

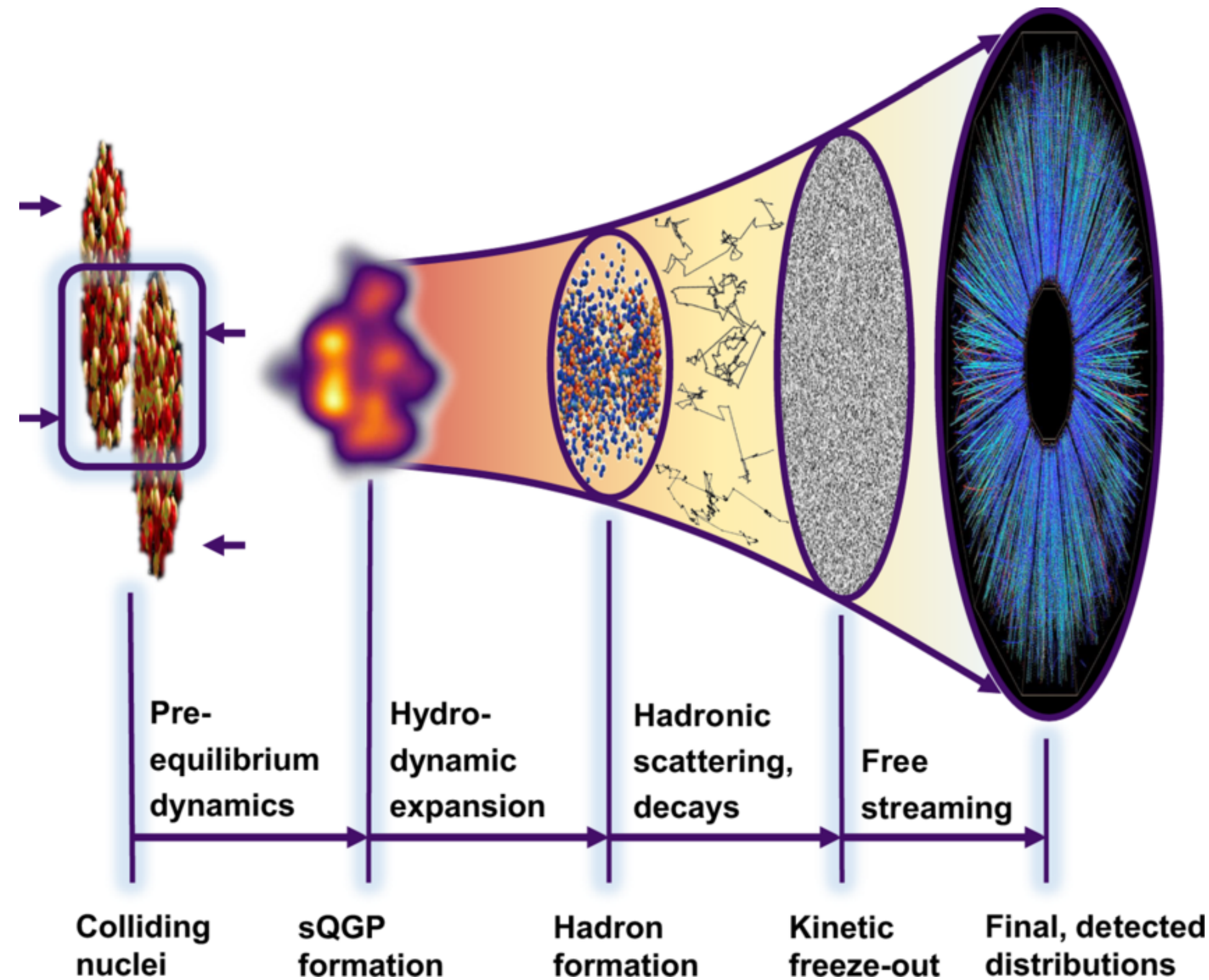
Rotational Symmetry Restoration in MC Glauber Models and Impacts on Final-State Bulk Observables

Speaker: Yanru Bao[暴彦汝]

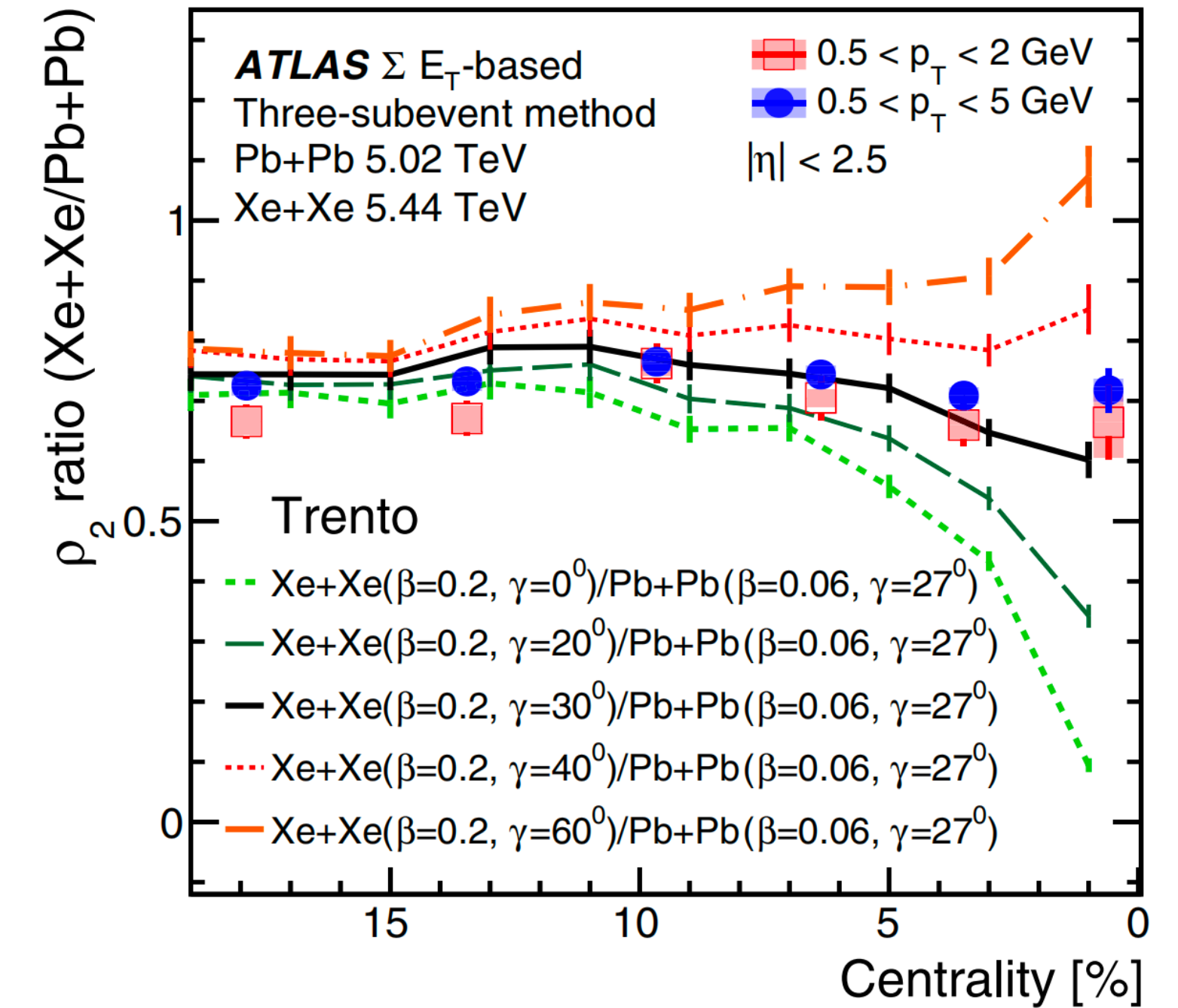
Collaborator: Weiyao Ke[柯伟尧] School: Central China Normal University

Spicy Gluons [胶麻] Workshop, 10/17-10/20, 2026, Wuhan, Hubei





- Traditional aim of HIC research: investigating QGP properties through model-to-data comparisons.

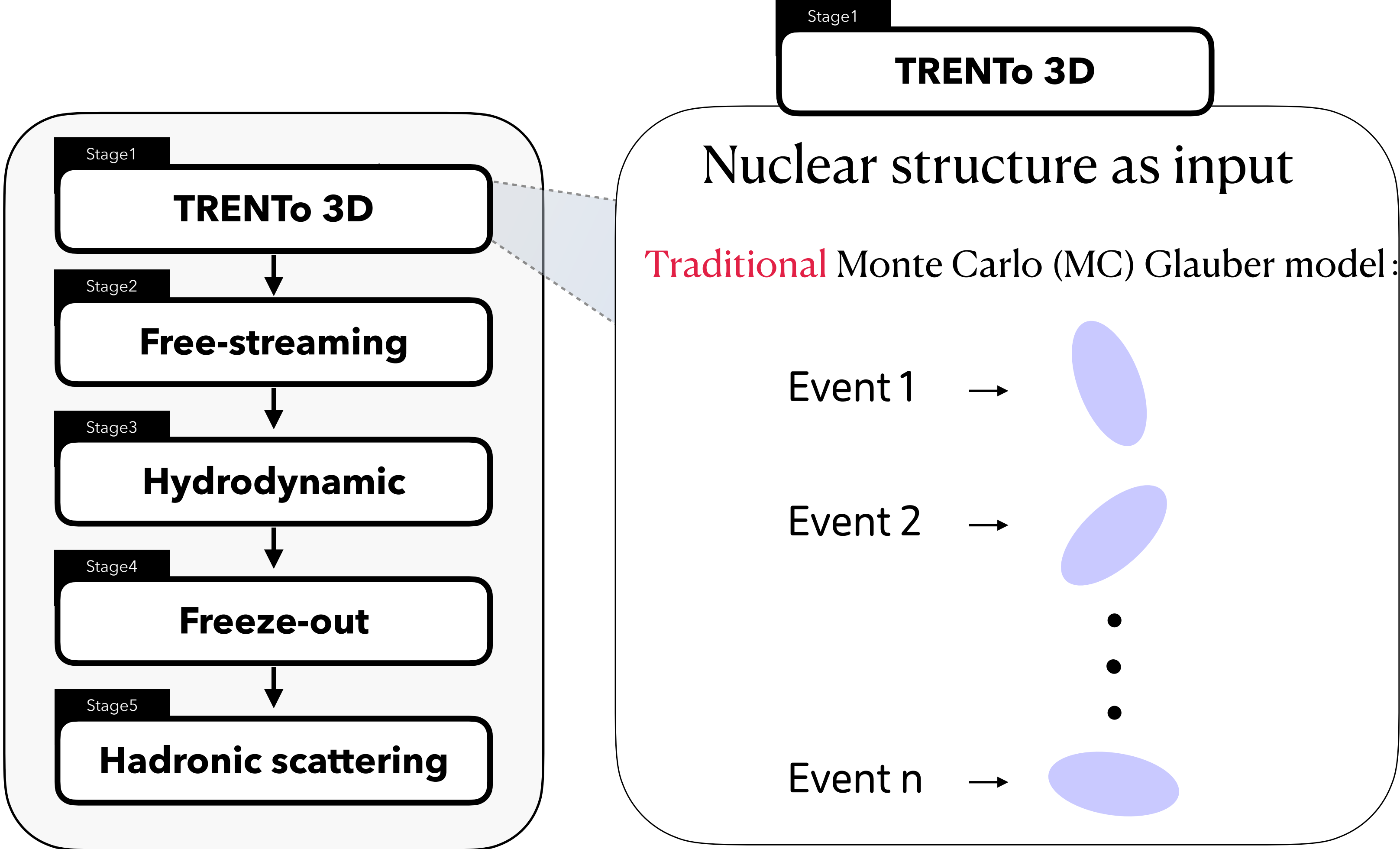
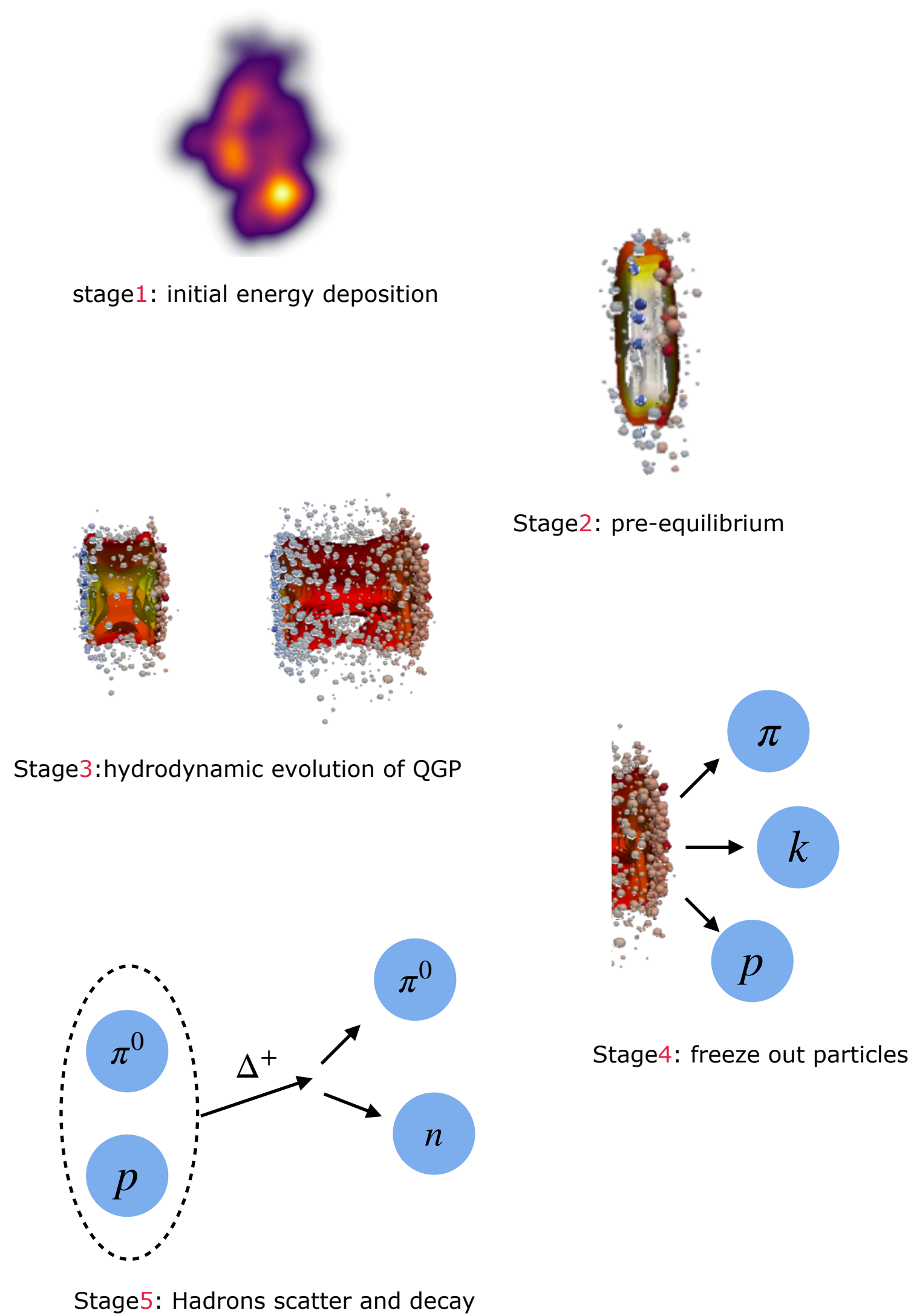


ATLAS Collaboration, Phys. Rev. C 107, 054910 (2023)

- Final-state observables are sensitive to nuclear structure, meaning that they are not only probes for QGP but also important tools for constraining nuclear shape parameters.
- This requires a more precise description of nuclear structure.

How is nuclear deformation incorporated into HIC simulations?

