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Collective Recoil as a Saturation Observable at the EIC

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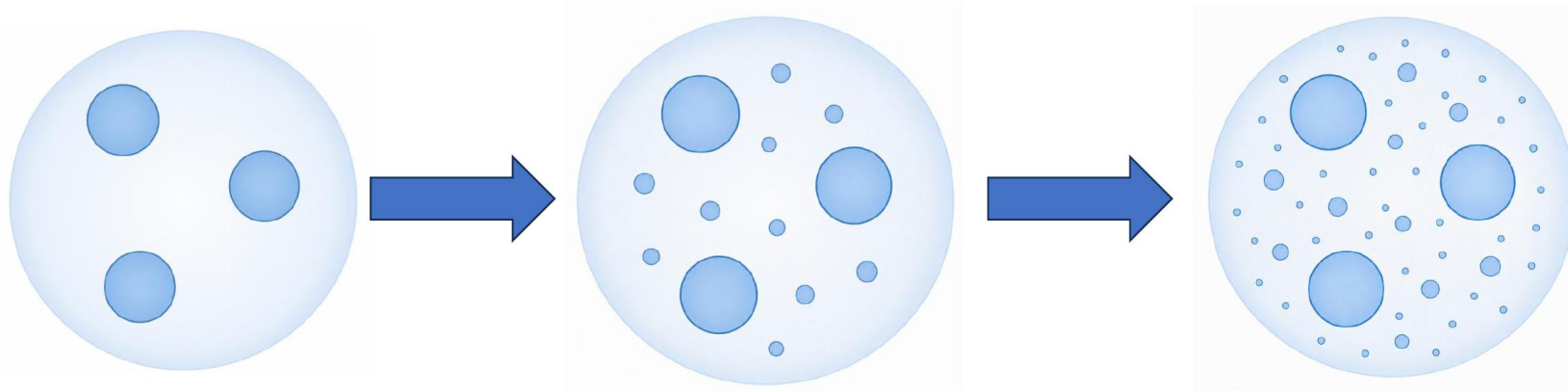
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Gluon Saturation in Nuclear Targets



Radiation at small x

$$g \longrightarrow g + g$$

Gluon density grows
as x decreases.

Fusion at high density

$$g + g \longrightarrow g$$

Recombination slows growth
when gluons overlap.

Saturation begins when gluon fusion competes with radiation.

Saturation Scale and Gluon Transverse Momentum

$$Q_s^2(x, A) \propto A^{1/3} x^{-\lambda}, \quad \lambda > 0$$

Schematic scaling. Smaller x raises density. Larger A increases nuclear thickness.

Transverse scale

$$r_\perp \sim \frac{1}{k_T}, \quad k_{T,\text{char}} \sim Q_s$$

Gluons at low momentum overlap across larger transverse distances.

Fusion slows growth at low momentum. A larger Q_s broadens the distribution.

Inputs at gluon $x = 0.01$

$$\mathcal{N}_A(x_{\text{sw}}, k_T) \propto \exp \left[-\frac{k_T^2}{Q_{s0,A}^2} \right]$$

Proton: $Q_{s0}^2 = 0.64 \text{ GeV}^2$

Gold: $Q_{s0}^2 = 4.00 \text{ GeV}^2$

Nonlinear GLR evolves these profiles toward smaller x .

Hard-Pair Imbalance in DIS

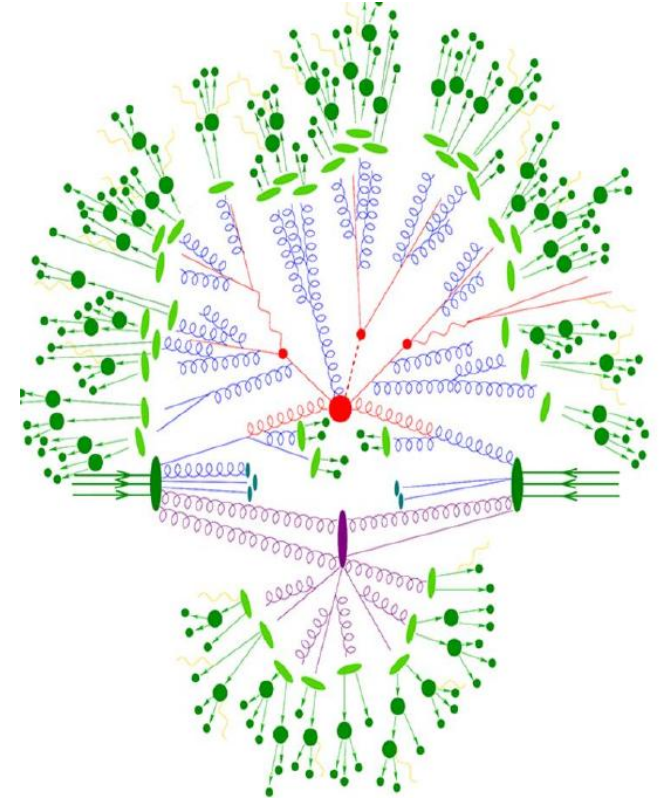
Before showering, in the Breit frame

$$\gamma^* + g^* \longrightarrow q + \bar{q}$$

$$\mathbf{k}_T = \mathbf{p}_{T,q} + \mathbf{p}_{T,\bar{q}}$$

Hard pairs inherit the incoming gluon recoil.

