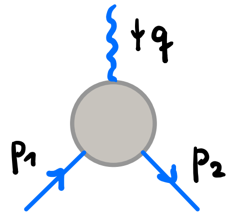
VI. The Sudakov Form Factor in SCET

We now return to the case of the off-shell Sudakov form factor in QCD,



$|q^2| \gg |p_i^2|$

$(m_i = 0)$

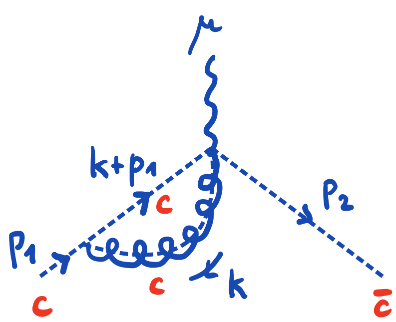
this time performing the calculation in SCET (and not ignoring the numerator terms). We thus evaluate the quark matrix element

$$C_V(Q^2) \; \langle q(p_2) | \, (\bar{\xi}_{\bar{n}} W_{\bar{c}})(0) \, \gamma_\perp^\mu \, (W_c^\dagger \xi_n)(0) \, | q(p_1) \rangle$$

at one-loop order, where $C_V(Q^2) = 1 + \mathcal{O}(\alpha_s)$ is the hard matching coefficient.

Collinear contribution:

The SCET Feynman rules allow for a single one-loop diagram:



(we work in Feynman gauge $\xi = 1$)

We find:

$$D_c = \int \frac{d^Dk}{(2\pi)^D} \, \bar{u}_{\bar{n}}(p_2) \, \gamma_\perp^\mu \, \frac{\not{n}}{2} \, \frac{i\,\bar{n}\cdot(k+p_1)}{(k+p_1)^2 + i0}$$

$$\times \, i g_s t_a \left(n^\alpha + \frac{\gamma_\perp^\alpha \not{p}_{1\perp}}{\bar{n}\cdot p_1} + \frac{(\not{k}_\perp + \not{p}_{1\perp})\gamma_\perp^\alpha}{\bar{n}\cdot(k+p_1)} - \bar{n}^\alpha \frac{(\not{k}_\perp + \not{p}_{1\perp})\not{p}_{1\perp}}{\bar{n}\cdot(k+p_1)\;\bar{n}\cdot p_1} \right) \frac{\not{\bar{n}}}{2} u_n(p_1)$$

$$\times \underbrace{(-g_s t_b) \frac{\bar{n}^\beta}{\bar{n}\cdot k}}_{\langle k | W_c^\dagger | 0 \rangle} \frac{(-i g_{\alpha\beta})}{k^2 + i0} \, \delta_{ab}$$

(cf. p.7, Lecture 4)

$$= -i C_F g_s^2 \, \bar{u}_{\bar{n}}(p_2) \underbrace{\gamma_\perp^\mu \frac{\not{n}\not{\bar{n}}}{4} u_n(p_1)}_{= \gamma_\perp^\mu} \underbrace{n\cdot\bar{n}}_{2}$$

$$\times \int \frac{d^Dk}{(2\pi)^D} \frac{\bar{n}\cdot(k+p_1)}{(k^2+i0)\,[(k+p_1)^2 + i0]\;\bar{n}\cdot k}$$

If we multiply numerator and denominator by $n\cdot p_2$ we get:

$$\frac{\bar{n}\cdot(k+p_1)}{\bar{n}\cdot k} \longrightarrow \frac{\bar{n}\cdot(k+p_1)\; n\cdot p_2}{\bar{n}\cdot k \; n\cdot p_2} = \frac{2k\cdot p_{2+} + 2p_{1-}\cdot p_{2+}}{2k\cdot p_{2+}}$$

Apart from the factor $2k\cdot p_{2+}$ in the numerator (which we had previously ignored), the loop integral coincides with the integral I_c for the collinear region on page 3 of Lecture 2. Evaluating the integral leads to:

$$D_c = \bar{u}_{\bar{n}}(p_2)\, \gamma_\perp^\mu\, u_n(p_1)\, \frac{C_F \alpha_s}{4\pi} \times \left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln\frac{\mu^2}{p_1^2} + \frac{2}{\epsilon} + \ln^2\frac{\mu^2}{p_1^2} + 2\ln\frac{\mu^2}{p_1^2} + 4 - \frac{\pi^2}{6} \right]$$

We work in the $\overline{MS}$ scheme, where $\mu^{2\epsilon} \longrightarrow \mu^{2\epsilon}(4\pi)^{-\epsilon} e^{\epsilon \gamma_E}$.