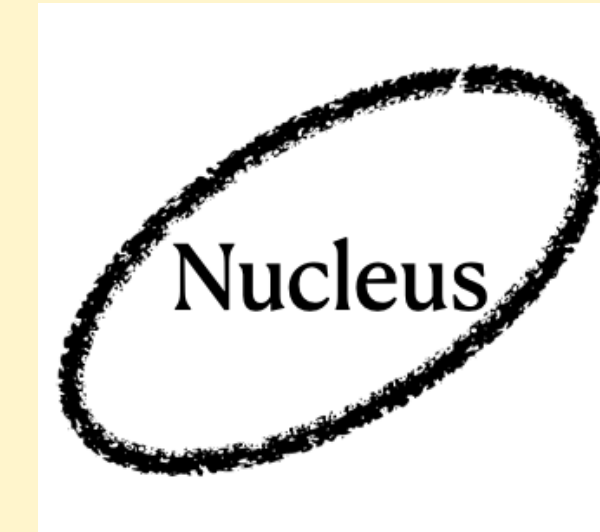
Corresponding physical process | multistage simulation



The ground state of deformed nuclei

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- For deformed nuclei with $J = 0$ (^{20}Ne , ^{238}U), describing their nuclear structure in the intrinsic frame induce that nuclear configuration breaks rotational symmetry.



Broken
rotational
symmetry

Nuclear shape in intrinsic frame

VS

- But $J = 0$ means the nuclear wave function must be rotationally invariant. Therefore, the physical ground state should be a quantum superposition of all possible configurations with rotational symmetry restored. The deformation information encoded in many-body correlations.



rotational
symmetry
restored

Nuclear shape after quantum superposition
of all possible deformed configuration.



- A single intrinsic-frame configuration cannot satisfy the rotational symmetry required for a $J = 0$ nucleus.

We therefore introduce the quantum-rotor into **an extended MC Glauber model** to restore rotational symmetry.



Step 1: Nuclear ground state in Quantum Mechanics

In quantum mechanics, the nuclear ground state is a superposition of all possible configurations.

Here, we approximate it as a superposition of all possible orientations.

$$|A\rangle = N_A^{-1} \int d\Omega |A, \Omega\rangle$$



Here Ω denotes one possible orientation of nucleus A.

Step 2: collision in Quantum Mechanics

Quantum nucleus–nucleus inelastic collision probability using $|A\rangle = N_A^{-1} \int d\Omega |A, \Omega\rangle$:

$$P_{AB,Q}^{\text{inel}}(\mathbf{b}) = \int d\Omega_A d\Omega'_A \frac{\langle \Omega_A | \Omega'_A \rangle}{|N_A|^2} \times \int d\Omega_B d\Omega'_B \frac{\langle \Omega_B | \Omega'_B \rangle}{|N_B|^2} \times \left\{ 1 - \left[1 - P_{\Omega_A \Omega'_A; \Omega_B \Omega'_B}^{NN}(\mathbf{b}) \right]^{AB} \right\}$$

For both A and B, depend on two orientation Ω and Ω' .

Step 2: collision in Quantum Mechanics

Classical nucleus–nucleus inelastic collision probability:

$$P_{AB,C}^{\text{inel}}(\mathbf{b}) = \int \frac{d\Omega_A}{4\pi} \int \frac{d\Omega_B}{4\pi} \left\{ 1 - \left[1 - P_{\Omega_A; \Omega_B}^{NN}(\mathbf{b}) \right]^{AB} \right\}$$

For both A and B, depend on **one** orientation Ω .

Quantum nucleus–nucleus inelastic collision probability using $|A\rangle = N_A^{-1} \int d\Omega |A, \Omega\rangle$:

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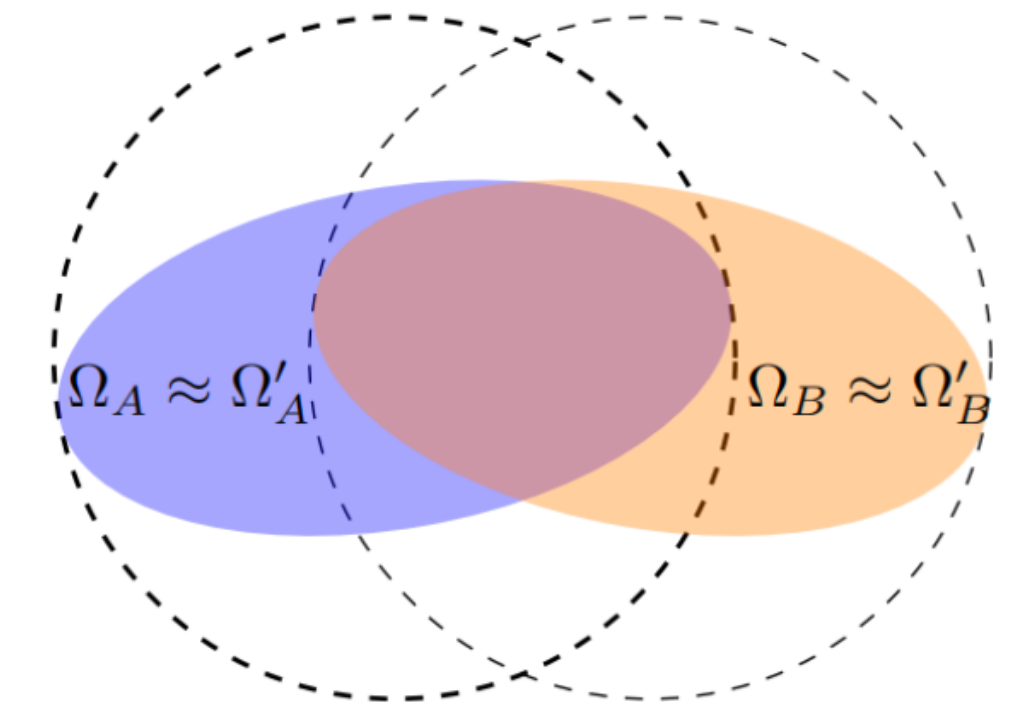
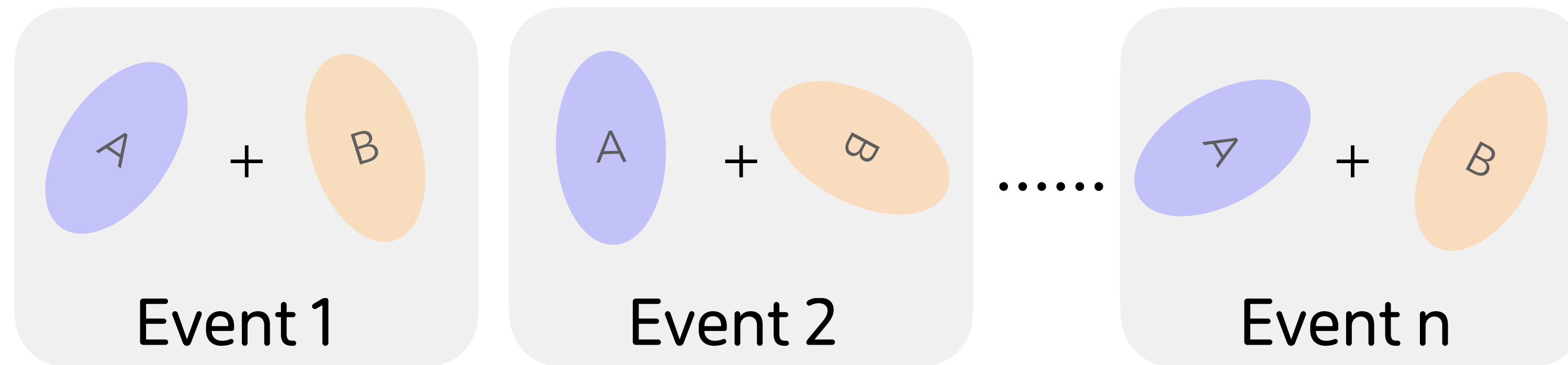
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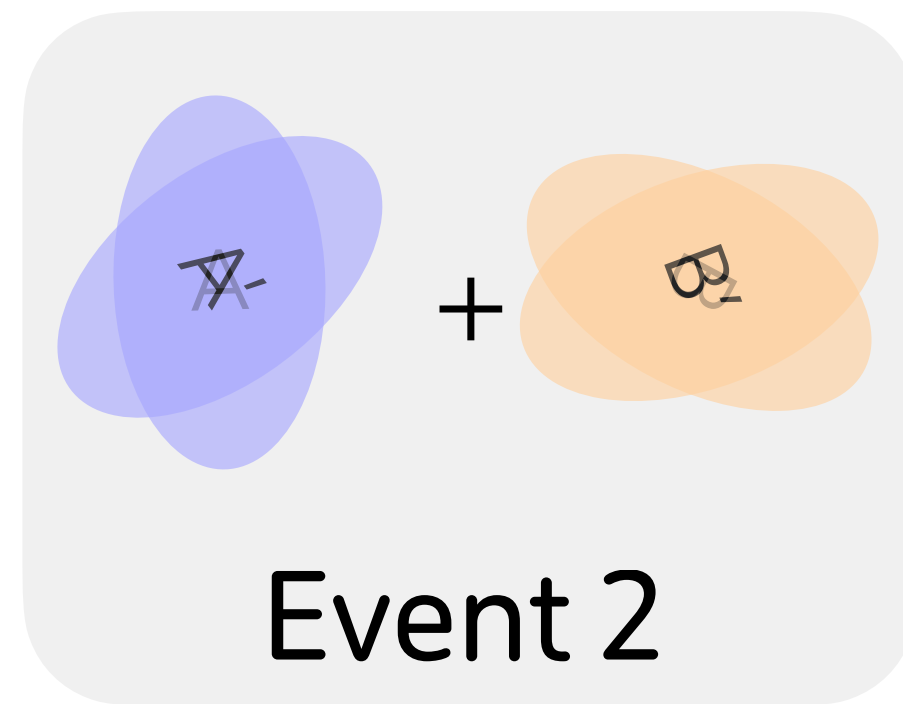
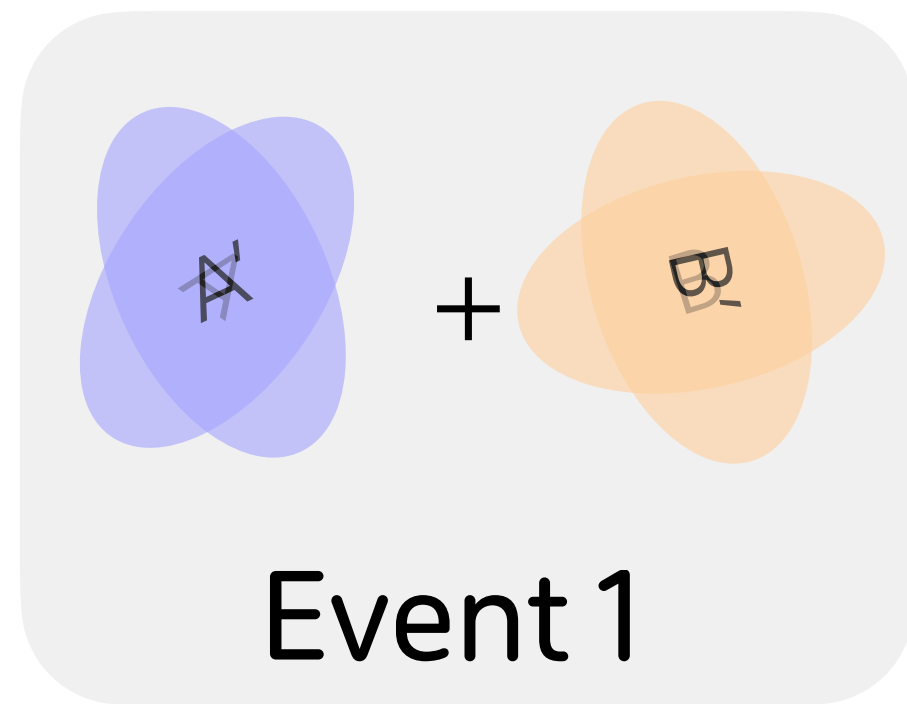


W. Ke, arXiv:2509.09549 (2025)

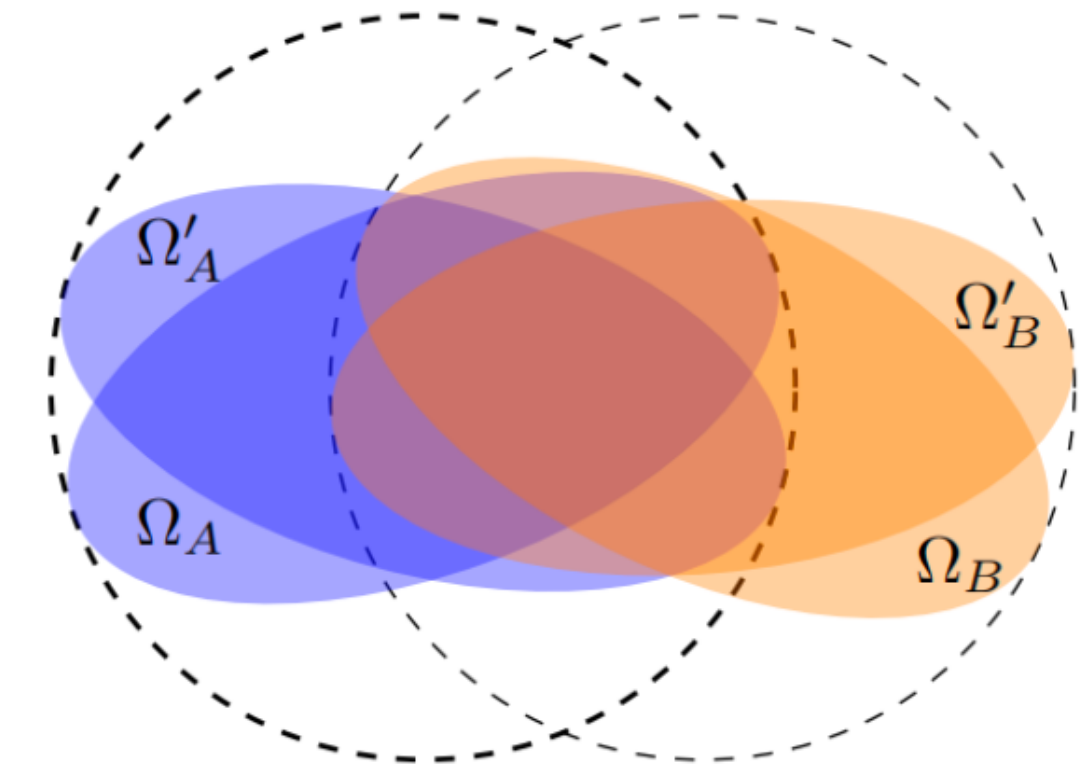
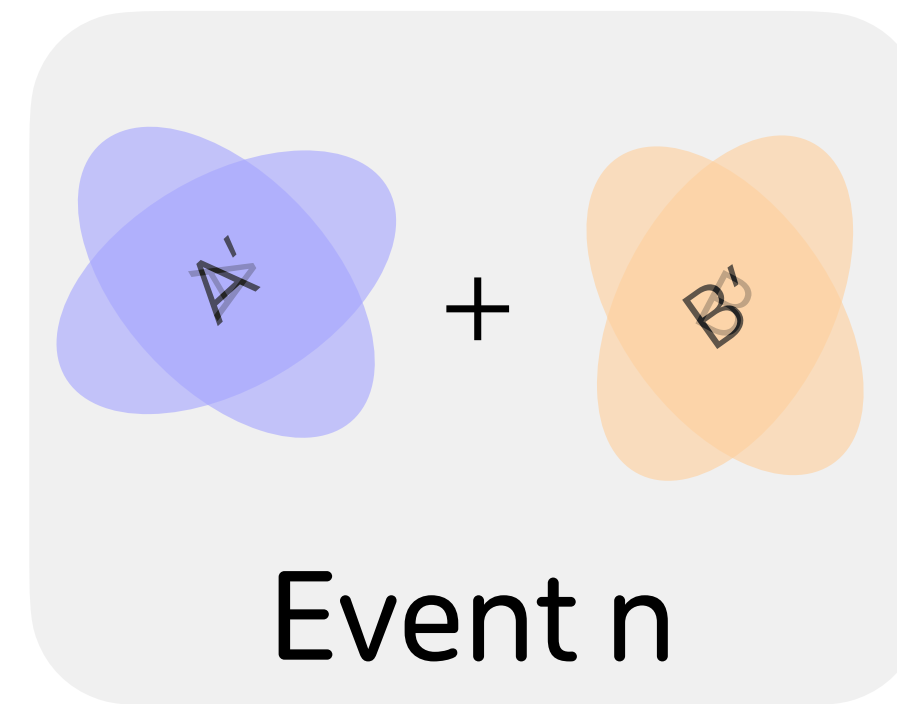
Improvement: restoring rotational symmetry in the MC Glauber model

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Step 2: collision in Quantum Mechanics



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W. Ke, arXiv:2509.09549 (2025)

Quantum nucleus–nucleus inelastic collision probability using $|A\rangle = N_A^{-1} \int d\Omega |A, \Omega\rangle$:

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For both A and B, depend on **two** orientation Ω and Ω' .