Showers and hadronization redistribute it.



Which hadronic observable retains this information?

Nonlinear Initial-State Shower

Folded GLR evolution

$$\eta = \ln(x_{\text{sw}}/x_g), \quad x_{\text{sw}} = 0.01$$

$$\partial_\eta \mathcal{N}_A = \underbrace{\mathcal{R}_A}_{\text{real emission}} - \underbrace{\Gamma_{\text{virtual}} \mathcal{N}_A}_{\text{virtual loss}} - \underbrace{C_{\text{nl}} \bar{\alpha}_s \mathcal{N}_A^2}_{\text{gluon fusion}}$$

Density and survival

$$\Gamma_A = \Gamma_{\text{virtual}} + C_{\text{nl}} \bar{\alpha}_s \mathcal{N}_A$$

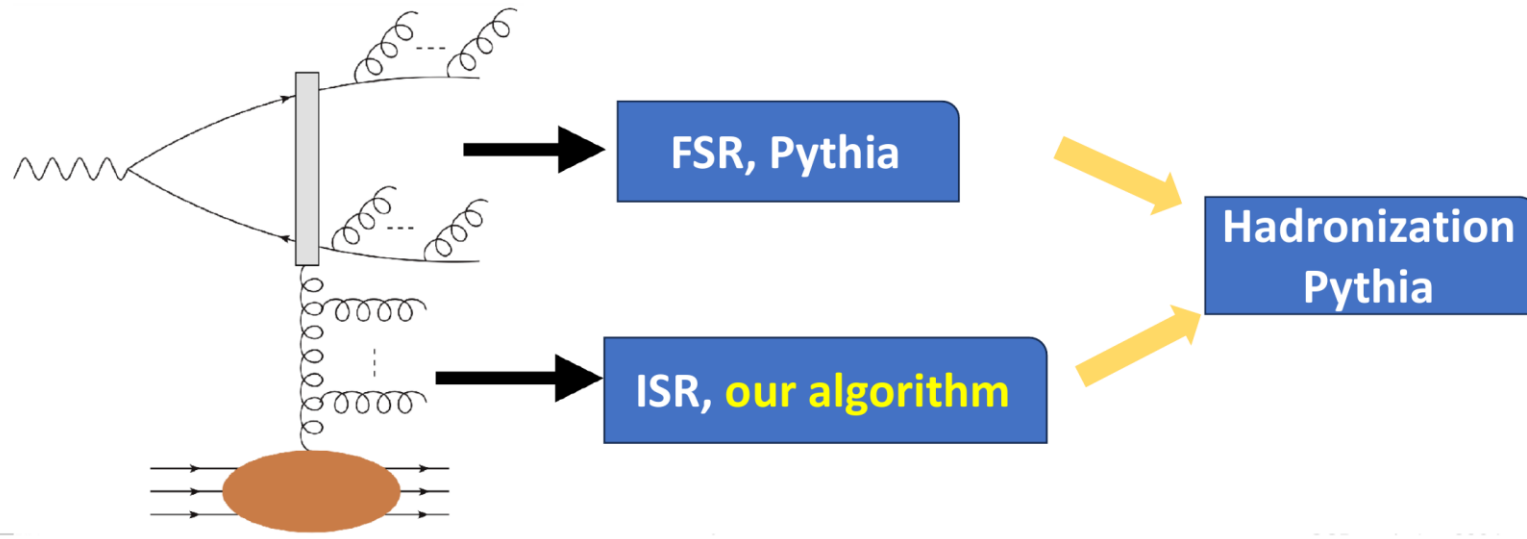
$$\Delta_A(\eta_2, \eta_1; k) = \exp \left[- \int_{\eta_1}^{\eta_2} d\xi \Gamma_A(\xi, k) \right]$$

**Fusion enters the loss rate
and the survival factor.**

Backward ISR generation

1. Evolve the density by target.
Sample the hard gluon from it.
2. Trace ISR back to larger x.
Use survival and density ratio.
3. At $x = 0.01$, switch to DGLAP.
Continue without overlap.