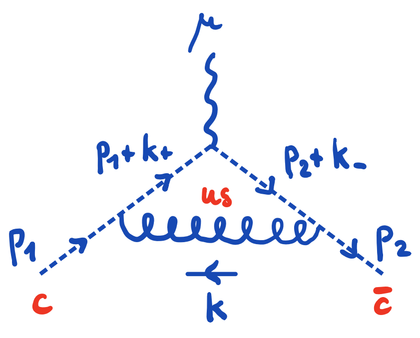
Anti-collinear contribution:

An analogous calculation leads to:

$$D_{\bar{c}} = \bar{u}_{\bar{n}}(p_2)\, \gamma_\perp^\mu\, u_n(p_1)\, \frac{C_F \alpha_s}{4\pi} \times \left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln\frac{\mu^2}{p_2^2} + \frac{2}{\epsilon} + \ln^2\frac{\mu^2}{p_2^2} + 2\ln\frac{\mu^2}{p_2^2} + 4 - \frac{\pi^2}{6} \right]$$

Ultra-soft contribution:

The SCET Feynman rules allow for a single one-loop diagram:



(we work in Feynman gauge $\xi = 1$)

Recall that 4-momentum is not conserved at these vertices.

We find:

$$D_{us} = \int \frac{d^Dk}{(2\pi)^D}\; \bar{u}_{\bar{n}}(p_2)\, i g_s t_a \bar{n}^\alpha \frac{\not{n}}{2}\frac{\not{\bar{n}}}{2} \frac{i\, n\cdot(\overbrace{k_-}^{=0}+p_2)}{(k_-+p_2)^2+i0}\, \gamma_\perp^\mu$$

$$\times \frac{\not{n}}{2} \frac{i\,\overbrace{\bar{n}\cdot(k_+}^{=0}+p_1)}{(k_++p_1)^2+i0}\, i g_s t_b\, n^\beta \frac{\not{\bar{n}}}{2}\, u_n(p_1)\, \frac{(-i g_{\alpha\beta})}{k^2+i0}\,\delta_{ab}$$

$$= -i C_F g_s^2\; \bar{u}_{\bar{n}}(p_2)\, \overbrace{\frac{\not{n}\not{\bar{n}}}{4}\gamma_\perp^\mu \frac{\not{n}\not{\bar{n}}}{4}}^{\gamma_\perp^\mu}\, u_n(p_1)\, \overbrace{n\cdot\bar{n}}^{2}$$

$$\times \int \frac{d^Dk}{(2\pi)^D} \frac{\bar{n}\cdot p_1\; n\cdot p_2}{(k^2+i0)(2k_+\cdot p_1 + p_1^2+i0)(2k_-\cdot p_2 + p_2^2 + i0)}$$

$$\uparrow$$

$$\frac{2\, p_{1-}\cdot p_{2+}}{(k^2+i0)(2k\cdot p_{1-} + p_1^2+i0)(2k\cdot p_{2+}+p_2^2+i0)}$$

This expression coincides with the integral I_{us} for the ultra-soft region on page 6 of Lecture 2. Evaluating the integral one finds:

$$D_{us} = \bar{u}_{\bar{n}}(p_2)\,\gamma_\perp^\mu\, u_n(p_1)\,\frac{C_F\alpha_s}{4\pi}$$

$$\times\left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\frac{\mu^2Q^2}{p_1^2p_2^2} - \ln^2\frac{\mu^2Q^2}{p_1^2p_2^2} - \frac{\pi^2}{2}\right]$$

Wave-function renormalization:

$$Z_q^{1/2} \, Z_{\bar q}^{1/2} \; \bar u_{\bar n}(p_2) \, \gamma_\perp^\mu \, u_n(p_1)$$

$\uparrow$ same as in QCD

In Feynman gauge, the WFR factor for an off-shell quark with momentum p^μ is:

$$Z_q = 1 + \frac{C_F \alpha_s}{4\pi} \left(-\frac{1}{\epsilon} - \ln \frac{\mu^2}{-p^2 - i0} + 1 \right)$$

We thus obtain:

$$\bar u_{\bar n}(p_2) \, \gamma_\perp^\mu \, u_n(p_1) \left\{ 1 + \frac{C_F \alpha_s}{4\pi} \left[-\frac{1}{\epsilon} - \frac{1}{2} \left(\ln \frac{\mu^2}{P_1^2} + \ln \frac{\mu^2}{P_2^2} \right) + 1 \right] \right\}$$

One-loop SCET matrix element:

Adding up all pieces, we find at one-loop order:

$$\langle q(p_2) | \, (\bar\xi_{\bar n} W_{\bar c})(0) \, \gamma_\perp^\mu \, (W_c^\dagger \xi_n)(0) \, | q(p_1) \rangle$$

$$= \bar u_{\bar n}(p_2) \, \gamma_\perp^\mu \, u_n(p_1)$$

$$\times \left\{ 1 + \frac{C_F \alpha_s}{4\pi} \left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \left(\ln \frac{\mu^2}{P_1^2} + \ln \frac{\mu^2}{P_2^2} - \ln \frac{\mu^2 Q^2}{P_1^2 P_2^2} \right) + \frac{3}{\epsilon} \right. \right.$$

$$+ \ln^2 \frac{\mu^2}{P_1^2} + \ln^2 \frac{\mu^2}{P_2^2} - \ln^2 \frac{\mu^2 Q^2}{P_1^2 P_2^2}$$

$$\left. \left. + \frac{3}{2} \ln \frac{\mu^2}{P_1^2} + \frac{3}{2} \ln \frac{\mu^2}{P_2^2} + 9 - \frac{5\pi^2}{6} \right] \right\}$$