Step 2: collision in Quantum Mechanics

The overlap function $\frac{\langle \Omega_X | \Omega'_X \rangle}{|N_X|^2}$ serves as the weight for each orientation pair $\langle \Omega_X | \Omega'_X \rangle$.

Quantum nucleus–nucleus inelastic collision probability using $|A\rangle = N_A^{-1} \int d\Omega |A, \Omega\rangle$:

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Step 2: collision in Quantum Mechanics

The overlap function $\frac{\langle \Omega_X | \Omega'_X \rangle}{|N_X|^2}$ serves as the weight for each orientation pair $\langle \Omega_X | \Omega'_X \rangle$.

The vertical axis shows another expression for the unnormalized overlap function:

$$\langle X, \hat{z} | e^{-i\theta \hat{J}_y} | X, \hat{z} \rangle = \langle \Omega_X | \Omega'_X \rangle$$

- ① As shown in the figure, the overlap depends on the nuclear mass number and deformation.
- ② Our results show reasonably good agreement with theoretical calculations.

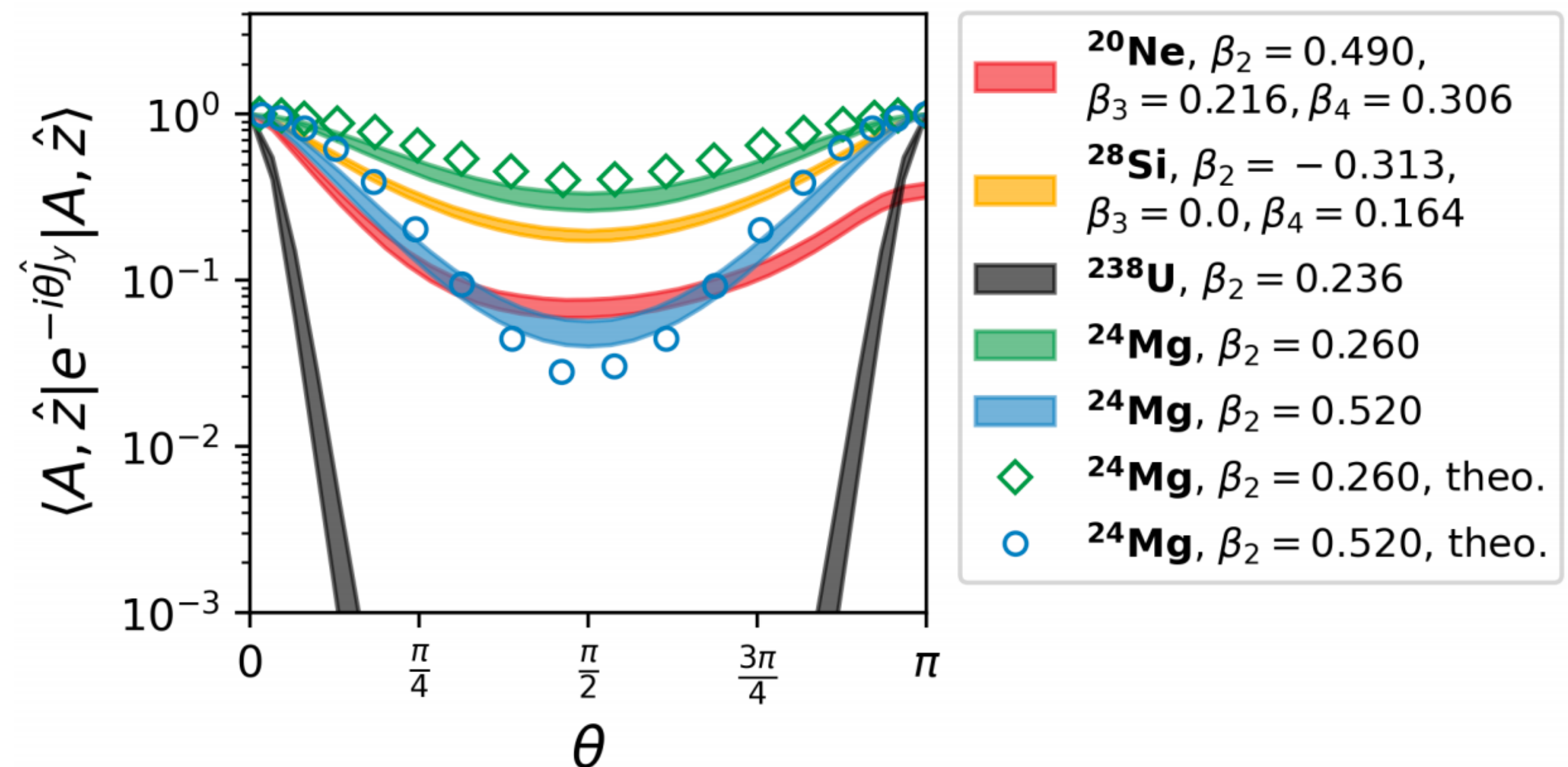


Figure: Yanru Bao and Weiyao Ke (submitted)

Open symbols: J. M. Yao et al., Phys. Rev. C 79, 044312 (2009).

Step 2: collision in Quantum Mechanics

The overlap function $\frac{\langle \Omega_X | \Omega'_X \rangle}{|N_X|^2}$ serves as the weight for each orientation pair $\langle \Omega_X | \Omega'_X \rangle$.

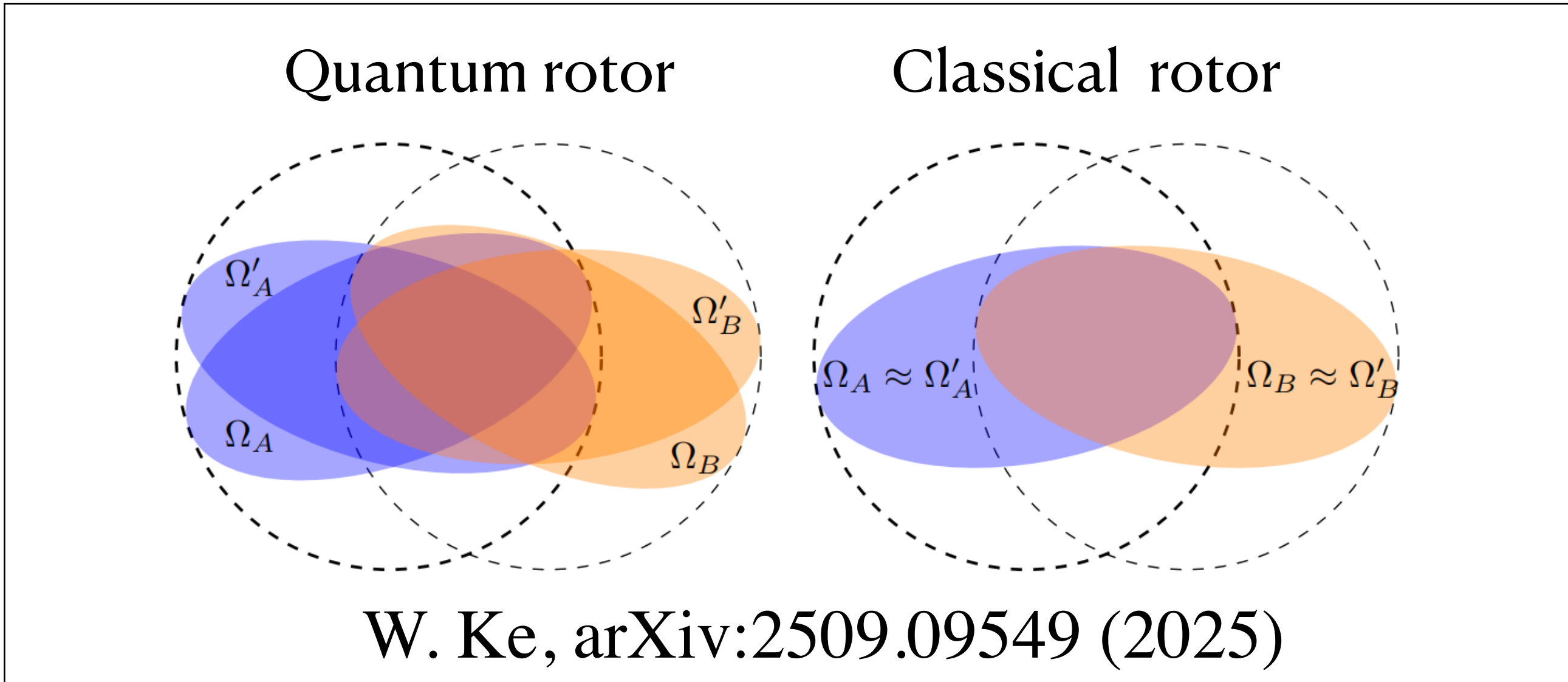
In the classical limit:

$$\frac{\langle \Omega_X | \Omega'_X \rangle}{|N_X|^2} \longrightarrow \frac{1}{4\pi} \delta^{(2)}(\Omega_X - \Omega'_X)$$

$$P_{AB,Q}^{\text{inel}}(\mathbf{b}) \longrightarrow P_{AB,C}^{\text{inel}}(\mathbf{b})$$

Ω, Ω'

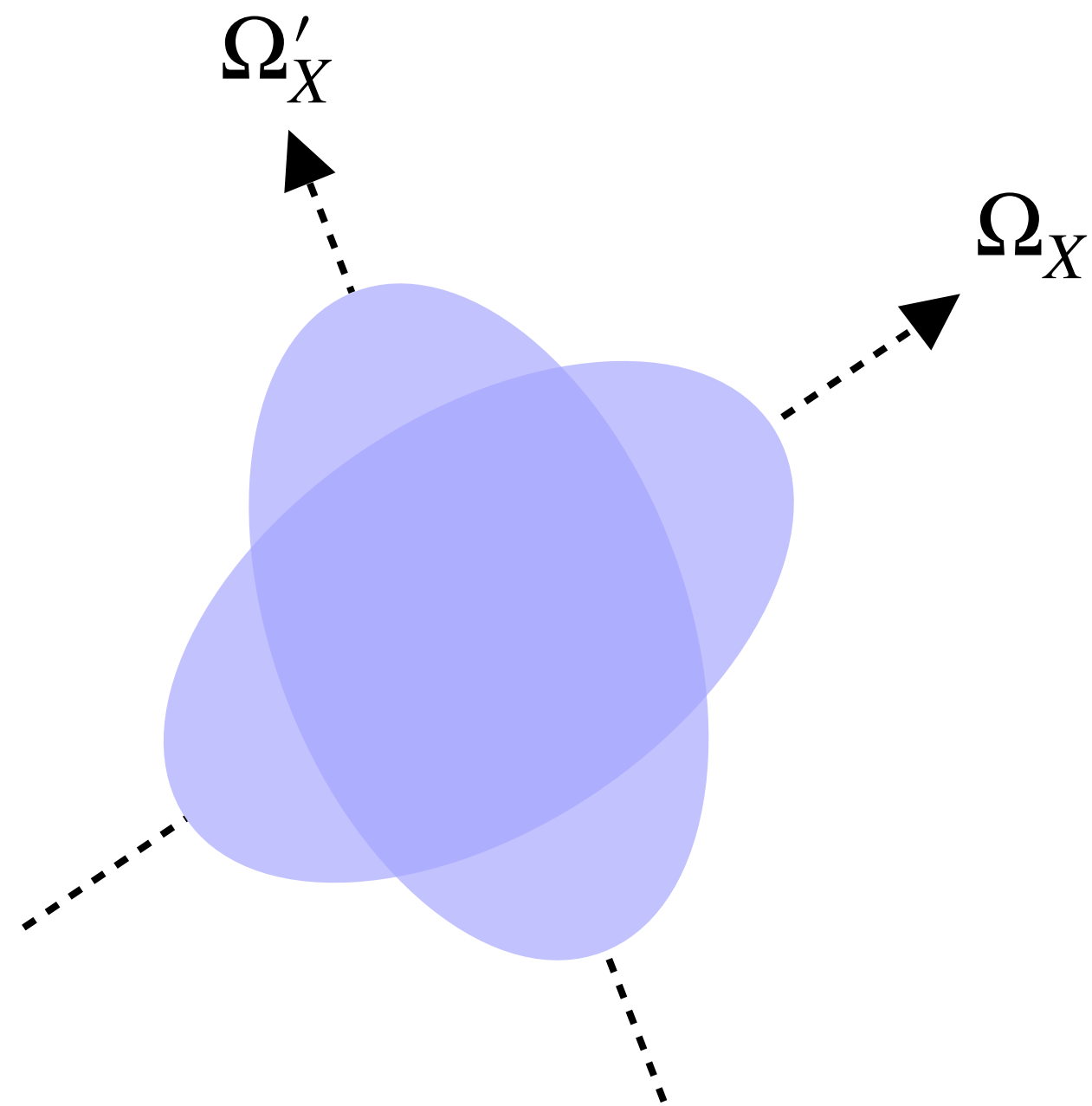
Ω



The **extended** Monto Carlo Glauber model can describe both quantum and classical rotors.

Step 3: Nucleon phase-space distribution via geometric averaging:

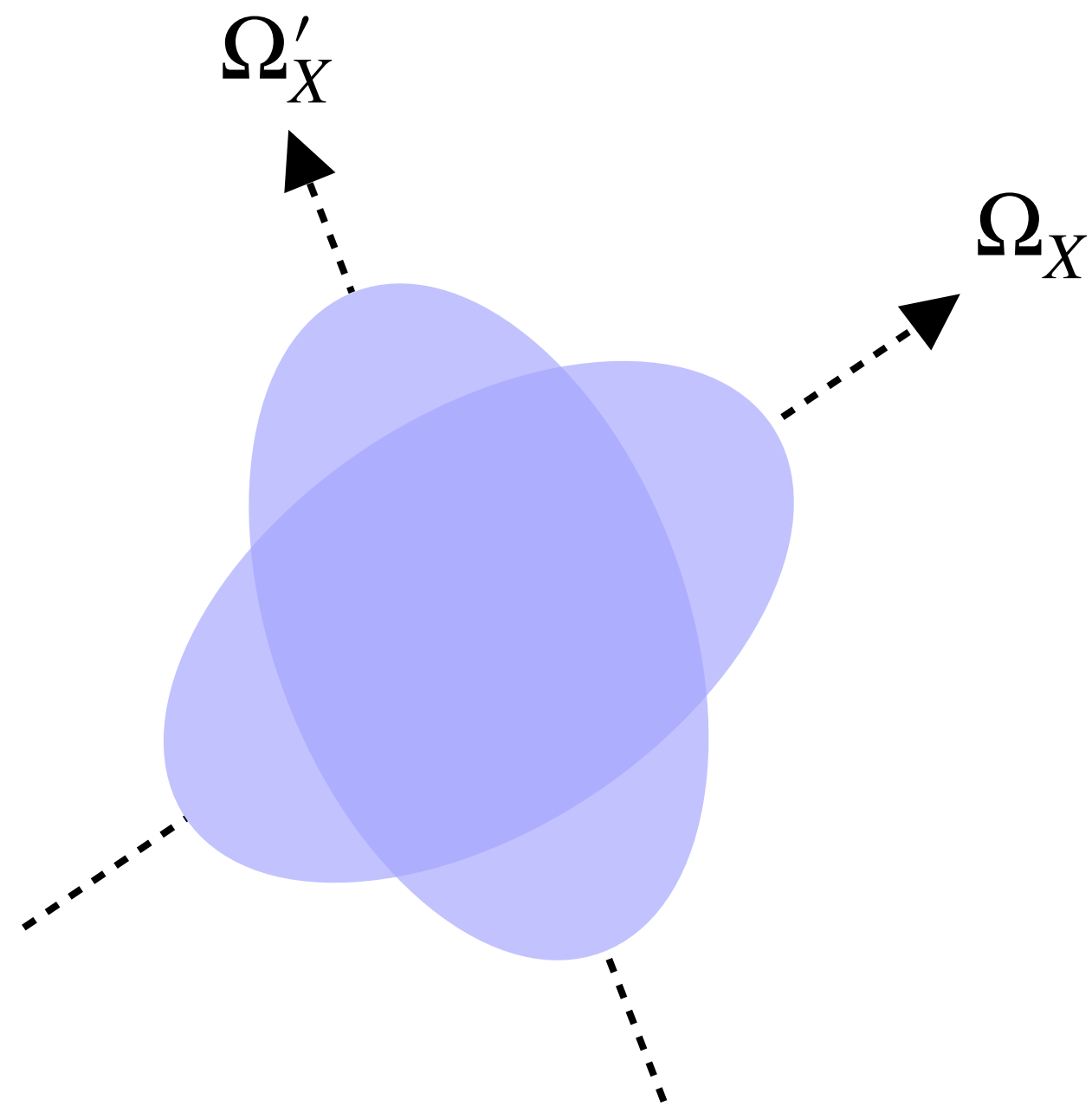
$$\mathcal{F}_X(\mathbf{r}, \mathbf{p}; \Omega_X, \Omega'_X) \equiv \frac{\sqrt{f_X(\mathbf{r}, \mathbf{p}; \Omega_X) f_X(\mathbf{r}, \mathbf{p}; \Omega'_X)}}{(2\pi)^3}$$



