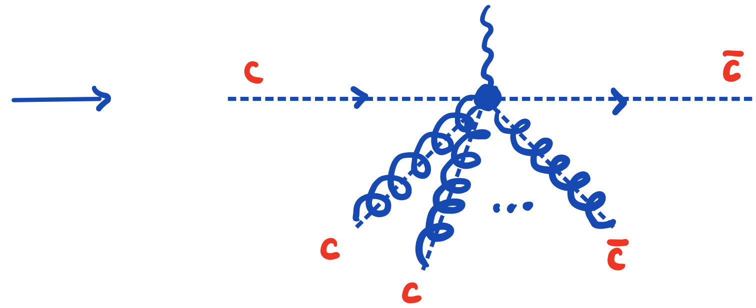
$$(\bar\xi_{\bar n} W_{\bar c})\,\gamma_\perp^\mu\,(W_c^\dagger \xi_n)$$

The (anti-) collinear Wilson lines are required by gauge invariance, but what do they represent physically? In fact, they account for an infinite set of QCD graphs of the type:

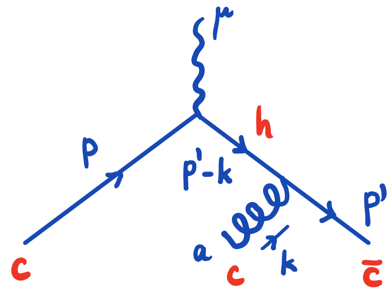


etc.

Despite the many hard propagators, these diagrams contain leading-power pieces in λ, which in the EFT are accounted for by the Wilson lines.



To see how this works, consider the attachment of a single collinear gluon to an anti-collinear quark:



$p' \sim (1, \lambda^2, \lambda)$

$k \sim (\lambda^2, 1, \lambda)$

$\epsilon \sim (\lambda^2, 1, \lambda)$

$$= \bar{u}(p')\, i g_s\, \not\epsilon(k)\, t_a \frac{i(\not p' - \not k)}{(p'-k)^2}\, \gamma^\mu u(p)$$

$$\underset{\lambda^2}{p'^2} + \underset{\lambda^2}{k^2} - \big(\underset{\lambda^0}{n\cdot p'}\, \underset{\lambda^0}{\bar{n}\cdot k} + \underset{\lambda^2}{\bar{n}\cdot p'}\, \underset{\lambda^2}{n\cdot k} + 2\underset{\lambda}{p'_\perp}\cdot \underset{\lambda}{k_\perp}\big)$$

$$= \bar{u}(p')\, i g_s \left(\underset{\lambda^0}{\bar{n}\cdot\epsilon}\, \frac{\not n}{2} + \underset{\lambda^2}{n\cdot\epsilon}\, \frac{\not{\bar{n}}}{2} + \underset{\lambda}{\not\epsilon_\perp} \right) t_a \frac{i}{(p'-k)^2}$$

$$\times \left(\underset{\lambda^2\ \lambda^0}{\bar{n}\cdot(p'-k)}\, \frac{\not n}{2} + \underset{\lambda^0\ \lambda^2}{n\cdot(p'-k)}\, \frac{\not{\bar{n}}}{2} + \underset{\lambda}{(\not p' - \not k)_\perp} \right) \gamma^\mu u(p)$$

$$= \bar{u}(p')\, \underbrace{\frac{\not n \not{\bar{n}}}{4}}_{P_n = \overline{P}_{\bar{n}}} (-g_s t_a) \frac{\bar{n}\cdot\epsilon\ \cancel{n\cdot p'}}{-\cancel{n\cdot p'}\ \bar{n}\cdot k}\, \gamma^\mu u(p) + \text{power-suppressed terms}$$

γ^μ: $\not n \frac{\bar{n}^\mu}{2} + \cancel{\not{\bar{n}} \frac{n^\mu}{2}} + \gamma_\perp^\mu$; 0

power-suppressed terms: drop

drop, since $\not{n}\, u(p) \approx 0$ is suppressed

$$= \underbrace{\bar{u}(p')\,\overline{P_{\bar{n}}}}_{\text{from } \bar{\xi}_{\bar{n}} = \bar{\psi}_{\bar{c}}\,\overline{P_{\bar{n}}}} \left(\frac{g_s t_a\, \bar{n}\cdot\epsilon(k)}{\bar{n}\cdot k} \right) \gamma_\perp^\mu \underbrace{P_n\, u_n(p)}_{\text{from } \xi_n = P_n \psi_c} + \dots$$

The highlighted factor is nothing but the one-gluon matrix element of the collinear Wilson line $W_c^\dagger$:

$$\langle 0 |\, W_c^\dagger(0)\, | \epsilon, k \rangle = \langle 0 |\, \underset{\substack{\uparrow \\ \text{opposite ordering than for } W_c}}{\bar{P}} \exp\left(-i g_s \int_{-\infty}^{0} dt\ \bar{n}\cdot A_c(t\bar{n}) \right) | \epsilon, k \rangle$$

$$= -i g_s t_a\, \bar{n}\cdot\epsilon(k) \int_{-\infty}^{0} dt\ e^{-it\,\bar{n}\cdot k} = \frac{g_s t_a\, \bar{n}\cdot\epsilon(k)}{\bar{n}\cdot k} \quad \checkmark$$

Hard matching corrections beyond tree level:

The 2-jet current carries a hard momentum transfer $q^2 = -Q^2$. Unlike for the SCET Lagrangian, we thus expect that there should be a non-trivial Wilson coefficient $C_V(Q^2)$ accounting for the effects of hard gluons, which have been integrated out, i.e.:

$$\boxed{\bar{\psi}\gamma^\mu\psi(0) \;\longrightarrow\; \underset{\substack{\uparrow \\ \text{hard quantum fluctuations}}}{C_V(Q^2)}\, (\bar{\xi}_{\bar{n}} W_{\bar{c}})(0)\, \gamma_\perp^\mu\, (W_c^\dagger \xi_n)(0)}$$