Complete Hadronic Events in eHIJING



Event assembly

Hard pair and ISR

Remnant recoil and
color connections

PYTHIA FSR, then
hadronization

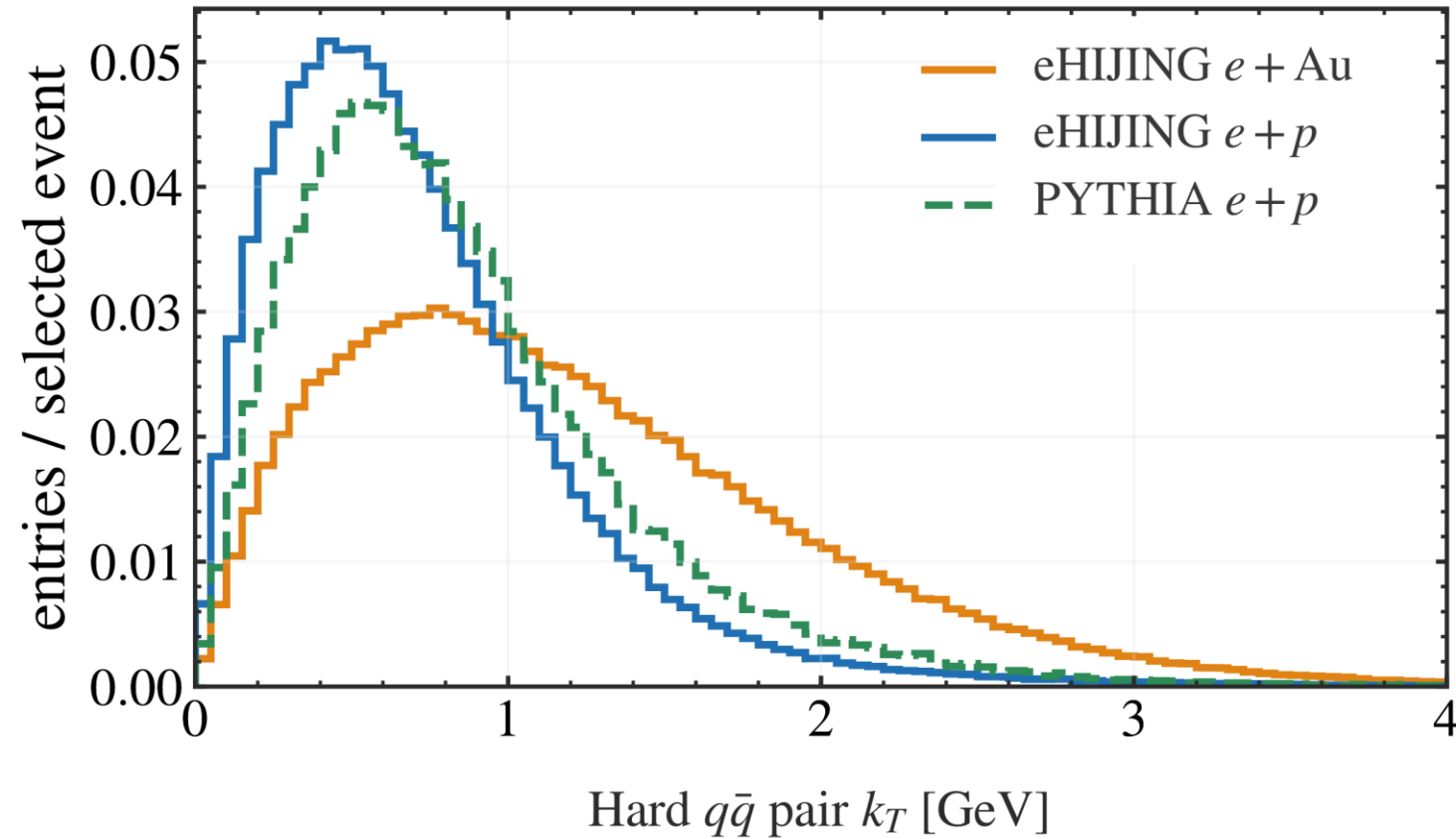
Electron beam: 18 GeV.

$Q^2 \geq 2 \text{ GeV}^2$, gluon $x < 0.01$.

Hadron beam: 110 GeV per nucleon.

Nominal PYTHIA FSR and hadronization.