One possible sampled quantum-rotor

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One possible sampled quantum-rotor

$$f_X(\mathbf{r}, \mathbf{p}; \Omega_X) = \frac{1}{1 + e^{[\epsilon_p - \epsilon_{F,X}(\mathbf{r})]/\Delta}}$$

where: $\epsilon_p = \frac{p^2}{2m_N}$, $\epsilon_F(\mathbf{r}) = \frac{p_F^2(\mathbf{r})}{2m_N}$

Nucleon mass : $m_N = 0.938 \text{ GeV}$

Fermi-surface smearing parameter: $\Delta = 0.5 \text{ MeV}$

$$g \int \frac{d^3p}{(2\pi)^3} f_X(\mathbf{r}, \mathbf{p}; \Omega_X) = \rho_X(\mathbf{r}; \Omega_X)$$

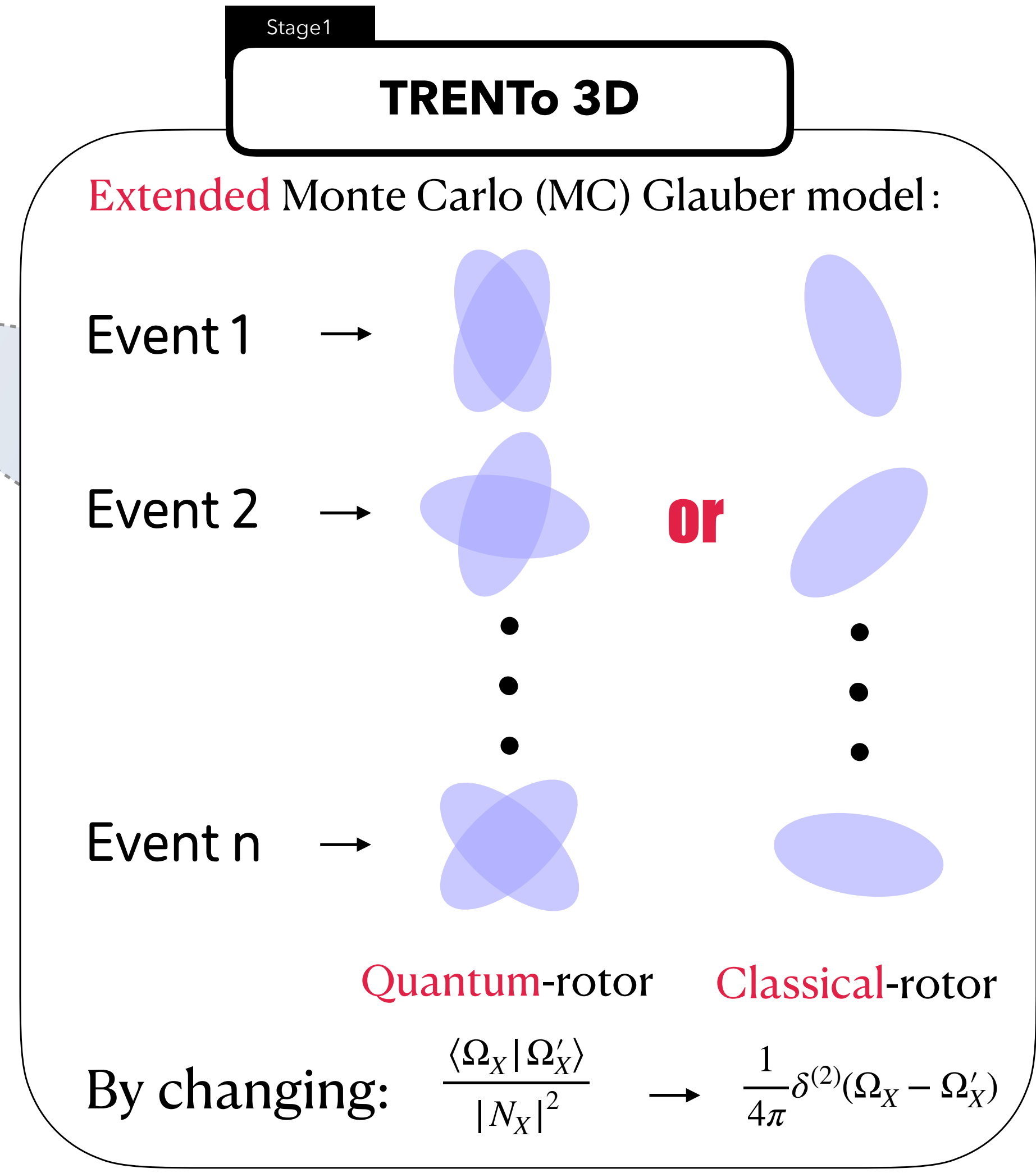
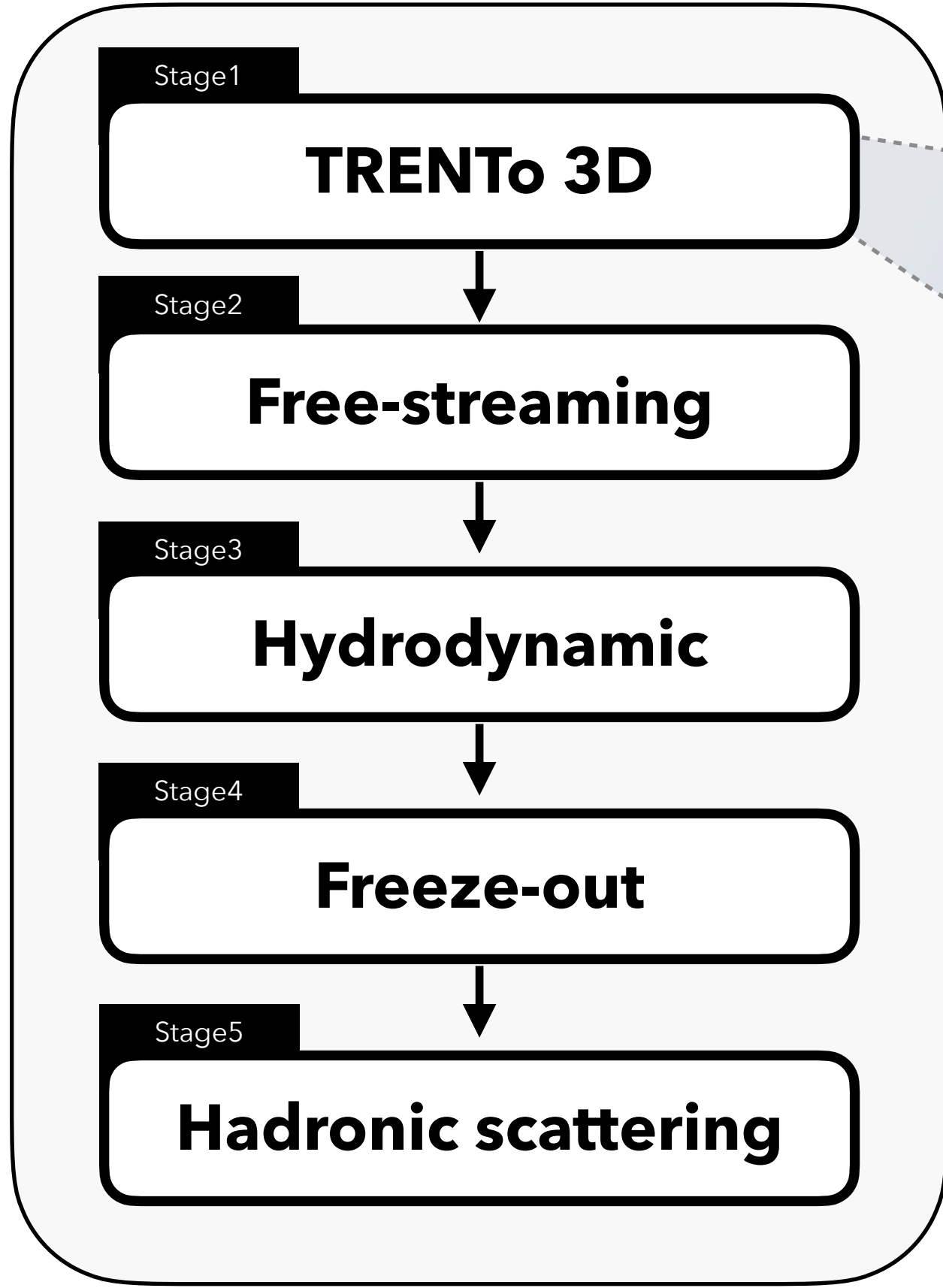
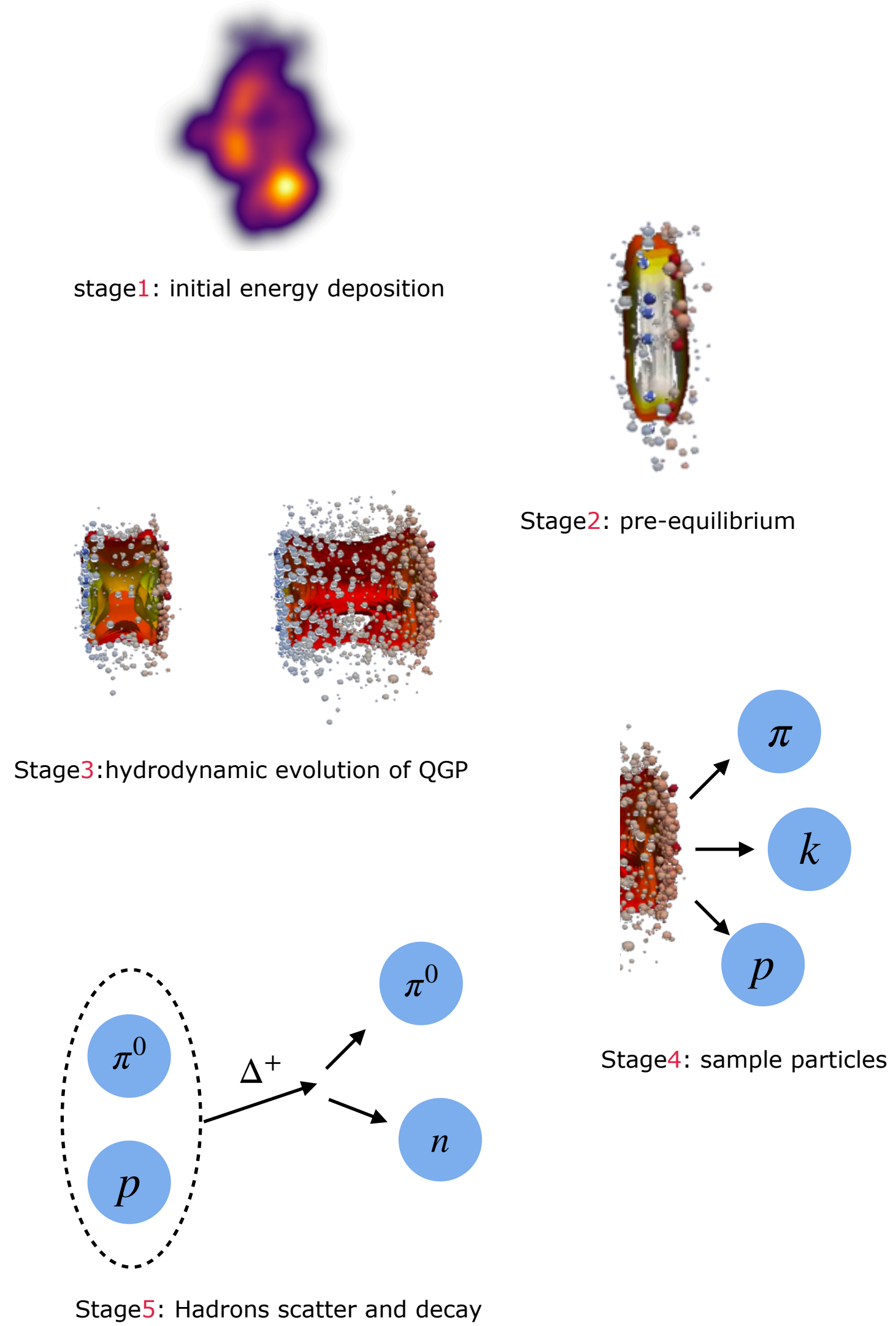
Woods-Saxon density: $\rho_X(\mathbf{r}; \Omega_X) = \frac{\rho_{0,X}}{1 + \exp\left[\frac{r - R_X(\theta, \phi)}{a_X}\right]}$

Nuclear radius: $R_X(\theta, \phi) = R_{0,X} [1 + \beta_{2,X} Y_{20}(\theta, \phi) + \beta_{3,X} Y_{30}(\theta, \phi) + \dots]$

How is nuclear deformation incorporate into HIC simulations?

Corresponding physical process

multistage simulation

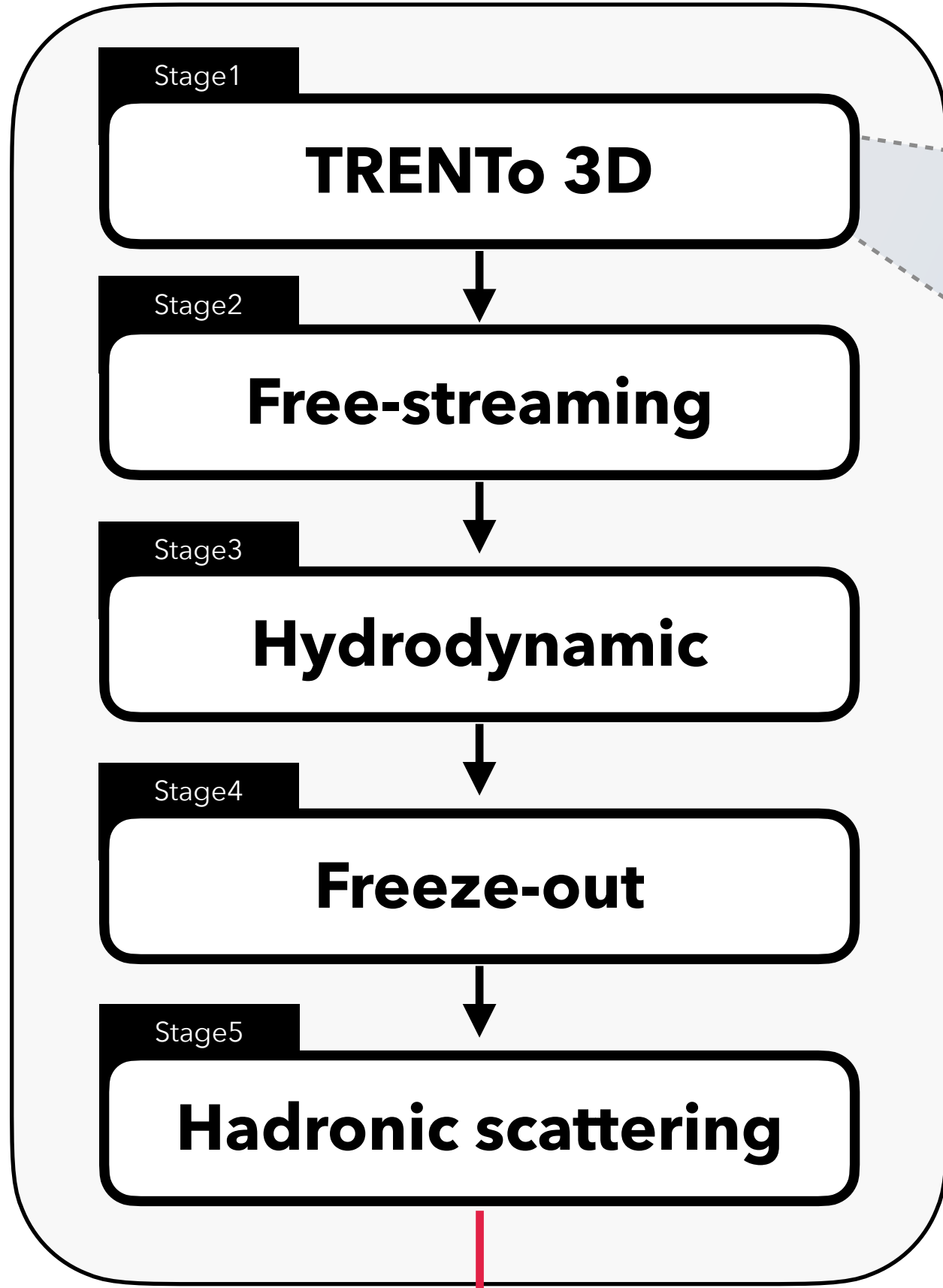
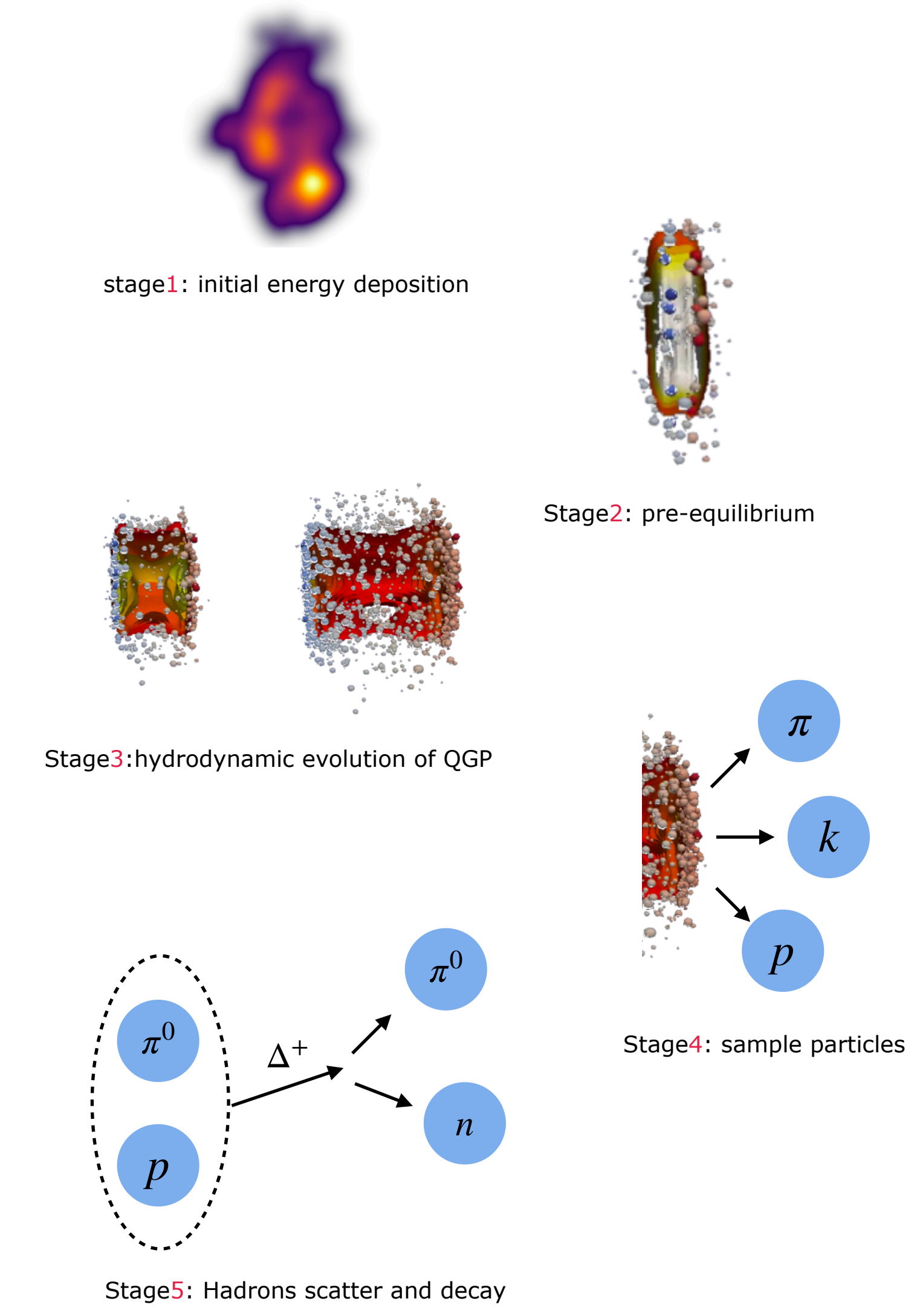


W. Ke, arXiv:2509.09549 (2025)

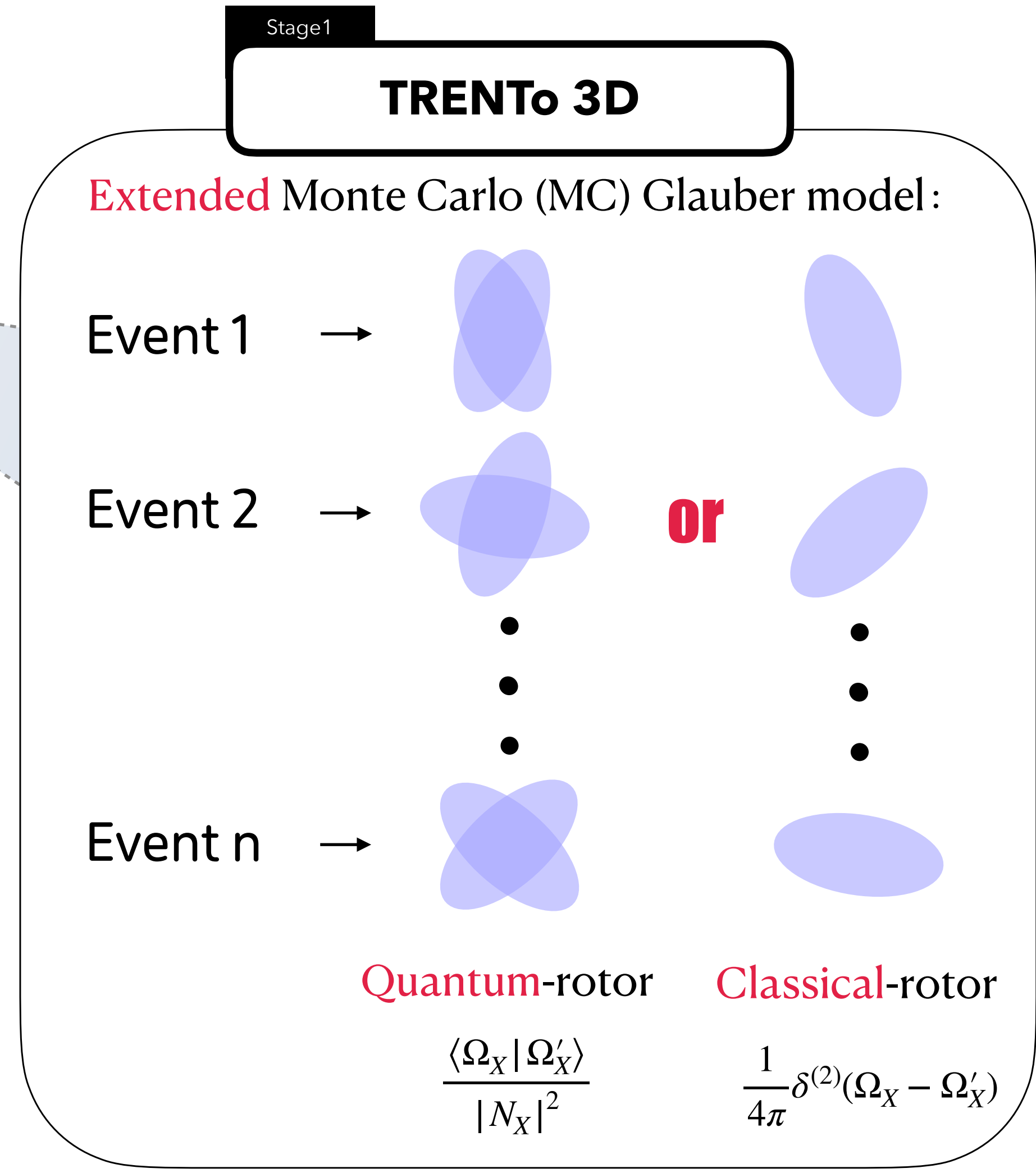
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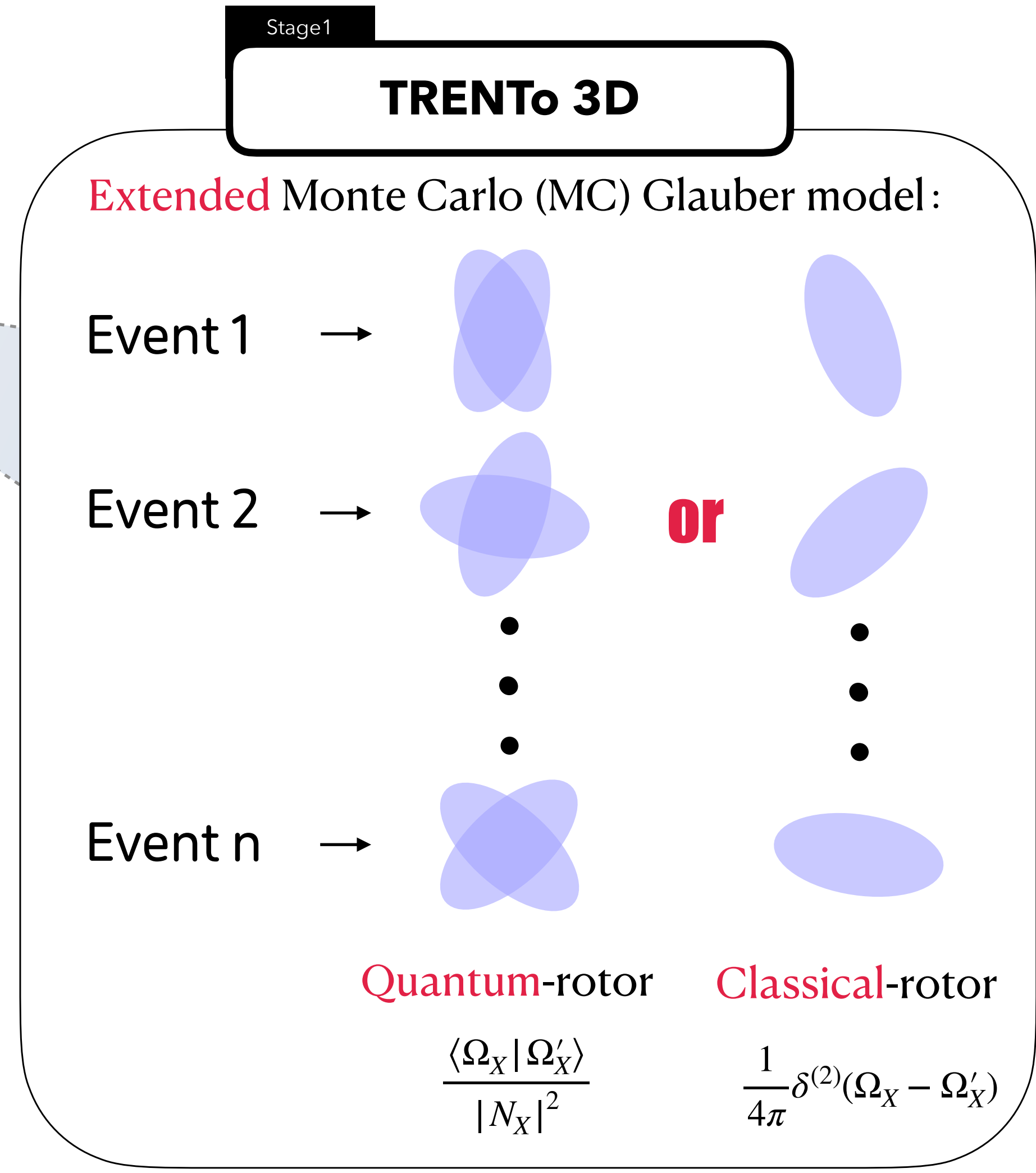
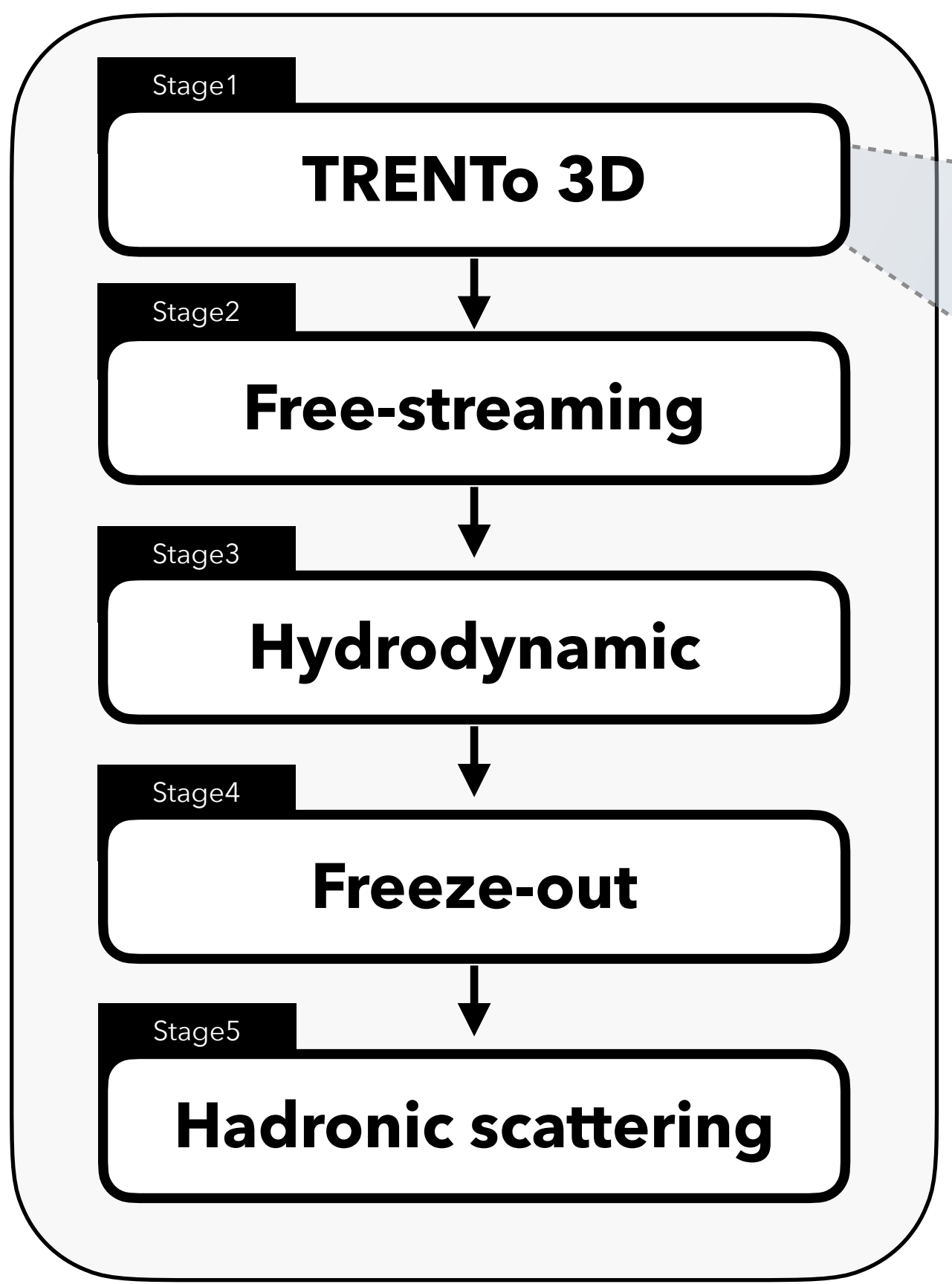
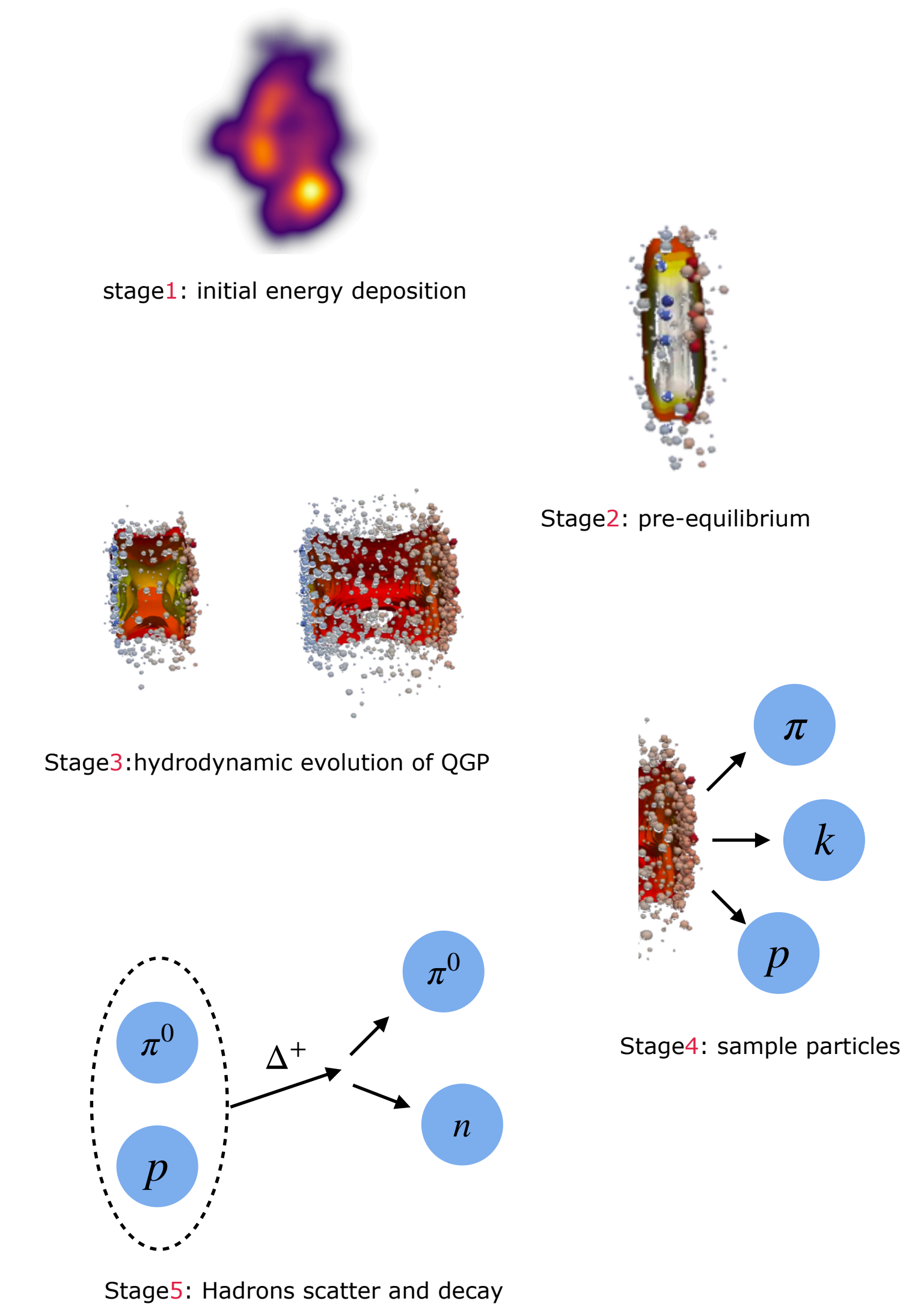
v_n^Q and v_n^C