Hard-Pair Transverse Momentum

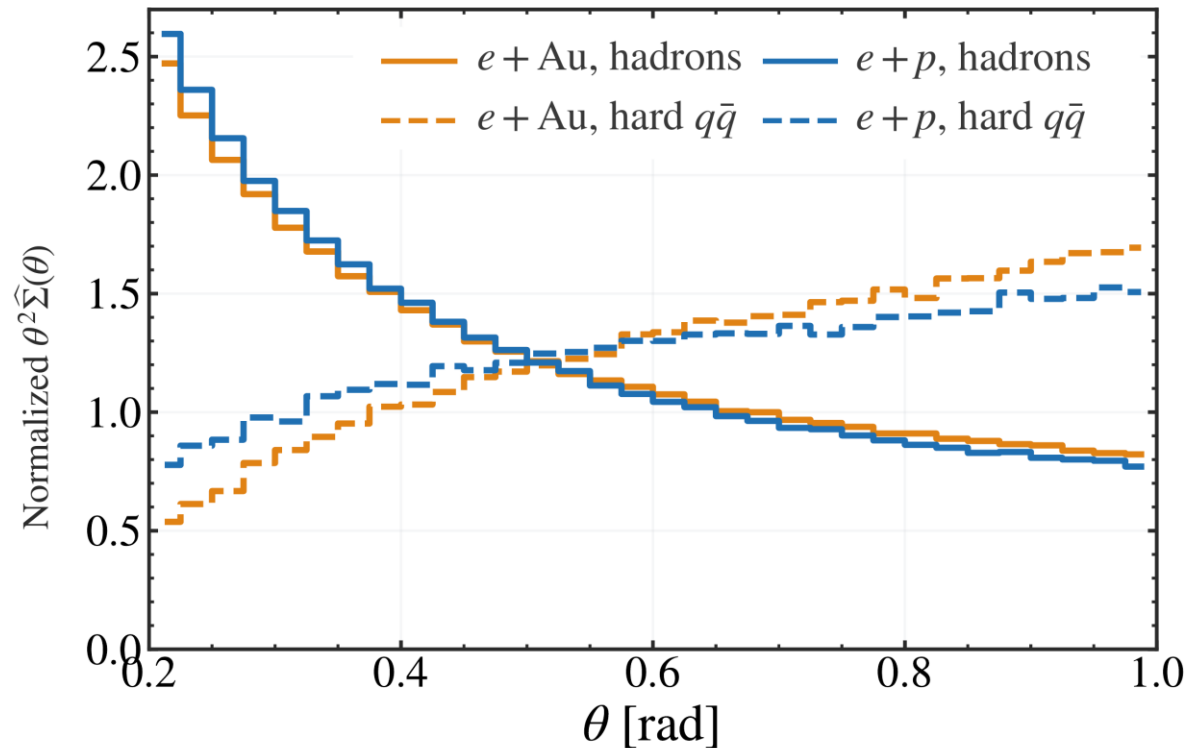


**Gold is broader
before showering.**

Lower peak.
Stronger tail.

Breit frame. eHIJING: pre-shower hard pairs.
PYTHIA: showered partons, before hadronization.

One-Point Energy Correlator



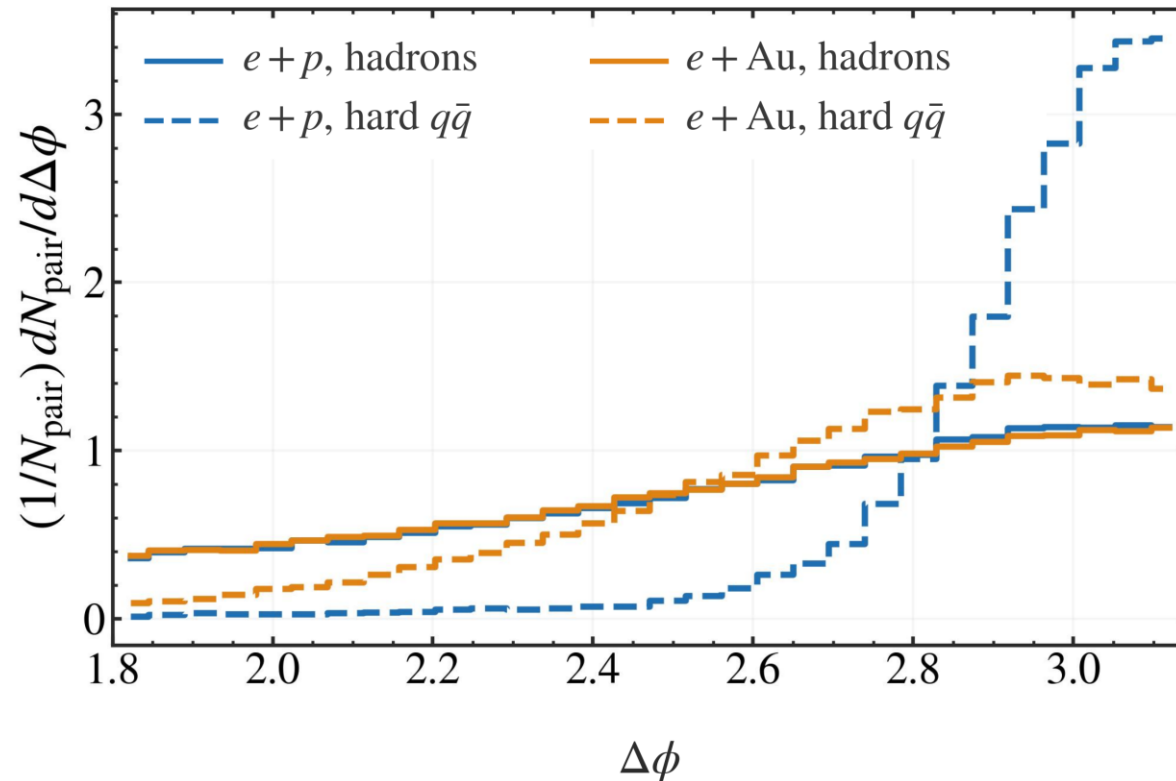
**Weaker contrast
at hadron level.**

Remnant fragments
also contribute.