How is nuclear deformation incorporated into HIC simulations?

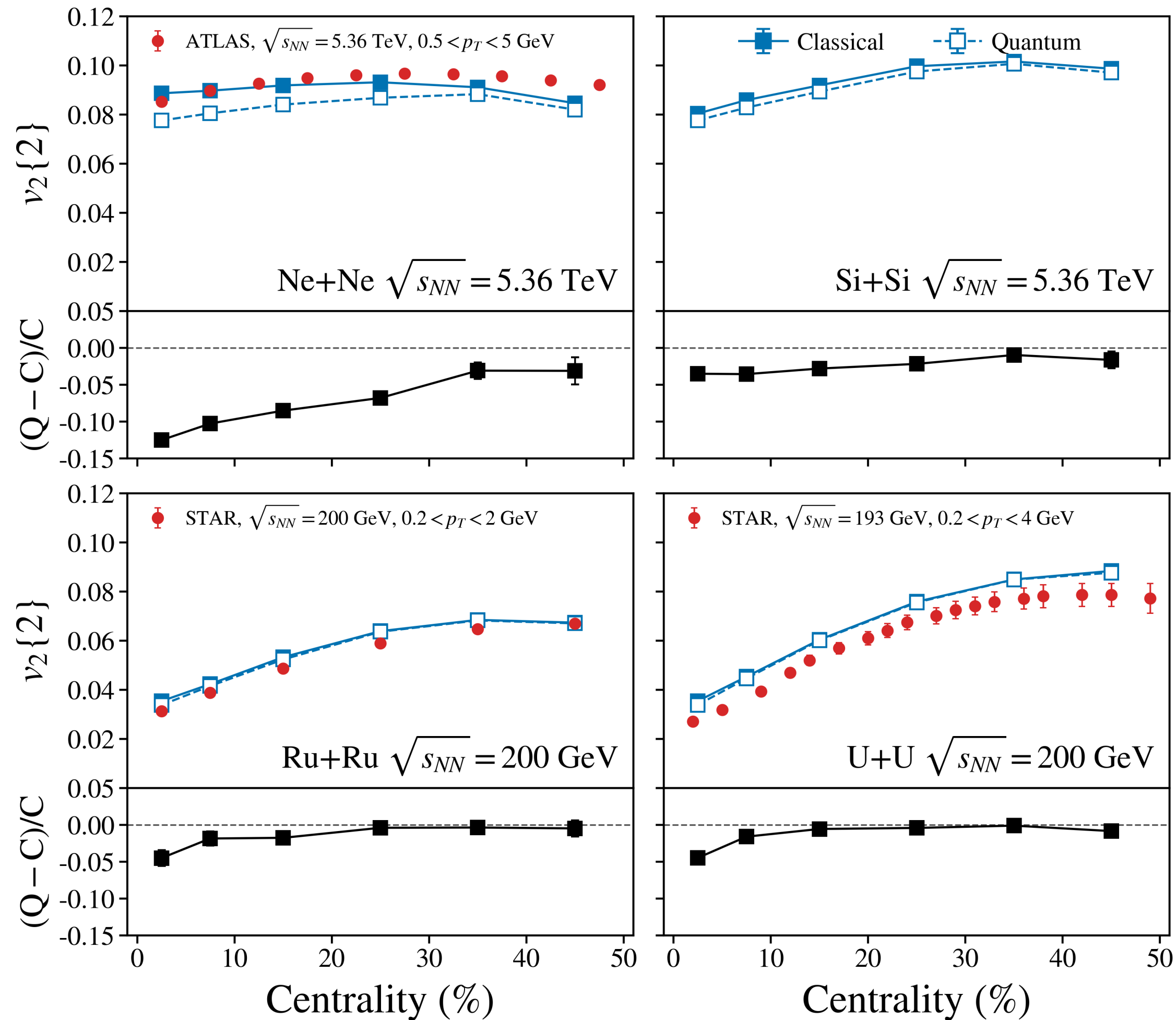
Corresponding physical process

multistage simulation



How does restoring rotational symmetry in the quantum description of nuclei affect final-state observables?

Results: Impact on anisotropic flow across centrality bins



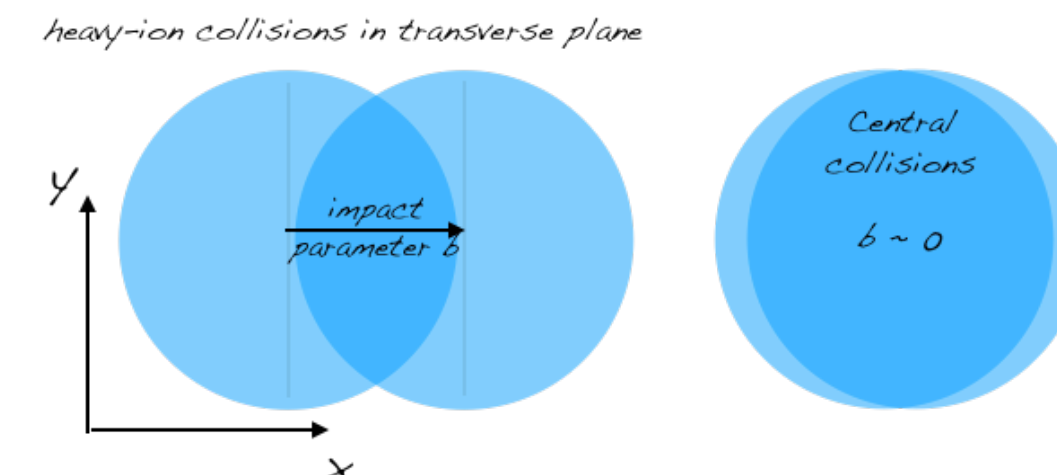
Nucleus X	A	$\beta_{2,X}$	$\beta_{3,X}$	$\beta_{4,X}$	$R_{0,X}$ (fm)	a_X (fm)
^{16}O	16	0	0	0	2.608	0.513
^{20}Ne	20	0.4899	0.2160	0.3055	2.724	0.498
^{28}Si	28	-0.313	0	0.164	3.34	0.58
^{96}Ru	96	0.16	0	0	5.065	0.485
^{197}Au	197	0	0	0	6.38	0.535
^{238}U	238	0.236	0	0.098	6.8054	0.605

Table: nuclear deformed parameters in intrinsic frame.

relative change:

$$(Q - C)/C \equiv (v_2^Q - v_2^C)/v_2^C$$

- ① In both light- and heavy-ion collisions, the quantum effect is observed, but its magnitude varies across different systems.
- ② The quantum effect is most pronounced in central collisions, where flow is most sensitive to nuclear structure.



M. Ploskon, *arXiv:1808.01411* (2018)

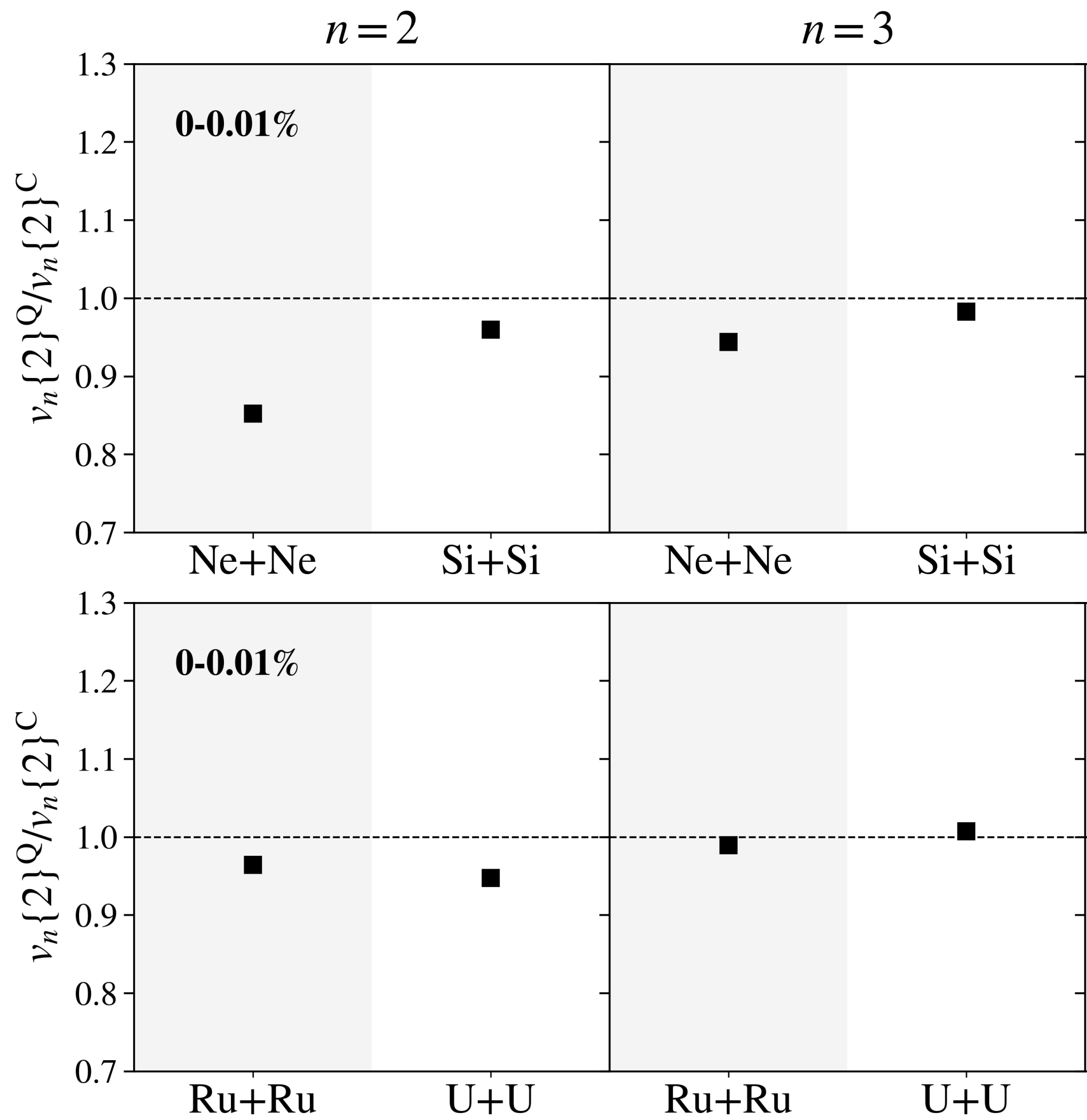
Experimental data:

Ne+Ne [1]; Ru+Ru [2]; U+U [3]

[1] *Phys. Rev. C* **113**, 045205 (2026).

[2] *Phys. Rev. C* **105**, 014901 (2022).

[3] *Phys. Rev. Lett.* **129**, 252301 (2022).



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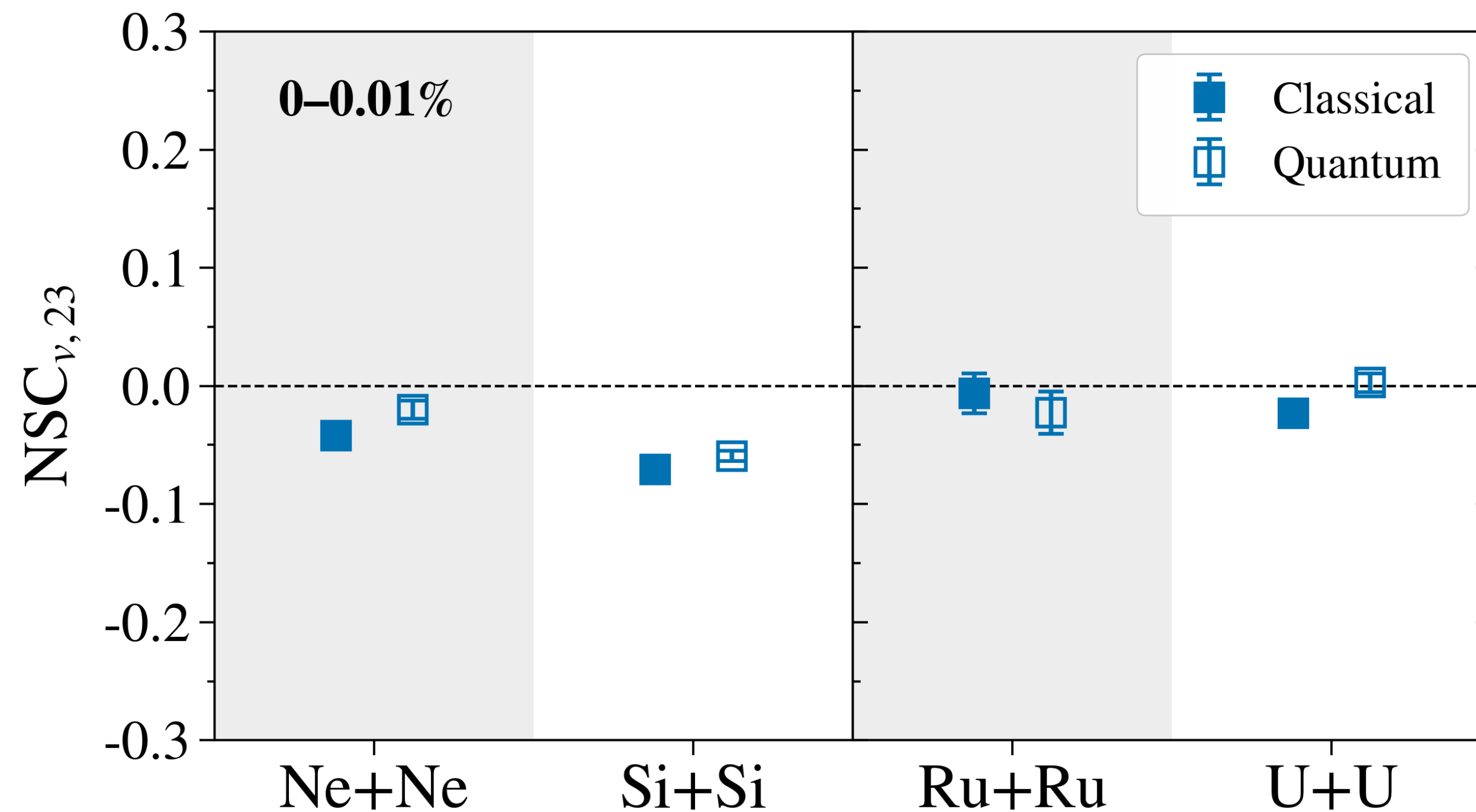
Table 1: intrinsic deformation parameters of the colliding nuclei.

- ① For $v_2\{2\}$, Ne+Ne exhibits the largest suppression, consistent with the strong quadrupole deformation ($\beta_2 = 0.4899$) of ^{20}Ne . This suggests that quantum effects may be particularly important for strongly deformed light nuclei.
- ② For $(v_3\{2\})$, Ne+Ne also shows the largest suppression, while the other systems are nearly unaffected. Since v_3 is mainly driven by initial-state fluctuations, we suggest that the larger suppression in Ne+Ne may be related to its nonzero octupole deformation, β_3 .

Results: Impact on Normalized Symmetric Cumulant in ultra-central collisions

Normalized Symmetric Cumulant:

$$\text{NSC}(2,3) = \frac{\langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle}{\langle v_2^2 \rangle \langle v_3^2 \rangle}$$



$\text{NSC}(2,3) > 0$:

Positive event-by-event correlation between v_2^2 and v_3^2 .

$\text{NSC}(2,3) < 0$:

Negative event-by-event correlation between v_2^2 and v_3^2 .

- ① Except for Ru+Ru, where the negative correlation becomes stronger, quantum effect generally weakens the correlations in the other systems, bringing their NSC values closer to zero.

Pearson correlation coefficient is sensitive to initial overlap geometry

Pearson correlation coefficient characterize the correlation between v_n^2 and the event-averaged transverse momentum $[p_T]$:

$$\rho_{n, \text{pcc}} = \frac{\text{Cov}(v_n^2, [p_T])}{\sqrt{\text{Var}(v_n^2)} \sqrt{\text{Var}([p_T])}}$$

Pearson correlation coefficient is sensitive to initial overlap geometry.

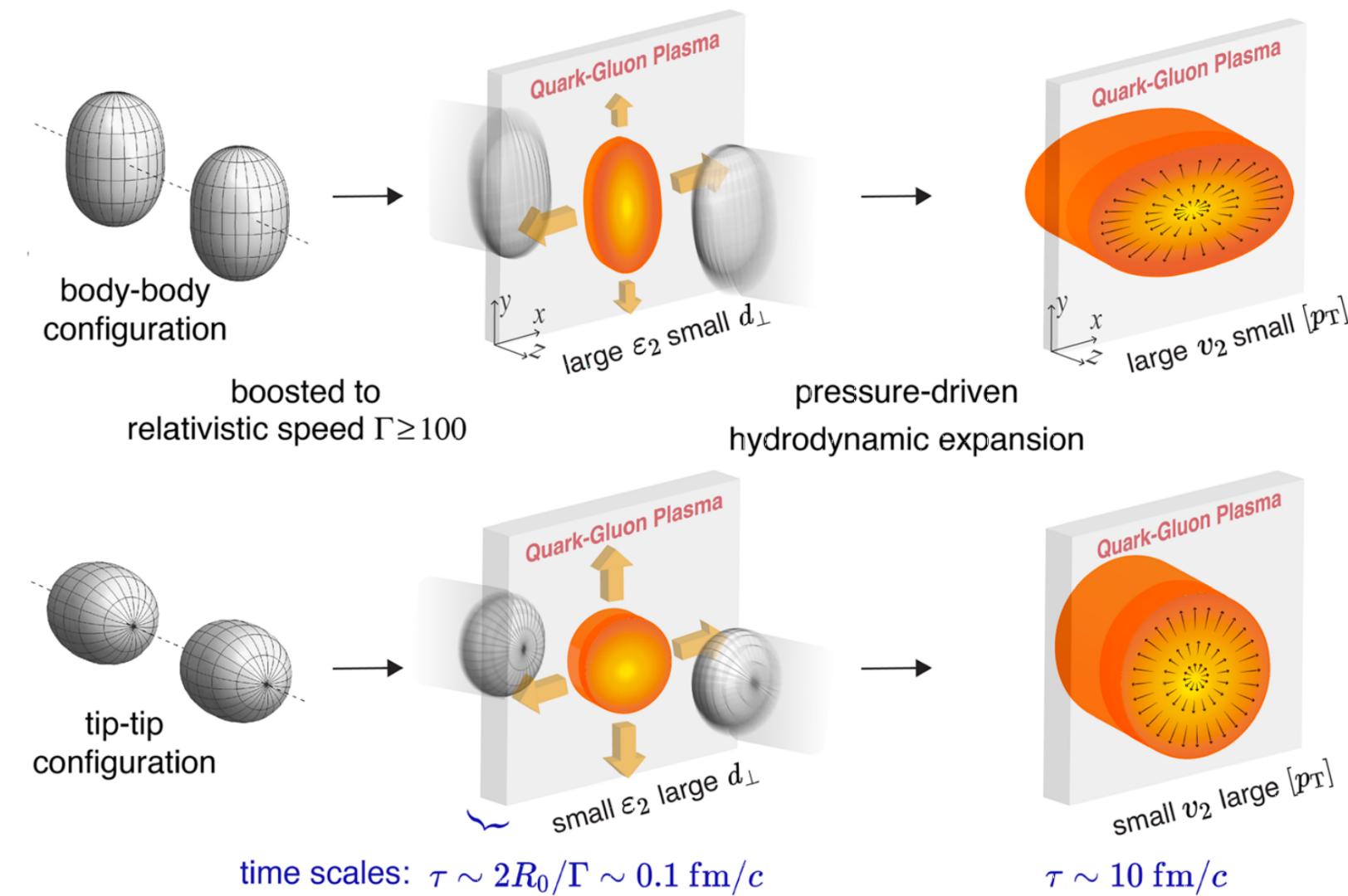


Figure reproduced from: G. Giacalone, Nucl. Sci. Tech. 35, 218 (2024).

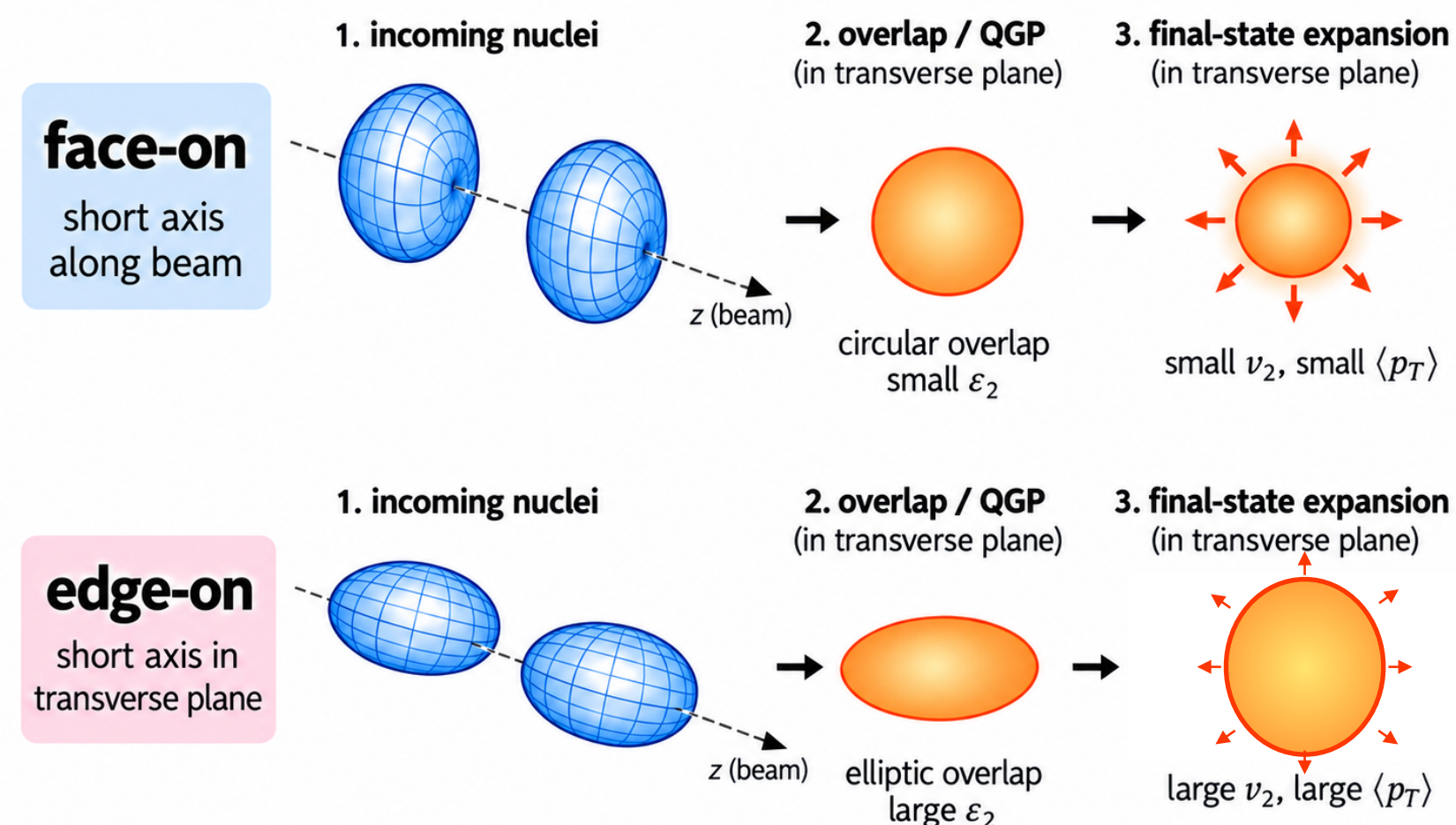


Figure created with AI assistance

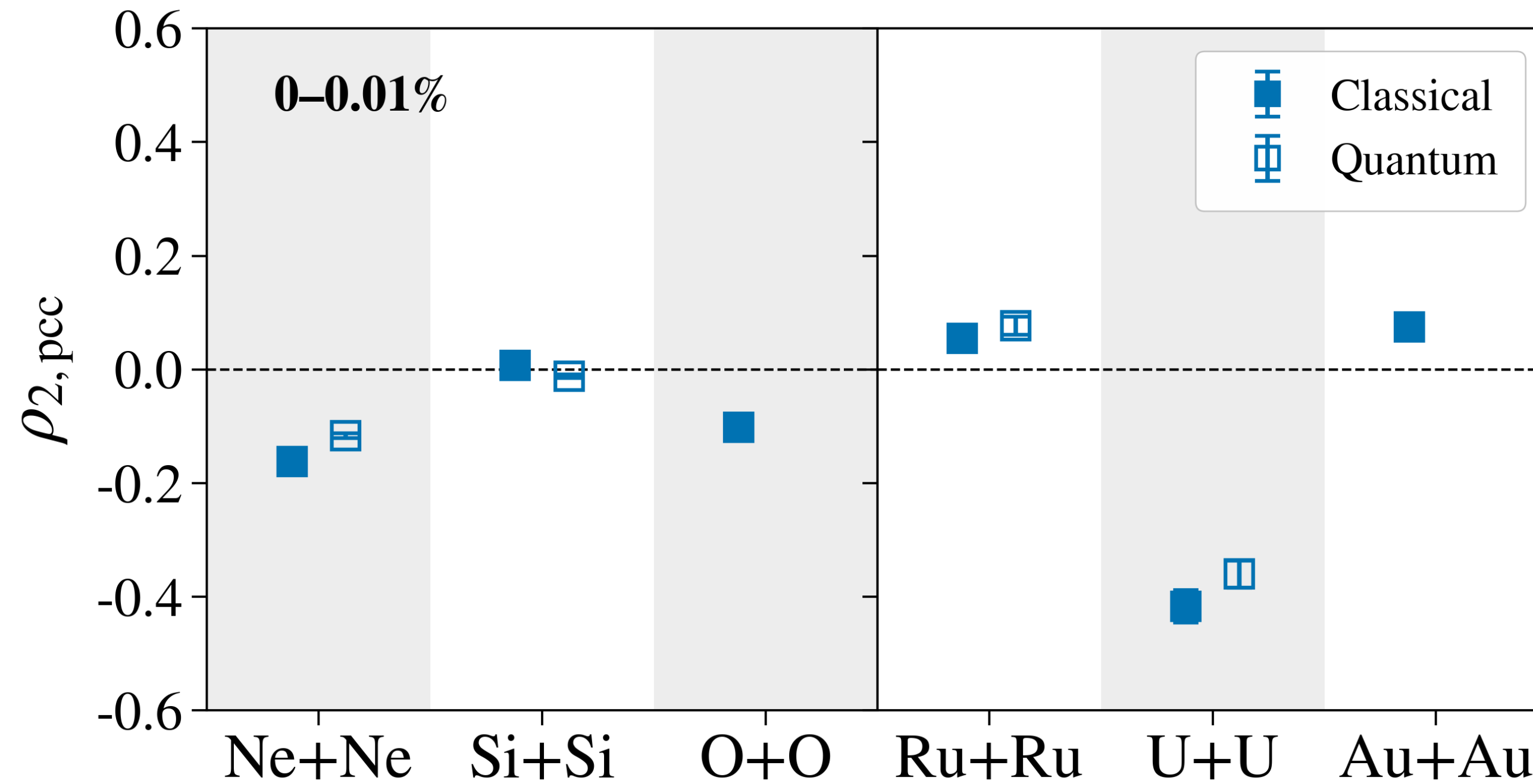
prolate ($\beta_2 > 0$):

Negative event-by-event correlation

obolate ($\beta_2 < 0$):

Positive event-by-event correlation

Results: Impact on Pearson correlation coefficient in ultra-central collisions



We use the spherical O+O and Au+Au systems as approximate fluctuation baselines for light- and heavy-ion collisions, respectively.