Bjorken x : 0.002–0.004. Q^2 : 15–23 GeV².
 $0.2 \leq \theta < 1.0$ rad. Normalized angular shapes.
 θ from the incoming hadron.

$$\hat{\Sigma}(\theta) = \frac{1}{N_{\text{evt}}} \sum_{\text{events}} \sum_{h \in \text{event}} \frac{E_h}{E_N} \delta(\theta^2 - \theta_h^2).$$

Leading-Dihadron Azimuthal Correlations



**Target separation
is strongly diluted.**

Hard pairs:
broader gold peak.

Leading hadrons:
shapes nearly match.

Breit frame. Hadron cuts: 1.0 / 0.6 GeV.

Hard-pair cuts: 3.0 / 2.0 GeV.

Transverse-momentum thresholds. Unit area: $1.8 \leq \Delta\phi \leq \pi$.

Global Hadronic Transverse Recoil

$$\mathbf{r}_T = \sum_{h: |y_h| < 2} \mathbf{p}_{T,h}, \quad r_T = |\mathbf{r}_T|$$

BREIT-FRAME RAPIDITY

$y < -2$
Current-going region

$-2 < y < 2$
All stable hadrons

$y > 2$
Target-going region

Collect redistributed recoil from all stable hadrons.

The $|y| < 2$ cut reduces direct ISR and remnant contributions.

Finite acceptance preserves a statistical relation to the hard imbalance.

Hadronic Correlation and Target Difference

Recoil second moment

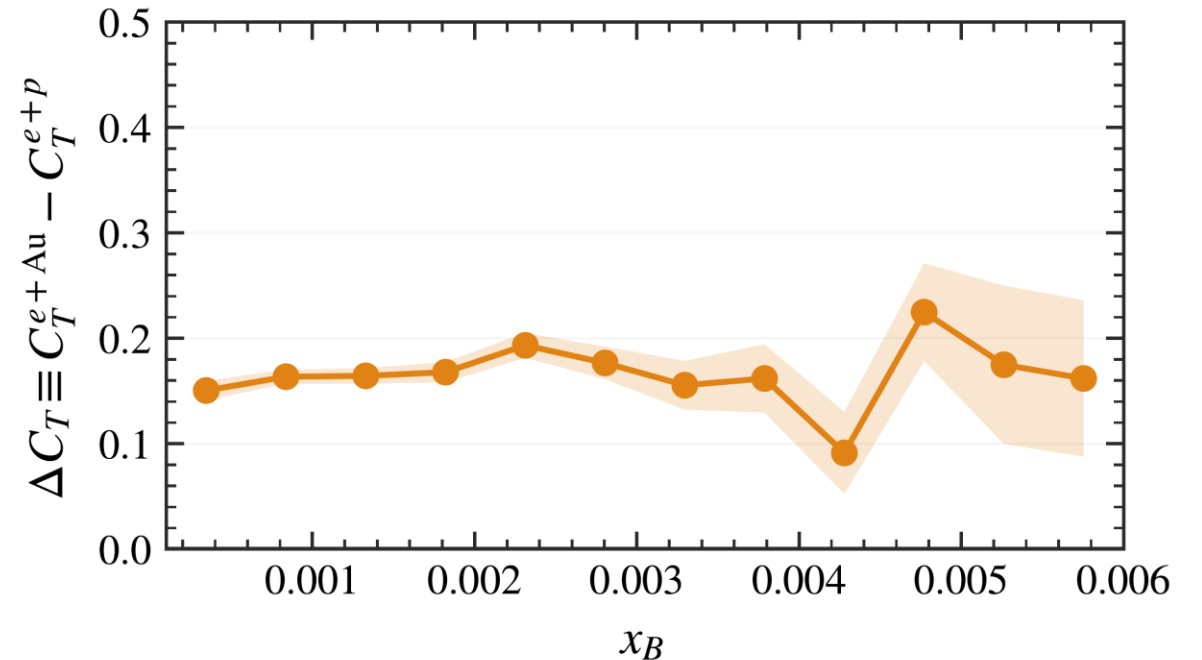
$$C_T = \frac{\langle r_T^2 - \sum_h p_{T,h}^2 \rangle}{\langle \sum_h p_{T,h}^2 \rangle}$$

Subtract self-correlations.
Keep distinct hadron pairs.

$C_T > 0$: net alignment

$C_T < 0$: net anti-alignment

$$\Delta C_T = C_T^{e+Au} - C_T^{e+p} > 0$$



All stable hadrons, $|y| < 2$ (Breit frame).
Band: statistical 1σ . Particle level.

Gold shifts the net pair correlation toward alignment.

Proton Response for Gold Reconstruction

$$X = (x_g, k_T), \quad Y = (x_B, Q^2, r_T)$$

Keep gluon x: the recoil response depends on the evolution interval.

Proton calibration

75%: response, prior and binning

25%: independent closure

$$R_{YX}^p = P_p(Y | X)$$



Gold hadronic input

Gold counts in Y

Test response transfer

$$P_{\text{Au}}(Y | X) \simeq P_p(Y | X)$$

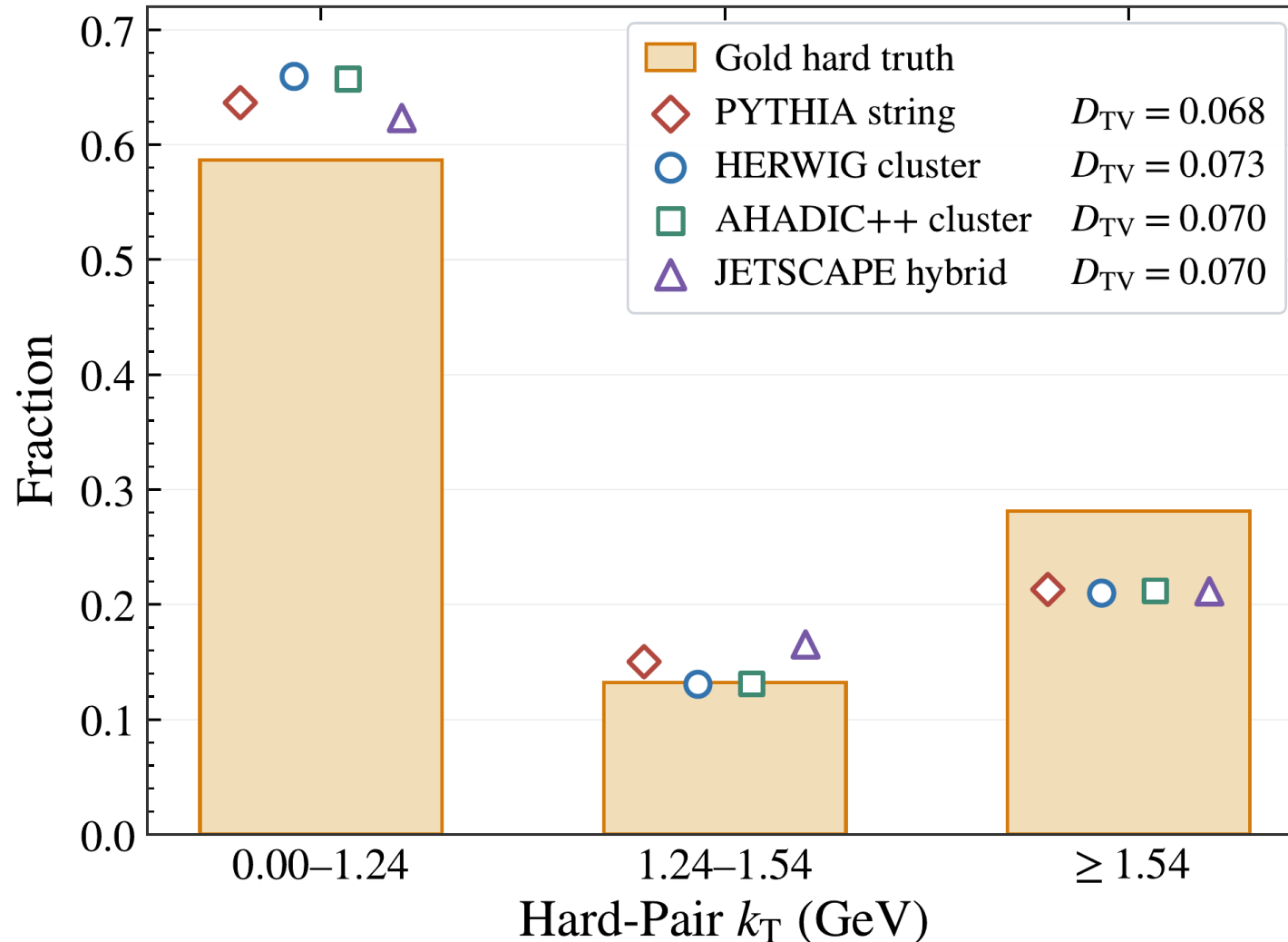


Joint Bayesian unfolding

$$\hat{N}_{\text{Au}}(x_g, k_T) \longrightarrow \hat{N}_{\text{Au}}(k_T)$$

Gold hard truth enters only the validation.

Gold Reconstruction Across Four Models



**Similar results
in coarse bins.
Residual bias.**

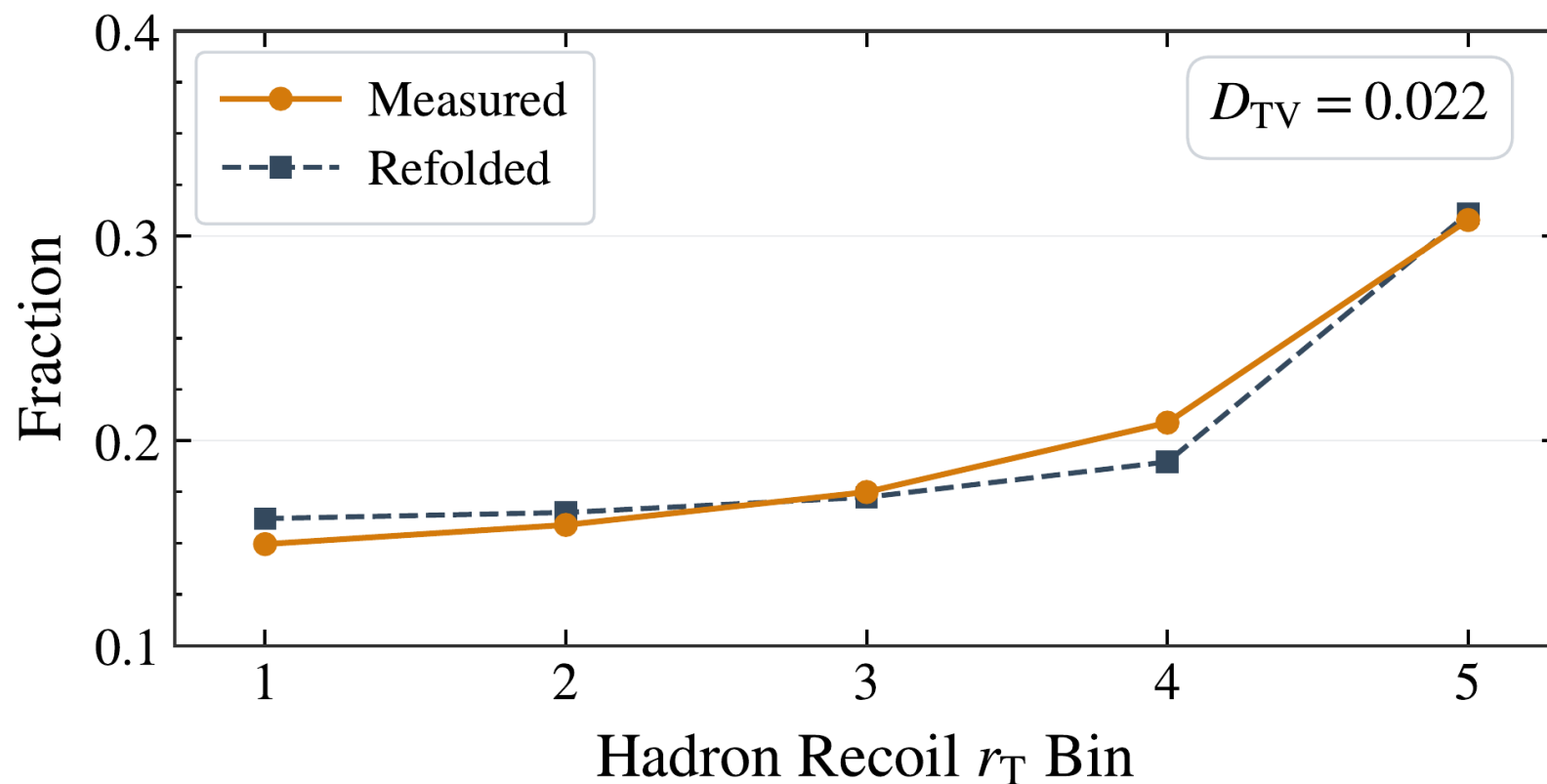
Common PYTHIA FSR.
Proton response
calibrated per model.

Total variation distance

$$D_{TV} = \frac{1}{2} \sum_i |P_i - Q_i|$$

P and Q are normalized.
0: identical. 1: no overlap.

Recoil Refolding Test



“Measured”: simulated recoil marginal.
PYTHIA string hadronization.
Horizontal axis: recoil-bin index.

Refolding matches the input shape.

Refolding

Proton response maps unfolded joint counts to hadronic recoil.

Complements the hard-truth comparison.

Summary and Outlook

Final-state dilution

Target differences shrink in one-point energy flow and leading-dihadron correlations.

Global recoil

A target difference survives at hadron level.

Proton response

Approximate reconstruction of gold hard-pair transverse momentum in three bins.

Outlook: a benchmark for cross-model and detector-level studies

Quark-Pair Production Cross Section

$$\gamma^* + g^* \longrightarrow q + \bar{q}$$

Hard weight from the evolved gluon TMD

$$\frac{d\sigma}{dQ^2 dx_B dz_q d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T}} = \frac{\alpha_{\text{em}}^2 N_c S_{\perp,A}}{2\pi^4 x_B Q^2} \sum_f e_f^2 \mathcal{H}_f(Q^2, x_B, z_q, \mathbf{k}_{1T}, \mathbf{k}_{2T}) \frac{\mathcal{N}_A(\eta, \mathbf{k}_T)}{\mathbf{k}_T^2}.$$

$$D_i = k_{iT}^2 + z_q(1 - z_q)Q^2 + m_f^2, \quad i = 1, 2$$

$$\begin{aligned} \mathcal{H}_f = & \left(1 - y + \frac{y^2}{2}\right) \left\{ [z_q^2 + (1 - z_q)^2] (\mathbf{k}_{1T}/D_1 + \mathbf{k}_{2T}/D_2)^2 + m_f^2 (1/D_1 - 1/D_2)^2 \right\} \\ & + 4(1 - y) z_q^2 (1 - z_q)^2 Q^2 \left(\frac{1}{D_1} - \frac{1}{D_2} \right)^2 \end{aligned}$$

Numerical weight keeps quark masses in kinematics and denominators.
The separate mass-squared term in the hard factor is omitted.

Backward Sampling and Kinematic Constraint

$$\eta = \ln(x_{\text{sw}}/x_g), \quad x_{\text{sw}} = 0.01, \quad C_{\text{nl}} = 5$$

Backward no-branching probability

$$B_A(\eta, \eta_c; k) = \Delta_A(\eta_c, \eta; k) \frac{\mathcal{N}_A(\eta, k)}{\mathcal{N}_A(\eta_c, k)}$$

The density ratio conditions the shower on the selected hard gluon.

Accepted emission and parent state

$$k_p = k_c - l, \quad \eta_p = \eta_v + \ln \frac{k_c^2}{k_c^2 + l^2}$$

c: current gluon v: trial branching p: parent gluon

A boundary-crossing emission remains GLR.

DGLAP continues from the handoff. No-emission handoff adds no gluon.

Selections and Observable Definitions

Observable	Selection	Definition
Hard pairs	Breit frame	Vector imbalance before showering
One-point flow	Bjorken x: 0.002–0.004 Q^2 : 15–23 GeV ² θ : 0.2–1.0 rad	Normalized shape within each level
Dihadron	Hadron: 1.0 / 0.6 GeV Hard: 3.0 / 2.0 GeV	Independent cuts; unit area
Recoil	All stable hadrons Breit: $-2 < y < 2$	Vector sum; zero recoil retained
Pair correlation	Bjorken x: 0.0001–0.006 Q^2 integrated	Ensemble moment ratio; gold – proton difference

One-point: energy-weighted angular distribution · Dihadron: independent hard/hadron cuts

Joint Response and Bayesian Updates

6 truth cells (2 × 3) and 40 particle cells (4 × 2 × 5)

α : hard cell m : particle cell 20 Bayesian iterations

$$R_{YX}^p = P_p(Y \mid X)$$

$$B_{\alpha m}^{(n)} = \frac{R_{m\alpha}^p \pi_{\alpha}^{(n)}}{\sum_{\beta} R_{m\beta}^p \pi_{\beta}^{(n)}}$$

Proton: 75% calibration / 25% holdout
Columns sum to one. Unit efficiency.
Zero-recoil events are retained.

$$\hat{N}_{\alpha}^{(n+1)} = \sum_m d_m B_{\alpha m}^{(n)},$$

$$\pi_{\alpha}^{(n+1)} = \frac{\hat{N}_{\alpha}^{(n+1)}}{\sum_{\beta} \hat{N}_{\beta}^{(n+1)}}$$

Marginalize over gluon x

$$\hat{N}(k_T^b) = \sum_a \hat{N}(x_g^a, k_T^b)$$

Refold into particle space

$$d_m^{\text{pred}} = \sum_{\alpha} R_{m\alpha}^p \hat{N}_{\alpha}^{(20)}$$

Hadronization Variations and Validation

Common PYTHIA FSR. Model-specific hadronization and proton response.

$$D_{\text{TV}}(P, Q) = \frac{1}{2} \sum_i |P_i - Q_i|$$

0: identical shapes

κ: response-matrix condition number

Test	Distribution / scope	Value
Proton closure	Joint (xg, kT), four models	0.0023–0.0045
Gold reconstruction	kT marginal, four models	0.068–0.073
PYTHIA refolding	Five-bin rT marginal	0.022
Response conditioning	Four hadronization models	κ = 13.0–16.2

Four-model ranges. PYTHIA recoil refolding is shown separately.

Separate nominal sample

Proton joint: 0.0078 · Gold joint: 0.068 · Full-Y refold: 0.031 · κ: 14.0