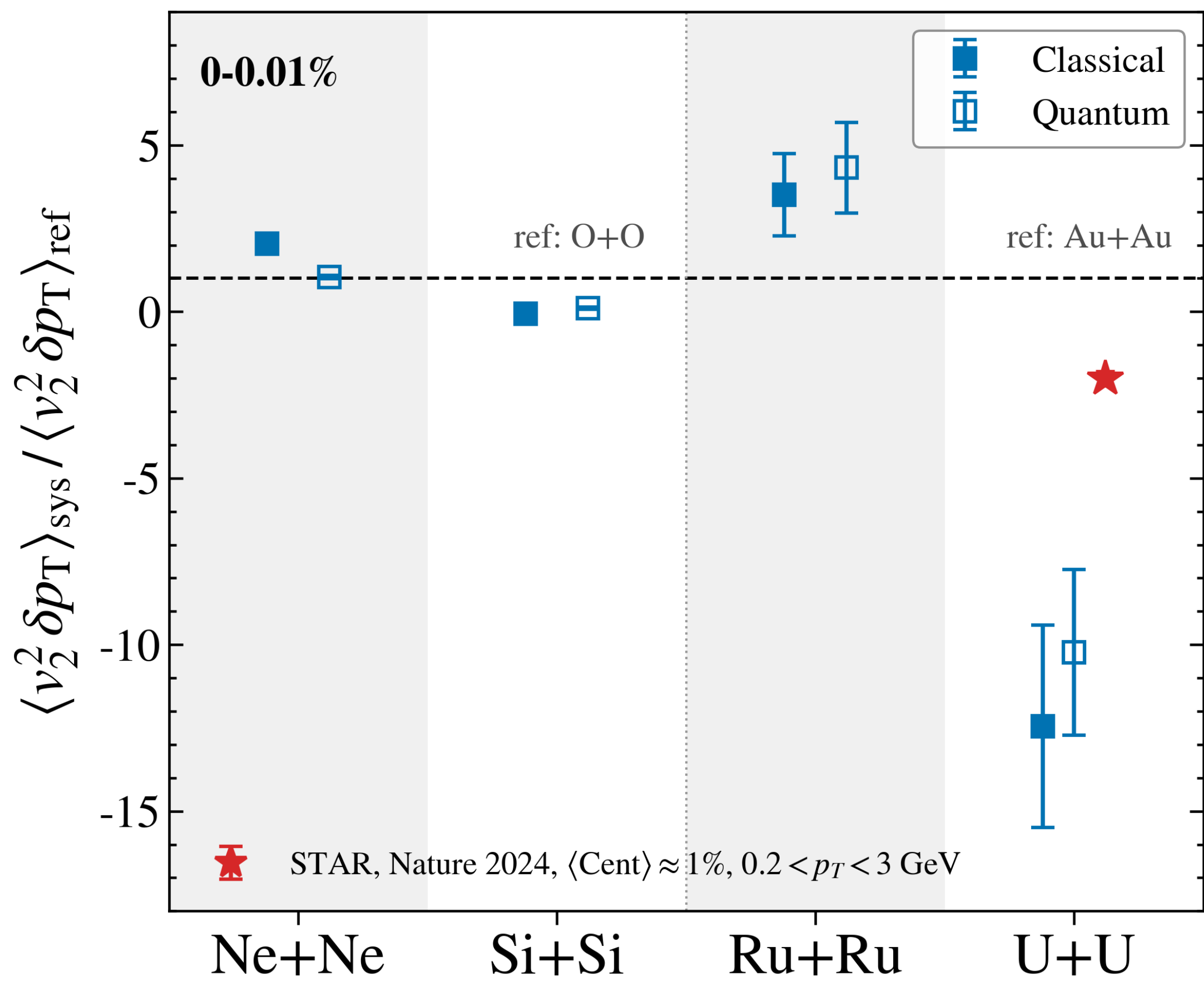
- ① Quantum superposition brings the correlations closer to the spherical baselines, indicating a weaker contribution from nuclear deformation.

- An empirical approximation describes the dependence of the Pearson correlation coefficient on nuclear quadrupole deformation (β_2) :

$$\rho(v_2^2, [p_T]) \simeq b_1 + b_2 \beta_2,$$

- b_1 : Fluctuation baseline for spherical nuclei.
- $b_2 \beta_2$: Deformation-dependent contribution ($b_2 < 0$).

Results: Impact on cross-system ratio $\langle v_2^2 \delta p_T \rangle_{\text{sys}} / \langle v_2^2 \delta p_T \rangle_{\text{ref}}$ in ultra-central collisions 24/26



The cross-system ratio of the covariance numerator:

$$R \equiv \frac{\langle v_2^2 \delta p_T \rangle_{\text{sys}}}{\langle v_2^2 \delta p_T \rangle_{\text{ref}}} = \frac{\text{Cov}(v_2^2, [p_T])_{\text{sys}}}{\text{Cov}(v_2^2, [p_T])_{\text{ref}}}$$

- ① Quantum superposition affects the cross-system ratios differently across collision systems.
- ② For U+U/Au+Au, quantum superposition reduces the absolute magnitude of the ratio.
- ③ Therefore, our results suggest that quantum effects may influence the extraction of nuclear deformation parameters.

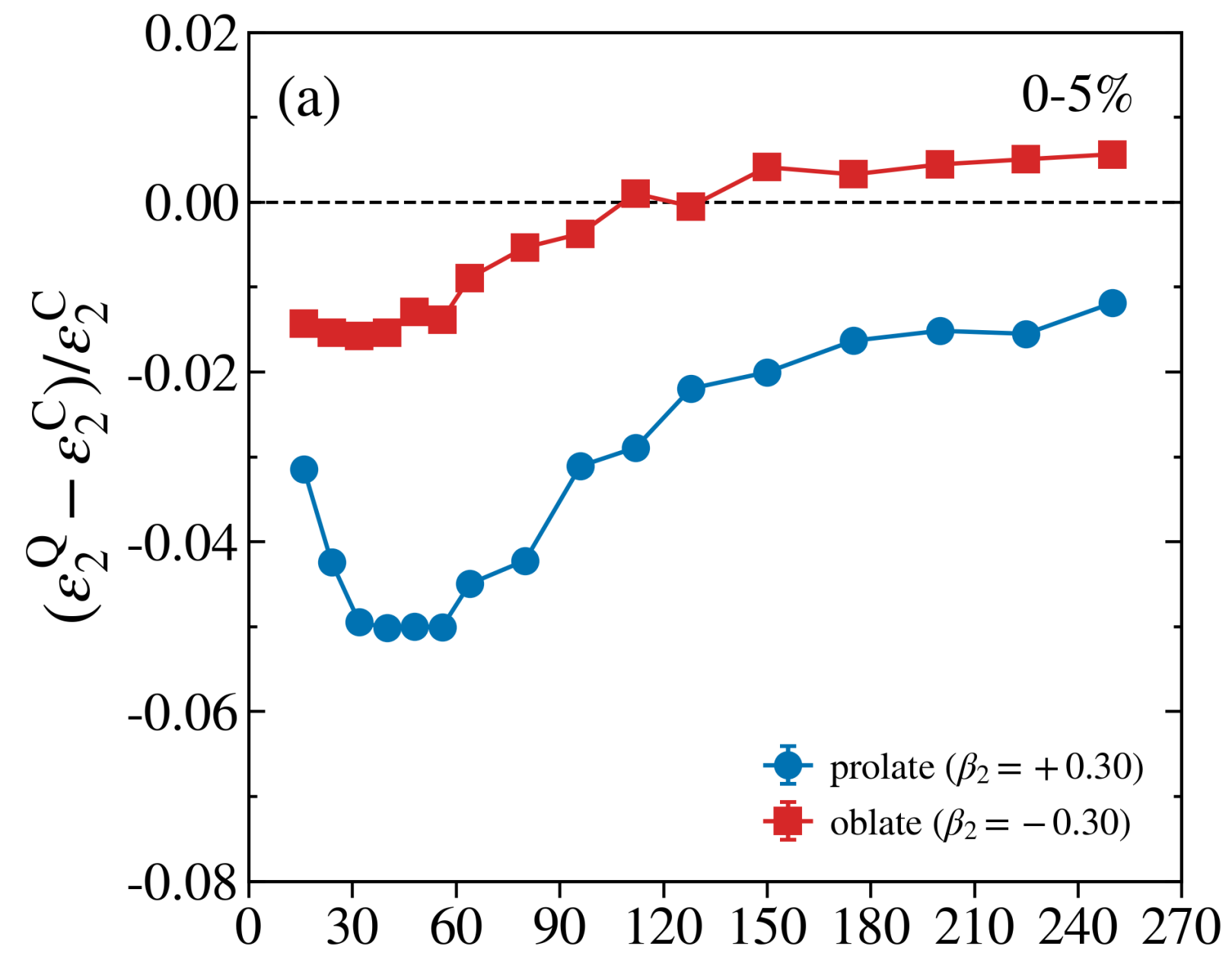
- I. We **extend** the MC Glauber model to generate both quantum-rotor and classical-rotor and evolve them through hydrodynamics. We systematically study the impact of rotational symmetry restoration on the final-state observables in both light- and heavy-ion collisions.
- II. We focus on the **0–0.01%** ultra-central collisions, where observables are particularly sensitive to nuclear structure. We find that quantum superposition affects collision systems to different degrees, with the largest effect observed in Ne+Ne collisions.
- III. Furthermore, the changes in cross-system ratios for both light- and heavy-ion collisions highlight the **importance of considering quantum nuclear configurations** when extracting nuclear deformation parameters.



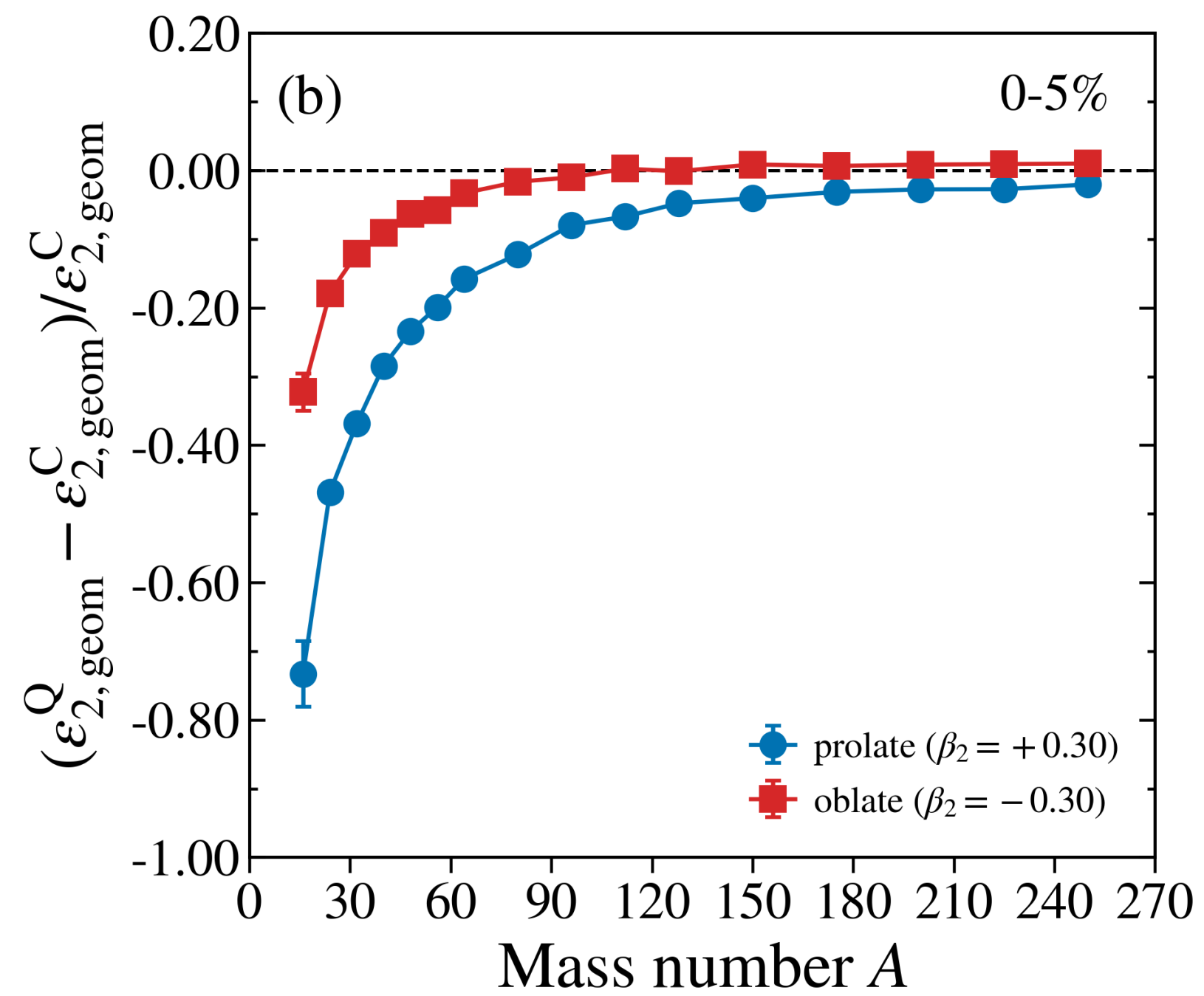
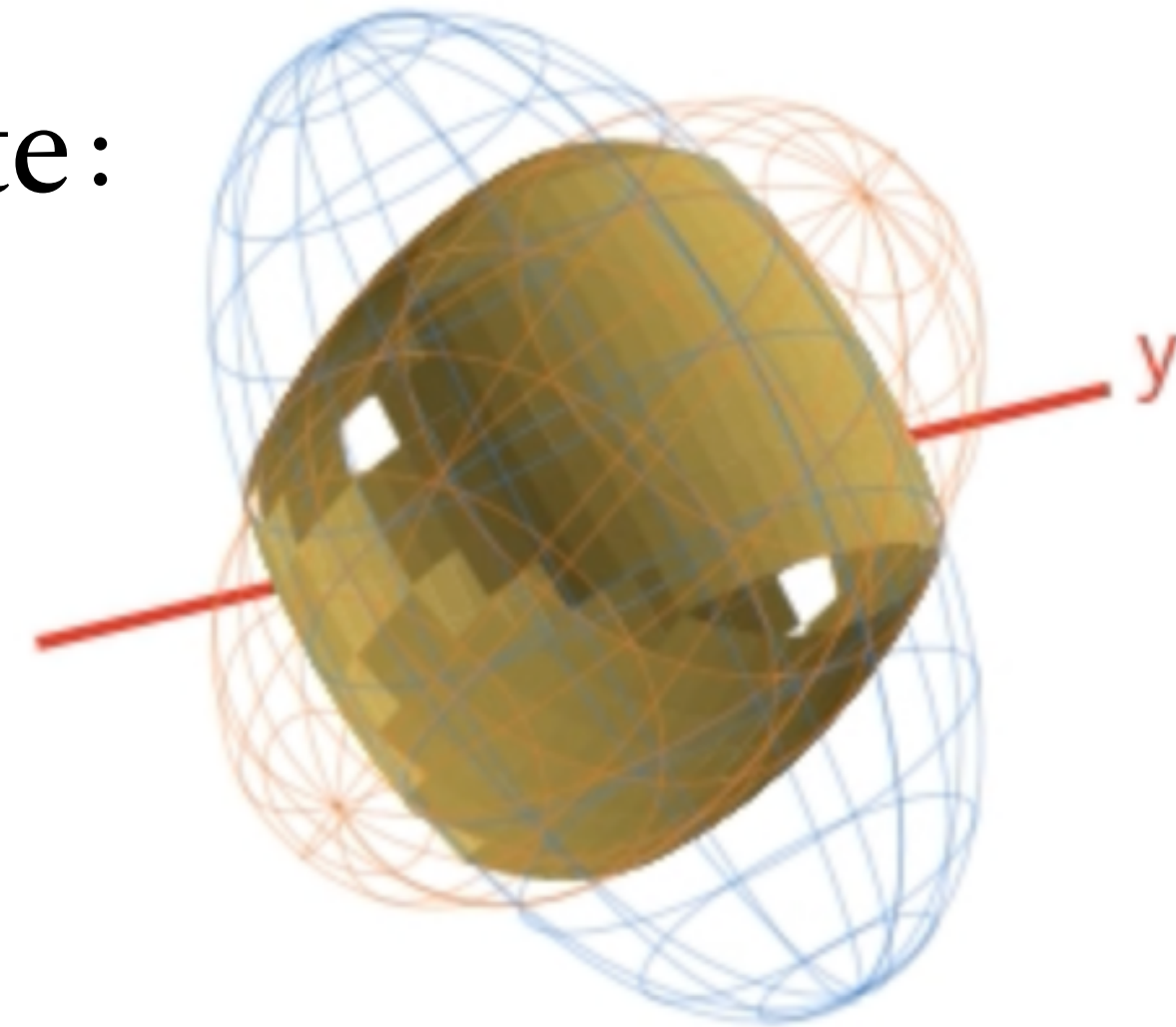
Thank you !!!

Yanru Bao (暴彦汝) 2026. 10. 19

Spicy Gluons [胶麻] Workshop, 10/17-10/20, 2026, Wuhan, Hubei



prolate:



oblate:

